Lesson 34
Transforms of Derivatives

Specific Objectives: At the end of the lesson, the students are expected to:

- Find the Laplace transforms of the derivatives of functions

Let f(t) be a continuous function for t $\geq$ 0 , then,

$$L\{F^{(n)}(t)\} = s^n L\{f(t)\} - [s^{n-1}f(0) + s^{n-2}f'(0) + s^{n-3}f''(0) + ... + f^{(n-1)}(0)]$$

$$= s^n L\{f(t)\} - \sum_{k=0}^{n-1} s^{n-1-k} f(0)$$

Ex. 1. Find $L\{f'''(e^{2t})\}$

$$\begin{array}{ll} f(t) = e^{2t} & f(0) = 1 \\ f'(t) = 2e^{2t} & f'(0) = 2e^{2(0)} = 2 \\ f'(t) = 4e^{2t} & f''(0) = 4 \end{array}$$

Thus, $L\{f'''(e^{2t})\} = s^3 L\{e^{2t}\} - [s^2 f(0) + sf'(0) + f'(0)]$

$$= s^3[\frac{1}{s-2}] - [s^2(1) + s(2) + 4]$$

$$= \frac{s^3}{s-2} - s^2 - 2s - 4$$

Ex. 2. Find $L\{f''(\sin^2 t)\}$

$$\begin{array}{ll} f(t) = \sin^2 t & f(0) = 0 \\ f'(t) = 2\sin t \cos t = \sin 2t & f'(0) = 0 \end{array}$$

Thus, $L\{f''(\sin^2 t) = s^2 L\{\sin^2 t\} - sf(0) - f'(0)$

$$= s^2 L\{1/2(1 - \cos 2t)\} - s(0) - 0$$

$$= \frac{s^2}{2} L\{1 - \cos 2t\}$$

$$= \frac{s^2}{2}\left[L\{1\} - L\{\cos 2t\}\right]$$

$$= \frac{s^2}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]$$

$$= \underline{\frac{2s}{s^2+4}}$$

Seatwork: Perform the indicated operations:

1. $L\{f''(t\sin wt)\}$
2. $L\{f'''(\cos wt)\}$

Homework: Perform the indicated operations:

1. $L\{f'''(\sin wt)\}$
2. $L\{f''(t\cos wt)\}$