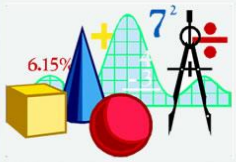
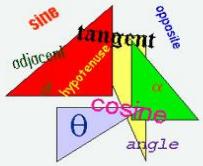
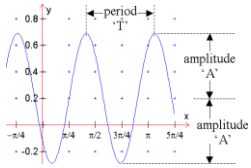
TRIGONOMETRY

[ma+(hema)+ic]s



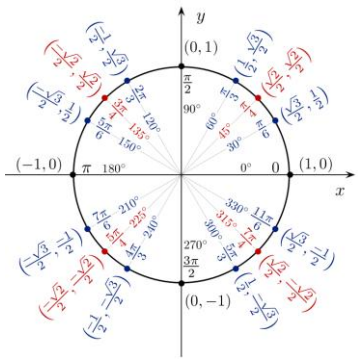


Graph of Trigonometric Functions



- Graph of Sine and Cosine Function
- Graph of Tangent and Cotangent Function
- Graph of Cosecant and Secant Function
  - Domain and Range
  - Amplitude and Period
  - Vertical and Horizontal Shift

Mapua Institute of Technology
Prepared By: Albert Griño Jr



Mapua Institute of Technology
Prepared By: Albert Griño Jr

Graph of Trigonometric Functions

DEFINITION

A function f is called **periodic** if there is a positive number p such that, whenever θ is in the domain of f, so is $\theta + p$, and

$$f(\theta + p) = f(\theta)$$

If there is a smallest such number p, this smallest value is called the **(fundamental) period** of f.

Periodic Properties

$\sin(\theta + 2\pi) = \sin\theta$	$\cos(\theta + 2\pi) = \cos\theta$	$\tan(\theta + \pi) = \tan\theta$
$\csc(\theta + 2\pi) = \csc\theta$	$\sec(\theta + 2\pi) = \sec\theta$	$\cot(\theta + \pi) = \cot\theta$

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Graph of Trigonometric Functions

THEOREM

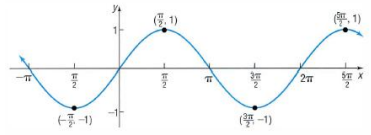
Even–Odd Properties

$\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$	$\tan(-\theta) = -\tan\theta$
$\csc(-\theta) = -\csc\theta$	$\sec(-\theta) = \sec\theta$	$\cot(-\theta) = -\cot\theta$

Trigonometry

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Graph of Trigonometric Functions



Graph of Sine Function

Properties of the Sine Function $y = \sin x$

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The sine function is an odd function, as the symmetry of the graph with respect to the origin indicates.
4. The sine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$; the y-intercept is 0.
6. The maximum value is 1 and occurs at $x = \ldots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$; the minimum value is -1 and occurs at $x = \ldots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$.

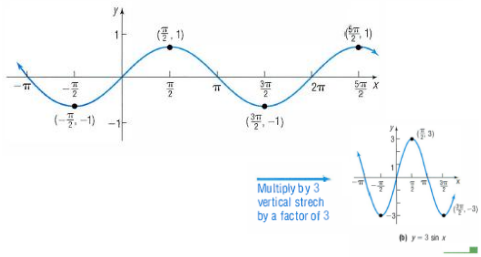
Trigonometry

Mapua Institute of Technology
Prepared By: Albert Griño Jr

Graph of Trigonometric Functions

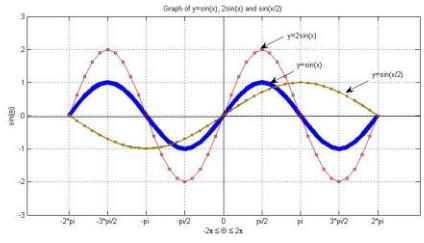
Graphing Functions of the Form $y = A\sin(\omega x)$ Using Transformations

Graph $y = 3\sin x$ using transformations.

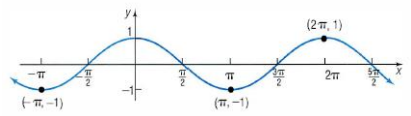


Trigonometry

Mapua Institute of Technology
Prepared By: Albert Griño Jr



Graph of Trigonometric Functions

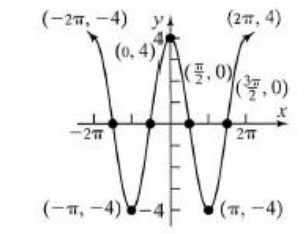


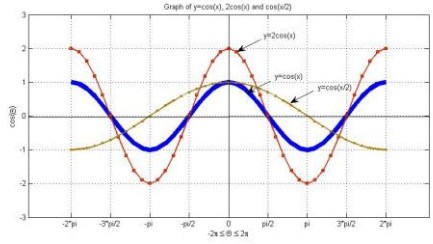
Graph of Cosine Function

Properties of the Cosine Function

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from −1 to 1, inclusive.
3. The cosine function is an even function, as the symmetry of the graph with respect to the y-axis indicates.
4. The cosine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$; the y-intercept is 1.
6. The maximum value is 1 and occurs at $x = \ldots, -2\pi, 0, 2\pi, 4\pi, 6\pi, \ldots$; the minimum value is −1 and occurs at $x = \ldots, -\pi, \pi, 3\pi, 5\pi, \ldots$.

Graph y=4 cos x





Graph of Trigonometric Functions

THEOREM

If $b > 0$, the amplitude and period of $y = A\sin(bx)$ and $y = A\cos(bx)$ are given by

$$\text{Amplitude} = |A| \qquad \text{Period} = T = \frac{2\pi}{b} \qquad (1)$$

Finding the Amplitude and Period of a Sinusoidal Function

Determine the amplitude and period of $y = 3\sin(4x)$.

Graph of Trigonometric Functions

Exercise 5.3 Determine the amplitude and the period of each function. Then sketch the graph over the interval $-2\pi \le x \le 2\pi$.

a) $y = 2\cos x$

b) $y = 3\sin 2x$

c) $y = 3\cos \frac{1}{2}x$

d) $y = -2\sin x$

Graph of Trigonometric Functions

5.4 Graph of $y = a\sin b(x + c) + d$ or $y = a\cos b(x + c) + d$

For $y = a\sin b(x + c) + d$ and $y = a\cos b(x + c) + d$

The amplitude is $|a|$. The period is $\frac{2\pi}{|b|}$

The phase shift from $y = a\sin bx$ or $y = a\cos bx$ is c units to the left if $c > 0$, and $|c|$ units to the right if $c < 0$.

The vertical shift from $y = a\sin bx$ or $y = a\cos bx$ is d units upward if $d > 0$, and $|d|$ units downward if $d < 0$.

Remarks:

1. The distance by which the graph is shifted to the right or to the left is called **phase shift.**
2. The distance by which the graph is shifted up or down is called the **vertical shift.**

Graph of Trigonometric Functions

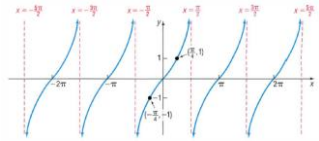
Example 5.4 Determine the amplitude, period, phase shift, and vertical shift of each function. Sketch the graph over the interval $0 \le x \le 4\pi$.

a) $y = 2\sin\left(x - \frac{\pi}{2}\right) + 1$

b) $y = 3\cos\left(x + \frac{\pi}{4}\right) - 2$

Graph of Trigonometric Functions

5.5 Graph of Tangent and Cotangent Functions



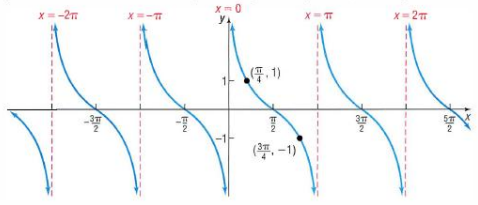
Graph of Tangent Function

Properties of the Tangent Function

1. The domain is the set of all real numbers, except odd multiples of $\frac{\pi}{2}$.
2. The range is the set of all real numbers.
3. The tangent function is an odd function, as the symmetry of the graph with respect to the origin indicates.
4. The tangent function is periodic, with period π.
5. The x-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$; the y-intercept is 0.
6. Vertical asymptotes occur at $x = \ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$.

Graph of Trigonometric Functions

Figure 96
$y = \cot x, -\infty < x < \infty, x$ not equal to integral multiples of $\pi, -\infty < y < \infty$.



Graph of Cotangent Function

Graph of Trigonometric Functions

5.6 Graph of y = a tan b(x + c) + d or y = a cot b(x + c) + d

For $y = a \tan b(x + c) + d$ and $y = a \cot b(x + c) + d$

The constant a vertically stretches or shrinks the graph compared to the graph of $y = \tan x$ or $y = \cot x$.

The period is $\frac{\pi}{|b|}$.

The phase shift from $y = a \tan bx$ or $y = a \cot bx$ is c units to the left if $c > 0$, and $|c|$ units to the right if $c < 0$.

The vertical shift from $y = a \tan bx$ or $y = a \cot bx$ is d units upward if $d > 0$, and $|d|$ units to the right if $d < 0$.

Asymptotes have these equations:

For tangent functions, $b(x + c) = \left(\frac{2n+1}{2}\right)\pi$, if n is an integer.

For cotangent functions, $b(x + c) = n\pi$, I f n is an integer.

Graph of Trigonometric Functions

5.7 Graph of Secant and Cosecant Functions

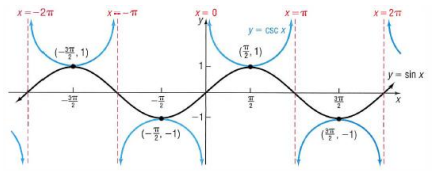
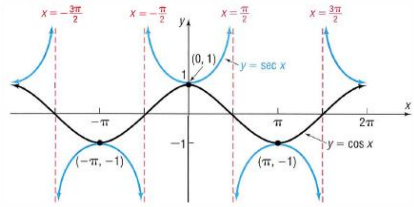


Figure 97
$y = \csc x, -\infty < x < \infty, x$ not equal to integer multiples of $\pi, |y| \geq 1$

Graph of Trigonometric Functions

5.7 Graph of Secant and Cosecant Functions

Figure 98
$y = \sec x, -\infty < x < \infty, x$ not equal to odd multiples of $\frac{\pi}{2}, |y| \geq 1$



Mapua Institute of Technology
Prepared By: Albert Grino Jr

Graph of Trigonometric Functions

5.8 Graph of $y = a \sec b(x + c) + d$ or $y = a \csc b(x + c) + d$

For $y = a \sec b(x + c) + d$ and $y = a \csc b(x + c) + d$

The constant a vertically stretches or shrinks the graph compared to the graph of $y = \sec x$ or $y = \csc x$.

The period is $\frac{\pi}{|b|}$.

The phase shift from $y = a \sec bx$ or $y = a \csc bx$ is c units to the left if $c > 0$, and $|c|$ units to the right if $c < 0$.

The vertical shift from $y = a \sec bx$ or $y = a \csc bx$ is d units upward if $d > 0$, and $|d|$ units to the right if $d < 0$.

Asymptotes have these equations:

For secant functions, $b(x + c) = \left(\frac{2n+1}{2}\right)\pi$, if n is an integer.

For cosecant functions, $b(x + c) = n\pi$, I f n is an integer.

Trigonometry

Mapua Institute of Technology
Prepared By: Albert Grino Jr

Graph of Trigonometric Functions

Exercises 5.8.1 Determine the period, the phase shift, and the vertical shift, if any, of each function. Describe any vertical stretching or shrinking.

a) $y = \tan 3(x - \pi)$

b) $y = \cot\left(x - \frac{3\pi}{4}\right)$

c) $y = 3\tan\left(x - \frac{4\pi}{3}\right) - 1$

d) $y = \sec\left(x - \frac{\pi}{4}\right)$

e) $y = \csc\left(3x + \frac{2\pi}{3}\right) - 2$

f) $y = \sec\left(2x + \frac{\pi}{2}\right) - 1$

Exercises 5.8.2 Graph each of the following over a two-period interval:

a) $y = \cot 2x$

b) $y = \tan(x - \pi)$

c) $y = 3 \sec 2x$

d) $y = \csc 2\left(x + \frac{2\pi}{3}\right)$