Lesson 35
Solution of a Differential Equation by Laplace Transforms
(Initial Value Problems)

Specific Objectives: At the end of the lesson, the students are expected to:

- Evaluate differential equations by Laplace transforms

The Laplace Operator will transform a linear D.E. with constant coefficients into an algebraic equation in the transformed function (Raindille + Bedient).
This method is easily used if the initial conditions are given.

Ex. Solve $y''-3y'+2y=e^{3t}$; $y(0)=y'(0)=0$

Find the laplace transforms of both sides.

$$L\{y''\}-3L\{y'\}+2L\{y\}=L\{e^{3t}\}$$
$$s^2L\{y\}-[sy(0)+y'(0)]-3[sL\{y\}-y(0)]+2L\{y\}=\frac{1}{s-3}$$
$$s^2L\{y\}-s(0)-0-3sL\{y\}+3(0)+2L\{y\}=\frac{1}{s-3}$$
$$(s^2-3s+2)L\{y\}=\frac{1}{s-3}$$
$$L\{y\}=\frac{1}{(s-3)(s^2-3s+2)}$$
$$y=L^{-1}\{\frac{1}{(s-3)(s^2-3s+2)}\}$$

By Partial Fractions:

$$\frac{1}{(s-3)(s-2)(s-1)}=\frac{A}{s-3}+\frac{B}{s-2}+\frac{C}{s-1}$$

$$1=A(s-2)(s-1)+B(s-3)(s-1)+C(s-3)(s-2)$$

If s=3, $A=\frac{1}{2}$

If s=2, B=-1

If s=1, C=$\frac{1}{2}$

Thus,

$$y = L^{-1}\{\frac{-1/2}{s-3} - \frac{1}{s-2} + \frac{1/2}{s-1}\}$$

$$\underline{y = \frac{1}{2}e^{3t} - e^{2t} + \frac{1}{2}e^{t}}$$

Seatwork: Perform the indicated operations:

1. $y'' + 2y' + 5y = \sin(3t)$; $y(0) = 1, y'(0) = -1$

Ans: $y(t) = e^{-t}\cos(2t) + \frac{3}{26}\left(-\cos(3t) - \frac{2}{3}\sin(3t) + e^{-2}\cos(2t) + \frac{3}{2}e^{-t}\sin(2t)\right)$

2. $y' + 3y = 10\sin t$; $y(0) = 0$

Ans: $e^{-3t} - \cos t + 3\sin t$

Homework: Perform the indicated operations:

1. $y'' - y = t$; $y(0) = 1, y'(0) = 1$

2. $y'' + 5y' + 10y = t$; $y(0) = y'(0) = 1$