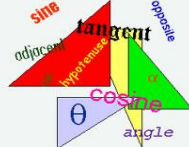
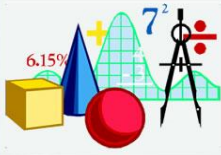
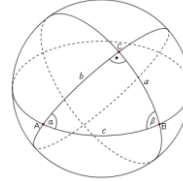


TRIGONOMETRY

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Spherical Trigonometry



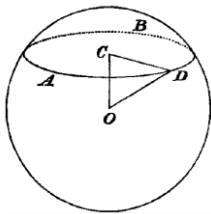
- Great Circle
- Spherical Triangle (Right & Oblique)
- Law of Sine and Cosine
- Napier's Analogies
- Terrestrial Sphere

Trigonometry

Spherical Trigonometry

Sphere is a solid bounded by a surface whose every point is equally distant from a fixed point which is called the **center**.

- The intersection of a plane with a sphere is a circle.



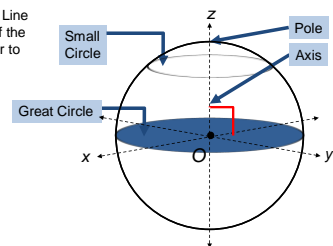
Trigonometry

Spherical Trigonometry

- **Great Circle** – If the plane passes through the center of the sphere (Radius of the great circle = Radius of the sphere)
- **Small Circle** – If the plane does not pass through the center of the sphere (Radius of the small circle < Radius of the sphere)

- **Axis of the Circle** – Line through the center of the sphere perpendicular to the plane of a circle

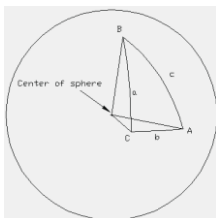
- **Poles** – two points where the axis pierces the sphere



Trigonometry

Spherical Trigonometry

Spherical triangle is defined when three planes pass through the surface of a sphere and through the sphere's center of volume.



- A spherical triangle has three surface angles and three central angles.

In the figure, 'A', 'B', and 'C' label the surface angles while 'a', 'b', and 'c' label the central angles

Trigonometry

Spherical Trigonometry

- The sum of the interior angles of a spherical is greater than 180° and less than 540° .

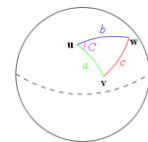
$$540^\circ \geq A+B+C > 180^\circ$$

$$\text{Area} = \frac{\pi R^2 E}{180}$$

$$E = A + B + C - 180$$

$$\tan \frac{1}{4} E = \sqrt{\tan \frac{1}{2} s \left[\tan \frac{1}{2} s - a \right] \left[\tan \frac{1}{2} s - b \right] \left[\tan \frac{1}{2} s - c \right]}$$

$$s = \frac{1}{2} a + b + c$$



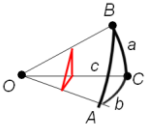
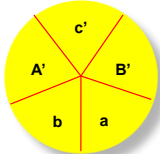
Trigonometry

Spherical Trigonometry

Right Spherical Triangle

The Napier's circle indicates the sides and angle of The triangle in consecutive order, not including the right angle.

Where: $A' = 90 - A$
 $B' = 90 - B$
 $c' = 90 - c$

Trigonometry

Spherical Trigonometry

Napier's Rules

1. In the Napier's circle, the sine of any middle part is Equal to product of the cosines of its opposite parts.

SIN-COOP RULE.

If we take b as the middle part, its opposite parts are c' and B' , then

$$\sin b = \cos c' (\cos B')$$

But: $\cos c' = \cos(90 - c) = \sin c$
 $\cos B' = \sin B$
 $\sin b = \sin c (\sin B)$

Trigonometry

Spherical Trigonometry

Napier's Rules

2. In the Napier's circle, the sine of any middle part is Equal to the product of the tangents of its adjacent parts.

SIN-TAAD RULE.

If we take A' as the middle part, then its adjacent parts are c' and b , then

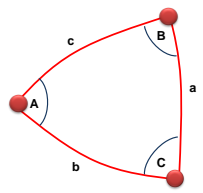
$$\sin A' = \tan c' (\tan b)$$

$$\cos A = \cot c (\tan b)$$

Trigonometry

Spherical Trigonometry

•Oblique Spherical Triangle



Laws of Sines

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Laws of Cosines for Sides

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

Laws of Cosines for Angles

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

$$\cos B = -\cos C \cos A + \sin C \sin A \cos b$$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

Trigonometry

Spherical Trigonometry

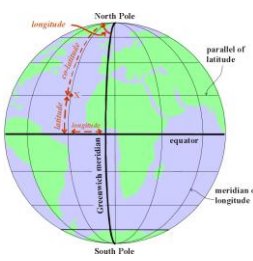
Terrestrial Sphere

Meridian is a great circle Passing through the North And South Poles.

Equator is a great circle Perpendicular to the meridians.

Small circles parallel to equator are **parallels of latitude**.

The zero-point is the meridian which passes through Greenwich (also called the **Prime Meridian**)




Trigonometry

Spherical Trigonometry

1. A spherical triangle ABC has an angle $C = 90^\circ$ and sides $a = 50^\circ$ and $c = 80^\circ$. Find the value of b in degrees.

Ans. $b = 74.33^\circ$


 **Trigonometry**

Spherical Trigonometry

2. Given a right spherical triangle whose given parts are $a=82^\circ$, $b=62^\circ$ and $C=90^\circ$. What is the value of the side opposite the right angle?

Ans. $c=86^\circ 15'$

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 **Trigonometry**

Spherical Trigonometry

3. Solve the remaining side of the spherical triangle whose given parts are $A=B=80^\circ$ and $a=b=89^\circ$.

Ans. $c=168^\circ 31'$

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 **Trigonometry**

Spherical Trigonometry

Exercise:

1. Solve for side b of a right spherical ABC whose parts are $a=46^\circ$, $c=75^\circ$ and $C=90^\circ$. **Ans. $b=68^\circ 07'$**

2. Determine the value of the angle B of an isosceles spherical triangle ABC whose given parts are $b=c=54^\circ 28'$ and $a=92^\circ 30'$. **Ans. $B=41^\circ 45'$**

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