TRIGONOMETRY

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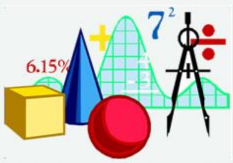

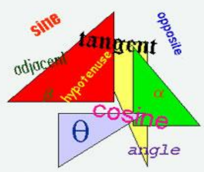

Trigonometry

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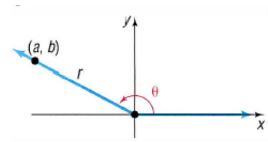
Trigonometric Functions of General Angles



- Evaluation trigonometric functions of any angle
  - Signs of Trigonometric Functions
  - Trigonometric Ratios of Quadrant Angles
  - Reference Angle

Trigonometry

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Trigonometric Functions of General Angles

Find the Exact Values of the Trigonometric Functions for General Angles

DEFINITION

Let θ be any angle in standard position, and let (a, b) denote the coordinates of any point, except the origin $(0, 0)$, on the terminal side of θ. If $r = \sqrt{a^2 + b^2}$ denotes the distance from $(0, 0)$ to (a, b), then the **six trigonometric functions of θ** are defined as the ratios

$$\sin\theta = \frac{b}{r} \qquad \cos\theta = \frac{a}{r} \qquad \tan\theta = \frac{b}{a}$$

$$\csc\theta = \frac{r}{b} \qquad \sec\theta = \frac{r}{a} \qquad \cot\theta = \frac{a}{b}$$

provided no denominator equals 0. If a denominator equals 0, that trigonometric function of the angle θ is not defined.

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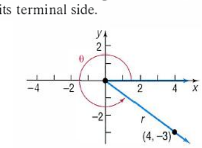
Trigonometric Functions of General Angles

Finding the Exact Values of the Six Trigonometric Functions of θ, Given a Point on the Terminal Side

Find the exact value of each of the six trigonometric functions of a positive angle θ if $(4, -3)$ is a point on its terminal side.



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$$\sin\theta = \frac{b}{r} = -\frac{3}{5} \qquad \cos\theta = \frac{a}{r} = \frac{4}{5} \qquad \tan\theta = \frac{b}{a} = -\frac{3}{4}$$

$$\csc\theta = \frac{r}{b} = -\frac{5}{3} \qquad \sec\theta = \frac{r}{a} = \frac{5}{4} \qquad \cot\theta = \frac{a}{b} = -\frac{4}{3}$$

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Finding the Exact Values of the Six Trigonometric Functions of Quadrantal Angles

Find the exact values of each of the six trigonometric functions of

(a) $\theta = 0 = 0°$ (b) $\theta = \frac{\pi}{2} = 90°$ (c) $\theta = \pi = 180°$ (d) $\theta = \frac{3\pi}{2} = 270°$

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Trigonometric Functions of General Angles

θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	0°	0	1	0	Not defined	1	Not defined
$\frac{\pi}{2}$	90°	1	0	Not defined	1	Not defined	0
π	180°	0	−1	0	Not defined	−1	Not defined
$\frac{3\pi}{2}$	270°	−1	0	Not defined	−1	Not defined	0

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Determine the Signs of the Trigonometric Functions of an Angle in a Given Quadrant

If θ is not a quadrantal angle, then it will lie in a particular quadrant. In such a case, the signs of the x-coordinate and y-coordinate of a point (a, b) on the terminal side of θ are known. Because $r = \sqrt{a^2 + b^2} > 0$, it follows that the signs of the trigonometric functions of an angle θ can be found if we know in which quadrant θ lies.

Quadrant of θ	$\sin\theta, \csc\theta$	$\cos\theta, \sec\theta$	$\tan\theta, \cot\theta$
I	Positive	Positive	Positive
II	Positive	Negative	Negative
III	Negative	Negative	Positive
IV	Negative	Positive	Negative

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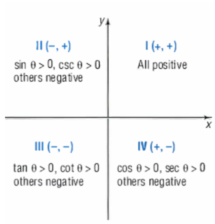

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Trigonometric Functions of General Angles

Finding the Quadrant in Which an Angle Lies

If $\sin\theta < 0$ and $\cos\theta < 0$, name the quadrant in which the angle θ lies.

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Find the Reference Angle of a General Angle

DEFINITION

Let θ denote an angle that lies in a quadrant. The acute angle formed by the terminal side of θ and the x-axis is called the **reference angle** for θ.

Figure 53

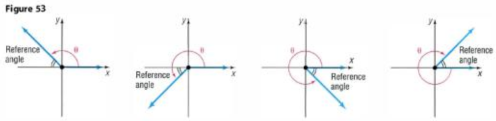

Figure 53 illustrates the reference angle for some general angles θ. Note that a reference angle is always an acute angle. That is, a reference angle has a measure between 0° and 90°.

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Trigonometric Functions of General Angles

Finding Reference Angles

Find the reference angle for each of the following angles:

(a) $150°$ (b) $-45°$ (c) $\frac{9\pi}{4}$ (d) $-\frac{5\pi}{6}$

Trigonometry

Trigonometric Functions of General Angles

Use a Reference Angle to Find the Exact Value of a Trigonometric Function

THEOREM

Reference Angles

If θ is an angle that lies in a quadrant and if α is its reference angle, then

$$\sin\theta = \pm\sin\alpha \qquad \cos\theta = \pm\cos\alpha \qquad \tan\theta = \pm\tan\alpha$$
$$\csc\theta = \pm\csc\alpha \qquad \sec\theta = \pm\sec\alpha \qquad \cot\theta = \pm\cot\alpha \qquad (2)$$

where the + or − sign depends on the quadrant in which θ lies.

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Trigonometric Functions of General Angles

Using the Reference Angle to Find the Exact Value of a Trigonometric Function

Find the exact value of each of the following trigonometric functions using reference angles.

(a) $\sin 135°$ (b) $\cos 600°$ (c) $\cos\frac{17\pi}{6}$ (d) $\tan\left(-\frac{\pi}{3}\right)$

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Trigonometric Functions of General Angles

Finding the Values of the Trigonometric Functions of a General Angle

If the angle θ is a quadrantal angle, draw the angle, pick a point on its terminal side, and apply the definition of the trigonometric functions.

If the angle θ lies in a quadrant:

1. Find the reference angle α of θ.
2. Find the value of the trigonometric function at α.
3. Adjust the sign (+ or −) according to the sign of the trigonometric function in the quadrant where θ lies.

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Finding the Exact Values of Trigonometric Functions

Given that $\cos\theta = -\frac{2}{3}$, $\frac{\pi}{2} < \theta < \pi$, find the exact value of each of the remaining trigonometric functions.

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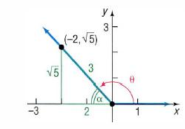
Trigonometric Functions of General Angles

Figure 63

$\cos\alpha = \frac{2}{3}$



$\sin\theta = \frac{\sqrt{5}}{3}$ $\quad \cos\theta = -\frac{2}{3}$ $\quad \tan\theta = -\frac{\sqrt{5}}{2}$

$\csc\theta = \frac{3\sqrt{5}}{5}$ $\quad \sec\theta = -\frac{3}{2}$ $\quad \cot\theta = -\frac{2\sqrt{5}}{5}$

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Finding the Exact Values of Trigonometric Functions

If $\tan\theta = -4$ and $\sin\theta < 0$, find the exact value of each of the remaining trigonometric functions of θ.

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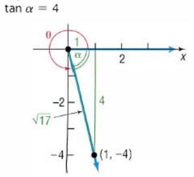
Trigonometric Functions of General Angles

Figure 64

$\tan\alpha = 4$



$\sin\theta = -\frac{4\sqrt{17}}{17}$ $\quad \cos\theta = \frac{\sqrt{17}}{17}$ $\quad \tan\theta = -4$

$\csc\theta = -\frac{\sqrt{17}}{4}$ $\quad \sec\theta = \sqrt{17}$ $\quad \cot\theta = -\frac{1}{4}$

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In Problems 59–88, use the reference angle to find the exact value of each expression. Do not use a calculator.

59. $\sin 150°$	**60.** $\cos 210°$	**61.** $\cos 315°$	**62.** $\sin 120°$	**63.** $\sin 510°$
65. $\cos(-45°)$	**66.** $\sin(-240°)$	**67.** $\sec 240°$	**68.** $\csc 300°$	**69.** $\cot 330°$
71. $\sin\frac{3\pi}{4}$	**72.** $\cos\frac{2\pi}{3}$	**73.** $\cot\frac{7\pi}{6}$	**74.** $\csc\frac{7\pi}{4}$	**75.** $\cos\frac{13\pi}{4}$
77. $\sin\left(-\frac{2\pi}{3}\right)$	**78.** $\cot\left(-\frac{\pi}{6}\right)$	**79.** $\tan\frac{14\pi}{3}$	**80.** $\sec\frac{11\pi}{4}$	**81.** $\csc(-315°)$
83. $\sin(8\pi)$	**84.** $\cos(-2\pi)$	**85.** $\tan(7\pi)$	**86.** $\cot(5\pi)$	**87.** $\sec(-3\pi)$

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In Problems 89–106, find the exact value of each of the remaining trigonometric functions of θ.

89. $\sin\theta = \frac{12}{13}$, θ in Quadrant II

90. $\cos\theta = \frac{3}{5}$, θ in Quadrant IV

92. $\sin\theta = -\frac{5}{13}$, θ in Quadrant III

93. $\sin\theta = \frac{5}{13}$, $90° < \theta < 180°$

95. $\cos\theta = -\frac{1}{3}$, $180° < \theta < 270°$

96. $\sin\theta = -\frac{2}{3}$, $180° < \theta < 270°$

98. $\cos\theta = -\frac{1}{4}$, $\tan\theta > 0$

99. $\sec\theta = 2$, $\sin\theta < 0$

101. $\tan\theta = \frac{3}{4}$, $\sin\theta < 0$

102. $\cot\theta = \frac{4}{3}$, $\cos\theta < 0$

104. $\sec\theta = -2$, $\tan\theta > 0$

105. $\csc\theta = -2$, $\tan\theta > 0$