TRIGONOMETRY

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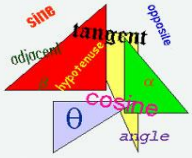


Definition:

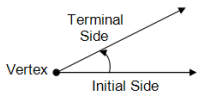
Definition 1.1 Trigonometry

Trigonometry is the branch of mathematics that deals with the measurement of triangle, that is, the sides and angles of a triangle. Trigonometry was derived from two Greek words *trigonon*, which means triangle and *metron*, which means measurement.

Angle Measure

Definition 1.2 .1 Angle

An **angle** is defined as the amount of rotation to move a ray (a halfline with an endpoint) from one position to another. The original position of the ray is called the **initial side** of the angle, and the final position of the ray is called the **terminal side**. The point about which the rotation occurs and at which the initial and terminal side of the angle intersect is called the **vertex.**



Angle Measure

An angle θ is said to be in **standard position** if its vertex is at the origin of a rectangular coordinate system and its initial side coincides with the positive x-axis. See Figure 3.

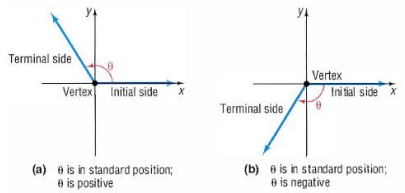
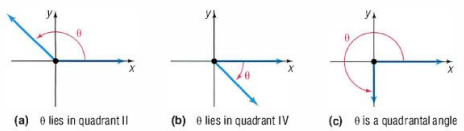


Figure 3

Angle Measure

When an angle θ is in standard position, the terminal side will lie either in a quadrant, in which case we say that θ **lies in that quadrant,** or θ will lie on the x-axis or the y-axis, in which case we say that θ is a **quadrantal angle.**



Angle Measure

Definition 1.2.2 Angle Measurement

Angles can be measured in degrees, radians or grads. One rotation measures 360°, 2π radians

There are three methods of measuring angles. These are:

- **Sexagesimal system** where the basic unit is **degree** (1°). The degree can be further divided into 60 equal parts called *minutes*, and each minute can be divided into 60 equal parts called *seconds*.

  $1^\circ = 60'$

  $1' = 60''$

- **Revolution system** where the angle measure is given as the number of rotations from the initial side to the terminal side.

  $1 \text{ rev} = 360^\circ$ $\quad \frac{1}{2} \text{ rev} = 180^\circ$ $\quad \frac{1}{4} \text{ rev} = 90^\circ$

  $\frac{1}{6} \text{ rev} = 60^\circ$ $\quad \frac{1}{8} \text{ rev} = 45^\circ$ $\quad \frac{1}{12} \text{ rev} = 30^\circ$

- **Radian system** where angle measure is in radians. A **radian** (rad) is the angle that intercepts an arc equal to the radius of the circle.

  $2\pi \text{ rad} = 360^\circ$ $\quad$ *or* $\quad \pi \text{ rad} = 180^\circ$

  thus, $1^\circ = \frac{\pi}{180}$ $\quad$ *and* $\quad 1 \text{ rad} = \frac{180^\circ}{\pi}$

Angle Measure

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Remarks:

1. To express a given number of degrees in radians, multiply the number of degrees by $\frac{\pi}{180}$.
2. To express a given number of radians in degrees, multiply the number of radians by $\frac{180}{\pi}$.

Example 1.2.1 Convert the following angles measured in degrees, minutes and seconds to angles measured to the nearest hundredth of a degree:

a) 64°24' 38"
b) 228° 23' 10"
c) 145° 11' 56"
d) 356° 09' 34"

Example 1.2.2 Convert the following angles measured in degrees to angles measured to the nearest minute:

a) 56.39°
c) 273.8°
c) 323.28°
d) 163.18°

Angle Measure

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Example 1.2.3 Find the degree measure of the angle for each rotation:

a) $\frac{3}{4}$ rotation, counterclockwise
b) $\frac{1}{3}$ rotation, clockwise
c) $\frac{4}{3}$ rotation, counterclockwise
d) $2\frac{1}{2}$ rotation, clockwise

Example 1.2.4 Express each angle measure in degrees:

a) $\frac{4\pi}{3}$
b) $\frac{5\pi}{4}$
c) $-\frac{18\pi}{3}$
d) $\frac{7\pi}{12}$
e) $\frac{11\pi}{6}$
f) $\frac{16\pi}{5}$

Example 1.2.5 Express each angle measure in radians. Give answer in terms of π.

a) 120°
b) 335°
c) -310°
d) 28°
e) 1035°
f) 450°

Angle Measure

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Definition 1.2.3 Coterminal Angles

Coterminal angles are angles in standard position whose initial and terminal sides are the same.

Remark:

To find angles coterminal to a given angle, add or subtract multiples of 360° to it.

Angle Measure

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Example 1.2.6 Draw the following angles and find two angles (one positive and one negative) coterminal with each.

a) 55°
b) 70°
c) 153°
d) 219°

Example 1.2.7 For each of the following angles, find a coterminal angle with measure β such that $0° \le \beta < 360°$.

a) -100°
b) 524°
c) 900°
d) 1250°