Multi-variable Functions

Question 1:

Draw the domains of the following multi-variable functions. For curves, use solid lines to include the curve as part of the domain, and use dashed lines to exclude the curve from the domain.

- $f(x,y) = \frac{\sqrt{-x^2+4x}}{\sqrt{9-x^2-y^2}}$
- $f(x,y) = \ln(x+y-x^2)$
- $f(x,y) = \frac{\ln(x)}{xy+2x-3y-6}$

Question 2:

Compute the following limits:

- $\lim_{t\to -1} \begin{bmatrix} \sqrt{t+3} \\ \frac{t^2}{t+2} \\ \ln(t+5) \end{bmatrix}$
- $\lim_{(x,y)\to(-1,-2)} \frac{\sqrt{x+y+5}}{x+y+4}$

Question 3:

Define the two-variable function $f(x,y) = \frac{xy}{x^2+y^2}$.

part 3a:

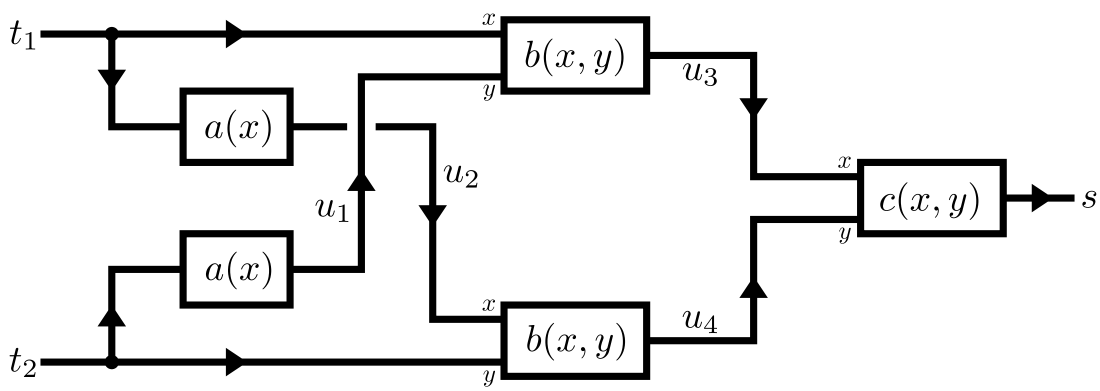
Compute the following partial derivatives:

$$\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \quad \frac{\partial^2 f}{\partial x^2} \quad \frac{\partial^2 f}{\partial y^2} \quad \frac{\partial^2 f}{\partial y \partial x}$$

part 3b:

Compute the gradient ∇f at the point $(x_0, y_0) = (3, 1)$. Compute the equation of the tangent plane to the surface $z = f(x,y)$ that passes through the point $(x_0, y_0, f(x_0, y_0))$. Given a direction of $\mathbf{v} = \langle 3, -4 \rangle$, what is the "directional derivative" of $f(x,y)$ at (x_0, y_0) in the direction of $\mathbf{v}$?

Question 4:



In the flow-chart (arithmetic circuit) above, the output quantity s is being computed from input quantities t_1 and t_2. There are the internal variables u_1, u_2, u_3, and u_4.

part 4a:

Build expressions for u_1, u_2, u_3, u_4, and s from the input parameters t_1 and t_2, and the functions $a(x)$, $b(x, y)$, and $c(x, y)$.

part 4b:

Without any knowledge of $a(x)$, $b(x, y)$, or $c(x, y)$, derive expressions for the following partial derivatives: $\frac{\partial u_1}{\partial t_1}$, $\frac{\partial u_1}{\partial t_2}$, $\frac{\partial u_2}{\partial t_1}$, and $\frac{\partial u_2}{\partial t_2}$.

Derive expressions for the following partial derivatives: $\frac{\partial u_3}{\partial t_1}$, $\frac{\partial u_3}{\partial t_2}$, $\frac{\partial u_4}{\partial t_1}$, and $\frac{\partial u_4}{\partial t_2}$ in terms of the partial derivatives computed previously.

Derive expressions for the following partial derivatives: $\frac{\partial s}{\partial t_1}$, and $\frac{\partial s}{\partial t_2}$ in terms of the partial derivatives computed previously.

part 4c:

Now let $a(x) = 1 - x$, $b(x, y) = xy$, and $c(x, y) = x + y - xy$. Compute all first-order derivatives: $\frac{da}{dx}$, $\frac{\partial b}{\partial x}$, $\frac{\partial b}{\partial y}$, $\frac{\partial c}{\partial x}$, and $\frac{\partial c}{\partial y}$.

part 4d:

From the results of the previous sections, compute at $(t_1, t_2) = (3/4, 1/4)$ the output s, as well as the partial derivatives $\frac{\partial s}{\partial t_1}$ and $\frac{\partial s}{\partial t_2}$.

Question 5:

For each of the following two variable functions $f(x, y)$, find and classify all of the critical points:

- $f(x, y) = -11x^2 + 6xy - 19y^2 + 78x - 94y - 211$

- $f(x,y) = \frac{1}{3}x^3 - \frac{7}{18}x^2 - \frac{2}{3}xy + y^2 - \frac{10}{9}x - \frac{8}{3}y + \frac{16}{9}$
- $f(x,y) = \frac{1}{3}x^3 + \frac{7}{16}x^2 - \frac{3}{2}xy - y^2 - \frac{3}{8}x + \frac{7}{2}y - \frac{49}{16}$
- $f(x,y) = \frac{1}{3}y^3 - x^2 - \frac{3}{2}xy + \frac{7}{16}y^2 - \frac{5}{2}x - \frac{39}{8}y - \frac{25}{16}$
- $f(x,y) = \frac{5}{4}x^4 - \frac{1}{3}x^3 - 2x^2y - 2x^2 + y^2 + 4x$