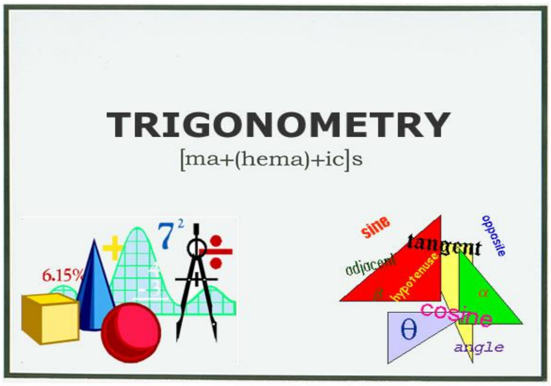

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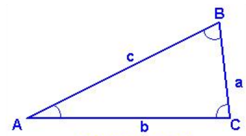
Oblique Triangle



Oblique Triangle

Solutions of Oblique Triangle
- Law of Sines (Case 1 and 2)
  - Derivation and Application
- Law of Cosines (Case 3 and 4)
  - Derivation and Application

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Oblique Triangle

To **solve an oblique triangle** means to find the lengths of its sides and the measurements of its angles. To do this, we shall need to know the length of one side* along with (i) two angles; (ii) one angle and one other side; or (iii) the other two sides. There are four possibilities to consider:

Case 1: One side and two angles are known (ASA or SAA).
Case 2: Two sides and the angle opposite one of them are known (SSA).
Case 3: Two sides and the included angle are known (SAS).
Case 4: Three sides are known (SSS).

Figure 10

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Oblique Triangle

Law of Sines

For a triangle with sides a, b, c and opposite angles A, B, C, respectively,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad (1)$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \frac{\sin A}{a} = \frac{\sin C}{c} \qquad \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$A + B + C = 180° \qquad (2)$$

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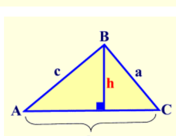
Oblique Triangle

LAW OF SINE (DERIVATION)

Triangle ABC at the right does not contain a right angle. A perpendicular is dropped from vertex B. It can now be observed that:



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Oblique Triangle

LAW OF SINE (DERIVATION)

$$\sin\sphericalangle A = \frac{h}{c} \quad \Rightarrow \quad h = c\sin\sphericalangle A$$

$$\sin\sphericalangle C = \frac{h}{a} \quad \Rightarrow \quad h = a\sin\sphericalangle C$$

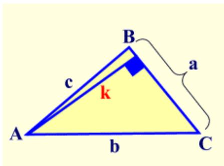
$$h = c\sin\sphericalangle A = a\sin\sphericalangle C$$

Since $c\sin\sphericalangle A = a\sin\sphericalangle C$, we have $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$.

Oblique Triangle

LAW OF SINE (DERIVATION)

Now, drop a perpendicular from vertex A. It can be observed that:



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Oblique Triangle

LAW OF SINE (DERIVATION)

$$\sin \measuredangle B = \frac{k}{c} \Rightarrow k = c \sin \measuredangle B$$

$$\sin \measuredangle C = \frac{k}{b} \Rightarrow k = b \sin \measuredangle C$$

$$k = c \sin \measuredangle B = b \sin \measuredangle C$$

Since $c \sin \measuredangle B = b \sin \measuredangle C$, we have $\frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

True for ALL triangles!

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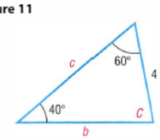
Oblique Triangle

EXAMPLE 1 Using the Law of Sines to Solve a SAA Triangle

Solve the triangle: $A = 40°, B = 60°, a = 4$

Figure 11



$b = \frac{4 \sin 60°}{\sin 40°} \approx 5.39$ $c = \frac{4 \sin 80°}{\sin 40°} \approx 6.13$

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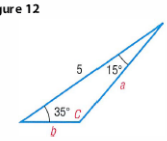
Oblique Triangle

EXAMPLE 2 Using the Law of Sines to Solve an ASA Triangle

Solve the triangle: $A = 35°, B = 15°, c = 5$

Figure 12



$a = \frac{5 \sin 35°}{\sin 130°} \approx 3.74$ $b = \frac{5 \sin 15°}{\sin 130°} \approx 1.69$

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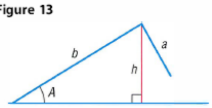
Oblique Triangle

2 Solve SSA Triangles

Case 2 (SSA), which applies to triangles for which two sides and the angle opposite one of them are known, is referred to as the **ambiguous case,** because the known information may result in one triangle, two triangles, or no triangle at all.

Figure 13



$\sin A = \frac{h}{b}$

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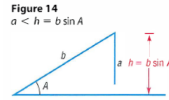
No Triangle If $a < h = b \sin A$, then side a is not sufficiently long to form a triangle. See Figure 14.

Figure 14
$a < h = b \sin A$



Two Triangles If $h = b \sin A < a$, and $a < b$ two distinct triangles can be formed from the given information. See Figure 16.

One Triangle If $a \geq b$, only one triangle can be formed. See Figure 17.

Figure 16
$b \sin A < a$ and $a < b$

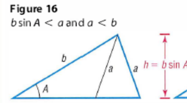
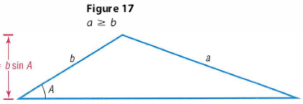


Figure 17
$a \geq b$



Oblique Triangle

EXAMPLE 3 Using the Law of Sines to Solve a SSA Triangle (One Solution)

Solve the triangle: $a = 3, b = 2, A = 40°$

Figure 18(a)

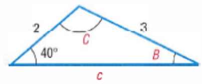
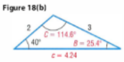


Figure 18(b)



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Oblique Triangle

EXAMPLE 4 Using the Law of Sines to Solve a SSA Triangle (Two Solutions)

Solve the triangle: $a = 6, b = 8, A = 35°$

Figure 19(a)

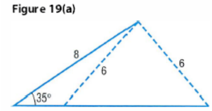
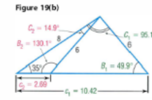


Figure 19(b)



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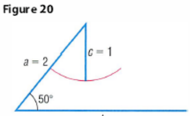
EXAMPLE 5 Using the Law of Sines to Solve a SSA Triangle (No Solution)

Solve the triangle: $a = 2, c = 1, C = 50°$

Figure 20



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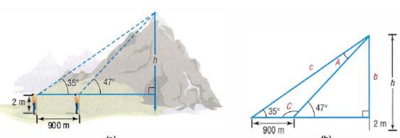
Oblique Triangle

EXAMPLE 6 Finding the Height of a Mountain

To measure the height of a mountain, a surveyor takes two sightings of the peak at a distance 900 meters apart on a direct line to the mountain.* See Figure 21(a) on page 680. The first observation results in an angle of elevation of 47°, and the second results in an angle of elevation of 35°. If the transit is 2 meters high, what is the height h of the mountain?

Figure 21



The height of the peak from ground level is approximately 1816 + 2 = 1818 meters.

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Oblique Triangle

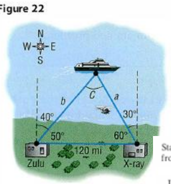
EXAMPLE 7 Rescue at Sea

Coast Guard Station Zulu is located 120 miles due west of Station X-ray. A ship at sea sends an SOS call that is received by each station. The call to Station Zulu indicates that the bearing of the ship from Zulu is N40°E (40° east of north). The call to Station X-ray indicates that the bearing of the ship from X-ray is N30°W (30° west of north).

(a) How far is each station from the ship?

(b) If a helicopter capable of flying 200 miles per hour is dispatched from the nearest station to the ship, how long will it take to reach the ship?

Figure 22



Station Zulu is about 111 miles from the ship, and Station X-ray is about 98 miles from the ship.

It will take about 29 minutes for the helicopter to reach the ship.

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Oblique Triangle

Law of Cosine

In the previous section, we used the Law of Sines to solve Case 1 (SAA or ASA) and Case 2 (SSA) of an oblique triangle. In this section, we derive the Law of Cosines and use it to solve the remaining cases, 3 and 4.

CASE 3: Two sides and the included angle are known (SAS).
CASE 4: Three sides are known (SSS).

THEOREM

Law of Cosines

For a triangle with sides a, b, c and opposite angles A, B, C, respectively,

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad (1)$$
$$b^2 = a^2 + c^2 - 2ac\cos B \qquad (2)$$
$$a^2 = b^2 + c^2 - 2bc\cos A \qquad (3)$$

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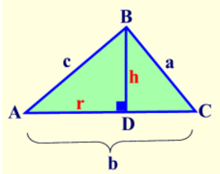
Oblique Triangle

Triangle *ABC* at the right does not contain a right angle. A perpendicular is dropped from vertex *B*. It can now be observed that:



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Oblique Triangle

$$\sin \sphericalangle A = \frac{h}{c} \Rightarrow h = c\sin \sphericalangle A$$

$$\cos \sphericalangle A = \frac{r}{c} \Rightarrow r = c\cos \sphericalangle A$$

Using the Pythagorean Theorem in triangle *CBD*, we have: $a^2 = h^2 + (b-r)^2$.
Substituting for *h* and *r* we have:

$$a^2 = (c\sin A)^2 + (b - c\cos A)^2$$
$$a^2 = c^2\sin^2 A + b^2 - 2bc\cos A + c^2\cos^2 A$$
$$a^2 = c^2\sin^2 A + b^2 - 2bc\cos A + c^2\cos^2 A$$
$$a^2 = c^2\sin^2 A + b^2 - 2bc\cos A + c^2\cos^2 A$$
$$a^2 = c^2(\sin^2 A + \cos^2 A) + b^2 - 2bc\cos A$$
$$a^2 = c^2(1) + b^2 - 2bc\cos A$$
$$a^2 = c^2 + b^2 - 2bc\cos A$$

This same process could be used to produce other lettered statements of this law.

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

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Oblique Triangle

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

A generalization of the Pythagorean Theorem. If angle C were a right angle, the cosine of angle C would be zero and the Pythagorean Theorem would result.

Trigonometry

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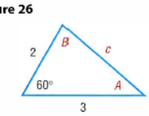
Oblique Triangle

EXAMPLE 1 Using the Law of Cosines to Solve a SAS Triangle

Solve the triangle: $a = 2,\ b = 3,\ C = 60°$

Figure 26



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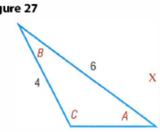
Oblique Triangle

EXAMPLE 2 Using the Law of Cosines to Solve a SSS Triangle

Solve the triangle: $a = 4, b = 3, c = 6$

Figure 27



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Oblique Triangle

EXAMPLE 3 Correcting a Navigational Error

A motorized sailboat leaves Naples, Florida, bound for Key West, 150 miles away. Maintaining a constant speed of 15 miles per hour, but encountering heavy crosswinds and strong currents, the crew finds, after 4 hours, that the sailboat is off course by 20°.

(a) How far is the sailboat from Key West at this time?
(b) Through what angle should the sailboat turn to correct its course?
(c) How much time has been added to the trip because of this? (Assume that the speed remains at 15 miles per hour.)

The sailboat is about 96 miles from Key West.

$\theta = 180° - A \approx 180° - 147.2° = 32.8°$

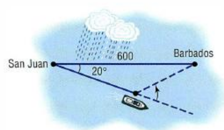
t=0.4 hr or 24 minutes

Oblique Triangle

35. **Avoiding a Tropical Storm** A cruise ship maintains an average speed of 15 knots in going from San Juan, Puerto Rico, to Barbados, West Indies, a distance of 600 nautical miles. To avoid a tropical storm, the captain heads out of San Juan in a direction of 20° off a direct heading to Barbados. The captain maintains the 15-knot speed for 10 hours, after which time the path to Barbados becomes clear of storms.

(a) Through what angle should the captain turn to head directly to Barbados?

(b) Once the turn is made, how long will it be before the ship reaches Barbados if the same 15-knot speed is maintained?



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