Silvia Cabellos
PHYS 247

2)

b) Show that $\Omega(N)$ can be found recursively using the following equation:

$$\Omega(N) = A \times A \times \ldots \times A \times \Omega(1) = A^{N-1} \times \Omega(1).$$

Proof: from (1) we know

$$\Omega(N) = \begin{pmatrix} \Omega(0,N) \\ \Omega(1,N) \\ \vdots \\ \Omega(9,N) \end{pmatrix} = \begin{pmatrix} (A\,\Omega(N-1))_{01} \\ (A\,(\Omega(N-1))_{11} \\ \vdots \\ (A\,(\Omega(N-1))_{91} \end{pmatrix} = A\,\Omega(N-1)$$

We know that $\Omega(1) = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}_{10\times 1}$ → Which represents all the "moves" ~~we~~ w can do from each number in the move pad of longitud 0. (or How many numbers of 1-digit we can get.)

Now, is $\Omega(2) = A\,\Omega(1)$? → When we multiply by $\Omega(1)$, which we do add all the elements of each row in A, we can obtain results in the number of 2-digits we can obtain proving that $\Omega(2) = A\,\Omega(1)$.

Let's suppose this is true for $\Omega(k) = A^{(k-1)}\,\Omega(1)$ (2)

then $\underset{(1)}{\Omega(k+1) = A\,\Omega(k)} \underset{(2)}{=} A \cdot A^{k-1}\,\Omega(1) = A^{k}\,\Omega(1)$

Q.E.D.