Double and Triple Integrals

Question 1:

Find the minimum and maximum values of the function:

$$f(x, y) = x + y$$

subject to the constraint:

$$x^2 + 4y^2 - 4x - 16y = -16$$

Question 2:

Find the minimum and maximum values of the function:

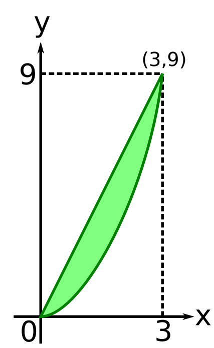
$$f(x, y) = 2x + y$$

subject to the constraint:

$$x^2 + y^2 - 2x - 2y = 2$$

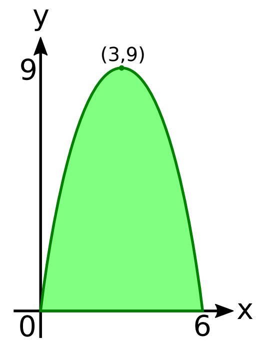
Question 3:

The leaf shaped region σ on the right is bounded from below by a parabola and from above by a straight line. Express σ as a Type I Cartesian region; a Type II Cartesian region; and a Polar region. In addition, given an arbitrary function $f(x, y)$, or $f(r, \theta)$ in polar coordinates, express the double integral $\iint_\sigma f(x, y)dA$ as a nested (iterated) integral using each of the 3 different forms of σ. Lastly, use all of the 3 forms to compute the double integral $\iint_\sigma x dA$.



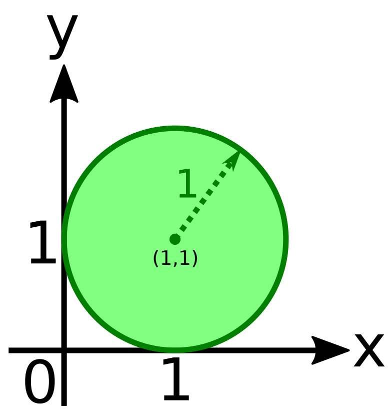
Question 4:

For the parabolic region σ on the right, express σ as: a Type I Cartesian region; a Type II Cartesian region; and a Polar region. In addition, given an arbitrary function $f(x, y)$, or $f(r, \theta)$ in polar coordinates, express the double integral $\iint_\sigma f(x, y)dA$ as a nested (iterated) integral using each of the 3 different forms of σ. Lastly, choose one of the 3 forms to compute the double integral $\iint_\sigma \frac{dA}{x}$.



Question 5:

For the circular region σ on the right, express σ as a polar region, and then express the double integral $\iint_\sigma f(r, \theta)dA$ as a nested integral. Lastly, evaluate the double integral $\iint_\sigma \frac{\sqrt{\sin(2\theta)} \cdot dA}{r}$.



Question 6:

Given the polar nested integral:

$$\int_{\theta=-\text{atan}(2)}^{\pi/4} \int_{r=0}^{\frac{2\cos\theta+\sin\theta}{\cos^2\theta}} r^2 \cos\theta dr d\theta$$

Sketch the region covered by this double integral, convert it to Cartesian coordinates, and lastly evaluate the integral.

Question 7:

Given the polar nested integral:

$$\int_{\theta=0}^{\pi/2} \int_{r=0}^{\frac{6}{2\cos\theta+3\sin\theta}} r^2 \cos\theta dr d\theta$$

Sketch the region covered by this double integral, convert it to Cartesian coordinates, and lastly evaluate the integral.

Question 8:

Given the Cartesian nested integral:

$$\int_{x=-2}^{2} \int_{y=0}^{\sqrt{4-x^2}} y\sqrt{x^2+y^2} \cdot dydx$$

Evaluate this integral directly, and then convert this integral to polar coordinates and evaluate that integral to demonstrate that you get the same result.

Question 9:

Compute the volume between the two surfaces $z_1(r,\theta) = \sqrt{R^2 - r^2}$ and $z_2(r,\theta) = -\sqrt{R^2 - r^2}$ over the region $\sigma = \{(r,\theta)|0 \leq \theta \leq 2\pi \text{ and } 0 \leq r \leq R\}$ where $R > 0$ is a fixed constant. What is the significance of this volume?

Question 10:

Given the volume $\Omega = \{(x,y,z)|0 \leq x \leq 1 \text{ and } 0 \leq y \leq 2x \text{ and } 0 \leq z \leq 3y\}$, compute the triple integral:

$$\iiint_{\Omega} xyzdV$$

Question 11:

For the tetrahedron Ω on the right, compute the center of mass assuming a uniform mass density m. The center of mass is the weighted average position $\langle x, y, z \rangle$ of the points in Ω where the "weight" assigned to each point is the density:

$$\mathbf{r}_{\text{CM}} = \frac{\iiint_{\Omega} m \langle x, y, z \rangle \, dV}{\iiint_{\Omega} m dV} = \frac{\iiint_{\Omega} \langle x, y, z \rangle \, dV}{\iiint_{\Omega} dV}$$