

Decision Boundary

In order to get our discrete 0 or 1 classification, we can translate the output of the hypothesis function as follows:

$$\begin{aligned} &h_{\theta}(x) \geq 0.5 \rightarrow y = 1 \\ &h_{\theta}(x) < 0.5 \rightarrow y = 0 \end{aligned}$$

The way our logistic function g behaves is that when its input is greater than or equal to zero, its output is greater than or equal to 0.5:

$$g(z) \geq 0.5 \text{ when } z \geq 0$$

Remember.

$$\begin{aligned} &z=0, e^0=1 \rightarrow g(z)=1/2 \\ &z \rightarrow -\infty, e^{-\infty} \rightarrow 0 \rightarrow g(z)=0 \\ &z \rightarrow \infty, e^{\infty} \rightarrow 1 \rightarrow g(z)=1 \end{aligned}$$

So if our input to g is $\theta^T X$, then that means:

$$h_{\theta}(x) = g(\theta^T x) \geq 0.5 \text{ when } \theta^T x \geq 0$$

From these statements we can now say:

$$\begin{aligned} &\theta^T x \geq 0 \rightarrow y = 1 \\ &\theta^T x < 0 \rightarrow y = 0 \end{aligned}$$

The **decision boundary** is the line that separates the area where $y = 0$ and where $y = 1$. It is created by our hypothesis function.

Example:

$$\begin{aligned} &\theta = \begin{bmatrix} 5 \\ -1 \end{bmatrix} \\ &y = 1 \text{ if } 5 + (-1)x_1 + 0x_2 \geq 0 \text{ and } 5 - x_1 \geq 0 \\ &\text{and } -x_1 \geq -5 \text{ and } x_1 \leq 5 \end{aligned}$$

In this case, our decision boundary is a straight vertical line placed on the graph where $x_1 = 5$, and everything to the left of that denotes $y = 1$, while everything to the right denotes $y = 0$.

Again, the input to the sigmoid function $g(z)$ (e.g. $\theta^T X$) doesn't need to be linear, and could be a function that describes a circle (e.g. $z = \theta_0 + \theta_1 x_1^2 + \theta_2 x_2^2$) or any shape to fit our data.