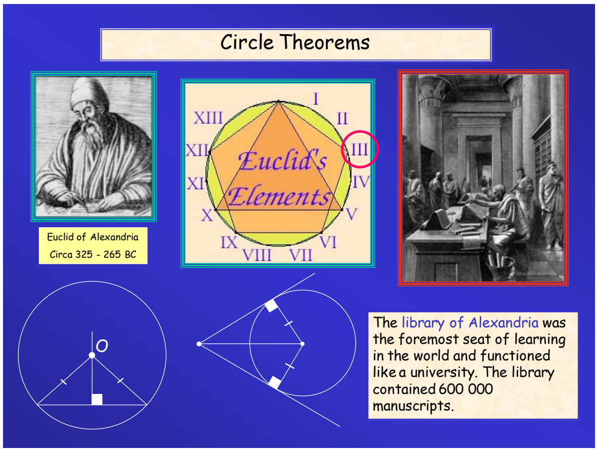
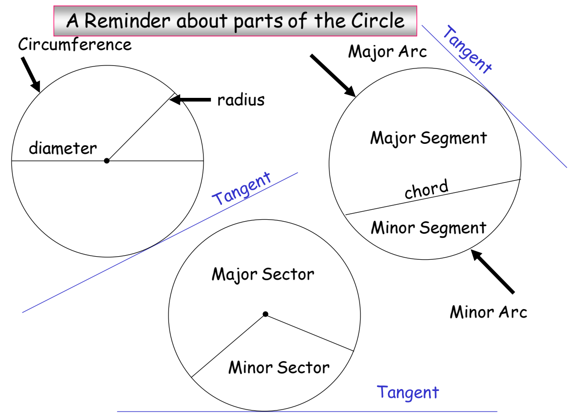
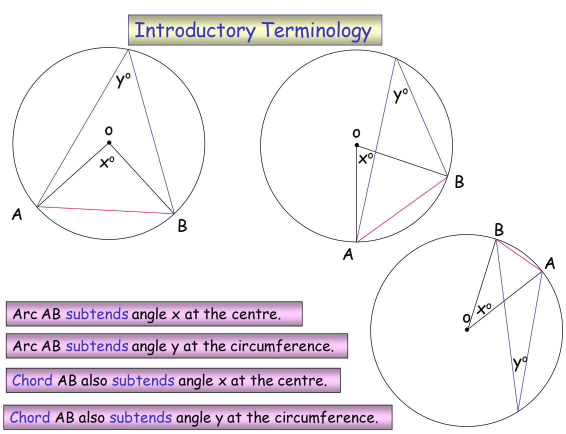
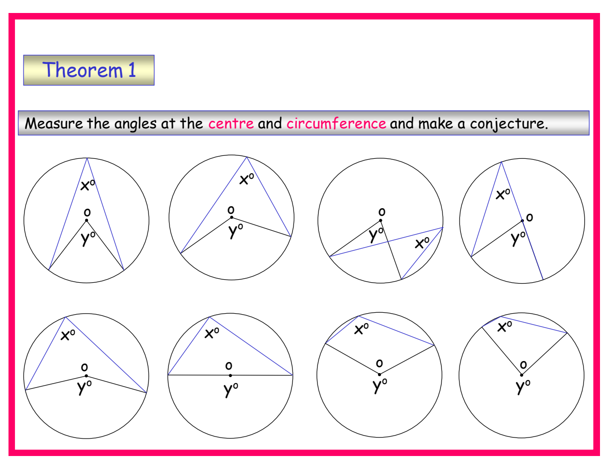

Circle Theorems
Euclid of Alexandria
Circa 325 - 265 BC
Euclid's Elements
I
II
III
IV
V
VI
VII
VIII
IX
X
XI
XII
XIII
O
The library of Alexandria was the foremost seat of learning in the world and functioned like a university. The library contained 600 000 manuscripts.


A Reminder about parts of the Circle
Circumference
radius
diameter
Tangent
Major Arc
Tangent
Major Segment
chord
Minor Segment
Minor Arc
Major Sector
Minor Sector
Tangent

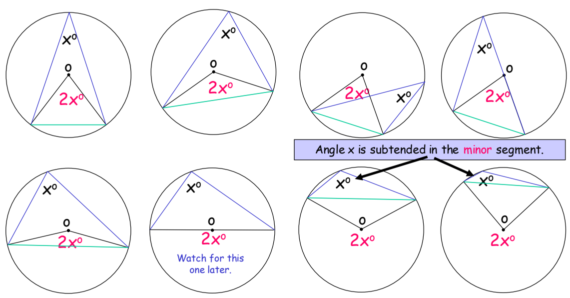

Introductory Terminology
y°
o
x°
A
B
Arc AB subtends angle x at the centre.
Arc AB subtends angle y at the circumference.
Chord AB also subtends angle x at the centre.
Chord AB also subtends angle y at the circumference.


Theorem 1
Measure the angles at the centre and circumference and make a conjecture.
x°
o
y°

Theorem 1

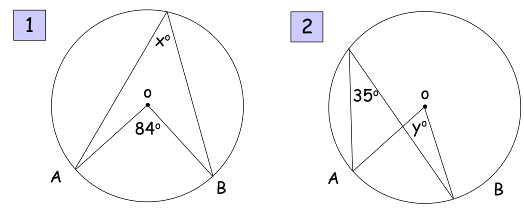
The angle subtended at the centre of a circle (by an arc or chord) is twice the angle subtended at the circumference by the same arc or chord. *(angle at centre)*

Measure the angles at the centre and circumference and make a conjecture.



Example Questions

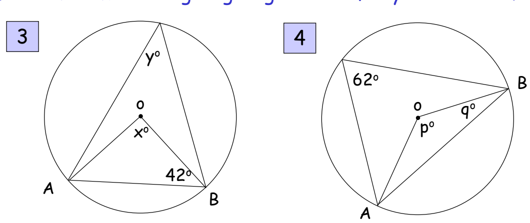
Find the unknown angles giving reasons for your answers.



angle x =	42° (Angle at the centre).
angle y =	70°(Angle at the centre)

Example Questions

Find the unknown angles giving reasons for your answers.

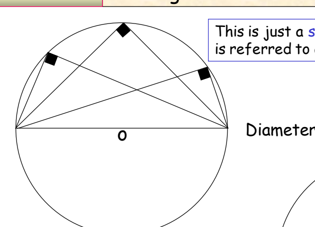


angle x =	(180 – 2 x 42) = 96° (Isos triangle/angle sum triangle).
angle y =	48° (Angle at the centre)

angle p =	124° (Angle at the centre)
angle q =	(180 – 124)/2 = 28° (Isos triangle/angle sum triangle).

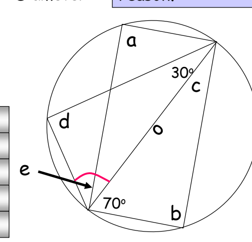
Theorem 2 The angle in a semi-circle is a right angle.

This is just a special case of Theorem 1 and is referred to as a theorem for convenience.



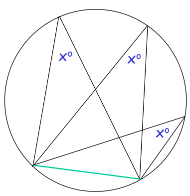
Find the unknown angles below stating a reason.

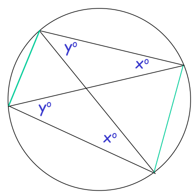
angle a =	90° angle in a semi-circle
angle b =	90° angle in a semi-circle
angle c =	20° angle sum triangle
angle d =	90° angle in a semi-circle
angle e =	60° angle sum triangle



Theorem 3

Angles subtended by an arc or chord in the same segment are equal.

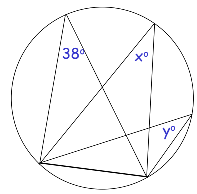


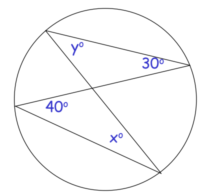


Theorem 3

Angles subtended by an arc or chord in the same segment are equal.

Find the unknown angles in each case





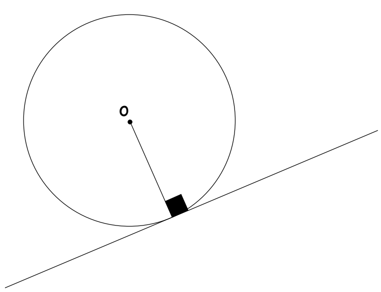
Angle x = angle y = 38°

Angle x = 30°

Angle y = 40°

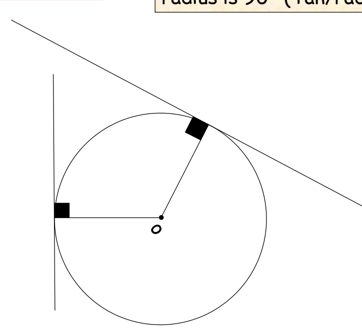
Theorem 4

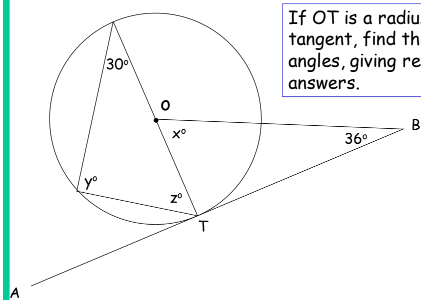
The angle between a tangent and a radius is 90°. (Tan/rad)



Theorem 4

The angle between a tangent and a radius is 90°. (Tan/rad)



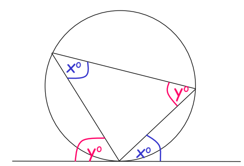


If OT is a radius and AB is a tangent, find the unknown angles, giving reasons for your answers.

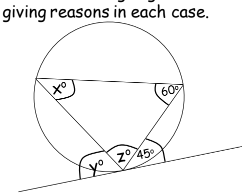
angle x =	180 – (90 + 36) = 54° Tan/rad and angle sum of triangle.
angle y =	90° angle in a semi-circle
angle z =	60° angle sum triangle

Theorem 5 The Alternate Segment Theorem.

The angle between a tangent and a chord through the point of contact is equal to the angle subtended by that chord in the alternate segment.



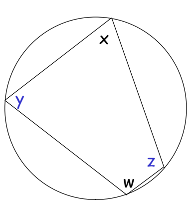
Find the missing angles below giving reasons in each case.

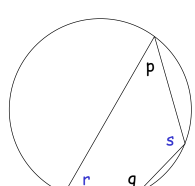


angle x =	45° (Alt Seg)
angle y =	60° (Alt Seg)
angle z =	75° angle sum triangle

Theorem 6 Cyclic Quadrilateral Theorem.

The opposite angles of a cyclic quadrilateral are supplementary. (They sum to 180°)





Angles x + w = 180°

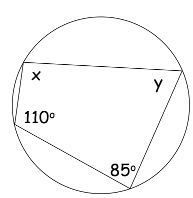
Angles y + z = 180°

Angles p + q = 180°

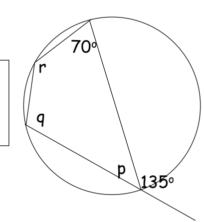
Angles r + s = 180°

Theorem 6 Cyclic Quadrilateral Theorem.

The opposite angles of a cyclic quadrilateral are supplementary. (They sum to 180°)



Find the missing angles below given reasons in each case.

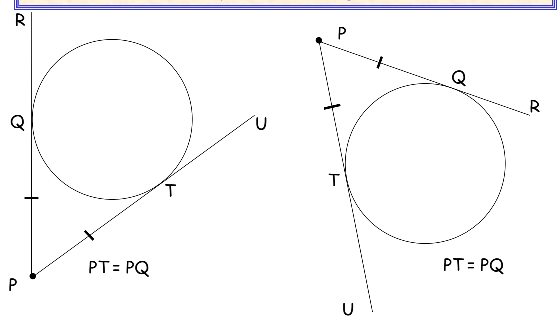


angle x =	180 – 85 = 95° (cyclic quad)
angle y =	180 – 110 = 70° (cyclic quad)

angle p =	180 – 135 = 45° (straight line)
angle q =	180 – 70 = 110° (cyclic quad)
angle r =	180 – 45 = 135° (cyclic quad)

Theorem 7 — Two Tangent Theorem.

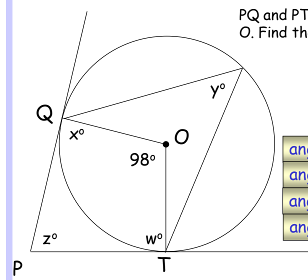
From any point outside a circle only two tangents can be drawn and they are equal in length.



Theorem 7 — Two Tangent Theorem.

From any point outside a circle only two tangents can be drawn and they are equal in length.

PQ and PT are tangents to a circle with centre O. Find the unknown angles giving reasons.

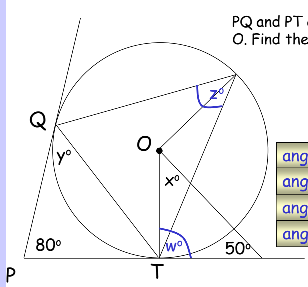


angle w =	90° (tan/rad)
angle x =	90° (tan/rad)
angle y =	49° (angle at centre)
angle z =	360° – 278 = 82° (quadrilateral)

Theorem 7 — Two Tangent Theorem.

From any point outside a circle only two tangents can be drawn and they are equal in length.

PQ and PT are tangents to a circle with centre O. Find the unknown angles giving reasons.

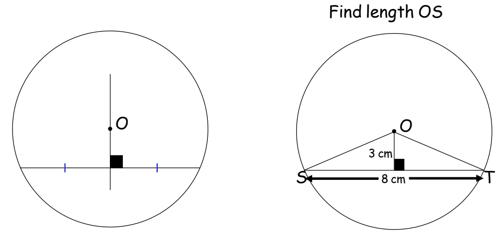


angle w =	90° (tan/rad)
angle x =	180 – 140 = 40° (angles sum tri)
angle y =	50° (isos triangle)
angle z =	50° (alt seg)

Theorem 8 — Chord Bisector Theorem.

A line drawn perpendicular to a chord and passing through the centre of a circle, bisects the chord..

Find length OS

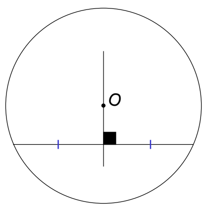


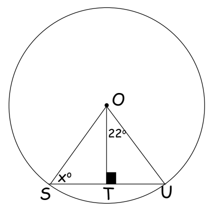
OS = 5 cm (pythag triple: 3,4,5)

Theorem 8 — Chord Bisector Theorem.

A line drawn perpendicular to a chord and passing through the centre of a circle, bisects the chord..

Find angle x

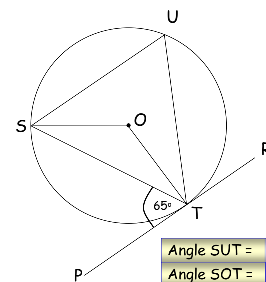




Angle SOT = 22° (symmetry/congruenncy)

Angle x = 180 – 112 = 68° (angle sum triangle)

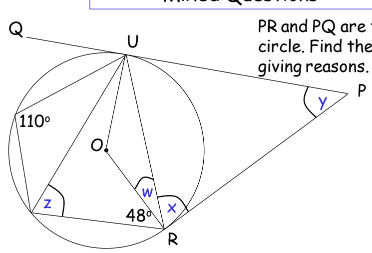
Mixed Questions



PTR is a tangent line to the circle at T. Find angles SUT, SOT, OTS and OST.

Angle SUT =	65° (Alt seg)
Angle SOT =	130° (angle at centre)
Angle OTS =	25° (tan rad)
Angle OST =	25° (isos triangle)

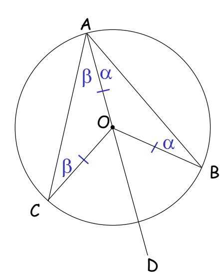
Mixed Questions



PR and PQ are tangents to the circle. Find the missing angles giving reasons.

Angle w =	22° (cyclic quad)
Angle x =	68° (tan rad)
Angle y =	44° (isos triangle)
Angle z =	68° (alt seg)

To Prove that the angle subtended by an arc or chord at the centre of a circle is twice the angle subtended at the circumference by the same arc or chord.



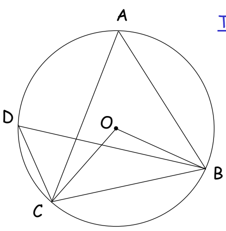
To prove that angle COB = 2 x angle CAB

- Extend AO to D
- AO = BO = CO (radii of same circle)
- Triangle AOB is isosceles(base angles equal)
- Triangle AOC is isosceles(base angles equal)
- Angle AOB = 180 - 2α (angle sum triangle)
- Angle AOC = 180 - 2β (angle sum triangle)
- Angle COB = 360 – (AOB + AOC)(<'s at point)
- Angle COB = 360 – (180 - 2α + 180 - 2β)
- Angle COB = 2α + 2β = 2(α+ β) = 2 x < CAB

Theorem 1 and 2

QED

To Prove that angles subtended by an arc or chord in the same segment are equal.



To prove that angle CAB = angle BDC

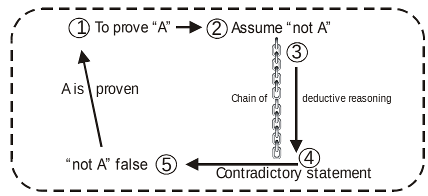
- With centre of circle O draw lines OB and OC.
- Angle COB = 2 x angle CAB (Theorem 1).
- Angle COB = 2 x angle BDC (Theorem 1).
- 2 x angle CAB = 2 x angle BDC
- Angle CAB = angle BDC

QED

Theorem 3

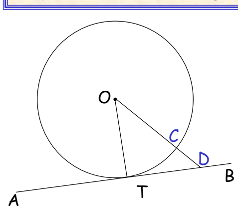
To prove that the angle between a tangent and a radius drawn to the point of contact is a right angle.

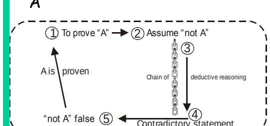
The type of proof that follows is a little different and is known as "Reducto ad absurdum" It was first exploited with great success by ancient Greek mathematicians. The idea is to assume that the premise is not true and then apply a deductive argument that leads to an absurd or contradictory statement. The contradictory nature of the statement means that the "not true" premise is false and so the premise is proven true.



To prove that the angle between a tangent and a radius drawn to the point of contact is a right angle.

To prove that OT is perpendicular to AB



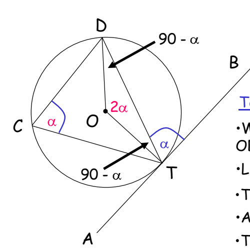


- Assume that OT is not perpendicular to AB
- Then there must be a point, D say, on AB such that OD is perpendicular to AB.
- Since ODT is a right angle then angle OTD is acute (angle sum of a triangle).
- But the greater angle is opposite the greater side therefore OT is greater than OD.
- But OT = OC (radii of the same circle) therefore OC is also greater than OD, the part greater than the whole which is impossible.
- Therefore OD is not perpendicular to AB.
- By a similar argument neither is any other straight line except OT.
- Therefore OT is perpendicular to AB.

QED

Theorem 4

To prove that the angle between a tangent and a chord through the point of contact is equal to the angle subtended by the chord in the alternate segment.



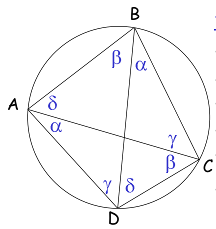
To prove that angle BTD = angle TCD

- With centre of circle O, draw straight lines OD and OT.
- Let angle DTB be denoted by α.
- Then angle DTO = 90 - α (Theorem 4 tan/rad)
- Also angle TDO = 90 - α (Isos triangle)
- Therefore angle TOD = 180 –(90 - α + 90 - α) = 2α (angle sum triangle)
- Angle TCD = α (Theorem 1 angle at the centre)
- Angle BTD = angle TCD

QED

Theorem 5

To prove that the opposite angles of a cyclic quadrilateral are supplementary (Sum to 180°).



To prove that angles A + C and B + D = 180°

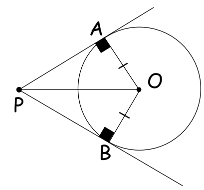
- Draw straight lines AC and BD
- Chord DC subtends equal angles α (same segment)
- Chord AD subtends equal angles β (same segment)
- Chord AB subtends equal angles γ (same segment)
- Chord BC subtends equal angles δ (same segment)
- $2(\alpha + \beta + \gamma + \delta) = 360°$ (Angle sum quadrilateral)
- $\alpha + \beta + \gamma + \delta = 180°$

Angles A + C and B + D = 180° QED

Theorem 6

α	β	γ	δ
alpha	beta	gamma	delta

To prove that the two tangents drawn from a point outside a circle are of equal length.

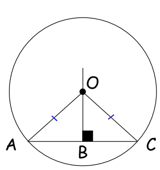


To prove that AP = BP.

- With centre of circle at O, draw straight lines OA and OB.
- OA = OB (radii of the same circle)
- Angle PAO = PBO = 90° (tangent radius).
- Draw straight line OP.
- In triangles OBP and OAP, OA = OB and OP is common to both.
- Triangles OBP and OAP are congruent (RHS)
- Therefore AP = BP. QED

Theorem 7

To prove that a line, drawn perpendicular to a chord and passing through the centre of a circle, bisects the chord.



To prove that AB = BC.

- From centre O draw straight lines OA and OC.
- In triangles OAB and OCB, OC = OA (radii of same circle) and OB is common to both.
- Angle OBA = angle OBC (angles on straight line)
- Triangles OAB and OCB are congruent (RHS)
- Therefore AB = BC QED

Theorem 8

COURSE CALENDAR AND SUMMARY SCORE SHEET - SUBJECT
Second Qtr SY 2010 - 2011

Surname: ____________ Firstname ______________ MI _____
Section: _______ Time : ________ Room: ________ Seat Number: ________ Contact Number: ____________

WEEK #	DATE	TOPICS	READINGS	HOMEWORKS	SCORES	
1		Orientation			S	A
2		Points, Lines, Planes, Angles, Polygons	pp. 1-58	Ex.2 -# 1,2,5,6, 8		15
		Triangles, Quadrilaterals & Circles		Ex.3 - #2,3,9,10		
				Ex. 4 - #3,4,5,6,7,9		
3						
4		LONG QUIZ 1	SCORE LQ1: /100			
5						
6						
7		LONG QUIZ 2	SCORE LQ2: /100			
8						
9						
10		LONG QUIZ 3	SCORE LQ3: /100			
		FINAL EXAM				

Faculty in Charge: Prof. Albert Griño Jr
Chairman, Mathematics Department: **Dr. Gerry Talisic**

Textbook: Planes & Solid Mensuration -Earnhart & Bejasa -

Mapua Institute of Technology - Mathematics Department

Student Signature: ____________________

HW Format: (Portrait- Short Bond Paper)