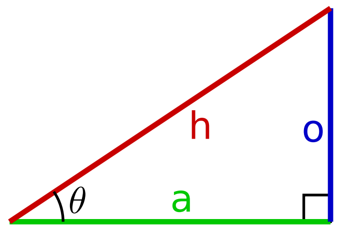


Problem 1 (2 marks): Given a right triangle that has a named angle of $\theta = 65°$, and an adjacent of $a = 67.22$, compute both the opposite o and the hypotenuse h to 4 significant digits.

Solution:

$o = (67.22)\tan 65° \approx 144.2$ and $h = (67.22)\sec 65° \approx 159.1$

Problem 2 (2 marks): Given a right triangle that has a named angle of $\theta = 40°$, and a hypotenuse of $h = 3.500 \times 10^5$, compute both the adjacent a and the opposite o to 4 significant digits.

Solution:

$a = (3.500 \times 10^5)\cos 40° \approx 2.681 \times 10^5$ and $o = (3.500 \times 10^5)\sin 40° \approx 2.250 \times 10^5$

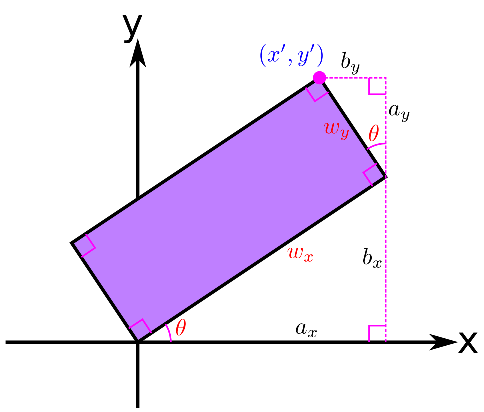
Problem 3 (2 marks): Given a right triangle that has a named angle of $\theta = 50°$, and an opposite of $o = 9.988 \times 10^{-3}$, compute both the adjacent a and the hypotenuse h to 4 significant digits.

Solution:

$a = (9.988 \times 10^{-3})\cot 50° \approx 8.381 \times 10^{-3}$ and $h = (9.988 \times 10^{-3})\csc 50° \approx 1.304 \times 10^{-2}$

Problem 4 (2 marks):

In the x, y coordinate system, is a box of width w_x and height w_y. The bottom left corner of the box is anchored to the origin $(0, 0)$ point. Initially the box's width w_x is along the positive x axis, and the box's height w_y is along the positive y axis. The box is rotated counterclockwise by an angle of θ with the bottom left corner still anchored to the origin point. Find the new x and y coordinates (x', y') of the top right corner of the box after the rotation. **Give x' and y' as expressions involving only the quantities w_x, w_y, and θ.**



Solution:

$a_x = w_x \cos\theta$, $b_x = w_x \sin\theta$, $a_y = w_y \cos\theta$, and $b_y = w_y \sin\theta$. Therefore:
$x' = a_x - b_y = w_x \cos\theta - w_y \sin\theta$ and $y' = b_x + a_y = w_x \sin\theta + w_y \cos\theta$.

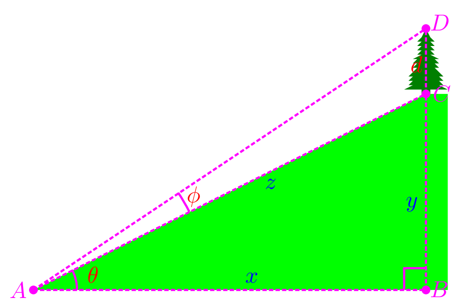
$$x' = w_x \cos\theta - w_y \sin\theta \quad \text{and} \quad y' = w_x \sin\theta + w_y \cos\theta$$

Problem 5 (15 marks): Fill in the table below:
For each empty cell, insert one of the following values or ranges:
0, 1, -1, $\pm\infty$, $(0,1)$, $(-1,0)$, $(0,+\infty)$, $(-\infty,0)$, $(1,+\infty)$, $(-\infty,-1)$
The chosen range must be tight: the range $(0,+\infty)$ is the incorrect choice when the range is actually $(1,+\infty)$.

θ	$\cos\theta$	$\sin\theta$	$\tan\theta$	$\sec\theta$	$\cot\theta$	$\csc\theta$
$\theta=0$	1	0	0	1	$\pm\infty$	$\pm\infty$
$\theta\in(0,\pi/2)$	$(0,1)$	$(0,1)$	$(0,+\infty)$	$(1,+\infty)$	$(0,+\infty)$	$(1,+\infty)$
$\theta=\pi/2$	0	1	$\pm\infty$	$\pm\infty$	0	1
$\theta\in(\pi/2,\pi)$	$(-1,0)$	$(0,1)$	$(-\infty,0)$	$(-\infty,-1)$	$(-\infty,0)$	$(1,+\infty)$
$\theta=\pi$	-1	0	0	-1	$\pm\infty$	$\pm\infty$
$\theta\in(\pi,3\pi/2)$	$(-1,0)$	$(-1,0)$	$(0,+\infty)$	$(-\infty,-1)$	$(0,+\infty)$	$(-\infty,-1)$
$\theta=3\pi/2$	0	-1	$\pm\infty$	$\pm\infty$	0	-1
$\theta\in(3\pi/2,2\pi)$	$(0,1)$	$(-1,0)$	$(-\infty,0)$	$(1,+\infty)$	$(-\infty,0)$	$(-\infty,-1)$
$\theta=2\pi$	1	0	0	1	$\pm\infty$	$\pm\infty$

Problem 6 (bonus 6 marks):
In the image of the right, a tree with a known height of $d = 10.00$m is on top of a hill with an incline of $\theta = 20°$. From the bottom of the hill, the "angular size" of the tree is $\phi = 5°$. Compute all of:

- x, the **horizontal** distance of the tree from the observer.
- y, the altitude of the base of the tree.
- z, the distance of the base of the tree from the observer.



Solution:
In the image on the right, there are 4 labeled points: Point A is at the base of the hill. Point B is at ground level beneath the tree. Point C is at the base of the tree. Point D is at the top of the tree. There are two right triangles. Triangle Δ_1 is formed by points A, B, and C, and is essentially the bulk of the hill. Triangle Δ_2 is formed by points A, B, and D. The named angles of Δ_1 and Δ_2 are both located at A. The named angle of Δ_1 is θ. The named angle of Δ_2 is $\theta+\phi$. The difference between the opposites of Δ_2 and Δ_1 is d. The opposite of Δ_1 is $x\tan\theta$. The opposite of Δ_2 is $x\tan(\theta+\phi)$. Therefore:

$$x\tan(\theta+\phi) - x\tan\theta = d \iff x(\tan(\theta+\phi)-\tan\theta) = d \iff x = \frac{d}{\tan(\theta+\phi)-\tan\theta}$$

From the value of x,

$$y = x\tan\theta = \frac{d\tan\theta}{\tan(\theta+\phi)-\tan\theta} \quad \text{and} \quad z = x\sec\theta = \frac{d\sec\theta}{\tan(\theta+\phi)-\tan\theta}$$

Substituting in the values of $d = 10.00$m, $\theta = 20°$, and $\phi = 5°$, gives:

$$x = \frac{10.00\text{m}}{\tan 25° - \tan 20°} \approx 97.72\text{m} \quad \text{and} \quad y = \frac{(10.00\text{m})\tan 20°}{\tan 25° - \tan 20°} \approx 35.57\text{m}$$

$$\text{and} \quad z = \frac{(10.00\text{m})\sec 20°}{\tan 25° - \tan 20°} \approx 104.0\text{m}$$