Edwards-Venn diagrams

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Polar coordinates with longitude θ and latitude ϕ. Arc distance from the equator is s and height above the equarorial plane is h.

$$
\begin{aligned}
s &= r\frac{\phi}{2\pi} \\
h &= r\sin\phi
\end{aligned}
$$

Project down onto the equatorial plane (a polar stereographic projection).

$$
\begin{aligned}
\rho &= \frac{\cos\phi}{1-\sin\phi}2r \\
x &= \rho\cos\theta \\
y &= \rho\sin\theta
\end{aligned}
$$

A Mercator projection onto the (?) equatorial cylinder

$$
\begin{aligned}
x &= r\cos\phi\cos\theta \\
y &= r\cos\phi\sin\theta
\end{aligned}
$$

$$
\begin{aligned}
x &= rtheta \\
y &= h
\end{aligned}
$$

In a Mercator projection the Smith functions are

$$
h = \frac{\cos(2^{n-2}\theta)}{2^{n-2}}
$$

Let

$$
\begin{aligned}
T_n &= \frac{1}{2^n}\cos 2^n x \\
&= \frac{1}{2^n}\cos\frac{1}{2}2^{n+1}x \\
2^{2n}T_n^2 &= \frac{1}{2}\left(1+2^{n+1}T_{n+1}\right) \\
T_{n+1} &= 2^n T_n^2 - \frac{1}{2^{n+1}}
\end{aligned}
$$

So $T_{n+1} = 0$ when $T_n = \pm 2^{-n}1/\sqrt{(2)}$; $2^n x = \pi/4 + (p/2)2\pi$.

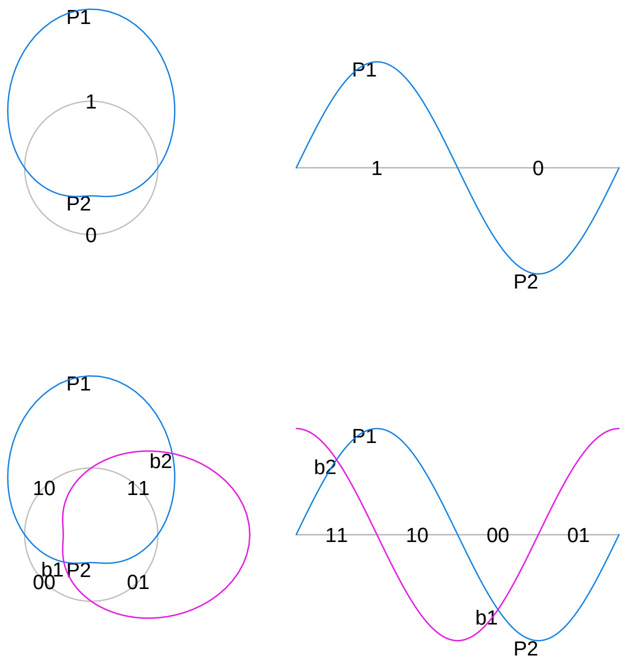
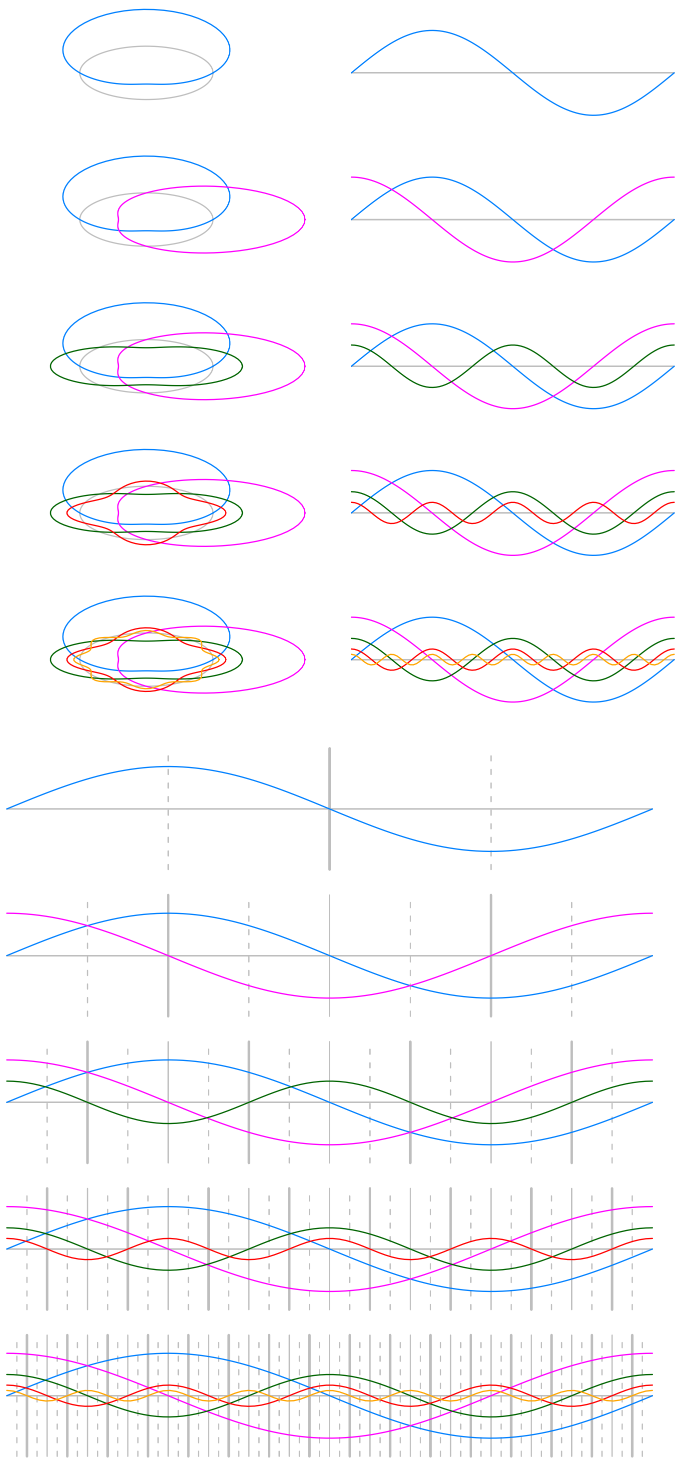
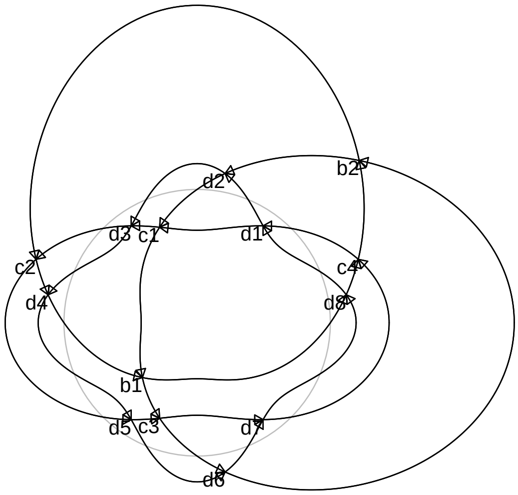


Figure 1: Additional nodes P1 and P2 introduced for n=1 and n=2 to avoid nasty edge condition s=0 to s=1 and to ensure no more than one directed edge between any pair of nodes.





References