TRIGONOMETRY

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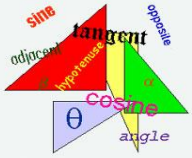
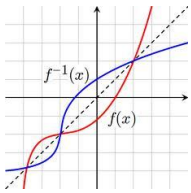


Inverse Trigonometric Functions

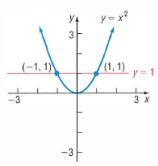


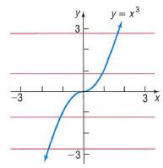
Inverse Trigonometric Functions

- Definition
- Graph
- Evaluation

Inverse Trigonometric Functions

If a function f is one-to-one, then it has an inverse function f^{-1}.



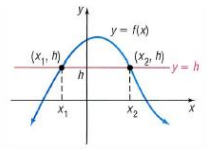


Inverse Trigonometric Functions

THEOREM

Horizontal-line Test

If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.



$f(x_1) = f(x_2) = h$ and $x_1 \neq x_2$; f is not a one-to-one function.

Inverse Trigonometric Functions

The Inverse Sine Function

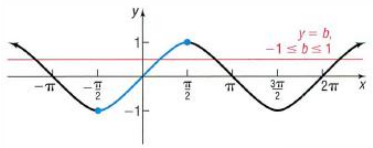


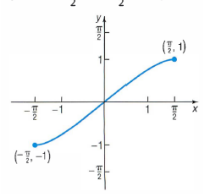
Figure 1
$y = \sin x, -\infty < x < \infty, -1 \leq y \leq 1$

Inverse Trigonometric Functions

DEFINITION

$$y = \sin^{-1} x \quad \text{means} \quad x = \sin y$$
$$\text{where} \quad -1 \leq x \leq 1 \quad \text{and} \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \qquad (1)$$

$y = \sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, -1 \leq y \leq 1$



Inverse Trigonometric Functions

EXAMPLE 1 Finding the Exact Value of an Inverse Sine Function

Find the exact value of: $\sin^{-1} 1$

EXAMPLE 2 Finding the Exact Value of an Inverse Sine Function

Find the exact value of: $\sin^{-1}\left(-\frac{1}{2}\right)$

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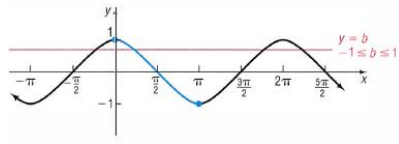
Inverse Trigonometric Functions

The Inverse Cosine Function



DEFINITION

$$y = \cos^{-1} x \quad \text{means} \quad x = \cos y \qquad (3)$$
$$\text{where} \quad -1 \le x \le 1 \quad \text{and} \quad 0 \le y \le \pi$$

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Inverse Trigonometric Functions

Find the exact value of: $\cos^{-1} 0$

Find the exact value of: $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

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Inverse Trigonometric Functions

DEFINITION

$$y = \tan^{-1} x \quad \text{means} \quad x = \tan y \qquad (5)$$
$$\text{where} \quad -\infty < x < \infty \quad \text{and} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

Find the exact value of:

(a) $\tan^{-1} 1$ (b) $\tan^{-1}\left(-\sqrt{3}\right)$

Solve the equation: $3 \sin^{-1} x = \pi$

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Inverse Trigonometric Functions

Find the exact value of: $\sin\left(\tan^{-1}\frac{1}{2}\right)$

$$\sin\left(\tan^{-1}\frac{1}{2}\right) = \sin\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

Find the exact value of: $\cos\left[\sin^{-1}\left(-\frac{1}{3}\right)\right]$

$$\cos\left[\sin^{-1}\left(-\frac{1}{3}\right)\right] = \cos\theta = \frac{2\sqrt{2}}{3}$$

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Inverse Trigonometric Functions

DEFINITION

$$y = \sec^{-1} x \quad \text{means} \quad x = \sec y \qquad (1)$$
$$\text{where} \quad |x| \ge 1 \quad \text{and} \quad 0 \le y \le \pi, \quad y \ne \frac{\pi}{2}$$

$$y = \csc^{-1} x \quad \text{means} \quad x = \csc y \qquad (2)$$
$$\text{where} \quad |x| \ge 1 \quad \text{and} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}, \quad y \ne 0$$

$$y = \cot^{-1} x \quad \text{means} \quad x = \cot y \qquad (3)$$
$$\text{where} \quad -\infty < x < \infty \quad \text{and} \quad 0 < y < \pi$$

Find the exact value of: $\csc^{-1} 2$

$$\csc^{-1} 2 = \frac{\pi}{6}.$$

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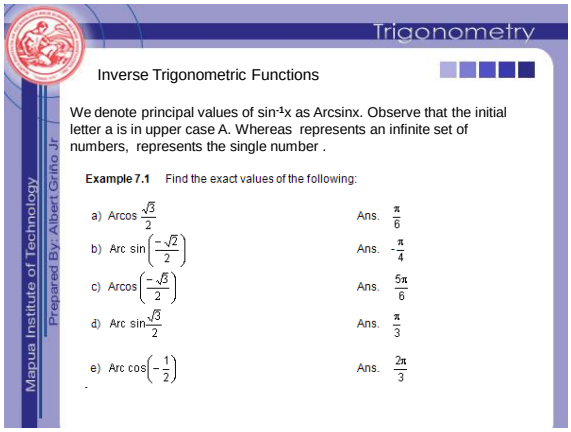
Trigonometry
Inverse Trigonometric Functions
We denote principal values of sin⁻¹x as Arcsinx. Observe that the initial letter a is in upper case A. Whereas represents an infinite set of numbers, represents the single number .
Example 7.1 Find the exact values of the following:
a) Arcos $\frac{\sqrt{3}}{2}$ Ans. $\frac{\pi}{6}$
b) Arc sin $\left(\frac{-\sqrt{2}}{2}\right)$ Ans. $-\frac{\pi}{4}$
c) Arcos $\left(\frac{-\sqrt{3}}{2}\right)$ Ans. $\frac{5\pi}{6}$
d) Arc sin $\frac{\sqrt{3}}{2}$ Ans. $\frac{\pi}{3}$
e) Arc cos $\left(-\frac{1}{2}\right)$ Ans. $\frac{2\pi}{3}$
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