

At the center of the program is class `multi_array`. This structure is a simple multi-dimensional array that is indexed by dimensions with different names. The dimensions do not have any particular ordering, as each dimension is referenced by its unique name. In the program, the dimensions are sorted in lexicographic order for clarity.

The basic operations involving multi-arrays are two types of extensions, the addition of dimensions, and two types of restrictions, the removal of dimensions.

There are two ways dimensions are added to an array:

- **Approach 1:** The array is cylindrically extended along the new dimensions, with the existing entries repeated along each new dimension.
- **Approach 2:** A specific index assignment is given for the new dimensions. All entries of the extended array, except for the entries whose indices agree with the given assignment for the new dimensions, are set to a default value. For entries whose indices agree with the given assignment, the entry is assigned the corresponding entry from the unextended array.

There are two approaches for removing dimensions from an array:

- **conditioning:** A specific index assignment is given for the to be removed dimensions. All entries whose indices do not agree with the given indices are removed. The array is reduced to a “slice” where the removed dimensions are restricted to their assigned indices.
- **marginalization:** Dimensions are eliminated by summing along the to be removed dimensions while holding all other dimensions constant. When a dimension is eliminated, all entries that now have the same indices are added together.

When a binary operator (such as addition) is performed on multi-arrays with different dimensions, each array is extended so that the arrays cover the same dimensions, and then the resultant array is formed by applying the binary operator to each pair of corresponding entries.

From these basic operators, the subroutines that expand a total joint probability distribution into a layered probability model are implemented.