

Data X

Data as a Signal and Correlation Data, Signals, and Systems

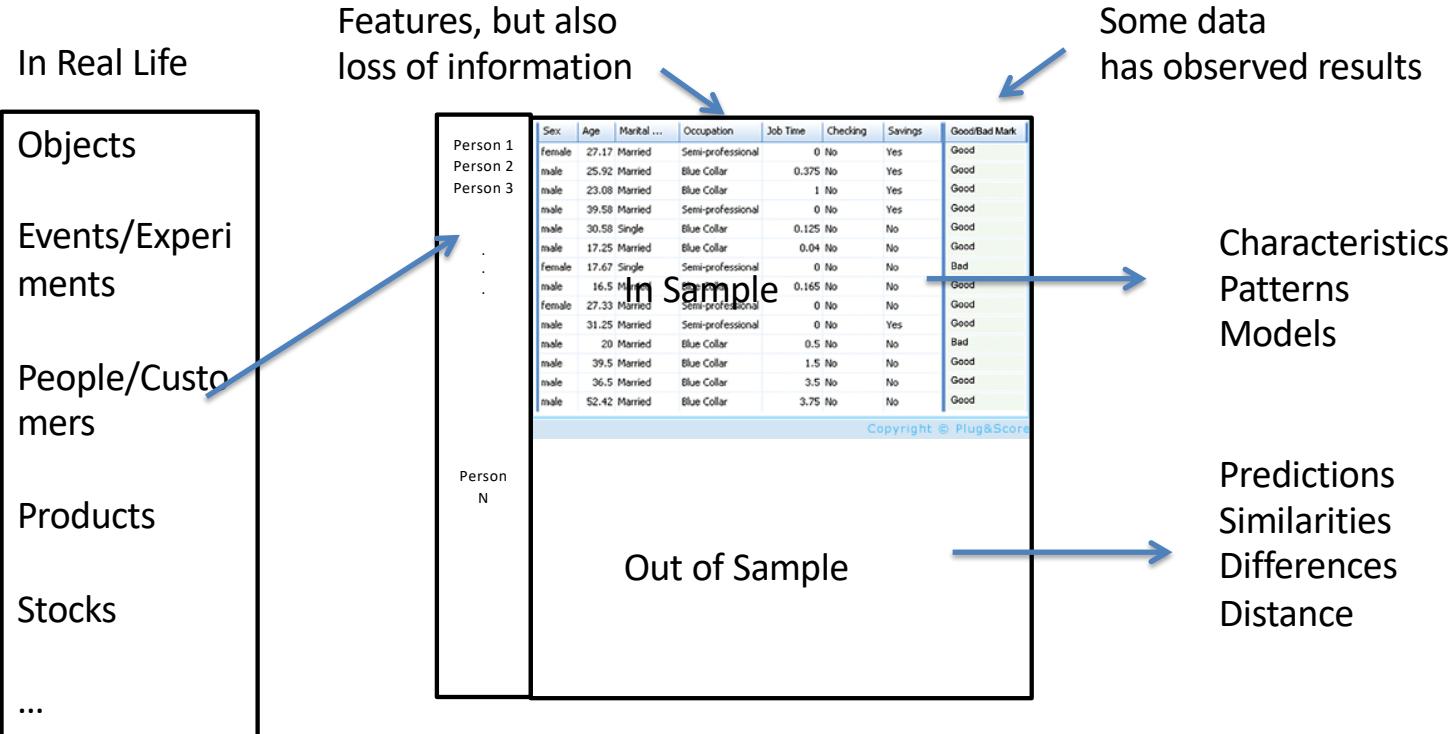
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Chief Scientist & Founding Director,
Sutardja Center for Entrepreneurship & Technology
IEOR Emerging Area Professor Award, UC Berkeley

Converting From Time Sequence Data to Features

Of course, not all data has a time property, but lets start with this type.
For example(key1, value 1),(key 2, value 2)... in this case, the keys are indexed by time.



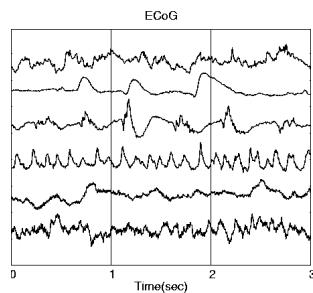
A High Level Framework



Converting From Time Sequence Data to Features

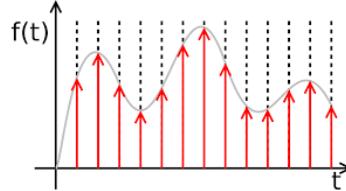
Many Types of data
are signals in time

- Stock market
- Temperature
- Instrument readings



Continuous signals
 $x(t)$

Sometimes we
sample them,
record at intervals
of T



Sampled signals (data)
 $x(nT)$

We get a
list in a table,
array, or vector

Rec	Observed
1	60.323
2	61.122
3	60.171
4	61.187
5	63.221
6	63.639
7	64.989
8	63.761
9	66.019
10	67.857
11	68.169
12	66.513
13	68.655
14	69.564
15	69.331
16	70.551



What we want
(for now):
features and
characteristics

For example:

- Means
- Variances
- Pattern matches
- Changes
- accumulation
- Frequency

Discrete data
 $x_n = x_1, x_2, x_3, \dots$

(might lose time reference)



This leads to tables where the time is an index and each column is a different time sequence (eg stock price, sensor readings, etc)

	A	B	C	D
Date	Ozone ($\mu\text{g}/\text{m}^3$)	Temperature ($^\circ\text{C}$)	Relative humidity (%)	n deaths
1 Jan 2002	4.59	-0.2	75.7	199
2 Jan 2002	4.88	0.1	77.5	231
3 Jan 2002	4.71	0.9	81.3	210
4 Jan 2002	4.14	0.5	85.4	203
5 Jan 2002	2.01	4.3	93.5	224
6 Jan 2002	2.4	7.1	96.4	198
7 Jan 2002	4.08	5.2	93.5	180
8 Jan 2002	3.13	3.5	81.5	188
9 Jan 2002	2.05	3.2	88.3	168
10 Jan 2002	5.19	5.3	85.4	194
11 Jan 2002	3.59	3.0	92.6	223
12 Jan 2002	12.87	4.8	94.2	201

$\text{Cor}(A, B)$ is _____

Example rows of time series data from the London dataset showing daily levels of environmental variables

https://www.researchgate.net/figure/Example-rows-of-time-series-data-from-the-London-dataset-showing-daily-levels-of_tbl1_237840362



What is the Correlation of the table?

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Data X

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Example rows of time series data from the London dataset showing daily levels of environmental variables

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Leads to question:

What does it mean for one row to be similar to another?

Is what is the Correlation (A, B)



Correlation and Correlation Matrices

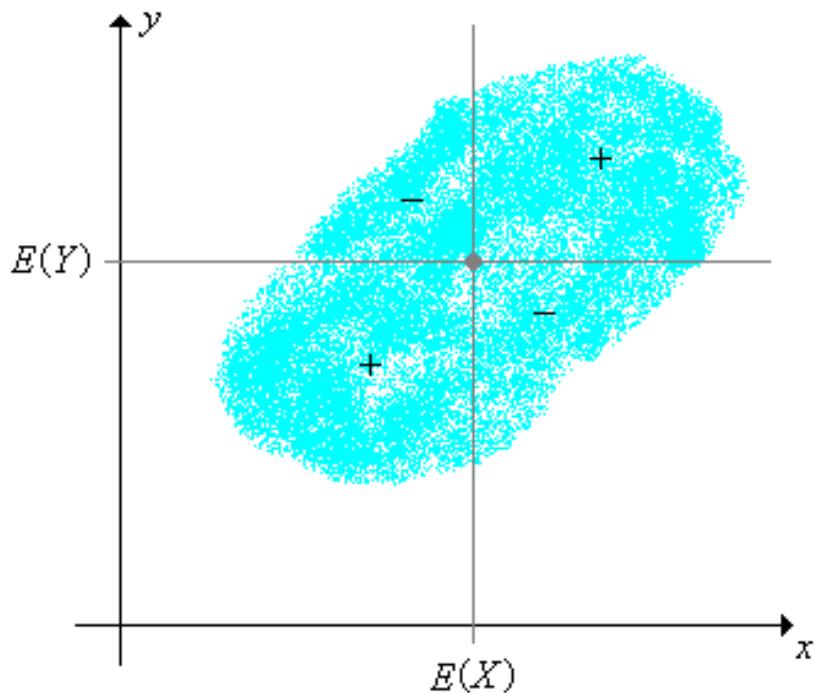
Data X



Correlation and Covariance: A practical Example

Data Table

X	Y
5	7
8	10
14	7
15	12
...	...
...	...
...	...
...	...
...	...



$$\text{cov}(X, Y) = \mathbb{E}([X - \mathbb{E}(X)][Y - \mathbb{E}(Y)])$$

$$\rho_{X,Y} = \text{corr}(X, Y) = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y} :$$

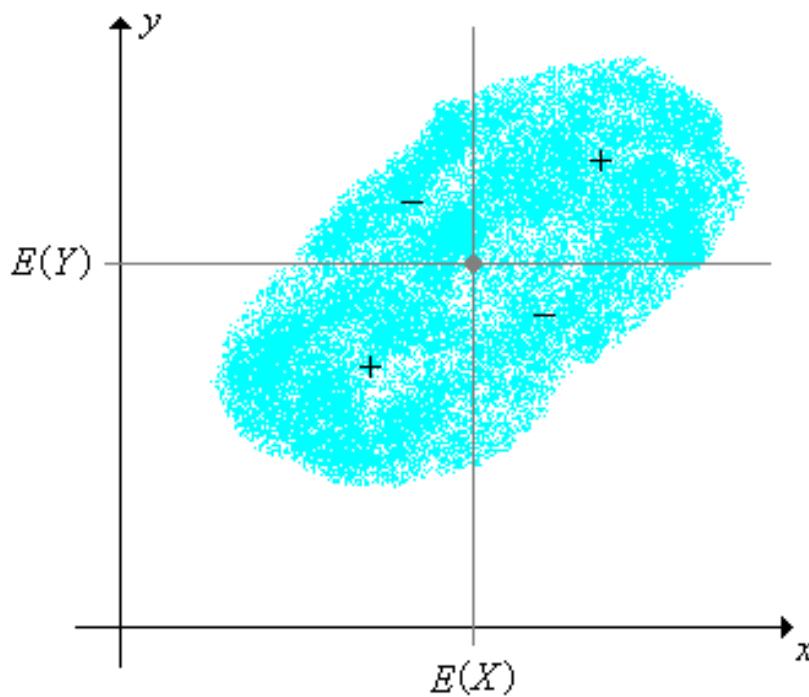
$$= \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y},$$



Correlation: A practical Example

Data Table

X	Y
5	7
8	10
14	7
15	12
...	...
...	...
...	...
...	...
...	...



$$\rho_{X,Y} = \text{corr}(X, Y) = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y} :$$

$$= \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y},$$

$$|Cov(X, Y)|^2 \leq Var(X)Var(Y)$$

$$\therefore |Cov(X, Y)| \leq \sqrt{Var(X)Var(Y)}$$

plug this result from the Cauchy-Schwarz

$$|\rho| = \left| \frac{Cov(X, Y)}{\sqrt{Var(X)Var(Y)}} \right| \leq \frac{\sqrt{Var(X)Var(Y)}}{\sqrt{Var(X)Var(Y)}} = 1$$



Data X

Example: What is the correlation of X,Y
How Do we Find It?

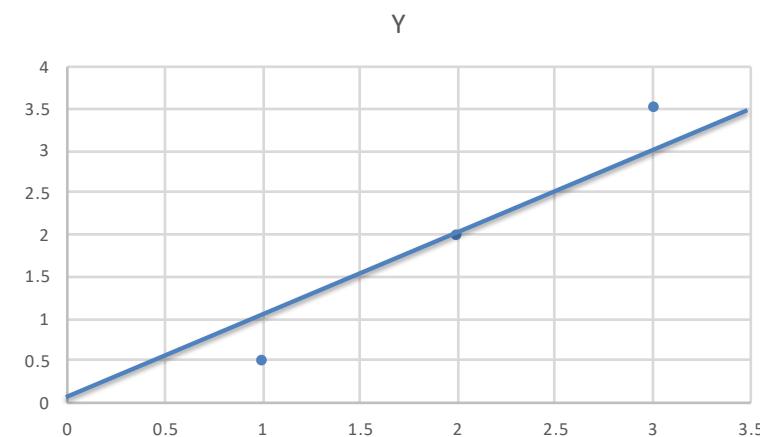
X	Y
1	0.5
2	2
3	3.5

Data X



Example: What is the correlation of X,Y How Do we Find It?

X	Y
1	0.5
2	2
3	3.5



Data X

Example: What is the correlation of X,Y How Do we Find It?

	X	Y
	1	0.5
	2	2
	3	3.5
mean	2.00	2.00
stdev	1	1.5
var	1	2.25

Data X



Example: What is the correlation of X,Y

One way to do it:

Example				
	X	Y	X*Y	E[X]E[Y]
	1	0.5	0.5	4
	2	2	4	4
	3	3.5	10.5	4
mean	2.00	2.00	5.00	4.00
stdev	1	1.5		
var	1	2.25		

$$\text{Cor}(X,Y) = \frac{E[(XY)] - E[X]E[Y]}{\text{stdev}(X) * \text{stdev}(Y)}$$
$$= E[(XY)] - E[X]E[Y] / 1.5$$
$$= 5 - 4 / (1.5)$$
$$= 1/1.5 = .67$$

Data X



Example: What is the correlation of X,Y The other way to do it

	X	Y	X-ux	Y-uy	(X-ux)(Y-uy)
1	1	0.5	-1	-1.5	1.5
2	2	2	0	0	0
3	3	3.5	1	1.5	1.5
mean	2.00	2.00	0.00	0.00	1.00
stdev	1	1.5			

$$\text{Cor}(X,Y) = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y},$$

$$= E[(X-ux)(Y-uy)] / 1.5$$

$$= \frac{[(1-2)(0.5-2) + (2-2)(2-2) + (3-2)(3.5-2)]/3}{1.5}$$

$$= 1.5 + 0 + 1 * 1.5 / (3 * 1.5) = 1/1.5 = .67$$



Correlation and Covariance

$$\rho_{X,Y} = \text{corr}(X, Y) = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y},$$

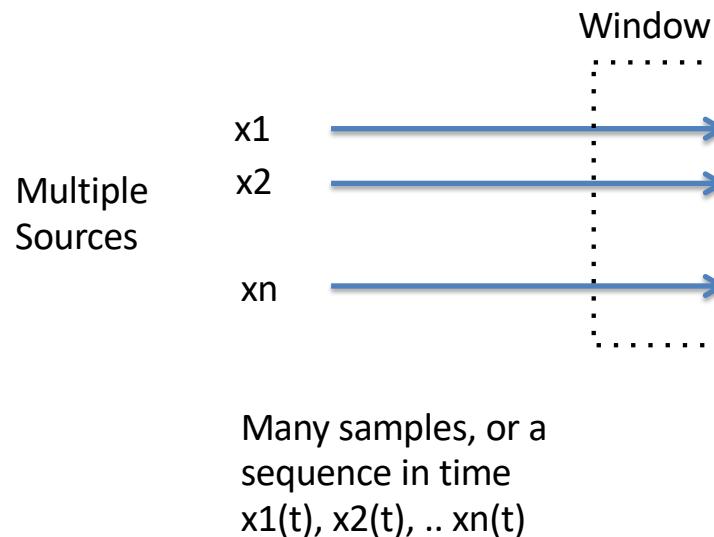
Properties

- a. $-1 \leq \text{cor}(X, Y) \leq 1$
- b. $-\text{sd}(X)\text{sd}(Y) \leq \text{cov}(X, Y) \leq \text{sd}(X)\text{sd}(Y)$
- c. $\text{cor}(X, Y) = 1$ if and only if Y is a linear function of X with positive slope.
- d. $\text{cor}(X, Y) = -1$ if and only if Y is a linear function of X with negative slope.

<http://www.math.uah.edu/stat/expect/Covariance.html>



Correlation Matrix



Table

Samples	x_1	x_2	...	x_n
1				
2				
3				
n				
.				
N+W				

Samples from Window of W

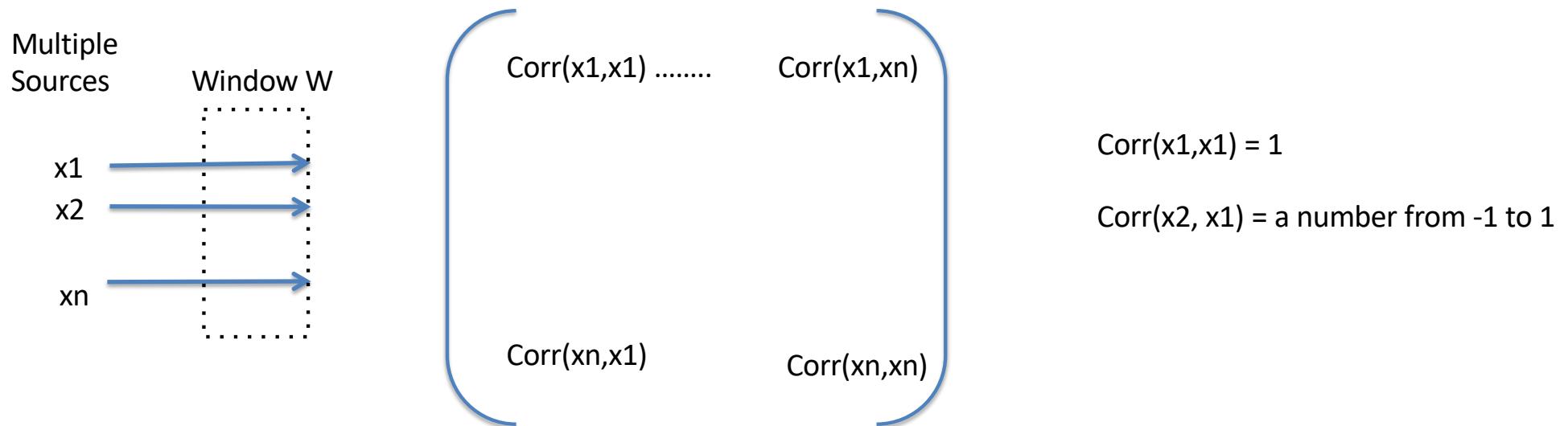
Detailed description: This diagram shows a table representing the Correlation Matrix. The columns are labeled x1, x2, ..., xn and the rows are labeled Samples 1, 2, 3, n, ., N+W. A dashed rectangle highlights the area under the heading 'Samples from Window of W', indicating that only a subset of the total samples is used for the matrix calculation.

To estimate from data:

- Use all samples ever collected
- Use window size of W samples of each to estimate a recent Corr Matrix



Correlation Matrix



To estimate from data:

- Use all samples ever collected
- Use window size of W samples of each to estimate recent Corr Matrix



Code Examples: Correlation of Rows with NumPy

```
Import numpy as np

# ignore line formatting
x = np.array(
    [[0.1, .32, .2, 0.4, 0.8],
     [.23, .18, .56, .61, .12],
     [.9, .3, .6, .5, .3],
     [.34, .75, .91, .19, .21]])

np.corrcoef(x)
Out[4]: array([
 [ 1.          , -0.35153114, -0.74736506, -0.48917666],
 [-0.35153114,  1.          ,  0.23810227,  0.15958285],
 [-0.74736506,  0.23810227,  1.          , -0.03960706],
 [-0.48917666,  0.15958285, -0.03960706,  1.          ],
 ])
```

Here each row is a vector of length 5
There are 4 vectors

Correlation matrix is 4×4

If you want the correlation of the columns,
just use transpose

```
np.corrcoef ( np.transpose(x) )
```

For a window, use a slice:
window = x[0:4,3:5] for the last
two columns



Correlation of Features from Different Sources

Mazda RX4
Mazda RX4 Wag
Datsun 710
Hornet 4 Drive
Hornet Sportabout
Valiant

	mpg	disp	hp	drat	wt	qsec
Mazda RX4	21.0	160	110	3.90	2.620	16.46
Mazda RX4 Wag	21.0	160	110	3.90	2.875	17.02
Datsun 710	22.8	108	93	3.85	2.320	18.61
Hornet 4 Drive	21.4	258	110	3.08	3.215	19.44
Hornet Sportabout	18.7	360	175	3.15	3.440	17.02
Valiant	18.1	225	105	2.76	3.460	20.22



Pandas Table
Use corr() method →
dataframe.corr()

pandas.DataFrame.corr

DataFrame.corr(method='pearson', min_periods=1)

[source]

Compute pairwise correlation of columns (excluding NA/null values)

Parameters:

- method : {'pearson', 'kendall', 'spearman'}
 - pearson : standard correlation coefficient
 - kendall : Kendall Tau correlation coefficient
 - spearman : Spearman rank correlation

min_periods : int, optional

Minimum number of observations required per pair of columns to have a valid result.
Currently only available for pearson and spearman correlation

Returns:

y : DataFrame

	mpg	disp	hp	drat	wt	qsec
mpg	1.00	-0.85	-0.78	0.68	-0.87	0.42
disp	-0.85	1.00	0.79	-0.71	0.89	-0.43
hp	-0.78	0.79	1.00	-0.45	0.66	-0.71
drat	0.68	-0.71	-0.45	1.00	-0.71	0.09
wt	-0.87	0.89	0.66	-0.71	1.00	-0.17
qsec	0.42	-0.43	-0.71	0.09	-0.17	1.00



Correlation Types: Pearson, Kendal, Spearman

pandas.DataFrame.corr

```
DataFrame.corr(method='pearson', min_periods=1)
```

Compute pairwise correlation of columns, excluding NA/null values

Parameters:

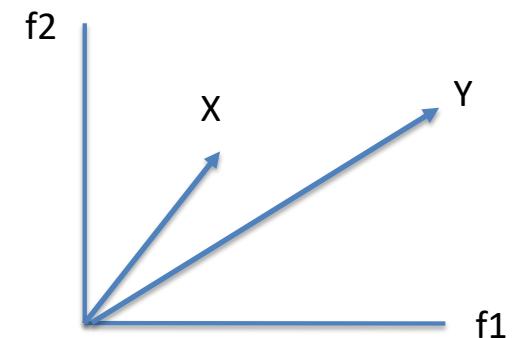
- method** : {‘pearson’, ‘kendall’, ‘spearman’}
 - pearson : standard correlation coefficient
 - kendall : Kendall Tau correlation coefficient
 - spearman : Spearman rank correlation
- min_periods** : int, optional
Minimum number of observations required per pair.
Currently only available for pearson and spearman.

Returns: **y** : DataFrame

Data Table

X	Y

5	7
8	10
14	7
15	12
...	...
...	...
...	...
...	...
...	...



$$X \bullet Y = |X| |Y| \cos \Theta$$

Pearson: Understanding
Correlation in a different way

Use n dimensions



Pandas will create a correlation matrix with “columns”

```
In [15]: frame = pd.DataFrame(np.random.randn(1000, 5), columns=[ 'a', 'b', 'c', 'd', 'e']

In [16]: frame.ix[::2] = np.nan

# Series with Series
In [17]: frame['a'].corr(frame['b'])
Out[17]: 0.013479040400098775

In [18]: frame['a'].corr(frame['b'], method='spearman')
Out[18]: -0.0072898851595406371

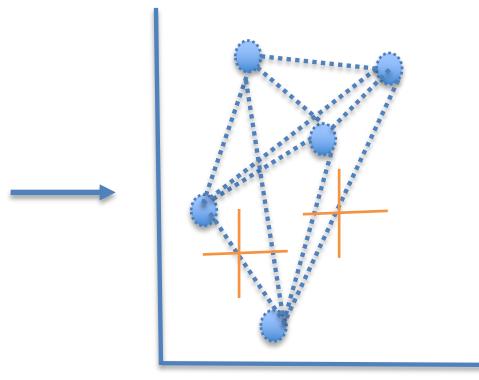
# Pairwise correlation of DataFrame columns
In [19]: frame.corr()
Out[19]:
      a          b          c          d          e
a  1.000000  0.013479 -0.049269 -0.042239 -0.028525
b  0.013479  1.000000 -0.020433 -0.011139  0.005654
c -0.049269 -0.020433  1.000000  0.018587 -0.054269
d -0.042239 -0.011139  0.018587  1.000000 -0.017060
e -0.028525  0.005654 -0.054269 -0.017060  1.000000
```



Kendall Correlation

List of (x,y) points

No	X	Y
1	2	3
2	4	6
3	3	8
4	9	12



N points
N(n-1)/2 pairs of x,y points

Concordant pairs: for (x_i, y_i) and (x_j, y_j) , where $i \neq j$,
 $x_i > x_j$ and $y_i > y_j$ or $x_i < x_j$ and $y_i < y_j$

Disconcordant pairs: when the above is not true
if $x_i > x_j$ and $y_i < y_j$
or if $x_i < x_j$ and $y_i > y_j$

The Kendall τ coefficient is defined as:

$$\tau = \frac{(\text{number of concordant pairs}) - (\text{number of discordant pairs})}{n(n - 1)/2}$$



$$r_s = \rho_{rg_X, rg_Y} = \frac{\text{cov}(rg_X, rg_Y)}{\sigma_{rg_X} \sigma_{rg_Y}}$$

where

- ρ denotes the usual Pearson correlation coefficient, but applied to the rank variables.
- $\text{cov}(rg_X, rg_Y)$ is the covariance of the rank variables.
- σ_{rg_X} and σ_{rg_Y} are the standard deviations of the rank variables.

Data ($x=IQ, y=TV$)

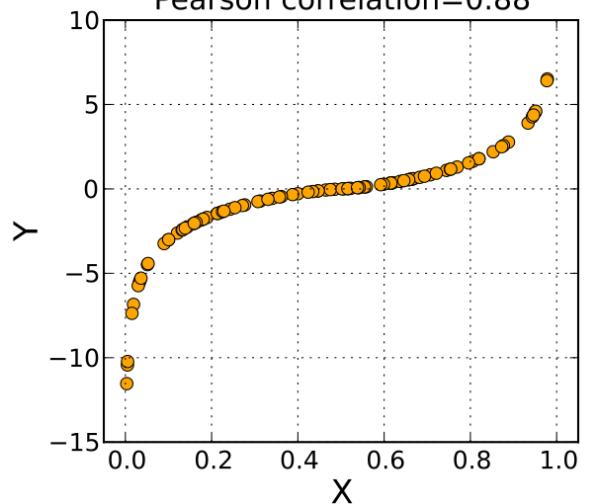
IQ, X_i	Hours of TV per week, Y_i
106	7
86	0
100	27
101	50
99	28
103	29
97	20
113	12
112	6
110	17

Order rows by X and
Index X and Y in
increasing order

X	y	rgx	rgy
97	20	2	6
99	28	3	8
100	27	4	7
101	50	5	10
103	29	6	9
106	7	7	3
110	17	8	5
112	6	9	2
113	12	10	4

Spearman Correlation

Spearman correlation=1
Pearson correlation=0.88



Then find
Pearson Correlation
of (rgx,rgy)

A Spearman correlation of 1 results when the two variables being compared are monotonically related, even if their relationship is not linear. This means that all data-points with greater x-values than that of a given data-point will have greater y-values as well. In contrast, this does not give a perfect Pearson correlation.

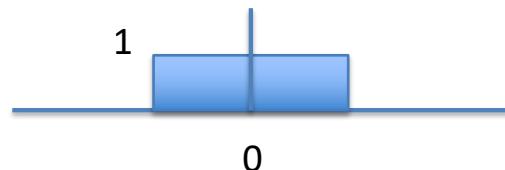
Wikipedia



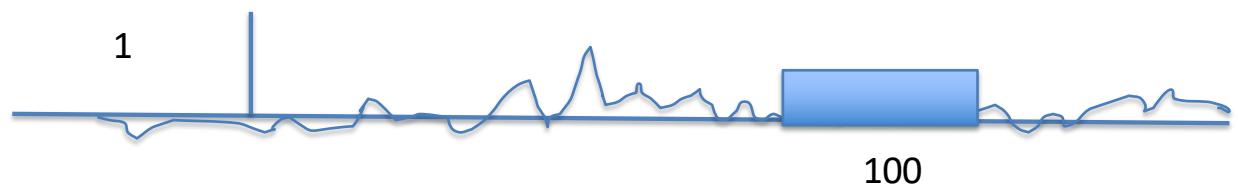
Data X

Correlation Matrix with multiple sources and time segments

Suppose this is x_1
as an array of numbers 0 0 1 1..1 0 0 0



Suppose this is x_2



What is `np.corr(x1,x2[n:n+w])`?



Approaches to the Data Sequences from Multiple Sources in Tables

A
.
.
D

Discrete data for each source
A, B, C..
 $x_n = x_1, x_2, x_3, \dots$



One row for each source
A, B, C, D..

Eg Numpy arrays

Time ->

A:	x1, x2, x3, ...
B:	x1, x2, x3, ...
C:	x1, x2, x3, ...
D:	x1, x2, x3, ...

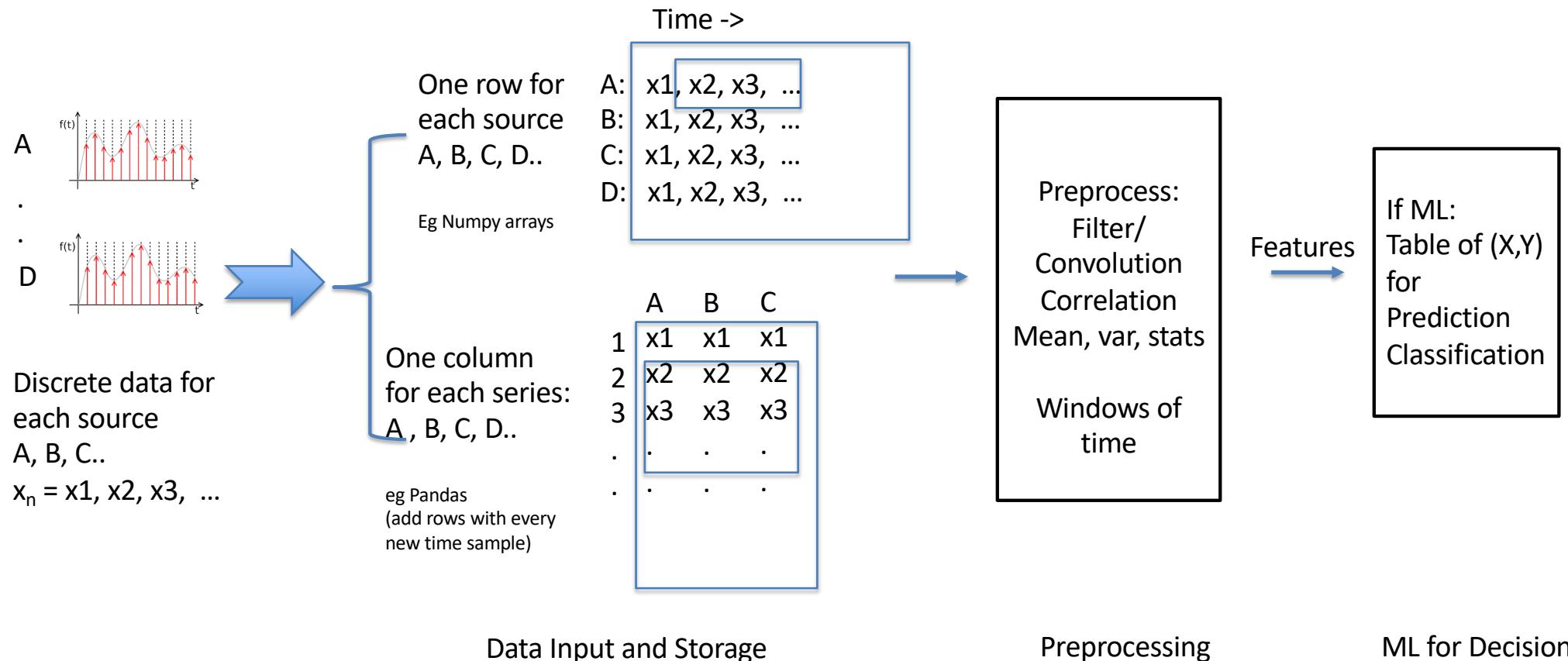
One column for each series:
A, B, C, D..

eg Pandas
(add rows with every new time sample)

	A	B	C
1	x1	x1	x1
2	x2	x2	x2
3	x3	x3	x3
.	.	.	.
.	.	.	.



Approaches to the Data Sequences in Tables



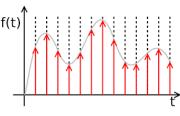
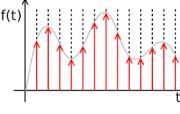
Data Input and Storage

Preprocessing

ML for Decisions



Approaches to the Data Sequences in Tables

A 
.
.
D 

Discrete data for each source
A, B, C..
 $x_n = x_1, x_2, x_3, \dots$

One row for each source
A, B, C, D..

Eg Numpy arrays

One column for each series:
A, B, C, D..

Eg Pandas
(add rows with every new time sample)

Time ->				
A:	x1, x2, x3, ...			
B:	x1, x2, x3, ...			
C:	x1, x2, x3, ...			
D:	x1, x2, x3, ...			

	A	B	C	
1	x1	x1	x1	
2	x2	x2	x2	
3	x3	x3	x3	
.	.	.	.	

	Cor(w/A)	Cor(w/B)	St.Dev.	Cor(w/A[n-20])	Labeled Condition
A	#	#	#	#	Danger
B	#	#	#	#	Safe
C	#	#	#	#	Safe
D	#	#	#	#	Warning

= number obtained from preprocessing

Example: pre-processed statistics can be used for in ML predictions

Data Input and Storage



End of Section

0 0 0 1 0 1 0 0 1 1 1 1 0 0 0 0 0 0 1 0 0 1 0 0 1 1 1 1 0 0
1 0 1 1 0 0 0 1 0 0 1 0 0 0 1 0 1 0 0 1 1 1 1 0 0 1 0 0 1 0 0
1 Data X 0 0 1 0 0 1 0 0 1 0 0 0 1 0 1 1 1 1 0 0 0 1 0 0 1 0 0 0 0 0 1 0 0