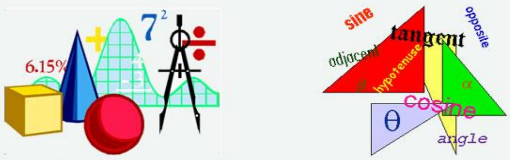
TRIGONOMETRY

[ma+(hema)+ic]s



Trigonometry

Mapua Institute of Technology

Prepared By: Albert Griño Jr

Exponential & logarithmic functions

Exponential Function
- Definition, Graph, Natural

Logarithmic Functions
- Definition
- Logarithmic and Exponential Form
- Properties, Graph
- Common/Briggsian
- Natural/Naplerian
- Laws of Logarithm
- Expanding and Combining
- Change of Base Formula

Trigonometry

Mapua Institute of Technology

Prepared By: Albert Griño Jr

Exponential & logarithmic functions

Laws of Exponents

If s, t, a, and b are real numbers with $a > 0$ and $b > 0$, then

$$a^s \cdot a^t = a^{s+t} \qquad (a^s)^t = a^{st} \qquad (ab)^s = a^s \cdot b^s$$
$$1^s = 1 \qquad a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s \qquad a^0 = 1 \qquad \textbf{(1)}$$

Exercise: Simplify the following:

a) $(3x^4)^2$ ans. $9x^8$

b) $(32x^{10})^{\frac{2}{5}}$ ans. $4x^4$

Trigonometry

Mapua Institute of Technology

Prepared By: Albert Griño Jr

Exponential & logarithmic functions

To solve exponential equations, the following property can be used:

$b^m = b^n$ *if and only if* $m = n$ *and* $b > 0$, $b \neq 1$

Exercise: Solve for x:

a) $3^{x+4} = 3^{2x-1}$ ans. 5

b) $4^{2x+1} = 8^{x+5}$ ans. 13

c) $\left(\frac{1}{2}\right)^{x+2} = 16^{-x}$ ans. $\frac{2}{3}$

d) $3^{\sqrt{x}+4} = 27^{\sqrt{x}}$ ans. 4

Trigonometry

Mapua Institute of Technology

Prepared By: Albert Griño Jr

Exponential & logarithmic functions

DEFINITION

An **exponential function** is a function of the form

$$f(x) = a^x$$

where a is a positive real number ($a > 0$) and $a \neq 1$. The domain of f is the set of all real numbers.

The following are examples of exponential functions:

a) $y = 3^x$ b) $f(x) = 6^x$ c) $f(x) = 2^x$

Trigonometry

Mapua Institute of Technology

Prepared By: Albert Griño Jr

Exponential & logarithmic functions

Evaluate the function $y = 4^x$ at the given values of x.

a) x = 2 b) x = -3 c) x = 0

Trigonometry

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Exponential & logarithmic functions

The following display summarizes the information that we have about $f(x) = a^x, a > 1$.

Properties of the Exponential Function $f(x) = a^x$, $a > 1$

1. The domain is the set of all real numbers; the range is the set of positive real numbers.
2. There are no x-intercepts; the y-intercept is 1.
3. The x-axis ($y = 0$) is a horizontal asymptote as $x \to -\infty$.
4. $f(x) = a^x, a > 1$, is an increasing function and is one-to-one.
5. The graph of f contains the points $\left(-1, \frac{1}{a}\right)$, $(0, 1)$, and $(1, a)$.
6. The graph of f is smooth and continuous, with no corners or gaps.

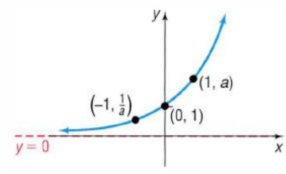
Trigonometry

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Exponential & logarithmic functions

Figure 21

$f(x) = a^x, a > 1$



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
y-intercept: (0,1)
x-intercept: none
Horizontal asymptote: x-axis

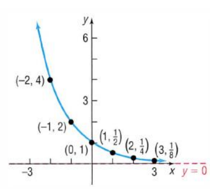
Trigonometry

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Exponential & logarithmic functions

Graph the exponential function: $f(x) = \left(\frac{1}{2}\right)^x$

x	$f(x) = \left(\frac{1}{2}\right)^x$
−10	$\left(\frac{1}{2}\right)^{-10} = 1024$
−3	$\left(\frac{1}{2}\right)^{-3} = 8$
−2	$\left(\frac{1}{2}\right)^{-2} = 4$
−1	$\left(\frac{1}{2}\right)^{-1} = 2$
0	$\left(\frac{1}{2}\right)^{0} = 1$
1	$\left(\frac{1}{2}\right)^{1} = \frac{1}{2}$
2	$\left(\frac{1}{2}\right)^{2} = \frac{1}{4}$
3	$\left(\frac{1}{2}\right)^{3} = \frac{1}{8}$
10	$\left(\frac{1}{2}\right)^{10} \approx 0.00098$



Trigonometry

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Exponential & logarithmic functions

Properties of the Exponential Function $f(x) = a^x$, $0 < a < 1$

1. The domain is the set of all real numbers; the range is the set of positive real numbers.
2. There are no x-intercepts; the y-intercept is 1.
3. The x-axis ($y = 0$) is a horizontal asymptote as $x \to \infty$.
4. $f(x) = a^x, 0 < a < 1$, is a decreasing function and is one-to-one.
5. The graph of f contains the points $\left(-1, \frac{1}{a}\right)$, $(0, 1)$, and $(1, a)$.
6. The graph of f is smooth and continuous, with no corners or gaps.

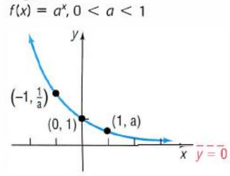
Trigonometry

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Exponential & logarithmic functions

Figure 25

$f(x) = a^x, 0 < a < 1$



Trigonometry

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Exponential & logarithmic functions

DEFINITION

The **logarithmic function to the base a,** where $a > 0$ and $a \neq 1$, is denoted by $y = \log_a x$ (read as "y is the logarithm to the base a of x") and is defined by

$$\boxed{y = \log_a x \quad \text{if and only if} \quad x = a^y}$$

The domain of the logarithmic function $y = \log_a x$ is $x > 0$.

Example:

Express in exponential form:

a) $\log_4 64 = 3$ ans. $4^3 = 64$

b) $\log_2 32 = 5$ ans. $2^5 = 32$

c) $\log_{\frac{1}{2}} 16 = -4$ ans. $\left(\frac{1}{2}\right)^{-4} = 16$

d) $\log_5 0.04 = -2$ ans. $5^{-2} = 0.04$

Trigonometry

Mapua Institute of Technology

Prepared By: Albert Griño Jr

Exponential & logarithmic functions

Express in logarithmic form:

a) $6^3 = 216$ ans. $\log_6 216 = 3$

b) $16^{\frac{5}{4}} = 32$ ans. $\log_{16} 32 = \frac{5}{4}$

c) $27^{-\frac{4}{3}} = \frac{1}{81}$ ans. $\log_{27} \frac{1}{81} = -\frac{4}{3}$

d) $49^{\frac{1}{2}} = 7$ ans. $\log_{49} 7 = \frac{1}{2}$

Trigonometry

Exponential & logarithmic functions

Solve the following:

a) $\log_x \frac{8}{27} = 3$ ans. $x = \frac{2}{3}$

b) $\log_4 x = \frac{5}{2}$ ans. $x = 32$

c) $\log_{\frac{2}{5}} \frac{4}{25} = \frac{x}{4}$ ans. $x = 8$

d) $\log_{\frac{3}{8}} \frac{9}{64} = 2x + 2$ ans. $x = 0$

Trigonometry

Exponential & logarithmic functions

Properties of the Logarithmic Function $f(x) = \log_a x$

1. The domain is the set of positive real numbers; the range is the set of all real numbers.
2. The x-intercept of the graph is 1. There is no y-intercept.
3. The y-axis ($x = 0$) is a vertical asymptote of the graph.
4. A logarithmic function is decreasing if $0 < a < 1$ and increasing if $a > 1$.
5. The graph of f contains the points $(1, 0)$, $(a, 1)$, and $\left(\frac{1}{a}, -1\right)$.
6. The graph is smooth and continuous, with no corners or gaps.

Trigonometry

Exponential & logarithmic functions

Properties of Logarithms

If *M*, *N* and *b* (*b* ≠ *1*) are positive real numbers, and *r* is any real number, then

$$\log_b MN = \log_b M + \log_b N$$
$$\log_b \frac{M}{N} = \log_b M - \log_b N$$
$$\log_b N^r = r \log_b N$$
$$\log_b b = 1$$
$$\log_b 1 = 0$$

Remark:

Since logarithmic function is continuous and one-to-one, every positive real number has a unique logarithm to the base *b*. Therefore,

$\log_b N = \log_b M$ *if and only if* $N = M$

Trigonometry

Exponential & logarithmic functions

Express the following in expanded form:

a) $\log_3 xyz$

b) $\log_6 (3x)^2$

c) $\log_4 \frac{mnp}{q^2}$

d) $\log_2 x^4 y^3$

e) $\log_5 \sqrt{\frac{x^2}{y^5 p^3}}$

Express as a single logarithm:

a) $\log_2(x+2) + \log_2 x - \log_2 3$

b) $2\log_a m - 3\log_a n$

c) $(\log_5 2 + 3\log_5 m + 2\log_5 n) - \frac{2}{3}\log_5 p$

Trigonometry

Exponential & logarithmic functions

Solve the following:

a) $\log_5(x^2 - 25) - \log_5(x - 5) = 2$ ans. 20

b) $\log_2(x+6) - \log_2(x+2) = \log_2 x$ ans. 2

c) $\log_7(x-5) + \log_7(x+1) = 1$ ans. 6

Trigonometry

Exponential & logarithmic functions

Natural Logarithm

Natural logarithms are to the base *e*. The symbol *ln x* is used for natural logarithms.

$$\ln x = \log_e x$$

Exercise:

Solve for x :

a) $\ln e^{\ln x} - \ln(x-3) = \ln 2$ ans. 6

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Trigonometry

Exponential & logarithmic functions

Change-of- Base Formula

$$\log_b x = \frac{\log_a x}{\log_a b}$$

The reference base is a.

Exercise:

Use common logarithms to find each logarithm to four decimal places:

a) $\log_5 65$

b) $\log_2 0.1$

c) $\log_5 17$

d) $\log_{0.8} 70$

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Trigonometry

Exponential & logarithmic functions

Applications: Exponential and Logarithmic Equations

Exponential and logarithmic equations can be used to describe exponential growth and decay, learning curves, logistic growth, etc.

Example #1

The growth rate for a particular bacterial culture can be calculated using the formula $B = 900(2)^{\frac{t}{50}}$, where *B* is the number of bacteria and *t* is the elapsed time in hours. How many bacteria will be present after 5 hours?
Ans. 965

How many hours will it take for there to be 18,000 bacteria present in the culture in example (a)? ans. 216

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Trigonometry

Exponential & logarithmic functions

Example #2

A fossil that originally contained 100 mg of carbon-14 now contains 75 mg of the isotope. Determine the approximate age of the fossil, to the nearest 100 years, if the half-life of carbon-14 is 5570 years. (Use the Exponential Decay Formula)

$$A = A_o 2^{-\frac{t}{k}}$$

where: A = present amount of isotope
A_o = original amount of isotope
t = time it takes to reduce original amount of isotope to present amount
k = Half-life of the isotope

ans. 2300

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Trigonometry

Exponential & logarithmic functions

Example #3

In a town of 15,000 people, the spread of a rumor that the local transit company would go on strike was such that *t* hours after the rumor started, *f(t)* persons heard the rumor, where experience over time has shown that

$$f(t) = \frac{15{,}000}{1 + 7499e^{-0.8t}}$$

1. How many people started the rumor? Ans. 2
2. How many people heard the rumor after 5 hours? Ans. 108

Mapua Institute of Technology
Prepared By: Albert Griño Jr