Cartesian and Polar Double Integrals

Question 1:

part 1a:

For the following double integrals, convert the function that is being integrated over (the integrand) from a function of Cartesian coordinates to a function of Polar coordinates:

- $\iint_\sigma (x+y)dA$
- $\iint_\sigma xydA$
- $\iint_\sigma \frac{y}{x}dA$
- $\iint_\sigma \frac{dA}{(x^2+y^2)^{3/2}}$
- $\iint_\sigma \frac{x^2-y^2}{2xy}dA$

part 1b:

For the following double integrals, convert the function that is being integrated over (the integrand) from a function of Polar coordinates to a function of Cartesian coordinates:

- $\iint_\sigma r\cos\theta dA$
- $\iint_\sigma r^2\sin\theta dA$
- $\iint_\sigma r^{-3}dA$
- $\iint_\sigma \frac{\cos\theta-3\sin\theta}{2\cos\theta+\sin\theta}dA$
- $\iint_\sigma r\tan\theta dA$
- $\iint_\sigma \cos(2\theta)dA$
- $\iint_\sigma \sin(2\theta)dA$
- $\iint_\sigma \tan(2\theta)dA$

Question 2:

part 2a:

For the following regions characterized using Cartesian coordinates, express these regions using Polar coordinates:

- $\sigma = \{(x,y) | -2 \le x \le 2 \text{ and } 0 \le y \le \sqrt{4-x^2}\}$

- $\sigma = \{(x,y)|0 \leq x \leq 2 \text{ and } -\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2}\}$
- $\sigma = \{(x,y)|0 \leq x \leq 1 \text{ and } -\frac{x}{\sqrt{3}} \leq y \leq x\}$
- $\sigma = \{(x,y)| -2 \leq x \leq 2 \text{ and } 0 \leq y \leq 4-x^2\}$
- $\sigma = \{(x,y)|0 \leq y \leq 3 \text{ and } -5+(5/3)y \leq x \leq 0\}$
- $\sigma = \{(x,y)| -1 \leq x \leq 1 \text{ and } 1-\sqrt{1-x^2} \leq y \leq 1+\sqrt{1-x^2}\}$
- $\sigma = \{(x,y)|1-\sqrt{2} \leq x \leq 1+\sqrt{2} \text{ and } 1-\sqrt{-x^2+2x+1} \leq y \leq 1+\sqrt{-x^2+2x+1}\}$
- $\sigma = \{(x,y)| -6 \leq x \leq 0 \text{ and } 2x^2+12x \leq y \leq 0\}$

part 2b:

For the following regions characterized using Polar coordinates, express these regions using Cartesian coordinates:

- $\sigma = \left\{(r,\theta)\middle| -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{4} \text{ and } 0 \leq r \leq \frac{3}{2\cos\theta - \sin\theta}\right\}$
- $\sigma = \left\{(r,\theta)\middle|0 \leq \theta \leq \frac{\pi}{4} \text{ and } 0 \leq r \leq 2\sin\theta\right\}$
- $\sigma = \left\{(r,\theta)\middle|-\text{atan}(2) \leq \theta \leq \frac{\pi}{4} \text{ and } 0 \leq r \leq \frac{2\cos\theta+\sin\theta}{\cos^2\theta}\right\}$
- $\sigma = \left\{(r,\theta)\middle| -\frac{\pi}{2} \leq \theta \leq \text{atan}(\frac{3}{2}) \text{ and } 0 \leq r \leq \frac{\sin\theta+\sqrt{1+3\cos^2\theta}}{2\cos^2\theta}\right\}$
- $\sigma = \left\{(r,\theta)\middle| -\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6} \text{ and } 2\cos\theta - \sqrt{4\cos^2\theta - 3} \leq r \leq 2\cos\theta + \sqrt{4\cos^2\theta - 3}\right\}$

Question 3:

For the following iterated integrals, reverse the order of integration:

- $\int_{x=0}^{2} \int_{y=x^2}^{2x} f(x,y)dydx$
- $\int_{y=0}^{3} \int_{x=-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x,y)dxdy$
- $\int_{x=-5}^{1} \int_{y=-4}^{-x^2-4x+1} f(x,y)dydx$