

Examples and Intuitions II

The $\Theta^{\{1\}}$ matrices for AND, NOR, and OR are:

```
\begin{align*} \text{AND:} \\\Theta^{\{1\}} \\&= \begin{bmatrix} -30 & 20 & 20 \end{bmatrix} \\ \text{NOR:} \\\Theta^{\{1\}} &= \\ \begin{bmatrix} 10 & -20 & -20 \end{bmatrix} \\ \text{OR:} \\\Theta^{\{1\}} &= \begin{bmatrix} -10 & 20 \\ & 20 \end{bmatrix} \end{align*}
```

We can combine these to get the XNOR logical operator (which gives 1 if x_1 and x_2 are both 0 or both 1).

```
\begin{align*} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \\ \rightarrow \begin{bmatrix} a_1^{\{2\}} \\ a_2^{\{2\}} \end{bmatrix} \\ \rightarrow \begin{bmatrix} a^{\{3\}} \end{bmatrix} \\ \rightarrow h_{\Theta(x)} \end{align*}
```

For the transition between the first and second layer, we'll use a $\Theta^{\{1\}}$ matrix that combines the values for AND and NOR:

```
\Theta^{\{1\}} = \begin{bmatrix} -30 & 20 & 20 \\ 10 & -20 & -20 \end{bmatrix}
```

For the transition between the second and third layer, we'll use a $\Theta^{\{2\}}$ matrix that uses the value for OR:

```
\Theta^{\{2\}} = \begin{bmatrix} -10 & 20 & 20 \end{bmatrix}
```

Let's write out the values for all our nodes:

```
\begin{align*} &a^{\{2\}} = g(\Theta^{\{1\}} \cdot x) \\ &a^{\{3\}} = g(\Theta^{\{2\}} \cdot a^{\{2\}}) \\ &h_{\Theta(x)} = a^{\{3\}} \end{align*}
```

And there we have the XNOR operator using a hidden layer with two nodes! The following summarizes the above algorithm:

