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Homework 2.

Problem 1:

- Section c). Prove explicitly that the Expected value of λ is $1/\alpha$. Hint: $E(\lambda) = \int_0^\infty \lambda P(\lambda) d\lambda$.

Proof: From section above, we know that $P(\lambda) = \alpha e^{-\alpha\lambda}$ following an exponential distribution. this distribution is supported on the interval $[0, \infty)$.

So we are going to prove $E[\lambda] = 1/\alpha$ solving.

$$\int_0^\infty \alpha P(\lambda) d\lambda = \int_0^\infty \alpha \lambda e^{-\alpha\lambda} d\lambda \underset{\substack{\downarrow \\ \alpha \text{ constant}}}{=} \alpha \int_0^\infty \lambda e^{-\alpha\lambda} d\lambda$$

<u>Integration by parts</u>

$u = \lambda \qquad dv = e^{-\alpha\lambda} d\lambda$

$\frac{du}{d\lambda} = 1 \qquad v = \int dv = \int e^{-\alpha\lambda} d\lambda = \frac{e^{-\alpha\lambda}}{-\alpha}$

$$u\,dv = uv - \int v\,du$$

$$\alpha\left[\left.\frac{\lambda e^{-\alpha\lambda}}{-\alpha}\right]_0^\infty - \left(\int_0^\infty \frac{e^{-\alpha\lambda}}{-\alpha} d\lambda\right)\right] \underset{\substack{\downarrow \\ (-)\text{signs}}}{=} \cancel{\alpha}\left(-\frac{\lambda e^{-\alpha\lambda}}{\cancel{\alpha}} + \frac{1}{\cancel{\alpha}}\int e^{-\alpha\lambda} d\lambda\right) =$$

$$= -\lambda e^{-\alpha\lambda}\Big]_0^\infty + \int_0^\infty e^{-\alpha\lambda} d\lambda = -\lambda e^{-\alpha\lambda}\Big]_0^\infty - \left.\frac{e^{-\alpha\lambda}}{-\alpha}\right]_0^\infty \underset{\substack{\downarrow \\ (\text{signs})}}{=} -\lambda e^{-\alpha\lambda}\Big]_0^\infty - \left.\frac{e^{-\alpha\lambda}}{\alpha}\right]_0^\infty$$

$$= [\underbrace{-\infty \cdot 0}_{0} - 0] - \left(\cancel{\frac{0}{\alpha}}^{\,0} - \frac{1}{\alpha}\right) = \frac{1}{\alpha} \Rightarrow E[\lambda] = \frac{1}{\alpha}$$

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Q.E.D.