Intro to Vector Calculus

Question 1:

Compute the following scalar line integrals:

- $\int_C (x+y)ds$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} t \\ 1-t \end{bmatrix}$ and $t \in [0,1]$
- $\int_C (x-y)ds$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} 4t \\ 3t \end{bmatrix}$ and $t \in [0,2]$
- $\int_C (x^2+y^2+z^2)ds$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} \sin(t) \\ \cos(t) \\ 8t \end{bmatrix}$ and $t \in [0,\pi/2]$
- $\int_C \sqrt{1+4y} \cdot ds$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} t \\ t^2 \end{bmatrix}$ and $t \in [0,2]$
- $\int_C xy^4 \cdot ds$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} 2\sin t \\ 2\cos t \end{bmatrix}$ and $t \in [0,\pi]$
- $\int_C y^2 \cdot ds$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} 3+2\cos t \\ 4+2\sin t \end{bmatrix}$ and $t \in [0,2\pi]$
- $\int_C \frac{ds}{x^2+y^2+z^2}$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} \cos t \\ \sin t \\ t \end{bmatrix}$ and $t \in [0,2\pi]$

Question 2:

Compute the following vector line integrals:

- $\int_C \begin{bmatrix} x^2+y \\ x^2-y \end{bmatrix} \cdot d\mathbf{r}$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} t \\ t^2 \end{bmatrix}$ and $t \in [0,1]$
- $\int_C \begin{bmatrix} -y/(x^2+y^2) \\ x/(x^2+y^2) \end{bmatrix} \cdot d\mathbf{r}$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} R\cos(t) \\ R\sin(t) \end{bmatrix}$ and $t \in [0,2\pi]$
- $\int_C \begin{bmatrix} 1/(x+y) \\ 1/(x+y) \end{bmatrix} \cdot d\mathbf{r}$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix}$ and $t \in [0,\pi/2]$
- $\int_C \begin{bmatrix} zy \\ x \\ z^2x \end{bmatrix} \cdot d\mathbf{r}$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} t \\ 2t \\ 3t \end{bmatrix}$ and $t \in [0,1]$

- $\int_C \begin{bmatrix} x \\ y \\ -5z \end{bmatrix} \cdot d\mathbf{r}$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} 2\cos(t) \\ 2\sin(t) \\ t \end{bmatrix}$ and $t \in [0, 2\pi]$
- $\int_C \begin{bmatrix} -(5y+4)/2 \\ \sqrt{2x} - 3 \end{bmatrix} \cdot d\mathbf{r}$ where C has the parameterization $C : \mathbf{r}_C(t) = \begin{bmatrix} t^2/2 \\ -2t \end{bmatrix}$ and $t \in [0, 3]$
- $\int_C \begin{bmatrix} y^2 \\ xy - x^2 \end{bmatrix} \cdot d\mathbf{r}$ where C has the parameterization $C : \mathbf{r}(t) = \begin{bmatrix} t \\ 3t \end{bmatrix}$ and $t \in [0, 1]$
- $\int_C \begin{bmatrix} y^2 \\ xy - x^2 \end{bmatrix} \cdot d\mathbf{r}$ where C has the parameterization $C : \mathbf{r}(t) = \begin{bmatrix} t^2/9 \\ t \end{bmatrix}$ and $t \in [0, 3]$

Question 3:

For each of the following vector fields $\mathbf{F}(\mathbf{r})$, determine if the field is conservative, and if so, find a scalar field $f(\mathbf{r})$ such that $\mathbf{F}(\mathbf{r}) = \nabla f(\mathbf{r})$.

- $\mathbf{F}(x,y) = \begin{bmatrix} F_x(x,y) \\ F_y(x,y) \end{bmatrix} = \begin{bmatrix} 2xy^3 \\ 3y^2x^2 \end{bmatrix}$
- $\mathbf{F}(x,y) = \begin{bmatrix} F_x(x,y) \\ F_y(x,y) \end{bmatrix} = \begin{bmatrix} -y + e^x\sin(y) \\ (x+2)e^x\cos(y) \end{bmatrix}$
- $\mathbf{F}(x,y) = \begin{bmatrix} F_x(x,y) \\ F_y(x,y) \end{bmatrix} = \begin{bmatrix} e^{2x}\sin(y) \\ e^{2x}\cos(y) \end{bmatrix}$
- $\mathbf{F}(x,y) = \begin{bmatrix} F_x(x,y) \\ F_y(x,y) \end{bmatrix} = \begin{bmatrix} 6x + 5y \\ 5x + 4y \end{bmatrix}$
- $\mathbf{F}(x,y) = \begin{bmatrix} F_x(x,y) \\ F_y(x,y) \end{bmatrix} = \begin{bmatrix} 2x\cos(y) - y\cos(x) \\ -x^2\sin(y) - \sin(x) \end{bmatrix}$
- $\mathbf{F}(x,y) = \begin{bmatrix} F_x(x,y) \\ F_y(x,y) \end{bmatrix} = \begin{bmatrix} 3x^2 - 4xy \\ 2x^2 - 4 \end{bmatrix}$
- $\mathbf{F}(x,y) = \begin{bmatrix} F_x(x,y) \\ F_y(x,y) \end{bmatrix} = \begin{bmatrix} 3x^2 + 4xy \\ 2x^2 - 4 \end{bmatrix}$
- $\mathbf{F}(x,y) = \begin{bmatrix} F_x(x,y) \\ F_y(x,y) \end{bmatrix} = \begin{bmatrix} e^{y^2-y} + e^x \\ x(2y-1)e^{y^2-y} \end{bmatrix}$
- $\mathbf{F}(x,y) = \begin{bmatrix} F_x(x,y) \\ F_y(x,y) \end{bmatrix} = \begin{bmatrix} -(x^4+y^2)^{-2} \cdot 4x^3 \\ -(x^4+y^2)^{-2} \cdot 2y \end{bmatrix}$