1 Estuarine Physics

1.1 Estuary Types

Estuaries can be classified into 3-types, salt-wedge, partially-mixed, and well-mixed, partitioned by the ratio of tidal prism, V, and volume of river discharge over a tidal cycle, R.

Type	**R / V**	**Mixing**
Salt-wedge	$R/V > 1$	low
Partially-mixed	$0.005 < R/V < 1$	moderate
Well-mixed	$R/V 0.005$	High

The *tidal prism* is defined at the difference in the volume of water in an estuary during low and high tide. This can also be thought of as the amount of water leaving the estuary during an ebb tide. A first order estimate of the tidal prism is

$$V = HA \tag{1}$$

where the tidal prism, V, is equal to the product of the average tidal range, H, and the average surface area, A, of the estuary or inlet. Let's consider Bellingham Bay for a moment, estimate it's tidal prism and calculate the river discharge over a tidal cycle for a couple different flow conditions.

Using this tool, https://www.daftlogic.com/projects-google-maps-area-calculator-tool.htm, I estimate an area of $1.7 \times 10^8\,\mathrm{m}^2$ if I include Bellingham and Samish Bays. The tide range in Bellingham is around 3-m, so the approximate volume water fluxing in and out of the bay during a tidal cycle is $5.1 \times 10^8\,\mathrm{m}^3$. Now what about the discharge of the Nooksack? Mean discharge in March is $85\,\mathrm{m}^3/\mathrm{s}$, while during a moderate flood event discharge may peak about 10x higher around $1000\,\mathrm{m}^3/\mathrm{s}$. How many seconds in a tidal cycle? A complete tidal cycle 24.8 hours, or $8.9 \times 10^4\,\mathrm{s}$, bringing the Nooksack flux during mean March conditions to $R = 7.6 \times 10^6\,\mathrm{m}^3$. Now taking the ratio, R/V, we end up with 0.015, which classifies as a partially-mixed estuary. During peak flood the ratio will be approximately 0.15, still partially-mixed, but closer to a salt-wedge estuary type.

So what does that look like? In Figure 1 you will see a strong halocline (line of constant salinity) separating fresh and salty waters in the vertical for the first type, the salt-wedge. Below in the partially mixed case this strong halocline begins to break down, creating a salinity gradient with mixing strongest at the turbidity maximum. With enough mixing, the vertical salinity gradient has nearly disappeared resulting in a horizontal gradient from salty (left) to fresh (right). In the well-mixed estuary there is much more tidal prism than fresh-water input. The large influx of waters in and out allow for extensive mixing.

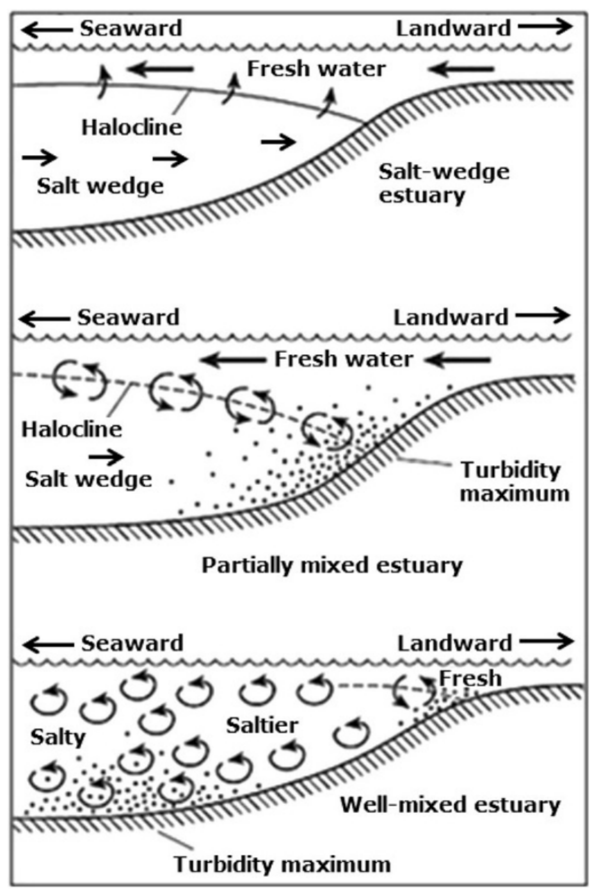


Figure 1: Estuary types, figure from Kumar et al 2014

1.1.1 Additional Resources:

- Estuary Types Video: https://www.youtube.com/watch?v=G5XcyxXVa60
- Salt water intrusion: http://www.coastalwiki.org/wiki/Seawater_intrusion_and_mixing_in_estuaries

1.2 Mixing

We observed that a large tidal prism drives a more mixed estuary, but how does mixing happen? And how do we include this in the our equations?

1.2.1 Eddy and molecular viscosity

So how do we include this in our momentum equations (or Navier-Stoke equations)? In our F terms we left out several physical components including external forcing such as tides and winds. Additionally, we also neglected mixing. Without going into too much detail, we can think of shear, such as in the illustration above, as having a stress and strain imparted on a fluid particle. This stress, τ, is proportional to the shear, such that

$$\frac{\tau}{\rho} = \nu \frac{\partial u}{\partial z} \tag{2}$$

where ρ is the density of water and ν is the molecular viscosity (ν=$1 \times 10^{-6} m^2/s$). This stress impart momentum to the water parcel if there is a spatial gradient. For example, if the stress of one side of the parcel is greater than the other the parcel can be squeezed, stretched or deformed. The stress on a fluid parcel is then

$$\frac{\partial u}{\partial t} \sim \frac{1}{\rho} \frac{\partial \tau_x}{\partial x} \tag{3}$$

and because we are in 3-dimensions, we have impacts from stress in each direction such that

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\nu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\nu \frac{\partial u}{\partial z}\right) \tag{4}$$

Typically, nu is constant and in vector notation this simplifies to

$$\frac{\partial u}{\partial t} = \nu \nabla^2 u \tag{5}$$

Molecular viscosity, ν, is quite small and represents the diffusion on a molecular scale, sometimes referred to as Brownian motion. However, a turbulent flow, typical in the ocean mixing and diffusion occurs much faster and is typically parameterized as *eddy viscosity*. A typical value of eddy viscosity, ν_e, is $1 \times 10^{-3} m^2/s$. Add this term now to our Navier-Stokes equation, in just the x-dimension we have

$$\hat{x} : \frac{Du}{Dt} = \frac{\partial u}{\partial t} + \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\text{Advective}} = \underbrace{-\frac{1}{\rho}\frac{\partial P}{\partial x}}_{\text{Pressure}} + \underbrace{fv}_{\text{Coriolis}} + \underbrace{\nu_e \nabla^2 u}_{\text{Mixing}} + F_x \tag{6}$$

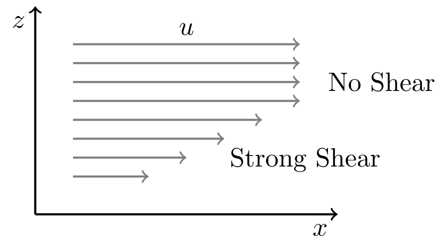
For more gritty detail, see

- https://marine.rutgers.edu/dmcs/ms501/2004/Notes/Friction2.htm

1.2.2 Vertical Mixing and Shear Production)

What controls mixing? Stratification acts to inhibit mixing, while shear acts to enhance it. Stratification can be quantified by the change in density in the vertical, $\frac{\partial \rho}{\partial z}$, where larger values indicate a greater stratification and smaller values vice-versa. In the larger ocean temperature tends to control vertical density, however, in an estuary salinity is the primary driver of density variation. When thinking about density variations inside an estuary, ask yourself first, where is the water fresher, where is it saltier?

What about shear, what even is shear? Velocity shear is when there is a rapid change in velocities over a short distance. For example, vertical shear, is quantified as, $\frac{\partial u}{\partial z}$, and is large when the change in velocity is large across the vertical. Visually,



you can see where strong shear is indicated by the rapid change in velocity with depth (z). This is typical of flow across the ocean bed, where friction with the ocean floor reduces the velocity of horizontal flow to zero. This shear gradient will produce turbulent mixing that is referred to as shear production.

1.2.3 Gradient Richardson number

So how can we quantify the impact of stratification and shear production on mixing? Often the non-dimensional Gradient Richardson number is used to classify the degree and type of vertical mixing. The Gradient Richardson number is defined as

$$R_i = \frac{-\frac{g}{\rho}\frac{\partial \rho}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \tag{7}$$

where the stratification is divided by the square of the shear. *Check to see if the units of R_i are indeed non-dimensional.* When $R_i < 0.25$ mixing is suppressed and stratification wins. In this case only a minor amount of mixing occurs through the process of breaking Holmboe Waves that entrain small amounts of fluid across the density gradient. When $R_i > 0.25$, Kelvin Helmhotz instabilities occur driving intense mixing. You may occasionally observe these instabilities in the clouds, when you see a very regular patter of what looks like breaking waves. See the videos below for examples of these processes.

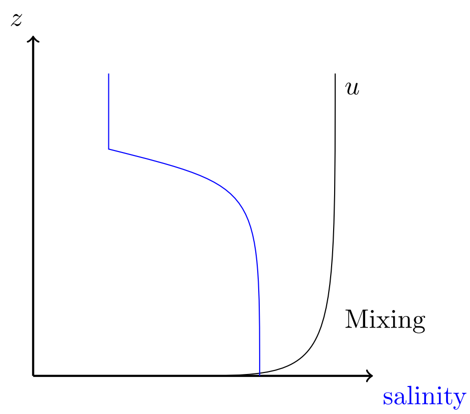
- Holmboe Instability, https://www.youtube.com/watch?v=Vl6CW1Txzoo
- Kelvin Helmholtz Instability, https://www.youtube.com/watch?v=mf_143gkKSQ

1.2.4 Bottom mixing and turbulence

In theory and models, the no-slip condition is typically imposed. The no-slip condition implies that at a boundary, such as the bed, there is no horizontal flow, or no “slip” at the interface. This means that the horizontal velocity is zero at the bed, and increases with height above the bottom. Within thin turbulent bottom boundary layer we generally prescribe a vertical velocity profile called the log-law, or “law of the wall”. This relation states that the flow velocity is proportional to the log of the distance to the “wall”, or as often the case in oceanography, the sea floor. For further details, you can consult this wiki link.

If there is significant tidal flow there will be strong shear near the bed, as in the previous diagram. The turbulent mixing, or *shear production*, will tend to destroy gradients in water properties such as salinity, mixing vertically fresh and salty waters. Shear production at the bed may or may not act to mix salinity gradients depending on the salinity structure and the height of the turbulent boundary layer. See below, for example, where mixing due to bottom-boundary layer shear production does (right) and does not (left) act to mix salinity gradients. Which panel (left of right) would you expect to observe in a salt wedge estuary?

The velocity magnitude and profile changes with the tidal ebb and flood, and with Spring and Neap tidal cycles. How do you expect the tidal prism to relate to the depth of the bottom boundary layer, or the depth of vertical shear present in the water column? These processes have significant impacts to the larger estuarine circulation and water structure (salinity, nitrates, oxygen, plankton).



Additional Resources

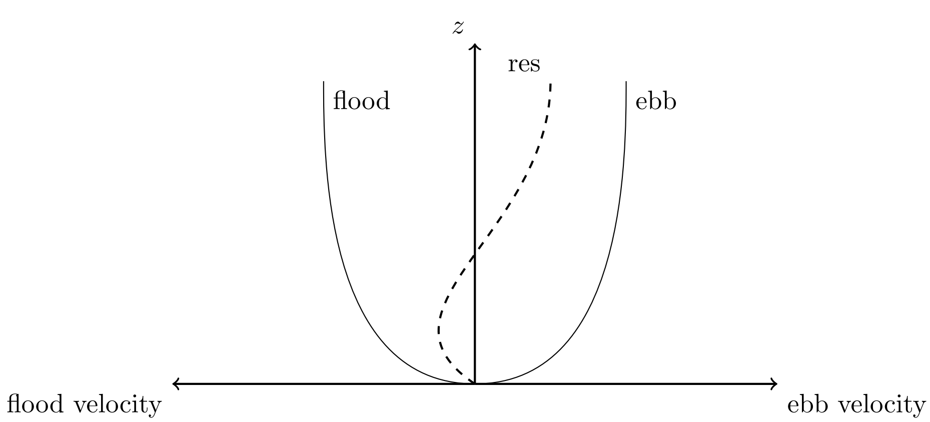
- The Estuarine Circulation by Geyer & MacCready. link

1.3 Estuarine Circulation

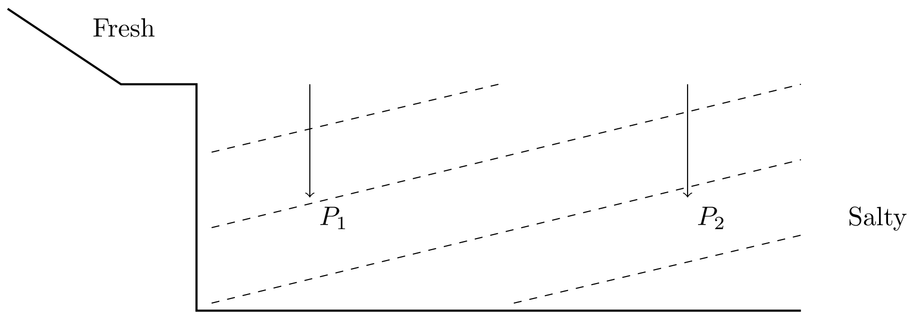
To first order, estuarine circulation is dominated by tidal forcing at the boundary. Flood and ebb tides work to flush and mix fresh-water and ocean inputs. This flow is driven by the *barotropic* pressure gradient setup as ocean boundary increases or decreases during the tidal cycle. Barotropic means one-layer flow that has no vertical structure. We described how a pressure gradient drives changes to velocities in Section **??**. In this case the pressure gradient is caused by the astronomical forces driving the tides.

1.3.1 Residual Circulation

While tidal forces day-to-day circulation, when one averages velocities over several tidal cycles a residual circulation pattern becomes apparent that has structure in the vertical. This residual circulation is described as *baroclinic*, meaning two layer flow. An example of this is shown below, where while flood and ebb tidal velocities are predominantly one-layer barotropic flow, once averaged over several tidal cycles, the residual two-layer flow becomes apparent. We saw this relatively clearly in modeled velocities in Wang and Yang 2015.



What drives this two-layer baroclinic flow? Again, we will find that it is a pressure gradient. This pressure gradient though is driven by density differences, instead of a difference in surface height. As we observed, the fresh-water input to the estuary creates a horizontal and vertical gradient of salinity across the estuary. Depending on the ratio of tidal prism to fresh-water discharge the salinity gradients are more horizontal or more vertical. In a partially-mixed estuary such as the Salish Sea there is a significant vertical salinity gradient that drives a horizontal pressure gradient at depth. Let's evaluate the pressure at the two locations below, where the dashed lines indicate isohalines.



At P_1 and P_2 we integrate the pressure from the surface down. The water above P_2 is saltier, and therefore denser and heavier. The pressure at P_2 is then larger than the pressure at P_1 and the pressure gradient $\frac{\partial P}{\partial x}$ is positive. A positive pressure gradient means acceleration and flux in the negative direction (Section **??**), which is flow directed into the estuary.

1.3.2 Knudsen Relation

Here we will show that with two straight-forward assumptions we can make some surprising progress towards estimating residual estuarine exchange flow. Suppose we have the relatively simple estuary schematized below, with river discharge, Q_R and surface flow out, Q_1, and bottom flow in Q_2. Our first assumption is

that the system is in steady-state. This means that the water flowing in to the system must equal the water flowing out, or mathematically

$$Q_1 = Q_R + Q_2\,. \tag{8}$$

Now we can also consider the salinity in the system. The volume averaged salinity is

$$V\frac{\partial S}{\partial t} = Q_2 S_2 - Q_1 S_1\,. \tag{9}$$

Another consequence of our steady-state assumption is that all time-derivatives are zero, or in other words, the salinity of the system is not changing. Therefore we have

$$Q_2 S_2 = Q_1 S_1 \tag{10}$$

With a little of algebra these two relations can be used to solve for Q_2, such that

$$Q_2 = \frac{S_1}{\Delta S} Q_R\,, \tag{11}$$

where $\Delta S = S_2 - S_1$. What this means is that we can estimate the exchange flow, Q_2 and Q_1 by simply measuring the vertical salinity profile and river discharge. This is advantageous as it is much easier to collect this data than to measure long-term velocity profiles.

1.3.3 Residence Time

Residence time is generally defined as the average time a water parcel spends in the estuary or basin before being ejected at the boundary. Residence time is an important metric to determine how much oxygen and nutrients will be available to estuarine ecosystems. One approximation to residence time (RT) is the total basin volume (V) divided by the flow in or out, or specifically

$$RT = \frac{V}{\sum \text{flux in/out}} \tag{12}$$

Additional Resources

- https://pasi.coastal.ufl.edu/lectures/lec02PASI.pdf
- http://www.coastalwiki.org/wiki/Estuarine_circulation