TRIGONOMETRY

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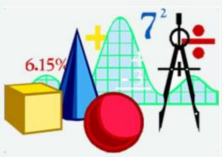

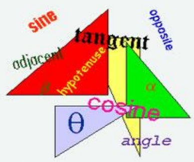

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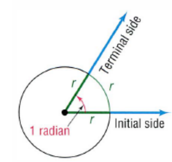
Angle Measure

Radians

A **central angle** is a positive angle whose vertex is at the center of a circle. The rays of a central angle subtend (intersect) an arc on the circle. If the radius of the circle is r and the length of the arc subtended by the central angle is also r, then the measure of the angle is **1 radian.**

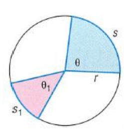


(a)

Angle Measure

2 Find the Arc Length of a Circle

Now consider a circle of radius r and two central angles, θ and θ_1, measured in radians. Suppose that these central angles subtend arcs of lengths s and s_1, respectively,



$$\frac{\theta}{\theta_1} = \frac{s}{s_1}$$

Suppose that $\theta_1 = 1$ radian

$$\frac{\theta}{1} = \frac{s}{r} \quad \text{or} \quad s = r\theta$$

Angle Measure

THEOREM

Arc Length

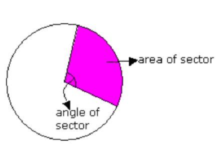
For a circle of radius r, a central angle of θ radians subtends an arc whose length s is

$$\boxed{s = r\theta \qquad (4)}$$

Finding the Length of an Arc of a Circle

Find the length of the arc of a circle of radius 2 meters subtended by a central angle of 0.25 radian.

Angle Measure



The area of a sector is a fraction of the area of the circle. This area is proportional to the angle of the sector. In other words, the bigger the angle, the larger is the area of the sector.

Formula

$$\frac{\theta}{2} r^2 \text{ (in radians)}$$

$$\frac{\theta}{360} \pi r^2 \text{ (in degrees)}$$

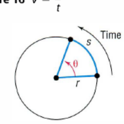
Angle Measure

Find the Linear Speed of an Object Traveling in Circular Motion

Suppose that an object moves around a circle of radius r at a constant speed. If s is the distance traveled in time t around this circle, then the **linear speed** v of the object is defined as

$$\boxed{v = \frac{s}{t} \qquad (9)}$$

Figure 16 $v = \frac{s}{t}$



As this object travels around the circle, suppose that θ (measured in radians) is the central angle swept out in time t. See Figure 16.

Angle Measure

The **angular speed** ω (the Greek letter omega) of this object is the angle θ (measured in radians) swept out, divided by the elapsed time t; that is,

$$\omega = \frac{\theta}{t} \qquad \textbf{(10)}$$

Angular speed is the way the turning rate of an engine is described. For example, an engine idling at 900 rpm (revolutions per minute) is one that rotates at an angular speed of

$$900\,\frac{\text{revolutions}}{\text{minute}} = 900\,\frac{\cancel{\text{revolutions}}}{\text{minute}}\cdot 2\pi\,\frac{\text{radians}}{\cancel{\text{revolution}}} = 1800\pi\,\frac{\text{radians}}{\text{minute}}$$

Angle Measure

There is an important relationship between linear speed and angular speed:

$$\text{linear speed} = v = \underset{(9)}{\frac{s}{t}} = \underset{s\,=\,r\theta}{\frac{r\theta}{t}} = r\underset{(10)}{\left(\frac{\theta}{t}\right)} = r\cdot\omega$$

So,

$$v = r\omega \qquad \textbf{(11)}$$

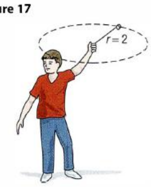
where ω is measured in radians per unit time.

Angle Measure

Finding Linear Speed

A child is spinning a rock at the end of a 2-foot rope at the rate of 180 revolutions per minute (rpm). Find the linear speed of the rock when it is released.

Figure 17



The linear speed of the rock when it is released is 2262 ft/min ≈ 25.7 mi/hr.

Exercise:

Motion on a Circle An object is traveling around a circle with a radius of 2 meters. If in 20 seconds the object travels 5 meters, what is its angular speed? What is its linear speed?

Exercise:

An automobile has a diameter of 48 inches. How many revolutions will the wheel make as the automobile travels 2 miles? (1mile=5280 ft)

Exercise:

An object is moving along a circular path of radius 16.2 meters with an angular velocity of 3.2 rad/sec. Find the linear velocity(km/hr) of the object. Note: 1km=1000m

Trigonometry

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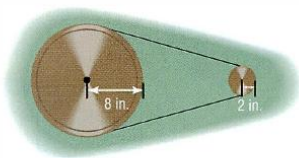
Exercise:

Pulleys Two pulleys, one with radius 2 inches and the other with radius 8 inches, are connected by a belt. (See the figure.) If the 2-inch pulley is caused to rotate at 3 revolutions per minute, determine the revolutions per minute of the 8-inch pulley.

[**Hint:** The linear speeds of the pulleys are the same; both equal the speed of the belt.]



Trigonometry

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Exercise:

Distance between Cities Memphis, Tennessee, is due north of New Orleans, Louisiana. Find the distance between Memphis (35°9′ north latitude) and New Orleans (29°57′ north latitude). Assume that the radius of Earth is 3960 miles.