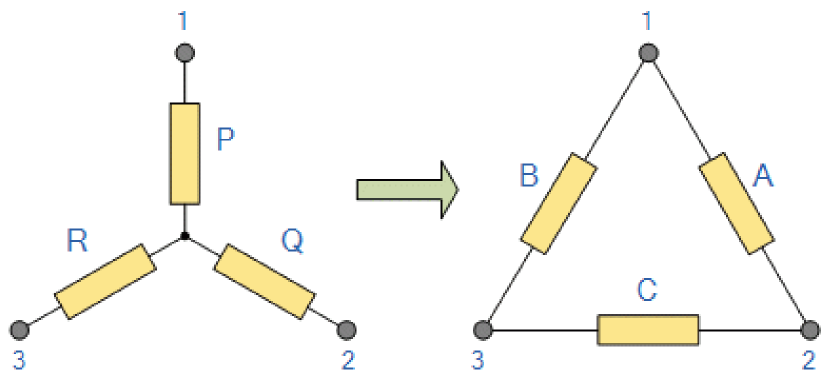


Star Delta Transformation

Star-Delta Transformations and Delta-Star Transformations allow us to convert impedances connected together in a 3-phase configuration from one type of connection to another

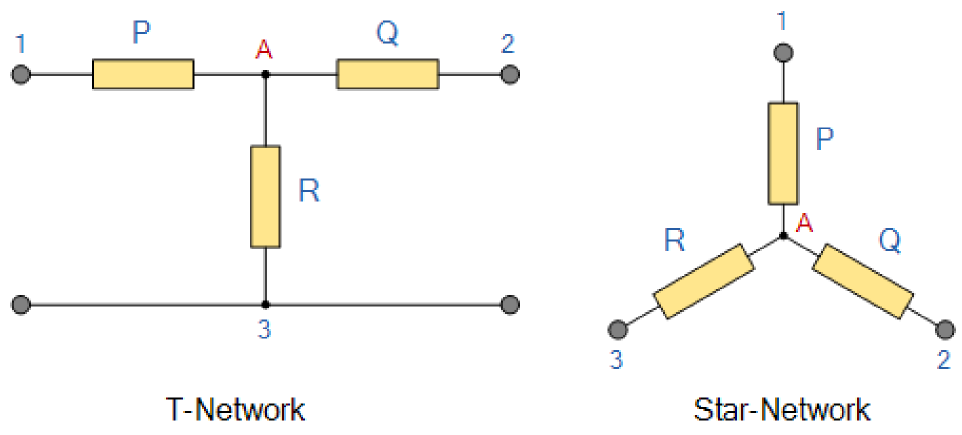
We can now solve simple series, parallel or bridge type resistive networks using **Kirchhoff´s Circuit Laws**, mesh current analysis or nodal voltage analysis techniques but in a balanced 3-phase circuit we can use different mathematical techniques to simplify the analysis of the circuit and thereby reduce the amount of math's involved which in itself is a good thing.

Standard 3-phase circuits or networks take on two major forms with names that represent the way in which the resistances are connected, a **Star** connected network which has the symbol of the letter, Υ (wye) and a **Delta** connected network which has the symbol of a triangle, Δ (delta).

If a 3-phase, 3-wire supply or even a 3-phase load is connected in one type of configuration, it can be easily transformed or changed it into an equivalent configuration of the other type by using either the **Star Delta Transformation** or **Delta Star Transformation** process.

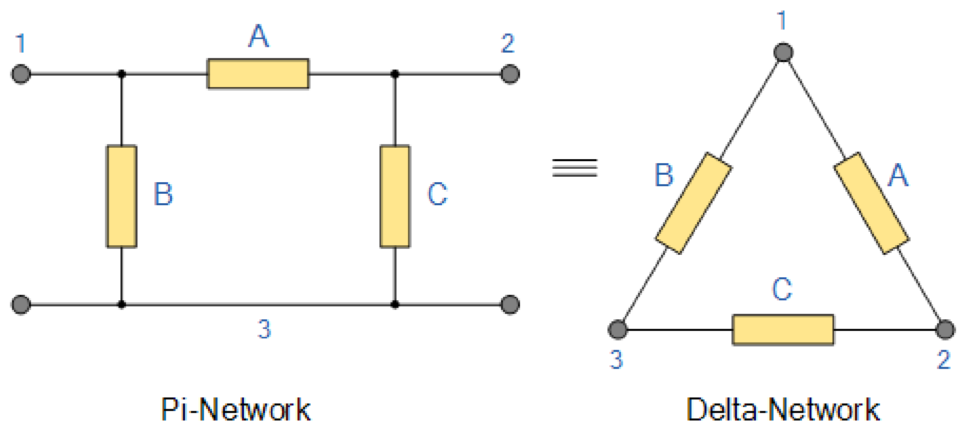
A resistive network consisting of three impedances can be connected together to form a T or "Tee" configuration but the network can also be redrawn to form a **Star** or Υ type network as shown below.

T-connected and Equivalent Star Network



As we have already seen, we can redraw the T resistor network above to produce an electrically equivalent **Star** or Y type network. But we can also convert a Pi or π type resistor network into an electrically equivalent **Delta** or Δ type network as shown below.

Pi-connected and Equivalent Delta Network



Having now defined exactly what is a **Star** and **Delta** connected network it is possible to transform the Y into an equivalent Δ circuit and also to convert a Δ into an equivalent Y circuit using a the transformation process.

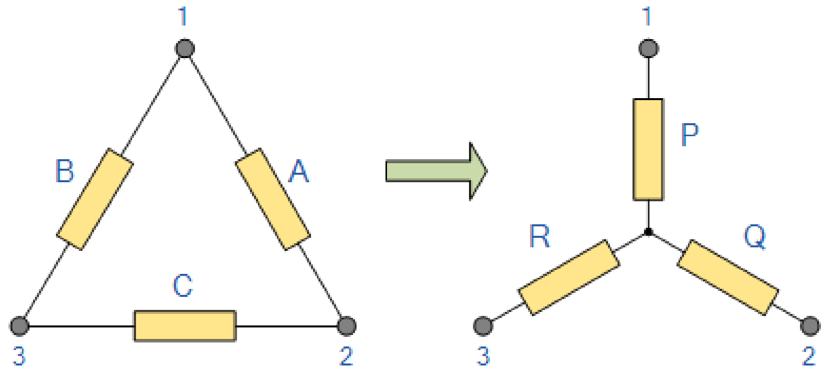
This process allows us to produce a mathematical relationship between the various resistors giving us a **Star Delta Transformation** as well as a **Delta Star Transformation**.

These circuit transformations allow us to change the three connected resistances (or impedances) by their equivalents measured between the terminals 1-2, 1-3 or 2-3 for either a star or delta connected circuit. However, the resulting networks are only equivalent for voltages and currents external to the star or delta networks, as internally the voltages and currents are different but each network will consume the same amount of power and have the same power factor to each other.

Delta Star Transformation

To convert a delta network to an equivalent star network we need to derive a transformation formula for equating the various resistors to each other between the various terminals. Consider the circuit below.

Delta to Star Network



Compare the resistances between terminals 1 and 2.

$$P + Q = A \text{ in parallel with } (B + C)$$

$$P + Q = \frac{A(B + C)}{A + B + C} \quad \text{...EQ1}$$

Resistance between the terminals 2 and 3.

$$Q + R = C \text{ in parallel with } (A + B)$$

$$Q + R = \frac{C(A + B)}{A + B + C} \quad \text{...EQ2}$$

Resistance between the terminals 1 and 3.

$$P + R = B \text{ in parallel with } (A + C)$$

$$P + R = \frac{B(A + C)}{A + B + C} \quad \text{...EQ3}$$

This now gives us three equations and taking equation 3 from equation 2 gives:

$$EQ3 - EQ2 = (P + R) - (Q + R)$$

$$P + R = \frac{B(A+C)}{A+B+C} - Q + R = \frac{C(A+B)}{A+B+C}$$

$$\therefore P - Q = \frac{BA+CB}{A+B+C} - \frac{CA+CB}{A+B+C}$$

$$\therefore \; P - Q = \frac{BA - CA}{A+B+C}$$

Then, re-writing Equation 1 will give us:

$$P + Q = \frac{AB + AC}{A + B + C}$$

Adding together equation 1 and the result above of equation 3 minus equation 2 gives:

$$(P - Q) + (P + Q)$$

$$= \frac{BA - CA}{A + B + C} + \frac{AB + AC}{A + B + C}$$

$$= 2P = \frac{2AB}{A + B + C}$$

From which gives us the final equation for resistor P as:

$$P = \frac{AB}{A+B+C}$$

Then to summarize a little about the above maths, we can now say that resistor P in a Star network can be found as Equation 1 plus (Equation 3 minus Equation 2) or Eq1 + (Eq3 – Eq2).

Similarly, to find resistor Q in a star network, is equation 2 plus the result of equation 1 minus equation 3 or Eq2 + (Eq1 – Eq3) and this gives us the transformation of Q as:

$$Q = \frac{AC}{A+B+C}$$

and again, to find resistor R in a Star network, is equation 3 plus the result of equation 2 minus equation 1 or Eq3 + (Eq2 – Eq1) and this gives us the transformation of R as:

$$R = \frac{BC}{A+B+C}$$

When converting a delta network into a star network the denominators of all of the transformation formulas are the same: A + B + C, and which is the sum of ALL the delta resistances. Then to convert any delta connected network to an equivalent star network we can summarized the above transformation equations as:

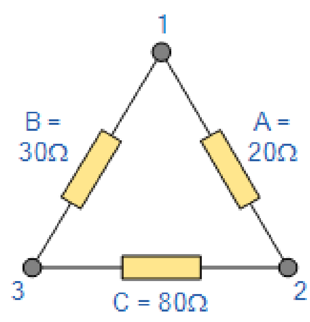
Delta to Star Transformations Equations

$$P = \frac{AB}{A+B+C} \qquad Q = \frac{AC}{A+B+C} \qquad R = \frac{BC}{A+B+C}$$

If the three resistors in the delta network are all equal in value then the resultant resistors in the equivalent star network will be equal to one third the value of the delta resistors. This gives each resistive branch in the star network a value of: $R_{STAR} = 1/3*R_{DELTA}$ which is the same as saying: $(R_{DELTA})/3$

Delta – Star Example No1

Convert the following Delta Resistive Network into an equivalent Star Network.



$$Q = \frac{AC}{A+B+C} = \frac{20\times 80}{130} = 12.31\Omega$$

$$P = \frac{AB}{A+B+C} = \frac{20\times 30}{130} = 4.61\Omega$$

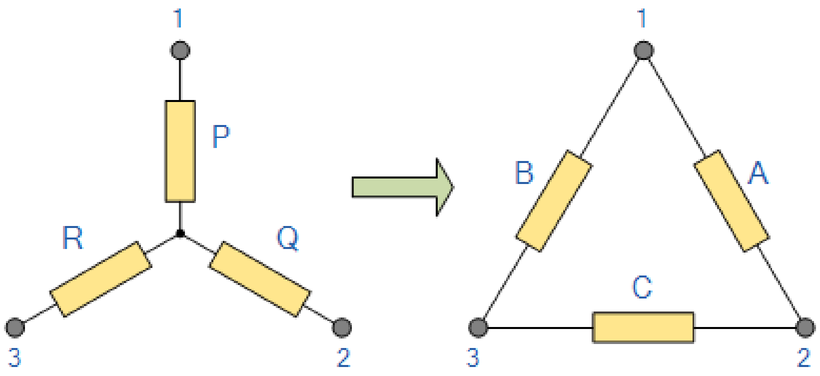
$$R = \frac{BC}{A+B+C} = \frac{30\times 80}{130} = 18.46\Omega$$

Star Delta Transformation

Star Delta transformation is simply the reverse of above. We have seen that when converting from a delta network to an equivalent star network that the resistor connected to one terminal is the product of the two delta resistances connected to the same terminal, for example resistor P is the product of resistors A and B connected to terminal 1.

By rewriting the previous formulas a little we can also find the transformation formulas for converting a resistive star network to an equivalent delta network giving us a way of producing a star delta transformation as shown below.

Star to Delta Transformation



The value of the resistor on any one side of the delta, Δ network is the sum of all the two-product combinations of resistors in the star network divide by the star resistor located "directly opposite" the delta resistor being found. For example, resistor A is given as:

$$A = \frac{PQ + QR + RP}{R}$$

with respect to terminal 3 and resistor B is given as:

$$B = \frac{PQ + QR + RP}{Q}$$

with respect to terminal 2 with resistor C given as:

$$C = \frac{PQ + QR + RP}{P}$$

with respect to terminal 1.

By dividing out each equation by the value of the denominator we end up with three separate transformation formulas that can be used to convert any Delta resistive network into an equivalent star network as given below.

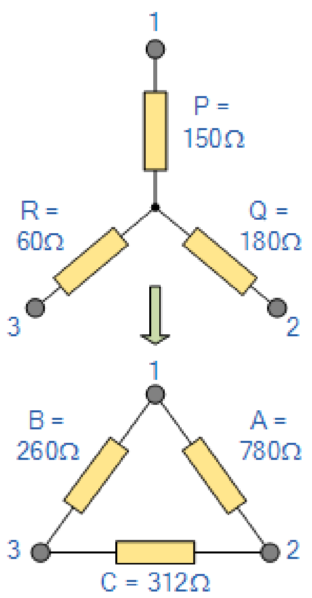
Star Delta Transformation Equations

$$A = \frac{PQ}{R} + Q + P \qquad B = \frac{RP}{Q} + P + R \qquad C = \frac{QR}{P} + Q + R$$

One final point about converting a star resistive network to an equivalent delta network. If all the resistors in the star network are all equal in value then the resultant resistors in the equivalent delta network will be three times the value of the star resistors and equal, giving: $R_{DELTA} = 3*R_{STAR}$

Star – Delta Example No2

Convert the following Star Resistive Network into an equivalent Delta Network.



$$A = \frac{QP}{R} + Q + P = \frac{180 \times 150}{60} + 180 + 150 = 780\Omega$$

$$B = \frac{RP}{Q} + R + P = \frac{60 \times 150}{180} + 60 + 150 = 260\Omega$$

$$C = \frac{QR}{P} + Q + R = \frac{180 \times 60}{150} + 180 + 60 = 312\Omega$$

Both **Star Delta Transformation** and **Delta Star Transformation** allows us to convert one type of circuit connection into another type in order for us to easily analyse the circuit. These transformation techniques can be used to good effect for either star or delta circuits containing resistances or impedances.