Series And Parallel Inductors (Formula & Example Problems)

October 29, 2021 by Electrical4U

Contents

What is an Inductor?

An inductor (also known as an electrical inductor) is defined as a two-terminal passive electrical element that stores energy in the form of a magnetic field when electric current flows through it. It is also called a coil, chokes, or reactor.

An inductor is simply a coil of wire. It usually consists of a coil of conducting material, typically insulated copper, wrapped into an iron core either of plastic or ferromagnetic material; thus, it is called an iron-core inductor.

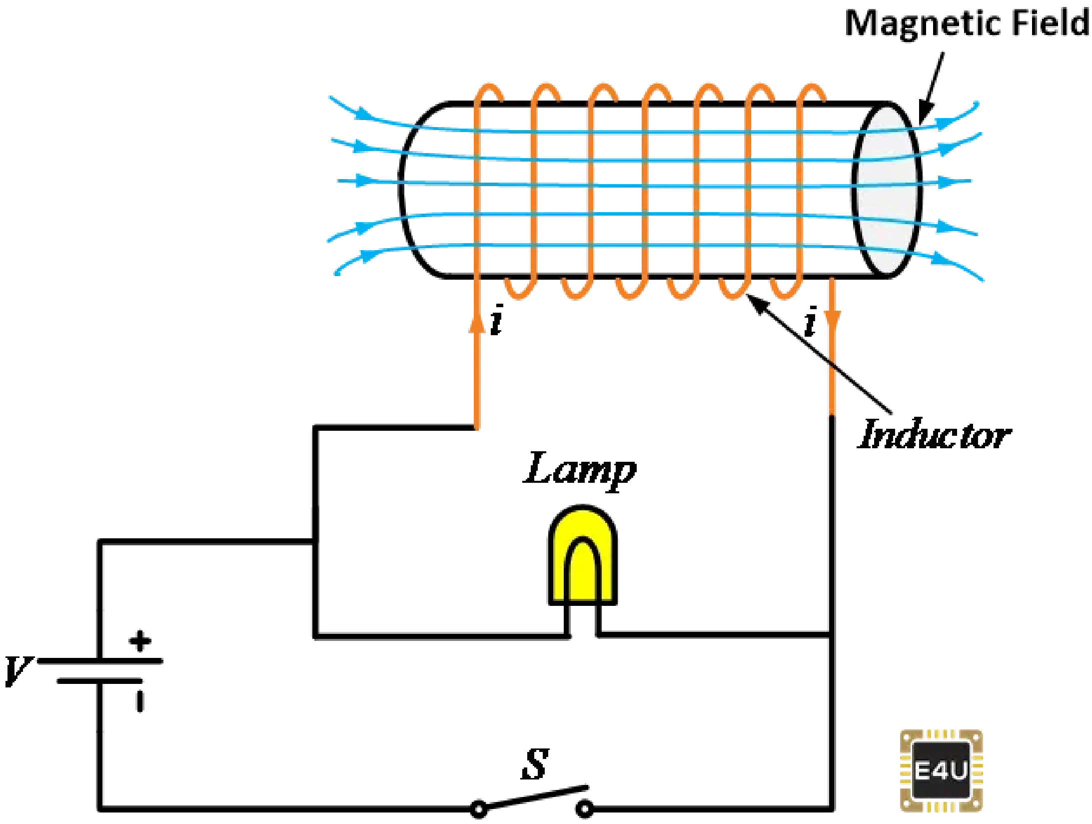
Inductors are typically available in the range from 1 μH (10^{-6} H) to 20 H. Many inductors have a magnetic core made of ferrite or iron inside the coil, which is used to increase the magnetic field and thus the inductor's inductance.

According's to Faraday's law of electromagnetic induction, when an electric current flowing through an inductor or coil changes, the time-varying magnetic field produces an e.m.f (electromotive force) or voltage in it. The induced voltage or e.m.f. across an inductor is directly proportional to the rate of change of the electric current flowing through the inductor.

Inductance (L) is a property of an inductor that opposes any change in magnitude or direction of current flowing through it. The larger an inductor's inductance, the greater the capacity to store electrical energy in the form of the magnetic field.

How Do Inductors Work?

The inductor in a circuit opposes changes in current flow through it by inducing a voltage across it which is proportional to the rate of change of current flow. To understand how the inductor work in a circuit, consider the image shown below.



Inductor Working in a Circuit

As shown, a lamp, a coil of wire (inductor), and a switch are connected to a battery. If we remove the inductor from the circuit, the lamp lights up normally. With the inductor, the circuit behaves completely differently.

The inductor or coil has much lower resistance compared to the lamp, thus when the switch is closed most of the current should starts flows through the coil as it provides a low-resistance path to the current. hence, we expect that lamp to glow very dimly.

But due to inductor behavior in the circuit, when we close the switch, the lamp glows brightly and then gets dimmer and when we open the switch, the bulb glows very brightly and then quickly goes out.

The reason is that, when voltage or potential difference is applied across an inductor, the electric current flowing through an inductor produces a magnetic field. This magnetic field again creates an induced electric current in the inductor but of opposite polarity, according's to Lenz's law.

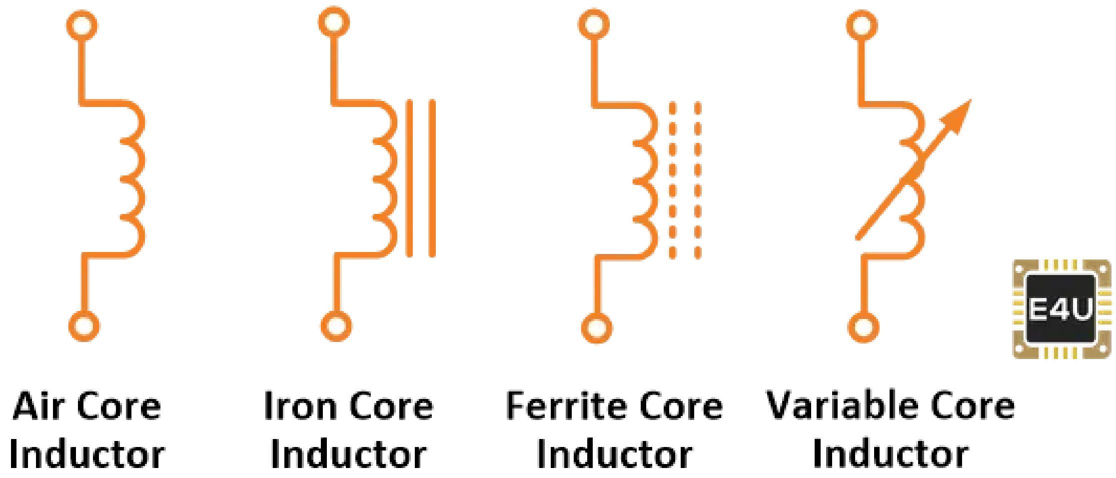
This induced current due to the magnetic field of the inductor tries to oppose any change, an increase or a decrease, in the current. Once the magnetic field is built, the current can flow normally.

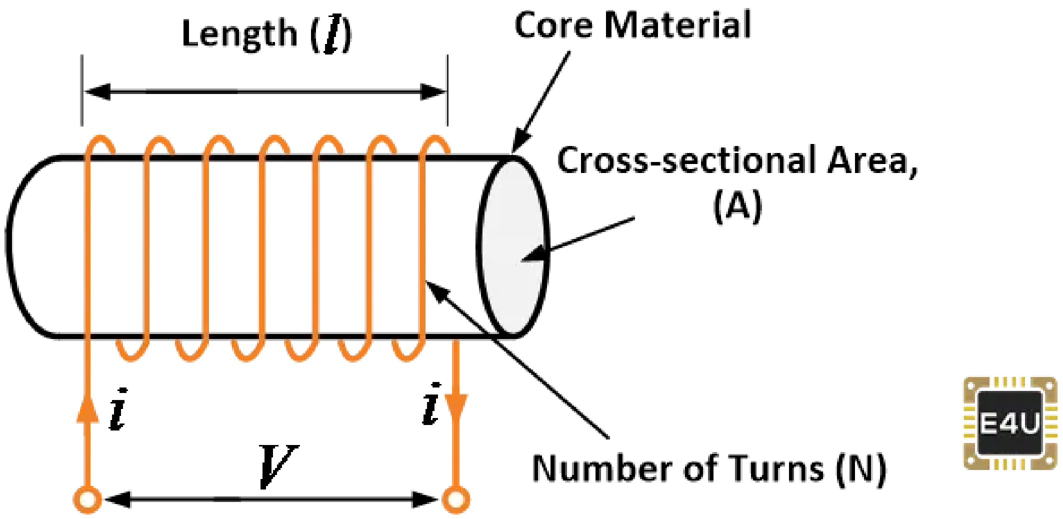
Now, when the switch is closed, the magnetic field around the inductor keeps current flowing in the inductor until the magnetic field collapses. This current keeps the lamp glowing for a certain amount of time even though the switch is open.

In other words, the inductor can store energy in the form of a magnetic field and it tries to oppose any change in the current flowing through it. Thus, the overall result of this is that the current through an inductor cannot change instantaneously.

Inductor Circuit Symbol

The schematic circuit symbol for an inductor is shown in the image below.





Inductor Symbol

Inductor Equation

Voltage Across an Inductor

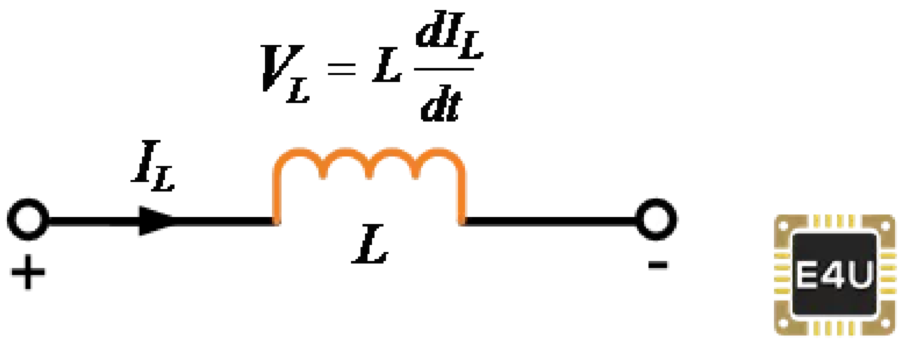
The voltage across an inductor is directly proportional to the rate of change of the electric current flowing through the inductor. Mathematically, the voltage across the inductor can be expressed as,

$$v_L = L\frac{di_L}{dt}$$

where, v_L = Instantaneous voltage across the inductor in Volts,

L = Inductance in Henry,

$\frac{di_L}{dt}$ = Rate of change of electric current in ampere per second



Voltage Across an Inductor

The voltage across an inductor is due to the energy stored in the magnetic field of the inductor.

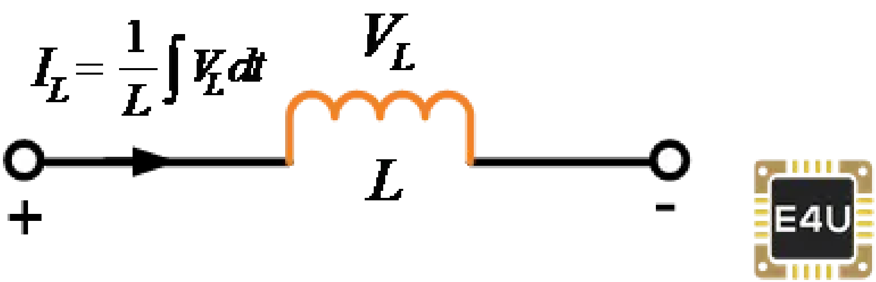
If d.c. current flows through the inductor $\frac{di_L}{dt}$ becomes zero as d.c. current is constant with respect to time. Hence, the voltage across the inductor becomes zero. Thus, as far as d.c. quantities are considered, in steady-state, the inductor acts as a short circuit.

Current Through an Inductor

We can express current through an inductor in terms of voltage developed across it as

$$i_L = \frac{1}{L}\int v_L dt$$

In the above equation, the limits of integration are decided by considering past history or initial conditions i.e., form $-\infty \ to \ t(0^-)$.



Current Through an Inductor

Now, assuming that switching action takes place at t=0, that means the switch is closed at t=0. We have the equation of current through an inductor as,

$$i_L = \frac{1}{L}\int v_L dt$$

We can split integration limits into two intervals as $-\infty\ to\ 0$ and $0\ to\ t$. we know that $0^-$is the instant just before switching action takes place, while 0^+ is the instant just after switching action takes place. Hence, we can write

$$i_L = \frac{1}{L}\int_{-\infty}^{t} v_L dt$$

Therefore,

$$i_L = \frac{1}{L}\int_{-\infty}^{0^-} v_L dt + \frac{1}{L}\int_{0^-}^{t} v_L dt$$

Here, the term $\frac{1}{L}\int_{-\infty}^{0^-} v_L dt$ indicate the value of inductor current in history period which is nothing but initial condition of i_L. Let it be denoted by $i_L(0^-)$.

$$i_L = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v_L dt$$

At $t = 0^+$, we can write,

$$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v_L dt$$

Initially, we assumed that switching action takes place at zero time. Thus, integration from 0^- to 0^+ is zero.

Therefore,

$$i_L(0^-) = i_L(0^+)$$

Thus, the current through the inductor cannot change instantaneously. That means the current through an inductor, before and after switching action is the same.

Inductor at t=0

Inductor at $t = 0$, i.e., at the time of switching the voltage across the inductor, is ideally ∞ as time interval dt is zero. Thus, at the time switching inductor acts as an open circuit. While in steady-state at $t = \infty$ it acts as a short circuit.

If the inductor carries an initial current I_0 before switching action, then at instant $t = 0^+$ it acts as a constant current source of value I_0, while in steady-state at $t = \infty$, it acts as a short circuit across a current source.

Series and Parallel Inductors

The inductors in series and parallel behave similarly to resistors in series and parallel. Consider two magnetically coupled coils 1 and 2 having self-inductance L_1 and L_2 respectively. Let M be the mutual inductance between two coils in henry.

The two inductors in an electrical circuit may be connected in different ways which give different values of equivalent inductance as discussed below.

Inductors in Series Formula

How to add inductors in series

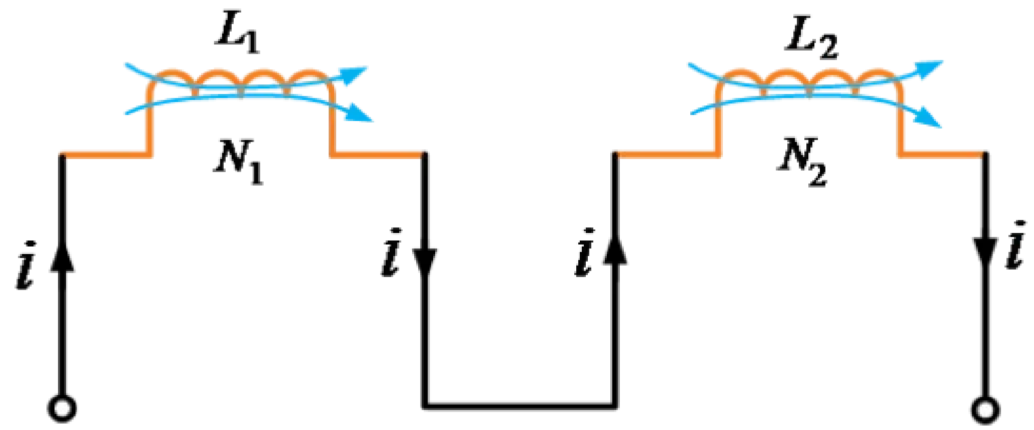
Consider a circuit containing two mutually coupled inductors or coils connected in series. There are two possible ways to connect the inductors in series.

- In a first way, the fluxes produced by the inductors act in the same direction. Then, such inductors are said to be connected in series-aiding or cumulatively.
- In a second way, if the current is reversed in the other inductor so that the fluxes produced by the inductors oppose each other, then such inductors are said to be connected in series-opposition or differentially.

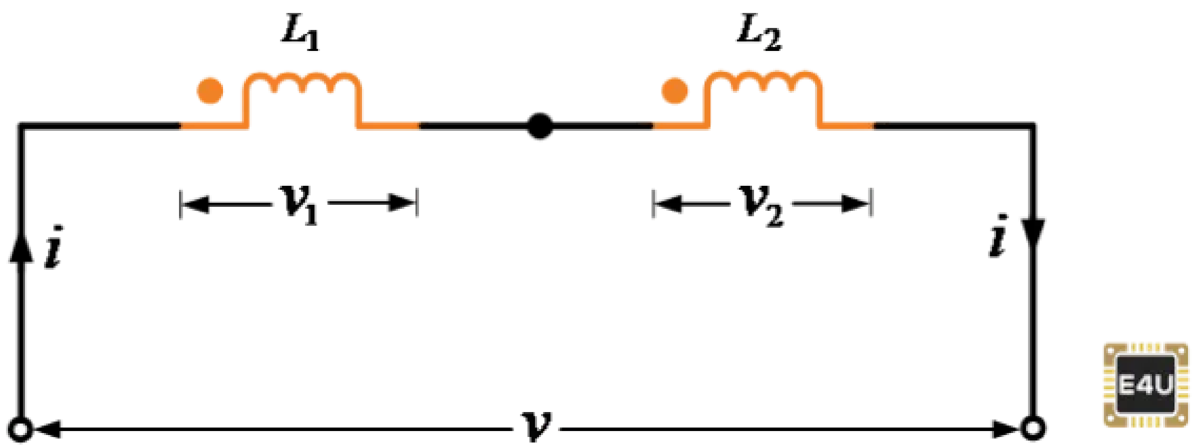
Let the self-inductance of inductor 1 be L_1 and that of inductor 2 be L_2. Both the inductors are coupled with the mutual inductance M.

Series-aiding (Cumulative) Connection (mutually induced emf assists the self-induced EMFs)

The two inductors or coils are connected in series-aiding or cumulatively, as shown in the image below.



Inductor Connected in Series-Aiding



Inductor Connected in Series-Aiding (Dot Notation)

In this connection, the self and mutual fluxes of both the inductors act in the same direction; thus, self and mutually induced e.m.f.s are also in the same direction.

Therefore,

- Self-induced e.m.f. in inductor 1, $e_{s1} = -L_1\frac{di}{dt}$
- Mutually induced e.m.f. in inductor 1, $e_{m1} = -M\frac{di}{dt}$
- Self-induced e.m.f. in inductor 2, $e_{s2} = -L_2\frac{di}{dt}$
- Mutually induced e.m.f. in inductor 1, $e_{m2} = -M\frac{di}{dt}$

Total induced e.m.f. in the combination,

$$e = -(L_1\frac{di}{dt} + M\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt})$$

$$e = -(L_1 + L_2 + 2M)\frac{di}{dt} \tag{1}$$

If $L_eq$is the equivalent inductance of the two inductors in a series-aiding connection, the e.m.f. induced in the combination is given by,

$$e = -L_{eq.}\frac{di}{dt} \tag{2}$$

Comparing equations (1) and (2), we get,

$$L_{eq.} = L_1 + L_2 + 2M \tag{3}$$

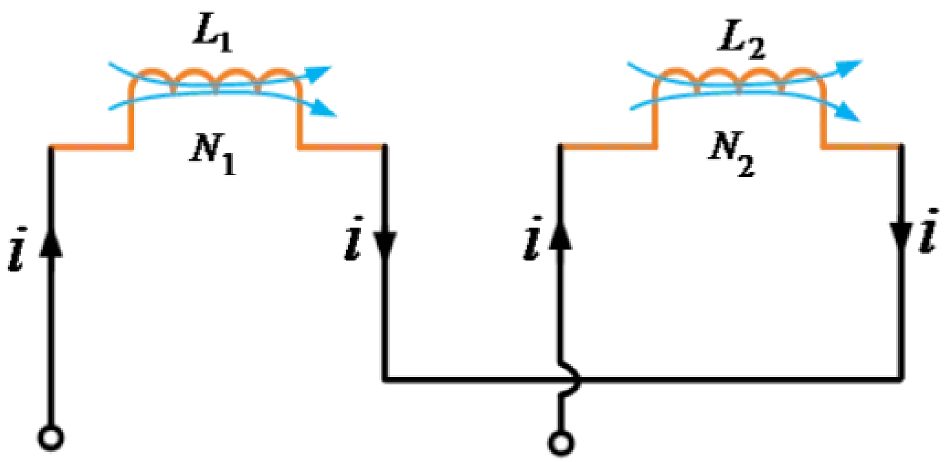
The above equation gives the equivalent inductance of two cumulatively or additively connected series inductors or coils.

If there is no mutual inductance between the two coils (i.e., M = 0), then,

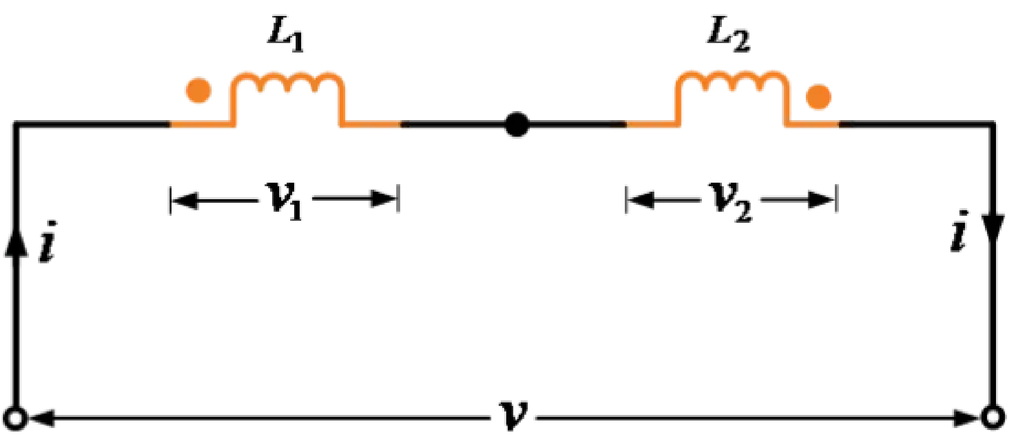
$$L_{eq.} = L_1 + L_2$$

Series Opposition (Differential) Connection (mutually induced emf opposes the self-induced EMFs)

Consider a circuit containing two mutually coupled inductors or coils connected in series such that the fluxes produced by the two inductors oppose each other, as shown in the image below.



Inductor Connected in Series-opposition



Inductor Connected in Series-opposition (Dot Notation)

As fluxes are in opposition, the sign for mutual-induced e.m.f. will be opposite to that of self-induced e.m.f.s. Therefore,

- Self-induced e.m.f. in inductor 1, $e_{s1} = -L_1\frac{di}{dt}$
- Mutually induced e.m.f. in inductor 1, $e_{m1} = +M\frac{di}{dt}$
- Self-induced e.m.f. in inductor 2, $e_{s2} = -L_2\frac{di}{dt}$
- Mutually induced e.m.f. in inductor 1, $e_{m2} = +M\frac{di}{dt}$

Total induced e.m.f. in the combination,

$$e = -(L_1\frac{di}{dt} - M\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt})$$

$$e = -(L_1 + L_2 - 2M)\frac{di}{dt} \tag{4}$$

If L_{eq} is the equivalent inductance of the two inductors in a series opposition connection, the e.m.f. induced in the combination is given by,

$$e = -L_{eq.}\frac{di}{dt} \tag{5}$$

Comparing equations (4) and (5), we get,

$$L_{eq.} = L_1 + L_2 - 2M \tag{6}$$

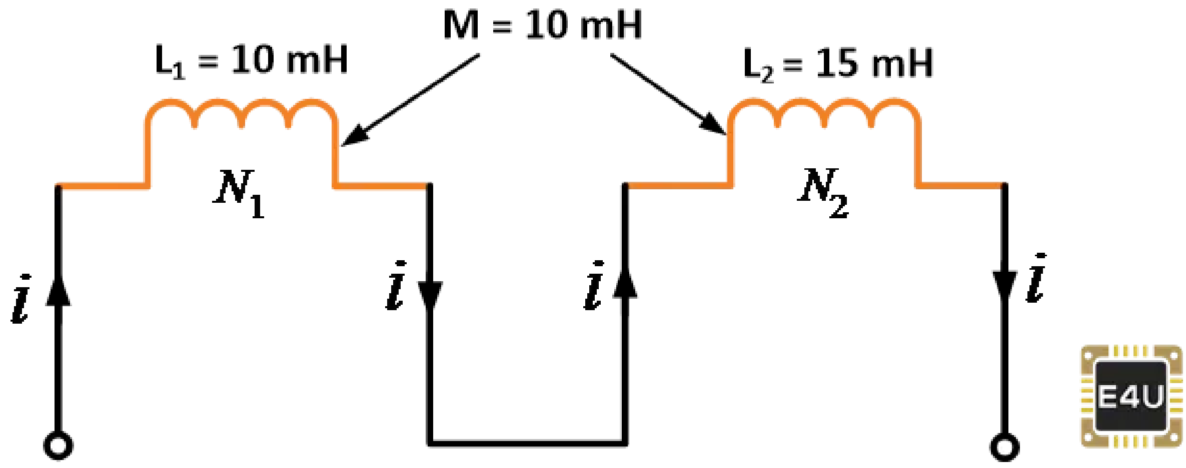
The above equation gives the equivalent inductance of two inductors connected in series opposition or differential connection.

If there is no mutual inductance between the two coils (i.e., M = 0), then,

$$L_{eq.} = L_1 + L_2$$

Example 1

Two coils have self-inductances of 10 mH and 15 mH and mutual inductance between two coils is 10 mH. Find the equivalent inductance when they are connected in series aiding.



Inductor Connected in Series-Aiding

Solution:

Given data: L_1 = 10 mH, L_2 = 15 mH and M = 10 mH

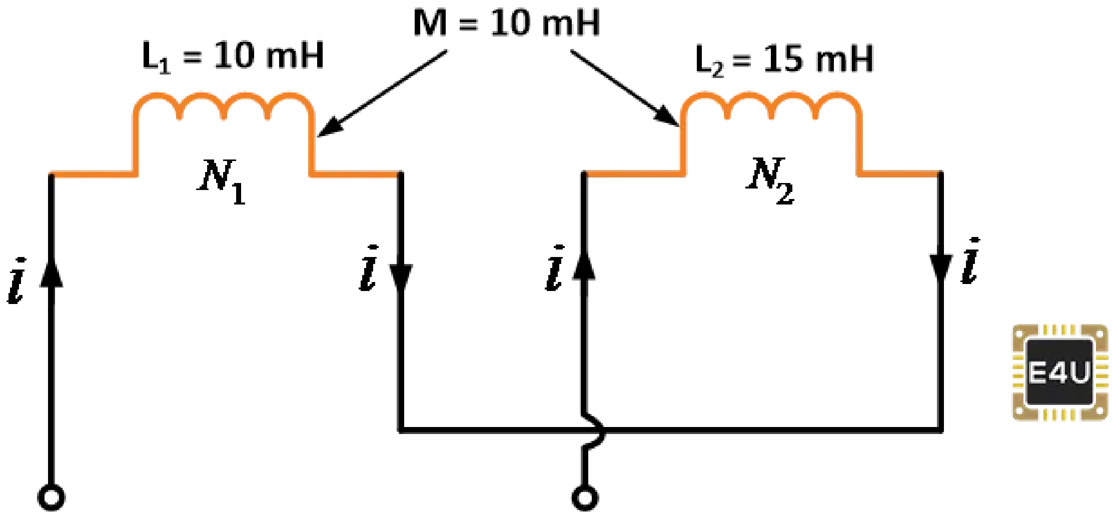
According to series aiding formula,

$$
\begin{aligned}
L_{eq.} &= L_1 + L_2 + 2M \\
&= 10 + 15 + 2(10) \\
&= 10 + 15 + 20 \\
L_{eq.} &= 45\ mH
\end{aligned}
$$

Thus, by using the equation, we get the equivalent inductance 45 mH when they are connected in series aiding.

Example 2

Two coils have self-inductances of 10 mH and 15 mH and mutual inductance between two coils is 10 mH. Find the equivalent inductance when they are connected in series opposing.



Inductor Connected in Series-opposition

Solution:

Given data: L_1 = 10 mH, L_2 = 15 mH and M = 10 mH

According to series opposition formula,

$$\begin{aligned} L_{eq.} &= L_1 + L_2 - 2M \\ &= 10 + 15 - 2(10) \\ &= 10 + 15 - 20 \\ &= 25 - 20 \\ L_{eq.} &= 5\ mH \end{aligned}$$

Thus, by using the equation, we get the equivalent inductance 5 mH when they are connected in series opposing.

Inductors in Parallel Formula

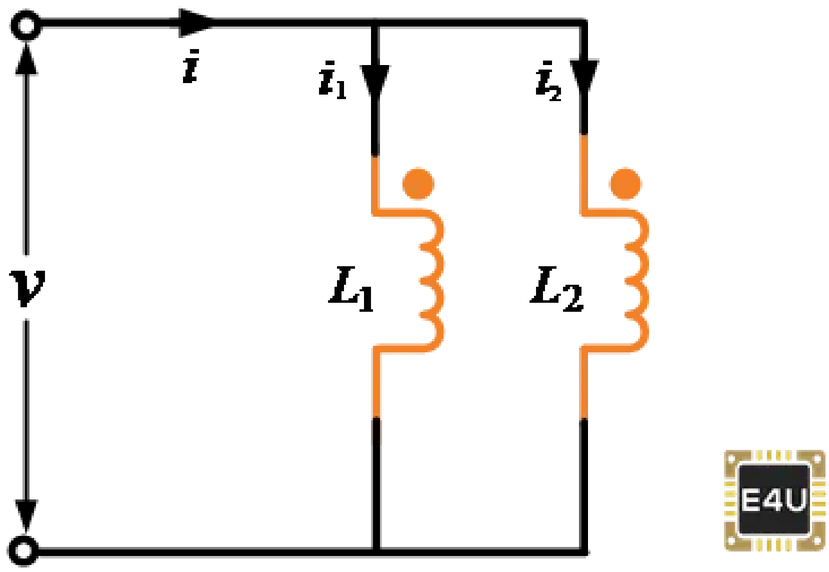
How to add inductors in parallel

The two inductors can be connected in parallel such that

- The mutually induced emf assists the self-induced EMFs i.e., parallel aiding connection
- The mutually induced emf opposes the self-induced EMFs i.e., parallel opposing connection

Parallel-aiding (Cumulative) Connection(mutually induced emf assists the self-induced EMFs)

When two inductors are connected in parallel aiding, the mutually induced emf assists the self-induced EMFs as shown in the figure below.



Inductor Connected in Parallel Aiding

Let i_1 and i_2 be the currents flowing through inductors L_1 and L_2 and I be the total current.

Thus,

$$i = i_1 + i_2 \tag{7}$$

Therefore,

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \tag{8}$$

In each inductor, there will be two EMFs induced. One due to self-induction and the other due to mutual induction.

Since the inductors are connected in parallel, the EMFs are equal.

Therefore,

$$L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \tag{9}$$

$$L_1\frac{di_1}{dt} - M\frac{di_1}{dt} = L_2\frac{di_2}{dt} - M\frac{di_2}{dt}$$

$$\frac{di_1}{dt}(L_1 - M) = \frac{di_2}{dt}(L_2 - M)$$

$$\frac{di_1}{dt} = (\frac{L_2 - M}{L_1 - M})\frac{di_2}{dt} \tag{10}$$

Now, put equation (9) in equation (8), we get,

$$\frac{di}{dt} = (\frac{L_2 - M}{L_1 - M})\frac{di_2}{dt} + \frac{di_2}{dt}$$

$$\frac{di}{dt} = (1 + \frac{L_2 - M}{L_1 - M}) + \frac{di_2}{dt} \tag{11}$$

If $L_{eq}.$ is the equivalent inductance of the parallel-connected inductors, the emf induced in it will be

$$e = L_{eq.}\frac{di}{dt} \tag{12}$$

This is equal to the emf induced in any one coil i.e.,

$$L_{eq.}\frac{di}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{1}{L_{eq.}}[L_1\frac{di_1}{dt} + M\frac{di_2}{dt}] \tag{13}$$

Substitute the value of $\frac{di_1}{dt}$ from equation (10) in to equation (13), we get,

$$\frac{di}{dt} = \frac{1}{L_{eq.}}[L_1(\frac{L_2 - M}{L_1 - M})\frac{di_2}{dt} + M\frac{di_2}{dt}]$$

$$\frac{di}{dt} = \frac{1}{L_{eq.}}[L_1(\frac{L_2 - M}{L_1 - M}) + M]\frac{di_2}{dt} \tag{14}$$

Now, equating equation (11) to equation (14),

$$1 + (\frac{L_2 - M}{L_1 - M})\frac{di_2}{dt} = \frac{1}{L_{eq.}}[L_1(\frac{L_2 - M}{L_1 - M}) + M]\frac{di_2}{dt}$$

$$\frac{L_1 + L_2 - 2M}{L_1 - M} = \frac{1}{L_{eq.}}[\frac{L_1L_2 - L_1M + L_1M - M^2}{L_1 - M}]$$

$$\frac{L_1 + L_2 - 2M}{L_1 - M} = \frac{1}{L_{eq.}}[\frac{L_1L_2 - M^2}{L_1 - M}]$$

$$L_{eq.} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} \tag{15}$$

The above equation gives the equivalent inductance of two inductors connected in parallel-aiding or cumulative connection.

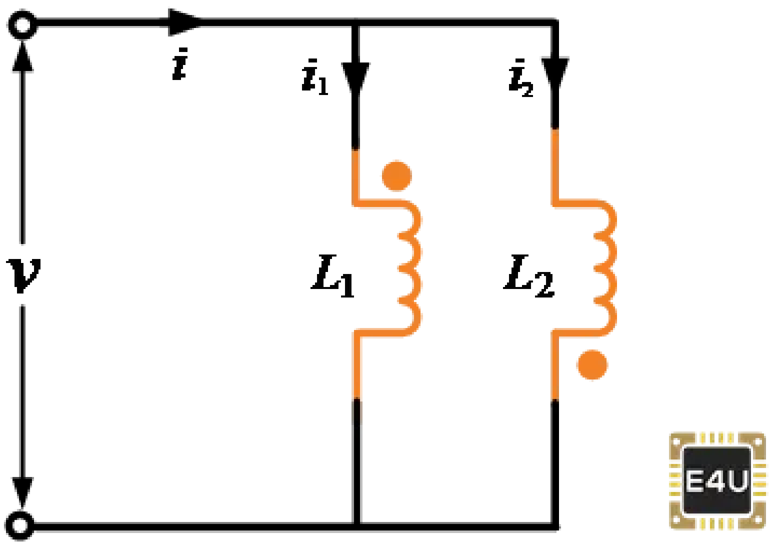
If there is no mutual inductance between the two coils (i.e., M = 0), then,

$$L_{eq.} = \frac{L_1 L_2 - (0)^2}{L_1 + L_2 - 2(0)} = \frac{L_1 L_2}{L_1 + L_2} = \frac{product}{sum}$$

Parallel Opposition (Differential) Connection (mutually induced emf opposes the self-induced EMFs)

When two inductors are connected in parallel opposition, the mutually induced emf opposes the self-induced EMFs.

As shown in the image below the two inductors are connected in parallel opposition or differentially.



Inductor Connected in Parallel Opposing

In a similar way to the parallel-aiding connection, it can be proved that,

$$L_{eq.} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \tag{16}$$

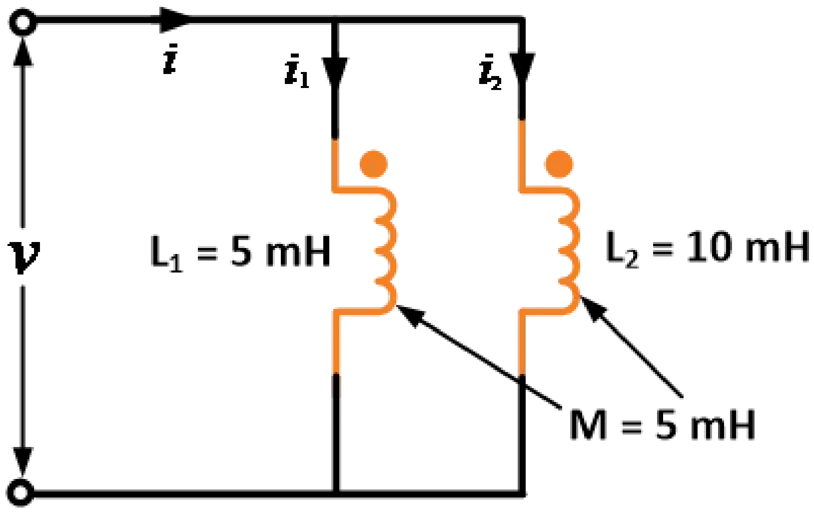
The above equation gives the equivalent inductance of two inductors connected in parallel opposition or differential connection.

If there is no mutual inductance between the two coils (i.e., M = 0), then,

$$L_{eq.} = \frac{L_1 L_2 - (0)^2}{L_1 + L_2 + 2(0)} = \frac{L_1 L_2}{L_1 + L_2} = \frac{product}{sum}$$

Example 1

Two inductors have self-inductances of 5 mH and 10 mH and mutual inductance between the two is 5 mH. Find the equivalent inductance when they are connected in parallel aiding.



Inductor Connected in Parallel Aiding

Solution:

Given data: $L_1 = 5$ mH, $L_2 = 10$ mH and M = 5 mH

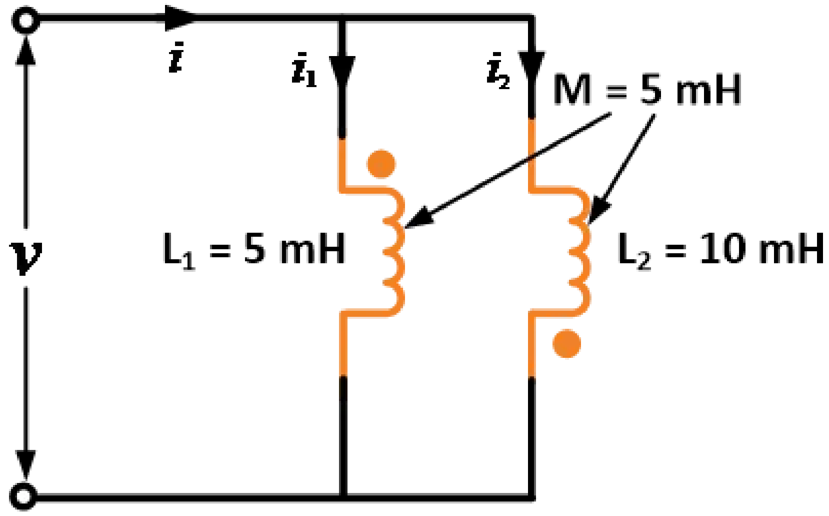
According to parallel aiding formula,

$$L_{eq.} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} ....if\ fluxes\ aid$$
$$= \frac{5 * 10 - (5)^2}{5 + 10 - 2(5)}$$
$$= \frac{50 - 25}{15 - 10}$$
$$= \frac{25}{5}$$
$$L_{eq.} = 5\ mH$$

Thus, by using the equation, we get the equivalent inductance 5 mH when they are connected in parallel aiding.

Example 2

Two inductors have self-inductances of 5 mH and 10 mH and mutual inductance between the two is 5 mH. Find the equivalent inductance when they are connected in parallel opposing.



Inductor Connected in Parallel Opposing

Solution:

Given data: $L_1 = 5$ mH, $L_2 = 10$ mH and M = 5 mH

According to parallel opposing formula,

$$L_{eq.} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \text{....}if\ fluxes\ oppose$$
$$= \frac{5 * 10 - (5)^2}{5 + 10 + 2(5)}$$
$$= \frac{50 - 25}{15 + 10}$$
$$= \frac{25}{25}$$
$$L_{eq.} = 1\ mH$$

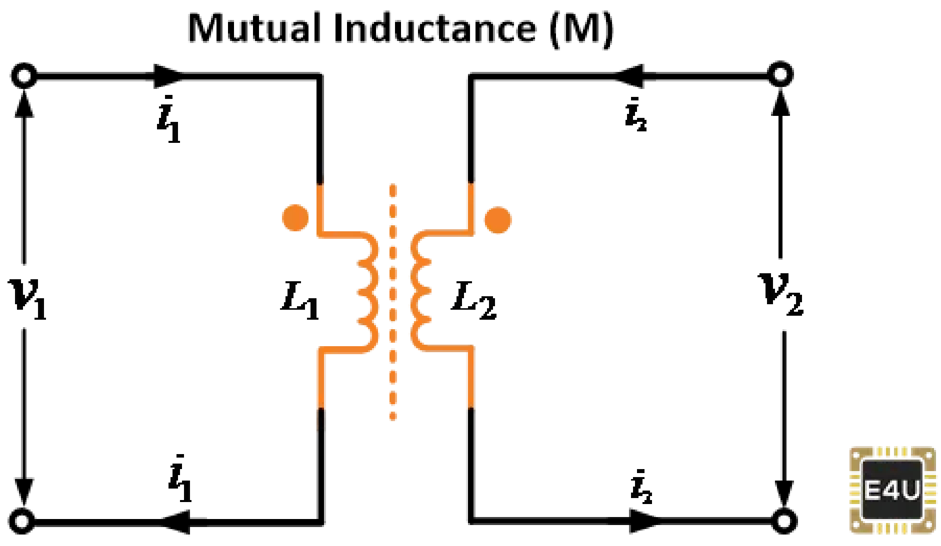
Thus, by using the equation, we get the equivalent inductance 1 mH when they are connected in parallel opposing.

Coupling Inductors

When the magnetic field of one inductor (coil) cuts or links the turns of another neighboring inductor, the two inductors are said to be magnetically coupled. Due to coupling inductors or coils, a mutual inductance exists between the two coils.

In coupled circuits, the energy transfer takes place from one circuit to another when either of the circuits is energized. A two-winding transformer, an autotransformer, and an induction motor are examples of magnetically coupled inductors or coils, or circuits.

Consider two magnetically coupled inductors or coils 1 and 2 having inductances L_1 and L_2 respectively. Let M be the mutual inductance between the two coils.



Mutually Coupled Inductor

The effect of mutual inductance is to either increase ($L_1 + M$ and $L_2 + M$) or decrease ($L_1 – M$ and $L_2 – M$) the inductance of the two coils, this is depending on the arrangement of the two coils or inductors.

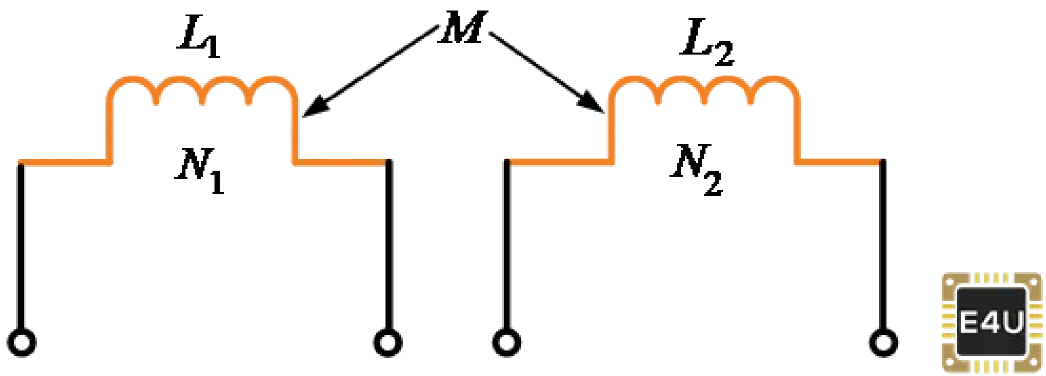
- When the two coils are so arranged that their fluxes aid each other, then the inductance of each coil is increased by M i.e., it becomes $L_1 + M$ for coil 1 and $L_2 + M$ for coil 2. It is because the total flux linking each coil is more than its own flux.
- When the two coils are so arranged that their fluxes oppose, then the inductance of each coil is decreased by M i.e., it becomes $L_1 – M$ for coil 1 and $L_2 – M$ for coil 2. It is because the total flux linking each coil is less than its own flux.

Mutual Inductance Formula

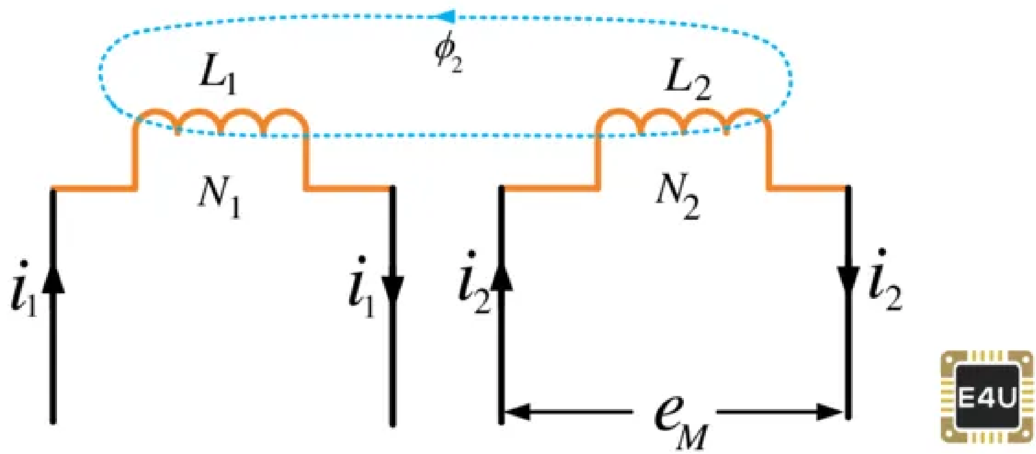
We know that any change of current in one coil is always accomplished by the production of mutually induced e.m.f. in the second coil.

Mutual inductance is defined as the ability of one coil (or circuit) to produce an e.m.f. in a nearby coil (or circuit) by induction when the current in the first coil changes.

In other words, the property of two coils by virtue of which each opposes any change of current flowing in the other is called mutual inductance between the two coils. This opposition occurs because a changing current in one coil produces mutually induced e.m.f. in the other coil which opposes a change of current in the first coil.



Mutual Inductance Between Two Coils



Mutual Inductance Between Two Coils

Mutual inductance (M) may be defined as the flux-linkages of a coil per unit current in the other coil.

Mathematically,

$$M = \frac{N_2 \phi_{12}}{I_1}$$

Where,

I_1 = Current in first coil

ϕ_{12} = Flux linking the second coil

N_2 = No. of turns on second coil

Mutual inductance between two coils is 1 henry if current changing at the rate of 1 ampere per second in one coil induces an e.m.f. of 1 V in the other coil.

Coefficient of Coupling

The coefficient of coupling (k) between two coils is defined as the fraction of magnetic flux produced by the current in one coil that links the other.

The coefficient of coupling is an important parameter for coupled circuits to determine the amount of coupling between the inductively coupled coils.

Mathematically, the coefficient of coupling can be expressed as,

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

Where,

L_1 is the self-inductance of the first coil

L_2 is the self-inductance of the second coil

M is the mutual inductance between two coils

The coupling coefficient is depending on the mutual inductance between two coils. If the coupling coefficient is higher so the mutual inductance will be higher. Two inductively coupled coils are linked using the magnetic flux.

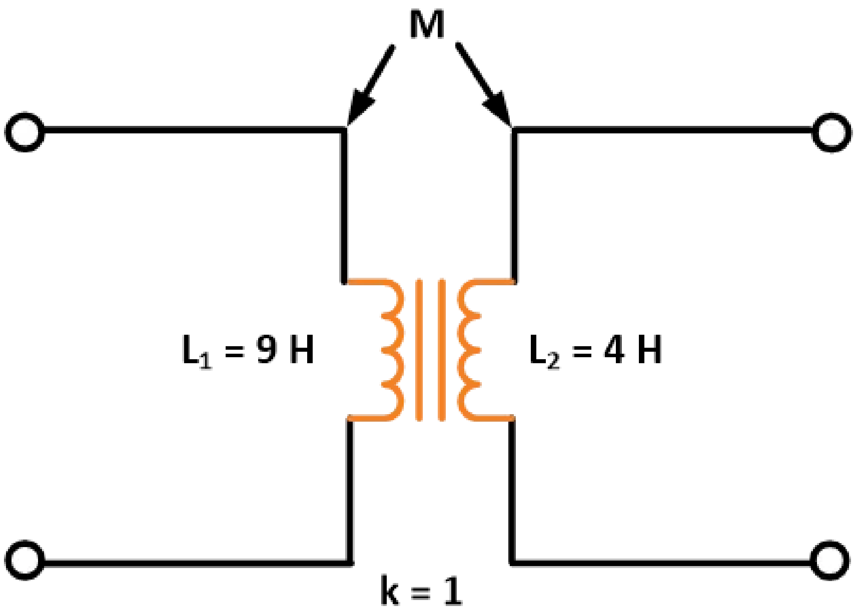
- When the entire flux of one coil links the other, the coefficient of coupling is 1 (i.e., 100%), then coils are said to be tightly coupled.
- If only half the flux set up in one coil links the other, the coefficient of coupling is 0.5 (i.e., 50%), then coils are said to be loosely coupled.
- If the flux of one coil does not at all link with the other coil, the coefficient of coupling is 0, the coils are said magnetically isolated from each other.

The coefficient of coupling will always be less than unity. It depends on the core materials used. For air core, the coupling coefficient can be 0.4 to 0.8 depending on the space between two coils and for iron or ferrite core it can be as high as 0.99.

Inductive Coupling Example Problem

Example 1

Calculate the mutual inductance between two unity-coupled coils of 9 H and 4 H.



Solution:

Given data: L_1 = 9 H, L_2 = 4 H and k = 1

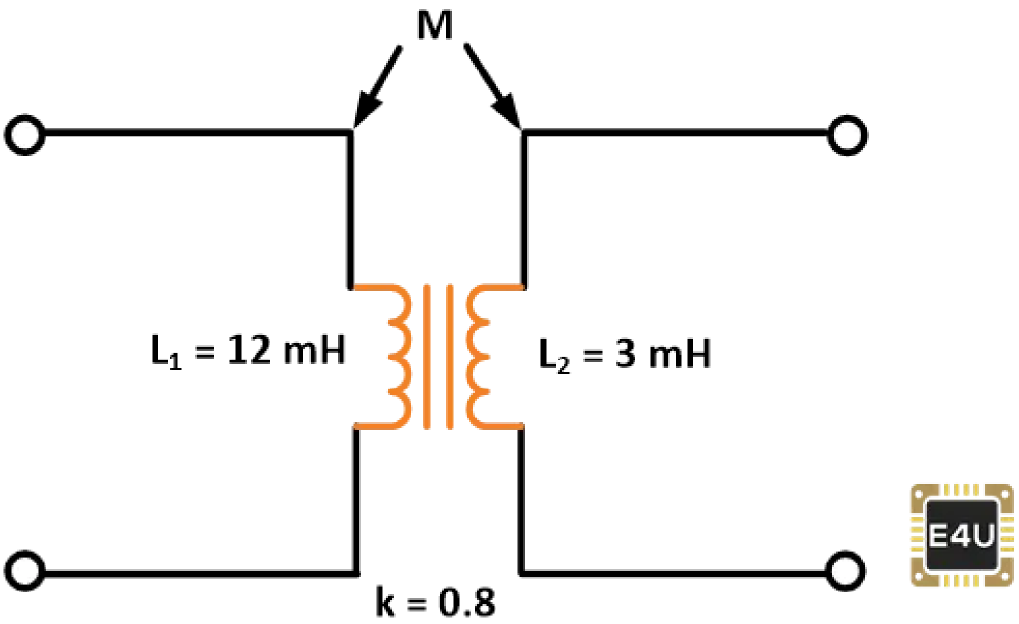
According's to formula,

$$M = k\sqrt{L_1 L_2}$$
$$= 1\sqrt{9 * 4}$$
$$= \sqrt{36}$$
$$M = 6\ mH$$

Thus, by using the formula, we get the mutual inductance between two unity-coupled coils is 6 H.

Example 2

Calculate the mutual inductance between two coils of the circuit shown below.



Solution:

Given data: L_1 = 12 mH, L_2 = 3 mH and k = 0.8

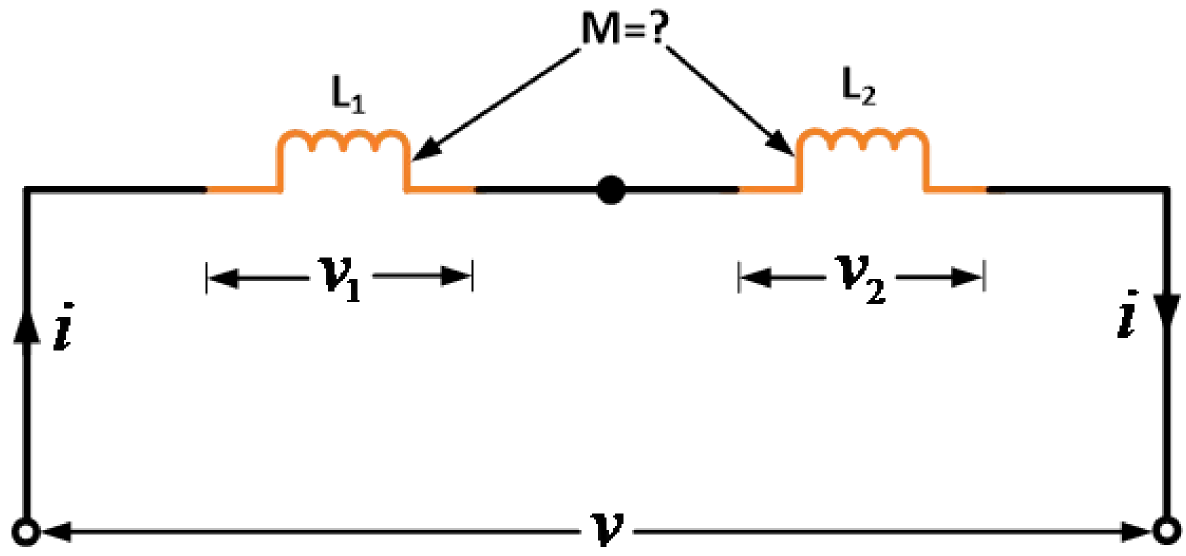
According's to formula,

$$
\begin{aligned}
M &= k\sqrt{L_1 L_2} \\
&= 0.8\sqrt{12 * 3} \\
&= 0.8\sqrt{36} \\
M &= 4.8\ mH
\end{aligned}
$$

Thus, by using the formula, we get the mutual inductance between two coupled coils is 4.8 mH.

Example 3

Two coupled coils connected in series have an equivalent inductance of 16 mH or 8 mH depending on their connection. Find the mutual inductance M between the coils.



Solution:

Given data: $L_{eq.}$ = 16 mH or 8 mH

Here, two coupled coils are connected in series, so according's to the series connection formula,

$$L_{eq.} = L_1 + L_2 \pm 2M$$

So,

$$16 = L_1 + L_2 + 2M \tag{17}$$

and

$$8 = L_1 + L_2 - 2M \tag{18}$$

Subtracting equation (18) from equation (17), we get

$$4M = 8$$

$$M = 2\ mH$$

Thus, by using the formula, we get the mutual inductance between two coupled coils is 2 mH.

What Are Inductors Used For

The applications of inductors include:

- Inductors are used for choking, filtering or smoothing, attenuating, and blocking high-frequency noise in an electrical circuit.

- Inductors are widely used in electronic equipment such as radio equipment in which they are used to allow DC to pass through it while AC to block. Inductors are designed for this purpose are known as chokes.
- Inductors are used in Hartley oscillators in which two inductive coils are connected in series with a parallel capacitor to form tuned oscillators or LC resonance tank circuits.
- Inductors are used in Colpitts oscillators in which two center-tapped capacitors are connected in series with a parallel inductor to form tuned oscillators or LC resonance tank circuits. (**Note that tuned oscillators circuits are used for transmitting or receiving microwave or radio-frequency signals**).
- Inductors are used to store and transfer electrical energy to output load or capacitor in power electronics converters (DC-DC or AC-DC) such as switch-mode power supplies.
- In power electronics, convertors inductors are used to filter the "ripple" current at the output. Higher values of inductance result in lower ripple current, which improves efficiency.
- Inductors are used for Impedance matching. Impedance matching involves matching the input or source impedance to the load impedance. Maximum power is transferred to the load from the source when the load impedance is matched to the source impedance, improving the efficiency of the circuit. Now if the load is capacitive compared to the source, then inductors can be used to counter the capacitance of the load and thus match the impedance.
- Inductors are used to limit switching currents and fault currents in the electrical transmission system.

Types of Inductors

Based on the core material used and mechanical construction, the inductors are classified into different types. Following are the main types.

- Air core inductor
- Variable core inductor
- Iron core inductor
- Powdered iron core inductor
- Ferrite core inductor
- Ferromagnetic core inductor
- Radio-frequency inductor
- Toroidal core inductor
- Multilayer ceramic inductor
- Film inductor
- Coupled inductor
- Molded inductor

Resistance of Inductors

Resistance of ideal inductor is zero, but practically inductors have some resistance because they are made out of wire and all wire has some resistance.

The Impedance of Inductors (Inductive Reactance of Inductors)

The impedance of an inductor (also called inductive reactance) is the measure of the opposition to the change of current flow. The formula of the impedance of the inductor is,

$$Z_L = j\omega L$$

Where,

Z_L is the impedance of the inductor,

ω is the angular frequency = $2\pi f$,

L is the inductance of the inductor

We can see that the impedance of the inductor is directly proportional to the frequency. That means if the frequency is zero the impedance is zero.

The impedance of the ideal inductor is positive for all frequencies as it is directly proportional to the frequency. The effective impedance of an inductor is depending on the frequency and it increases with frequency.

Inductors in DC Circuits vs AC Circuits

The behavior of inductors in DC and AC circuits is different. Let's discuss it in brief.

Inductors in DC Circuits

When DC (Direct Current) is applied across an inductor, the inductor behaves as a short circuit with zero resistance.

In DC, the rate of flow of current changes is zero, so there is no voltage is induced and hence the inductor does not oppose the flow of DC.

We know that the voltage across the inductor can be expressed as,

$$v_L = L\frac{di_L}{dt}$$

Where,

v_L = Instantaneous voltage across the inductor in Volts,

L = Inductance in Henry,

$\frac{di_L}{dt}$ = Rate of change of electric current in ampere per second

If d.c. current flows through the inductor, $\frac{di_L}{dt}$ becomes zero as d.c. current is constant with respect to time. Hence, the voltage across the inductor becomes zero.

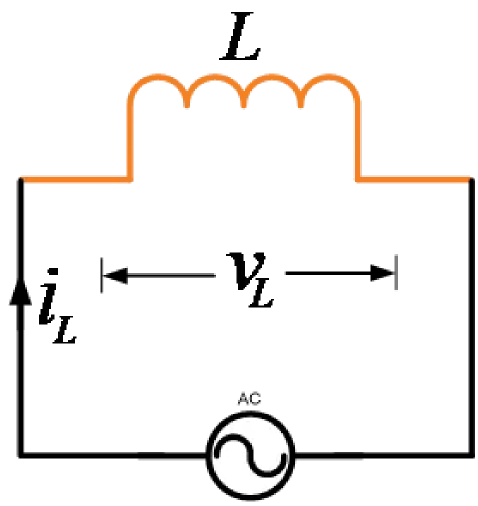
Thus, as far as d.c. quantities are considered, in steady-state, the inductor acts as a short circuit.

Inductors in AC Circuits

When AC (Alternating Current) is applied across the inductor, the AC changes the rate of current flow which is opposed by the inductor by increasing its reactance.

The higher the frequency of AC, the higher the rate of flow of current changes and hence the higher the blocking effect from the inductor.

As shown in the image below, the inductor is directly connected across the AC. The self-induced e.m.f. in the inductor increases and decreases with the increases and decreases in the supply voltage.



Inductor Connected Across an AC Supply

The self-induced e.m.f.s is directly proportional to the rate of change of the current through the inductor and it is greatest when the supply voltage crosses over from its positive half cycle to negative half cycle or vice versa along with the AC sinusoidal wave.

Do Inductors Have Polarity

The single inductor doesn't have polarity and it works equally in either direction. But if there is another inductor magnetically coupled to it, then the relative polarity of the inductors is important.

According's Lenz's law, the induced voltage across an inductor has a polarity (direction) that opposes the change in current that produced it. Hence, inductors oppose any change in current through it.

How Do Inductors Store Energy

Pure inductors do not dissipate or consume energy. The only resistance is capable of converting electrical energy to heat energy. Pure inductors only store energy when electric current flows through them. We can say that energy is stored in the magnetic field of the inductor.

When electrical energy is supplied to an inductor or coil, it is spent in two ways.

- Some part of it is spent to meet I^2R loss which is lost in the form of heat.
- The remaining part is used to create a magnetic field around the coil and is stored in the magnetic field.

Consider an inductor having inductance L and a small resistance R connected to a d.c. supply through a switch. When switch S is closed, the current increases in the inductor gradually and attains a steady value.

This increase in current is opposed by the self-induced e.m.f. produced in the inductor due to current change. To overcome opposition some energy is supplied by the source which is stored in the magnetic field of the inductor.

Now, when the switch is opened, the magnetic field collapses, and the stored energy is released and returned to the circuit and it is dissipated in the form of heat.

This is the similar potential energy of lifted weight. When a body of mass 'm' is lifted through a height of 'h' meter, then potential energy stored in it is "m*g*h". Work is done in lifting the body but once raised to a certain height, no further expenditure of energy is required to maintain it in that position.

This mechanical energy can be recovered by allowing the body to fall, similarly, the electrical energy stored in the magnetic field can be recovered by collapsing the magnetic field.

The Magnitude of Energy Stored in the Magnetic Field

The magnitude of energy stored in the magnetic field can be found in the following way.

Let at any instant, the current flowing through the inductor is I and is containing at the rate of $\frac{di}{dt}$.

The self-induced e.m.f. in the inductor, $e = L\frac{di}{dt}$

The instantaneous power,

$$p = e * i$$
$$= L\frac{di}{dt} * i$$
$$= Li\frac{di}{dt}$$

Now, energy is stored in the magnetic field of energy supplied to the inductor during a short interval of time dt.

$$dW = p * dt$$
$$= Li\frac{di}{dt} * dt$$
$$= Lidi$$

Now, the total energy stored in the magnetic field when the current rises from 0 to I (final value) is given by,

$$\int dW = \int_0^I L\ i\ di$$
$$W = L[\frac{i^2}{2}]_0^I$$
$$W = \frac{1}{2}LI^2\ joule$$

Where L is the inductance in henry,

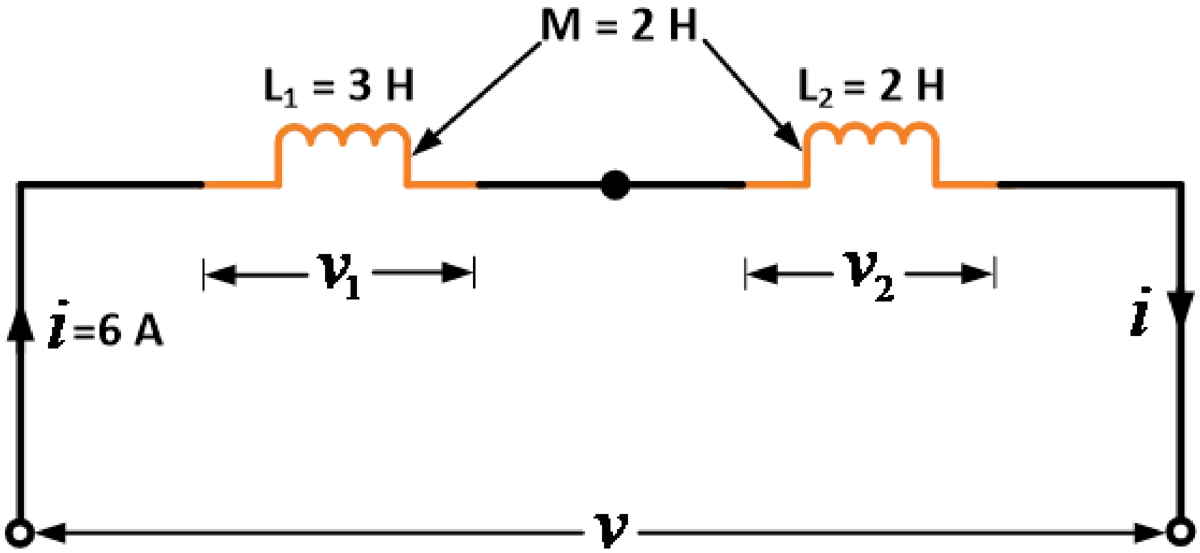
I is the current in ampere

The above equation represents the energy stored in the inductor.

Note that the inductor stores energy only during the time the current is increasing. When the current in the inductor decays to zero, the stored energy is returned to the source or dissipated in the resistance in the circuit.

Example

Two coils have self-inductance of 3 H and 2 H respectively and the mutual inductance is 2 H. They are connected in series and a current of 5 A is flowing through them. Calculate the energy stored in the magnetic field when coils are connected (i) cumulatively and (ii) differentially also find the coefficient of coupling.



Solution:

Given data: $L_1 = 3$ H, $L_2 = 2$ H, M = 2 H, I = 6 A

(i) For cumulative connection:

$$L_{eq.} = L_1 + L_2 + 2M$$
$$= 3 + 2 + 2(2)$$
$$= 5 + 4$$
$$L_{eq.} = 9\ H$$

$$Energy\ Stored\ W = \frac{1}{2} L_{eq.} I^2\ joule$$
$$= \frac{1}{2} * 9 * (6)^2$$
$$W = 162\ joule$$

(ii) For differential connection:

$$L_{eq.} = L_1 + L_2 - 2M$$
$$= 3 + 2 - 2(2)$$
$$= 5 - 4$$
$$L_{eq.} = 1\ H$$

$$Energy\ Stored\ W = \frac{1}{2} L_{eq.} I^2\ joule$$
$$= \frac{1}{2} * 1 * (6)^2$$
$$W = 18\ joule$$

(iii) Coefficient of Coupling: