# TRIGONOMETRY

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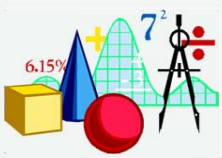


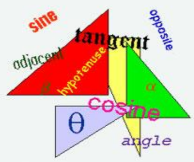


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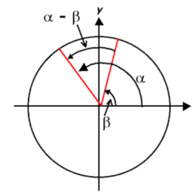
Mapua Institute of Technology
Prepared By: Albert Griño Jr

## Trigonometric Identities




- Fundamental Trigonometric Identities
  - Proving
  - Addition & Subtraction
  - Double, Half-Angle and Product-Sum

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## Trigonometric Identities

**SUMMARY** Sum and Difference Formulas

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

**Double-angle Formulas**

$$\sin(2\theta) = 2\sin\theta\cos\theta \quad \textbf{(1)}$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta \quad \textbf{(2)}$$

$$\cos(2\theta) = 1 - 2\sin^2\theta \quad \textbf{(3)}$$

$$\cos(2\theta) = 2\cos^2\theta - 1 \quad \textbf{(4)}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta} \quad \textbf{(5)}$$

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## Trigonometric Identities

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2} \quad \textbf{(6)}$$

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2} \quad \textbf{(7)}$$

$$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)} \quad \textbf{(8)}$$

**Establishing an Identity**

Write an equivalent expression for $\cos^4\theta$ that does not involve any powers of sine or cosine greater than 1.

$$= \frac{3}{8} + \frac{1}{2}\cos(2\theta) + \frac{1}{8}\cos(4\theta)$$

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## Trigonometric Identities

**Projectile Motion**

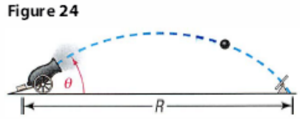
An object is propelled upward at an angle $\theta$ to the horizontal with an initial velocity of $v_0$ feet per second. See Figure 24. If air resistance is ignored, the **range** $R$, the horizontal distance that the object travels, is given by the function

$$R(\theta) = \frac{1}{16}v_0^2 \sin\theta\cos\theta$$

(a) Show that $R(\theta) = \frac{1}{32}v_0^2\sin(2\theta)$.

(b) Find the angle $\theta$ for which $R$ is a maximum.

**Figure 24**

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## Trigonometric Identities

**Use Half-angle Formulas to Find Exact Values**

$$\sin^2\frac{\alpha}{2} = \frac{1 - \cos\alpha}{2} \qquad \cos^2\frac{\alpha}{2} = \frac{1 + \cos\alpha}{2} \qquad \tan^2\frac{\alpha}{2} = \frac{1 - \cos\alpha}{1 + \cos\alpha} \quad \textbf{(9)}$$

**THEOREM**

**Half-angle Formulas**

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}} \quad \textbf{(10a)}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}} \quad \textbf{(10b)}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}} \quad \textbf{(10c)}$$

where the + or − sign is determined by the quadrant of the angle $\frac{\alpha}{2}$.

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Trigonometric Identities

**Finding Exact Values Using Half-angle Formulas**

Use a Half-angle Formula to find the exact value of:

(a) $\cos 15°$ (b) $\sin(-15°)$

$\frac{\sqrt{2+\sqrt{3}}}{2}$ $-\frac{\sqrt{2-\sqrt{3}}}{2}$

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Trigonometric Identities

**Finding Exact Values Using Half-angle Formulas**

If $\cos\alpha = -\frac{3}{5}, \pi < \alpha < \frac{3\pi}{2}$, find the exact value of:

(a) $\sin\frac{\alpha}{2}$ (b) $\cos\frac{\alpha}{2}$ (c) $\tan\frac{\alpha}{2}$

$\frac{2\sqrt{5}}{5}$ $-\frac{\sqrt{5}}{5}$ $-2$

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**Half-angle Formulas for $\tan\frac{\alpha}{2}$**

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha} \quad \textbf{(11)}$$

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Trigonometric Identities

**Product-to-Sum Formulas**

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)] \quad \textbf{(1)}$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)] \quad \textbf{(2)}$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)] \quad \textbf{(3)}$$

Proof:

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

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Trigonometric Identities

**Expressing Products as Sums**

Express each of the following products as a sum containing only sines or only cosines.

(a) $\sin(6\theta)\sin(4\theta)$ (b) $\cos(3\theta)\cos\theta$ (c) $\sin(3\theta)\cos(5\theta)$

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**Sum-to-Product Formulas**

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \quad \textbf{(6)}$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2} \quad \textbf{(7)}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \quad \textbf{(8)}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \quad \textbf{(9)}$$

**Proof**

$$2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \underbrace{2\cdot\frac{1}{2}}_{\text{Product-to-Sum Formula (3)}}\left[\sin\left(\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}\right) + \sin\left(\frac{\alpha+\beta}{2}-\frac{\alpha-\beta}{2}\right)\right]$$

$$= \sin\frac{2\alpha}{2} + \sin\frac{2\beta}{2} = \sin\alpha + \sin\beta$$

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Trigonometric Identities

**Expressing Sums (or Differences) as a Product**

Express each sum or difference as a product of sines and/or cosines.

(a) $\sin(5\theta) - \sin(3\theta)$ (b) $\cos(3\theta) + \cos(2\theta)$

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Trigonometric Identities

*In Problems 7–18, use the information given about the angle $\theta$, $0 \le \theta < 2\pi$, to find the exact value of*

*(a)* $\sin(2\theta)$ *(b)* $\cos(2\theta)$ *(c)* $\sin\frac{\theta}{2}$ *(d)* $\cos\frac{\theta}{2}$

**7.** $\sin\theta = \frac{3}{5}$ $0 < \theta < \frac{\pi}{2}$

**8.** $\cos\theta = \frac{3}{5}$ $0 < \theta < \frac{\pi}{2}$

**10.** $\tan\theta = \frac{1}{2}$ $\pi < \theta < \frac{3\pi}{2}$

**11.** $\cos\theta = -\frac{\sqrt{6}}{3}$ $\frac{\pi}{2} < \theta < \pi$

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Trigonometric Identities

**Laser Projection** In a laser projection system, the **optical** or **scanning angle** $\theta$ is related to the throw distance $D$ from the scanner to the screen and the projected image width $W$ by the equation $D = \dfrac{\frac{1}{2}W}{\csc\theta - \cot\theta}$.

(a) Show that the projected image width is given by $W = 2D\tan\frac{\theta}{2}$.

(b) Find the optical angle if the throw distance is 15 feet and the projected image width is 6.5 feet.

***Source:*** *Pangolin Laser Systems, Inc.*