

## Voltage Dividers

Voltage Divider circuits are used to produce different voltage levels from a common voltage source but the current is the same for all components in a series circuit

**Voltage Divider Circuits** are useful in providing different voltage levels from a common supply voltage. This common supply can be a single supply either positive or negative, for example, +5V, +12V, -5V or -12V, etc. with respect to a common point or ground, usually 0V, or it could be across a dual supply, for example  $\pm 5V$ , or  $\pm 12V$ , etc.

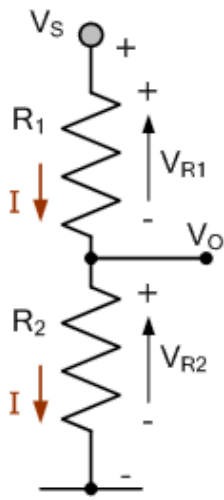
Voltage dividers are also known as potential dividers, because the unit of voltage, the “Volt” represents the amount of *potential difference* between two points. A voltage or potential divider is a simple passive circuit that takes advantage of the effect of voltages being dropped across components which are connected in series.

The potentiometer, which is a variable resistor with a sliding contact, is the most basic example of a voltage divider as we can apply a voltage across its terminals and produce an output voltage in proportion to the mechanical position of its sliding contact. But we can also make voltage dividers using individual resistors, capacitors and inductors as they are two-terminal components which can be connected together in series.

### Resistive Voltage Divider

The simplest, easiest to understand, and most basic form of a passive voltage divider network is that of two resistors connected together in series. This basic combination allows us to use the *Voltage Divider Rule* to calculate the voltage drops across each series resistor.

### Resistive Voltage Divider Circuit



Here the circuit consists of two resistors connected together in series:  $R_1$ , and  $R_2$ . Since the two resistors are connected in series, it must therefore follow that the same value of electric current must flow through each resistive element of the circuit as it has nowhere else to go. Thus providing an  $I \times R$  voltage drop across each resistive element.

With a supply or source voltage,  $V_S$  applied across this series combination, we can apply Kirchhoff's Voltage Law, (KVL) and also using Ohm's Law to find the voltage dropped across each resistor derived in terms of the common current,  $I$  flowing through them. So solving for the current ( $I$ ) flowing through the series network gives us:

$$V_S = V_{R1} + V_{R2} \quad (\text{KVL})$$

$$V_{R1} = I \times R_1 \quad \text{and} \quad V_{R2} = I \times R_2$$

$$\text{Then : } V_S = I \times R_1 + I \times R_2$$

$$\therefore V_S = I(R_1 + R_2)$$

$$\text{So : } I = \frac{V_S}{(R_1 + R_2)}$$

The current flowing through the series network is simply  $I = V/R$  following Ohm's Law. Since the current is common to both resistors, ( $I_{R1} = I_{R2}$ ) we can calculate the voltage dropped across resistor,  $R_2$  in the above series circuit as being:

$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{V_S}{(R_1 + R_2)}$$

$$\therefore V_{R2} = V_S \left( \frac{R_2}{R_1 + R_2} \right)$$

Likewise for resistor  $R_1$  as being:

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{V_S}{(R_1 + R_2)}$$

$$\therefore V_{R1} = V_S \left( \frac{R_1}{R_1 + R_2} \right)$$

## Voltage Divider Example No1

How much current will flow through a  $20\Omega$  resistor connected in series with a  $40\Omega$  resistor when the supply voltage across the series combination is 12 volts dc. Also calculate the voltage drop produced across each resistor.

$$R_T = R_1 + R_2 = 20 + 40 = 60\Omega$$

$$I = \frac{V_S}{R_T} = \frac{12}{60} = 0.2 \text{ [Amps, A] or } 200\text{mA}$$

$$V_{R1} = I \times R_1 = V_S \left( \frac{R_1}{R_1 + R_2} \right) = 12 \left( \frac{20}{20 + 40} \right) = 4\text{volts}$$

$$V_{R2} = I \times R_2 = V_S \left( \frac{R_2}{R_1 + R_2} \right) = 12 \left( \frac{40}{20 + 40} \right) = 8\text{volts}$$

Each resistance provides an  $I \times R$  voltage drop which is proportionally equal to its resistive value across the supply voltage. Using the voltage divider ratio rule, we can see that the largest resistor produces the largest  $I \times R$  voltage drop. Thus,  $R_1 = 4V$  and  $R_2 = 8V$ . Applying Kirchhoff's Voltage Law shows that the sum of the voltage drops around the resistive circuit is exactly equal to the supply voltage, as  $4V + 8V = 12V$ .

Note that if we use two resistors of equal value, that is  $R_1 = R_2$ , then the voltage dropped across each resistor would be exactly half the supply voltage for two resistances in series as the voltage divider ratio would equal 50%.

Another use of a voltage divider network is that to produce a variable voltage output. If we replace resistor  $R_2$  with a variable resistor (potentiometer), then the voltage dropped across  $R_2$  and therefore  $V_{OUT}$  can be controlled by an amount dependant on the position of the potentiometers wiper and therefore the ratio of the two resistive values as we have one fixed and one variable resistor. Potentiometers, trimmers, rheostats and variacs are all examples of variable voltage division devices.

We could also take this idea of variable voltage division one step further by replacing the fixed resistor  $R_2$  with a sensor such as a *light dependent resistor*, or LDR. Thus as the resistive value of the sensor changes with changes in light levels, the output voltage  $V_{OUT}$  also changes by a proportional amount. Thermistors and strain gauges are other examples of resistive sensors.

Since the two voltage division expressions above relate to the same common current, mathematically they must therefore be related to each other. So for any number of individual resistors forming a series network, the voltage dropped across any given resistor is given as:

## Voltage Divider Equation

$$V_{R(x)} = V_S \left( \frac{R_x}{R_T} \right)$$

Where:  $V_{R(x)}$  is the voltage drop across the resistor,  $R_X$  is the value of the resistor, and  $R_T$  is the total resistance of the series network. This voltage divider equation can be used for any number of series resistances connected together because of the proportional relationship between each resistance,  $R$  and its corresponding voltage drop,  $V$ . Note however, that this equation is given for an unloaded *voltage divider network* without any additional resistive load connected or parallel branch currents.

## Voltage Divider Example No2

Three resistive elements of  $6k\Omega$ ,  $12k\Omega$  and  $18k\Omega$  are connected together in series across a 36 volt supply. Calculate, the total resistance, the value of the current flowing around the circuit, and the voltage drops across each resistor.

Data given:  $V_S = 36$  volts,  $R_1 = 6k\Omega$ ,  $R_2 = 12k\Omega$  and  $R_3 = 18k\Omega$

$$R_T = R_1 + R_2 + R_3 = 6k\Omega + 12k\Omega + 18k\Omega = 36k\Omega$$

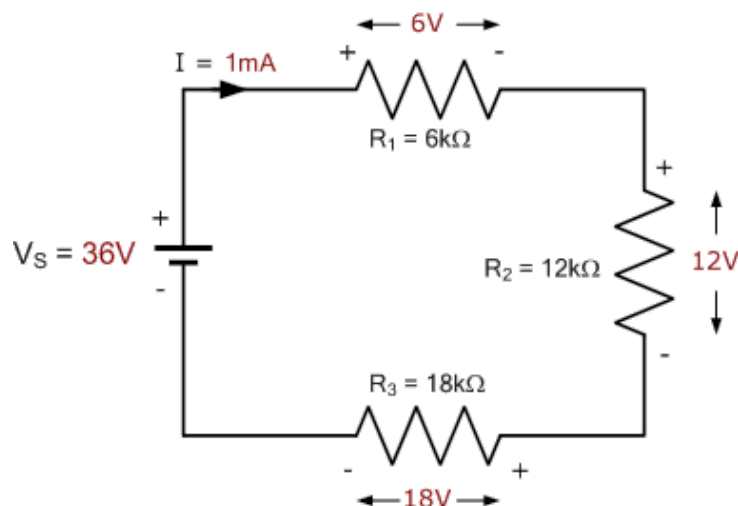
$$I = \frac{V_S}{R_T} = \frac{36}{36000} = 1mA$$

$$V_{R1} = V_S \left( \frac{R_1}{R_T} \right) = 36 \left( \frac{6000}{36000} \right) = 6 \text{volts}$$

$$V_{R2} = V_S \left( \frac{R_2}{R_T} \right) = 36 \left( \frac{12000}{36000} \right) = 12 \text{volts}$$

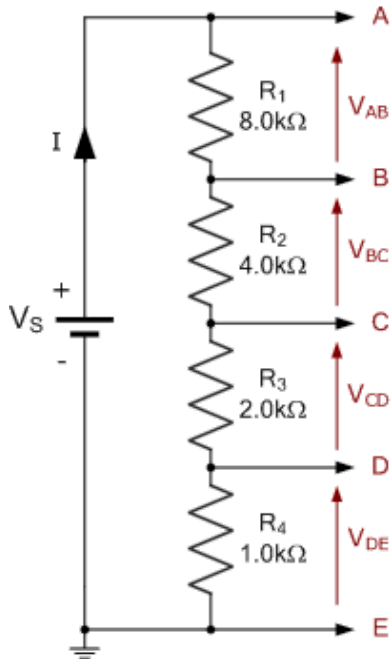
$$V_{R3} = V_S \left( \frac{R_3}{R_T} \right) = 36 \left( \frac{18000}{36000} \right) = 18 \text{volts}$$

## Voltage Divider Circuit



The voltage drops across all three resistors should add up to the supply voltage as defined by Kirchhoff's Voltage Law (KVL). So the sum of the voltage drops is:  $V_T = 6\text{ V} + 12\text{ V} + 18\text{ V} = 36.0\text{ V}$  the same value of the supply voltage,  $V_S$  and so is correct. Again notice that the largest resistor produces the largest voltage drop.

## Voltage Tapping Points in a Divider Network



Consider a long series of resistors connected to a voltage source,  $V_S$ . Along the series network there are different voltage tapping points, **A**, **B**, **C**, **D**, and **E**.

The total series resistance can be found by simply adding together the individual series resistance values giving a total resistance,  $R_T$  value of  $15\text{k}\Omega$ . This resistive value will limit the flow of current through the circuit produced by the supply voltage,  $V_S$ .

The individual voltage drops across the resistors are found using the equations above, so  $V_{R1} = V_{AB}$ ,  $V_{R2} = V_{BC}$ ,  $V_{R3} = V_{CD}$ , and  $V_{R4} = V_{DE}$ .

The voltage levels at each tapping point is measured with respect to ground (0V). Thus the voltage level at point **D** will be equal to  $V_{DE}$ , and the voltage level at point **C** will be equal to  $V_{CD} + V_{DE}$ . In other words, the voltage at point **C** is the sum of the two voltage drops across  $R_3$  and  $R_4$ .

So hopefully we can see that by choosing a suitable set of resistive values, we can produce a sequence of voltage drops which will have a proportional voltage value obtained from a single supply voltage. Note also that in this example each output voltage point will be positive in value because the negative terminal of the voltage supply,  $V_S$  is grounded.

## Voltage Divider Example No3

1. Calculate the no-load voltage output for each tapping point of the voltage divider circuit above if the series-connected resistive network is connected to a 15 volt DC supply.

$$R_T = R_1 + R_2 + R_3 + R_4 = 8k\Omega + 4k\Omega + 2k\Omega + 1k\Omega = 15k\Omega$$

$$V_{R1} = V_{AB} = V_S \left( \frac{R_1}{R_T} \right) = 15 \left( \frac{8000}{15000} \right) = 8 \text{volts}$$

$$V_{R2} = V_{BC} = V_S \left( \frac{R_2}{R_T} \right) = 15 \left( \frac{4000}{15000} \right) = 4 \text{volts}$$

$$V_{R3} = V_{CD} = V_S \left( \frac{R_3}{R_T} \right) = 15 \left( \frac{2000}{15000} \right) = 2 \text{volts}$$

$$V_{R4} = V_{DE} = V_S \left( \frac{R_4}{R_T} \right) = 15 \left( \frac{1000}{15000} \right) = 1 \text{volts}$$

2. Calculate the no-load voltage output from between points **B** and **E**.

$$R_T = R_1 + R_2 + R_3 + R_4 = 8k\Omega + 4k\Omega + 2k\Omega + 1k\Omega = 15k\Omega$$

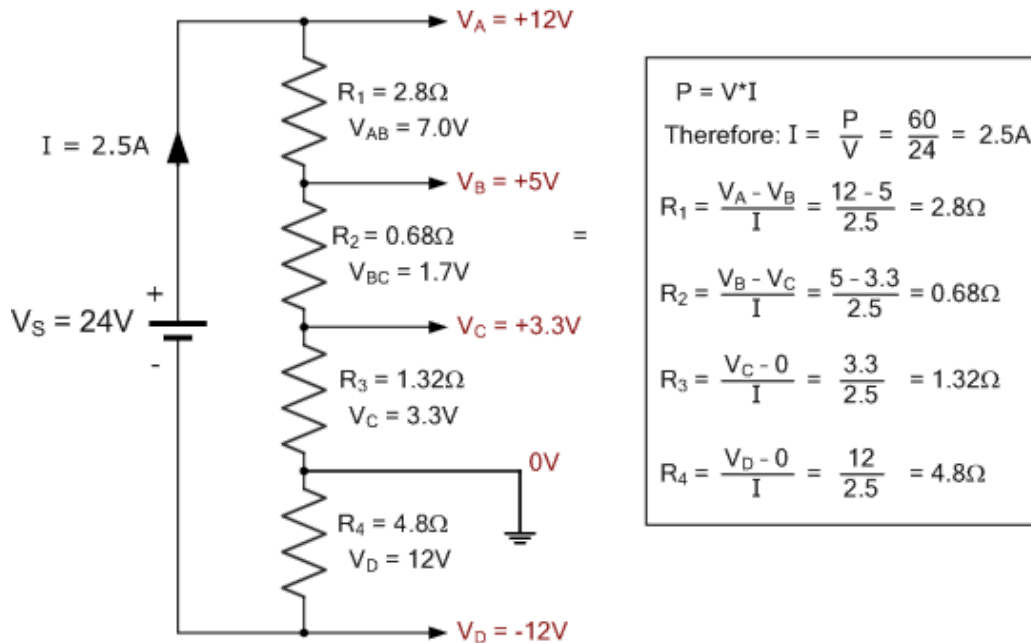
$$V_{BE} = V_S \left( \frac{R_2 + R_3 + R_4}{R_T} \right) = 15 \left( \frac{4k\Omega + 2k\Omega + 1k\Omega}{15k\Omega} \right) = 7 \text{volts}$$

## A Negative and Positive Voltage Divider

In the simple voltage divider circuit above all the output voltages are referenced from a common zero-voltage ground point, but sometimes it is necessary to produce both positive and negative voltages from a single source voltage supply. For example the different voltage levels from a computer PSU, -12V, +3.3V, +5V and +12V, with respect to a common reference ground terminal.

## Voltage Divider Example No4

Using Ohm's Law, find the values of resistors  $R_1$ ,  $R_2$ ,  $R_3$  and  $R_4$  required to produce the voltage levels of -12V, +3.3V, +5V and +12V if the total power supplied to the unloaded voltage divider circuit is 24 volts DC, 60 watts.



In this example, the zero-voltage ground reference point has been moved to produce the required positive and negative voltages, while maintaining the voltage divider network across the supply. Thus the four voltages are all measured with respect to this common reference point resulting in point D being at the required negative potential of -12V with respect to ground.

We have seen so far that series resistive circuits can be used to create a voltage divider, or potential divider network which can be widely used in electronic circuits. By selecting appropriate values for the series resistances, any value of output voltage can be obtained which is lower than the input or supply voltage. But as well as using resistances and a DC supply voltage to create a *resistive voltage divider network*, we can also use capacitors (C) and inductors (L), but with a sinusoidal AC supply as capacitors and inductors are reactive components, meaning that their resistance “reacts” against the flow of electric current.

## Capacitive Voltage Dividers

As the name suggests, **Capacitive Voltage Divider** circuits produce voltage drops across capacitors connected in series to a common AC supply. Generally capacitive voltage dividers are used to “step-down” very high voltages to provide a low voltage output signal which can then be used for protection or metering. Nowadays, high frequency capacitive voltage dividers are used more in display devices and touch screen technologies found in mobile phones and tablets.

Unlike resistive voltage divider circuits which operate on both AC and DC supplies, voltage division using capacitors is only possible with a sinusoidal AC supply. This is because the voltage division between series connected capacitors is calculated using the reactance of the capacitors,  $X_C$  which is dependent on the frequency of the AC supply.

We remember from our tutorials about **capacitors in AC circuits**, that capacitive reactance,  $X_C$  (measured in Ohms) is inversely proportional to both frequency and capacitance, and is therefore given by the following equation of:

## Capacitive Reactance Formula

$$X_C = \frac{1}{2\pi f C}$$

Where:

$X_C$  = Capacitive Reactance in Ohms, ( $\Omega$ )

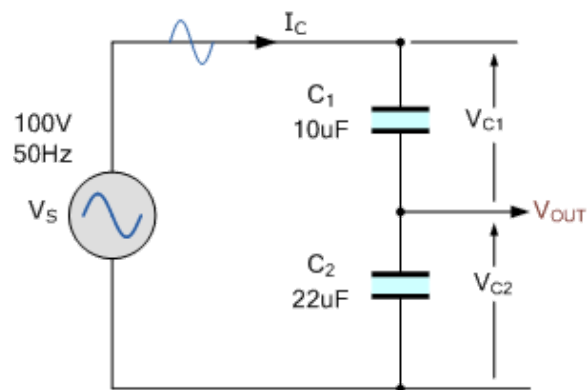
$\pi$  (pi) = a numeric constant of 3.142

$f$  = Frequency in Hertz, (Hz)

$C$  = Capacitance in Farads, (F)

Therefore by knowing the voltage and frequency of the AC supply, we can calculate the reactances of the individual capacitors, substitute them in the above equation for the resistive voltage divider rule, and obtain the corresponding voltage drops across each capacitor as shown.

### Capacitive Voltage Divider



Using the two capacitors of 10uF and 22uF in the series circuit above, we can calculate the rms voltage drops across each capacitor in terms of their reactance when connected to a 100 volts, 50Hz rms supply.



$$X_{C1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = 318.3\Omega$$

$$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi \times 50 \times 22 \times 10^{-6}} = 144.7\Omega$$

$$X_{CT} = X_{C1} + X_{C2} = 318.3\Omega + 144.7\Omega = 463\Omega$$

$$V_{C1} = V_S \left( \frac{X_{C1}}{X_{CT}} \right) = 100 \left( \frac{318.3}{463} \right) = 69 \text{ volts}$$

$$V_{C2} = V_S \left( \frac{X_{C2}}{X_{CT}} \right) = 100 \left( \frac{144.7}{463} \right) = 31 \text{ volts}$$

When using pure capacitors the sum of all the series voltage drops equals the source voltage, the same as for series resistances. While the amount of voltage drop across each capacitors is proportional to its reactance, it is inversely proportional to its capacitance.

As a result, the smaller 10uF capacitor has more reactance (318.3Ω) so therefore a greater voltage drop of 69 volts compared to the larger 22uF capacitor which has a reactance of 144.7Ω and a voltage drop of 31 volts respectively. The current in the series circuit,  $I_C$  will be 216mA, and is the same value for  $C_1$  and  $C_2$  as they are in series.

One final point about **capacitive voltage divider** circuits is that as long as there is no series resistance, purely capacitive, the two capacitor voltage drops of 69 and 32 volts will arithmetically be equal to the supply voltage of 100 volts as the two voltages produced by the capacitors are in-phase with each other. If for whatever reason the two voltages are out-of-phase with each other then we can not just simple add them together as we would using Kirchhoffs voltage law, but instead phasor addition of the two waveforms is required.

## Inductive Voltage Dividers

As its name suggests, **Inductive Voltage Dividers** create voltage drops across inductors or coils connected together in series to a common AC supply. An *inductive voltage divider* can consist of a single winding or coil which is divided into two sections where the output voltage is taken from across one of the section, or from two individual coils connected together. The most common example of an inductive voltage divider is the *auto-transformer* with multiple tapping points along its secondary winding.

When used with steady state DC supplies or with sinusoids having a very low frequency, approaching 0 Hz, inductors act as a short circuit. This is because their reactance is almost zero allowing any DC current to easily pass through them, so like the previous capacitive voltage divider network, we must perform any inductive voltage division using a sinusoidal AC supply. Inductive voltage division between series connected inductors can be calculated using the reactance of the inductors,  $X_L$  which like *capacitive inductance*, is dependent on the frequency of the AC supply.

In the tutorials about **inductors in AC circuits**, we saw that inductive reactance,  $X_L$  (also measured in Ohms) is proportional to both frequency and inductance so any increases in the supply frequency increases an inductors reactance. Thus *inductive reactance* is defined as:

## Inductive Reactance Formula

$$X_L = 2\pi fL$$

Where:

$X_L$  = Inductive Reactance in Ohms, ( $\Omega$ )

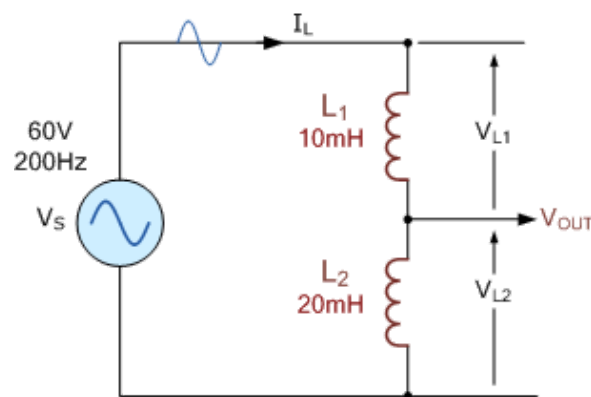
$\pi$  (pi) = a numeric constant of 3.142

$f$  = Frequency in Hertz, (Hz)

$L$  = Inductance in Henries, (H)

If we know the voltage and frequency of the AC supply, we can calculate the reactances of the two inductors and use them along with the voltage divider rule to obtain the voltage drops across each inductor as shown.

## Inductive Voltage Divider



Using the two inductors of 10mH and 20mH in the series circuit above, we can calculate the rms voltage drops across each capacitor in terms of their reactance when connected to a 60 volts, 200Hz rms supply.

$$X_{L1} = 2\pi fL_1 = 2\pi \times 200 \times 10 \times 10^{-3} = 12.56\Omega$$

$$X_{L2} = 2\pi fL_2 = 2\pi \times 200 \times 20 \times 10^{-3} = 25.14\Omega$$

$$X_{LT} = X_{L1} + X_{L2} = 12.56\Omega + 25.14\Omega = 37.7\Omega$$

$$V_{L1} = V_S \left( \frac{X_{L1}}{X_{LT}} \right) = 60 \left( \frac{12.56}{37.7} \right) = 20 \text{ volts}$$

$$V_{L2} = V_S \left( \frac{X_{L2}}{X_{LT}} \right) = 60 \left( \frac{25.14}{37.7} \right) = 40 \text{ volts}$$

Like the previous resistive and capacitive voltage division circuits, the sum of all the series voltage drops across the inductors will equal the source voltage, as long as there are no series resistances. Meaning a pure inductor. The amount of voltage drop across each inductor is proportional to its reactance.

The result is that the smaller 10mH inductor has less reactance ( $12.56\Omega$ ), so therefore less of a voltage drop at 30 volts compared to the larger 20mH inductor which has a reactance of  $25.14\Omega$  and a voltage drop of 40 volts respectively. The current,  $I_L$  in the series circuit is 1.6mA, and will be the same value for  $L_1$  and  $L_2$  as these two inductors are connected in series.

## Voltage Divider Summary

We have seen here that the voltage divider, or network is a very common and useful circuit configuration allowing us to produce different voltage levels from a single voltage supply, thus eliminating the need to have separate power supplies for different parts of a circuit operating at different voltage levels.

As its name suggests, a voltage or potential divider, “divides” a fixed voltage into precise proportions using resistors, capacitors or inductors. The most basic and commonly used voltage divider circuit is that of two fixed-value series resistors, but a potentiometer or rheostat can also be used for voltage division by simply adjusting its wiper position.

A very common application of a voltage divider circuit is to replace one of the fixed-value resistors with a sensor. Resistive sensores such as light sensores, temperature sensores, pressure sensores and strain guages, which change their resistive value as they respond to environmental changes can all be used in a voltage divider network to provide an analogue voltage output. The biasing of bipolar transistors and MOSFETs is also another common application of a **Voltage Divider**.