## Regularized Linear Regression

**Note:** [8:43 - It is said that X is non-invertible if  $m \le n$ . The correct statement should be that X is non-invertible if  $m \le n$ , and may be non-invertible if m = n.

We can apply regularization to both linear regression and logistic regression. We will approach linear regression first.

## **Gradient Descent**

We will modify our gradient descent function to separate out  $\theta_0$  from the rest of the parameters because we do not want to penalize  $\theta_0$ .

The term  $\frac{\lambda}{m}\theta_j$  performs our regularization. With some manipulation our update rule can also be represented as:

$$\theta_j := \theta_j (1 - \alpha \frac{\lambda}{m}) - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

The first term in the above equation,  $1-\alpha\frac{\lambda}{m}$  will always be less than 1. Intuitively you can see it as reducing the value of  $\theta_j$  by some amount on every update. Notice that the second term is now exactly the same as it was before.

## **Normal Equation**

Now let's approach regularization using the alternate method of the non-iterative normal equation.

To add in regularization, the equation is the same as our original, except that we add another term inside the parentheses:

L is a matrix with 0 at the top left and 1's down the diagonal, with 0's everywhere else. It should have dimension  $(n+1)\times(n+1)$ . Intuitively, this is the identity matrix (though we are not including  $x_0$ ), multiplied with a single real number  $\lambda$ .

Recall that if m < n, then  $X^TX$  is non-invertible. However, when we add the term  $\lambda \cdot L$ , then  $X^TX + \lambda \cdot L$  becomes invertible.