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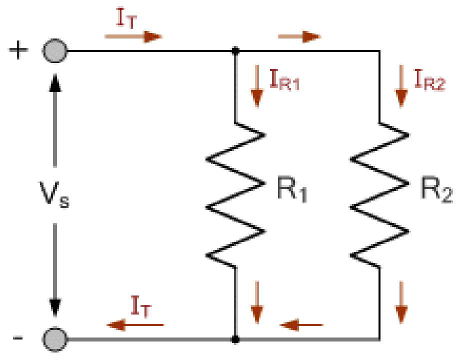


Current Dividers

Current Divider circuits have two or more parallel branches for currents to flow through but the voltage is the same for all components in the parallel cicruit

Current Divider Circuits are parallel circuits in which the source or supply current divides into a number of parallel paths. In a parallel connected circuit, all the components have their terminals connected together sharing the same two end nodes. This results in different paths and branches for the current to flow or pass along. However, the currents can have different values through each component.

The main characteristic of parallel circuits is that while they may produce different currents flowing through different branches, the voltage is common to all the connected paths. That is $V_{R1}= V_{R2} = V_{R3}$ … etc. Therefore the need to find the individual resistor voltages is eliminated allowing branch currents to be easily found with Kirchhoff's Current Law, (KCL) and of course Ohm's Law.

Resistive Voltage Divider

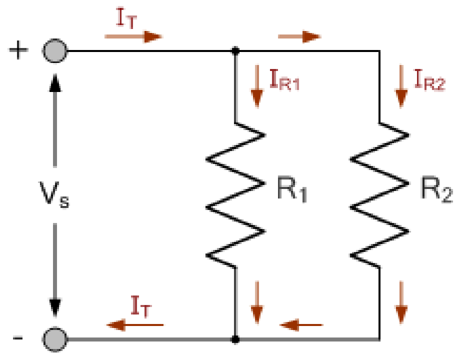
The easiest to understand, and most basic form of a passive current divider network is that of two resistors connected together in parallel. The *Current Divider Rule* allows us to calculate the current flowing through each parallel resistive branch as a percentage of the total current. Consider the circuit below.

Resistive Current Divider Circuit

Here this basic current divider circuit consists of two resistors: R_1, and R_2 in parallel which splits the supply or source current I_S between them into two separate currents I_{R1} and I_{R2} before joining together again and returning back to the source.

As the source or total current equals the sum of the individual branch currents, then the total current, I_T flowing in the circuit is given by Kirchoffs current law KCL as being:

$$I_T = I_{R1} + I_{R2}$$



As the two resistors are connected in parallel, for Kirchhoff's Current Law, (KCL) to hold true it must therefore follow that the current flowing through resistor R_1 will be equal to:

$$I_{R1} = I_T - I_{R2}$$

and the current flowing through resistor R_2 will be equal to:

$$I_{R2} = I_T - I_{R1}$$

As the same voltage, (V) is present across each resistive element, we can find the current flowing through each resistor in terms of this common voltage as it is simply V = I*R following Ohm's Law. So solving for the voltage (V) across the parallel combination gives us:

$$I_T = I_{R1} + I_{R2}$$

$$I_{R1} = \frac{V}{R_1} \quad \text{and} \quad I_{R2} = \frac{V}{R_2}$$

$$I_T = \frac{V}{R_1} + \frac{V}{R_2} = V\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$\therefore\ V = I_T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]^{-1} = I_T\left[\frac{R_1 R_2}{R_1 + R_2}\right]$$

Solving for I_{R1} gives:

$$I_{R1} = \frac{V}{R_1} = I_T\left[\frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}}\right]$$

$$\therefore\ I_{R1} = I_T\left(\frac{R_2}{R_1 + R_2}\right)$$

Likewise, solving for I_{R2} gives:

$$I_{R2} = \frac{V}{R_2} = I_T \left[\frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} \right]$$

$$\therefore I_{R2} = I_T \left(\frac{R_1}{R_1 + R_2} \right)$$

Notice that the above equations for each branch current has the opposite resistor in its numerator. That is to solve for I_1 we use R_2, and to solve for I_2 we use R_1. This is because each branch current is inversely proportional to its resistance resulting in the smaller resistance having the larger current.

Current Divider Example No1

A 20Ω resistor is connected in parallel with a 60Ω resistor. If the combination is connected across a 30 volts battery supply, find the current flowing through each resistor and the total current supplied by the source.

$$I_{R1} = \frac{V}{R_1} = \frac{30}{20} = 1.5\,\text{Amps}$$

$$I_{R2} = \frac{V}{R_2} = \frac{30}{60} = 0.5\,\text{Amps}$$

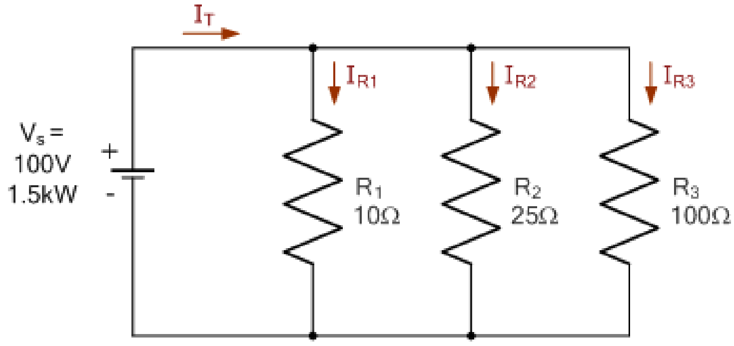
$$I_T = I_{R1} + I_{R2} = 1.5 + 0.5 = 2.0\,\text{Amperes}$$

Note that the smaller 20Ω resistor has the larger current because by its very nature, current will always flow through the path or branch of least resistance. This implies then that a short-circuit will produce maximum current flow, while an open-circuit will result in zero current flow. Remember also that the equivalent resistance, R_{EQ} of parallel connected resistors will always be less than the ohmic value of the smallest resistor with the equivalent resistance decreasing as more parallel resistances are added.

Sometimes it is not necessary to calculate all the branch currents, if the supply or total current, $I_T$is already known, then the final branch current can be found by simply subtracting the calculated currents from the total current as defined by Kirchhoffs current law.

Current Divider Example No2

Three resistors are connected together to form a current divider circuit as shown below. If the circuit is fed from a 100 volts 1.5kW power supply, calculate the individual branch currents using the current division rule and the equivalent circuit resistance.



1) Total circuit current I_T

$$P = V_S \times I_T$$

$$I_T = \frac{P}{V} = \frac{1500}{100} = 15\,\text{Amps}$$

2) Equivalent resistance R_{EQ}

$$R_{EQ} = \left[\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}\right]$$

$$R_{EQ} = \left[\frac{1}{\frac{1}{10} + \frac{1}{25} + \frac{1}{100}}\right]$$

$$R_{EQ} = \frac{1}{0.15} = 6.667\,\Omega$$

3) Branch currents I_{R1}, I_{R2}, I_{R3}

$$I_{R1} = I_T\left(\frac{R_{EQ}}{R_1}\right) = 15\left(\frac{6.667}{10}\right) = 10\text{ Amps}$$

$$I_{R2} = I_T\left(\frac{R_{EQ}}{R_2}\right) = 15\left(\frac{6.667}{25}\right) = 4\text{ Amps}$$

$$I_{R3} = I_T\left(\frac{R_{EQ}}{R_3}\right) = 15\left(\frac{6.667}{100}\right) = 1\text{ Amps}$$

We can check our calculations as according to Kirchhoff's Current Rule, all the branch currents will be equal to the total current, so: $I_T = I_{R1} + I_{R2} + I_{R3} = 10 + 4 + 1 = 15$ amperes, as expected. Thus we can see that the total current, I_T is divided according to a simple ratio determined by the branch resistances. Also, as the number of resistors connected in parallel increases, the supply ot total current, I_T will also increase for a given supply voltage, V_S as there are more parallel branches taking current.

Current Division using Conductances

Another simple method of finding the branch currents in a parallel circuit is to use the conductance method. In DC circuits, **Conductance** is the reciprocal of resistance, and is denoted by the letter "G". As conductance (G) is the reciprocal of resistance (R) which is measured in Ohm's (Ω), the reciprocal of Ohm's is called "mho" (℧), (an inverted ohm sign). Thus G = 1/R. The electrical units given to conductance is the Siemen (symbol S).

So for parallel connected resistors, the equavalent or total conductance, C_T will be equal to the sum of the individual conductances as shown.

Parallel Conductance

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{...etc}$$

$$G_T = G_1 + G_2 + G_3 + \text{...etc}$$

Therefore, if a resistance has a fixed value of 10Ω, it will have an equivalent conductance of 0.1S and so on. Because of the reciprocal, a high value of conductance represents a low value of resistance, and vice versa. We can also use prefixes in the form of *milli-Siemens*, mS, *micro-Siemens*, uS and even *nano-Siemens*, nS for very small conductances. So a resistor of 10kΩ will have a conductance of 100uS.

Using the Ohm's Law equation for current in which I = V/R, we can define the branch currents using conductance as being: I = V*G

In fact we can take this one step further by saying that the supply current to a our parallel resisive network above is:

$$I_S = G_T \times V_S$$

$$\therefore I_S = (G_1 + G_2 + G_3) \times V$$

But we know from above that for a parallel connected circuit, voltage is common to all components and as voltage equals current times resistance, V = I*R, we can therefore conclude that when using conductance, the voltage is equal to current divided by conductance. That is V = I/G.

Then we can express the above equations for the current divider rule in relationship to conductance (G), instead of the resistance (R) as being:

Current Divider Rule using Conductance

$$I_{R1} = G_1 \times V = G_1\left(\frac{I_T}{G_T}\right)$$

$$\therefore I_{R1} = I_T\left(\frac{G_1}{G_T}\right)$$

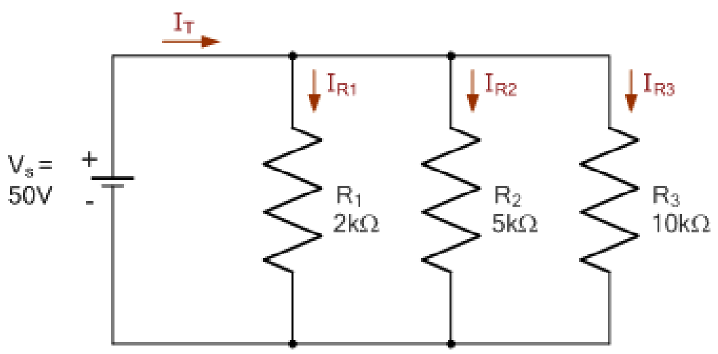
Likewise for the currents in parallel resistors R_2 and R_3 are given as:

$$I_{R2} = I_T\left(\frac{G_2}{G_T}\right); \qquad I_{R3} = I_T\left(\frac{G_3}{G_T}\right)$$

You may have noticed that unlike the equations above for resistance, each branch current has the same conductance in its numerator. That is to solve for I_1 we use G_1, and to solve for I_2 we use G_2. This is because the conductances are the reciprocals of the resistances.

Current Divider Example No3

Using the conductance method, find the individual branch currents, I_1, I_2 and I_3 of the following parallel resistive circuit.



Total conductance G_T

$$G_T = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2000} + \frac{1}{5000} + \frac{1}{10000}$$

$$\therefore\ G_T = \frac{1}{17000} = 800\,\mu S$$

Total supply current I_S

$$I_T = V_S \times G_T = 50 \times 0.0008 = 0.04\,A \text{ or } 40\,mA$$

$$G_1 = \frac{1}{2000} = 500\,\mu S$$

$$G_2 = \frac{1}{5000} = 200\,\mu S$$

$$G_3 = \frac{1}{10000} = 100\,\mu S$$

Individual branch currents I_1, I_2 and I_3

$$I_{R1} = I_T\left(\frac{G_1}{G_T}\right) = 0.04\left(\frac{0.0005}{0.0008}\right) = 25\,mA$$

$$I_{R2} = I_T\left(\frac{G_2}{G_T}\right) = 0.04\left(\frac{0.0002}{0.0008}\right) = 10\,mA$$

$$I_{R3} = I_T\left(\frac{G_3}{G_T}\right) = 0.04\left(\frac{0.0001}{0.0008}\right) = 5\,mA$$

As conductance is the reciprocal or inverse of resistance, the equivalent resistance value of the example circuit is simply 1/800uS which equals 1250Ω or 1.25kΩ, which is clearly less than the smallest resistor value of R_1 at 2kΩ.

Current Divider Summary

Current dividers or *current division* is the process of finding the individual branch currents in a parallel circuit were each parallel element has the same voltage. *Kirchhoff's current law*, (KCL) states that the algebraic sum of the individual currents entering a junction or node will equal the currents leaving it. That is the net result is zero.

Kirchhoff's current divider rule can also be used to find individual branch currents when the equivalent resistance and the total circuit current are known. When only two resistive branches are involved, the current in one branch will be some fraction of the total current I_T. If the two parallel resistive branches are of equal value, the current will divide equally.

In the case of three or more parallel branches, the equivalent resistance R_{EQ} is used to divide the total current into the fractional currents for each branch producing a current ratio which is equal to the inverse of their resistive values resulting in the smaller value resistance having the greatest share of the current. The supply or total current, I_T being the sum of all the individual branch currents. This then makes current dividers useful for use with current sources.

It is sometimes convenient to use conductance with parallel circuits as it can help reduce the maths required for determining the branch currents through individual circuit elements that are connected together in parallel. This is because for parallel circuits the total conductance is the sum of the individual conductance values. Conductance is the reciprocal or inverse of resistance as $G = 1/R$. The units for conductance are Siemens, S. The conductance of an element can also be used even if the supply voltage is DC or AC for *current dividers*.