Lines and Planes

Define the points $P(5,0,4)$; $Q(1,1,1)$; $R(7,3,1)$; and $S(-5,-1,1)$.

Question 1:

part 1a:

Compute the displacement of Q relative P: $\overrightarrow{PQ}$.

part 1b:

Compute the magnitude $\left|\overrightarrow{PQ}\right|$.

part 1c:

Compute a unit vector that shares the same direction of $\overrightarrow{PQ}$.

part 1d:

Compute parametric and implicit equations of a line L_{PQ} that contains P and Q.

Question 2:

part 2a:

Compute the angle $\angle RPQ$ using the dot product.

part 2b:

Compute the area of triangle ΔPQR using the cross product.

part 2c:

Compute an implicit equation of a plane M_{PQR} that contains P, Q and R.

part 2d:

Find the shortest distance between R and the line L_{PQ} that contains P and Q.

part 2e:

Find the intersection between the line L_{PQ} that contains P and Q and the line L_{RS} that contains R and S, if this intersection exists.

part 2f:

Find the volume of the parallelepiped bounded by $\overrightarrow{PQ}$, $\overrightarrow{PR}$, and $\overrightarrow{PS}$. Explain your result.

Question 3:

part 3a:

Given the line $L_1 : \begin{cases} x(t) = & 1+t \\ y(t) = & 2+3t \\ z(t) = & 1+t \end{cases}$ and the plane $M_1 : 2x - y + 7z = 9$, find the intersection between L_1 and M_1.

part 3b:

Given the plane $M_2 : x + y + z = 3$, find the intersection between M_1 and M_2.

part 3c:

Given the point $T(3, 3, 3)$, find the closest distance between T and M_1.

part 3d:

The plane $M_3 : -6x + 3y - 21z = -33$ is parallel to M_1. Find the separation between M_1 and M_3.

Question 4:

part a)

Given vector $\mathbf{u} = \begin{bmatrix} 2 \\ 5 \\ -7 \end{bmatrix}$ and vector $\mathbf{v} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$, find x and y such that $\mathbf{u}||\mathbf{v}$.

part b)

Given vector $\mathbf{u} = \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix}$ and vector $\mathbf{v} = \begin{bmatrix} -1 \\ 1 \\ z \end{bmatrix}$, find z such that $\mathbf{u} \perp \mathbf{v}$.