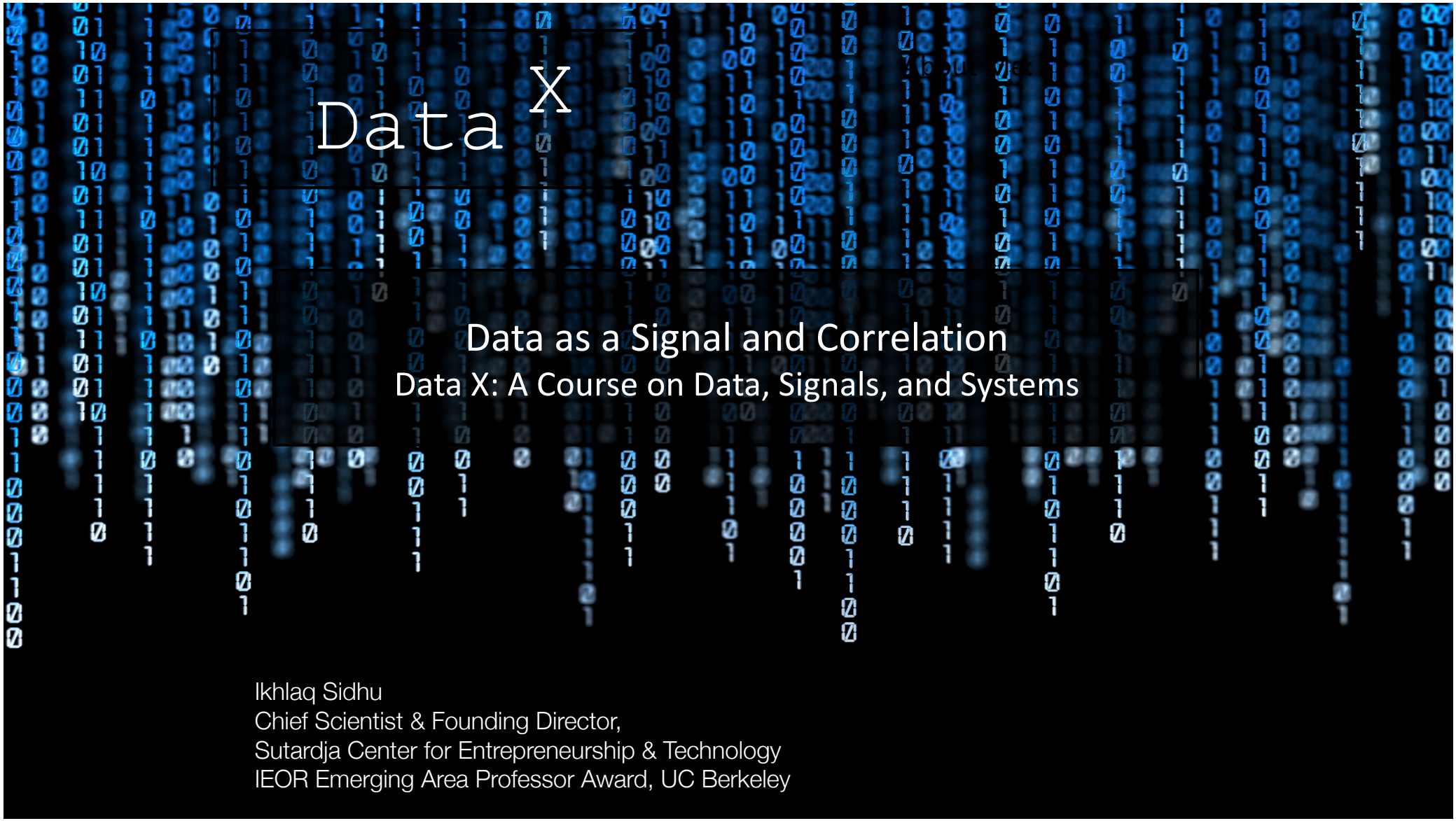
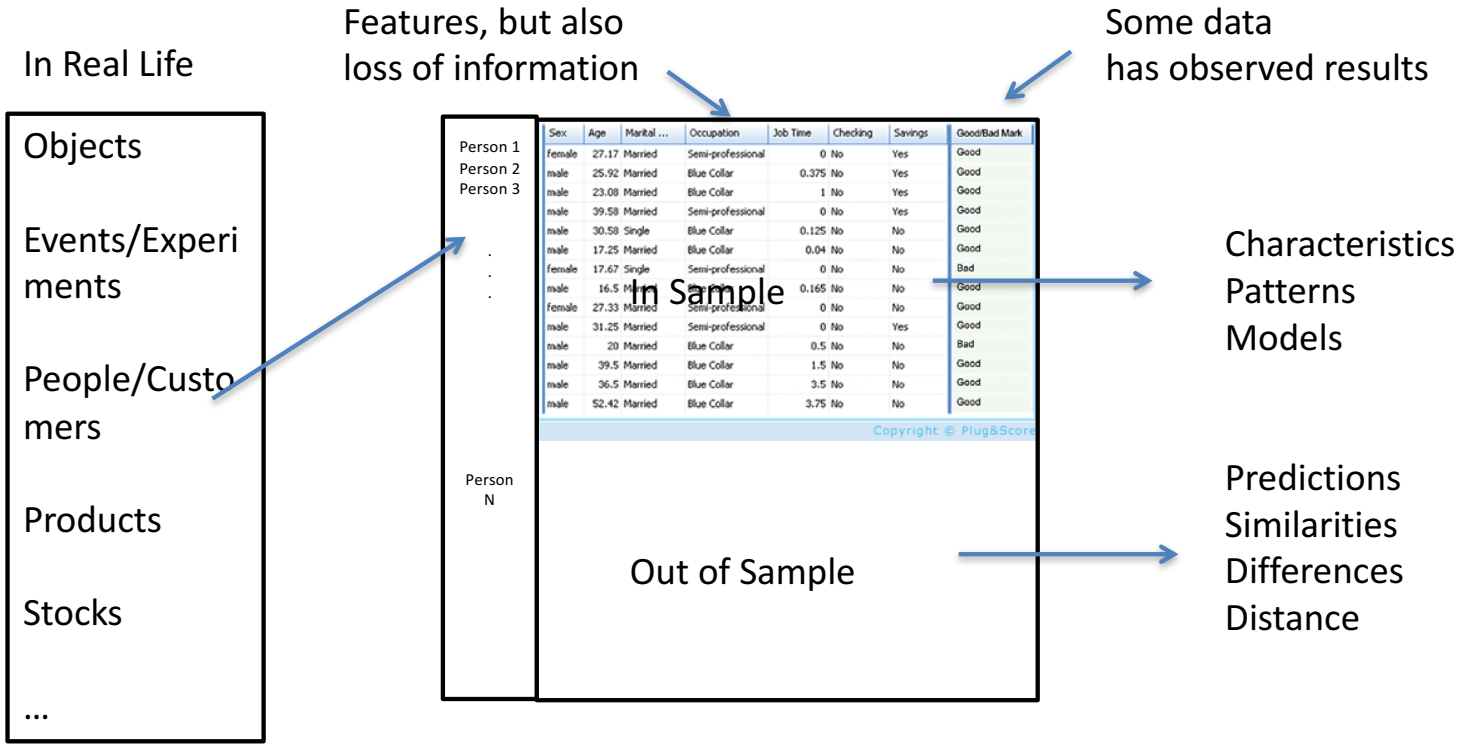


Data as a Signal and Correlation

Data X: A Course on Data, Signals, and Systems

Ikhlaq Sidhu
Chief Scientist & Founding Director,
Sutardja Center for Entrepreneurship & Technology
IEOR Emerging Area Professor Award, UC Berkeley

A High Level Framework



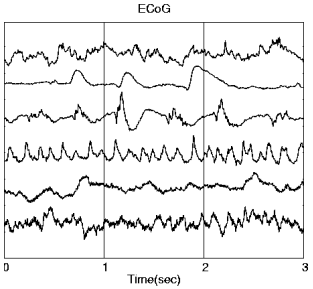
Converting From Time Sequence Data to Features

Of course, not all data has a time property, but lets start with this type.
For example(key1, value 1),(key 2, value 2)… in this case, the keys are indexed by time.

Converting From Time Sequence Data to Features

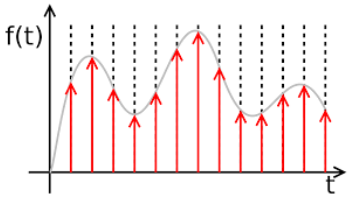
Many Types of data are signals in time

- Stock market
- Temperature
- Instrument readings



Continuous signals
x(t)

Sometimes we sample them, record at intervals of T



Sampled signals (data)
x(nT)

We get a list in a table, array, or vector

Rec	Observed
1	60.323
2	61.122
3	60.171
4	61.187
5	63.221
6	63.639
7	64.989
8	63.761
9	66.019
10	67.857
11	68.169
12	66.513
13	68.655
14	69.564
15	69.331
16	70.551

Discrete data
x_n = x1, x2, x3, …

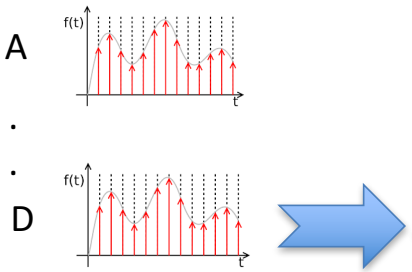
(might lose time reference)

What we want (for now): features and characteristics

For example:

- Means
- Variances
- Patten matches
- Changes
- accumulation
- Frequency

Approaches to the Data Sequences from Multiple Sources in Tables



Discrete data for each source
A, B, C..
x_n = x1, x2, x3, …

One row for each source
A, B, C, D..

Eg Numpy arrays

Time ->

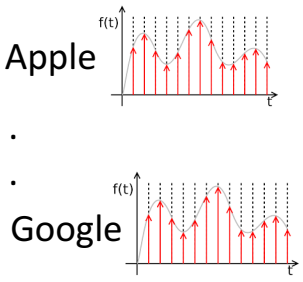
A:	x1, x2, x3, …
B:	x1, x2, x3, …
C:	x1, x2, x3, …
D:	x1, x2, x3, …

One column for each series:
A , B, C, D..

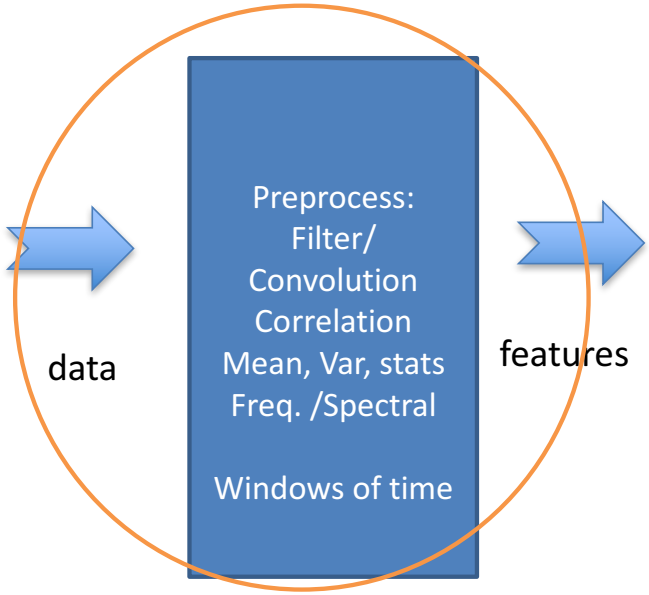
eg Pandas
(add rows with every new time sample)

	A	B	C
1	x1	x1	x1
2	x2	x2	x2
3	x3	x3	x3
.	.	.	.
.	.	.	.

Data Sequence in Tables Example



Discrete data for each source
A, B, C..
x_n = x1, x2, x3, …



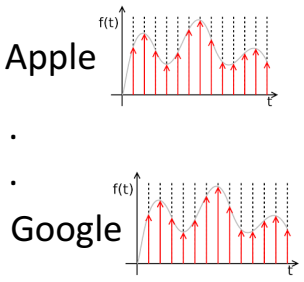
	Price	Price[n-20]	20 day MA	1 year average	Expected Price?
APPL	appl[n]	appl[n-20]	mva(appl, 20)	mva(appl, 200)	
FB	fb[n]	fb[n-20]	mva(fb,20)	mva(fb,200)	
GOOG	goog[n]	goog[n-20]	mva(goog, 20)	mva(goog, 200)	

Data Input and Temp Storage

Preprocess (and lose some information)

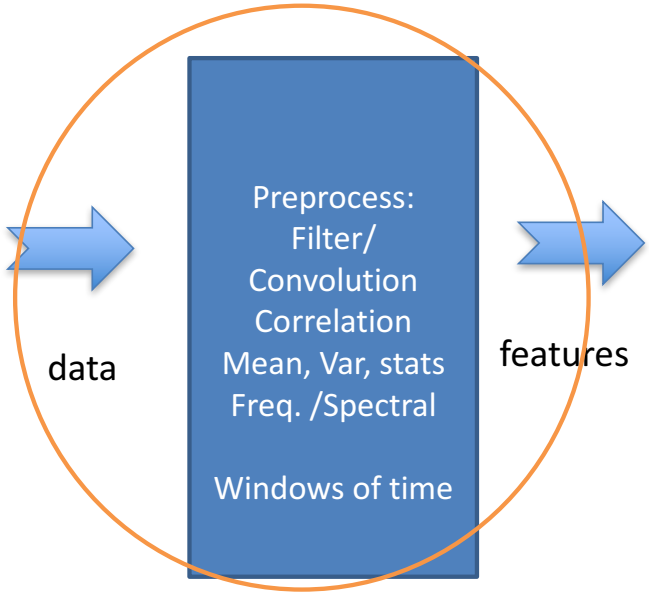
ML for Decisions / Predictions

Data Sequence in Tables Example



Discrete data for each source
A, B, C..
x_n = x1, x2, x3, …

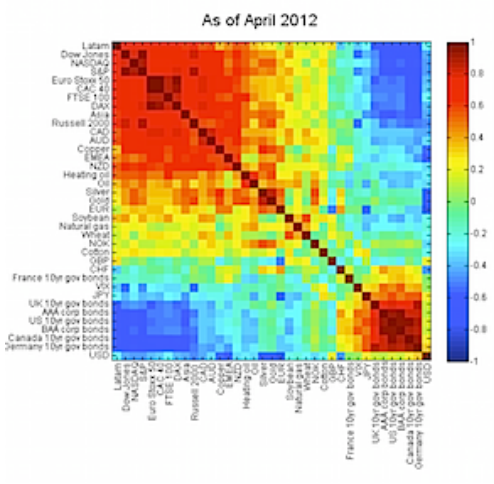
Data Input and Temp Storage



Preprocess (and lose some information)

Correlation Matrix: Daily % Change Correlation Over Last Ten Years

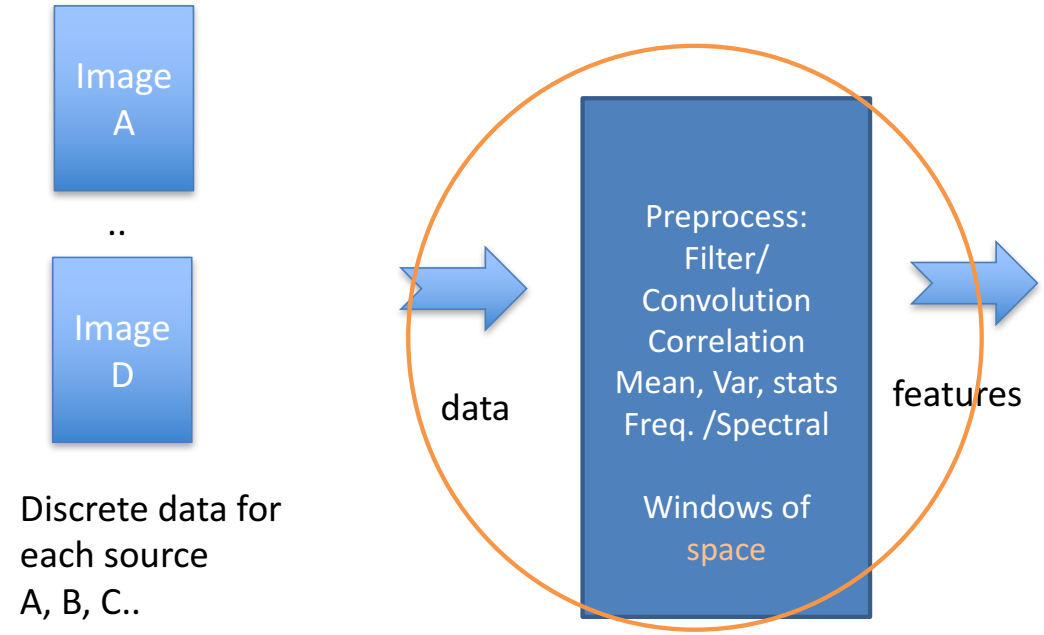
Ticker	S&P 500	C. Disc.	C. Stap	Energy	Finan.	H Care	Indust.	Mater.	Tech	Telcom	Utilit.	Oil	Gold	Dollar	L Bnd
S&P 500	1.00	0.94	0.85	0.81	0.88	0.86	0.94	0.89	0.90	0.78	0.76	0.24	-0.01	-0.17	-0.37
Cons. Disc.	0.94	1.00	0.81	0.68	0.82	0.78	0.90	0.82	0.85	0.72	0.67	0.16	-0.07	-0.12	-0.34
Cons. Stap.	0.85	0.81	1.00	0.65	0.67	0.81	0.78	0.71	0.70	0.69	0.71	0.12	-0.06	-0.10	-0.28
Energy	0.81	0.68	0.65	1.00	0.62	0.66	0.73	0.81	0.66	0.59	0.70	0.48	0.16	-0.28	-0.30
Financials	0.88	0.82	0.67	0.62	1.00	0.68	0.82	0.73	0.72	0.64	0.57	0.16	-0.06	-0.13	-0.31
H Care	0.86	0.78	0.81	0.66	0.68	1.00	0.78	0.71	0.73	0.68	0.69	0.14	-0.04	-0.12	-0.29
Industrials	0.94	0.90	0.78	0.73	0.82	0.78	1.00	0.87	0.84	0.70	0.68	0.21	-0.01	-0.18	-0.37
Materials	0.89	0.82	0.71	0.81	0.73	0.71	0.87	1.00	0.79	0.65	0.67	0.29	0.15	-0.27	-0.35
Technology	0.90	0.85	0.70	0.66	0.72	0.73	0.84	0.79	1.00	0.71	0.63	0.16	-0.04	-0.10	-0.35
Telecom	0.78	0.72	0.69	0.59	0.64	0.68	0.70	0.65	0.71	1.00	0.63	0.12	-0.05	-0.10	-0.25
Utilities	0.76	0.67	0.71	0.70	0.57	0.69	0.68	0.67	0.63	0.63	1.00	0.19	0.02	-0.15	-0.22
Oil	0.24	0.16	0.12	0.48	0.16	0.14	0.21	0.29	0.16	0.12	0.19	1.00	0.29	-0.30	-0.22
Gold	-0.01	-0.07	-0.06	0.16	-0.06	-0.04	-0.01	0.15	-0.04	-0.05	0.02	0.29	1.00	-0.43	0.07
Dollar	-0.17	-0.12	-0.10	-0.28	-0.13	-0.12	-0.18	-0.27	-0.10	-0.10	-0.15	-0.30	-0.43	1.00	-0.05
Long Bond	-0.37	-0.34	-0.28	-0.30	-0.31	-0.29	-0.37	-0.35	-0.35	-0.25	-0.22	-0.22	0.07	-0.05	1.00



Correlation Matrix

Descriptive Statistics

Data Sequence in Tables Example



Discrete data for each source
A, B, C..
x_n = 2-D Array

If ML: Table of (X,Y) for Prediction Classification

Data Input and Temp Storage

Preprocess (and lose some information)

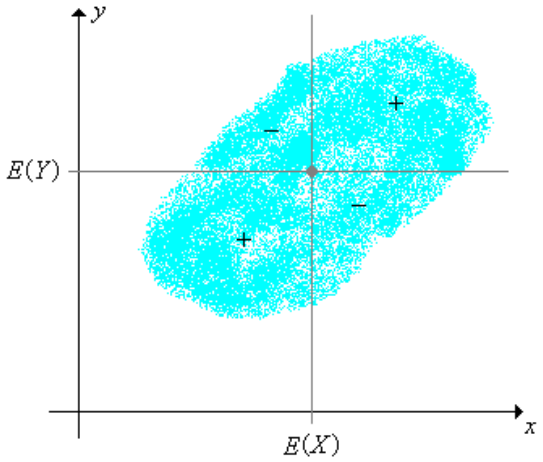
Correlation Matrices

Correlation and Covariance

1. The *covariance* of (X, Y) is defined by

$$\operatorname{cov}(X, Y) = \mathbb{E}\left([X - \mathbb{E}(X)]\,[Y - \mathbb{E}(Y)]\right)$$

and, assuming the variances are positive, the *correlation* of (X, Y) is defined by



$$\rho_{X,Y} = \operatorname{corr}(X, Y) = \frac{\operatorname{cov}(X, Y)}{\sigma_X \sigma_Y} =$$

$$= \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y},$$

http://www.math.uah.edu/stat/expect/Covariance.html

Correlation and Covariance

$$\rho_{X,Y} = \operatorname{corr}(X,Y) = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y},$$

Properties

a. $-1 \leq \operatorname{cor}(X,Y) \leq 1$

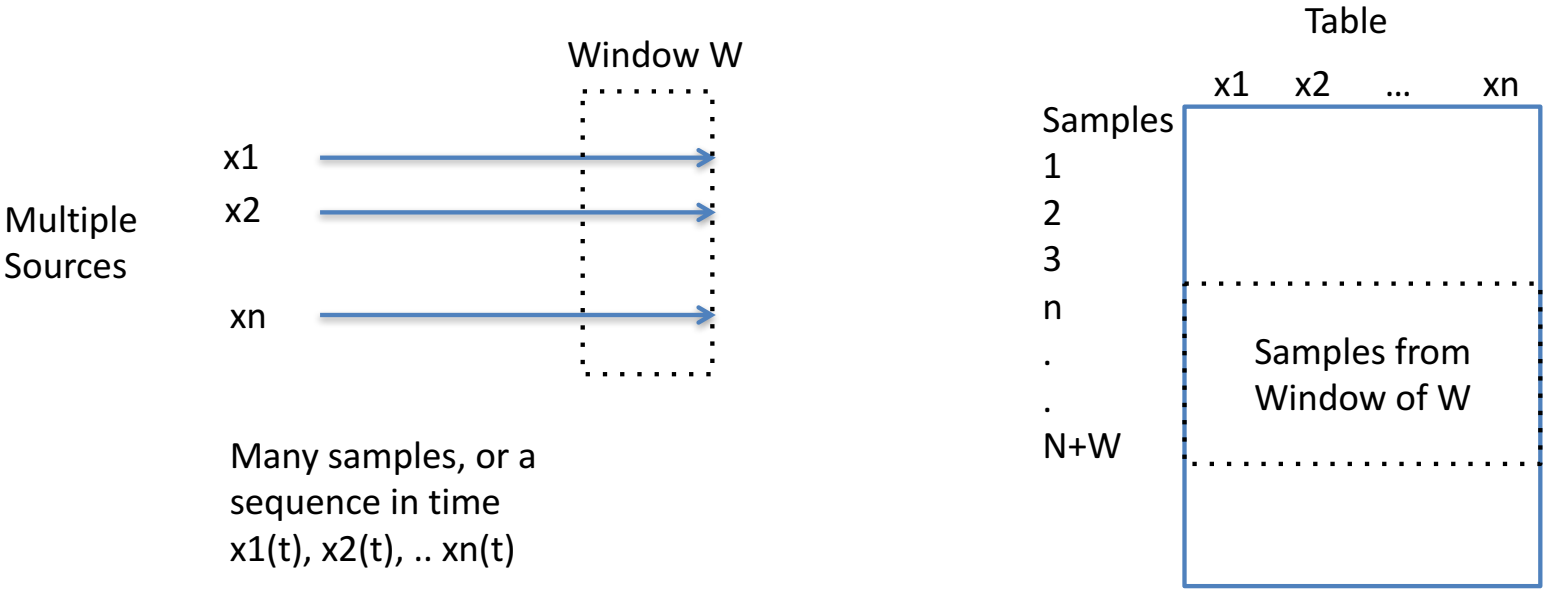
b. $-\operatorname{sd}(X)\operatorname{sd}(Y) \leq \operatorname{cov}(X,Y) \leq \operatorname{sd}(X)\operatorname{sd}(Y)$

c. $\operatorname{cor}(X,Y) = 1$ if and only if Y is a linear function of X with positive sloper.

d. $\operatorname{cor}(X,Y) = -1$ if and only if Y is a linear function of X with negative slope.

http://www.math.uah.edu/stat/expect/Covariance.html

Correlation Matrix

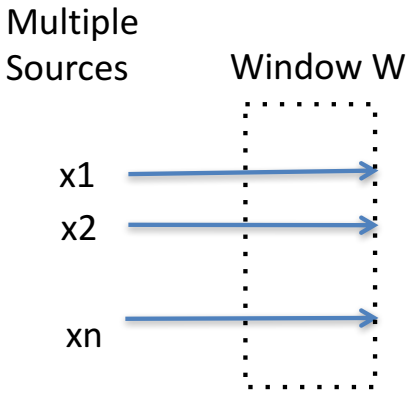


To estimate from data:

a) Use all samples ever collected

b) Use window size of W samples of each to estimate a recent Corr Matrix

Correlation Matrix



$$\begin{pmatrix} \text{Corr}(x1,x1) \ldots\ldots & \text{Corr}(x1,xn) \\ \text{Corr}(xn,x1) & \text{Corr}(xn,xn) \end{pmatrix}$$

Corr(x1,x1) = 1

Corr(x2, x1)
= E[x2 x1 – E[x2]E[x1]] / stdev(x1) stdev(x2)

You could even do this by hand:

x1	x2	E[x1 x2]	E[x1]	E[x2]
1	2	2	1.5	2.5
2	4			
1	1			
2	3			

To estimate from data:

a) Use all samples ever collected
b) Use window size of W samples of each to estimate recent Corr Matrix

Code Examples: Correlation of Rows with NumPy

```
Import numpy as np

# ignore line formatting
x  =  np.array(
       [[0.1,  .32,  .2,    0.4,  0.8],
        [.23,  .18,  .56,  .61,  .12],
        [.9,     .3,    .6,    .5,    .3],
        [.34,  .75,  .91,  .19,  .21]])


np.corrcoef(x)
Out[4]:  array([
[  1.            ,  -0.35153114,  -0.74736506,  -0.48917666],
[-0.35153114,    1.            ,    0.23810227,    0.15958285],
[-0.74736506,    0.23810227,    1.            ,  -0.03960706],
[-0.48917666,    0.15958285,  -0.03960706,    1.            ]
])
```

Here each row is a vector of length 5
There are 4 vectors

Correlation matrix is 4 x 4

If you want the correlation of the columns, just use transpose

np.corrcoef (np.transpose(x))

For a window, use a slice:
window = x[0:4,3:5] for the last two columns

Correlation of Features from Different Sources

	mpg	disp	hp	drat	wt	qsec
Mazda RX4	21.0	160	110	3.90	2.620	16.46
Mazda RX4 Wag	21.0	160	110	3.90	2.875	17.02
Datsun 710	22.8	108	93	3.85	2.320	18.61
Hornet 4 Drive	21.4	258	110	3.08	3.215	19.44
Hornet Sportabout	18.7	360	175	3.15	3.440	17.02
Valiant	18.1	225	105	2.76	3.460	20.22

Pandas Table
Use corr()

Ie dataframe.corr()

pandas.DataFrame.corr

`DataFrame.corr(method='pearson', min_periods=1)` [source]

Compute pairwise correlation of columns, excluding NA/null values

Parameters:

method *: {'pearson', 'kendall', 'spearman'}*

- pearson : standard correlation coefficient
- kendall : Kendall Tau correlation coefficient
- spearman : Spearman rank correlation

min_periods *: int, optional*

Minimum number of observations required per pair of columns to have a valid result. Currently only available for pearson and spearman correlation

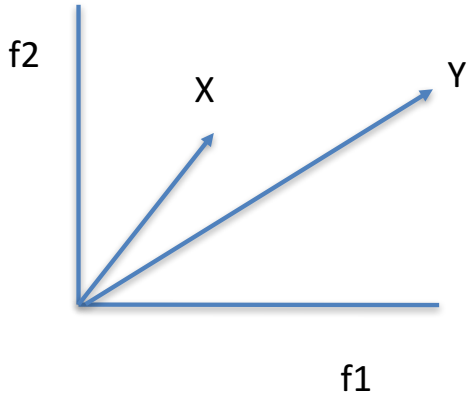
Returns:

y *: DataFrame*

	mpg	disp	hp	drat	wt	qsec
mpg	1.00	-0.85	-0.78	0.68	-0.87	0.42
disp	-0.85	1.00	0.79	-0.71	0.89	-0.43
hp	-0.78	0.79	1.00	-0.45	0.66	-0.71
drat	0.68	-0.71	-0.45	1.00	-0.71	0.09
wt	-0.87	0.89	0.66	-0.71	1.00	-0.17
qsec	0.42	-0.43	-0.71	0.09	-0.17	1.00

Correlation Types: Pearson Kendal, Spearman

Understanding Correlation in a different way



$X \cdot Y = |X|\ |Y| \cos \Theta$

pandas.DataFrame.corr

`DataFrame.corr(method='pearson', min_periods=1)`

Compute pairwise correlation of columns, excluding NA/null values

Parameters:

method : *{'pearson', 'kendall', 'spearman'}*

- pearson : standard correlation coefficient
- kendall : Kendall Tau correlation coefficient
- spearman : Spearman rank correlation

min_periods : *int, optional*

Minimum number of observations required per pa
Currently only available for pearson and spearma

Returns:

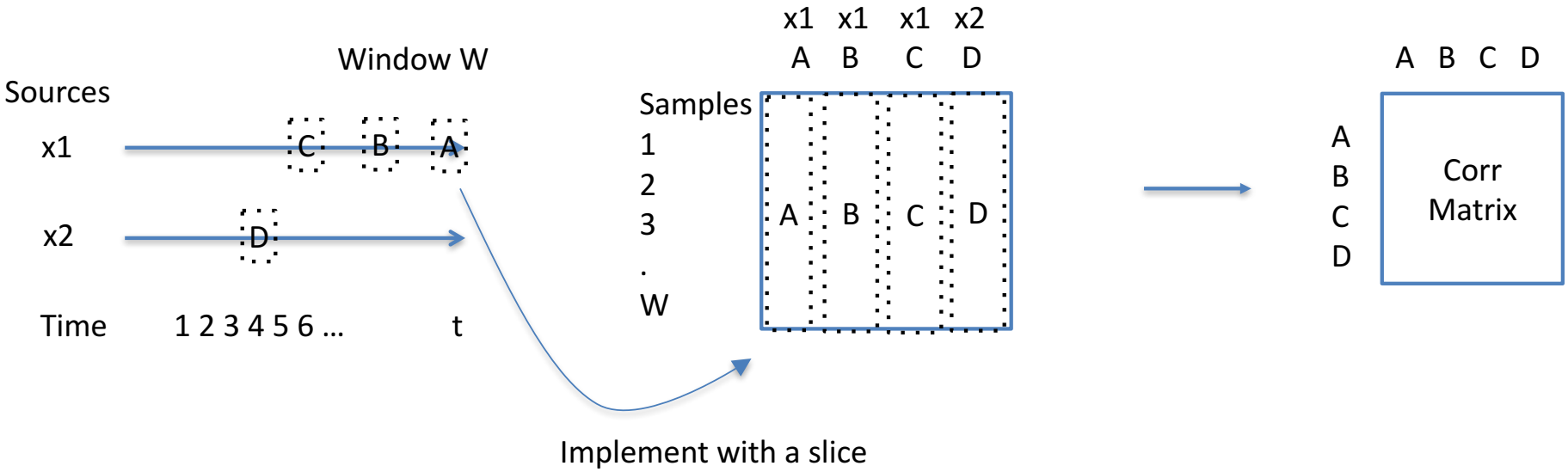
y : *DataFrame*

	mpg	disp	hp	drat	wt	qsec
mpg	1.00	-0.85	-0.78	0.68	-0.87	0.42
disp	-0.85	1.00	0.79	-0.71	0.89	-0.43
hp	-0.78	0.79	1.00	-0.45	0.66	-0.71
drat	0.68	-0.71	-0.45	1.00	-0.71	0.09
wt	-0.87	0.89	0.66	-0.71	1.00	-0.17
qsec	0.42	-0.43	-0.71	0.09	-0.17	1.00

Pandas will create a correlation matrix with “columns”

```
In [15]: frame = pd.DataFrame(np.random.randn(1000, 5), columns=['a', 'b', 'c', 'd', 'e

In [16]: frame.ix[::2] = np.nan

# Series with Series
In [17]: frame['a'].corr(frame['b'])
Out[17]: 0.013479040400098775

In [18]: frame['a'].corr(frame['b'], method='spearman')
Out[18]: -0.0072898851595406371

# Pairwise correlation of DataFrame columns
In [19]: frame.corr()
Out[19]:
          a         b         c         d         e
a  1.000000  0.013479 -0.049269 -0.042239 -0.028525
b  0.013479  1.000000 -0.020433 -0.011139  0.005654
c -0.049269 -0.020433  1.000000  0.018587 -0.054269
d -0.042239 -0.011139  0.018587  1.000000 -0.017060
e -0.028525  0.005654 -0.054269 -0.017060  1.000000
```

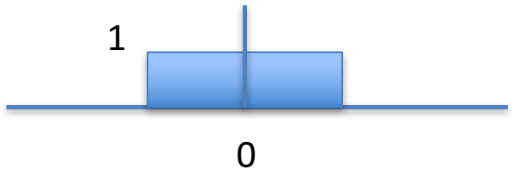
Correlation Matrix with multiple sources and time segments



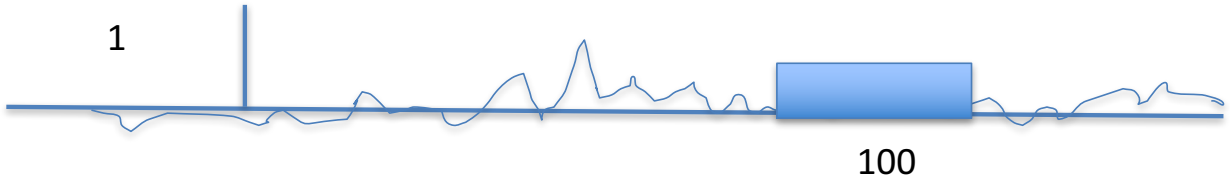
Why would you want a correlation matrix like this?

Correlation Matrix with multiple sources and time segments

Suppose this is x1
as an array of numbers 0 0 1 1..1 0 0 0

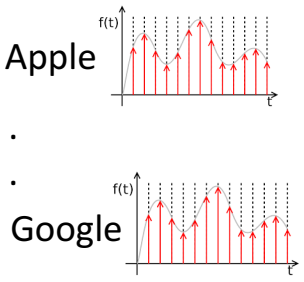


Suppose this is x2



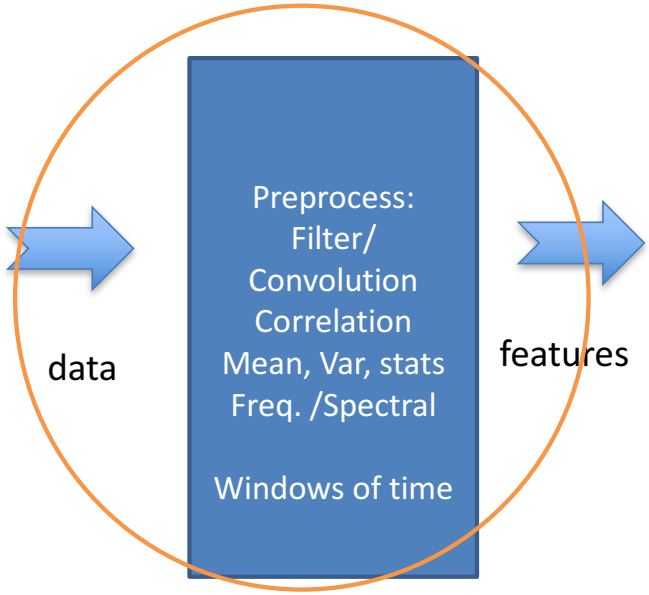
What is np.corr(x1,x2[n:n+w])?

Data Sequence in Tables Example



Discrete data for each source
A, B, C..
x_n = x1, x2, x3, …

Data Input and Temp Storage



Preprocess (and lose some information)

Correlation Matrix: Daily % Change Correlation Over Last Ten Years

Ticker	S&P 500	C. Disc.	C. Stap	Energy	Finan.	H Care	Indust.	Mater.	Tech	Telcom	Utilit.	Oil	Gold	Dollar	L Bnd
S&P 500	1.00	0.94	0.85	0.81	0.88	0.86	0.94	0.89	0.90	0.78	0.76	0.24	-0.01	-0.17	-0.37
Cons. Disc.	0.94	1.00	0.81	0.68	0.82	0.78	0.90	0.82	0.85	0.72	0.67	0.16	-0.07	-0.12	-0.34
Cons. Stap.	0.85	0.81	1.00	0.65	0.67	0.81	0.78	0.71	0.70	0.69	0.71	0.12	-0.06	-0.10	-0.28
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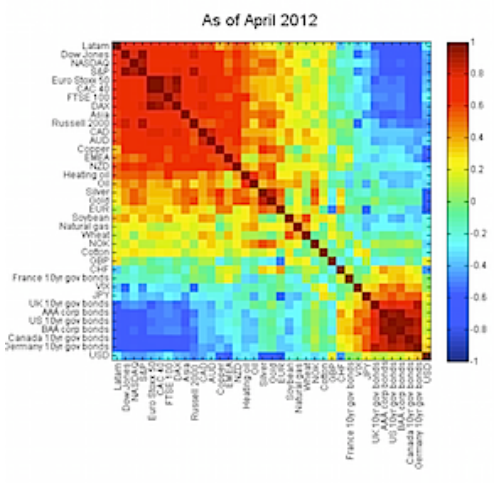
Financials	0.88	0.82	0.67	0.62	1.00	0.68	0.82	0.73	0.72	0.64	0.57	0.16	-0.06	-0.13	-0.31
H Care	0.86	0.78	0.81	0.66	0.68	1.00	0.78	0.71	0.73	0.68	0.69	0.14	-0.04	-0.12	-0.29
Industrials	0.94	0.90	0.78	0.73	0.82	0.78	1.00	0.87	0.84	0.70	0.68	0.21	-0.01	-0.18	-0.37
Materials	0.89	0.82	0.71	0.81	0.73	0.71	0.87	1.00	0.79	0.65	0.67	0.29	0.15	-0.27	-0.35
Technology	0.90	0.85	0.70	0.66	0.72	0.73	0.84	0.79	1.00	0.71	0.63	0.16	-0.04	-0.10	-0.35
Telecom	0.78	0.72	0.69	0.59	0.64	0.68	0.70	0.65	0.71	1.00	0.63	0.12	-0.05	-0.10	-0.25
Utilities	0.76	0.67	0.71	0.70	0.57	0.69	0.68	0.67	0.63	0.63	1.00	0.19	0.02	-0.15	-0.22
Oil	0.24	0.16	0.12	0.48	0.16	0.14	0.21	0.29	0.16	0.12	0.19	1.00	0.29	-0.30	-0.22
Gold	-0.01	-0.07	-0.06	0.16	-0.06	-0.04	-0.01	0.15	-0.04	-0.05	0.02	0.29	1.00	-0.43	0.07
Dollar	-0.17	-0.12	-0.10	-0.28	-0.13	-0.12	-0.18	-0.27	-0.10	-0.10	-0.15	-0.30	-0.43	1.00	-0.05
Long Bond	-0.37	-0.34	-0.28	-0.30	-0.31	-0.29	-0.37	-0.35	-0.35	-0.25	-0.22	-0.22	0.07	-0.05	1.00



Correlation Matrix

Descriptive Statistics

End of Section