Vector Valued Functions

Question 1:

part 1a:

Given the vector valued function $\mathbf{r}(t) = \begin{bmatrix} t^2 - 1/t \\ \sqrt{t+5} \\ t^3 \end{bmatrix}$, compute its derivative $\frac{d\mathbf{r}}{dt}$.

part 1b:

Given a vector valued function $\mathbf{r}_1(t)$ where $\mathbf{r}_1(2) = \begin{bmatrix} 2 \\ -7 \\ 1 \end{bmatrix}$ and $\frac{d\mathbf{r}_1}{dt}\big|_{t=2} = \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix}$; and a vector valued function $\mathbf{r}_2(t)$ where $\mathbf{r}_2(2) = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$ and $\frac{d\mathbf{r}_2}{dt}\big|_{t=2} = \begin{bmatrix} 0 \\ -1 \\ 9 \end{bmatrix}$; use the **product rule** to compute $\frac{d}{dt}(\mathbf{r}_1(t) \cdot \mathbf{r}_2(t))\big|_{t=2}$ and $\frac{d}{dt}(\mathbf{r}_1(t) \times \mathbf{r}_2(t))\big|_{t=2}$.

part 1c:

Given a vector valued function $\mathbf{r}(t)$ where $\frac{d\mathbf{r}}{dt}\big|_{t=3} = \begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix}$ and $\frac{d\mathbf{r}}{dt}\big|_{t=-5} = \begin{bmatrix} -7 \\ 2 \\ 8 \end{bmatrix}$; and a scalar valued function $f(t)$ where $f(3) = -5$ and $\frac{df}{dt}\Big|_{t=3} = 4$; use the **chain rule** to compute $\frac{d}{dt}(\mathbf{r}(f(t)))\big|_{t=3}$.

part 1d:

Given the vector valued function $\mathbf{r}(t) = \begin{bmatrix} t\sqrt{t^2+1} \\ 1/t \\ \sin^2(t) \end{bmatrix}$, compute the definite integral $\int_{t=1}^{2} \mathbf{r}(t)dt$.

Question 2:

Consider the curve $\mathbf{r}(t) = \begin{bmatrix} e^{-t}\cos(t) \\ e^{-t}\sin(t) \\ 0 \end{bmatrix}$.

Compute the velocity $\mathbf{v}(t)$; the acceleration $\mathbf{a}(t)$; the speed $u(t)$; the unit length tangent vector $\mathbf{T}(t)$; the curvature $\kappa(t)$; and the unit length normal vector $\mathbf{N}(t)$.

Question 3:

For the curve $\mathbf{r}(t) = \begin{bmatrix} 3t^2 \\ 5t^2 \\ -4t^2 \end{bmatrix}$, compute the arc-length parameterization.

Question 4 (hard):

For the spiral $\mathbf{r}(t) = \begin{bmatrix} (1+t^2)\cos(\ln(1+t^2)) \\ (1+t^2)\sin(\ln(1+t^2)) \\ 0 \end{bmatrix}$, compute the arc-length parameterization.