TRIGONOMETRY

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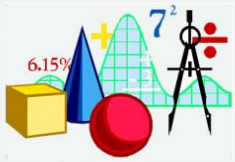
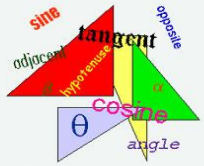
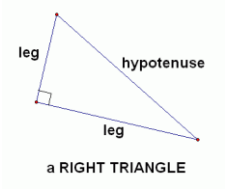




Right Triangle Trigonometry



- Trigonometric Ratios
- Trigonometric Ratios for Special Angles
- Solution of Right Triangles
- Angle of Elevation/Depression
- Bearing and Course

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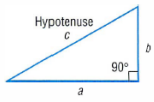
Prepared By: Albert Griño Jr

Right Triangle Trigonometry

A triangle in which one angle is a right angle (90 degrees) is called a **right triangle**.

Figure 18



The **Pythagorean Theorem** states that the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$c^2 = a^2 + b^2$$

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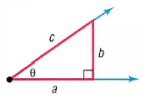
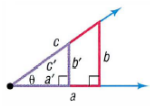
Right Triangle Trigonometry

Now, suppose that θ is an **acute angle;** that is, $0° < \theta < 90°$ (if θ is measured in degrees) and $0 < \theta < \frac{\pi}{2}$ (if θ is measured in radians).



(b) Right triangle

$$\frac{b}{c}, \frac{a}{c}, \frac{b}{a}, \frac{c}{b}, \frac{c}{a}, \frac{a}{b}$$



(c) Similar triangles

$$\frac{b}{c} = \frac{b'}{c'} \quad \frac{a}{c} = \frac{a'}{c'} \quad \frac{b}{a} = \frac{b'}{a'} \quad \frac{c}{b} = \frac{c'}{b'} \quad \frac{c}{a} = \frac{c'}{a'} \quad \frac{a}{b} = \frac{a'}{b'}$$

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Right Triangle Trigonometry

Because the ratios depend only on the angle θ and not on the triangle itself, we give each ratio a name that involves θ: sine of θ, cosine of θ, tangent of θ, cosecant of θ, secant of θ, and cotangent of θ.

DEFINITION

The six ratios of a right triangle are called **trigonometric functions of acute angles** and are defined as follows:

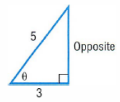
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{c} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{c} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{b}{a}$$
$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{c}{b} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{c}{a} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{a}{b} \qquad (1)$$

Right Triangle Trigonometry

Finding the Value of Trigonometric Functions

Find the value of each of the six trigonometric functions of the angle θ in Figure 21.

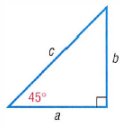
Figure 21



Right Triangle Trigonometry

Finding the Exact Values of the Trigonometric Functions of $\frac{\pi}{4} = 45°$

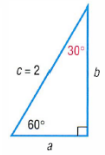
Find the exact values of the six trigonometric functions of $\frac{\pi}{4} = 45°$.



Right Triangle Trigonometry

Finding the Exact Values of the Trigonometric Functions of $\frac{\pi}{6} = 30°$ and $\frac{\pi}{3} = 60°$

Find the exact values of the six trigonometric functions of $\frac{\pi}{6} = 30°$ and $\frac{\pi}{3} = 60°$.

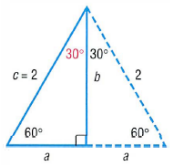


(a)

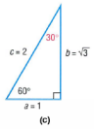
Form a right triangle in which one of the angles is $\frac{\pi}{6} = 30°$. It then follows that the third angle is $\frac{\pi}{3} = 60°$. Figure 28(a) illustrates such a triangle with hypotenuse of length 2. Our problem is to determine a and b.

Right Triangle Trigonometry

We begin by placing next to the triangle in Figure 28(a) another triangle congruent to the first, as shown in Figure 28(b).



Right Triangle Trigonometry



$$\sin 30^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2} \qquad \cos \frac{\pi}{3} = \cos 60^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} \qquad \sin \frac{\pi}{3} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \cot \frac{\pi}{3} = \cot 60^\circ = \frac{\sqrt{3}}{3}$$

$$\csc 30^\circ = \frac{1}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2 \qquad \sec \frac{\pi}{3} = \sec 60^\circ = 2$$

$$\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \qquad \csc \frac{\pi}{3} = \csc 60^\circ = \frac{2\sqrt{3}}{3}$$

$$\cot 30^\circ = \frac{1}{\tan 30^\circ} = \frac{1}{\frac{\sqrt{3}}{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} \qquad \tan \frac{\pi}{3} = \tan 60^\circ = \sqrt{3}$$

Right Triangle Trigonometry

Table 3 summarizes the information just derived for the angles $\frac{\pi}{6} = 30°, \frac{\pi}{4} = 45°$, and $\frac{\pi}{3} = 60°$.

Table 3

θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Right Triangle Trigonometry

Example 1.8.1 Each of the following is a point on the terminal side of an angle in standard position. Determine the value to four decimal places of all six trigonometric functions of the angle.

a) $(7,12)$
b) $(9,8)$
c) $(4,\sqrt{5})$
d) $(7.4,11.8)$

Example 1.8.2 Find the other five functions of the acute angle A, given that::

a) $\tan A = \frac{3}{4}$
b) $\sec A = \sqrt{2}$
c) $\cos A = \frac{\sqrt{3}}{2}$
d) $\sin A = \frac{2mn}{m^2 + n^2}$

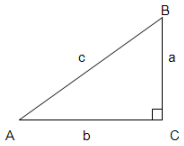
Right Triangle Trigonometry

1.9 Pythagorean Relations of the Trigonometric Functions

If θ is an acute angle of a right triangle, then by applying Pythagorean Theorem,

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

1.10 Functions of Complementary Angles



Right Triangle Trigonometry

By referring to the definitions of the trigonometric functions, for the acute angle B,

$$\sin B = \frac{b}{c} \qquad \csc B = \frac{c}{b}$$
$$\cos B = \frac{a}{c} \qquad \sec B = \frac{c}{a}$$
$$\tan B = \frac{b}{a} \qquad \cot B = \frac{a}{b}$$

Comparing these formulas with the formulas in 11.7, and making use of the fact that A and B are complementary angles (i.e. A + B = 90°), then:

sin B = sin (90° - A) = cos A
cos B = cos(90° - A) = sin A
tan B = tan (90°- A) = cot A
cot B = cot (90° - A) = tan A
sec B = sec (90° - A) = csc A
csc B = csc (90° - A) = sec A

Any function of the complement of an angle is equal to the c-function of the angle.

Right Triangle Trigonometry

Example 1.10.1 Express each of the following in terms of its cofunction:

a) sin 76°
b) csc 80°35'32"
c) cos 39°23'
d) tan (A + 15°)

Example 1.10.2 Determine the value of β that will satisfy the following:

a) $\tan(-5\beta) = \cot 14\beta$
b) $\csc(6\beta + 12°) = \sec 7\beta$
c) $\sec(4\beta - 5°) = \frac{1}{\sin(3\beta - 10°)}$
d) $\sin\left(\frac{\beta}{2} + 12°\right) = \cos\left(\frac{3\beta}{2} - 8°\right)$

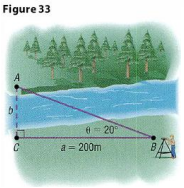
Right Triangle Trigonometry

Model and Solve Applied Problems Involving Right Triangles

Right triangles can be used to model many types of situations

Finding the Width of a River

Figure 33



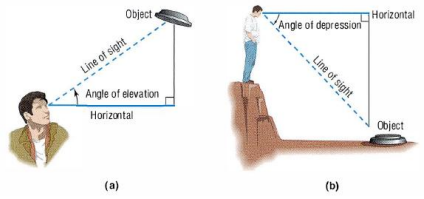
A surveyor can measure the width of a river by setting up a transit* at a point C on one side of the river and taking a sighting of a point A on the other side. Refer to Figure 33. After turning through an angle of 90° at C, the surveyor walks a distance of 200 meters to point B. Using the transit at B, the angle θ is measured and found to be 20°. What is the width of the river rounded to the nearest meter?

The width of the river is 73 meters, rounded to the nearest meter.

Right Triangle Trigonometry

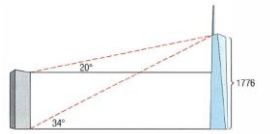
Vertical heights can sometimes be measured using either the *angle of elevation* or the *angle of depression*. If a person is looking up at an object, the acute angle measured from the horizontal to a line of sight to the object is called the **angle of elevation.** See Figure 34(a).

Figure 34



Right Triangle Trigonometry

The Freedom Tower The Freedom Tower is to be the centerpiece of the rebuilding of the World Trade Center in New York City. The tower will be 1776 feet tall (not including a broadcast antenna). The angle of elevation from the base of an office building to the top of the tower is 34°. The angle of elevation from the helipad on the roof of the office building to the top of the tower is 20°.



(a) How far away is the office building from the Freedom Tower? Assume the side of the tower is vertical. Round to the nearest foot.

(b) How tall is the office building? Round to the nearest foot.

Right Triangle Trigonometry

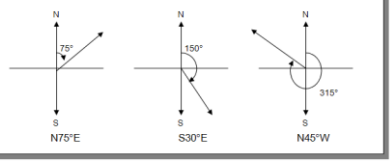
Definition 2.3.1 Bearing

Bearing of the point is the clockwise angle from north to the line of sight to appoint of reference.

Definition: 2.3.2 Course

In air or sea navigation, the angle measured clockwise from north to the line of travel is the **course** of the plane or the ship.

Course of 75°, 150° and 315° are illustrated below.

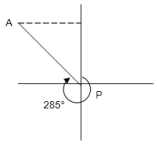


Right Triangle Trigonometry

The bearing from A to C is S 52°E. The bearing from A to B is N 84°E. The bearing from B to C is S38°W. A plane flying at 250 mph takes 2.4 hours to go from A to B. Find the distance from B to C.

Right Triangle Trigonometry

Example 2.3.2 From port Ilocandia to Camp Airstrip a plane flies on a course of 285°. If Camp Airstrip is 1525 km further north than Port Ilocandia, find the distance from Port Ilocandia to Camp Airstrip.

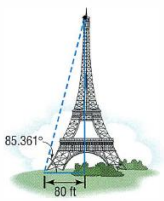


Right Triangle Trigonometry

A flagpole broken by the wind forms a right triangle with the ground. If the broken portion makes an angle of 58° with the ground and the distance from the tip of the pole to the foot is 45 ft, what is the original height of the pole?

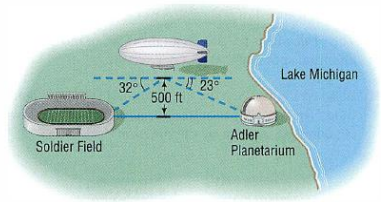
Right Triangle Trigonometry

The Eiffel Tower The tallest tower built before the era of television masts, the Eiffel Tower was completed on March 31, 1889. Find the height of the Eiffel Tower (before a television mast was added to the top) using the information given in the illustration.

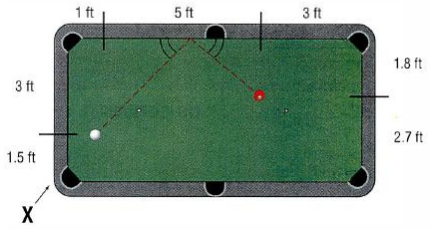


Right Triangle Trigonometry

Finding the Distance between Two Objects A blimp, suspended in the air at a height of 500 feet, lies directly over a line from Soldier Field to the Adler Planetarium on Lake Michigan (see the figure). If the angle of depression from the blimp to the stadium is 32° and from the blimp to the planetarium is 23°, find the distance between Soldier Field and the Adler Planetarium.



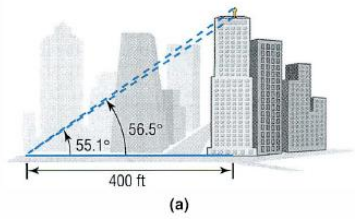
Calculating Pool Shots A Pool player located at **X** wants to shoot the white ball off the top cushion and hit the red ball dead center. He knows from physics that the white ball will come off a cushion at the same angle as it hits a cushion. Where on the top cushion should he hit the white ball?

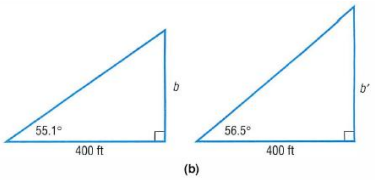


Finding the Height of a Statue on a Building

Adorning the top of the Board of Trade building in Chicago is a statue of Ceres, the Roman goddess of wheat. From street level, two observations are taken 400 feet from the center of the building. The angle of elevation to the base of the statue is found to be 55.1° and the angle of elevation to the top of the statue is 56.5°. See Figure 36(a). What is the height of the statue?

Figure 36





The height of the statue is approximately 604.33 − 573.39 = 30.94 feet ≈ 31 feet.