

Problem 1 - Linear Algebra

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PHYS 247.

a) show that the equation $\Omega(s, N) = \sum_{k \in \{j; A_{sj}=1\}} \Omega(k, N-1)$ can be written in a form of matrix such that $\Omega(s, N)$ is the s^{th} element of a 10×1 matrix $\Omega(N)$ where $\Omega(N) = A \times \Omega(N-1)$

Proof

So the matrix $\Omega(N)$ is such that s -element is $\Omega(s, N)$, i.e.:

$$\Omega(N) = \begin{pmatrix} \Omega(0, N) \\ \Omega(1, N) \\ \Omega(2, N) \\ \vdots \\ \Omega(9, N) \end{pmatrix}_{10 \times 1}$$

, therefore we can write:

$$\Omega(s, N) = \Omega_s^N = [\Omega(N)]_s \quad (\text{Notation})$$

starting from:

$$\Omega(s, N) = \sum_{k \in \{j; A_{sj}=1\}} \Omega(k, N-1) \Rightarrow$$

$$\Omega(s, N) = \sum_{k \in \{j; A_{sj}=1\}} \Omega(k, N-1) = \sum_{j=0}^9 A_{sj} \Omega(j, N-1) = \sum_{j=0}^9 A_{sj} [\Omega(N-1)]_j$$

$$= [A \cdot \Omega(N-1)]_s \Rightarrow \Omega(s, N) \text{ is the } s^{\text{th}} \text{ element of the}$$

matrix $A \cdot \Omega(N-1)$ Q.E.D.