Welfare Assessments with Heterogeneous Individuals*

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Abstract

This paper develops a new approach to make welfare assessments based on the notion of Dynamic Stochastic weights (DS-weights for short). For a large class of dynamic stochastic economies with heterogeneous individuals, we introduce an aggregate additive decomposition that satisfies desirable properties and that allows us to exactly decompose welfare assessments into four components: i) aggregate efficiency, ii) risk-sharing, iii) intertemporal-sharing, and iv) redistribution. We leverage DS-weights to i) revisit how welfarist (e.g., utilitarian) planners make interpersonal welfare comparisons and ii) formalize new welfare criteria that are exclusively based on one or several of the components that we identify.

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1 Introduction

Assessing whether a policy change is desirable in dynamic stochastic economies with rich individual heterogeneity and imperfect insurance is far from trivial. One significant challenge is to understand the channels — such as aggregate efficiency, intertemporal-sharing, risk-sharing, or redistribution — through which a particular normative criterion finds a policy change desirable. A different challenge is how to formally define welfare criteria that exclusively value one or several of the aforementioned channels but not others.[1]

This paper tackles both challenges by developing a new approach to making welfare assessments in dynamic stochastic economies. This approach is based on the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (Dynamic Stochastic weights or DS-weights, for short). The introduction of DS-weights accomplishes two main objectives. First, DS-weights allow us to decompose aggregate welfare assessments of policy changes into four distinct components: aggregate efficiency, intertemporal-sharing, risk-sharing, and redistribution, each capturing a different normative consideration. Second, DS-weights allow us to systematically formalize new welfare criteria that society may find appealing. In particular, we are able to define normative criteria that are exclusively based on one or several of the four normative considerations that we identify, potentially disregarding the others.

We introduce our results in a canonical dynamic stochastic environment with heterogeneous individuals. As a benchmark, we explicitly define in our environment i) Pareto-improving policies and ii) desirable policies for a welfarist planner. While Pareto improvements seem highly desirable, they are rare to find, which forces planners/policymakers to make interpersonal welfare comparisons. Such comparisons typically rely on a Social Welfare Function — this is the welfarist approach. While the welfarist approach is popular and widely applicable, it is not easy to understand how a welfarist planner exactly makes tradeoffs among individuals that are ex-ante heterogeneous, because of the ordinal nature of individual utilities. By reviewing these well-understood approaches and treating them as benchmarks, we set the stage for the introduction of DS-weights.

In our approach, it is not necessary to specify a social welfare objective that a planner maximizes. Instead, in order to make welfare assessments, a planner must simply specify DS-weights, which represent the value that society places on a marginal dollar of consumption by a particular individual i at a particular time t and along a particular history s^t. Equipped with these weights, we define a policy to be desirable when the weighted sum — using DS-weights — across all individuals, dates, and histories of the instantaneous consumption-equivalent effects of a policy is positive. By defining DS-weights marginally, we can define normative criteria that the welfarist approach cannot capture.

In order to understand how a DS-planner, that is, a planner who adopts DS-weights, carries out welfare assessments, we introduce two different decompositions. First, we introduce an individual

[1] Recently, the Federal Reserve seems to have explicitly included cross-sectional considerations in its policy-making process — see e.g., `https://www.nytimes.com/2021/04/19/business/economy/federal-reserve-politics.html`. The approach that we develop in this paper can plausibly be used to define a mandate for a monetary authority or other policymakers that explicitly incorporates or removes cross-sectional concerns from policy assessments.

multiplicative decomposition of DS-weights. We show that, in general, the DS-weights assigned to a given individual can be decomposed into i) an individual component, which is invariant across all dates and histories; ii) a dynamic component, which can vary across dates, but not across histories at a given date; and a stochastic component, which can vary across dates and histories. Moreover, we show that there exists a unique normalized individual multiplicative decomposition of DS-weights, which is easily interpretable and has desirable properties.

Having introduced DS-weights, we leverage them to characterize three sets of results. First, we develop an aggregate additive decomposition of welfare assessments (Section 3). Second, we introduce normalized welfarist planners that allow us to precisely describe how welfarist planners make interpersonal tradeoffs (Section 4). Third, we show how to use DS-weights to systematically formalize new welfare criteria (Section 5).

In our first set of results, we introduce an aggregate additive decomposition of welfare assessments. We show that, in dynamic stochastic environments, welfare assessments made by DS-planners can be exactly decomposed into four components: i) an aggregate efficiency component, ii) a risk-sharing component, iii) an intertemporal-sharing component, and iv) a redistribution component.[2] The aggregate efficiency component accounts for the change in aggregate consumption-equivalents across all individuals. The remaining three components of the decomposition are driven by the cross-sectional variation of each of the three elements (individual, dynamic, stochastic) of the individual multiplicative decomposition. In particular, the risk-sharing component adds up across all dates and histories the cross-sectional covariances between the stochastic component of the individual multiplicative decomposition and the change in normalized instantaneous utility at each date and history. Similarly, the intertemporal-sharing component adds up across all dates the covariances between the dynamic component of the individual multiplicative decomposition and the change in normalized net utility at each date. Finally, the redistribution component can be expressed as a single cross-sectional covariance between the individual components of the individual multiplicative decomposition and the change in individual lifetime marginal utility from the perspective of a DS-planner.

Next, we systematically present properties of the aggregate additive decomposition and its components for a general DS-planner. We show that a DS-planner who assigns DS-weights that do not vary across individuals at all dates and histories makes welfare assessments purely based on aggregate efficiency considerations. Similarly, different components of the aggregate additive decomposition may vanish depending on which specific components of the individual multiplicative decomposition of DS-weights are invariant across individuals: if the individual multiplicative component is constant across individuals, then the redistribution component of the aggregate decomposition is zero; if the dynamic multiplicative component is constant across individuals at all dates, then the intertemporal-sharing component of the aggregate decomposition is zero; if the stochastic multiplicative component

[2]The aggregate additive decomposition can be used to separate efficiency from redistribution considerations. The sum of the first three components of the decomposition — aggregate efficiency, risk-sharing, and intertemporal-sharing — defines a notion of efficiency.

is constant across individuals at all dates and histories, then the risk-sharing component of the aggregate decomposition is zero. We highlight four implications of these results with practical relevance. First, welfare assessments in single- or representative-agent economies are exclusively attributed to aggregate efficiency considerations. Second, welfare assessments in perfect-foresight economies (under normalized DS-weights) are never attributed to risk-sharing. Third, welfare assessments in economies in which all individuals are ex-ante identical (but not necessarily ex-post) are never attributed to intertemporal-sharing or redistribution. Fourth, welfare assessments in static economics (under normalized DS-weights) are exclusively attributed to aggregate efficiency or redistribution considerations. We also provide conditions on policies that imply that a subset of the components of the aggregate decompositions are zero. In particular, we show that, under normalized DS-weights, the risk-sharing, intertemporal-sharing, and redistribution components are zero whenever a given policy impacts all individuals identically. Finally, we show that the aggregate efficiency component of the aggregate decomposition is zero in endowment economies.

In our second set of results, given the importance of the welfarist approach in practice, we characterize how a welfarist DS-planner makes tradeoffs across periods and histories for a given individual, and across individuals.[3] Critically, we do so in easily interpretable consumption units. We formally characterize the unique normalized individual multiplicative decomposition of DS-weights implied by a given welfarist planner and discuss its implications. Armed with this decomposition, we characterize five new additional properties of the aggregate additive decomposition of welfare assessments for welfarist planners. In particular, we show that all normalized welfarist planners conclude that i) the risk-sharing and intertemporal-sharing components are zero when markets are complete, ii) the intertemporal-sharing component is zero when all individuals can freely trade a riskless bond, and iii) that different normalized welfarist planners — with different Social Welfare Functions — exclusively disagree on the redistribution component. We also show that iv) the efficiency components (aggregate efficiency, risk-sharing, and intertemporal-sharing) of the aggregate additive decomposition are invariant to monotonically increasing transformations of individual's lifetime utilities and positive affine (increasing linear) transformations of individual's instantaneous utilities and v) that all normalized welfarist planners conclude that Pareto improving policies increase efficiency, i.e., the sum of aggregate efficiency, risk-sharing, and intertemporal-sharing. To our knowledge, the aggregate additive decomposition of welfare assessments introduced in this paper is the first welfare decomposition for which these properties — which seem highly desirable — have been established.

In our third set of results, we describe how to use DS-weights to systematically formalize new welfare criteria that society may find appealing. We first introduce three sets of novel DS-planners: aggregate efficiency (AE) DS-planners, aggregate efficiency/risk-sharing (AR) DS-planners, and no-redistribution (NR) DS-planners, and characterize their properties.[4] The welfare assessments made

[3]Adopting a conventional Social Welfare Function (e.g., utilitarian) to make welfare assessments can be interpreted as selecting a particular set of DS-weights, which we show how to compute.

[4]For instance, the current "dual mandate" (stable prices and maximum employment) of the Federal Reserve (as defined by the 1977 Federal Reserve Act) seems to be better described by an aggregate efficiency (AE) DS-planner than

by these new planners purposefully set to zero particular components of the aggregate additive decomposition. Within each set of DS-planners, we identify a pseudo-welfarist planner as the one that represents the minimal departure relative to the normalized welfarist planner. We also introduce an $\boldsymbol{\alpha}$-DS-planner, a new planner that spans i) AE, ii) AR, and iii) NR pseudo-welfarist planners, as well as iv) the associated normalized welfarist planner. Finally, we explain why some new planners (AE and AR) are paternalistic, while others are not (NR).[5] We also discuss the implications of introducing new planners for policy mandates and institutional design.

Before presenting an application of our framework, we describe several additional results. First, we further decompose the components of the aggregate additive decomposition and then explain how to connect welfare assessments to measures of inequality. We explain how to make welfare assessments using DS-weights in recursive environments, and show how to implement welfare assessments via an instantaneous Social Welfare Function. We also show that each of the component of the aggregate decomposition, as well as aggregate welfare assessments, have a term structure, which allows us to distinguish transition from steady-state welfare gains and losses. Finally, we briefly describe additional results included in the Online Appendix. Among other results, we show how our approach nests the widely used consumption-equivalent approach introduced by Lucas (1987) and Alvarez and Jermann (2004).

At last, we illustrate the mechanics of our approach by conducting welfare assessments in a fully specified application. We explore two particular scenarios in single-good economies with no financial markets. Scenario 1 corresponds to an economy in which individuals with identical preferences face idiosyncratic shocks. We consider transfer policies that can potentially provide perfect consumption smoothing and carefully explain how, depending on the persistence of idiosyncratic risk, a normalized utilitarian planner can find such policies desirable for different reasons. In particular, when risks are transitory, the planner attributes most of the welfare gains of the policy to risk-sharing. When risks are very persistent, the planner attributes most of the gains to redistribution instead. Scenario 2 corresponds to an economy in which individuals with different risk preferences face aggregate shocks. We consider transfer policies that shift aggregate risk to the more risk-tolerant individuals and carefully explain how a normalized utilitarian planner finds such policies desirable for different reasons depending on the state of the economy in which welfare assessments take place.

This paper is accompanied by a code repository and user guides, which can be found at https://github.com/schaab-lab/DS-weights.

Related literature. This paper contributes to several literatures, specifically those on i) interpersonal welfare comparisons, ii) welfare decompositions, iii) welfare evaluation of policies in dynamic environments, and iv) institutional mandates.

Interpersonal welfare comparisons. The question of how to make interpersonal welfare comparisons to form aggregate welfare assessments has a long history in economics — see, among many others,

by a normalized utilitarian criterion that would care about risk-sharing, intertemporal-sharing, and redistribution.

[5] See Section 5.3 and Section G.3.1 of the Online Appendix for formal definitions of paternalism.

Kaldor (1939), Hicks (1939), Bergson (1938), Samuelson (1947), Harsanyi (1955), Sen (1970) or, more recently, Kaplow and Shavell (2001), Saez and Stantcheva (2016), Hendren (2020), Tsyvinski and Werquin (2020), and Hendren and Sprung-Keyser (2020). Formally, our approach based on endogenous welfare weights is most closely related to the work of Saez and Stantcheva (2016), who introduce Generalized Social Marginal Welfare Weights. Building on their terminology, in this paper we introduce the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (Dynamic Stochastic weights or DS-weights, for short). In static environments, our approach collapses to theirs, as we explain in Section G.3.4 of the Online Appendix. In dynamic stochastic environments, using DS-weights allows us to formalize a new, larger set of welfare criteria and to understand the normative implications for aggregate efficiency, risk-sharing, intertemporal-sharing, and redistribution of different welfare criteria, including the widely used welfarist criteria. In particular, Section 4 leverages DS-weights to provide a novel interpretation of how welfarist planners trade off welfare gains or losses across individuals in dynamic stochastic environments, a result at the heart of the question of how to make interpersonal welfare comparisons.[6]

Welfare decompositions. Our results, in particular the aggregate additive decomposition introduced in Proposition 1, contribute to the work that seeks to decompose welfare changes in models with heterogeneous agents. The most recent contribution to this literature is the work by Bhandari et al. (2021), who propose a decomposition of welfare changes when switching from a given policy to another that can be applied to a larger set of economies than the seminal contributions of Benabou (2002) and Floden (2001).[7] A fundamental difference between these papers and ours is that, in addition to decomposing the aggregate welfare effects of a policy change, our approach allows us to define a new set of normative criteria that can be used to endow a planner/policymaker with a specific mandate.

Purely from the perspective of the decomposition of welfare assessments, there are other significant differences between the approaches of Benabou (2002) and Bhandari et al. (2021) and ours, as we describe in Section G.3.6 of the Online Appendix. In particular, no existing decomposition satisfies Proposition 6, in which we show that all normalized welfarist planners conclude that the risk-sharing and intertemporal-sharing components are zero when markets are complete; Proposition 7, in which we show that all normalized welfarist planners conclude that intertemporal-sharing component

[6]The central insight in Saez and Stantcheva (2016) is that by using generalized weights it is possible to accommodate alternatives to welfarism, such as equality of opportunity, libertarianism, or Rawlsianism, among others. It should be evident that our approach, which nests theirs, can also accommodate these possibilities. We purposefully avoid studying these issues, since these normative approaches are rarely used in the study of dynamic stochastic economies.

[7]At an intuitive level, the decomposition proposed by Benabou (2002) and Floden (2001) is based on certainty equivalents, while the decomposition of Bhandari et al. (2021) is based on allocations. Our decomposition is instead based on marginal utilities. Note that Benabou (2002) states the following:

> "*I will also compute more standard social welfare functions, which are aggregates of (intertemporal) utilities rather than risk-adjusted consumptions. These have the clearly desirable property that maximizing such a criterion ensures Pareto efficiency. On the other hand, it will be seen that they cannot distinguish between the effects of policy that operate through its role as a substitute for missing markets, and those that reflect an implicit equity concern.*"

Our results show that it is actually possible to distinguish between the effects of policy that substitute for missing markets and those that reflect equity concerns when using standard social welfare functions.

is zero when individuals can freely trade a riskless bond; and Proposition 8, in which we show that different normalized welfarist planners exclusively disagree on the redistribution component. The decomposition proposed by Bhandari et al. (2021) does not satisfy Proposition 9, in which we show that the efficiency components (aggregate efficiency, risk-sharing, and intertemporal-sharing) of the aggregate additive decomposition are invariant to monotonically increasing transformations of individual's lifetime utilities and positive affine (increasing linear) transformations of individual's instantaneous utilities.

Welfare assessments in dynamic stochastic models. Our results are also related to the Lucas (1987) approach to making welfare assessments in dynamic environments, in particular to its marginal formulation introduced in Alvarez and Jermann (2004). Formally, as we show in Section G.3.4 of the Online Appendix, the marginal approach to making welfare assessments of Alvarez and Jermann (2004) corresponds to choosing a particular set of DS-weights. While both Lucas (1987) and Alvarez and Jermann (2004) study representative-agent environments, others have used a similar approach in environments with heterogeneity; see e.g., Atkeson and Phelan (1994), Krusell and Smith (1999), or Krusell et al. (2009), among many others. However, as highlighted by these papers, a well-known downside of the Lucas (1987) approach is that it does not aggregate meaningfully because individual welfare assessments are reported as constant shares of individual consumption. In this paper, we show that normalized welfarist planners — which we introduce — are able to meaningfully aggregate welfare assessments among heterogeneous individuals.

Institutional mandates. Finally, our results contribute to the literature that studies policymakers' mandates. For instance, Woodford (2003) shows in a representative agent economy that endowing a monetary authority with the objective to minimize inflation and output gaps maximizes instantaneous welfare. Relatedly, Rogoff (1985) shows that choosing a particular planner (a conservative central banker) may be desirable in some circumstances. However, the literature on institutional mandates has eschewed cross-sectional considerations. We hope that the approach we develop in this paper opens the door to future disciplined discussions on policy-making mandates, in particular when trading off efficiency and redistribution motives in dynamic stochastic environments.

Outline. Section 2 introduces the baseline environment and describes conventional approaches used to make welfare assessments. Section 3 introduces the notion of DS-weights, defines an individual multiplicative decomposition of DS-weights, an aggregate additive decomposition of welfare assessments, and provides general properties of such decompositions. Section 4 studies how welfarist planners make welfare assessments through the lens of DS-weights, characterizing properties of the aggregate additive decomposition in that case. Section 5 formalizes new welfare criteria that isolate different components of the aggregate additive decomposition and discusses the implications of such planners for institutional design. Section 6 further decomposes the components of the aggregate additive decomposition, explains how to connect welfare assessments to measures of inequality, describes how to make welfare assessments in recursive environments, shows how to make welfare assessments via an instantaneous Social Welfare Function, and introduces a term structure of welfare assessments. Section 7 illustrates how to employ the approach introduced in this paper in the context of a fully specified dynamic stochastic model. All proofs and derivations are in the Appendix. The Online Appendix also includes several extensions and additional results.

2 Environment and Benchmarks

In this section, we first describe our baseline environment, which encompasses a wide variety of dynamic stochastic models with heterogeneous individuals. Subsequently, we describe the conventional approaches to making welfare assessments, setting the stage for the introduction of DS-weights in Section 3.

2.1 Baseline Environment

Our notation closely follows that of Ljungqvist and Sargent (2018). We consider an economy populated by individuals, indexed by $i \in I$. For simplicity, we assume that there is a unit measure of individuals, so $\int di = 1$, although our results apply unchanged to economies with a finite number of individuals. At each date $t \in \{0, \ldots, T\}$, where $T \leq \infty$, there is a realization of a stochastic event $s_t \in S$. We denote the history of events up to and until date t by $s^t = (s_0, s_1, \ldots, s_t)$. We denote the unconditional probability of observing a particular sequence of events s^t by $\pi_t \left(s^t \middle| s_0 \right)$. We assume that the initial value of s_0 is predetermined, so $\pi_0 \left(s^0 | s_0 \right) = 1$.

There is a single nonstorable consumption good — which serves as numeraire — at all dates and histories. Each individual i derives utility from consumption and (dis)utility from working, with a lifetime utility representation, starting from s_0, given by

$$V_i \left(s_0 \right) = \sum_{t=0}^{T} \left(\beta_i \right)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0 \right) u_i \left(c_t^i \left(s^t \right), n_t^i \left(s^t \right) \right), \tag{1}$$

where $c_t^i \left(s^t \right)$ and $n_t^i \left(s^t \right)$ respectively denote the consumption and hours worked by individual i at history s^t; $u_i \left(\cdot \right)$ corresponds to individual i's instantaneous utility, potentially non-separable between consumption and hours; and $\beta_i \in [0, 1)$ denotes individual i's discount factor.[8] Note that Equation (1) corresponds to the time-separable expected utility preferences with exponential discounting and homogeneous beliefs commonly used in dynamic macroeconomics and finance. Note also that we purposefully allow for individual-specific preferences.

We assume that preferences are well-behaved and, for now, directly impose that $c_t^i \left(s^t \right)$ and $n_t^i \left(s^t \right)$ are smooth functions of a primitive parameter $\theta \in [0, 1]$, so

$$\frac{dc_t^i \left(s^t \right)}{d\theta} \quad \text{and} \quad \frac{dn_t^i \left(s^t \right)}{d\theta}$$

[8] Following Acemoglu (2009), we refer to $V_i \left(\cdot \right)$ as *lifetime* utility and to $u_i \left(\cdot \right)$ as *instantaneous* utility. As in Ljungqvist and Sargent (2018), we use a subscript i to refer to $V_i \left(\cdot \right)$, β_i, and $u_i \left(\cdot \right)$, and a superscript i to refer to individual variables indexed by time or histories.

are well-defined. We interpret changes in θ as policy changes although, at this level of generality, our approach is valid for any change in primitives. This formulation allows us to consider a wide range of policies, as we illustrate in our applications.[9] In those applications — and more generally — the mapping between outcomes, $c_t^i(s^t)$ and $n_t^i(s^t)$, and policy, θ, emerges endogenously, and typically accounts for general equilibrium effects. However, for most of the paper, we can proceed without further specifying endowments, budget constraints, equilibrium notions, etc.

In the Online Appendix, we extend our results to more general environments. In particular, in Section F.1, we describe how to account for heterogeneous beliefs. In Sections F.2 and F.3, we show how our approach extends to richer preference specifications, in particular, the widely used Epstein-Zin preferences. In Section F.3 we show how our results extend to economies with multiple commodities. In Section F.4, we describe how to extend our approach to environments in which preferences and probabilities directly depend on θ. Finally, in Section F.5 we describe how to allow for births, deaths, and related intergenerational considerations.

2.2 Benchmarks: Conventional Approaches to Welfare Assessments

Before introducing DS-weights, we first define in our environment i) Pareto-improving policies and ii) desirable policies for a welfarist planner. To that end, it is useful to characterize the change in the lifetime utility of an individual i induced by a marginal policy change, $\frac{dV_i(s_0)}{d\theta}$.

Lifetime utility effect of policy change. Starting from Equation (1), $\frac{dV_i(s_0)}{d\theta}$, which is measured in utils (utility units), can be expressed as

$$\frac{dV_i(s_0)}{d\theta} = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0 \right) \frac{\partial u_i(s^t)}{\partial c_t^i} \frac{du_{i|c}(s^t)}{d\theta}, \tag{2}$$

where we respectively denote individual i's marginal utilities of consumption and hours worked at history s^t by

$$\frac{\partial u_i(s^t)}{\partial c_t^i} = \frac{\partial u_i\left(c_t^i(s^t), n_t^i(s^t)\right)}{\partial c_t^i(s^t)} \quad \text{and} \quad \frac{\partial u_i(s^t)}{\partial n_t^i} = \frac{\partial u_i\left(c_t^i(s^t), n_t^i(s^t)\right)}{\partial n_t^i(s^t)},$$

and where we denote the *instantaneous consumption-equivalent effect* of the policy at history s^t by

$$\frac{du_{i|c}(s^t)}{d\theta} = \frac{\frac{du_i\left(c_t^i(s^t), n_t^i(s^t)\right)}{d\theta}}{\frac{\partial u_i(s^t)}{\partial c_t^i}} = \frac{dc_t^i(s^t)}{d\theta} + \frac{\frac{\partial u_i(s^t)}{\partial n_t^i}}{\frac{\partial u_i(s^t)}{\partial c_t^i}} \frac{dn_t^i(s^t)}{d\theta}. \tag{3}$$

[9] The fact that θ is one-dimensional is not restrictive, since θ can be interpreted as the scale of an arbitrary policy variation that can differ across individuals, dates, and histories. An advantage of formulating our approach in terms of marginal welfare assessments is that there is no ambiguity about how to make welfare assessments in units of a numeraire for a single individual — see Schlee (2013) for a formal proof. We explain how to use our approach to consider global assessments in Sections G.3.4 and G.5 of the Online Appendix.

Equation (2) shows that the impact of a policy change on the lifetime utility of individual i is given by a particular combination of instantaneous consumption-equivalent effects, which, importantly, are expressed in consumption units at a specific history. The relevance of each of these effects for $\frac{dV_i(s_0)}{d\theta}$ is determined by $(\beta_i)^t \pi_t \left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i}$, that is, by how far in the future and how likely a given history is, and by how much individual i values (in utils) a marginal unit of consumption at that particular history. Equation (3) highlights that the instantaneous consumption-equivalent effect at a given history depends on how consumption and hours worked respond to the policy change, as well as on the rate at which an individual trades off both variables, captured by the individual marginal rate of substitution between consumption and hours worked, given by $\frac{\partial u_i(s^t)}{\partial n_t^i} / \frac{\partial u_i(s^t)}{\partial c_t^i}$.[10]

Pareto-improving policy change. Equation (2) allows us to determine whether an individual is better or worse off after a policy change. That is, when $\frac{dV_i(s_0)}{d\theta} > (<) \, 0$, individual i perceives to be better (worse) off after a policy change. Hence, it is possible to define a Pareto-improving policy change as follows.

Definition 1. *(Pareto-improving policy change) A policy change is strictly (weakly) Pareto-improving if every individual i perceives to be strictly (weakly) better off after the policy change. Hence, a policy change is strictly Pareto-improving when $\frac{dV_i(s_0)}{d\theta} > 0$, $\forall i$, and weakly Pareto-improving when $\frac{dV_i(s_0)}{d\theta} \geq 0$, $\forall i$.*

Note that the notion of Pareto improvement does not involve interpersonal welfare comparisons, and simply exploits the ordinal nature of utility. While Pareto improvements seem highly desirable, they are rare to find, which forces planners/policymakers to make interpersonal welfare comparisons, as we describe next.[11]

Desirable policy change for a welfarist planner. The conventional approach in economics to balance welfare gains or losses among different individuals is based on individualistic social welfare functions (SWF). As in Kaplow (2011) or Saez and Stantcheva (2016), we refer to this approach — typically traced back to Bergson (1938) and Samuelson (1947) — as the *welfarist* approach. For a welfarist planner, social welfare is a real-valued function of individuals' lifetime utilities, which we formally denote in our environment by

$$\mathcal{W}\left(\{V_i(s_0)\}_{i \in I}\right), \qquad \text{(welfarist planner)} \tag{4}$$

[10] Note that the definition of the instantaneous consumption-equivalent effect in Equation (3) does not make use of individual optimality (i.e., the envelope theorem). However, in specific applications, exploiting individual optimality conditions can yield simple expressions for $\frac{du_{i|c}(s^t)}{d\theta}$.

[11] As shown by Mas-Colell, Whinston and Green (1995) or Ljungqvist and Sargent (2018), among others, by varying the welfare weights assigned to different individuals, a planner who maximizes a utilitarian social welfare function can fully trace the Pareto frontier whenever a utility possibility set is convex, and partially when it is not. Even though characterizing Pareto frontiers is a valuable exercise, we seek to study welfare assessments generally, even away from the Pareto frontier. Moreover, the aggregate additive decomposition of welfare assessments that we introduce in this paper can also be used at the Pareto frontier.

where $V_i(s_0)$ is defined in Equation (1) and where typically $\frac{\partial \mathcal{W}}{\partial V_i} \geq 0$, $\forall i$. As carefully explained in Kaplow (2011), the critical restriction implied by the welfarist approach is that the social welfare function $\mathcal{W}(\cdot)$ cannot depend on any model outcomes besides individual utility levels.

Different welfarist social welfare functions $\mathcal{W}(\cdot)$ have different implications for the assessment of policies. In particular, the utilitarian SWF, which adds up a weighted sum of individual utilities, is given by

$$\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right) = \int \overline{\lambda}_i V_i(s_0)\, di, \qquad \text{(utilitarian planner)} \tag{5}$$

where $\overline{\lambda}_i$ are a set of predetermined individual-specific scalars, commonly referred to as Pareto weights. While the utilitarian SWF is by far the most used in practice, there exist other well-known SWF's, such as isoelastic (Atkinson, 1970) and maximin/Rawlsian (Rawls, 1971, 1974), among others, as we describe in Section G.3.1 of the Online Appendix.

Next, we formally define when a policy change is desirable for a welfarist planner.

Definition 2. *(Desirable policy change for a welfarist planner) A welfarist planner finds a policy change desirable if and only if* $\frac{dW^{\mathcal{W}}(s_0)}{d\theta} > 0$*, where*

$$\begin{aligned}\frac{dW^{\mathcal{W}}(s_0)}{d\theta} &= \int \lambda_i(s_0) \frac{dV_i(s_0)}{d\theta} di \\ &= \int \lambda_i(s_0) \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i} \frac{du_{i|c}(s^t)}{d\theta} di,\end{aligned} \tag{6}$$

where $\lambda_i(s_0) = \frac{\partial \mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right)}{\partial V_i}$*, and where* $\frac{dV_i(s_0)}{d\theta}$ *is defined in Equation (2).*

The properties of the welfarist approach have been widely studied.[12] In particular, a welfarist planner is non-paternalistic, since aggregate welfare assessments are based on individual welfare assessments, and Paretian when $\frac{\partial \mathcal{W}}{\partial V_i} \geq 0$, $\forall i$, since every Pareto-improving policy is desirable. Moreover, when individuals are ex-ante homogeneous, i.e., they have identical preferences and face an identical environment from the perspective of s_0, all welfarist planners agree on whether a policy change is desirable or not, even if individuals experience different shocks ex-post.[13]

However, because of the ordinal nature of individual utilities, it is not easy to understand how a welfarist planner exactly makes tradeoffs among individuals that are ex-ante heterogeneous along some dimension. For instance, a welfarist planner would mechanically put more weight on the gains or losses of an individual whose lifetime utility is multiplied by a positive constant factor, even though, since individual utility is ordinal, this has no impact on allocations. Relatedly, it is not clear how a welfarist planner trades off the welfare gains or losses of individuals with different preferences, endowments, or life-cycle profiles; who have access to different insurance opportunities; or who face

[12] See e.g., Mas-Colell, Whinston and Green (1995), Kaplow (2011), or Adler and Fleurbaey (2016) for recent textbook treatments. Somewhat surprisingly, dynamic and stochastic considerations are not central to the literature on policy assessments.

[13] Even in this case, it is not obvious to determine whether a welfarist planner finds a policy change desirable because of aggregate efficiency or risk-sharing considerations, as we illustrate in Section 4.

shocks driven by different stochastic processes.

By introducing Dynamic Stochastic weights, we are able to systematically i) provide a new transparent interpretation of how a particular planner (including all welfarist planners, but also other non-welfarist planners) implicitly trade off gains or losses across individuals, dates, and histories, and ii) define new welfare criteria that capture normative objectives that society may find appealing.

3 Dynamic Stochastic Weights

In this section, we introduce a new approach to assess the desirability of policy changes, based on the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (Dynamic Stochastic weights or DS-weights, for short).

3.1 Definition of DS-weights

We begin by formally defining when a policy change is desirable for a planner who adopts DS-weights, a "DS-planner."

Definition 3. *(Desirable policy change for a DS-planner/Definition of DS-weights) A DS-planner, that is, a planner who adopts DS-weights, finds a policy change desirable if and only if* $\frac{dW^{DS}(s_0)}{d\theta} > 0$*, where*

$$\frac{dW^{DS}(s_0)}{d\theta} = \int \sum_{t=0}^{T} \sum_{s^t} \omega_t^i \left(s^t \middle| s_0 \right) \frac{du_{i|c}(s^t)}{d\theta} di, \tag{7}$$

where $\frac{du_{i|c}(s^t)}{d\theta}$ *denotes the instantaneous consumption-equivalent effect of the policy at history* s^t*, defined in Equation (3), and where* $\omega_t^i (s^t | s_0) > 0$*, which can be a function of all the possible paths of outcomes, denotes the DS-weight assigned to individual* i *at history* s^t *for a welfare assessment that takes place at* s_0*.*

Equation (7) shows that, in order to carry out a welfare assessment, a DS-planner must i) know the instantaneous consumption-equivalent effect of a policy for each individual at all dates and histories, that is, $\frac{du_{i|c}(s^t)}{d\theta}$, $\forall i$, $\forall t$, $\forall s^t$, which is measured in consumption units; and ii) specify DS-weights $\omega_t^i (s^t | s_0)$ for each individual at all dates and histories, that is, $\omega_t^i (s^t | s_0)$, $\forall i$, $\forall t$, $\forall s^t$.[14] Hence, $\frac{du_{i|c}(s^t)}{d\theta}$ and $\omega_t^i (s^t | s_0)$ are sufficient statistics for welfare analysis, which makes the computation of welfare assessments conceptually straightforward. Intuitively, a DS-planner computes the impact of a policy change in consumption units at each history for every individual and then weights those changes to form an aggregate welfare assessment.[15] Different choices of DS-weights $\omega_t^i (s^t | s_0)$ will have different normative implications, as the remainder of this paper will show.

[14] To simplify the exposition, we focus on the case in which DS-weights are strictly positive for all individuals and histories. It is possible to extend our results to the case in which some DS-weights can be zero.

[15] Throughout the paper we use consumption as the numeraire for welfare assessments. In Section F.3 of the Online Appendix we explain how to define DS-planners based on other numeraires and how this may impact welfare assessments.

It is worth highlighting four features that define a DS-planner. First, note that DS-weights can be functions of model outcomes, which are typically endogenous variables. For instance, by comparing Equations (6) and (7), it follows that every welfarist planner can be interpreted as a DS-planner with DS-weights given by

$$\omega_t^i\left(s^t\middle|\, s_0\right) = \lambda_i\left(s_0\right)\left(\beta_i\right)^t \pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}, \tag{8}$$

where $\lambda_i\left(s_0\right) = \frac{\partial \mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right)}{\partial V_i}$. Second, by making s_0 an explicit argument of $\frac{dW^{DS}(s_0)}{d\theta}$, we emphasize that welfare assessments in dynamic stochastic economies are contingent on the state in which the assessment takes place. This observation may lead to time-inconsistency of welfare assessments, a topic we revisit in Section 6.3. Third, note that we define the welfare assessment of a DS-planner in marginal form, i.e, DS-weights are marginal welfare weights. This contrasts with the welfarist approach, which takes a lifetime social welfare function as primitive.[16] In Section 6.4, we show how a DS-planner can be equivalently defined in terms of an instantaneous social welfare function with generalized (endogenous) welfare weights. Finally, note that Equation (7) allows us to define a local optimum for a DS-planner as a value of θ for which $\frac{dW^{DS}(s_0)}{d\theta} = 0$. We explain how to conduct non-marginal welfare assessments in Section G.5 of the Online Appendix

3.2 Individual Multiplicative Decomposition of DS-weights

In Lemma 1, we introduce an individual multiplicative decomposition of DS-weights into i) individual, ii) dynamic, and iii) stochastic components.[17] This individual multiplicative decomposition of DS-weights is useful to i) provide a meaningful economic interpretation of how a planner trades off welfare gains or losses across individuals, dates, and histories, given a set of DS-weights; ii) formally define and study the aggregate additive decomposition of welfare assessments, as we show in Section 3.3; and iii) formalize welfare criteria by defining DS-weights in terms of each of its components, as we illustrate in Section 4. We also define a normalized decomposition, which is unique and has desirable properties, as we show throughout the paper.

Lemma 1. *(DS-weights: individual multiplicative decomposition; unique normalized decomposition)*

a) The DS-weights that a DS-planner assigns to an individual i can be multiplicatively decomposed into three different components, up to a choice of units, as follows:

$$\omega_t^i\left(s^t\middle|\, s_0\right) = \underbrace{\tilde{\omega}^i\left(s_0\right)}_{individual}\underbrace{\tilde{\omega}_t^i\left(s_0\right)}_{dynamic}\underbrace{\tilde{\omega}_t^i(s^t|s_0)}_{stochastic}, \qquad \text{where} \tag{9}$$

i) $\tilde{\omega}^i\left(s_0\right)$ corresponds to an individual component, which is invariant across all dates and histories;

[16] As we show in Section 4, it is of course possible to compute the DS-weights implied by a welfarist planner.

[17] This individual multiplicative decomposition is inspired by Alvarez and Jermann (2005) and Hansen and Scheinkman (2009), who multiplicatively decompose pricing kernels into permanent and transitory components.

ii) $\tilde{\omega}_t^i(s_0)$ *corresponds to a dynamic component, which can vary across dates, but not across histories at a given date; and*

iii) $\tilde{\omega}_t^i(s^t|\, s_0)$ *corresponds to a stochastic component, which can vary across dates and histories.*

b) For any set of DS-weights, there exists a unique "normalized" individual multiplicative decomposition, such that

i) stochastic components add up to 1 *at every date, that is,* $\sum_{s^t} \tilde{\omega}_t^i(s^t|\, s_0) = 1$, $\forall t$, $\forall i$;

ii) dynamic components add up to 1 *across all dates, that is,* $\sum_{t=0}^T \tilde{\omega}_t^i(s_0) = 1$, $\forall i$; *and*

iii) individual components add up to 1 *across individuals, that is,* $\int \tilde{\omega}^i(s_0)\, di = 1$.

We refer to planners who adopt this decomposition as "normalized" DS-planners.

The components of the individual multiplicative decomposition define social marginal rates of substitution for a DS-planner across individuals, dates, and histories. The stochastic component, $\tilde{\omega}_t^i(s^t|\, s_0)$, which has the interpretation of a risk-neutral measure at date t when $\sum_{s^t} \tilde{\omega}_t^i(s^t|\, s_0) = 1$, determines how a DS-planner values units of consumption good across different histories s^t at date t for a given individual. The dynamic component, $\tilde{\omega}_t^i(s_0)$, which has the interpretation of a normalized discount factor when $\sum_{t=0}^T \tilde{\omega}_t^i(s_0) = 1$, determines how a DS-planner values consumption across different dates for a given individual.[18] The individual component determines how a DS-planner trades off permanent gains or losses across individuals. In the case of the normalized decomposition, when $\int \tilde{\omega}^i(s_0)\, di = 1$, it defines the units in which $\frac{dW^{DS}(s_0)}{d\theta}$ is expressed. In particular, the individual component of individual i, $\tilde{\omega}^i(s_0)$, exactly determines the weight that a DS-planner gives to a permanent transfer of one unit of consumption good at all dates and histories to individual i, measured in units of a permanent transfer of one unit of consumption good to all individuals at all dates and histories.

It is worth highlighting that the sign of $\frac{dW^{DS}(s_0)}{d\theta}$ — and hence whether a policy change is desirable or not — is fully determined by the value of the DS-weights as a whole and not by any individual multiplicative decomposition. However, we will show that the normalized individual multiplicative decomposition is associated with desirable properties in the context of the aggregate additive decomposition that we introduce next, while unnormalized decompositions typically are not. The normalized decomposition guarantees that its components, as well as $\frac{dW^{DS}(s_0)}{d\theta}$, have a meaningful interpretation in terms of units of consumption across specific histories, dates, and individuals. In general, once the units of $\omega_t^i(s^t|\, s_0)$ and its components are defined, every individual multiplicative decomposition is unique. See Section 4, and Section G.1 of the Online Appendix for further details.

[18]Risk-neutral measures are widely used in finance (Duffie, 2001; Cochrane, 2005), while normalized discount factors are common in the study of repeated games (Fudenberg and Tirole, 1991; Mailath and Samuelson, 2006).

For instance, a possible individual multiplicative decomposition for an (unnormalized) welfarist planner is given by

$$\tilde{\omega}^i(s_0) = \lambda_i(s_0), \quad \tilde{\omega}^i_t(s_0) = (\beta_i)^t, \quad \text{and} \quad \tilde{\omega}^i_t\left(s^t \middle| s_0\right) = \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c^i_t}, \tag{10}$$

where $\lambda_i(s_0) = \frac{\partial \mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right)}{\partial V_i}$. This decomposition, because it is expressed in utils, cannot be used to understand how a planner makes tradeoffs in terms of consumption units. In Section 4, we instead introduce the individual multiplicative decomposition of a normalized welfarist planner, which allows us to describe how a welfarist DS-planner precisely makes tradeoffs in consumption units. In that section, we also show that using a normalized individual multiplicative decomposition of DS-weights is associated with desirable properties for the aggregate additive decomposition of welfare assessments, which we introduce next.

3.3 Aggregate Additive Decomposition of Welfare Assessments

Armed with the individual multiplicative decomposition of DS-weights, we now introduce an exact additive decomposition of the welfare assessments made by a DS-planner. This decomposition shows that the welfare assessment of a policy change $d\theta$ made by a DS-planner is driven by exactly four considerations: aggregate efficiency, risk-sharing, intertemporal-sharing, and redistribution.[19]

Proposition 1. *(Welfare assessments: aggregate additive decomposition) The aggregate welfare assessment of a DS-planner, $\frac{dW^{DS}(s_0)}{d\theta}$, can be decomposed into four components: i) an aggregate efficiency component, ii) a risk-sharing component, iii) an intertemporal-sharing component, and iv) a redistribution component, as follows:*

[19] We have chosen the term risk-sharing and the (less conventional) term intertemporal-sharing to highlight that both components of the aggregate additive decomposition are driven by cross-sectional differences, via interpersonal sharing. Alternative terms, such as insurance, consumption smoothing, or intertemporal smoothing, do not have such connotation, since they could be applied to a single individual.

$$
\begin{aligned}
\frac{dW^{DS}(s_0)}{d\theta} = & \underbrace{\sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]\sum_{s^t}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)\right]\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{AE}\ \textit{(Aggregate Efficiency)}} \\
& + \underbrace{\sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]\sum_{s^t}\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right),\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{RS}\ \textit{(Risk-sharing)}} \\
& + \underbrace{\sum_{t=0}^{T}\mathbb{C}ov_i\left[\tilde{\omega}_t^i(s_0),\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{IS}\ \textit{(Intertemporal-sharing)}} \\
& + \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{RD}\ \textit{(Redistribution)}},
\end{aligned}
\tag{11}
$$

where $\mathbb{E}_i[\cdot]$ *and* $\mathbb{C}ov_i[\cdot,\cdot]$ *respectively denote cross-sectional expectations and covariances, where the history-specific term that determines the aggregate efficiency component,* $\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right]$*, is given by*

$$
\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right] = \int\frac{dc_t^i(s^t)}{d\theta}di + \int\frac{\frac{\partial u_i(s^t)}{\partial n_t^i}}{\frac{\partial u_i(s^t)}{\partial c_t^i}}\frac{dn_t^i(s^t)}{d\theta}di, \tag{12}
$$

and where, without loss of generality, we have assumed that $\mathbb{E}_i\left[\tilde{\omega}^i(s_0)\right] = \int\tilde{\omega}^i(s_0)\,di = 1$*.*

The first component of the aggregate additive decomposition is the aggregate efficiency component, Ξ_{AE}. This component accounts for the aggregate instantaneous consumption-equivalent effect of the policy, expressed in consumption units. As shown in Equation (12), Ξ_{AE} adds up — after appropriately discounting — the changes in consumption-equivalents resulting from the marginal policy change across all dates and histories. Because Ξ_{AE} can be computed using exclusively cross-sectional averages of $\tilde{\omega}_t^i(s_0)$, $\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)$, and $\frac{du_{i|c}(s^t)}{d\theta}$, we refer to the this term as aggregate efficiency.[20]

[20]Note that Equation (12) can be rewritten as

$$
\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right] = \int\frac{dc_t^i(s^t)}{d\theta}\tau_t^i\left(s^t\right)di = \mathbb{E}_i\left[\frac{dc_t^i(s^t)}{d\theta}\right]\mathbb{E}_i\left[\tau_t^i\left(s^t\right)\right] + \mathbb{C}ov\left[\frac{dc_t^i(s^t)}{d\theta},\tau_t^i\left(s^t\right)\right],
$$

where $\tau_t^i\left(s^t\right) = 1 + \frac{\frac{\partial u_i(s^t)}{\partial n_t^i}}{\frac{\partial u_i(s^t)}{\partial c_t^i}}\frac{\frac{dn_t^i(s^t)}{d\theta}}{\frac{dc_t^i(s^t)}{d\theta}}$, which shows that aggregate efficiency is tightly connected to labor wedges. Intuitively, policies that increases aggregate consumption contribute more to aggregate efficiency when the aggregate labor wedge is greater than 1, i.e., when $\mathbb{E}_i\left[\tau_t^i\left(s^t\right)\right] > 1$. Alternatively, policies that do not change aggregate consumption can contribute to aggregate efficiency if they increase the consumption of those individuals with higher individual labor wedges by more. More generally, in production economies, the aggregate efficiency component is

The remaining three components of the aggregate additive decomposition are driven by the cross-sectional variation of each of the three elements (individual, dynamic, stochastic) of the individual multiplicative decomposition of DS-weights. In particular, the risk-sharing component, Ξ_{RS}, adds up across all dates and histories the covariances between the stochastic component, $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, and the instantaneous consumption-equivalent effect at each date and history. Similarly, the intertemporal-sharing component, Ξ_{IS}, adds up across all dates the covariances between the dynamic component, $\tilde{\omega}_t^i(s_0)$, and the (expected, under the risk-neutral measure interpretation of stochastic weights) instantaneous consumption-equivalent effect at each date. Finally, the redistribution component, Ξ_{RD}, consists of a single cross-sectional covariance between the individual component, $\tilde{\omega}^i\left(s^0\right)$, and the present discounted value — using the dynamic and stochastic components — of instantaneous consumption-equivalent effects that a DS-planner assigns to a particular individual.

Before we discuss the properties of this decomposition below, it is worth making three remarks. First, the aggregate additive decomposition is exact for any marginal policy change and does not rely on any approximations. Relatedly, the decomposition can be computed using only the individual multiplicative decomposition of DS-weights — typically a function of model outcomes — and instantaneous consumption-equivalent effects.

Second, the aggregate additive decomposition is based on cross-sectional averages and covariances, and does not include covariances over future periods or histories. In Section 6.1, we further decompose the aggregate efficiency and redistribution components along those lines, developing a stochastic decomposition — see Propositions 12 and 14. There, we also provide an alternative decomposition of the risk-sharing and intertemporal-sharing components still based on cross-sectional averages and covariances.[21]

Finally, one can interpret the aggregate additive decomposition as first separating efficiency and redistribution, and then further decomposing efficiency into aggregate efficiency, risk-sharing, and intertemporal-sharing. Formally, $\frac{dW^{DS}(s_0)}{d\theta}$ can be written as

$$\frac{dW^{DS}(s_0)}{d\theta} = \underbrace{\Xi_E}_{\text{Efficiency}} + \underbrace{\Xi_{RD}}_{\text{Redistribution}}, \quad \text{where} \quad \Xi_E = \Xi_{AE} + \Xi_{RS} + \Xi_{IS}.$$

This distinction will become clear in Section 4.2, in which we show that differences in welfare assessments among normalized welfarist planners are exclusively based on the redistribution component Ξ_{RD} and that Pareto improving policies must necessarily feature $\Xi_E > 0$.

3.4 General Properties of the Aggregate Additive Decomposition

The merits of the aggregate additive decomposition introduced in Proposition 1 lie in its properties. Similarly, the names we attribute to each of the components, Ξ_{AE} through Ξ_{RD}, are only meaningful

tightly linked to production efficiency, a relation that we plan to explore in future work.

[21] The aggregate decomposition introduced in Proposition 1 is appealing because it systematically treats each of the components of the individual multiplicative decomposition. That is, Ξ_{RS} is directly determined by $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, Ξ_{IS} by $\tilde{\omega}_t^i(s_0)$, and Ξ_{RD} by $\tilde{\omega}^i(s_0)$.

insofar as they satisfy desirable properties. Hence, in the remainder of this section, we present properties of the aggregate additive decomposition and its components for a general DS-planner.

First, in Proposition 2, we identify conditions on DS-weights and their components under which the welfare assessments of a DS-planner i) are purely based on aggregate efficiency considerations or ii) are such that the risk-sharing, intertemporal-sharing, or redistribution components are zero.

Proposition 2. *(Properties of aggregate additive decomposition: individual-invariant DS-weights)*

a) If DS-weights $\omega_t^i \left(s^t \middle| s_0 \right)$ are constant across all individuals at all dates and histories, then the welfare assessment of a DS-planner is exclusively based on aggregate efficiency considerations, i.e., $\Xi_{RS} = \Xi_{IS} = \Xi_{RD} = 0$.

b) If the stochastic component of DS-weights is constant across all individuals at all dates and histories, then $\Xi_{RS} = 0$.

c) If the dynamic component of DS-weights is constant across all individuals at all dates, then $\Xi_{IS} = 0$.

d) If the individual component of DS-weights is constant across all individuals, then $\Xi_{RD} = 0$.

Proposition 2 shows that a DS-planner who assigns DS-weights that do not vary across individuals at all dates and histories makes welfare assessments purely based on aggregate efficiency considerations. This result bears a resemblance to the classic question of defining a normative representative consumer — see e.g., Mas-Colell, Whinston and Green (1995) or Acemoglu (2009). In particular, Proposition 2a) implies that the risk-sharing, intertemporal-sharing, and redistribution components are zero in single-agent or representative-agent economies in which all individuals have the same DS-weights, i.e., DS-weights are symmetric. Parts b) through d) of Proposition 2 also show that, depending on which specific components of the individual multiplicative decomposition of DS-weights are invariant across individuals, it may be that $\Xi_{RS} = 0$, $\Xi_{IS} = 0$, or $\Xi_{RD} = 0$. These results highlight the cross-sectional nature of the risk-sharing, intertemporal-sharing, and redistribution components. Moreover, parts c) and d) of Proposition 2 respectively imply that the intertemporal-sharing and the redistribution components are always zero when individuals are ex-ante identical.

Given their practical importance, we highlight several immediate implications of Proposition 2 in four corollaries.[22]

Corollary 1. *(Representative-agent economies) Welfare assessments in single- or representative-agent economies in which DS-weights are symmetric are exclusively attributed to aggregate efficiency considerations, i.e., $\Xi_{RS} = \Xi_{IS} = \Xi_{RD} = 0$.*

[22]We say that DS-weights are symmetric when two individuals with identical preferences and identical paths for consumption and hours are assigned identical DS-weights. This is a natural restriction when making welfare assessments — see e.g., Mas-Colell, Whinston and Green (1995) for a discussion of symmetry. Corollaries 2 and 4 require a normalized individual multiplicative decomposition so that the choice of units of $\omega_t^i \left(s^t \middle| s_0 \right)$ and $\omega_t^i (s_0)$ does not generate meaningless cross-sectional variation when computing Ξ_{RS} and Ξ_{IS}.

Corollary 2. *(Perfect-foresight economies) Welfare assessments in perfect-foresight economies in which the individual multiplicative decomposition of DS-weights is normalized are never attributed to risk-sharing, i.e.,* $\Xi_{RS} = 0$.

Corollary 3. *(Economies with ex-ante identical individuals) Welfare assessments in economies in which all individuals are ex-ante identical (but not necessarily ex-post) and DS-weights are symmetric are never attributed to intertemporal-sharing or redistribution, i.e.,* $\Xi_{IS} = \Xi_{RD} = 0$.

Corollary 4. *(Static economies) Welfare assessments in static economics in which the individual multiplicative decomposition of DS-weights is normalized are exclusively attributed to aggregate efficiency or redistribution considerations, i.e.,* $\Xi_{RS} = \Xi_{IS} = 0$.

In Proposition 3, we identify conditions on policies under which the welfare assessments of a DS-planner i) are purely based on aggregate efficiency considerations or ii) are such that the risk-sharing, or the risk-sharing and the intertemporal-sharing components are zero. Generically, a policy change will affect all four components of the aggregate additive decomposition. Hence, to guarantee that some components of the aggregate decomposition are zero, Proposition 3 identifies policies that impact all individuals identically along certain dimensions.

Proposition 3. *(Properties of aggregate additive decomposition: individual-invariant policies) Suppose that the individual multiplicative decomposition of DS-weights is normalized, so* $\sum_{s^t} \tilde{\omega}_t^i \left(s^t \middle| s_0\right) = 1$, $\forall t$, $\forall i$, *and* $\sum_{t=0}^{T} \tilde{\omega}_t^i (s_0) = 1$, $\forall i$. *If the instantaneous consumption-equivalent effect of a policy change,* $\frac{du_{i|c}(s^t)}{d\theta}$, *is identical across individuals*

- *a) at all dates and histories, then the welfare assessment of a DS-planner is exclusively based on aggregate efficiency, i.e.,* $\Xi_{RS} = \Xi_{IS} = \Xi_{RD} = 0$.
- *b) at all histories on a date, for all dates, then the welfare assessment of a DS-planner is based on aggregate efficiency and redistribution, i.e.,* $\Xi_{RS} = \Xi_{IS} = 0$.
- *c) conditional on a date and history, for all dates and histories, then the welfare assessment of a DS-planner is based on aggregate efficiency, intertemporal sharing, and redistribution, i.e.,* $\Xi_{RS} = 0$.

Proposition 3a) shows that a policy change that affects all individuals identically across all dates and histories can only affect aggregate welfare via aggregate efficiency considerations. Proposition 3b) shows that a policy change that varies over time but affects all agents identically across all histories at a given date can affect aggregate welfare via aggregate efficiency and redistribution, but not risk-sharing or intertemporal-sharing. Proposition 3c) shows that a policy change that affects all individuals identically conditional on a history taking place but that can vary across dates and individuals will have no risk-sharing component. It should be evident that, for generic DS-weights, the converse of these results also holds. That is, policy changes must affect different individuals differently if they load on the risk-sharing, intertemporal-sharing, or redistribution components of the aggregate additive decomposition.

Proposition 3 critically relies on considering a normalized individual multiplicative decomposition of (the dynamic and stochastic components of) DS-weights. As highlighted above, such normalization guarantees that the components of the individual multiplicative decomposition have meaningful units, which makes it possible to derive conditions on how policies affect individuals in terms of consumption. See Section G.1 of the Online Appendix for further details.

Finally, we show in Proposition 4 that, in an endowment economy, aggregate efficiency considerations play no role for a DS-planner when making normative assessments. We use the term endowment economy to refer to economies in which all consumption comes from predetermined endowments of the consumption good at each date and history, and individuals' instantaneous utility exclusively depends on consumption. If individual utility depends on other variables, Proposition 4 remains valid only when the sum of instantaneous consumption-equivalent effects is zero.

Proposition 4. *(Properties of aggregate additive decomposition: endowment economies) In an endowment economy in which the aggregate endowment of the consumption good is invariant to policy, the aggregate efficiency component of the welfare assessment of a DS-planner is zero for any set of DS-weights, i.e.,* $\Xi_{AE} = 0$.

Altogether, Propositions 2 through 4 as well as Corollaries 1 through 4 show that the additive aggregate decomposition satisfies desirable properties for any DS-planner.

4 Normalized Welfarist Planners

One of the challenges of the welfarist approach is to understand how a particular planner makes tradeoffs among heterogeneous individuals, because of the ordinal nature of individual utilities. In Section 4.1, we first show how to systematically characterize — critically, in easily interpretable consumption units — how a welfarist DS-planner makes such tradeoffs across periods and histories for a given individual, and across individuals. Next, in Section 4.2, we characterize new additional properties of the aggregate additive decomposition of welfare assessments for normalized welfarist planners.

We focus on defining and studying normalized welfarist planners because virtually all applied work uses a welfarist approach and because the welfare assessments of normalized welfarist planners satisfy highly desirable properties. In particular, we show that all normalized welfarist planners conclude that the risk-sharing and intertemporal-sharing components are zero when markets are complete, that the intertemporal-sharing component is zero when individuals can freely trade a riskless bond, and that different normalized welfarist planners — with different SWF's $\mathcal{W}(\cdot)$ — exclusively disagree on the redistribution component. We also show that the efficiency components (aggregate efficiency, risk-sharing, and intertemporal-sharing) of the aggregate additive decomposition are invariant to ordinal utility transformations and that Pareto improving policies always increase efficiency.

4.1 Individual Multiplicative Decomposition for Normalized Welfarist Planners

Proposition 5 characterizes the unique normalized individual multiplicative decomposition of DS-weights for a given welfarist planner, i.e., for a given SWF, $\mathcal{W}(\cdot)$, defined in Equation (4). By computing normalized DS-weights, we can explicitly determine how a welfarist DS-planner makes tradeoffs — critically, in easily interpretable consumption units —across periods and histories for a given individual, and across individuals.

Proposition 5. *(Normalized welfarist planners: individual multiplicative decomposition) The unique normalized individual multiplicative decomposition of DS-weights for a welfarist planner with SWF, $\mathcal{W}(\cdot)$, is given by*

$$\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\, s_0\right) = \frac{\pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\sum_{s^t}\pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}} \tag{13}$$

$$\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right) = \frac{\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\sum_{t=0}^{T}\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}} \tag{14}$$

$$\tilde{\omega}^{i,\mathcal{W}}\left(s_0\right) = \frac{\lambda_i\left(s_0\right)\sum_{t=0}^{T}\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\int\lambda_i\left(s_0\right)\sum_{t=0}^{T}\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}di}, \tag{15}$$

where $\lambda_i\left(s_0\right) = \frac{\partial\mathcal{W}\left(\left\{V_i\left(s_0\right)\right\}_{i\in I}\right)}{\partial V_i}$.

This normalization precisely describes how a welfarist planner makes tradeoffs. First, note that the instantaneous consumption-equivalent effect of the policy at date t and history s^t, $\frac{du_{i|c}\left(s^t\right)}{d\theta}$, is expressed in units of the consumption good (dollars) at such a history. The stochastic component, $\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\, s_0\right)$, can consequently be interpreted as the marginal rate of substitution between a dollar in history s^t and a dollar across all possible histories at date t for individual i from the planner's perspective. Formally, the denominator of Equation (13) corresponds to the marginal value of transferring one dollar in every possible history at date t. For instance, if the stochastic component is 0.4 for a given individual, history, and date, a welfarist planner equally values a one-dollar transfer at that particular history and a transfer of 0.4 dollars to the same individual in each history at that date.

The dynamic component, $\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)$, can similarly be interpreted as a marginal rate of substitution between a dollar at date t and a permanent dollar (i.e., a dollar paid at all dates, irrespective of histories) for individual i from the planner's perspective. Formally, the denominator of Equation (14) corresponds to the marginal value of permanently transferring one dollar across all dates and histories. For instance, if the dynamic component is 0.3 for a given individual and date, a welfarist planner equally values — for that individual — a one-dollar permanent transfer across all histories at that particular date and a transfer of 0.3 dollars at all dates, irrespective of histories. Both the stochastic and the dynamic components are thus useful because they allow the planner to meaningfully compare the welfare impact of policy changes across dates and histories for a given individual i.

Finally, the individual component, $\tilde{\omega}_t^{i,\mathcal{W}}(s_0)$, can be interpreted as the weight that a welfarist planner assigns to welfare changes for a given individual, expressed in terms of a permanent dollars. Formally, the denominator of Equation (15) corresponds to the marginal value of permanently transferring one marginal dollar to each individual in the economy across all dates and histories. For instance, if the individual component is 0.2 for a given individual, a welfarist planner equally values a one-dollar permanent transfer to that individual across all dates and histories and a permanent transfer of 0.2 dollars to all individuals across all dates and histories. It follows from Equation (15) that a welfarist planner gives more weight to individuals who are more patient, whose utility function has more curvature, who have lower consumption, and for whom $\lambda_i(s_0)$ is lower.

Several implications follow from Proposition 5. First, the welfare assessment of a normalized welfarist planner has a cardinal interpretation, since it is measured in dollars at all dates and histories for all individuals. In other words, if $\frac{dW^{\mathcal{W}}}{d\theta} = 0.1$, a normalized welfarist planner concludes that a marginal policy change is equivalent to a permanent transfer to all individuals at all dates and histories of 0.1 dollars.

Second, it is possible to reformulate the dynamic and stochastic normalized components as

$$\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\, s_0\right) = \frac{q_t^i\left(s^t|s_0\right)}{\sum_{s^t} q_t^i\left(s^t|s_0\right)} = \frac{\text{individual } i \text{ date-0 state-price of history } s^t}{\text{individual } i \text{ date-0 price of date-}t\text{ zero coupon bond}} \tag{16}$$

$$\tilde{\omega}_t^{i,\mathcal{W}}(s_0) = \frac{\sum_{s^t} q_t^i\left(s^t|s_0\right)}{\sum_{t=0}^{T}\sum_{s^t} q_t^i\left(s^t|s_0\right)} = \frac{\text{individual } i \text{ date-0 price of date-}t\text{ zero coupon bond}}{\text{individual } i \text{ date-0 price of } T\text{-consol bond}}, \tag{17}$$

where $q_t^i\left(s^t|s_0\right)$ denotes the state-price over history s^t from the perspective of individual i at date 0, given by[23]

$$q_t^i\left(s^t|s_0\right) = (\beta_i)^t\,\pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\Big/\frac{\partial u_i\left(s^0\right)}{\partial c_0^i}. \tag{18}$$

Equations (16) and (17) highlight that a welfarist planner makes tradeoffs across dates and histories for a given individual exclusively using the individual's own stochastic discount factor. This is a natural result, since welfarist planners are non-paternalistic.

Third, we can reformulate the individual normalized components as

$$\tilde{\omega}^{i,\mathcal{W}}(s_0) = \frac{\lambda_i(s_0)\frac{\partial u_i(s^0)}{\partial c_0^i}\sum_{t=0}^{T}\sum_{s^t} q_t^i\left(s^t|s_0\right)}{\int \lambda_i(s_0)\frac{\partial u_i(s^0)}{\partial c_0^i}\sum_{t=0}^{T}\sum_{s^t} q_t^i\left(s^t|s_0\right)di}, \tag{19}$$

where $q_t^i\left(s^t|s_0\right)$ is defined in Equation (19). In contrast to Equations (13) and (16), the exact form of the SWF $\mathcal{W}(\cdot)$ does impact the normalized individual components, a fact that is critical to show that welfarist planners exclusively disagree about the redistribution — see Proposition 8 below.[24]

[23]Consol bonds are typically defined as fixed-income securities with no maturity date. Since we consider economies that may have a finite horizon, we define a T-consol bond as a bond that pays at every date. When $T = \infty$, the conventional definition and ours coincide.

[24]Interestingly, as we discuss in Section G.3.4 of the Online Appendix, a planner who uses a date-0 normalization

Fourth, we typically expect all four components of the aggregate additive decomposition to be non-zero for a normalized welfarist planner, at least when markets are incomplete — see Proposition 6 below.

Finally, aggregate welfare assessments made by a particular welfarist planner (e.g., with a particular $\mathcal{W}(\cdot)$) are directionally invariant to whether we consider a normalized or an unnormalized individual multiplicative decomposition. That is, both decompositions agree on whether a policy is desirable or not. However, only the normalized individual multiplicative decomposition will have desirable properties, as we describe next.

4.2 Properties of Aggregate Additive Decomposition for Normalized Welfarist Planners

Since welfarist planners are particular DS-planners, every result established in Section 3 immediately applies to normalized welfarist planners. However, we can further exploit the characterization of the individual multiplicative decomposition introduced in Proposition 5 to identify new desirable properties of the aggregate decomposition that apply to normalized planners.

In particular, we show that i) all normalized welfarist planners conclude that the risk-sharing and intertemporal-sharing components are zero when markets are complete, ii) the intertemporal-sharing component is zero when individuals can freely trade a riskless bond, iii) different normalized welfarist planners exclusively disagree on the redistribution component, iv) the efficiency components (aggregate efficiency, risk-sharing, and intertemporal-sharing) of the aggregate additive decomposition are invariant to monotonically increasing transformations of individual's lifetime utilities and positive affine (increasing linear) transformations of individual's instantaneous utilities, and v) all normalized welfarist planners conclude that Pareto improving policies improve efficiency, i.e., the sum of aggregate efficiency, risk-sharing, and intertemporal-sharing. To our knowledge, the aggregate additive decomposition of welfare assessments introduced in this paper is the first welfare decomposition for which these properties — which seem highly desirable — have been established.

It seems natural to conjecture that the intertemporal-sharing and risk-sharing components of the aggregate additive decomposition depend critically on the ability of individuals to smooth consumption intertemporally and across histories. For the purposes of Proposition 6, we say that markets are complete when the marginal rates of substitution across all dates and histories in terms of the numeraire are equalized across agents — this condition is endogenously satisfied in any equilibrium model in which individuals can freely trade claims that pay in terms of consumption goods spanning all possible contingencies.[25]

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in which $\lambda_i(s_0)\frac{\partial u_i(s^0)}{\partial c_0^i} = 1$, implicitly assigns higher individual weights to those with higher willingness to pay for T-consol bonds, since $\tilde{\omega}^{i,\mathcal{W}}(s_0) = \frac{\sum_{t=0}^{T}\sum_{s^t} q_t^i(s^t|s_0)}{\int \sum_{t=0}^{T}\sum_{s^t} q_t^i(s^t|s_0)di}$. This may seem desirable in particular circumstances.

[25] It is important that we define complete markets in terms of the numeraire. Propositions 6 and 7 imply that the natural commodity to choose as numeraire is the commodity on which financial claims are written (e.g., dollars). In Section F.3 of the Online Appendix, we expand on the implications of the choice of numeraire for welfare assessments.

Proposition 6. *(Properties of normalized welfarist planners: complete markets) When the marginal rates of substitution across all dates and histories are equalized across individuals — a condition that complete market economies satisfy — the intertemporal-sharing and the risk-sharing components of the aggregate welfare decomposition for a normalized welfarist planner are zero, that is,* $\Xi_{RS} = \Xi_{IS} = 0$*. Hence, in that case, welfare assessments made by a normalized welfarist planner are exclusively driven by aggregate efficiency and redistribution.*

When markets are complete, $\tilde{\omega}_t^{i,NU}\left(s^t \middle| s_0\right)$ and $\tilde{\omega}_t^{i,NU}\left(s_0\right)$ become identical across individuals, as shown by the fact that there is a unique stochastic discount factor, so $q_t^i = q_t$, $\forall i$ in Equations (16) and (17). Combined with Proposition 2b), this immediately implies that $\Xi_{RS} = \Xi_{IS} = 0$. Intuitively, a normalized welfarist planner perceives that no policy can entail welfare gains or losses coming from risk-sharing or intertemporal-sharing among individuals, since individuals can perfectly share risks and substitute intertemporally.[26]

Proposition 7. *(Properties of normalized welfarist planners: riskless borrowing/saving) When the marginal rates of substitution across dates are equalized across individuals — a condition that is satisfied when all individuals are able to borrow and save freely at all times — the intertemporal-sharing component of the aggregate welfare decomposition for a normalized welfarist planner is zero, that is,* $\Xi_{IS} = 0$*.*

When all individuals are able to borrow and save freely at all times, $\tilde{\omega}_t^{i,NU}\left(s_0\right)$ becomes identical across individuals. This follows directly from Equation (16), since in that case $\sum_{s^t} q_t^i\left(s^t | s_0\right)$ is constant for all individuals. Intuitively, a normalized welfarist planner perceives that no policy can entail welfare gains or losses coming from intertemporal-sharing among individuals, since individuals can perfectly transfer resources across periods. Proposition 7 immediately implies that constraints to borrowing or saving are needed for the intertemporal-sharing component to be non-zero.

Proposition 8. *(Properties of normalized welfarist planners: welfarist planners only disagree about redistribution) For a given policy, the efficiency components (aggregate efficiency, risk-sharing, and intertemporal-sharing) of the aggregate additive decomposition are identical for all normalized welfarist planners. Hence, differences in welfare assessments among normalized welfarist planners are exclusively based on how they assess redistribution.*

Proposition 8 follows from the fact that the individual component of the individual multiplicative decomposition, $\tilde{\omega}^{i,\mathcal{W}}\left(s_0\right)$, is the only component that depends on the exact form of $\mathcal{W}\left(\cdot\right)$. Therefore, differences in welfare assessments between welfarist planners can always be traced back to differences in the redistribution component of the aggregate additive decomposition.[27] This result crucially

[26]Proposition 6 suggests that the cross-sectional dispersions of the dynamic and stochastic components of DS-weights, $\mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^t \middle| s_0\right)\right]$ and $\mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^0\right)\right]$, may be natural candidates to measure the potential welfare gains from completing markets for a normalized welfarist planner — see also Proposition 15 below.

[27]Note that Proposition 8, when combined with Corollary 3 rationalizes why all normalized welfarist planners directionally agree on welfare assessments when individuals are ex-ante identical. In that case, Corollary 3 implies that the redistribution component is zero, and Proposition 8 shows that Ξ_{AE}, Ξ_{RS}, and Ξ_{IS} (and consequently Ξ_{E}) are identical for normalized welfarist planners.

hinges on the fact that welfarist planners are non-paternalistic, that is, welfarist planners use individual lifetime utilities as inputs into their aggregate welfare calculations. In the next section, we introduce new "pseudo-welfarist" planners for which this property does not hold — see also Section G.3.1 of the Online Appendix.

Proposition 9. *(Properties of normalized welfarist planners: invariance of efficiency components to utility transformations) The efficiency components (aggregate efficiency, risk-sharing, and intertemporal-sharing) of the aggregate additive decomposition are invariant to i) monotonically increasing transformations of individual's lifetime utilities and ii) positive affine (increasing linear) transformations of individual's instantaneous utilities, for all normalized welfarist planners.*

As we described in Section 2.2, a welfarist planner mechanically puts more weight on the gains or losses of an individual whose lifetime utility experiences a monotonically increasing transformation or whose instantaneous utility experiences a positive affine transformation, even though this has no impact on allocations. Proposition 9 shows that this allegedly undesirable feature of the welfarist approach is fully confined to the redistribution component of the aggregate decomposition. Hence, Proposition 9 implies that the potential arbitrariness of the welfare assessments of a welfarist planner due to the choice of utility units is exclusively due to the redistribution component.[28]

Altogether, Propositions 8 and 9 have profound implications for the use in practice of Social Welfare Functions. First, the fact that every welfarist planner agrees on Ξ_{AE}, Ξ_{RS}, and Ξ_{IS} implies that there should be no disagreement over the efficiency gains of any policy. Second, the fact that only the redistribution component is sensitive to the choice of SWF and utility units implies that the redistributional welfare implications of a policy are simply a function of judiciously choosing the individual component of the individual multiplicative decomposition of DS-weights.

While proving the converse to Propositions 6 through 9 — that is, that the aggregate additive decomposition for normalized welfarist planners is the only one that satisfies such properties — is outside of the scope of this paper, it should be evident why using normalized weights is critical.[29] In particular, note that Equations (13) and (14) (equivalently, (16) and (17)) are expressed as *ratios* of individual marginal utilities or individual valuations. Since individual valuations of particular claims are i) invariant to the considered utility transformations and ii) identical among individuals when markets are complete or a riskless asset can be freely traded, Propositions 6 through 9 follow. Hence, any other decomposition that satisfies these properties will have to rely on ratios of marginal utilities.

Finally, we show that normalized welfarist planners always conclude that Pareto improving policies improve efficiency. This is another desirable property that the aggregate additive decomposition satisfies.

[28]We say potential arbitrariness because using different transformations of individual utilities or different social welfare functions simply corresponds to choosing specific individual normalized components, $\tilde{\omega}^{i,\mathcal{W}}(s_0)$, as defined in Equation (15).

[29]It is straightforward to show that there are slight variations of normalized weights that satisfy Propositions 6 through 10. See, for instance, the discussion of date-0 normalizations in Section G.1 of the Online Appendix.

Proposition 10. *(Properties of normalized welfarist planners: Pareto improvements increase efficiency) If a policy change is a (strict or weak) Pareto improvement, then the sum of the efficiency components (aggregate efficiency, risk-sharing, and intertemporal-sharing) must be strictly positive, that is,* $\Xi_E = \Xi_{AE} + \Xi_{IS} + \Xi_{RS} > 0$.

Proposition 10 shows that every Pareto improvement must improve efficiency. Interestingly, even when one or two of the efficiency components are negative, as long as the sum of the three is strictly positive, there is scope for the policy considered to be a Pareto improvement. However, policies for which $\Xi_{AE} + \Xi_{IS} + \Xi_{RS} < 0$ cannot be a Pareto improvement.

Proposition 10 provides a necessary but not a sufficient condition for a policy to be a Pareto improvement since there are scenarios in which $\Xi_{AE} + \Xi_{IS} + \Xi_{RS} > 0$ that are not Pareto improvements. However, converse results can be obtained in specific cases. For instance, in economies with ex-ante identical individuals, policies for which $\Xi_{AE} + \Xi_{IS} > 0$ are necessarily Pareto improvements. Also, in economies in which a planner can set permanent individual-specific transfers that cannot be conditioned on time or histories, it can be shown that all policies for which $\Xi_{AE} + \Xi_{IS} + \Xi_{RS} > 0$ are Pareto improvements.

5 New Welfare Criteria

A central objective of this paper is to provide a framework to systematically formalize new welfare criteria to assess and conduct policy. In this section, we describe how to use DS-weights to formalize new welfare criteria that capture particular normative objectives that society may find appealing. These results have the potential to allow for disciplined discussions about the mandates of independent technocratic institutions (central banks, financial regulators, other regulatory agencies, etc.).[30]

5.1 AE/AR/NR DS-Planners

In this subsection, we formally introduce novel DS-planners that only value some normative considerations but not others. By doing this, we are able to define new welfare criteria that set to 0 particular components of the aggregate additive decomposition. We refer to these planners as i) aggregate efficiency (AE) DS-planners, ii) aggregate efficiency/risk-sharing (AR) DS-planners, and iii) no-redistribution (NR) DS-planners. In principle, there exists a family of DS-planners that sets to 0 particular components of the aggregate additive decomposition. Within each family of DS-planners, we identify a pseudo-welfarist planner as the one that represents the minimal departure relative to the normalized welfarist planner.

By introducing these new planners we are able to formalize new welfare criteria that, for instance, isolate aggregate efficiency as the sole welfare objective, or that remove the desire to redistribute

[30]For instance, in ongoing work, we explore whether it is possible to implement the timeless Ramsey solution of a utilitarian planner, which requires commitment, in an environment in which a central banker chooses monetary policy under discretion using one of the new welfare criteria introduced in this section.

across individuals, among other goals. These new DS-planners are helpful not only to provide analytical characterizations, but also to characterize and compute optimal policy solutions guided by particular normative considerations.

Definition 4. *(AE/AR/NR DS-planners: definition)*

a) (Aggregate efficiency DS-planners) An aggregate efficiency (AE) DS-planner, that is, a planner who exclusively values aggregate efficiency, is a DS-planner for whom the individual, dynamic, and stochastic components of DS-weights are constant across all individuals at all dates and histories. A pseudo-welfarist AE DS-planner, who values aggregate efficiency as a normalized welfarist planner, has DS-weights $\omega_t^{i,\mathcal{W},AE}\left(s^t\middle|\, s_0\right)$ *defined by*

$$\tilde{\omega}^{i,\mathcal{W},AE}\left(s_0\right)=1,\;\; \tilde{\omega}_t^{i,\mathcal{W},AE}\left(s_0\right)=\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)\right],\;\text{ and }\;\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\middle|\, s_0\right)=\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\, s_0\right)\right]. \tag{20}$$

b) (Aggregate efficiency/risk-sharing DS-planners) An aggregate efficiency/risk-sharing (AR) DS-planner, that is, a planner who exclusively values aggregate efficiency and risk-sharing, is a DS-planner for whom the individual and dynamic components of DS-weights are constant across all individuals at all dates. A pseudo-welfarist AR DS-planner, who values aggregate efficiency and risk-sharing as a normalized welfarist planner, has DS-weights $\omega_t^{i,\mathcal{W},AR}\left(s^t\middle|\, s_0\right)$ *defined by*

$$\tilde{\omega}^{i,\mathcal{W},AR}\left(s_0\right)=1,\;\; \tilde{\omega}_t^{i,\mathcal{W},AR}\left(s_0\right)=\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)\right],\;\text{ and }\;\tilde{\omega}_t^{i,\mathcal{W},AR}\left(s^t\middle|\, s_0\right)=\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\, s_0\right). \tag{21}$$

c) (No-redistribution DS-planners) A no-redistribution (NR) DS-planner, that is, a planner who exclusively values aggregate efficiency, risk-sharing, and intertemporal-sharing, but disregards redistribution, is a DS-planner for whom the individual component of DS-weights is constant across all individuals. A pseudo-welfarist AR DS-planner, who values aggregate efficiency, risk-sharing, and intertemporal-sharing as a normalized welfarist planner, has DS-weights $\omega_t^{i,\mathcal{W},NR}\left(s^t\middle|\, s_0\right)$ *defined by*

$$\tilde{\omega}^{i,\mathcal{W},NR}\left(s_0\right)=1,\;\; \tilde{\omega}_t^{i,\mathcal{W},NR}\left(s_0\right)=\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right),\;\text{ and }\;\tilde{\omega}_t^{i,\mathcal{W},NR}\left(s^t\middle|\, s_0\right)=\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\, s_0\right). \tag{22}$$

Formally, an AE DS-planner adopts components of the individual multiplicative decomposition of DS-weights that are individual invariant. The pseudo-welfarist AE DS-planner sets these components exactly equal to the cross-sectional average of those used by a normalized welfarist planner.[31] An AR DS-planner only makes the individual and dynamic components individual invariant, while the

[31] It is straightforward to consider other AE DS-planners that are not pseudo-welfarist. For instance, one could choose the following weights:

$$\tilde{\omega}^{i,AE}\left(s_0\right)=1,\quad \tilde{\omega}_t^{i,AE}\left(s_0\right)=\overline{\beta}^t,\quad\text{and}\quad \tilde{\omega}_t^{i,AE}\left(s^t\middle|\, s_0\right)=\pi_t\left(s^t\middle|\, s_0\right),$$

for some $\overline{\beta}$, plausibly $\overline{\beta}=\int\beta_i di$. This is helpful because, in some applications, DS-planners that are not pseudo-welfarist may be easier to operationalize.

Table 1: New Welfare Criteria: Summary

DS-Planners	Ξ_{AE} Aggregate Efficiency	Ξ_{RS} Risk-sharing	Ξ_{IS} Intertemporal-sharing	Ξ_{RD} Redistribution
Aggregate Efficiency (AE)	✓	$=0$	$=0$	$=0$
Aggregate Efficiency/Risk-Sharing (AR)	✓	✓	$=0$	$=0$
No-Redistribution (NR)	✓	✓	✓	$=0$
Welfarist ($\mathcal{W}$)	✓	✓	✓	✓

Note: Table 1 summarizes the properties of the aggregate additive decomposition for the DS-planners introduced in Definition 4. These properties follow from Proposition 11.

pseudo-welfarist AR DS-planner further preserves the stochastic component used by the normalized welfarist planner. A NR DS-planner only makes the individual component individual invariant, while the pseudo-welfarist NR DS-planner further preserves the dynamic and stochastic components used by the normalized welfarist planner.

We formalize the properties of these new planners for the components of the aggregate additive decomposition in Proposition 11. Table 1 summarizes its results.

Proposition 11. *(AE/AR/NR DS-planners: properties)*

a) For an AE DS-planner, the risk-sharing, intertemporal-sharing, and redistribution components of the aggregate additive decomposition are zero, that is, $\Xi_{RS} = \Xi_{IS} = \Xi_{RD} = 0$. The aggregate efficiency component, Ξ_{AE}, is identical for a pseudo-welfarist AE DS-planner and its associated normalized welfarist planner.

b) For an AR DS-planner, the intertemporal-sharing and redistribution components of the aggregate additive decomposition are zero, that is, $\Xi_{IS} = \Xi_{RD} = 0$. The aggregate efficiency and risk-sharing components, Ξ_{AE} and Ξ_{RS}, are identical for a pseudo-welfarist AR DS-planner and its associated normalized welfarist planner.

c) For a NR DS-planner, the redistribution component of the aggregate additive decomposition is zero, that is, $\Xi_{RD} = 0$. The aggregate efficient, risk-sharing, and intertemporal-sharing components, Ξ_{AE}, Ξ_{RS}, and Ξ_{IS}, are identical for a pseudo-welfarist NR DS-planner and its associated normalized welfarist planner.

Proposition 11 shows that the new DS-planners, by making the individual components of DS-weights invariant across individuals, dates, or histories, are defined to directly exploit the properties of the aggregate additive decomposition characterized in Proposition 2. Moreover, the pseudo-welfarist planners are defined so as to exactly preserve the value of their components relative to the associated welfarist planner along the dimensions in which they are not zero. This is useful in practice because it allows us to interpret specific sums of the components of the aggregate decomposition of a welfarist planner as the welfare assessment made by a pseudo-welfarist planner.

Given its practical importance, we formally state this result as Corollary 5.

Corollary 5. *(Pseudo-welfarist planners as components of welfarist aggregate additive decomposition) Specific sums of the components of the aggregate additive decomposition of welfare assessments for a given welfarist planner have the interpretation of welfare assessments for particular pseudo-welfarist DS-planners.*

Interestingly, it is not possible to define a new pseudo-welfarist planner for whom exclusively the risk-sharing and intertemporal-sharing components are zero, as we show in Section G.2 of the Online Appendix. To guarantee that $\Xi_{RS} = \Xi_{IS} = 0$, a planner would need $\omega_t^i(s_0)$ and $\tilde{\omega}_t^i(s^t|s_0)$ to be individual-invariant, which would interfere with ensuring that the value of Ξ_{RD} is the same as for a welfarist planner. A similar logic applies to other combinations of the different components. Nonetheless, it is certainly possible to define new planners that are not pseudo-welfarist but that for instance exclusively value aggregate efficiency and redistribution.

5.2 $\boldsymbol{\alpha}$-DS-planners

The new planners that we introduce in Definition 4 by no means exhaust the set of new planners that one can define using DS-weights. In particular, it is possible to define a new planner that spans i) AE, ii) AR, and iii) NR pseudo-welfarist planners, as well as iv) the associated normalized welfarist planner. We refer to this planner as an $\boldsymbol{\alpha}$-DS-planner.

Definition 5. *($\boldsymbol{\alpha}$-DS-planner: definition) An $\boldsymbol{\alpha}$-DS-planner is a DS-planner for whom the individual, dynamic, and stochastic components of DS-weights are linear combinations of the components of a normalized welfarist planner and the component of an AE pseudo-welfarist planner. An $\boldsymbol{\alpha}$-DS-planner has DS-weights $\omega_t^{i,\mathcal{W},\boldsymbol{\alpha}}(s^t|s_0)$ defined by*

$$\begin{aligned}
\tilde{\omega}_t^{i,\mathcal{W},\boldsymbol{\alpha}}\left(s^t\middle|s_0\right) &= (1-\alpha_2)\,\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\middle|s_0\right) + \alpha_2\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)\\
\tilde{\omega}_t^{i,\mathcal{W},\boldsymbol{\alpha}}(s_0) &= (1-\alpha_3)\,\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0) + \alpha_3\tilde{\omega}_t^{i,\mathcal{W}}(s_0)\\
\tilde{\omega}^{i,\mathcal{W},\boldsymbol{\alpha}}(s_0) &= (1-\alpha_4)\,\tilde{\omega}^{i,\mathcal{W},AE}(s_0) + \alpha_4\tilde{\omega}^{i,\mathcal{W}}(s_0)\,,
\end{aligned}$$

where $\boldsymbol{\alpha} = (\alpha_2, \alpha_3, \alpha_4)$, and where $\alpha_2 \in [0,1]$, $\alpha_3 \in [0,1]$, $\alpha_4 \in [0,1]$.

Depending on the value of $\boldsymbol{\alpha}$, an $\boldsymbol{\alpha}$-DS-planner behaves as a particular pseudo-welfarist planner or as a combination of pseudo-welfarist planners. In particular, as we show in Section G.2 of the Online Appendix, when $\boldsymbol{\alpha} = (0,0,0)$, we have an AE DS-planner; when $\boldsymbol{\alpha} = (1,0,0)$, we have an AR DS-planner; when $\boldsymbol{\alpha} = (1,1,0)$, we have a NR DS-planner; and when $\boldsymbol{\alpha} = (1,1,1)$, we have a welfarist planner.

By varying $\boldsymbol{\alpha}$, it is possible to model planners who care about the different components to different degrees. Moreover, estimating $\boldsymbol{\alpha}$ from actual policies in the context of a particular policy problem has the potential to uncover the weights that a particular policymaker attaches in practice to the different components of the aggregate additive decomposition.

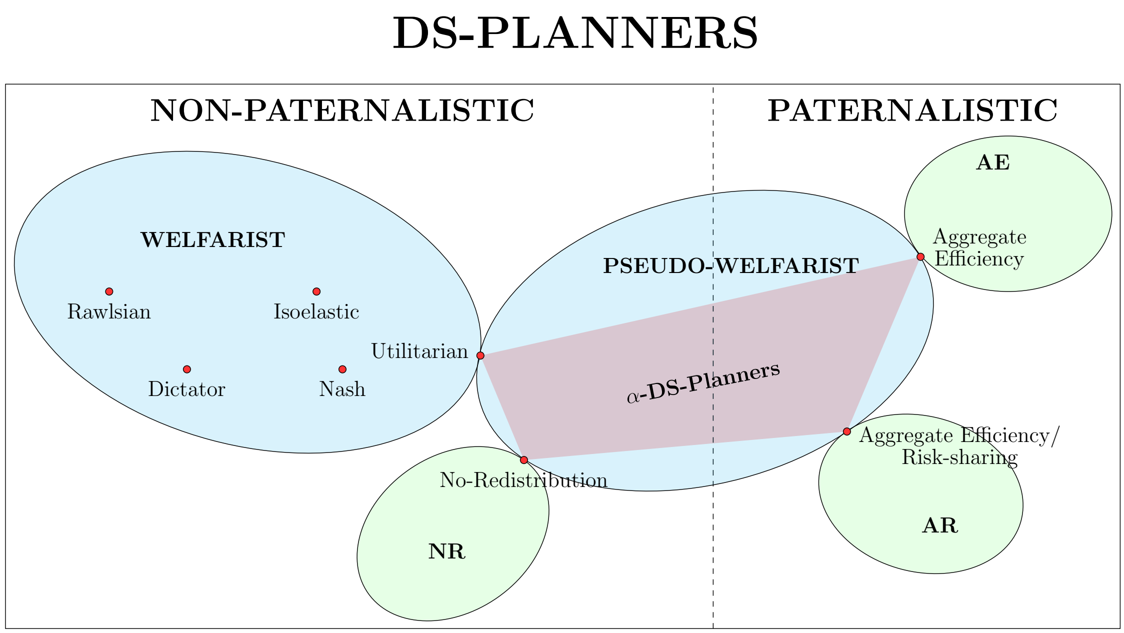

Figure 1: DS-Planners: Summary

Note: Figure 1 summarizes the relations between the different planners studied in Section 5 paper. The vertical dashed line separates non-paternalistic planners from paternalistic planners. All welfarist planners, as well as no-redistribution (NR) planners, are non-paternalistic. Aggregate efficiency (AE) and aggregate efficiency/risk-sharing (AR) planners are paternalistic. Some pseudo-welfarist planners are non-paternalistic (welfarist, NR), while others are paternalistic (AE, AR). In this figure, the $\boldsymbol{\alpha}$-DS-planners are pseudo-welfarist with respect to the utilitarian planner.

5.3 Paternalism and Institutional Design

In Figure 1, we summarize the relations between the different planners studied in this section. We conclude this section with two remarks.

Remark 1. *(Paternalistic vs. Non-paternalistic DS-planners; AE and AR planners are paternalistic)* It is important to highlight that AE and AR DS-planners are paternalistic, in the sense that their welfare assessments do not take as an input changes in the lifetime welfare assessments of individuals.[32] In these cases, a planner and an individual may have different assessments of whether a policy change is welfare improving or not for that individual. However, NR DS-planners are not paternalistic. Intuitively, the welfare assessments of any planner who respects individual preferences must value intertemporal-sharing and risk-sharing considerations as long as individuals do. Redistributional concerns are independent of whether a planner respects individuals' desires for

[32] As explained in Section G.3.1 of the Online Appendix, a non-paternalistic planner makes welfare assessments according to

$$\frac{dW^{NP}(s_0)}{d\theta} = \int \phi_i(s_0) \frac{dV_i(s_0)}{d\theta} di,$$

where $\phi_i(s_0)$ are functions of all possible paths of outcomes and where $\frac{dV_i(s_0)}{d\theta}$ is defined in Equation (2). The key distinction between a welfarist and a non-paternalistic planner is that, for welfarist planners $\phi_i(s_0)$ must take the particular form $\frac{\partial \mathcal{W}(\{V_i(s_0)\}_{i\in I})}{\partial V_i}$, where $\mathcal{W}(\cdot)$ is a SWF of the form described in Equation (4). Non-paternalistic planners can set $\phi_i(s_0)$ freely.

interpersonal sharing. Therefore, if a planner wants to make welfare assessments that do not value intertemporal-sharing or risk sharing, such a planner must necessarily be paternalistic.

Remark 2. *(Implications for policy mandates and institutional design)* The framework developed in this paper has the potential to guide the design of independent technocratic institutions. In practice, such institutions must be given a "mandate", much like defining a set of DS-weights. Therefore, a society may want to consider designing independent technocratic institutions that have some normative considerations in their mandate but not others, along the lines of the logic we have developed in this section. For instance, the current "dual mandate" (stable prices and maximum employment) of the Federal Reserve (as defined by the 1977 Federal Reserve Act) seems to be better described by an aggregate efficiency DS-planner, rather than a welfarist planner, which would care about cross-sectional considerations. Alternatively, an institution like the Federal Emergency Management Agency (FEMA) has as part of its mandate to "support the Nation in a risk-based, comprehensive emergency management system", which unavoidably involves risk-sharing considerations.

6 Additional Results

In this section, we include additional results. First, we further decompose the components of the aggregate additive decomposition and then explain how to connect welfare assessments to measures of inequality. Next, we explain how to make welfare assessments using DS-weights in recursive environments, and show how to implement welfare assessments via an instantaneous Social Welfare Function. Finally, we describe how to compute a term structure for aggregate welfare assessments and for each of the components of the aggregate additive decomposition and then briefly describe additional results included in the Online Appendix.

6.1 Decomposing the Components of the Aggregate Additive Decomposition

Here, we further decompose and provide additional insights into the four components of the aggregate additive decomposition. For the aggregate efficiency and the redistribution components, we provide new stochastic decompositions. For the risk-sharing and intertemporal-sharing components, we provide alternative cross-sectional decompositions.

Aggregate efficiency (Ξ_{AE}). It is important to highlight that the aggregate efficiency component Ξ_{AE} includes aggregate valuation considerations. We formalize this insight by further decomposing the aggregate efficiency component of the aggregate additive decomposition into an expected aggregate efficiency component and an aggregate smoothing component.

Proposition 12. *(Aggregate efficiency component: stochastic decomposition) The aggregate efficiency component of the aggregate additive decomposition, Ξ_{AE}, can be decomposed into i) an expected aggregate efficiency component, Ξ_{EAE}, and ii) an aggregate smoothing component, Ξ_{AM}, as* *follows:*

$$\Xi_{AE}=\underbrace{\sum_{t=0}^{T}\overline{\omega}_{t}\left(s_{0}\right)\mathbb{E}_{0}\left[\overline{\omega}_{t}^{\pi}\left(s^{t}\middle|\,s_{0}\right)\right]\mathbb{E}_{0}\left[\frac{d\overline{u}_{i|c}\left(s^{t}\right)}{d\theta}\right]}_{=\Xi_{EAE}\text{ (Expected Aggregate Efficiency)}}+\underbrace{\sum_{t=0}^{T}\overline{\omega}_{t}\left(s_{0}\right)\mathbb{C}ov_{0}\left[\overline{\omega}_{t}^{\pi}\left(s^{t}\middle|\,s_{0}\right),\frac{d\overline{u}_{i|c}\left(s^{t}\right)}{d\theta}\right]}_{=\Xi_{AM}\text{ (Aggregate Smoothing)}},\tag{23}$$

where we define $\overline{\omega}_{t}\left(s_{0}\right)=\mathbb{E}_{i}\left[\tilde{\omega}_{t}^{i}\left(s_{0}\right)\right]$, $\overline{\omega}_{t}^{\pi}\left(s^{t}\middle|\,s_{0}\right)=\frac{\mathbb{E}_{i}\left[\tilde{\omega}_{t}^{i}\left(s^{t}|s_{0}\right)\right]}{\pi_{t}\left(s^{t}|s_{0}\right)}$, *and* $\frac{d\overline{u}_{i|c}\left(s^{t}\right)}{d\theta}=\mathbb{E}_{i}\left[\frac{du_{i|c}\left(s^{t}\right)}{d\theta}\right]$, *and where* $\mathbb{E}_{0}\left[\cdot\right]$ *and* $\mathbb{C}ov_{0}\left[\cdot,\cdot\right]$ *denote expectations and covariances conditional on* s_{0}.

The expected aggregate efficiency component, Ξ_{EAE}, captures the discounted expectation over time and histories of the aggregate instantaneous consumption-equivalent effect of the policy change. The aggregate smoothing component, Ξ_{AM}, captures whether aggregate efficiency gains take place in histories that a DS-planner values more in aggregate terms. It should be evident that aggregate smoothing, Ξ_{AM}, based on aggregate covariances over histories, is logically different from the risk-sharing and intertemporal-sharing components, Ξ_{RS} and Ξ_{IS}, based on cross-sectional covariances.

In practical terms, the welfare gains associated with eliminating aggregate business cycles in a representative-agent economy, as in the policy experiment of Lucas (1987), fully arise from aggregate smoothing considerations, that is, Ξ_{AM}. Note that both the expected aggregate efficiency and the aggregate smoothing components incorporate discounting via $\overline{\omega}_{t}\left(s_{0}\right)$, so policy changes that front-load gains from expected aggregate efficiency or aggregate smoothing are more desirable.

Risk-sharing and intertemporal-sharing components (Ξ_{RS} and Ξ_{IS}). While Propositions 2 through 4 establish desirable properties of the aggregate additive decomposition, it is possible to provide alternative formulations of the risk-sharing and intertemporal-sharing components. In Proposition 13 we further decompose the intertemporal-sharing component into a pure intertemporal-sharing component, a weight concentration component, and a policy-weights coskewness component. We also show a new identity that the sum of the risk-sharing and intertemporal-sharing components, $\Xi_{RS}+\Xi_{IS}$, must satisfy.

Proposition 13. *(Risk-sharing/intertemporal-sharing components: alternative cross-sectional decompositions)*

a) The intertemporal-sharing component of the aggregate additive decomposition, Ξ_{IS}*, can be decomposed into i) a pure intertemporal-sharing component,* Ξ_{PIS}*, ii) a weight concentration* *component, Ξ_{WC} and iii) a policy-weights coskewness component, Ξ_{PC} as follows:*

$$
\begin{aligned}
\Xi_{IS} = & \underbrace{\sum_{t=0}^{T} \sum_{s^t} \mathbb{E}_i \left[\tilde{\omega}_t^i \left(s^t \middle| s_0 \right)\right] \mathbb{C}ov_i \left[\tilde{\omega}_t^i (s_0), \frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{PIS} \text{ (Pure Intertemporal-sharing)}} \\
& + \underbrace{\sum_{t=0}^{T} \sum_{s^t} \mathbb{C}ov_i \left[\tilde{\omega}_t^i (s_0), \tilde{\omega}_t^i \left(s^t \middle| s_0 \right)\right] \mathbb{E}_i \left[\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{WC} \text{ (Weight Concentration)}} \\
& + \underbrace{\sum_{t=0}^{T} \sum_{s^t} \mathbb{E}_i \left[\left(\frac{du_{i|c}(s^t)}{d\theta} - \mathbb{E}_i \left[\frac{du_{i|c}(s^t)}{d\theta}\right]\right) \left(\tilde{\omega}_t^i (s_0) - \mathbb{E}_i \left[\tilde{\omega}_t^i (s_0)\right]\right) \left(\tilde{\omega}_t^i \left(s^t \middle| s_0 \right) - \mathbb{E}_i \left[\tilde{\omega}_t^i \left(s^t \middle| s_0 \right)\right]\right)\right]}_{=\Xi_{PC} \text{ (Policy-weights Coskewness)}} .
\end{aligned}
\tag{24}
$$

b) The sum of the risk-sharing and the intertemporal-sharing components, $\Xi_{RS} + \Xi_{IS}$, can be decomposed into i) a weight concentration component, Ξ_{WC} and ii) an interpersonal-sharing component, Ξ_{IPS} as follows:

$$
\begin{aligned}
\Xi_{RS} + \Xi_{IS} = & \underbrace{\sum_{t=0}^{T} \sum_{s^t} \mathbb{C}ov_i \left[\tilde{\omega}_t^i (s_0), \tilde{\omega}_t^i \left(s^t \middle| s_0 \right)\right] \mathbb{E}_i \left[\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{WC} \text{ (Weight Concentration)}} \\
& + \underbrace{\sum_{t=0}^{T} \sum_{s^t} \mathbb{C}ov_i \left[\tilde{\omega}_t^i (s_0) \tilde{\omega}_t^i \left(s^t \middle| s_0 \right), \frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{IPS} \text{ (Interpersonal-sharing)}},
\end{aligned}
\tag{25}
$$

where $\Xi_{IPS} = \Xi_{RS} + \Xi_{PIS} + \Xi_{PC}$.

The first component of Ξ_{IS} introduced in Proposition 13a), Ξ_{PIS}, can be interpreted as capturing pure intertemporal-sharing considerations. The major difference between Ξ_{IS} and Ξ_{PIS} is that the former is based on cross-sectional covariances of the dynamic component of DS-weights with the expected — interpreting the stochastic weights as probabilities — instantaneous consumption-equivalent effect of the policy at a given date. The latter, on the other hand, is based on the expectation of cross-sectional covariances of the dynamic component of DS-weights with the actual instantaneous consumption-equivalent effect of the policy. Formally, the difference between Ξ_{IS} and Ξ_{PIS} is captured by the remaining two components, which we describe next.

The second component of Ξ_{IS} introduced in Proposition 13a), Ξ_{WC}, can be interpreted as capturing the welfare gain (loss) associated with policies that increase aggregate instantaneous consumption-equivalent when the dynamic and stochastic components of DS-weights are positively (negatively) correlated across individuals. While one may consider including Ξ_{WC} in the aggregate efficiency component, there are two good reasons not to do so. First, it would require knowledge of the cross-section of the dynamic and stochastic components of DS-weights, which goes against expressing

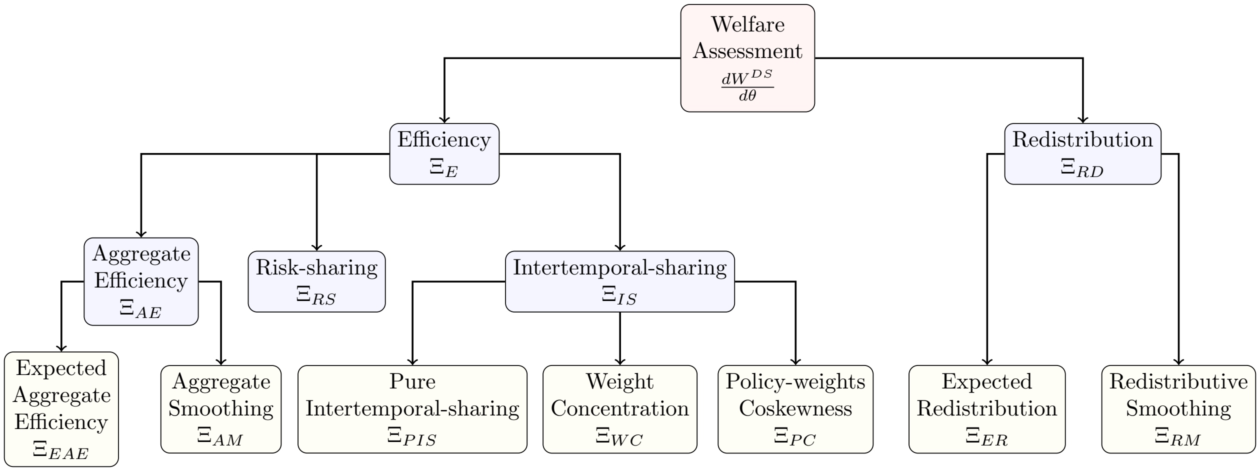

Figure 2: Aggregate additive decomposition

Note: Figure 2 illustrates the aggregate additive decomposition of welfare assessments for a general DS-planner, and how its four components can be further decomposed. See Propositions 1, 12, 13, and 14 for formal definitions of each of the terms.

the aggregate efficiency component exclusively as a function of aggregate statistics. Second, as implied by Proposition 6, for the case of welfarist planners, $\Xi_{WC} = 0$ when markets are complete. This fact highlights that Ξ_{WC} necessarily relies on imperfect smoothing across individuals, which makes this term unsuitable to capture aggregate efficiency considerations.

The third component of Ξ_{IS} introduced in Proposition 13a), Ξ_{PC}, is exactly based on the coskewness between i) the dynamic component of DS-weights, ii) the stochastic component of DS-weights, and iii) the instantaneous consumption-equivalent effect of a policy. Coskewness is a measure of how much three random variables jointly change. For instance, note that Ξ_{PC} could be non-zero even when $\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right), \tilde{\omega}_t^i\left(s^t\middle| s_0\right)\right] = 0$ and, consequently, $\Xi_{WC} = 0$. Also, coskewness is zero when the random variables are multivariate normal (Bohrnstedt and Goldberger, 1969), so it relies on higher-order moments.[33] Note also that if one of $\tilde{\omega}_t^i\left(s_0\right)$, $\tilde{\omega}_t^i\left(s^t\middle| s_0\right)$, or $\frac{du_{i|c}\left(s^t\right)}{d\theta}$ is constant across all individuals, then $\Xi_{WC} = 0$.

Proposition 13b) simply provides an alternative decomposition of the sum of risk-sharing and intertemporal-sharing. Its first component is exactly the weight concentration component just described, Ξ_{WC}, while the second component corresponds to the sum of risk-sharing, Ξ_{RS}, pure intertemporal-sharing, Ξ_{PIS}, and policy-weights coskewness, Ξ_{PC}. At times, this alternative decomposition may provide additional insights relative to the one in Proposition 1.

Redistribution component (Ξ_{RD}). Similarly to the aggregate efficiency component, the redistribution component Ξ_{RD} is shaped by valuation considerations, in this case at the individual level. Here, we decompose the redistribution component of the aggregate additive decomposition

[33] We expect these terms to be in important in models that emphasize higher moments of the distribution of individual risks (e.g., Guvenen, Ozkan and Song (2014)).

into an expected redistribution component and a redistributive smoothing component.

Proposition 14. *(Redistribution component: stochastic decomposition) The redistribution component of the aggregate additive decomposition, Ξ_{RD}, can be decomposed into i) an expected redistribution component, Ξ_{ER}, and ii) a redistributive smoothing component, Ξ_{RM}, as follows:*

$$\Xi_{RD} = \underbrace{\mathbb{C}ov_i \left[\tilde{\omega}^i \left(s^0 \right), \sum_{t=0}^{T} \tilde{\omega}_t^i \left(s_0 \right) \mathbb{E}_0 \left[\tilde{\omega}_t^{i,\pi} \left(s^t \middle| s_0 \right) \right] \mathbb{E}_0 \left[\frac{du_{i|c} \left(s^t \right)}{d\theta} \right] \right]}_{= \Xi_{ER} \text{ (Expected Redistribution)}}$$
$$+ \underbrace{\mathbb{C}ov_i \left[\tilde{\omega}^i \left(s^0 \right), \sum_{t=0}^{T} \tilde{\omega}_t^i \left(s_0 \right) \mathbb{C}ov_0 \left[\tilde{\omega}_t^{i,\pi} \left(s^t \middle| s_0 \right), \frac{du_{i|c} \left(s^t \right)}{d\theta} \right] \right]}_{= \Xi_{RM} \text{ (Redistributive Smoothing)}},$$

where we define $\overline{\omega}_t^{i,\pi} \left(s^t \middle| s_0 \right) = \frac{\tilde{\omega}_t^i \left(s^t | s_0 \right)}{\pi_t \left(s^t | s_0 \right)}$, and where $\mathbb{E}_0 \left[\cdot \right]$ and $\mathbb{C}ov_0 \left[\cdot, \cdot \right]$ denote expectations and covariances conditional on s_0.

The expected redistribution component, Ξ_{ER}, captures the perceived gains for a DS-planner from changes in the expected instantaneous consumption-equivalent effect of the policy change. When individuals with a high individual component of DS-weights, $\tilde{\omega}^i \left(s^0 \right)$, have higher expected instantaneous consumption-equivalent effect, a planner attributes this to the redistribution component. The redistributive smoothing component, Ξ_{RM}, captures whether individual gains from the policy change take place in histories that are more desirable for individuals with a higher individual component of DS-weights, $\tilde{\omega}^i \left(s^0 \right)$. In practical terms, the redistributive smoothing component will be non-zero when a policy improves individual smoothing for individuals with a higher individual component of DS-weights.[34]

6.2 Inequality, Bounds, and Welfare Assessments

Concerns related to inequality often take a prominent role when assessing policies. Our aggregate additive decomposition provides direct insights into which particular forms of inequality matter for the determination of aggregate welfare assessments and each of their components. Formally, in Proposition 15, we provide bounds for the risk-sharing component, the intertemporal-sharing component, and the redistribution component defined in Proposition 1 based on the cross-sectional dispersion of DS-weights and policy effects.[35] These bounds are helpful in practice because they can be computed using univariate statistics, i.e., cross-sectional standard deviations, and do not require the joint distribution of DS-weights and normalized consumption-equivalent effects, which are necessary to compute cross-sectional covariances (a multivariate statistic).

[34] Note that the redistribution component, Ξ_{RD}, can be positive or negative for Pareto-improving policies. This can occur if different individuals are differentially affected by the policy and if a DS-planner has different individual multiplicative components for different individuals.

[35] It should be clear that cross-sectional variances and standard deviations can only bound the welfare effect of policies. Equation (11) shows that cross-sectional covariances exactly determine each of the components of the aggregate additive decomposition.

Proposition 15. *(Cross-sectional dispersion bounds) The value of the risk-sharing, the intertemporal-sharing, and the redistribution components defined in Proposition 1 satisfy the following bounds:*

$$|\Xi_{RS}| \leq \sum_{t=0}^{T} \mathbb{E}_i \left[\tilde{\omega}_t^i \left(s^0\right)\right] \sum_{s^t} \mathbb{SD}_i \left[\tilde{\omega}_t^i \left(s^t \middle| s_0\right)\right] \times \mathbb{SD}_i \left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] \tag{26}$$

$$|\Xi_{IS}| \leq \sum_{t=0}^{T} \mathbb{SD}_i \left[\tilde{\omega}_t^i \left(s^0\right)\right] \times \mathbb{SD}_i \left[\sum_{s^t} \tilde{\omega}_t^i \left(s^t \middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta}\right] \tag{27}$$

$$|\Xi_{RD}| \leq \mathbb{SD}_i \left[\tilde{\omega}^i \left(s^0\right)\right] \times \mathbb{SD}_i \left[\sum_{t=0}^{T} \tilde{\omega}_t^i \left(s^0\right) \sum_{s^t} \tilde{\omega}_t^i \left(s^t \middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta}\right], \tag{28}$$

where $\mathbb{SD}_i [\cdot]$ denotes a cross-sectional standard deviation.

Proposition 15 shows that the magnitude of each of the three components considered here is determined (bounded above) by i) the cross-sectional dispersion of the different components of DS-weights, $\mathbb{SD}_i \left[\tilde{\omega}_t^i \left(s^t \middle| s^0\right)\right]$, $\mathbb{SD}_i \left[\tilde{\omega}_t^i \left(s^0\right)\right]$, and $\mathbb{SD}_i \left[\tilde{\omega}^i \left(s^0\right)\right]$, as well as ii) the cross-sectional dispersion of the instantaneous consumption-equivalent effect of the policy, effectively $\mathbb{SD}_i \left[\frac{du_{i|c}(s^t)}{d\theta}\right]$. Consequently, inequality considerations do matter for the aggregate assessments of policies via the cross-sectional dispersion of DS-weights or the impact of a policy by itself.

Proposition 15 is helpful for three reasons. First, it shows that normative criteria with highly dispersed DS-weights have the potential to generate a large welfare effect of policies via risk-sharing, intertemporal-sharing, and redistribution. Second, by computing the cross-sectional dispersion of the different components of DS-weights for a given criterion, it shows that it is possible to understand the potential scope that inequality may play when determining the risk-sharing, intertemporal-sharing, and redistribution components of aggregate welfare assessments. Finally, Proposition 15 shows that the risk-sharing, intertemporal-sharing and redistribution components depend on the extent to which policies impact different individuals differently. That is, the more $\frac{du_{i|c}(s^t)}{d\theta}$ varies across individuals, dates, or histories, the more likely dispersion in DS-weights matters for welfare assessments.

6.3 Recursive Formulation

Up to now, we have defined DS-weights for a sequence formulation of a dynamic stochastic economy. Here, we describe how to operationalize DS-weights in recursive environments, which are widely used in practice. As in Ljungqvist and Sargent (2018), we denote possible recursive states by s and s'.[36]

[36] Note that in recursive economies with idiosyncratic (and potentially aggregate) states (i.e., Aiyagari or Krusell-Smith style economies) individuals can be ex-ante heterogeneous at the time of making a welfare assessment for two different reasons. First, individuals can be heterogeneous ex-ante (e.g., individuals can have different time-invariant preferences or face shocks that come from different distributions). Second, individuals can be heterogenous ex-post (e.g., individuals can have different endowments or asset holdings at the time of the welfare assessment, even though they face identical problems starting from a given idiosyncratic state). This is an important observation to interpret correctly some of the results in this paper. For instance, Corollary 3 of Proposition 2 only applies when all individuals are identical because of predetermined reasons and when they all have the same initial state. Obviously, ex-post, individuals will also be heterogeneous if they experience different shocks. In the notation used in this section, ex-ante heterogeneity of either form is captured by the index i. See Section G.6 of the Online Appendix for a reformulation of

Proposition 16. *(Recursive formulation) Suppose that individual consumption and hours are exclusively a function of the current realization of s_t and do not depend on the full history leading to those outcomes, so that $c_t^i(s^t) = c^i(s_t) = c^i(s)$ and $n_t^i(s^t) = n^i(s_t) = n^i(s)$. Then, it is possible to express $\frac{dW^{DS}(s_0)}{d\theta}$, as defined in Equation (7), as follows:*

$$\frac{dW^{DS}(s_0)}{d\theta} = \int \omega_0^i \left(s^0|s_0\right) \frac{d\hat{V}_{i,0}^{DS}(s_0)}{d\theta} di, \tag{29}$$

where $\frac{d\hat{V}_{i,t}^{DS}(s)}{d\theta}$ has the following recursive representation:

$$\frac{d\hat{V}_{i,t}^{DS}(s)}{d\theta} = \frac{du_{i|c}(s)}{d\theta} + \hat{\beta}_{i,t} \sum_{s'} \hat{\pi}_{i,t}(s'|s) \frac{d\hat{V}_{i,t+1}^{DS}(s')}{d\theta}, \tag{30}$$

where $\hat{\beta}_{i,t}$ and $\hat{\pi}_{i,t}(s'|s)$ correspond to a twisted discount factor and a twisted set of transition probabilities of the form:

$$\hat{\beta}_{i,t} = \frac{\tilde{\omega}_{t+1}^i(s_0)}{\tilde{\omega}_t^i(s_0)} \qquad \text{and} \qquad \hat{\pi}_{i,t}(s'|\,s) = \frac{\tilde{\omega}_{t+1}^i(s'|\,s_0)}{\tilde{\omega}_t^i(s|\,s_0)}. \tag{31}$$

For Equation (30) to be a valid recursive representation, it must be that $\hat{\beta}_{i,t}$ is exclusively a function of time and s_0 and that $\hat{\pi}_{i,t}(s'|\,s)$ is exclusively a function of time, s, and s_0, but not of the full histories.

Proposition 16 shows that, in order to make a welfare assessment at a state s_0, a DS-planner must compute the date-0 DS-weights for all individuals, $\omega_0^i(s^0|s_0)$, as well as the value of $\frac{d\hat{V}_{i,0}^{DS}(s_0)}{d\theta} = \frac{\frac{dV_i^{DS}(s_0)}{d\theta}}{\tilde{\omega}_0^i(s_0)\tilde{\omega}_0^i(s^0|s_0)}$, which can be computed recursively following Equation (30). Intuitively, it is possible to find a recursive representation for $\frac{d\hat{V}_{i,t}^{DS}(s)}{d\theta}$, because it is expressed in units of consumption good at state s. In fact, $\frac{d\hat{V}_{i,t}^{DS}(s)}{d\theta}$ has the interpretation of an asset pricing equation for an asset that pays $\frac{du_{i|c}(s)}{d\theta}$ units of consumption good to individual i in state s.

It is worth highlighting that the set of DS-weights that admits a recursive representation is smaller than the set of DS-weights that can be expressed non-recursively. In particular, $\hat{\beta}_{i,t}$ and $\hat{\pi}_{i,t}(s'|\,s)$, which are ratios components of the individual decomposition of DS-weights cannot depend on histories, although they may be time-dependent. Interestingly, even in a fully recursive economy, the recursive representation of $\frac{d\hat{V}_{i,t}^{DS}(s)}{d\theta}$ is typically time-dependent, because the state in which the welfare assessment takes place will anchor the future values of the dynamics and stochastic components of the individual multiplicative decomposition for a DS-planner. Only in particular cases is it possible to find a time-independent recursive representation, as we discuss next.

As we show in the Online Appendix, when $\pi(s'|\,s)$ is Markov, we can express $\hat{\beta}_{i,t}$ and $\hat{\pi}_{i,t}(s'|\,s)$

our approach using notation that differentiates between idiosyncratic and aggregate states.

for a normalized welfarist planners as follows:

$$\hat{\beta}_{i,t}^{\mathcal{W}}=\beta_i\underbrace{\frac{\sum_{s'}\pi_{t+1}\left(s'\middle|\,s_0\right)\frac{\partial u_i(s')}{\partial c^i}}{\sum_{s}\pi_{t}\left(s\middle|\,s_0\right)\frac{\partial u_i(s)}{\partial c^i}}}_{=\text{ dynamic correction}}=\frac{1}{R_{i,t}^{f}}\quad\text{and}\quad\hat{\pi}_{i,t}^{\mathcal{W}}\left(s'\middle|\,s\right)=\pi\left(s'\middle|\,s\right)\underbrace{\frac{\frac{\frac{\partial u_i(s')}{\partial c^i}}{\sum_{s'}\pi_{t+1}(s'|s_0)\frac{\partial u_i(s')}{\partial c^i}}}{\frac{\frac{\partial u_i(s)}{\partial c^i}}{\sum_{s}\pi_{t}(s|s_0)\frac{\partial u_i(s)}{\partial c^i}}}}_{=\text{ stochastic correction}}=\pi_{i,t}^{\star}\left(s'\middle|\,s\right).\tag{32}$$

In this case, Equation (30) can be literally interpreted as a cum-dividend asset pricing equation, since $\hat{\beta}_{i,t}^{\mathcal{W}}=1/R_{i,t}^{f}$ has the interpretation of individual i's one-period forward rate between dates t and $t+1$, and $\hat{\pi}_{i,t}^{\mathcal{W}}\left(s'\middle|\,s\right)=\pi_{i,t}^{\star}\left(s'\middle|\,s\right)$ has the interpretation of individual i's risk-neutral probability between dates t and $t+1$. As we show in the Online Appendix, Equation (30) is time-independent for normalized welfarist planners and NR pseudo-welfarist planners.[37] However, Equation (30) is time-dependent for AR and AE pseudo-welfarist planners. In our application, which we formulate recursively, we further illustrate how to use DS-weights in recursive environments.

6.4 Instantaneous SWF Formulation

As explained in Section 2.2, the conventional approach to making welfare assessments relies on defining a Social Welfare Function that takes individual lifetime utilities as arguments. In this paper, we have shown that an approach based on generalized marginal DS-weights defined over instantaneous consumption-equivalents allows us to consider a larger class of normative objectives. In this section, we show that it is possible to interpret $\frac{dW^{DS}(s_0)}{d\theta}$, defined in Equation (7), as the derivative of a planner with a particular Social Welfare Function that i) takes as arguments individuals' instantaneous utilities, not lifetime utilities, and ii) features generalized (endogenous) welfare weights.

Formally, a linear instantaneous Social Welfare Function, which we denote by $\mathcal{I}\left(\cdot\right)$, is a linear function of individuals' instantaneous utilities, given by

$$\mathcal{I}\left(\left\{u_i\left(c_t^i\left(s^t\right),n_t^i\left(s^t\right)\right)\right\}_{i,t,s^t}\right)=\int\sum_{t=0}^{T}\sum_{s^t}\lambda_t^i\left(s^t\right)u_i\left(c_t^i\left(s^t\right),n_t^i\left(s^t\right)\right)di,\tag{33}$$

where the instantaneous Pareto weights $\lambda_t^i\left(s^t\right)$ define scalars that are individual-, date-, and history-specific.[38]Proposition 17 shows that welfare assessments made under DS-weights correspond to the

[37]Note that the product $\hat{\beta}_{i}^{\mathcal{W}}\left(s\right)\cdot\hat{\pi}_{i}^{\mathcal{W}}\left(s'\middle|\,s\right)$ corresponds to the state-price assigned at state s by individual i to state s':

$$\hat{\beta}_{i,t}^{\mathcal{W}}\cdot\hat{\pi}_{i,t}^{\mathcal{W}}\left(s'\middle|\,s\right)=\beta_i\pi\left(s'\middle|\,s\right)\frac{\partial u_i\left(s'\right)}{\partial c^i}/\frac{\partial u_i\left(s\right)}{\partial c^i}.$$

[38]At times, it may be more convenient to define a linear instantaneous SWF $\mathcal{I}\left(\cdot\right)$ as follows:

$$\mathcal{I}\left(\left\{u_i\left(c_t^i\left(s^t\right),n_t^i\left(s^t\right)\right)\right\}_{i,t,s^t}\right)=\int\sum_{t=0}^{T}\sum_{s^t}\left(\beta_i\right)^t\pi_t\left(s^t\middle|\,s_0\right)\lambda_t^i\left(s^t\right)u_i\left(c_t^i\left(s^t\right),n_t^i\left(s^t\right)\right)di.$$

Both formulations are fully exchangeable in the baseline environment considered in this paper.

derivative of a planner whose objective function is given by a particular linear instantaneous SWF. It also shows that any local optimum can be found as the first-order condition of a planner who maximizes a linear ISWF, where DS-weights are evaluated at the optimum.

Proposition 17. *(Linear instantaneous SWF formulation) For any set of DS-weights, there exist instantaneous Pareto weights $\{\lambda_t^i(s^t)\}_{i,t,s^t}$ such that $\frac{dW^{DS}(s_0)}{d\theta}$, defined in Equation (7), corresponds to the first-order condition of a planner who maximizes a linear instantaneous SWF $\mathcal{I}(\cdot)$ with instantaneous Pareto weights $\lambda_t^i(s^t) = \omega_t^i(s^t;\theta) / \frac{\partial u_i(s^t;\theta)}{\partial c_t^i}$. Moreover, at a local optimum, in which $\frac{dW^{DS}(s_0)}{d\theta} = 0$, there exist instantaneous Pareto weights $\{\lambda_t^i(s^t)\}_{i,t,s^t}$ such that the optimal policy satisfies the first-order condition formula of a linear instantaneous SWF $\mathcal{I}(\cdot)$, defined in Equation (33). The instantaneous Pareto weights in that case are evaluated at the optimum, so $\lambda_t^i(s^t) = \omega_t^i(s^t;\theta^\star) / \frac{\partial u_i(s^t;\theta^\star)}{\partial c_t^i}$, where $\theta^\star$ denotes the value of θ at the local optimum.*

Proposition 17 is helpful because it shows how to reverse-engineer Pareto weights of a linear instantaneous SWF from DS-weights, while guaranteeing that any local optimum can be interpreted as the solution to the maximization of a particular linear instantaneous SWF. Because the instantaneous Pareto weights $\lambda_t^i(s^t)$ are evaluated at the optimum $\theta^\star$, they are taken as fixed in the maximization of a linear instantaneous SWF. In practice, it is impossible to define the instantaneous Pareto weights $\lambda_t^i(s^t)$ without first having solved for the optimum using our approach that starts with DS-weights as primitives. Relatedly, it is typically impossible to translate DS-weights into instantaneous Pareto weights that are invariant to θ and the rest of the environment.[39]

6.5 Term Structure of Welfare Assessments: Transition vs. Steady-State Assessments

In this subsection, we show that the aggregate additive decomposition, and each of its four components, has a term structure. In other words, it is possible to attribute welfare gains or losses in the aggregate or for each of the components of the aggregate additive decomposition to particular dates in the future.[40]

Proposition 18. *(Term structure of welfare assessments and aggregate additive decomposition) The aggregate welfare assessment of a DS-planner, $\frac{dW^{DS}(s_0)}{d\theta}$, can be expressed as follows:*

$$\frac{dW^{DS}(s_0)}{d\theta} = \sum_{t=0}^{T} \mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right] \frac{dW_t^{DS}(s_0)}{d\theta},$$

where each of the date-specific assessments, $\frac{dW_t^{DS}(s_0)}{d\theta}$, can be decomposed into the same four

[39] For the purpose of showing that it is possible to define a DS-planner via a well-defined SWF with generalized (endogenous) weights, it is sufficient to consider *linear* instantaneous SWF's. There is scope to explore further the welfare implications of using more general instantaneous SWF, or even SWF's directly defined over consumption, hours, or other commodities.

[40] It is also possible to define term structures for all subdecompositions introduced in Section 6.1.

components of the aggregate additive decomposition introduced in Proposition 1:

$$\frac{dW_t^{DS}(s_0)}{d\theta} = \overbrace{\sum_{s^t} \mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t \middle| s_0\right)\right] \mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}^{=\Xi_{AE,t}} + \overbrace{\sum_{s^t} \mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t \middle| s_0\right), \frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}^{=\Xi_{RS,t}}$$
$$+ \underbrace{\mathbb{C}ov_i\left[\frac{\tilde{\omega}_t^i(s_0)}{\mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]}, \sum_{s^t} \tilde{\omega}_t^i\left(s^t \middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{IS,t}} + \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \frac{\tilde{\omega}_t^i(s_0)}{\mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]} \sum_{s^t} \tilde{\omega}_t^i\left(s^t \middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{RD,t}}.$$

Proposition 18 shows that a welfare assessment can be interpreted as the discounted sum of date-specific welfare assessments, where each of these date-specific assessments can also be decomposed into the same four components introduced in Proposition 1.[41] Interestingly, Proposition 18 shows that the appropriate discount factor is given by the cross-sectional average of the dynamic components, $\mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]$. In Section 7, we provide an illustration of the term structure of each of the components of a welfare assessment, as well as of the term structure of aggregate assessments.

Proposition 18 also allows us to decompose the transition and steady-state impact of policy changes for aggregate assessments and each of the components of the aggregate additive decomposition. Formally, under the assumption that an economy reaches a new steady-state at date $T^\star$, we can decompose welfare assessments into transition welfare effects and steady-state welfare effects:

$$\frac{dW^{DS}(s_0)}{d\theta} = \underbrace{\sum_{t=0}^{T^\star} \mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right] \frac{dW_t^{DS}(s_0)}{d\theta}}_{\text{transition welfare effects}} + \underbrace{\sum_{t=T^\star}^{T} \mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right] \frac{dW_t^{DS}(s_0)}{d\theta}}_{\text{steady-state welfare effects}}.$$

We illustrate this approach in Section 7, highlighting the fact that convergence to a new steady-state in terms of allocations does not guarantee convergence of DS-weights.[42]

6.6 Summary of Additional Results

In Section G of the Online Appendix, we discuss additional results. First, we provide a systematic dimensional analysis of DS-weights and their components, illustrating why the choice of units is critical to make meaningful welfare assessments. Second, we expand on how the approach that we

[41] Given the definition of $\Xi_{AE,t}$ in Proposition 18, we can express Ξ_{AE} as follows: $\Xi_{AE} = \sum_{t=0}^{T} \mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right] \Xi_{AE,t}$. The same applies to all the other components.

[42] When the economy converges to the new steady-state asymptotically, we define $T^\star$ as the first period in which a convergence criterion is satisfied. To facilitate comparisons, it seems more natural to report the value of steady-state welfare effects expressed in permanent dollars starting at $T^\star$, rather than starting at date-0, that is:

$$\underbrace{\frac{\sum_{t=T^\star}^{T} \mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right] \frac{dW_t^{DS}(s_0)}{d\theta}}{\sum_{t=T^\star}^{T} \mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]}}_{\text{steady-state gains welfare effect valued at } T^\star}.$$

develop in this paper relates to other approaches used to make welfare assessments. In particular, we i) revisit different welfarist SWF's; ii) describe how our results relate to Saez and Stantcheva (2016) and the Kaldor (1939)/Hicks (1939) compensation principle; iii) show how the consumption-equivalent approach of Lucas (1987) and Alvarez and Jermann (2004) can be seen as using a particular set of DS-weights that are related to the DS-weights used by welfarist planners but do not allow for aggregation; iv) explain how allowing for transfers can be interpreted as restricting or partially selecting a set of DS-weights; and v) discuss how our welfare decomposition relates to existing decompositions. We explain how to make use of DS-weights in optimal policy problems using both primal and dual methods, and discuss how to use our approach to make global welfare assessments. Finally, we show to reformulate our results using a notation that explicitly differentiates between idiosyncratic and aggregate states.

7 Application: Transfer Policies under Incomplete Markets

In this section, we illustrate how to make welfare assessments using DS-weights in a fully specified application. The purpose of this application is to illustrate the mechanics of our approach in a tractable dynamic stochastic environment.

After defining a common economic environment, we consider two different scenarios. Scenario 1 corresponds to an economy in which individuals with identical preferences face idiosyncratic risk. In this case, we consider a transfer policy that perfectly smooths consumption across individuals. An important takeaway from our results is that depending on primitives, such policy will be attributed to risk-sharing, intertemporal-sharing, and redistribution to different degrees. Scenario 2 corresponds to an economy in which individuals with different preferences face aggregate risk. In this case, we consider transfer policies that shift aggregate risk to the more risk-tolerant individuals. In both scenarios, we carefully explain the channels through which normalized welfarist planners find such policies desirable or not.

Common environment. We consider an economy with two types of individuals (individuals, for short), with each corresponding to half of the population. Both individuals have time-separable constant relative risk aversion (CRRA) preferences with exponential discounting. We formulate individual lifetime utility recursively as follows:

$$V_i(s) = u_i\left(c^i(s)\right) + \beta \sum_{s'} \pi(s'|s) V_i(s'), \quad \text{where} \quad u_i(c) = \frac{c^{1-\gamma_i}}{1-\gamma_i},$$

where $V_i(s)$ and $c^i(s)$ respectively denote the lifetime utility and the consumption of individual i in a given state s; s and s' denote possible states, and $\pi(s'|s)$ is a Markov transition matrix, described below; β is a discount factor, equal for both individuals; and $u_i(c)$ denotes the instantaneous utility function of an individual i. A higher CRRA coefficient γ_i is mechanically associated with a lower willingness to substitute consumption intertemporally.

Table 2: Summary of scenarios

	Uncertainty	Preferences	Endowment $y^i(s)$		Policy $T^i(s)$		Consumption $c^i(s)$	
			$y^1(s)$	$y^2(s)$	$T^1(s)$	$T^2(s)$	$c^1(s)$	$c^2(s)$
#1	Idiosyncratic	Common	$\overline{y}+\varepsilon(s)$	$\overline{y}-\varepsilon(s)$	$-\varepsilon(s)$	$\varepsilon(s)$	$\overline{y}+\varepsilon(s)(1-\theta)$	$\overline{y}-\varepsilon(s)(1-\theta)$
#2	Aggregate	Heterogeneous	$\overline{y}+\varepsilon(s)$	$\overline{y}+\varepsilon(s)$	$-\varepsilon(s)$	$\varepsilon(s)$	$\overline{y}+\varepsilon(s)(1-\theta)$	$\overline{y}+\varepsilon(s)(1+\theta)$

Note: Instantaneous utility for both individuals is given by $u_i(c)=\frac{c^{1-\gamma_i}}{1-\gamma_i}$. Our benchmark parameterization is given by $\beta=0.975$, $\overline{y}=1$, $\varepsilon(H)=0.25$, $\varepsilon(L)=-0.25$, and $\rho=0.95$. If preferences are common, $\gamma_1=\gamma_2=2$. If preferences are heterogeneous, we assume that individual 1 is more risk averse, so $\gamma_1>\gamma_2$, where $\gamma_1=5$ and $\gamma_2=2$.

There is a single nonstorable consumption good (dollar), which serves as numeraire. We consider an extreme form of incomplete markets: no financial markets. Hence, in the absence of policy transfers, individuals consume their endowments. The consumption of individual i at state s is given by their endowment $y^i(s)$, and a transfer, $\theta T^i(s)\gtreqless 0$, where $\theta\in[0,1]$ scales the size of the transfers at all dates and states. Hence, the budget constraint of individual i in state s is given by

$$c^i(s)=y^i(s)+\theta T^i(s), \tag{34}$$

where the form of $y^i(s)$ and $T^i(s)$ varies in each scenario considered. Given the lack of financial markets, the equilibrium definition is trivial, so Equation (34) also defines equilibrium consumption for individual i. We further assume that the transfers net out in the aggregate, so $T^1(s)+T^2(s)=0$. This assumption will immediately imply that aggregate efficiency is 0 for any policy.

Uncertainty in this economy is captured by a two-state Markov chain, with states denoted by $s=\{L,H\}$, standing for a low (L) and a high (H) realization of $y^1(s)$ (for individual 1) and a transition matrix given by

$$\Pi=\begin{pmatrix}\rho & 1-\rho\\ 1-\rho & \rho\end{pmatrix},$$

where $\rho\in[0,1]$. Table 2 summarizes the assumptions on $y^i(s)$ and $T^i(s)$ made in each scenario. In this model, since $\frac{du_{i|c}(s^t)}{d\theta}=T^i(s)$, welfare assessments are simply given by

$$\frac{dW^{DS}(s_0)}{d\theta}=\int\sum_{t=0}^{\infty}\sum_{s^t}\omega_t^i\left(s^t\middle|\,s_0\right)T^i(s)\,di.$$

7.1 Scenario 1: Idiosyncratic Risk, Homogeneous Preferences

Environment. In our first scenario, we assume i) that both individuals have identical preferences, so $\gamma_1=\gamma_2=\gamma$, and ii) that they exclusively face idiosyncratic risk. Formally, we assume that

$$y^1(s)=\overline{y}+\varepsilon(s)\quad\text{and}\quad y^2(s)=\overline{y}-\varepsilon(s),$$

where $\overline{y}>0$, and where $\varepsilon(L)=-\varepsilon(H)$. We consider the welfare assessment of a transfer policy that provides full consumption smoothing. Formally, we set $T^1(s)=-\varepsilon(s)$ and $T^2(s)=\varepsilon(s)$, so individual consumption takes the form

$$c^1(s) = \overline{y} + \varepsilon(s)(1-\theta) \quad \text{and} \quad c^2(s) = \overline{y} - \varepsilon(s)(1-\theta).$$

Under this policy, when $\theta = 1$, the consumption of both individuals is fully identical. Note that aggregate consumption does not depend on s or θ since $\int c^i(s)\,di = \overline{y}$.

Results. We adopt the following parameters: $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, and $\gamma_1 = \gamma_2 = 2$. Importantly, we make the endowment processes persistent, by setting $\rho = 0.975$ as our benchmark. In Figure 4, we compare how welfare assessments change when the endowment process is extremely persistent ($\rho = 0.999$) and fully transitory ($\rho = 0.5$).[43] As a benchmark, we consider a normalized utilitarian planner with equal weights. In Figure 5 we compare how welfare assessments change when we consider a normalized isoelastic planner.

Individual multiplicative decomposition of DS-weights. In Figure 3, we start by showing the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner for each of the individuals when $\theta = 0.25$. Several insights emerge.

First, Figure 3 clearly illustrates that the DS-weights have time-dependence, despite the fact that we consider a model that is recursive and stationary. This occurs because the shocks are persistent.

Second, the plots of the dynamic components show that a normalized utilitarian planner overweights earlier periods for those individuals who initially have a low endowment and high marginal utility. As reference we include the value of $(1-\beta)\beta^t = \beta^t / \sum_{t=0}^{\infty} \beta^t$, which corresponds to the dynamic weight for a hypothetical individual with linear marginal utility, i.e., when $u_i'(c^i(s)) = 1$. Importantly, since dynamic weights must add up to 1 over time, overweighting initial periods for individuals with low endowment and high marginal utility necessarily implies underweighting periods later in the future.

Third, the plots of the stochastic components show that a normalized utilitarian planner initially overweights those states that are more likely given the initial state, although eventually the impact of the initial state dissipates. More importantly, in the long run (although also in the short run), regardless of the initial state, the stochastic components are higher for those states in which an individual has a lower endowment and high marginal utility.

Fourth, the individual components of the DS-weights further capture the differences in the marginal valuation of transfers among individuals for different initial states. A normalized utilitarian planner values a hypothetical permanent transfer at all dates and states towards the individual with a low endowment at s_0 at 1.186, and towards the individual with a high endowment at 0.814. The plot of DS-weights multiplicatively combines the dynamic, stochastic, and individual components just discussed.

Aggregate additive decomposition of welfare assessments. In Figure 3, we show the components of the aggregate additive decomposition of welfare assessments for a normalized utilitarian planner for

[43] We use $\rho = 0.999$ since it makes for an easier illustration of the results. We could have used $\rho = 1$ instead.

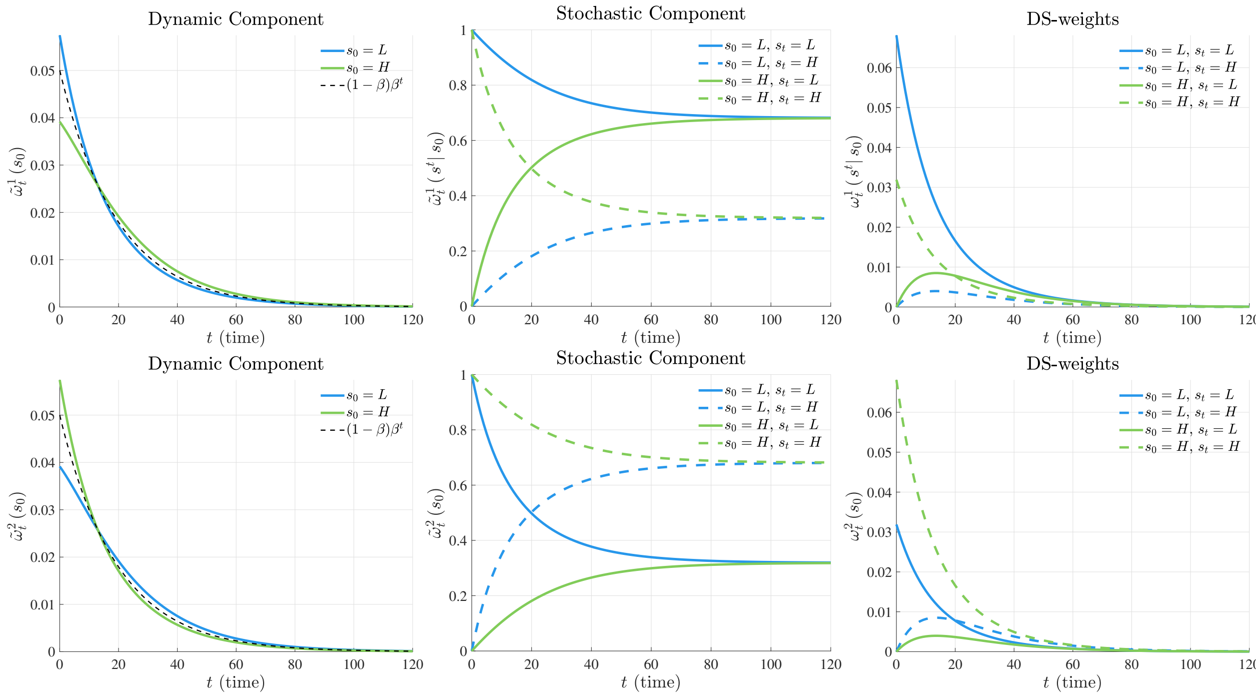

Figure 3: Individual multiplicative decomposition of DS-weights (Scenario 1)

Note: Figure 3 shows the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner, defined in Proposition 5. We assume that $\theta = 0.25$, although all figures are qualitatively similar when $\theta \in [0, 1)$. The top row shows each of the components for individual 1, while the bottom row shows them for individual 2. The left panels show the dynamic component, $\tilde{\omega}_t^i(s_0)$, for different values of t for different initial states, $s_0 = \{H, L\}$. For reference, we also show the dynamic weight for a hypothetical individual with linear marginal utility, given by $(1-\beta)\beta^t = \beta^t / \sum_t \beta^t$. Note that the sum under each of the curves adds up to 1. The middle panels show the stochastic component, $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, for different values of t, for different initial states, $s_0 = \{H, L\}$, and for different final states, $s_t = \{H, L\}$. The right panels show the actual DS-weights, $\omega_t^i\left(s^t \middle| s_0\right)$, also for different values of t, and different initial and final states: $s_0 = \{H, L\}$ and $s_t = \{H, L\}$. The parameters are $\theta = 0.25$, $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.975$, and $\gamma_1 = \gamma_2 = 2$. The individual component of DS-weights are $\tilde{\omega}^1(s_0 = L) = 1.186$ and $\tilde{\omega}^2(s_0 = L) = 0.814$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1(s_0 = H) = 0.814$ and $\tilde{\omega}^2(s_0 = H) = 1.186$ when the assessment takes place at $s_0 = H$.

three different parametrizations: $\rho = \{0.5, 0.975, 0.999\}$. We exclusively consider the initial state $s_0 = L$ since the aggregate welfare assessments are identical in both states.[44] A different set of insights emerge from the aggregate additive decomposition.

First, as formally shown in Proposition 3, the aggregate efficiency component is zero, that is, $\Xi_{AE} = 0$. This occurs because we study an endowment economy for which aggregate consumption is invariant to the policy.

Second, a normalized utilitarian planner always finds it optimal to increase transfers until $\theta = 1$, which corresponds to perfect consumption smoothing. Moreover, we show that all three remaining motives, risk-sharing, intertemporal-sharing, and redistribution contribute qualitatively

[44] If we had considered a welfare assessment at an ex-ante stage in which individuals are identical before the initial state $s_0 = L$ or $s_0 = H$ is realized our conclusions would be significantly different. In particular, as per Corollary 3, intertemporal-sharing and redistribution would be zero in that case. This fact underscores that the decomposition of welfare assessments critically depends on the state in which an assessment takes place.

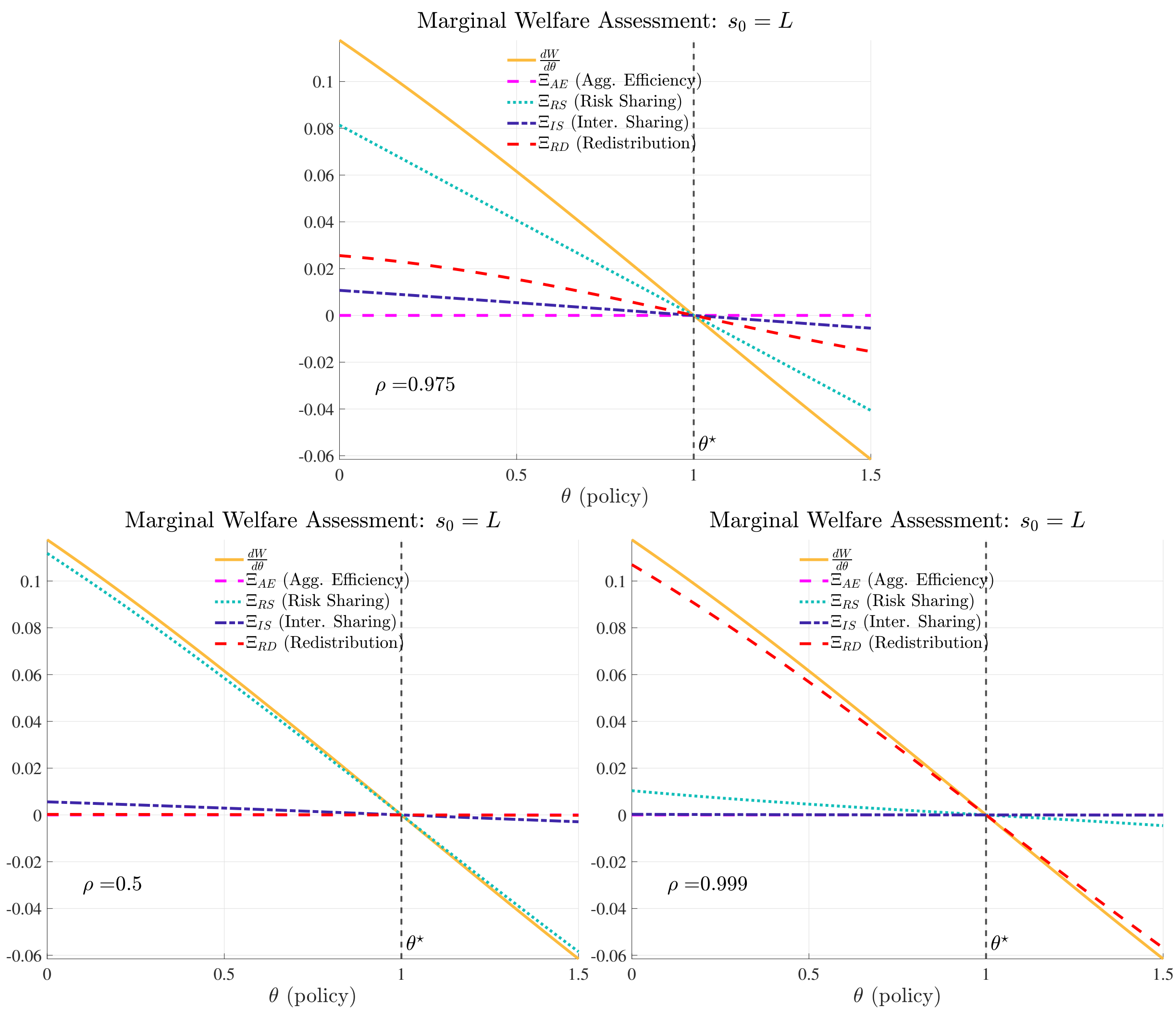

Figure 4: Aggregate additive decomposition of welfare assessments (Scenario 1)

Note: Figure 4 shows the marginal welfare assessment of a normalized utilitarian planner, $\frac{dW}{d\theta}$, and the components of its aggregate additive decomposition, as defined in Proposition 5, for three different scenarios: $\rho = 0.975$ (top panel; benchmark), $\rho = 0.5$ (bottom left panel), and $\rho = 0.999$ (bottom right panel), when $s_0 = L$. When shocks are transitory ($\rho = 0.5$), most welfare gains are attributed to risk-sharing, while when shocks are almost permanent ($\rho = 0.999$), most welfare gains come from redistribution. Intertemporal sharing peaks at intermediate levels of ρ. Note that $\frac{dW}{d\theta} = \Xi_{AE} + \Xi_{RS} + \Xi_{IS} + \Xi_{RD}$. In all three scenarios, the parameters are $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, and $\gamma_1 = \gamma_2 = 2$. This Figure illustrates that the smoothing policy considered here can be attributed to different components, depending on primitives.

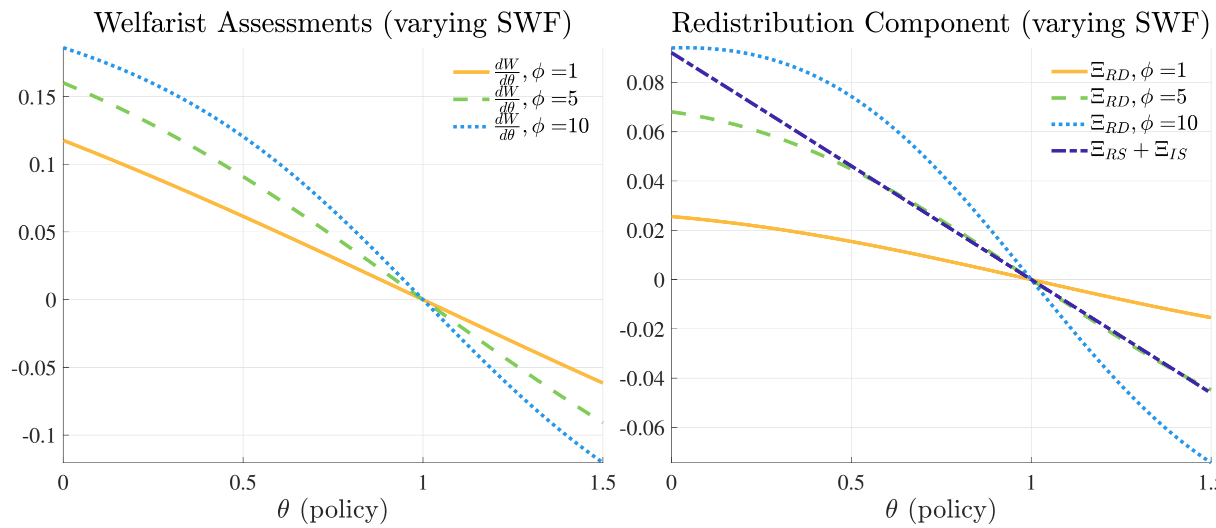

Figure 5: Aggregate additive decomposition; comparison of welfarist planners (Scenario 1)

Note: The left panel of Figure 5 shows the marginal welfare assessment of normalized welfarist planners with social welfare function

$$\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right) = \left(\int a_i\left(-V_i(s_0)\right)^{\phi} di\right)^{1/\phi},$$

for $\phi \in \{1, 5, 10\}$. The utilitarian benchmark corresponds to $\phi = 1$. The right panel of Figure 5 shows the redistribution component, Ξ_{RD}, for such planners, as well as the sum of the risk-sharing and intertemporal-sharing components for either of them, since $\Xi_{RS} + \Xi_{IS}$ is identical in all three cases. In this economy, $\Xi_{AE} = 0$ at all times. Consistently with Proposition 8, differences in welfare assessments among normalized welfarist planners are exclusively based on how they assess the redistribution component. The parameters are $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.975$, and $\gamma_1 = \gamma_2 = 2$.

to that conclusion. Hence, in this scenario, all pseudo-welfarist planners would agree on an optimal policy of $\theta^{\star} = 1$. When $\theta = 1$, markets are effectively complete, which implies that both risk and intertemporal-sharing components are zero, that is, $\Xi_{RS} = \Xi_{IS} = 0$. This is consistent with Propositions 6 and 7. When $\theta = 1$, both individuals have identical consumption paths, so $\Xi_{RD} = 0$. This is consistent with Corollary 3.

Third, the nature of endowment shocks, in particular whether such shocks are transitory or permanent, has a significant impact on the aggregate additive decomposition of welfare assessments. When shocks are transitory ($\rho = 0.5$), the planner attributes most of the welfare gains to risk-sharing, with intertemporal-sharing playing a much smaller role and redistribution being virtually zero. When shocks are persistent ($\rho = 0.975$), part of the welfare gains are now attributed to redistribution, which is now larger than intertemporal-sharing, although risk-sharing is still the most important component. When shocks are almost permanent ($\rho = 0.999$), the planner attributes most of the welfare gains to redistribution, with risk-sharing and intertemporal-sharing playing a much smaller role.

Finally, note that while $\theta^{\star} = 1$ is a global optimum in this economy, setting $\theta = 1$ is by no means a Pareto improvement relative to $\theta = 0$. When $s_0 = L$, there is a value of θ that is less than 1 at which individual 2 becomes worse off relative to $\theta = 0$.

Comparing Social Welfare Functions. In Figure 5, we show the marginal assessment of normalized welfarist planners for different values of the redistribution coefficient ϕ of an isoelastic social welfare function, given by

$$\mathcal{W}\left(\{V_i\left(s_0\right)\}_{i\in I}\right)=\left(\int a_i\left(-V_i\left(s_0\right)\right)^{\phi}di\right)^{1/\phi}.$$

We consider three cases: $\phi \in \{1, 5, 10\}$, where the utilitarian benchmark corresponds to $\phi = 1$.[45] Consistently with Proposition 8, differences in welfare assessments among normalized welfarist planners are exclusively based on how they assess the redistribution component. Intuitively, higher values of the curvature parameter ϕ are associated with more dispersed individual components of DS-weights, which in turn increase the redistribution component of the aggregate decomposition. Moreover, we show the value of the sum of the risk-sharing and intertemporal-sharing components, $\Xi_{RS} + \Xi_{IS}$, is invariant to the value of ϕ — in fact, it corresponds to the assessment of a pseudo-welfarist NR DS-planner. This figure illustrates an important conclusion of this paper, which is that the choice of SWF does not impact the aggregate efficiency, risk-sharing, and intertemporal-sharing components of a normalized welfarist DS-planner.

Term structure of welfare assessments. In Figure 6, we show the implied term structure of welfare assessments, based on the results introduced in Section 6.5. As in Figure 3, we illustrate the results when $\theta = 0.25$. The top plot in Figure 6 shows that the term structure of the aggregate welfare assessment, $\frac{dW_t^{DS}(s_0)}{d\theta}$, is mildly downward sloping, which implies that the welfare gains from the policy are higher in earlier periods.

While the overall gains do not vary substantially over time, each of the components features significant time-variation. The risk-sharing component, $\Xi_{RS,t}$, which is positive at all times and is 0 at $t = 0$, ends up concentrating all of the gains from the policy in the long run. This occurs because this policy has permanent risk-sharing benefits at all dates, since $\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right), \frac{du_{i|c}\left(s^t\right)}{d\theta}\right]$ is strictly positive at all times after $t = 0$.

On the contrary, both the intertemporal-sharing and redistribution components are significantly positive at $t = 0$, but they end up contributing negatively to the welfare assessments of the policy. The bottom two plots in Figure 6 justify the time-variation in $\Xi_{IS,t}$ and $\Xi_{RD,t}$. The bottom left plot shows that the social marginal valuation of the policy at future dates, $\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}$, is positive for the individual with the low endowment realization ($i = 1$) and positive for the other, and it converges to a positive value that is constant across individuals when $t \to \infty$. This implies that $\Xi_{IS,t}$ must converge to 0 in the long run. The date at which $\Xi_{IS,t}$ becomes negative is determined by the date in which the dynamic components of both individuals cross — this date is shown in Figure 3. The bottom right plot shows that the discounted normalized social marginal valuation of the welfare effect of a policy at a given date, $\frac{\tilde{\omega}_t^i(s_0)}{\mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]}\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}$, converges to positive values

[45] Our definition of isoelastic SWF is somewhat nonstandard since lifetime utilities are negative for CRRA individuals with $\gamma > 1$. Our formulation, in which $\phi \geq 1$, guarantees that the SWF is concave, implying that a planner prefers individual utilities to be less dispersed. See Section G.3.1 of the Online Appendix for further details.

for both individuals, but it is higher for the individual for which it was negative at $t = 0$.[46] Because this object does not converge to the same value for both individuals, the redistribution component is permanently non-zero. Intuitively, while the policy contributes positively to the flow utility of both individuals in the long run, this gain is valued more by the individual who started with a higher endowment, since this individual values more consumption in the future. This logic implies that $\Xi_{RD,t}$ must be negative in the long run. In general, the subtle patterns behind $\Xi_{IS,t}$ and $\Xi_{RD,t}$ are driven by the fact that the dynamic components of the DS-weights must cross, since they integrate to 1.

7.2 Scenario 2: Aggregate Risk, Heterogeneous Preferences

Environment. In our second scenario, we assume i) that some individuals are more risk-averse/unwilling to substitute intertemporally than others, and ii) that all endowment risk is aggregate. In particular, we assume that individual 1 is more risk averse than individual 2, so $\gamma_1 > \gamma_2$. Formally, we assume that

$$y^1(s) = \overline{y} + \varepsilon(s) \quad \text{and} \quad y^2(s) = \overline{y} + \varepsilon(s),$$

where $\overline{y} \geq 0$, and where $\varepsilon(L) = -\varepsilon(H)$. We consider the welfare assessment of a transfer policy that shifts the amount of risk borne by individual 1 to individual 2. Formally, we set $T^1(s) = -\varepsilon(s)$ and $T^2(s) = \varepsilon(s)$, so individual consumption takes the form

$$c^1(s) = \overline{y} + \varepsilon(s)(1-\theta) \quad \text{and} \quad c^2(s) = \overline{y} + \varepsilon(s)(1+\theta).$$

Under this policy, when $\theta = 1$, individual 1 is fully insured, at the expense of increasing the consumption fluctuations of individual 2 in response to aggregate shocks. In this scenario, aggregate consumption varies with the aggregate state, but not with θ, since $\int c^i(s)\,di = \overline{y} + \varepsilon(s)$.

Results. With the exception of risk aversion, set to $\gamma_1 = 5$ and $\gamma_2 = 2$, we use the same parameters as in Scenario 1: $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$. As in the benchmark parameterization of Scenario 1, we set $\rho = 0.975$, so endowment shocks are persistent. Once again, we consider a normalized utilitarian planner with equal weights.

Individual multiplicative decomposition of DS-weights. In Figure 7, we show the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner for each of the individuals when $\theta = 0.25$. This new scenario is associated with new insights.

First, the plots of the dynamic components show that a normalized utilitarian planner overweights earlier periods for *all* individuals when the aggregate endowment is low (graphically, the solid blue line is above the black dashed line for both individuals when $s_0 = L$; this is not the case in Scenario

[46] In this application, $\lim_{t\to\infty} \frac{\tilde{\omega}_t^1(s_0)}{\tilde{\omega}_t^2(s_0)} = 0.687$ when $s_0 = L$.

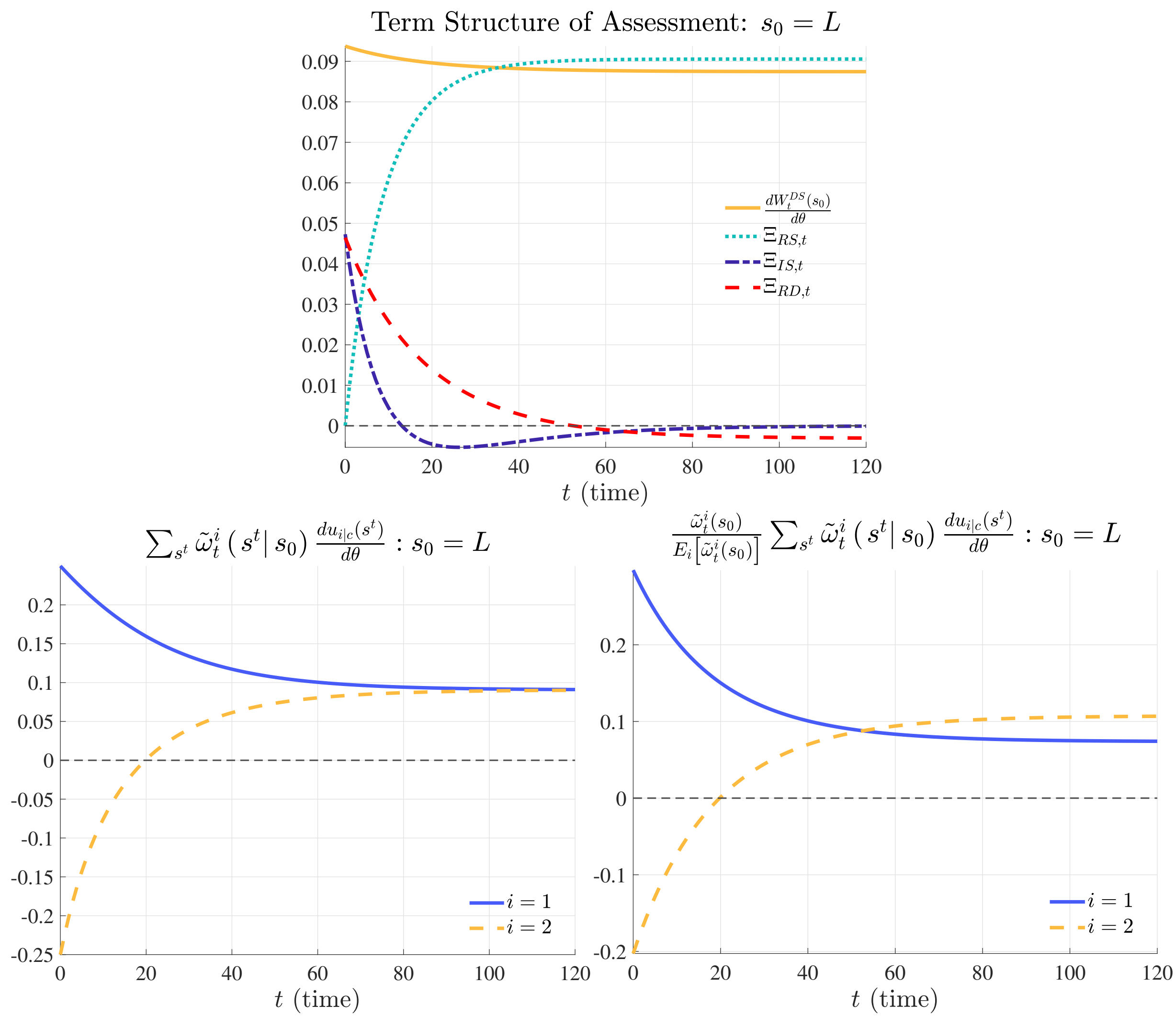

Figure 6: Term structure of aggregate welfare assessments and components (Scenario 1)

Note: The top panel of Figure 6 shows the term structure of marginal welfare assessments, $\frac{dW_t^{DS}(s_0)}{d\theta}$, and each of its non-zero components, $\Xi_{RS,t}$, $\Xi_{IS,t}$, and $\Xi_{RD,t}$ for a normalized utilitarian planner, as defined in Proposition 18, when $s_0 = L$. The bottom left panel corresponds to the term structure of the expected values $\sum_{s^t} \tilde{\omega}_t^i \left(s^t \middle| s_0 \right) \frac{du_{i|c}(s^t)}{d\theta}$, measured as of date t, for both individuals. The bottom right panel corresponds to the term structure of values — as of date 0 and normalized by $\mathbb{E}_i \left[\tilde{\omega}_t^i (s_0) \right]$ — of the impact on an individual at date t: $\frac{\tilde{\omega}_t^i(s_0)}{\mathbb{E}_i[\tilde{\omega}_t^i(s_0)]} \sum_{s^t} \tilde{\omega}_t^i \left(s^t \middle| s_0 \right) \frac{du_{i|c}(s^t)}{d\theta}$. The date at which $\Xi_{IS,t}$ turns negative is the date in which the dynamic components cross in Figure 3. The date at which $\Xi_{RD,t}$ turns negative is the date in which the lines cross in the bottom left plot. Note that $\frac{dW_t^{DS}(s_0)}{d\theta} = \Xi_{AE,t} + \Xi_{RS,t} + \Xi_{IS,t} + \Xi_{RD,t}$. The parameters are $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.975$, and $\gamma_1 = \gamma_2 = 2$.

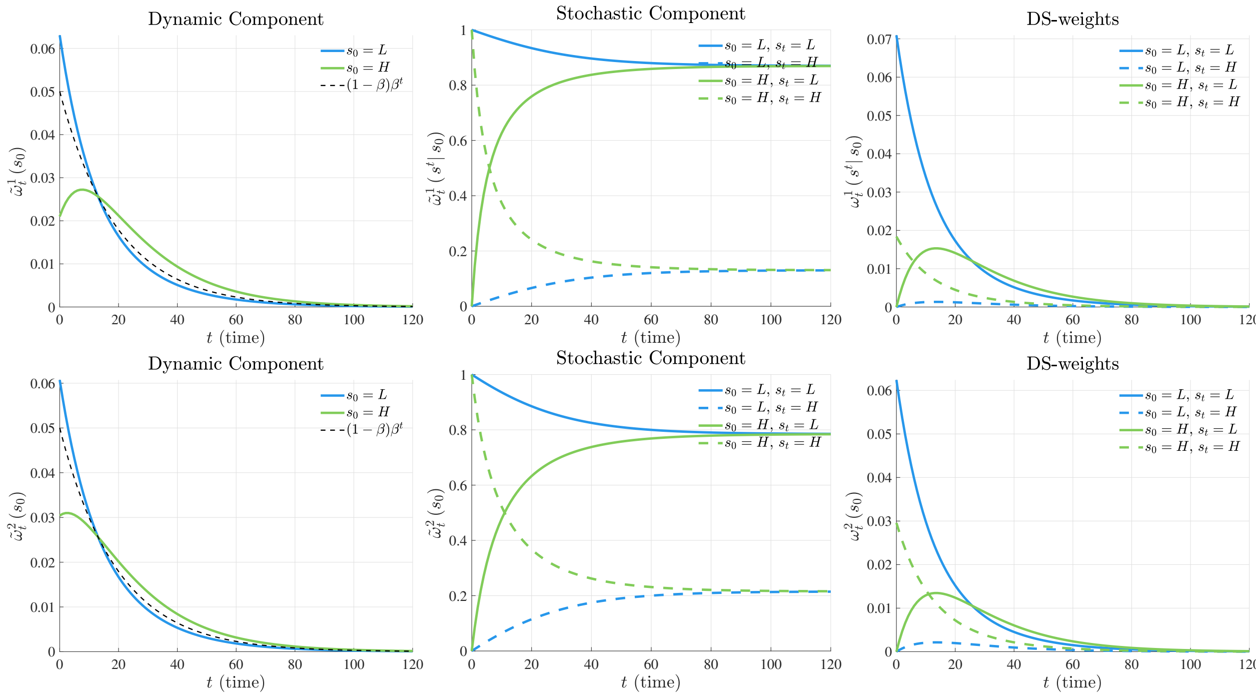

Figure 7: Individual multiplicative decomposition of DS-weights (Scenario 2)

Note: Figure 7 shows the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner, defined in Proposition 5. We assume that $\theta = 0.25$, although all figures are qualitatively similar when $\theta \in [0, 1)$. The top row shows each of the components for individual 1, while the bottom row shows them for individual 2. The left panels show the dynamic component, $\tilde{\omega}_t^i(s_0)$, for different values of t for different initial states, $s_0 = \{H, L\}$. For reference, we also show the dynamic weight for a hypothetical individual with linear marginal utility, given by $(1-\beta)\beta^t = \beta^t / \sum_t \beta^t$. Note that the area under each of the curves adds up to 1. The middle panels show the stochastic component, $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, for different values of t, for different initial states, $s_0 = \{H, L\}$, and for different final states, $s_t = \{H, L\}$. The right panels show the actual DS-weights, $\omega_t^i\left(s^t \middle| s_0\right)$, also for different values of t, and different initial and final states: $s_0 = \{H, L\}$ and $s_t = \{H, L\}$. The parameters are $\theta = 0.25$, $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.975$, $\gamma_1 = 5$, and $\gamma_2 = 2$. The individual component of DS-weights are $\tilde{\omega}^1(s_0 = L) = 1.125$ and $\tilde{\omega}^2(s_0 = L) = 0.875$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1(s_0 = H) = 1.027$ and $\tilde{\omega}^2(s_0 = H) = 0.973$ when the assessment takes place at $s_0 = H$.

1). As one would expect, it does so more for individual 1, with the higher curvature coefficient $\gamma_1 = 5$. Note, for instance, that $\tilde{\omega}_0^1(s_0 = L) > \tilde{\omega}_0^2(s_0 = L)$ and that $\tilde{\omega}_0^1(s_0 = H) < \tilde{\omega}_0^2(s_0 = H)$.

Second, as in Scenario 1, the plots of the stochastic components show that a normalized utilitarian planner overweights more likely states, given the initial state. More importantly, in the long run (although also in the short run), regardless of the initial state, the stochastic components give relatively more weight to those states in which an individual has a lower endowment and higher marginal utility, but differentially more for the individual 1, with the highest curvature coefficient $\gamma_1 = 5$. Note, for instance, that $\tilde{\omega}_\infty^1(s_t = L) > \tilde{\omega}_\infty^2(s_t = L)$ and that $\tilde{\omega}_\infty^1(s_t = H) < \tilde{\omega}_\infty^2(s_t = H)$.

Third, the individual components of the DS-weights still capture differences in the marginal valuation of permanent transfers among individuals for different initial states. However, in this scenario these differences are mostly driven by the differences in preferences between individuals. Unlike in scenario 1, a normalized utilitarian planner gives more value to a hypothetical permanent

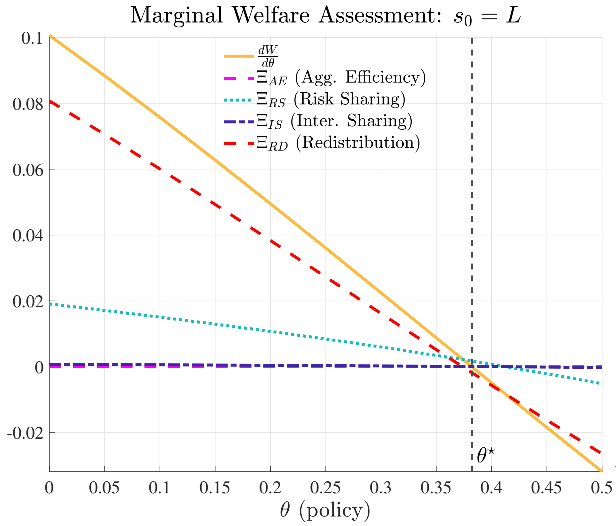

Figure 8: Aggregate additive decomposition of welfare assessments (Scenario 2)

Note: Figure 8 shows the marginal welfare assessment of a normalized utilitarian planner, $\frac{dW}{d\theta}$, and the components of its aggregate additive decomposition, as defined in Proposition 5. The left plot corresponds to the assessment when $s_0 = L$, while the right panel corresponds to the assessments when $s_0 = H$. Note that $\frac{dW}{d\theta} = \Xi_{AE} + \Xi_{RS} + \Xi_{IS} + \Xi_{RD}$. The parameters are $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.975$, and $\gamma_1 = 5 > \gamma_2 = 2$.

transfer towards individual 1 at all states, since $\tilde{\omega}^1(s_0 = L) > \tilde{\omega}^2(s_0 = L)$ and $\tilde{\omega}^1(s_0 = H) > \tilde{\omega}^2(s_0 = H)$. This result illustrates how by computing the individual component it is possible to determine the implicit desire for redistribution of a utilitarian planner.

*Aggregate additive decomposition of welfare assessment*s. In Figure 8, we show the components of the aggregate additive decomposition of welfare assessments for a normalized utilitarian planner. As in Scenario 1, because we study an endowment economy for which aggregate consumption is invariant to the policy, the aggregate efficiency component is zero, that is, $\Xi_{AE} = 0$. There is a new set of insights.

First, we show that a normalized utilitarian planner finds it optimal to increase transfers until some value of $\theta^{\star}$, regardless of whether the optimal policy is determined from $s_0 = L$ or $s_0 = H$. This should not be surprising, since transferring aggregate risk to the individual most willing to bear such a risk seems desirable. Interestingly, the reason for why a planner finds it desirable to increase θ until $\theta^{\star}$ varies with the initial state of the economy. When $s_0 = L$, we show that a normalized utilitarian planner mostly attributes welfare gains to redistribution (Ξ_{RD}), followed by risk-sharing (Ξ_{RS}), with intertemporal-sharing (Ξ_{IS}) barely playing a role. Instead, when $s_0 = H$, we show that a normalized utilitarian planner mostly attributes welfare gains to risk-sharing (Ξ_{RS}), followed by redistribution (Ξ_{RD}) and intertemporal-sharing (Ξ_{IS}).

These findings are intuitive. When $s_0 = L$, consumption is persistently lower, which amplifies differences in curvature between individuals on a persistent basis. This is reflected in the large redistribution component. Building on the insights of Proposition 15, one can trace these results to the cross-sectional dispersion of the different components of DS-weights. In particular, Figure

8 illustrates how the cross-sectional dispersion of the individual component is significantly higher when $s_0 = L$, which explains why the redistribution component is more important when $s_0 = L$. Alternatively, Figure 8 reflects that the cross-sectional dispersion of the dynamic and the stochastic components is higher when $s_0 = H$.

Finally, note that at the optimal $\theta^{\star}$ for both $s_0 = L$ and $s_0 = H$, the normalized utilitarian planner perceives Ξ_{RS} to be positive and Ξ_{RD} to be negative and greater in magnitude than Ξ_{RS}, which is also positive. This implies that both pseudo-utilitarian NS and NR DS-planners would choose a level of $\theta^{\star}$ higher than the normalized utilitarian planner, regardless of the state in which the assessment is made. This result illustrates that, in general, different pseudo-utilitarian DS-planners would disagree on the choice of optimal policies.

8 Conclusion

In this paper, we introduce the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (Dynamic Stochastic weights or DS-weights, for short) and explore their properties. We leverage DS-weights to characterize three sets of results. First, we develop an aggregate additive decomposition of welfare assessments into four distinct components: aggregate efficiency, intertemporal-sharing, risk-sharing, and redistribution. Second, we introduce normalized welfarist planners that allow us to precisely describe how welfarist planners make interpersonal tradeoffs. Third, we show how to use DS-weights to systematically formalize new welfare criteria.

Retrospectively, the aggregate additive decomposition and the definition of normalized welfarist planners introduced in this paper open the door to revisiting the exact rationales that have justified particular welfare assessments in existing work. Looking forward, we hope that our approach informs ongoing and future discussions on i) the desirability of particular policies and ii) the design of policy-making mandates, particularly when trading off efficiency and redistribution objectives.

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Appendix

A Proofs and Derivations: Section 3

Proof of Lemma 1. (DS-weights: individual multiplicative decomposition; unique normalized decomposition)

Proof. By offering a constructive proof of part b), we automatically show that it is always possible to construct an individual multiplicative decomposition, in particular a normalized one. Let us start with a set of DS-weights $\breve{\omega}_t^i\left(s^t\middle|\, s_0\right) > 0$, defined for each individual, date, and history. After multiplying and dividing by $\sum_{s^t} \breve{\omega}_t^i\left(s^t\middle|\, s_0\right)$, $\sum_{t=0}^{T}\sum_{s^t} \breve{\omega}_t^i\left(s^t\middle|\, s_0\right)$, and $\int \sum_{t=0}^{T}\sum_{s^t} \breve{\omega}_t^i\left(s^t\middle|\, s_0\right) di$, we reach the following identity:

$$\underbrace{\frac{\breve{\omega}_t^i\left(s^t\middle|\, s_0\right)}{\int \sum_{t=0}^{T}\sum_{s^t} \breve{\omega}_t^i\left(s^t\middle|\, s_0\right) di}}_{=\omega_t^i\left(s^t|s_0\right)} = \underbrace{\frac{\sum_{t=0}^{T}\sum_{s^t} \breve{\omega}_t^i\left(s^t\middle|\, s_0\right)}{\int \sum_{t=0}^{T}\sum_{s^t} \breve{\omega}_t^i\left(s^t\middle|\, s_0\right) di}}_{=\tilde{\omega}^i(s_0)} \underbrace{\frac{\sum_{s^t} \breve{\omega}_t^i\left(s^t\middle|\, s_0\right)}{\sum_{t=0}^{T}\sum_{s^t} \breve{\omega}_t^i\left(s^t\middle|\, s_0\right)}}_{=\tilde{\omega}_t^i(s_0)} \underbrace{\frac{\breve{\omega}_t^i\left(s^t\middle|\, s_0\right)}{\sum_{s^t} \breve{\omega}_t^i\left(s^t\middle|\, s_0\right)}}_{=\tilde{\omega}_t^i\left(s^t|s_0\right)},$$

which defines an individual multiplicative decomposition since $\omega_t^i\left(s^t\middle|\, s_0\right)$ and $\breve{\omega}_t^i\left(s^t\middle|\, s_0\right)$ are identical from the perspective of Definition 4, but for a normalization regarding the choice of units. It follows immediately that $\sum_{s^t} \tilde{\omega}_t^i\left(s^t\middle|\, s_0\right) = 1$, $\sum_{t=0}^{T} \tilde{\omega}_t^i\left(s_0\right) = 1$, and $\int \tilde{\omega}^i\left(s_0\right) di = 1$, which concludes the proof. □

Proof of Proposition 1. (Welfare assessments: aggregate additive decomposition)

Proof. Combining Equations (7) and (9), the definition of a desirable policy change for a DS-planner can be expressed as follows:

$$\frac{dW^{DS}\left(s_0\right)}{d\theta} = \int \tilde{\omega}^i\left(s_0\right) \frac{dV_i^{DS}\left(s_0\right)}{d\theta} di = \mathbb{E}_i\left[\tilde{\omega}^i\left(s^0\right) \frac{dV_i^{DS}\left(s_0\right)}{d\theta}\right], \tag{35}$$

where

$$\frac{dV_i^{DS}\left(s_0\right)}{d\theta} = \sum_{t=0}^{T} \tilde{\omega}_t^i\left(s_0\right) \sum_{s^t} \tilde{\omega}_t^i\left(s^t\middle|\, s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta}. \tag{36}$$

Hence, we can first decompose $\frac{dW^{DS}(s_0)}{d\theta}$ as follows:

$$\frac{dW^{DS}\left(s_0\right)}{d\theta} = \underbrace{\mathbb{E}_i\left[\tilde{\omega}^i\left(s^0\right)\right]}_{=1} \mathbb{E}_i\left[\frac{dV_i^{DS}\left(s_0\right)}{d\theta}\right] + \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \frac{dV_i^{DS}\left(s_0\right)}{d\theta}\right]}_{=\Xi_{RD}} \tag{37}$$

where we use the fact that — without loss of generality, but for the choice of units — we can set

$\mathbb{E}_i\left[\tilde{\omega}^i\left(s^0\right)\right]=\int\tilde{\omega}^i\left(s^0\right)di=1$, and where Ξ_{RD} satisfies

$$\Xi_{RD}=\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right].$$

Next, we can decompose $\mathbb{E}_i\left[\frac{dV_i^{DS}(s_0)}{d\theta}\right]$ as follows:

$$\begin{aligned}
\mathbb{E}_i\left[\frac{dV_i^{DS}\left(s_0\right)}{d\theta}\right]&=\mathbb{E}_i\left[\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\\
&=\sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]\mathbb{E}_i\left[\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]+\underbrace{\sum_{t=0}^{T}\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right),\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{IS}}\\
&=\sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]\sum_{s^t}\left(\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\right]\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]+\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\right)+\Xi_{IS}\\
&=\underbrace{\sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]\sum_{s^t}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\right]\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{AE}}\\
&\quad+\underbrace{\sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]\sum_{s^t}\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{RS}}+\Xi_{IS}\\
&=\Xi_{AE}+\Xi_{RS}+\Xi_{IS}.
\end{aligned}\tag{38}$$

Proposition 1 follows immediately after combining Equations (37) and (38). □

Proof of Proposition 2. (Properties of aggregate additive decomposition: individual-invariant DS-weights)

Proof. a) If DS-weights $\omega_t^i\left(s^t\middle|\,s_0\right)$ do not vary across individuals, parts b), c), and d) below are valid.
b) If the stochastic components, $\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)$, do not vary across individuals at all dates and histories, then

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]=0,\forall t,\forall s^t\implies\Xi_{RS}=0.$$

c) If the dynamic components, $\tilde{\omega}_t^i\left(s_0\right)$, do not vary across individuals at all dates, then

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right),\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]=0,\forall t\implies\Xi_{IS}=0.$$

d) If the individual components, $\tilde{\omega}^i\left(s_0\right)$, do not vary across individuals, then

$$\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]=0\implies\Xi_{RD}=0.$$

$\square$

Proof of Corollaries 1 through 4

Proof. Corollary 1 follows from part a). Corollary 2 follows from part b) since $\tilde{\omega}_t^i\left(s^t|s_0\right)=1$, $\forall t$, $\forall i$ in perfect foresight economies. Corollary 3 follows from part d). Corollary 4 follows from parts b) and c) since $\tilde{\omega}_t^i\left(s^t|s_0\right)=1$ and $\tilde{\omega}_t^i\left(s_0\right)=1$, $\forall s^t$, $\forall t$, $\forall i$ in static economies. $\square$

Proof of Proposition 3. (Properties of aggregate additive decomposition: individual-invariant policies)

Proof. Note that $\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)$ and $\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)=1$ imply that $\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)=1$.
a) If $\frac{du_{i|c}\left(s^t\right)}{d\theta}=g\left(\cdot\right)$, where $g\left(\cdot\right)$ does not depend on i, t, or s^t, then

$$\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\right]\frac{du_{i|c}\left(s^t\right)}{d\theta}=0\implies\Xi_{RD}=0.$$

And the results from parts b) and c) also apply.
b) If $\frac{du_{i|c}\left(s^t\right)}{d\theta}=g\left(t\right)$, where $g\left(t\right)$ may depend on t, but not on i or s^t, then

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right),\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\right]\frac{du_{i|c}\left(s^t\right)}{d\theta}=0\implies\Xi_{IS}=0.$$

And the result from part c) also applies.
c) If $\frac{du_{i|c}\left(s^t\right)}{d\theta}=g\left(t,s^t\right)$, where $g\left(t,s^t\right)$ may depend on t and s^t, but not on i, then

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]=0\implies\Xi_{RS}=0.$$

$\square$

Proof of Proposition 4. (Properties of aggregate additive decomposition: endowment economies)

Proof. In an endowment economy, Equation (11) simply corresponds to

$$\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]=\int\frac{dc_t^i\left(s^t\right)}{d\theta}di=0,$$

where the last equality follows from the fact that aggregate consumption is equal to the aggregate endowment, and hence fixed and invariant to θ, that is, $\frac{d\int c_t^i\left(s^t\right)di}{d\theta}=0$. $\square$

B Proofs and Derivations: Section 4

Proof of Proposition 5. (Normalized welfarist planners: individual multiplicative decomposition)

Proof. Starting from Equation (6), note that we can express $\frac{dV_i(s_0)}{d\theta}$ as follows:

$$
\begin{aligned}
\frac{dV_i(s_0)}{d\theta} &= \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i} \frac{du_{i|c}\left(s^t\right)}{d\theta} \\
&= \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i} \sum_{t=0}^{T} \frac{(\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i} \frac{du_{i|c}(s^t)}{d\theta}}{\sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i}} \\
&= \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i} \sum_{t=0}^{T} \underbrace{\frac{(\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i}}{\sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i}}}_{=\tilde{\omega}_t^i(s_0)} \sum_{s^t} \underbrace{\frac{(\beta_i)^t \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i}}{(\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i}}}_{=\tilde{\omega}_t^i\left(s^t \middle| s_0\right)} \frac{du_{i|c}\left(s^t\right)}{d\theta} \\
&= \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i} \sum_{t=0}^{T} \tilde{\omega}_t^i(s_0) \sum_{s^t} \tilde{\omega}_t^i\left(s^t \middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta},
\end{aligned} \tag{39}
$$

where we define dynamic and stochastic components of DS-weights as in Equations (13) and (14). Hence, we can express $\frac{dW^{\mathcal{W}}(s_0)}{d\theta}$ — with appropriately normalized units — as follows:

$$
\frac{\frac{dW^{\mathcal{W}}(s_0)}{d\theta}}{\int \lambda_i(s_0) \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i} di} = \int \tilde{\omega}^i(s_0) \sum_{t=0}^{T} \tilde{\omega}_t^i(s_0) \sum_{s^t} \tilde{\omega}_t^i\left(s^t \middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta} di,
$$

where we define the individual component as in Equation (15):

$$
\tilde{\omega}^i(s_0) = \frac{\lambda_i(s_0) \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i}}{\int \lambda_i(s_0) \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i} di}.
$$

It is straightforward to verify that $\sum_{s^t} \tilde{\omega}_t^i\left(s^t \middle| s_0\right) = 1$, $\forall t$, $\forall i$; that $\sum_{t=0}^{T} \tilde{\omega}_t^i(s_0) = 1$, $\forall i$; and that $\int \tilde{\omega}^i(s_0)\, di = 1$, which concludes the proof. Note that by multiplying and dividing the dynamic and stochastic components of a given individual by his marginal utility of consumption at 0, we recover Equations (16) and (17):

$$
\begin{aligned}
\tilde{\omega}_t^i(s_0) &= \frac{(\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i} / \frac{\partial u_i(s^0)}{\partial c_0^i}}{\sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i} / \frac{\partial u_i(s^0)}{\partial c_0^i}} = \frac{q_t^i\left(s^t | s_0\right)}{\sum_{s^t} q_t^i\left(s^t | s_0\right)} \\
\tilde{\omega}_t^i\left(s^t \middle| s_0\right) &= \frac{\pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i} / \frac{\partial u_i(s^0)}{\partial c_0^i}}{\sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i} / \frac{\partial u_i(s^0)}{\partial c_0^i}} = \frac{\sum_{s^t} q_t^i\left(s^t | s_0\right)}{\sum_{t=0}^{T} \sum_{s^t} q_t^i\left(s^t | s_0\right)}.
\end{aligned}
$$

□

Proof of Proposition 6. (Properties of normalized welfarist planners: complete markets)

Proof. When markets are complete, there is a unique stochastic discount factor, which implies that $q_t^i\left(s^t|s_0\right) = q_t\left(s^t|s_0\right)$, $\forall i$. From Equations (16) and (17), it follows immediately that $\tilde{\omega}_t^i\left(s_0\right)$ and $\tilde{\omega}_t^i\left(\left.s^t\right|s_0\right)$ are invariant across all individuals at all dates and histories. Hence, parts b) and c) of Proposition 2 guarantee that $\Xi_{RS} = \Xi_{IS} = 0$. $\square$

Proof of Proposition 7. (Properties of normalized welfarist planners: riskless borrowing/saving)

Proof. When individuals can freely borrow and save, it must be the case that the valuation of a riskless bond is identical for all individuals, which implies that $\sum_{s^t} q_t^i\left(s^t|s_0\right)$ is identical across individuals. Hence, from Equation (17), it follows immediately that $\tilde{\omega}_t^i\left(s_0\right)$ is invariant across all individuals at all dates. Hence, Part c) Proposition 2 guarantees that $\Xi_{IS} = 0$. $\square$

Proof of Proposition 8. (Properties of normalized welfarist planners: welfarist planners only disagree about redistribution)

Proof. Note that Equations (13) and (14) do not depend on $\mathcal{W}\left(\cdot\right)$, while Equation (15) does. This fact, along with Proposition 1, immediately imply that Ξ_{AE}, Ξ_{RS}, and Ξ_{IS} identical for all welfarist planner, but Ξ_{RD} is not. $\square$

Proof of Proposition 9. (Properties of normalized welfarist planners: invariance of efficiency components to utility transformations)

Proof. It follows immediately from Equations (13) and (14) that $\tilde{\omega}_t^i\left(s_0\right)$ and $\tilde{\omega}_t^i\left(\left.s^t\right|s_0\right)$ are invariant to the transformations considered, which multiply numerator and denominator by constant factors. $\square$

Proof of Proposition 10. (Properties of normalized welfarist planners: Pareto improvements increase efficiency)

Proof. From Equation (38), it immediately follows that

$$\Xi_{AE} + \Xi_{RS} + \Xi_{IS} = \mathbb{E}_i\left[\frac{\frac{dV_i(s_0)}{d\theta}}{\sum_{t=0}^{T}\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(\left.s^t\right|s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}}\right],$$

where $\frac{dV_i(s_0)}{d\theta}$ is defined in Equation (39). If a policy is a strict Pareto improvement, $\frac{dV_i(s_0)}{d\theta} > 0$, which implies that $\Xi_{AE} + \Xi_{RS} + \Xi_{IS}$ must be strictly positive, since $\sum_{t=0}^{T}\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(\left.s^t\right|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i} > 0$. The same logic applies to weak Pareto improvements, since at least one individual must have $\frac{dV_i(s_0)}{d\theta} > 0$. $\square$

C Proofs and Derivations: Section 5

Proof of Proposition 11 (AE/AR/NR DS-planners: properties)

Proof. a) This result follows from part a) of Proposition 3, since $\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)$, $\tilde{\omega}_t^i\left(s_0\right)$, and $\tilde{\omega}^i\left(s_0\right)$ do not vary across individuals. Note that Ξ_{AE} is identical for the pseudo-welfarist AE DS-planner and its associated normalized welfarist planner, since

$$\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s_0\right)\right]=\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)\right] \text{ and } \mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\middle|\, s_0\right)\right]=\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\, s_0\right)\right].$$

b) This result follows from parts c) and d) of Proposition 3, since $\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)$ and $\tilde{\omega}_t^i\left(s_0\right)$ do not vary across individuals. Note that Ξ_{AE} and Ξ_{RS} are identical for the pseudo-welfarist AR DS-planner and its associated normalized welfarist planner, since

$$\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W},AR}\left(s_0\right)\right]=\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)\right] \text{ and } \tilde{\omega}_t^{i,\mathcal{W},AR}\left(s^t\middle|\, s_0\right)=\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\, s_0\right).$$

c) This result follows from part d) of Proposition 3, since the individual components $\tilde{\omega}^i\left(s_0\right)$ do not vary across individuals. Note that Ξ_{AE}, Ξ_{RS}, and Ξ_{IS} are identical for the pseudo-welfarist NR DS-planner and its associated normalized welfarist planner, since

$$\tilde{\omega}_t^{i,\mathcal{W},NR}\left(s_0\right)=\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right) \text{ and } \tilde{\omega}_t^{i,\mathcal{W},NR}\left(s^t\middle|\, s_0\right)=\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\, s_0\right).$$

□

Online Appendix

Section D of this Online Appendix includes proofs and derivations for Section 6. Section E includes additional results for Application 1. Section F includes several extensions and Section G contains additional results.

D Proofs and Derivations: Section 6

Proof of Proposition 12. (Aggregate efficiency component: stochastic decomposition)

Proof. Starting from the definition of the aggregate efficiency component in Equation (11), we can express Ξ_{AE} as follows:

$$\begin{aligned}\Xi_{AE} &= \sum_{t=0}^{T} \mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right] \sum_{s^t} \mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t \middle| s_0\right)\right] \mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] \\ &= \sum_{t=0}^{T} \overline{\omega}_t \sum_{s^t} \overline{\omega}_t\left(s^t \middle| s_0\right) \frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta},\end{aligned}$$

where we define $\overline{\omega}_t\left(s_0\right) = \mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]$, $\overline{\omega}_t\left(s^t\middle| s_0\right) = \mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle| s_0\right)\right]$, and $\frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta} = \mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]$. Multiplying and dividing by $\pi_t\left(s^t\middle| s_0\right)$ at every history, we can express and decompose Ξ_{AE} as follows:

$$\begin{aligned}\Xi_{AE} &= \sum_{t=0}^{T} \overline{\omega}_t\left(s_0\right) \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\overline{\omega}_t\left(s^t\middle| s_0\right)}{\pi_t\left(s^t\middle| s_0\right)} \frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta} = \sum_{t=0}^{T} \overline{\omega}_t\left(s_0\right) \mathbb{E}_0\left[\overline{\omega}_t^{\pi}\left(s^t \middle| s_0\right) \frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta}\right] \\ &= \underbrace{\sum_{t=0}^{T} \overline{\omega}_t\left(s_0\right) \mathbb{E}_0\left[\overline{\omega}_t^{\pi}\left(s^t \middle| s_0\right)\right] \mathbb{E}_0\left[\frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{EAE}} + \underbrace{\sum_{t=0}^{T} \overline{\omega}_t\left(s_0\right) \mathbb{C}ov_0\left[\overline{\omega}_t^{\pi}\left(s^t \middle| s_0\right), \frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{AM}},\end{aligned}$$

which corresponds to Equation (23) in the text. □

Proof of Proposition 13. (Risk-sharing/intertemporal-sharing components: alternative cross-sectional decompositions)

Proof. Here we make use of the following property of covariances (Bohrnstedt and Goldberger, 1969):

$$\mathbb{C}ov\left[X, YZ\right] = \mathbb{E}\left[Y\right] \mathbb{C}ov\left[X, Z\right] + \mathbb{E}\left[Z\right] \mathbb{C}ov\left[X, Y\right] + \mathbb{E}\left[\left(X - \mathbb{E}\left[X\right]\right)\left(Y - \mathbb{E}\left[Y\right]\right)\left(Z - \mathbb{E}\left[Z\right]\right)\right],$$

where X, Y, and Z denote random variables. Applying this property to $\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right), \tilde{\omega}_t^i\left(s^t\middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta}\right]$, we find that

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i(s_0),\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}(s^t)}{d\theta}\right]=\underbrace{\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\right]\mathbb{C}ov_i\left[\tilde{\omega}_t^i(s_0),\frac{du_{i|c}(s^t)}{d\theta}\right]}_{\sim\Xi_{PIS}}$$

$$+\underbrace{\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right]\mathbb{C}ov_i\left[\tilde{\omega}_t^i(s_0),\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\right]}_{\sim\Xi_{WC}}$$

$$+\underbrace{\mathbb{E}_i\left[\left(\frac{du_{i|c}(s^t)}{d\theta}-\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right]\right)\left(\tilde{\omega}_t^i(s_0)-\mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]\right)\left(\tilde{\omega}_t^i\left(s^t\middle|s_0\right)-\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\right]\right)\right]}_{\sim\Xi_{PC}},$$

which immediately yields Equation (24) in the text after adding up over dates and histories. Equation (25) follows immediately after using once again the same property of covariances on $\mathbb{C}ov_i\left[\tilde{\omega}_t^i(s_0)\tilde{\omega}_t^i(s^t|s_0),\frac{du_{i|c}(s^t)}{d\theta}\right]$. □

Proof of Proposition 14. (Redistribution component: stochastic decomposition)

Proof. We can express $\frac{dV_i^{DS}(s_0)}{d\theta}$, defined in Equation (36), as follows:

$$\frac{dV_i^{DS}(s_0)}{d\theta}=\sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\mathbb{E}_0\left[\frac{\tilde{\omega}_t^i(s^t|s_0)}{\pi_t(s^t|s_0)}\frac{du_{i|c}(s^t)}{d\theta}\right]$$
$$=\underbrace{\sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\mathbb{E}_0\left[\tilde{\omega}_t^{i,\pi}\left(s^t\middle|s_0\right)\right]\mathbb{E}_0\left[\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\frac{dV_i^{DS,ER}(s_0)}{d\theta}}+\underbrace{\sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\mathbb{C}ov_0\left[\left[\tilde{\omega}_t^{i,\pi}\left(s^t\middle|s_0\right)\right],\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\frac{dV_i^{DS,RM}(s_0)}{d\theta}}.$$

Hence, we can express Ξ_{RD} as follows:

$$\Xi_{RD}=\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\frac{dV_i^{DS}(s_0)}{d\theta}\right]=\underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\frac{dV_i^{DS,ER}(s_0)}{d\theta}\right]}_{\Xi_{ER}}+\underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\frac{dV_i^{DS,RM}(s_0)}{d\theta}\right]}_{\Xi_{RM}},$$

which corresponds to Equation (15) in the text. □

Proof of Proposition 15. (Cross-sectional dispersion bounds)

Proof. Equations (26) through (28) follow from applying the Cauchy-Schwarz inequality, which states that $|\mathbb{C}ov[X,Y]|\leq\sqrt{\mathbb{V}ar[X]}\sqrt{\mathbb{V}ar[Y]}$ for any pair of square integrable random variables X and Y.

When applied to the relevant elements of Ξ_{RS}, Ξ_{IS}, and Ξ_{RD}, we find that

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\leq\sqrt{\mathbb{V}ar_i\left[\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\right]}\sqrt{\mathbb{V}ar_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}$$

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right),\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\leq\sqrt{\mathbb{V}ar_i\left[\tilde{\omega}_t^i\right]}\sqrt{\mathbb{V}ar_i\left[\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}$$

$$\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\leq\sqrt{\mathbb{V}ar_i\left[\tilde{\omega}^i\left(s^0\right)\right]}\sqrt{\mathbb{V}ar_i\left[\sum_{t=0}^{T}\sum_{s^t}\tilde{\omega}_t^i\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}.$$

These three inequalities, when combined with the definitions of Ξ_{RS}, Ξ_{IS}, and Ξ_{RD} in Equation (11), immediately imply Equations (26) through (28) in the text. □

Proof of Proposition 16. (Recursive formulation)

Proof. Starting from Equation (35), note that we can express $\frac{dW^{DS}(s_0)}{d\theta}$ as follows:

$$\begin{aligned}\frac{dW^{DS}\left(s_0\right)}{d\theta}&=\int\tilde{\omega}^i\left(s_0\right)\frac{dV_i^{DS}\left(s_0\right)}{d\theta}di\\&=\int\underbrace{\tilde{\omega}^i\left(s_0\right)\tilde{\omega}_0^i\left(s_0\right)\tilde{\omega}_0^i\left(s^0\middle|\,s_0\right)}_{=\omega_0^i\left(s^0|s_0\right)}\underbrace{\frac{\frac{dV_i^{DS}(s_0)}{d\theta}}{\tilde{\omega}_0^i\left(s_0\right)\tilde{\omega}_0^i\left(s^0\middle|\,s_0\right)}}_{=\frac{d\hat{V}_{i,0}^{DS}(s_0)}{d\theta}}di\\&=\int\omega_0^i\left(s^0\middle|\,s_0\right)\frac{d\hat{V}_{i,0}^{DS}\left(s_0\right)}{d\theta}di.\end{aligned}$$

Note that we can also express $\frac{d\hat{V}_{i,0}^{DS}(s_0)}{d\theta}$ as follows:

$$\begin{aligned}\frac{d\hat{V}_{i,0}^{DS}\left(s_0\right)}{d\theta}&=\sum_{t=0}^{T}\frac{\tilde{\omega}_t^i\left(s_0\right)}{\tilde{\omega}_0^i\left(s_0\right)}\sum_{s^t}\frac{\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)}{\tilde{\omega}_0^i\left(s^0\middle|\,s_0\right)}\frac{du_{i|c}\left(s_t\right)}{d\theta}\\&=\underbrace{\frac{\tilde{\omega}_0^i\left(s_0\right)}{\tilde{\omega}_0^i\left(s_0\right)}}_{=1}\underbrace{\frac{\tilde{\omega}_0^i\left(s^0\middle|\,s_0\right)}{\tilde{\omega}_0^i\left(s^0\middle|\,s_0\right)}}_{=1}\frac{du_{i|c}\left(s_0\right)}{d\theta}+\sum_{t=1}^{T}\frac{\tilde{\omega}_t^i\left(s_0\right)}{\tilde{\omega}_0^i\left(s_0\right)}\sum_{s^t}\frac{\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)}{\tilde{\omega}_0^i\left(s^0\middle|\,s_0\right)}\frac{du_{i|c}\left(s_t\right)}{d\theta}\\&=\frac{du_{i|c}\left(s_0\right)}{d\theta}+\frac{\tilde{\omega}_1^i\left(s_0\right)}{\tilde{\omega}_0^i\left(s_0\right)}\left(\underbrace{\frac{\tilde{\omega}_1^i\left(s_0\right)}{\tilde{\omega}_1^i\left(s_0\right)}}_{=1}\sum_{s^1}\frac{\tilde{\omega}_1^i\left(s^1\middle|\,s_0\right)}{\tilde{\omega}_0^i\left(s^0\middle|\,s_0\right)}\frac{du_{i|c}\left(s_1\right)}{d\theta}+\sum_{t=2}^{T}\frac{\tilde{\omega}_t^i\left(s_0\right)}{\tilde{\omega}_1^i\left(s_0\right)}\sum_{s^t}\frac{\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)}{\tilde{\omega}_0^i\left(s^0\middle|\,s_0\right)}\frac{du_{i|c}\left(s_t\right)}{d\theta}\right)\\&=\frac{du_{i|c}\left(s_0\right)}{d\theta}+\frac{\tilde{\omega}_1^i\left(s_0\right)}{\tilde{\omega}_0^i\left(s_0\right)}\left(\sum_{s^1}\frac{\tilde{\omega}_1^i\left(s^1\middle|\,s_0\right)}{\tilde{\omega}_0^i\left(s^0\middle|\,s_0\right)}\underbrace{\left(\frac{du_{i|c}\left(s_1\right)}{d\theta}+\sum_{t=2}^{T}\frac{\tilde{\omega}_t^i\left(s_0\right)}{\tilde{\omega}_1^i\left(s_0\right)}\sum_{s^t|s^1}\frac{\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)}{\tilde{\omega}_1^i\left(s^1\middle|\,s_0\right)}\frac{du_{i|c}\left(s_t\right)}{d\theta}\right)}_{=\frac{d\hat{V}_{i,1}^{DS}(s_1)}{d\theta}}\right),\end{aligned}$$

which immediately implies Equation (30) in the text, since this derivation is valid starting from any state s_0.

The definitions of $\hat{\beta}_{i,t}^{\mathcal{W}}$ and $\hat{\pi}_{i,t}^{\mathcal{W}}$ follow immediately after combining Equations (13) and (14) with Equation (31). Note that the product $\hat{\beta}_{i}^{\mathcal{W}}(s) \cdot \hat{\pi}_{i}^{\mathcal{W}}(s' | s)$ corresponds to the state-price assigned at state s by individual i to state s':

$$\hat{\beta}_{i,t}^{\mathcal{W}} \cdot \hat{\pi}_{i,t}^{\mathcal{W}}(s' | s) = \beta_i \pi (s' | s) \frac{\partial u_i(s')}{\partial c^i} / \frac{\partial u_i(s)}{\partial c^i},$$

and that this state-price is time-independent. This observation, combined with the definition of the pseudo-utilitarian NR planner, implies the claim that Equation (30) is time invariant for welfarist and pseudo-welfarist NR planners. □

Proof of Proposition 17 (Linear instantaneous SWF formulation)

Proof. Note that, for a planner with a linear instantaneous SWF, it must be that

$$\frac{d\mathcal{I}(\cdot)}{d\theta} = \int \sum_{t=0}^{T} \sum_{s^t} \lambda_t^i \left(s^t\right) \frac{\partial u_i(s^t)}{\partial c_t^i} \frac{du_{i|c}(s^t)}{d\theta} di, \tag{OA1}$$

where $\frac{du_{i|c}(s^t)}{d\theta}$ is defined in Equation (3). The results for both the marginal welfare assessment and the optimum follow immediately by comparing Equation (7) to Equation (OA1), where the following relation must be satisfied:

$$\lambda_t^i \left(s^t\right) = \frac{\omega_t^i(s^t)}{\frac{\partial u_i(s^t)}{\partial c_t^i}}.$$

□

E Application: Additional Figures

Figures OA-1 and OA-2 are the counterparts of Figure 3 in the text when $\rho = 0.999$ and $\rho = 0.5$. When $\rho = 0.999$, the components of the individual multiplicative decompositions evolve extremely slowly. Given the extreme persistence of the shocks, all of the welfare gains from increasing θ arise from redistribution (Ξ_{RD}). When $\rho = 0.5$, endowments shocks are fully transitory, and the components of the individual multiplicative decomposition barely have any time-dependence. In this case, the welfare gains from increasing θ arise mostly from risk-sharing. The gains from redistribution are nonzero, but very small, since they are only driven by marginal utility differences at $t = 0$.

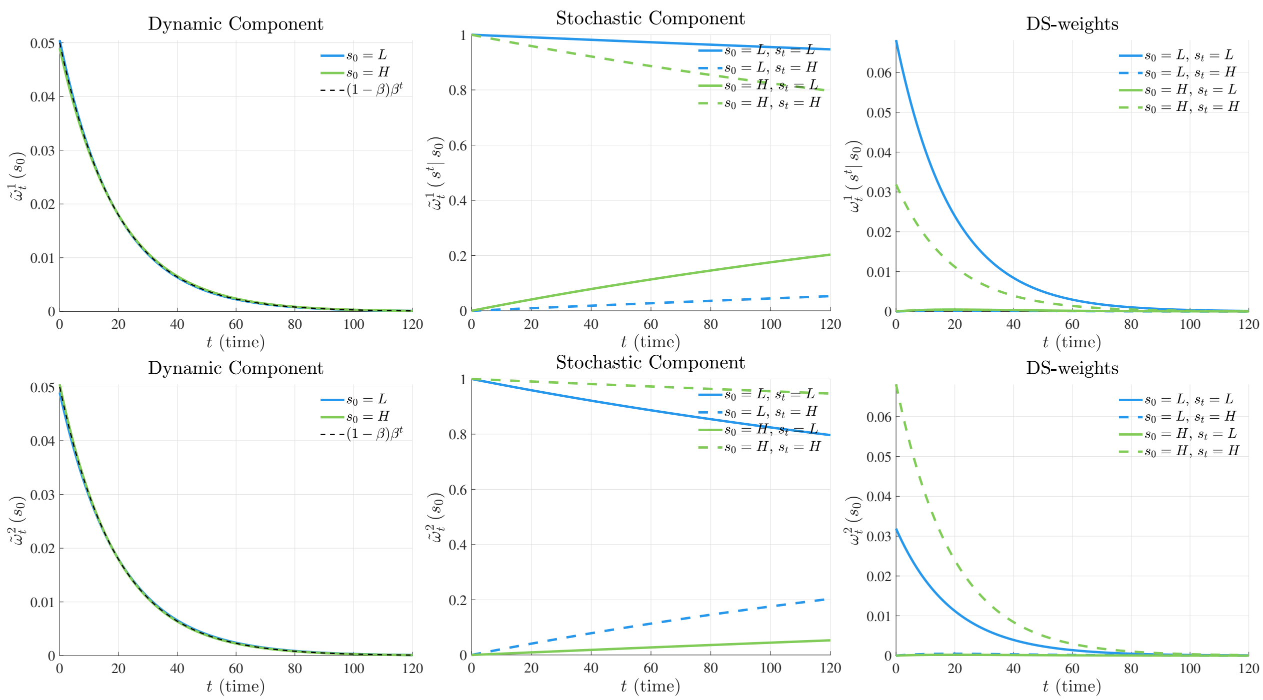

Figure OA-1: Individual multiplicative decomposition of DS-weights (Scenario 1; $\rho = 0.999$)

Note: Figure OA-1 is the counterpart of Figure 3 in the text when endowment shocks are extremely persistent ($\rho = 0.999$). The individual component of DS-weights in this case are $\tilde{\omega}^1\,(s_0 = L) = 1.349$ and $\tilde{\omega}^2\,(s_0 = L) = 0.651$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1\,(s_0 = H) = 0.651$ and $\tilde{\omega}^2\,(s_0 = H) = 1.349$ when the assessment takes place at $s_0 = H$.

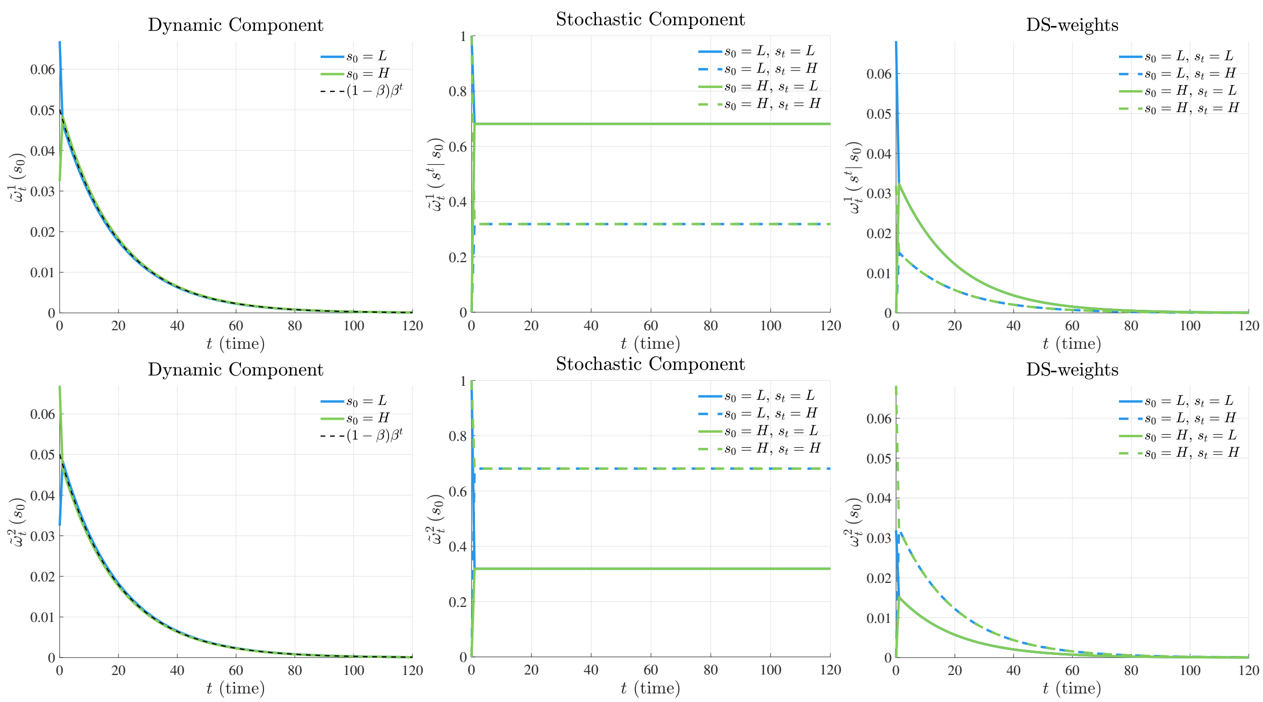

Figure OA-2: Individual multiplicative decomposition of DS-weights (Scenario 1; $\rho = 0.5$)

Note: Figure OA-2 is the counterpart of Figure 3 in the text when endowment shocks are fully temporary ($\rho = 0.5$). The individual component of DS-weights in this case are $\tilde{\omega}^1(s_0 = L) = 1.018$ and $\tilde{\omega}^2(s_0 = L) = 0.928$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1(s_0 = H) = 0.982$ and $\tilde{\omega}^2(s_0 = H) = 1.018$ when the assessment takes place at $s_0 = H$.

F Extensions

F.1 Heterogeneous beliefs

In this section, we show how to use DS-weights to make paternalistic and non-paternalistic welfare assessments in environments with heterogeneous beliefs.[47] Note that the notion of paternalism used here is fully consistent with the formal definition given in Footnote 32. To model heterogeneous beliefs, instead of Equation (1), we assume instead that individual preferences take the form

$$V_i\left(s_0\right)=\sum_{t=0}^{T}\left(\beta_i\right)^t \sum_{s^t} \pi_t^i\left(s^t \middle| s_0\right) u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right), \tag{OA2}$$

where $\pi_t^i\left(s^t \middle| s_0\right)$, denotes the beliefs held by individual i over histories, which are now individual-specific.

In this case, a non-paternalistic planner would substitute $\pi_t^i\left(s^t \middle| s_0\right)$ for $\pi_t\left(s^t \middle| s_0\right)$ whenever it appears in Equations (8) through (22). Alternatively, a paternalistic planner who imposes a single-belief would substitute some planner's belief, $\pi_t^P\left(s^t \middle| s_0\right)$, which is invariant across individuals, for $\pi_t\left(s^t \middle| s_0\right)$ whenever it appears in Equations (8) through (22).[48]

F.2 Recursive utility: Epstein-Zin preferences

In this section, we show how to use DS-weights in the context of economies with recursive preferences. In particular, we consider the widely used Epstein-Zin preferences, which we define recursively as follows:

$$V_i(s)=\left(\left(1-\beta_i\right)\left(u_i\left(c^i(s), n^i(s)\right)\right)^{1-\frac{1}{\psi_i}}+\beta_i\left(\sum_{s'} \pi\left(s'|s\right)\left(V^i\left(s'\right)\right)^{1-\gamma_i}\right)^{\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}}\right)^{\frac{1}{1-\frac{1}{\psi_i}}},$$

where γ_i modulates risk aversion and ψ_i modulates intertemporal substitution. We use s and s' to denote any two recursive states (Ljungqvist and Sargent, 2018).

In this case, we can recursively express the welfare effect of a policy change, measured in lifetime

[47] A recent literature has explored how to make normative assessments in environments with heterogeneous beliefs. See, among others, Brunnermeier, Simsek and Xiong (2014), Gilboa, Samuelson and Schmeidler (2014), Dávila (2020), Blume et al. (2018), Caballero and Simsek (2019), and Dávila and Walther (2021).

[48] At times, it makes sense to reinterpret heterogeneous beliefs as state-dependent preferences. In that case, $V_i\left(s_0\right)$ can be expressed as

$$V_i\left(s_0\right)=\sum_{t=0}^{T}\left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right); s^t\right).$$

All our results remain valid in the case of state-dependent preferences.

utils (utility units), as follows:

$$\frac{dV_i(s)}{d\theta} = \frac{\partial V_i(s)}{\partial c^i(s)}\frac{du_{i|c}(s)}{d\theta} + \sum_{s'}\frac{\partial V_i(s)}{\partial V^i(s')}\frac{dV_i(s')}{d\theta}, \tag{OA3}$$

where

$$\frac{\partial V_i(s)}{\partial c^i(s)} = (1-\beta_i)(V_i(s))^{\frac{1}{\psi}}(u_i(s))^{-\frac{1}{\psi_i}}\frac{\partial u_i(s)}{\partial c^i}$$

$$\frac{\partial V_i(s)}{\partial V^i(s')} = \beta_i (V_i(s))^{\frac{1}{\psi}}\left(\sum_{s'}\pi(s'|s)\left(V^i(s')\right)^{1-\gamma_i}\right)^{\frac{\gamma_i-\frac{1}{\psi_i}}{1-\gamma_i}}\pi(s'|s)\left(V^i(s')\right)^{-\gamma_i},$$

and where $\frac{du_{i|c}(s)}{d\theta}$ is defined as in Equation (3). The structure of Equation (OA3) immediately implies that $\frac{dV_i(s)}{d\theta}$ can be expressed as a linear transformation of instantaneous consumption-equivalent effects, $\frac{du_{i|c}(s)}{d\theta}$, which in turn guarantees that the definition of a DS-planner in Equation (6) can also be used in the context of economies with recursive preferences.

Note that it is straightforward to define normalized DS-weights when considering normalized welfarist planners, as in Section 4. In particular, Equations (16), (17), and (19) remain valid, and the one-period version of Equation (18), from which it is straightforward to compute state-prices for any date and state, becomes

$$q^i(s'|s) = \frac{\frac{\partial V_i(s)}{\partial c^i(s')}}{\frac{\partial V_i(s)}{\partial c^i(s)}} = \frac{\frac{\partial V_i(s)}{\partial V^i(s')}\frac{\partial V^i(s')}{\partial c^i(s')}}{\frac{\partial V_i(s)}{\partial c^i(s)}} = \beta_i\pi(s'|s)\left(\frac{V_i(s')}{H(s)}\right)^{\frac{1}{\psi}-\gamma_i}\left(\frac{c^i(s')}{c^i(s)}\right)^{-\frac{1}{\psi_i}}\frac{\frac{\partial u_i(s')}{\partial c^i}}{\frac{\partial u_i(s)}{\partial c^i}},$$

where $H(s) = \left(\sum_{s'}\pi(s'|s)\left(V^i(s')\right)^{1-\gamma_i}\right)^{\frac{1}{1-\gamma_i}}$. It is straightforward to define DS-weights for even more general preferences, including preferences that are not time-separable or recursive, as we do next.

F.3 General utility with multiple commodities

In the baseline model, we already illustrate how to make welfare assessments when there are multiple goods/commodities, since we consider an environment with two commodities: consumption and hours. Here we consider a more abstract scenario, in which $i \in I$ individuals have general preferences over a set of commodities $\ell \in L$, which can also be indexed by dates $t \in T$ and histories s^t. In this case, the lifetime utility of individual i is given by

$$V_i(s_0) = U_i\left(\left\{x_t^{i,\ell}\left(s^t\right)\right\}_{t,s^t,\ell}\right).$$

At this level of generality, the different commodities can represent hours worked, as in the baseline environment, different consumption goods, flow utility from housing, or any variable that directly impacts instantaneous utility. Hence, we can express the lifetime utility effect of a policy change for individual i as follows:

$$\frac{dV_i\left(s_0\right)}{d\theta} = \sum_{t=0}^{T} \sum_{s^t} \sum_{\ell} \frac{\partial U_i}{\partial x_t^{i,\ell}\left(s^t\right)} \frac{dx_t^{i,\ell}\left(s^t\right)}{d\theta}.$$

Consequently, we can generalize the definition of DS-planner (in Definition 3) to a general environment, by assuming that $\frac{dW^{DS}(s_0)}{d\theta}$ takes the form

$$\frac{dW^{DS}\left(s_0\right)}{d\theta} = \int \sum_{t=0}^{T} \sum_{s^t} \sum_{\ell} \omega_t^{i,\ell}\left(s^t \middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta} di,$$

where $\omega_t^{i,\ell}\left(s^t \middle| s_0\right)$ is a DS-weight defined for each specific commodity ℓ, for each date t, at each history s^t, and for each individual i.[49]

Hence, paralleling Lemma 1, we can define a multiplicative decomposition of the form

$$\omega_t^{i,\ell}\left(s^t \middle| s_0\right) = \underbrace{\tilde{\omega}^i\left(s_0\right)}_{\text{individual}} \underbrace{\tilde{\omega}_t^i\left(s_0\right)}_{\text{dynamic}} \underbrace{\tilde{\omega}_t^i(s^t|s_0)}_{\text{stochastic}} \underbrace{\tilde{\omega}_t^{i,\ell}(s^t|s_0)}_{\text{commodity}},$$

where the choice of $\tilde{\omega}_t^{i,\ell}(s^t|s_0)$ is shaped by the choice of numeraire. Throughout the paper, we assume that consumption is the numeraire good, so with $\ell \in \{c, n\}$, we have that

$$\tilde{\omega}_t^{i,c}(s^t|s_0) = 1$$

$$\tilde{\omega}_t^{i,n}(s^t|s_0) = \frac{\frac{\partial u_i\left(s^t\right)}{\partial n_t^i}}{\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}.$$

By doing this, we guarantee that $\sum_{\ell} \omega_t^{i,\ell}\left(s^t \middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta}$ is measured in units of consumption good at history s^t. These results highlight how welfare assessments also rely on the choice of numeraire. However, more generally we can consider any bundle of goods $\left\{\psi^{\ell}\right\}_{\ell \in L}$ as numeraire, that is we could set

$$\tilde{\omega}_t^{i,\ell}(s^t|s_0) = \frac{\frac{\partial U_i}{\partial x_t^{i,\ell}(s^t)}}{\sum_{\ell} \psi^{\ell} \frac{\partial U_i}{\partial x_t^{i,\ell}(s^t)}},$$

where by choosing a unit vector for some commodity ℓ we choose a single commodity as numeraire. For the purpose of making meaningful welfare assessments, this normalization/choice of numeraire must be consistent across all individuals. Welfare assessments are typically not invariant to the choice of numeraire, but there are good reasons to choose some numeraires (e.g., consumption, some particular consumption bundle, or dollars) over others.[50]

[49] In this case, the generalization of the lifetime and instantaneous Social Welfare Functions is a "commodity Social Welfare Function", given by

$$W\left(\left\{x_t^{i,\ell}\left(s^t\right)\right\}_{t,s^t,\ell,i}\right).$$

[50] One could potentially pick different numeraires in different dates or histories, but it seems natural to choose a consistent numeraire to yield easily interpretable results.

In the case of a normalized welfarist planner, it is straightforward to characterize commodity-DS-weights. Using the first commodity ($\ell = 1$) as numeraire, and defining $\lambda_i(s_0) = \frac{\partial \mathcal{W}(\{V_i(s_0)\}_{i\in I})}{\partial V_i}$, it follows that

$$\begin{aligned}
\frac{dW^{DS}(s_0)}{d\theta} &= \int \lambda_i(s_0) \sum_{t=0}^{T} \sum_{s^t} \sum_{\ell} \frac{\partial U_i}{\partial x_t^{i,\ell}(s^t)} \frac{dx_t^{i,\ell}\left(s^t\right)}{d\theta} di \\
&= \int \lambda_i(s_0) \sum_{t=0}^{T} \sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)} \sum_{\ell} \frac{\frac{\partial U_i}{\partial x_t^{i,\ell}(s^t)}}{\frac{\partial U_i}{\partial x_t^{i,1}(s^t)}} \frac{dx_t^{i,\ell}\left(s^t\right)}{d\theta} di \\
&= \int \lambda_i(s_0) \sum_{t=0}^{T} \sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)} \sum_{s^t} \frac{\frac{\partial U_i}{\partial x_t^{i,1}(s^t)}}{\sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)}} \sum_{\ell} \frac{\frac{\partial U_i}{\partial x_t^{i,\ell}(s^t)}}{\frac{\partial U_i}{\partial x_t^{i,1}(s^t)}} \frac{dx_t^{i,\ell}\left(s^t\right)}{d\theta} di \\
&= \int \lambda_i(s_0) \sum_{t} \sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)} \sum_{t=0}^{T} \frac{\sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)}}{\sum_t \sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)}} \sum_{s^t} \frac{\frac{\partial U_i}{\partial x_t^{i,1}(s^t)}}{\sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)}} \sum_{\ell} \frac{\frac{\partial U_i}{\partial x_t^{i,\ell}(s^t)}}{\frac{\partial U_i}{\partial x_t^{i,1}(s^t)}} \frac{dx_t^{i,\ell}\left(s^t\right)}{d\theta} di \\
&= \int \lambda_i(s_0) \sum_{t} \sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)} di \int \underbrace{\frac{\lambda_i(s_0) \sum_t \sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)}}{\int \lambda_i(s_0) \sum_t \sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)} di}}_{=\tilde{\omega}^i(s_0)} \sum_{t=0}^{T} \underbrace{\frac{\sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)}}{\sum_t \sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)}}}_{=\tilde{\omega}_t^i(s_0)} \sum_{s^t} \underbrace{\frac{\frac{\partial U_i}{\partial x_t^{i,1}(s^t)}}{\sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)}}}_{=\tilde{\omega}_t^i(s^t|s_0)} \underbrace{\sum_{\ell} \underbrace{\frac{\frac{\partial U_i}{\partial x_t^{i,\ell}(s^t)}}{\frac{\partial U_i}{\partial x_t^{i,1}(s^t)}}}_{=\tilde{\omega}_t^{i,\ell}(s^t|s_0)} \frac{dx_t^{i,\ell}\left(s^t\right)}{d\theta}}_{=\frac{du_{i|c}(s^t)}{d\theta}} di.
\end{aligned}$$

so we can write

$$\frac{\frac{dW^{DS}(s_0)}{d\theta}}{\int \lambda_i(s_0) \sum_t \sum_{s^t} \frac{\partial U_i}{\partial x_t^{i,1}(s^t)} di} = \int \tilde{\omega}^i(s_0) \sum_{t=0}^{T} \tilde{\omega}_t^i(s_0) \sum_{s^t} \tilde{\omega}_t^i\left(s^t | s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta} di.$$

This derivation highlights that once we choose a numeraire, the dynamic, stochastic, and individuals components of DS-weights are expressed in terms of such numeraire — it is straightforward to use a bundle-numeraire of the form $\sum_\ell \psi^\ell \frac{\partial U_i}{\partial x_t^{i,\ell}(s^t)}$. Hence, for Proposition 6 to be valid, the natural commodity to choose as numeraire is the commodity on which financial claims are written on.

Finally, note that it is also possible to introduce multiple commodities in the baseline model with time-separable expected utility preferences. To do so, we define a generalized version of Equation (1), which includes multiply commodities, indexed by $\ell \in L$, as follows:

$$V_i(s_0) = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i\left(\left\{c_t^{i,\ell}\left(s^t\right)\right\}_{\ell\in L}\right).$$

Without loss, we treat commodity 1 as the numeraire for the purpose of making welfare assessments, so we can express $\frac{dV_i(s_0)}{d\theta}$ as follows:

$$\frac{dV_i(s_0)}{d\theta} = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^{i,1}} \sum_{\ell\in L} \frac{du_{i|c^1}\left(s^t\right)}{d\theta},$$

where $\frac{\partial u_i(s^t)}{\partial c_t^{i,\ell}} = \frac{\partial u_i\left(\left\{c_t^{i,\ell}(s^t)\right\}_{\ell\in L}\right)}{\partial c_t^{i,\ell}(s^t)}$ and where the instantaneous commodity-1-equivalent effect of the policy at history s^t, is given by $\frac{du_{i|c^1}(s^t)}{d\theta}$, where

$$\frac{du_{i|c^1}(s^t)}{d\theta} = \frac{\frac{du_i\left(\left\{c_t^{i,\ell}(s^t)\right\}_{\ell\in L}\right)}{d\theta}}{\frac{\partial u_i(s^t)}{\partial c_t^{i,1}}} = \frac{dc_t^{i,1}(s^t)}{d\theta} + \sum_{\ell\in L} \frac{\frac{\partial u_i(s^t)}{\partial c_t^{i,\ell}}}{\frac{\partial u_i(s^t)}{\partial c_t^{i,1}}} \frac{dc_t^{i,\ell}(s^t)}{d\theta}.$$

Once again, when there are multiple commodities, it is necessary to account for the marginal rates of substitutions between those commodities and the commodity chosen as numeraire. Note that the choice of numeraire will not change the directional welfare assessment of a welfarist planner, but it can have an impact on the units of such assessment, as well as on the value of the components of the aggregate additive decomposition.

F.4 Policy changes that affect probabilities

In this section, we describe how to use DS-weights in environments in which policy changes affect probabilities. Starting from Equation (2), note that we can express $\frac{dV_i(s_0)}{d\theta}$ as follows

$$\frac{dV_i(s_0)}{d\theta} = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t\middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i} \left(\frac{du_{i|c}(s^t)}{d\theta} + \frac{d\ln \pi_t(s^t|\, s_0)}{d\theta} \frac{u_i\left(c_t^i(s^t), n_t^i(s^t)\right)}{\frac{\partial u_i(s^t)}{\partial c_t^i}}\right).$$

Hence, we can use the following definition of a DS-planner in this case.

Definition 6. *(Desirable policy change for a DS-planner) A DS-planner, that is, a planner who adopts DS-weights, finds a policy change desirable in an environment in which policies can also affect probabilities if and only if* $\frac{dW(s_0)}{d\theta} > 0$*, where*

$$\frac{dW^{DS}(s_0)}{d\theta} = \int \sum_{t=0}^{T} \sum_{s^t} \omega_t^i\left(s^t\middle| s_0\right) \left(\frac{du_{i|c}(s^t)}{d\theta} + \frac{d\ln \pi_t(s^t|\, s_0)}{d\theta} \frac{u_i\left(c_t^i(s^t), n_t^i(s^t)\right)}{\frac{\partial u_i(s^t)}{\partial c_t^i}}\right) di,$$

where $\frac{d\ln \pi_t(s^t|s_0)}{d\theta} = \frac{\frac{d\pi_t(s^t|s_0)}{d\theta}}{\pi_t(s^t|s_0)}$.

Identical results apply in the case in which policy changes directly affect preferences. See Dávila and Goldstein (2021) for an application of the results of this paper to an environment in which policy changes have a discontinuous impact on payoffs.

F.5 Intergenerational considerations

In this section, we describe how to use DS-weights in environments with births, deaths, bequest motives, and related considerations, which non-trivially affect welfare assessments — see Calvo and

Obstfeld (1988), Farhi and Werning (2010), Heathcote, Storesletten and Violante (2017), or Phelan and Rustichini (2018). The most direct way of extending our baseline environment, is to interpret the set of individuals I considered in the baseline model as the set all individuals i) alive or ii) yet-to-be-born from the perspective of s_0. Under that interpretation, $\frac{du_{i|c}(s^t)}{d\theta}$ is non-zero only for those alive at a given history, so Definition 3 applies unchanged.[51]

Bequest motives, altruism, warm-glow preferences, social discounting or similar considerations only impact welfare assessments via the choice of DS-weights. For instance, a welfarist planner who values future generations directly placing a positive weight on their welfare and that in turn perceives an effective social discount rate lower than the private one, can be modeled by choosing a particular set of DS-weights. While do not explore that possibility in this paper, there is scope to use the law of total covariance to internationally decompose the cross-sectional components of the aggregate additive decomposition.

[51]An important practical consideration is that Proposition 7 will never apply to economies with births, since yet-to-be-born individuals cannot freely trade with alive individuals. These ideas deserve further exploration.

G Additional Results

G.1 Dimensional analysis

This paper puts great emphasis on the units in which different variables are defined. In this section, we carefully describe the units of the different components of individual multiplicative decomposition for a normalized welfarist planner and for a general DS-planner

Welfarist planners. As we discuss in the text, the units of our formulation of DS-weights for the case of the normalized welfarist planner have a clear interpretation in terms of dollars at different dates and histories. Here, we provide a systematic dimensional analysis (de Jong, 1967) of the welfare assessments made by a normalized welfarist planner. We denote the units of a specific variable by $\dim(\cdot)$, where, for instance, $\dim\left(c_t^i\left(s^t\right)\right) =$ dollars at history s^t ,where we interchangeably use dollars and units of the consumption good.

First, note that the units of $\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)$, $\tilde{\omega}_t^i\left(s_0\right)$, and $\tilde{\omega}^i\left(s_0\right)$ for a welfarist planner, as defined in Equations (13), (14), and (15), are respectively given by

$$\dim\left(\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\, s_0\right)\right) = \frac{\frac{\text{instantaneous utils at } s_0 \text{ for individual } i}{\text{dollars at history } s^t}}{\frac{\text{instantaneous utils at } s_0 \text{ for individual } i}{\text{dollars at date } t}} = \frac{\text{dollars at date } t}{\text{dollars at history } s^t}$$

$$\dim\left(\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)\right) = \frac{\frac{\text{instantaneous utils at } s_0 \text{ for individual } i}{\text{dollars at date } t}}{\frac{\text{instantaneous utils at } s_0 \text{ for individual } i}{\text{dollars at all dates and histories}}} = \frac{\text{dollars at all dates and histories}}{\text{dollars at date } t}$$

$$\dim\left(\tilde{\omega}^{i,\mathcal{W}}\left(s_0\right)\right) = \frac{\frac{\text{instantaneous utils at } s_0 \text{ for individual } i}{\text{dollars at all dates and histories}}}{\frac{\text{instantaneous utils at } s_0}{\text{dollars at all dates and histories for all individuals}}}$$

$$= \frac{\text{dollars at all dates and histories for all individuals}}{\text{dollars at all dates and histories}},$$

where the last cancellation accounts for the implicit comparability of utility units among individuals.[52] The term $\left(\beta_i\right)^t \pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}$, which defines the numerator of $\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)$, is measured in instantaneous utils at s_0 per dollars at history s^t for individual i, since

$$\dim\left(\left(\beta_i\right)^t\right) = \frac{\text{instantaneous utils at } s_0 \text{ for individual } i}{\text{instantaneous utils at history } s^t \text{ for individual } i}$$

$$\dim\left(\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\right) = \frac{\text{instantaneous utils at history } s^t \text{ for individual } i}{\text{dollars at history } s^t},$$

and probabilities, like $\pi_t\left(s^t\middle|\, s_0\right)$, are unitless. The same logic applies to the remaining elements of $\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right)$, $\tilde{\omega}_t^i\left(s_0\right)$, and $\tilde{\omega}^i\left(s_0\right)$.

[52]From the perspective of aggregation of lifetime utilities, which takes places through the individual component $\tilde{\omega}^i\left(s_0\right)$, any welfarist planner has $|I|+1$ degrees of freedom: the planner can give different weights to each of the $|I|$ individual assessments, and can further normalize the units of aggregate welfare.

Consequently, it follows that

$$\dim\left(\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\,s_0\right)\right)=\dim\left(\tilde{\omega}^i\left(s_0\right)\tilde{\omega}_t^i\left(s_0\right)\tilde{\omega}_t^i\left(s^t\middle|\,s_0\right)\right)\tag{OA4}$$
$$=\frac{\text{dollars at all dates and histories for all individuals}}{\text{dollars at history } s^t}.$$

Hence, the DS-weights $\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\,s_0\right)$ translates dollars at history s^t into $\lambda_i\left(s_0\right)$ dollars at all dates and histories for all individuals.

Second, note that the units of $\frac{du_{i|c}\left(s^t\right)}{d\theta}$ are given by

$$\dim\left(\frac{du_{i|c}\left(s^t\right)}{d\theta}\right)=\frac{\frac{\text{instantaneous utils at history } s^t \text{ for individual } i}{\text{unit of policy change}}}{\frac{\text{instantaneous utils at history } s^t \text{ for individual } i}{\text{dollars at history } s^t}}=\frac{\text{dollars at history } s^t}{\text{unit of policy change}},\tag{OA5}$$

which follows directly from Equation (14).

Finally, combining Equations (OA4) and (OA5), it follows that

$$\dim\left(\frac{dW^{\mathcal{W}}\left(s_0\right)}{d\theta}\right)=\dim\left(\omega_t^i\left(s^t\middle|\,s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right)=\frac{\text{dollars at all dates and histories for all individuals}}{\text{unit of policy change}}.\tag{OA6}$$

Hence, the units of $W^{\mathcal{W}}$ for a normalized welfarist planner are dollars paid to all individuals at all dates and histories. That is, if $\frac{dW^{NU}}{d\theta}=7$, the welfare gain associated with a marginal policy change is equivalent to paying 7 dollars to all individuals in the economy at all dates and histories.

General DS-planners. The dimensional analysis in the case of general planners is similar. In this case, the welfare units of $\tilde{\omega}_t^{i,DS}\left(s^t\middle|\,s_0\right)$ can be directly computed as

$$\dim\left(\tilde{\omega}_t^{i,DS}\left(s^t\middle|\,s_0\right)\right)=\frac{\text{units of } W^{DS}}{\text{dollars at history } s^t}.$$

In this case, it is also possible to compute the units of each of the components of the individual multiplicative decomposition as we just did for welfarist planners. By doing so, it becomes clear that the units of each of the components of the individual multiplicative decomposition for any normalized DS-planner (including those who are not welfarist) are identical.

Undesirable properties of unnormalized decompositions. As briefly explained in the text, using unnormalized individual multiplicative decompositions of DS-weights can be problematic in the context of the aggregate additive decomposition, since unnormalized decompositions are expressed in utils. This is not the case for normalized decompositions since these always make tradeoffs in dollar units.

For instance, if one were to set $\lambda_i\left(s_0\right)=1$, $\forall i$, in the decomposition presented in Equation (10), the redistribution component of the aggregate additive decomposition would be zero, $\Xi_{RD}=0$. This result captures the fact that an unnormalized equal-weighted utilitarian planner is indifferent between redistribution across individuals in utility terms. Hence, by directly adding up utils, we would fail to capture the idea that a utilitarian planner wants to redistribute resources (in consumption units) towards individuals with low marginal utility — see e.g., Salanie (2011). Similarly, if individual discount factors are identical, that is, $\beta_i = \beta$, $\forall i$, a welfarist planner under the decomposition presented in Equation (10) will conclude that intertemporal-sharing is zero, that is, $\Xi_{IS} = 0$, regardless of the form of the policy under consideration. Equally important, the dynamic and stochastic weights for a welfarist planner defined as in Equation (10) need not add up to 1. Hence, according to Proposition 3, even when the instantaneous consumption-equivalent effect of a policy change is identical across individuals at all dates and histories, an unnormalized utilitarian planner would typically find non-zero intertemporal-sharing components and redistribution components of the aggregate additive decomposition. This is is another undesirable property of the unnormalized utilitarian welfare criterion.

An alternative date-0 normalization. One of the contributions of this paper is to introduce the notion of a normalized planner — see Lemma 1 — as one for which the stochastic, dynamic, and individual components of the multiplicative decomposition add up to 1 across specific dimensions. However, these is an alternative normalization that seems reasonable: one may consider normalizing the individual welfare effect of a policy change by date-0 marginal utility. In that case, it is possible to decompose the DS-weights of a welfarist planner as follows:

$$\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|\, s_0\right) = \frac{\pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\sum_{s^t}\pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}} = \frac{q_t^i\left(s^t|s_0\right)}{\sum_{s^t} q_t^i\left(s^t|s_0\right)}$$

$$\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right) = \frac{\left(\beta_i\right)^t\sum_{s^t}\pi_t\left(s^t\middle|\, s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\frac{\partial u_i\left(s^0\right)}{\partial c_t^i}} = \sum_{t=0}^{T}\sum_{s^t} q_t^i\left(s^t|s_0\right)$$

$$\tilde{\omega}^{i,\mathcal{W}}\left(s_0\right) = \frac{\lambda_i\left(s_0\right)\frac{\partial u_i\left(s^0\right)}{\partial c_t^i}}{\int\lambda_i\left(s_0\right)\frac{\partial u_i\left(s^0\right)}{\partial c_t^i}di}.$$

This decomposition satisfies $\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|\, s_0\right) = 1$, $\forall t$, $\forall i$, and $\int\tilde{\omega}^i\left(s_0\right)di = 1$, but it is clear that $\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right) \neq 1$. Instead, in this decomposition, $\tilde{\omega}_0^{i,\mathcal{W}}\left(s_0\right) = 1$, for all individuals. In terms of units, this decomposition adds up individual welfare effects according to $\tilde{\omega}^{i,\mathcal{W}}\left(s_0\right)$, once they are expressed in date-0 dollars, which may seem reasonable or even desirable in some circumstances. However, in this case Proposition 3a) will not be valid if using this normalization. In particular, the redistribution component of the aggregate decomposition will not be zero for policies that are invariant across all individuals at all dates and histories. In this case, the component Ξ_{RD} captures redistribution from a date-0 perspective, note a lifetime perspective.

G.2 $\boldsymbol{\alpha}$-DS-Planners

After substituting the definition of the components of the DS-weights, we can explicitly express welfare assessments for a $\boldsymbol{\alpha}$-DS-planner as follows:

$$\begin{aligned}\frac{dW^{\mathcal{W},\boldsymbol{\alpha}}(s_0)}{d\theta} &= \underbrace{\sum_{t=0}^{T}\mathbb{E}_i\left[\left(1-\alpha_3^i\right)\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0)+\alpha_3^i\tilde{\omega}_t^{i,\mathcal{W}}(s_0)\right]\sum_{s^t}\mathbb{E}_i\left[\left(1-\alpha_2\right)\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\middle|s_0\right)+\alpha_2\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)\right]\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{AE}\text{ (Aggregate Efficiency)}} \\ &+\underbrace{\sum_{t=0}^{T}\mathbb{E}_i\left[\left(1-\alpha_3^i\right)\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0)+\alpha_3^i\tilde{\omega}_t^{i,\mathcal{W}}(s_0)\right]\sum_{s^t}\mathbb{C}ov_i\left[\left(1-\alpha_2\right)\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\middle|s_0\right)+\alpha_2\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{RS}\text{ (Risk-sharing)}} \\ &+\underbrace{\sum_{t=0}^{T}\mathbb{C}ov_i\left[\left(1-\alpha_3^i\right)\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0)+\alpha_3^i\tilde{\omega}_t^{i,\mathcal{W}}(s_0),\sum_{s^t}\left(\left(1-\alpha_2\right)\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\middle|s_0\right)+\alpha_2\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{IS}\text{ (Intertemporal-sharing)}} \\ &+\underbrace{\mathbb{C}ov_i\left[\left(1-\alpha_4\right)\tilde{\omega}^{i,\mathcal{W},AE}(s_0)+\alpha_4\tilde{\omega}^{i,\mathcal{W}}(s_0),X\right]}_{=\Xi_{RD}\text{ (Redistribution)}},\end{aligned} \tag{OA7}$$

where

$$X=\sum_{t=0}^{T}\left(\left(1-\alpha_3\right)\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0)+\alpha_3\tilde{\omega}_t^{i,\mathcal{W}}(s_0)\right)\sum_{s^t}\left(\left(1-\alpha_2\right)\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\middle|s_0\right)+\alpha_2\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}.$$

Note that the notion of $\boldsymbol{\alpha}$-DS-planner introduced in Definition 4 is designed so that the following properties are satisfied:

$$\begin{aligned}\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W},\boldsymbol{\alpha}}\left(s^t\middle|s_0\right)\right]&=\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\middle|s_0\right)=\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)\right]\\ \mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W},\boldsymbol{\alpha}}(s_0)\right]&=\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0)=\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}(s_0)\right]\\ \mathbb{E}_i\left[\tilde{\omega}^{i,\mathcal{W},\boldsymbol{\alpha}}(s_0)\right]&=\tilde{\omega}^{i,\mathcal{W},AE}(s_0)=\mathbb{E}_i\left[\tilde{\omega}^{i,\mathcal{W}}(s_0)\right].\end{aligned}$$

Hence, Equation (OA7) implies that when $\boldsymbol{\alpha}=(0,0,0)$, we have an AE pseudo-welfarist DS-planner; when $\boldsymbol{\alpha}=(1,0,0)$, we have an AR pseudo-welfarist DS-planner; when $\boldsymbol{\alpha}=(1,1,0)$, we have a NR pseudo-welfarist DS-planner; and when $\boldsymbol{\alpha}=(1,1,1)$, we have a welfarist planner. We summarize this results in Table OA-1.

Table OA-1: $\boldsymbol{\alpha}$-DS-planner: Special cases

$(\alpha_2,\alpha_3,\alpha_4)$	$\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$	$\tilde{\omega}_t^i(s_0)$	$\tilde{\omega}^i\left(s^0\right)$	Planner
$(1,1,1)$	$\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)$	$\tilde{\omega}_t^{i,\mathcal{W}}(s_0)$	$\tilde{\omega}^{i,\mathcal{W}}(s_0)$	$\mathcal{W}$
$(1,1,0)$	$\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)$	$\tilde{\omega}_t^{i,\mathcal{W}}(s_0)$	$\tilde{\omega}^{i,\mathcal{W},AE}(s_0)$	NR
$(1,0,0)$	$\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)$	$\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0)$	$\tilde{\omega}^{i,\mathcal{W},AE}(s_0)$	NS
$(0,0,0)$	$\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\middle|s_0\right)$	$\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0)$	$\tilde{\omega}^{i,\mathcal{W},AE}(s_0)$	AE

Note: Note that all the $\boldsymbol{\alpha}$-DS-planners considered in this table are pseudo-welfarist.

However, note that there are other possible extreme combinations of $\boldsymbol{\alpha}$ that one may want to consider, these are the following:

$$\{(1,0,1),(0,1,0),(0,1,1),(0,0,1)\}. \tag{OA8}$$

The problem with the $\boldsymbol{\alpha}$'s in Equation (OA8) is that, as long as one of the first two elements of $\boldsymbol{\alpha}$ are 0, the redistribution component will be different from the redistribution component of the relevant welfarist planner. Hence, these choices of $\boldsymbol{\alpha}$ are not pseudo-welfarist. Hence, those $\boldsymbol{\alpha}$-DS-planners will not be pseudo-welfarist, despite being perfectly valid DS-planners.

G.3 Relation to existing work

G.3.1 Welfarist Social Welfare Functions

In addition to the utilitarian SWF, defined in Equation (5), there are other welfarist SWF's that are at times used in specific applications — see e.g., Mas-Colell, Whinston and Green (1995), Kaplow (2011), or Adler and Fleurbaey (2016) for details. Here we briefly described those.

The *isoelastic* SWF, commonly traced back to Atkinson (1970), is given by

$$\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right)=\left(\int a_i\left(V_i(s_0)\right)^{\phi}di\right)^{1/\phi},$$

where the (inequality) coefficient ϕ is typically restricted to lie in $[-\infty,1]$, so that $\mathcal{W}(\cdot)$ is concave when $V_i(s_0)>0$, and where it is typically assumed that $\int a_i di=1$, and that $a_i\geq 0$, $\forall i$.[53] Limiting cases of the isoelastic SWF correspond to the other four widely used SWF's. First, when $\phi\to 1$, the isoelastic SWF becomes the conventional *utilitarian* SWF. In that case:

$$\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right)=\int a_i V_i(s_0)\,di.$$

Second, when $\phi\to 0$, the isoelastic SWF becomes the *Nash* (Cobb-Douglas) SWF. In that case:

$$\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right)=\int \left(V_i(s_0)\right)^{a_i}di.$$

Third, when $\phi\to-\infty$, the isoelastic SWF becomes the *Rawlsian/maximin* (Leontief) SWF. In that case:

$$\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right)=\min\left\{\ldots,\frac{V_i(s_0)}{a_i},\ldots\right\}.$$

Finally, when the isoelastic SWF gives positive weight to a single individual, it can be interpreted

[53] Note that, for an isoelastic SWF, $\frac{\partial\mathcal{W}}{\partial V_i}=a_i\left(\frac{V_i}{\mathcal{W}}\right)^{\phi-1}$. More importantly $\frac{\frac{\partial\mathcal{W}}{\partial V_i}}{\frac{\partial\mathcal{W}}{\partial V_j}}=\frac{a_i}{a_j}\left(\frac{V_i}{V_j}\right)^{\phi-1}$. When lifetime utilities are negative, it is possible to define an isoelastic SWF of the form

$$\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right)=\left(\int a_i\left(-V_i(s_0)\right)^{\phi}di\right)^{1/\phi},$$

by considering $\phi\in[1,\infty]$.

as a *dictatorial* SWF. In that case:

$$\mathcal{W}\left(\{V_i\left(s_0\right)\}_{i\in I}\right) = V_1\left(s_0\right).$$

Note that all of these SWF are Paretian, although the Rawlsian/maximin and the dictatorial SWF's are not strictly Paretian.[54]

G.3.2 Relation to Saez and Stantcheva (2016)

It is straightforward to define welfare assessments in our framework that are based on the approach introduced by Saez and Stantcheva (2016).

Definition 7. *(Desirable policy change for a planner who uses generalized social marginal welfare weights (Saez and Stantcheva, 2016)) A planner who uses generalized social marginal welfare weights finds a policy change desirable if and only if* $\frac{dW^{SS}(s_0)}{d\theta} > 0$*, where*

$$\frac{dW^{SS}\left(s_0\right)}{d\theta} = \int h_i\left(\cdot\right)\frac{dV_i\left(s_0\right)}{d\theta}di, \tag{OA9}$$

where $h_i\left(\cdot\right) > 0$*,* $\forall i \in I$*, are a collection of individual-specific positive functions, and where* $\frac{dV_i(s_0)}{d\theta}$ *is defined in Equation (2).*

By comparing Equation (OA9) with Equation (6), it is evident that the approach based on generalized social marginal welfare weights introduced in Saez and Stantcheva (2016) is more general than the welfarist approach. The key difference between the two approaches is that for welfarist planners the functions $h_i\left(\cdot\right)$ are restricted to take the form

$$h_i\left(\cdot\right) = \frac{\partial\mathcal{W}\left(\{V_i\left(s_0\right)\}_{i\in I}\right)}{\partial V_i},$$

while $h_i\left(\cdot\right)$ can take many other values under the Saez and Stantcheva (2016) approach. Saez and Stantcheva (2016) show that their approach can capture alternatives to welfarism, such as libertarianism or equality of opportunity. It is also evident from definition 7 that a planner who uses generalized social marginal welfare weights is not paternalistic, since welfare assessments are based on individual lifetime welfare effects, $\frac{dV_i(s_0)}{d\theta}$.

In static economies, the individual component of the individual multiplicative decomposition of DS-weights introduced in Lemma 1 exactly corresponds to the notion of generalized welfare weights introduced in Saez and Stantcheva (2016). In other words, in static environments, the contribution of our paper is only to introduce the aggregate additive decomposition of welfare assessments in aggregate efficiency and redistribution, but not to introduce the notion of generalized individual weights for particular individuals, which is already in Saez and Stantcheva (2016).

[54] A planner with an isoelastic SWF is strictly Paretian when $\phi > -\infty$ if $a_i > 0$, $\forall i$.

G.3.3 Relation to Kaldor/Hicks principle

The classic Kaldor/Hicks (Kaldor, 1939; Hicks, 1939) compensation principle can be formalized in marginal form in static environments by equal individual generalized weights among individuals, see e.g., Hendren (2020). This observation implies that the Kaldor/Hicks welfare criterion can also be formalized as a particular DS-planner.

In dynamic environments, there is some ambiguity on when and how to compensate different individuals. When the Kaldor/Hicks compensation is defined in permanent dollars (dollars across all dates and histories), the Kaldor/Hicks welfare criterion exactly correspond to the pseudo-welfarist NR planner introduced in Section 5, in which

$$\tilde{\omega}^i(s_0) = 1, \ \forall i. \quad \text{(Kaldor-Hicks)}$$

Intuitively, if a welfarist planner has access to permanent lump-sum transfers among individuals, an optimality condition for such a planner is that

$$\lambda_i(s_0) \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0 \right) \frac{\partial u_i(s^t)}{\partial c_t^i}$$

must be equal across all agents, implying that $\tilde{\omega}^i(s_0) = 1$.[55] This is the sense in which $\tilde{\omega}^i(s_0) = 1$ has the interpretation of a Kaldor-Hicks planner. However, while allowing for lump-sum transfers implies that $\tilde{\omega}^i(s_0) = 1$, the converse is not true, that is, it is possible to make welfare assessments using $\tilde{\omega}^i(s_0) = 1$ as individual weights even when no transfers at all are made in the background. We further elaborate on the role of transfers in Section G.3.5.

G.3.4 Relation to Lucas (1987) and Alvarez and Jermann (2004)

It is common in papers that study the welfare consequences of policies in dynamic and stochastic environments to compute welfare gains or losses of policies as in Lucas (1987), who measures the welfare gains associated with a policy change — specifically, the welfare gains associated with eliminating business cycles. Since our approach is built on marginal arguments, we connect instead our results to those in Alvarez and Jermann (2004), who provide a marginal formulation of the approach in Lucas (1987).

While the Lucas (1987) and Alvarez and Jermann (2004) approach is easily interpretable in representative agent economies, it has the pitfall that it cannot be meaningfully aggregated when there are heterogeneous individuals. See, for instance, how Atkeson and Phelan (1994), Krusell and Smith (1999), or Krusell et al. (2009) carefully avoid aggregating welfare gains or losses for different individuals.

To illustrate these arguments, here we consider a policy change for a given individual i, who

[55] Alternatively, as discussed in Footnote 24, a date-0 Kaldor-Hicks normalization, so that $\lambda_i(s_0) \frac{\partial u_i(s^0)}{\partial c_0^i} = 1$, is equivalent to assigning a higher individual weight to individuals with higher willingness to pay for T-consol bonds.

could be a representative agent or not. Formally, we consider a special case of the environment laid out in Section 3, in which an individual i has preferences given by

$$V_i(s_0) = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i\left(c_t^i\left(s^t\right)\right).$$

We suppose that the consumption of individual i at date t and history s^t can be written as

$$c_t^i\left(s^t\right) = (1-\theta)\,\underline{c_t^i}\left(s^t\right) + \theta\overline{c_t^i}\left(s^t\right),$$

where both $\underline{c_t^i}(s^t)$ and $\overline{c_t^i}(s^t)$ are sequences measurable with respect to history s^t. The sequence $\underline{c_t^i}(s^t)$ can be interpreted as a given initial consumption path (when $\theta = 0$) and the sequence $\overline{c_t^i}(s^t)$ can be interpreted as a final consumption path (when $\theta = 1$). In the case of Lucas (1987), $\theta = 1$ corresponds to fully eliminating business cycles.

First, we compute the marginal gains from marginally reducing business cycles, as in Alvarez and Jermann (2004). Next, we compute the marginal gains from marginally reducing business cycles using an additive compensation.

Multiplicative compensation. Lucas (1987) proposes using a time-invariant equivalent variation, expressed multiplicatively as a constant fraction of consumption at each date and history as follows

$$\sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i\left(c_t^i\left(s^t\right)(1+\lambda(\theta))\right) = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i\left((1-\theta)\,\underline{c_t^i}\left(s^t\right) + \theta\overline{c_t^i}\left(s^t\right)\right), \tag{OA10}$$

where $\lambda(\theta)$ implicitly defines the welfare gains associated with a policy indexed by θ; the exact definition in Lucas (1987) exactly corresponds to solving for $\lambda(\theta = 1)$.[56]

Following Alvarez and Jermann (2004), we can compute the derivative of the RHS of Equation (OA10) as follows:

$$\frac{d\left((\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i\left((1-\theta)\,\underline{c_t^i}(s^t) + \theta\overline{c_t^i}(s^t)\right)\right)}{d\theta} = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i'\left((1-\theta)\,\underline{c_t^i}(s^t) + \theta\overline{c_t^i}(s^t)\right)\frac{dc_t^i(s^t)}{d\theta} \tag{OA11}$$

where here $\frac{dc_t^i(s^t)}{d\theta} = \overline{c_t^i}(s^t) - \underline{c_t^i}(s^t)$.

[56] Note that one could also define an alternative compensating variation as

$$\sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i\left(c_t^i\left(s^t\right)\right) = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i\left(\left(\overline{c_t^i}\left(s^t\right) + \theta\overline{\Delta c_t^i}\left(s^t\right)\right)(1+\lambda(\theta))\right).$$

Analogously, we can also compute the derivative of the LHS of Equation (OA10) as follows:

$$\frac{d\left(\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i\left(c_t^i\left(s^t\right)(1+\lambda(\theta))\right)\right)}{d\theta}=\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(c_t^i\left(s^t\right)(1+\lambda(\theta))\right)c_t^i\left(s^t\right)\lambda'(\theta)\,. \tag{OA12}$$

Hence, combining Equations (OA11) and (OA12) and solving for $\frac{d\lambda}{d\theta}=\lambda'(\theta)$, yields the marginal cost of business cycles, as defined in Alvarez and Jermann (2004). Formally, we can express $\frac{d\lambda}{d\theta}$ as follows

$$\begin{aligned}\frac{d\lambda}{d\theta}=\lambda'(\theta)&=\frac{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left((1-\theta)\underline{c_t^i}\left(s^t\right)+\theta\overline{c_t^i}\left(s^t\right)\right)\frac{dc_t^i\left(s^t\right)}{d\theta}}{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(c_t^i\left(s^t\right)(1+\lambda(\theta))\right)c_t^i\left(s^t\right)}\\&=\sum_{t=0}^{T}\sum_{s^t}\omega_t^i\left(s^t\middle|\,s_0\right)\frac{dc_t^i\left(s^t\right)}{d\theta},\end{aligned} \tag{OA13}$$

where the second line shows how to reformulate $\frac{d\lambda}{d\theta}$ in terms of DS-weights given by

$$\omega_t^i\left(s^t\middle|\,s_0\right)=\frac{(\beta_i)^t\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(\overline{c_t^i}\left(s^t\right)+\theta\overline{\Delta c_t^i}\left(s^t\right)\right)}{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(c_t^i\left(s^t\right)(1+\lambda(\theta))\right)c_t^i\left(s^t\right)}. \tag{OA14}$$

Additive compensation. Here, we would like to contrast the approach in Lucas (1987) to one that relies on a time-invariant compensating variation, expressed additively in terms of consumption at each date and history as follows:

$$\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i\left(c_t^i\left(s^t\right)+\lambda(\theta)\right)=\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i\left((1-\theta)\underline{c_t^i}\left(s^t\right)+\theta\overline{c_t^i}\left(s^t\right)\right).$$

In this case, we can follow the same steps as above to find the counterpart of Equation (OA13), which is given by

$$\begin{aligned}\frac{d\lambda}{d\theta}=\lambda'(\theta)&=\frac{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left((1-\theta)\underline{c_t^i}\left(s^t\right)+\theta\overline{c_t^i}\left(s^t\right)\right)\frac{dc_t^i\left(s^t\right)}{d\theta}}{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(c_t^i\left(s^t\right)+\lambda(\theta)\right)c_t^i\left(s^t\right)}\\&=\sum_{t=0}^{T}\sum_{s^t}\omega_t^i\left(s^t\middle|\,s_0\right)\frac{dc_t^i\left(s^t\right)}{d\theta},\end{aligned} \tag{OA15}$$

where the second line shows how to reformulate $\frac{d\lambda}{d\theta}$ in terms of DS-weights given by

$$\omega_t^i\left(s^t\middle|\,s_0\right)=\frac{(\beta_i)^t\pi_t\left(s^t\middle|\,s_0\right)u_i'\left((1-\theta)\underline{c_t^i}\left(s^t\right)+\theta\overline{c_t^i}\left(s^t\right)\right)}{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|\,s_0\right)u_i'\left(c_t^i\left(s^t\right)+\lambda(\theta)\right)}. \tag{OA16}$$

Comparison and implications. We focus on comparing Equations (OA13) and (OA15) in the case of $\theta=0$ — similar insights emerge when $\theta\neq 0$. When $\theta=0$, Equations (OA14) and (OA16) become

$$\omega_t^i\left(s^t\middle|s_0\right)=\frac{(\beta_i)^t\,\pi_t\left(s^t\middle|s_0\right)u_i'\left(\underline{c}_t^i\left(s^t\right)\right)}{\sum_{t=0}^T(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)u_i'\left(\underline{c}_t^i\left(s^t\right)\right)\underline{c}_t^i\left(s^t\right)}\quad\text{(multiplicative}\Rightarrow\text{Lucas/Alvarez-Jermann)}\tag{OA17}$$

$$\omega_t^i\left(s^t\middle|s_0\right)=\frac{(\beta_i)^t\,\pi_t\left(s^t\middle|s_0\right)u_i'\left(\underline{c}_t^i\left(s^t\right)\right)}{\sum_{t=0}^T(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)u_i'\left(\underline{c}_t^i\left(s^t\right)\right)}.\quad\text{(additive}\Rightarrow\text{normalized welfarist DS-planner)}\tag{OA18}$$

Two major insights emerge from Equations (OA17) and (OA18). First, the DS-weights defined for the additive case in Equation (OA18) exactly correspond to the dynamic and stochastic components of DS-weights for a normalized utilitarian planner, as defined in Equations (13) and (14). Second, note that the denominator of the DS-weights in the multiplicative case includes $\underline{c}_t^i\left(s^t\right)$ at all dates and histories. This captures the fact that the welfare assessment is computed as a fraction of consumption at each date and history, not in units of the consumption good. The presence of $\underline{c}_t^i\left(s^t\right)$ in the denominator is what complicates the aggregation of welfare assessments using the Lucas (1987) approach.

Relation to EV, CV, and CS. Finally, note that the analysis in this section illustrates how the marginal approach relates to the conventional approaches in classic demand theory: equivalent variation (EV), compensating variation (CV), and consumer surplus (CS).

The approach of Lucas (1987) and Alvarez and Jermann (2004), and the alternative version described in Footnote 56 are the dynamic counterpart of compensating and equivalent variations, expressed in proportional terms, in a dynamic stochastic environment. Hence, the analysis of this section shows that a DS-planner can be used to operationalize the counterpart of all three notions — either proportionally or additively — in dynamic stochastic environments. As expected, these considerations only matter away from the $\theta=0$ case. However, the consumer surplus approach yields the most straightforward approach to making global assessments, as explained in Section G.5.

G.3.5 Relation to welfare assessments that involve transfers

Finally, it is worth discussing how having the ability to costlessly transfer resources across individuals impact the welfare assessments of a DS-planner. To do so, we consider an environment in which a DS-planner has access to a set of transfers $T_i^i\left(s^t\right)$, so that individual's budget constraints have the form

$$c_t^i\left(s^t\right)=T_i^i\left(s^t\right)+\ldots.$$

In that case, it follows immediately that

$$\frac{dW^{DS}\left(s_0\right)}{dT_i^i\left(s^t\right)}=\omega_t^i\left(s^t\middle|s_0\right).$$

Hence, having transfers available will endogenously impose restrictions across the DS-weights of different individuals. For instance, a welfarist planner who can transfer resources freely across all individuals, at all dates and histories will equalize the DS-weights across all individuals, at all dates and histories. Given Proposition 2, this implies that this planner will only value aggregate efficiency. Similar conclusions can be reached when a DS-planner only has access to a subset of transfers.

G.3.6 Relation to existing welfare decompositions

Our paper is not the first to introduce a decomposition of welfare assessments in different components. In fact, most of the existing literature that applies welfare decompositions to specific environments follows versions of the decompositions introduced by Benabou (2002) and Floden (2001). There is also the more recent decomposition introduced by Bhandari et al. (2021). We discussed how our approach is related to both of these next.

Benabou (2002)/Floden (2001) The starting point for the Benabou (2002)/Floden (2001) approach is the (incorrect) presumption that the welfarist approach cannot distinguish the effects of policy that operate via efficiency, missing markets, and redistribution. Benabou (2002) explicitly writes:[57]

> "*I will also compute more standard social welfare functions, which are aggregates of (intertemporal) utilities rather than risk-adjusted consumptions. These have the clearly desirable property that maximizing such a criterion ensures Pareto efficiency. On the other hand, it will be seen that they cannot distinguish between the effects of policy that operate through its role as a substitute for missing markets, and those that reflect an implicit equity concern.*"

In this paper, we have shown that it is possible to distinguish — using standard Social Welfare Functions — the effects of policy that operate through efficiency, including in economies with missing markets, and redistribution/equity. As Benabou (2002) points out, his non-welfarist approach may conclude that Pareto-improving policies are undesirable. When staying within the welfarist class, our approach is trivially Paretian. When consider DS-planners outside of the welfarist class, our approach is precise in the way in which specific departures take place.

In terms of properties, it is evident that the Benabou (2002)/Floden (2001) approach does not satisfy Proposition 6, in which we show that all normalized welfarist planners conclude that the risk-sharing and intertemporal-sharing components are zero when markets are complete; Proposition 7, in which we show that all normalized welfarist planners conclude that intertemporal-sharing component is zero when individuals can freely trade a riskless bond; and Proposition 8, in which we show that different normalized welfarist planners exclusively disagree on the redistribution component. Their approach satisfies Proposition 9, in which we show that the efficiency components (aggregate

[57] The Benabou (2002)/Floden (2001) approach is based on first computing certainty-equivalent consumption levels for individuals and then building measures of inequality from the distribution of such certainty-equivalents.

efficiency, risk-sharing, and intertemporal-sharing) of the aggregate additive decomposition are invariant to monotonically increasing transformations of individual's lifetime utilities and positive affine (increasing linear) transformations of individual's instantaneous utilities. However, the Benabou (2002)/Floden (2001) approach satisfies Proposition 9 only because it is defined for environments in which all individuals have identical preferences, which are highly restrictive.

Bhandari et al. (2021) The approach introduced by Bhandari et al. (2021), considers the case of a utilitarian planner with arbitrary weights α_i. although it seems obvious to apply to general Social Welfare Functions. In contrast to Benabou (2002)/Floden (2001), the approach of Bhandari et al. (2021) is defined for a general dynamic stochastic economies in which individuals may have different preferences.

For simplicity, we consider a scenario in which there is a single consumption good. In this environment, Bhandari et al. (2021) propose to first decompose the consumption of a given individual at a given date and history as

$$c_t^i\left(s^t\right) = C \times w_i \times \left(1 + \varepsilon_t^i\left(s^t\right)\right), \tag{OA19}$$

where C captures aggregate consumption, w_i captures the share of individual i's consumption relative to the aggregate and $1+\varepsilon_t^i\left(s^t\right)$ captures any residual variation. While Equation (OA19) may resemble the individual multiplicative decomposition introduced in Lemma 1, it is conceptually different. First, and most importantly, the decomposition in Equation (OA19) decomposes consumption, $c_t^i\left(s^t\right)$, while the individual multiplicative decomposition introduced in Lemma 1 decomposes DS-weights, i.e., social marginal valuations, $\omega_t^i\left(s^t\right)$. Second, the term w_i in Equation (OA19) can heuristically be mapped to the individual component of our individual multiplicative decomposition, while the term $1+\varepsilon_t^i\left(s^t\right)$ can be heuristically mapped to both the dynamic and stochastic components.

Bhandari et al. (2021) then introduce a second-order Taylor expansion around a midpoint to write welfare differences (partially adopting the notation in that paper) as follows:

$$\mathcal{W}^B - \mathcal{W}^A \simeq \underbrace{\int \phi_i \Gamma di}_{\text{agg. efficiency}} + \underbrace{\int \phi_i \Delta_i di}_{\text{redistribution}} + \underbrace{\int \phi_i \gamma_i \Lambda_i di}_{\text{insurance}}, \tag{OA20}$$

where $\phi_i = \alpha_i \sum_t \sum_{s^t} \frac{\partial u_i\left(s^t\right)}{\partial c_t^i} c_t^i\left(s^t\right)$ denotes quasi-weights — using the terminology in Bhandari et al. (2021) — and γ_i is a measure of risk-aversion, $-c_t^i\left(s^t\right) \frac{\partial^2 u_i\left(s^t\right)}{\partial\left(c_t^i\right)^2} / \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}$, and where $\Gamma = \ln C^B - \ln C^A$, $\Delta_i = \ln w_i^B - \ln w_i^A$, and $\Lambda_i = -\frac{1}{2}\left[\mathbb{V}ar_i\left[\ln c_i^B\right] - \mathbb{V}ar_i\left[\ln c_i^A\right]\right]$. It is then possible to decompose $\mathcal{W}^B - \mathcal{W}^A$ into three terms as follows:

$$1 = \underbrace{\frac{\int \phi_i \Gamma di}{\mathcal{W}^B - \mathcal{W}^A}}_{\text{agg. efficiency}} + \underbrace{\frac{\int \phi_i \Delta_i di}{\mathcal{W}^B - \mathcal{W}^A}}_{\text{redistribution}} + \underbrace{\frac{\int \phi_i \gamma_i \Lambda_i di}{\mathcal{W}^B - \mathcal{W}^A}}_{\text{insurance}}. \tag{OA21}$$

Bhandari et al. (2021) establish three properties of the decomposition in Equation (OA21): a) a welfare change that affects aggregate consumption C but not $\{w_i, \varepsilon_i\}_i$ is exclusively attributed to aggregate efficiency; b) a welfare change that affects expected shares $\{w_i\}_i$ but not C and $\{\varepsilon_i\}_i$ is exclusively attributed to redistribution; c) a welfare change that affects $\{\varepsilon_i\}_i$ but not C and $\{w_i\}_i$ is exclusively attributed to insurance.[58] These properties are conceptually the counterpart of Proposition 3, since they consider properties of a decomposition for particular policy changes. However, it should be evident that properties a), b), and c) in Bhandari et al. (2021) neither imply nor are implied by the properties that we establish in Proposition 3. This occurs because properties a), b), and c) consider proportional changes while Proposition 3 considers changes in levels of consumption, with both the proportional and level approaches being different but reasonable.[59]

However, more importantly, the decomposition of Bhandari et al. (2021) does not satisfy the counterparts of Proposition 6, in which we show that all normalized welfarist planners conclude that the risk-sharing and intertemporal-sharing components are zero when markets are complete; Proposition 7, in which we show that all normalized welfarist planners conclude that intertemporal-sharing component is zero when individuals can freely trade a riskless bond; Proposition 8, in which we show that different normalized welfarist planners exclusively disagree on the redistribution component; and Proposition 9, in which we show that the efficiency components (aggregate efficiency, risk-sharing, and intertemporal-sharing) of the aggregate additive decomposition are invariant to monotonically increasing transformations of individual's lifetime utilities and positive affine (increasing linear) transformations of individual's instantaneous utilities.

That is, it is possible to consider complete market economies in which the decomposition of Bhandari et al. (2021) attributes welfare changes to their insurance component. Also, it should be evident from Equation (OA21) that changing the Pareto weights α_i that a utilitarian planner assigns to an individual or simply multiplying the lifetime utility of a single individual by a constant factor — a transformation that has no impact on allocations — will change all three elements (aggregate efficiency, redistribution, insurance) of the decomposition introduced by Bhandari et al. (2021).[60] The are two choices that explain why the decomposition in Equation (OA21) does not

[58]The insurance component in Bhandari et al. (2021) is heuristically related to the risk-sharing and intertemporal-sharing components in our paper. Bhandari et al. (2021) also establish a fourth property, reflexivity, which our approach also satisfies.

[59]Formally, note that by writing $c_t^i\left(s^t\right) = C \times w_i \times \left(1 + \varepsilon_t^i\left(s^t\right)\right)$, we can express $\frac{du_{i|c}\left(s^t\right)}{d\theta}$ as follows:

$$\frac{du_{i|c}\left(s^t\right)}{d\theta} = \frac{dc_t^i\left(s^t\right)}{d\theta} = \frac{dC}{d\theta} \times w_i \times \left(1 + \varepsilon_t^i\left(s^t\right)\right) + C \times \frac{dw_i}{d\theta} \times \left(1 + \varepsilon_t^i\left(s^t\right)\right) + C \times w_i \times \frac{d\left(1 + \varepsilon_t^i\left(s^t\right)\right)}{d\theta}.$$

In this case, even when $\frac{dw_i}{d\theta} = \frac{d\left(1+\varepsilon_t^i\left(s^t\right)\right)}{d\theta} = 0$, a change in $\frac{dC}{d\theta}$, by virtue of being *proportional* to existing consumption, does not imply a uniform change in $\frac{du_{i|c}\left(s^t\right)}{d\theta}$ across individuals, dates, and histories, which are the changes considered in Proposition 3a). A similar logic applies to changes in $\frac{dw_i}{d\theta}$ and $\frac{d\varepsilon_t^i\left(s^t\right)}{d\theta}$. More generally, the decompositions yield different conclusions. For instance, the decomposition in Bhandari et al. (2021) attributes welfare gains associated to smoothing business cycles in a representative agent economy — as in Lucas (1987) — to insurance, while our decomposition attributes such gains to the aggregate insurance subcomponent of aggregate efficiency, as described in Section 6.1.

[60]Formally, it follows from the definition of ϕ_i above that a change in α_i or a linear transformation of utilities will

satisfy Propositions 6 through 9, which are central properties of our aggregate additive decomposition. First, the decomposition in Equation (OA19) does not ensure that the insurance component vanishes when individuals marginal rates of substitution are equalized across dates/states. Second, $\mathcal{W}^B - \mathcal{W}^A$ in Equation (OA20) (as well as ϕ_i) is expressed in utils, not consumption units.[61] Hence, changes in Pareto weights or utility transformations directly affect all the components of the decomposition, including aggregate efficiency and insurance in Equation (OA21). By introducing normalized DS-weights for welfarist planners, our approach confines the impact of varying SWF's or considering utility transformation to the redistribution component. Alternatively, directly specifying the individual component of DS-weights allows a DS-planner to directly modulate how the redistribution component is determined.

G.4 Optimal policy problems using DS-weights

Throughout most of the paper we have focused on how to make welfare assessments. Here, we show how it is straightforward to use DS-weights in the context of optimal policy problems, both in primal and in dual forms. To do so, we consider an environments in which a planner chooses a set of policy instruments $\boldsymbol{\tau}$ to maximize social welfare, which depends on allocations $\boldsymbol{X}(\boldsymbol{\tau})$. We consider two possibilities.

First, we consider a primal problem, in which a planner maximizes social welfare $W(\boldsymbol{X}(\boldsymbol{\tau}))$, subject to a set of implementability conditions, $\boldsymbol{H}(\boldsymbol{X}, \boldsymbol{\tau})$.[62] Consistent with Section 6.4, we assume that $W(\boldsymbol{X}(\boldsymbol{\tau}))$ corresponds to an instantaneous SWF. In this case, the planner solves

$$\min_{\boldsymbol{\lambda}} \max_{\boldsymbol{X},\boldsymbol{\tau}} W(\boldsymbol{X}) + \boldsymbol{\lambda} \boldsymbol{H}(X, \boldsymbol{\tau}),$$

with optimality conditions for $\boldsymbol{\tau}$ given by

$$\frac{\partial W}{\partial \boldsymbol{X}} + \boldsymbol{\lambda} \frac{\partial \boldsymbol{H}}{\partial \boldsymbol{X}} = 0. \tag{OA22}$$

Second, we consider a dual problem, in which a planner maximizes social welfare $W(\boldsymbol{X}^\star(\boldsymbol{\tau}))$, where $\boldsymbol{X}^\star(\tau)$ denotes the equilibrium mapping implicitly defined as $\boldsymbol{H}(\boldsymbol{X}^\star(\boldsymbol{\tau}), \boldsymbol{\tau}) = 0$. In this case, the planner solves

$$\max_{\boldsymbol{\tau}} W(\boldsymbol{X}^\star(\boldsymbol{\tau})),$$

with optimality conditions for $\boldsymbol{\tau}$ given

$$\frac{\partial W}{\partial \boldsymbol{X}} \frac{d\boldsymbol{X}^\star}{d\boldsymbol{\tau}} = 0. \tag{OA23}$$

change ϕ^i and consequently each of the three elements on the right-hand side of Equation (OA20).

[61] Bhandari et al. (2021) explain how $\mathcal{W}^B - \mathcal{W}^A$ is measured in utils as follows:

> *"Quasi-weights $\{\phi_i\}_i$ convert percent changes $\{\Gamma, \Delta_i, \Lambda_i\}_i$ that into a welfare change $\mathcal{W}^B - \mathcal{W}^A$, measured in utils."*

[62] While social welfare is a scalar, bold variables can be vectors/matrices.

In both cases, it is necessary to characterize $\frac{\partial W}{\partial \boldsymbol{X}}$ to find optimal policies. Hence, by defining $\frac{\partial W}{\partial \boldsymbol{X}}$ as in Definition 3, it is straightforward to find optimal policies for different DS-planners. As a final remark, note that, consistently with Section 6.4, it is important to understand that one cannot define a conventional SWF from the onset, DS-weights must be introduced at the marginal level in Equations (OA22) and (OA23).

G.5 Global welfare assessments

In the body of the paper, we have focused on marginal welfare assessments because there is no ambiguity about the welfare gains or losses of a policy when measured in units of a particular numeraire — see Schlee (2013) for a formal proof.[63] But one may still be interested in exploring the impact of non-marginal welfare assessments. It is well understood that even for a single individual there is no unambiguous approach to measure welfare gains or losses for non-marginal changes — see e.g., Silberberg (1972) or Mas-Colell, Whinston and Green (1995) — with the same logic extending to every component of the aggregate additive decomposition. This phenomenon is typically illustrated by the discrepancy between consumer surplus, equivalent variation, and compensating variation in classic demand theory. Despite this unavoidable hurdle, it is possible to make judicious global welfare assessments.

In practice, the easiest approach to study global policy changes is to parameterize policies using a line integral, as we illustrate in Scenarios 1 and 2 in Section 7. Assuming that policy changes can be scaled by $\theta \in [0,1]$, where $\theta = 0$ corresponds to the status-quo and $\theta = 1$ corresponds to a global non-marginal change, it is possible to define a non-marginal welfare change as follows:

$$W^{DS}\left(s_0; \theta = 1\right) - W^{DS}\left(s_0; \theta = 0\right) = \int_0^1 \frac{dW^{DS}\left(s_0; \theta\right)}{d\theta} d\theta,$$

where θ is an explicit argument of $\frac{dW^{DS}(s_0;\theta)}{d\theta}$, which is given by

$$\frac{dW^{DS}\left(s_0; \theta\right)}{d\theta} = \int \sum_{t=0}^{T} \sum_{s^t} \omega_t^i \left(s^t \middle| s_0; \theta\right) \frac{du_{i|c}\left(s^t; \theta\right)}{d\theta} di. \tag{OA24}$$

That is, by recomputing $\frac{dW^{DS}(s_0;\theta)}{d\theta}$ along a particular path, it is possible to come up with a social welfare measure that is akin to consumer surplus, with the same logic applying to each of the components of the aggregate additive decomposition. While in principle using different paths may yield different answers when considering multidimensional policies even for identical start and end points, in practice it is often possible to find monotonic paths of integration, as defined by Zajac (1979) and Stahl (1984), which guarantees that the approach laid in Equation (OA24) yields globally consistent welfare assessments.

Two additional remarks are worth making. First, while the approach outlined here is the

[63] Schlee (2013) shows that the measures of consumer surplus, equivalent variation, and compensating variation are identical for marginal changes in a classical demand setup.

easiest to implement, it is possible to use the same methodology as Alvarez and Jermann (2004) to consider equivalent/compensating variation-like global assessments for welfarist planners within the DS-weights framework, although this will only be valid for aggregate assessments, not necessarily each of the components of the aggregate additive decomposition. Second, the potential for ambiguity of global assessments is not relevant if one is interested in using DS-planners to solve optimal policy problems, since $\frac{dW^{DS}(s_0)}{d\theta}$ is unambiguously defined for any policy perturbation. Hence, if there is a point at which $\frac{dW^{DS}(s_0)}{d\theta} = 0$ given the set of policy instruments, this will be a critical point and, under suitable second-order conditions, a local optimum. If there is single local optimum and it is possible to establish that the optimum is interior, this optimum will be global. If there are multiple local optima, one could use the value of the SWF to rank them in the welfarist case. So welfarist planners can unambiguously rank any two policies globally. Outside of the welfarist case, one can look for monotonic paths of integration (Zajac, 1979; Stahl, 1984) to rank different local optima, so it is only when this is not possible to find such paths that there may be some global ambiguity when ranking two particular policies.[64] In general, one can choose a set of reasonable policy paths (e.g., linear paths or bounded paths) and compare the predictions for the associated welfare assessments both in aggregate and for each of the elements of the aggregate additive decompositions.

G.6 Welfare assessments in economics with idiosyncratic/aggregate states

Until now, we have introduced our results in a canonical dynamic-stochastic model, following closely the notation of Chapter 8 in Ljungqvist and Sargent (2018). However, at times — in particular in Bewley-style economies — it is more convenient to work with a different notation that differentiates between idiosyncratic and aggregate states. We explain how to extend our framework to these environments, in which it is possible to derive new results. Our notation follows Krueger and Lustig (2010) whenever possible.

Environment We consider an economy populated by individuals that can be different for two different reasons at any point in time. First, we assume that individuals may be ex-ante heterogeneous, and we index this heterogeneity by i.[65] This form of heterogeneity is meant to capture immutable heterogeneity, for instance in terms of preferences. Second, we assume that individuals have different idiosyncratic states, so at a given point in time individuals that have in principle identical preferences may be different because they have a different idiosyncratic state.

In our economy there are aggregate and idiosyncratic states. We denote aggregate states by $z_t \in Z$ and idiosyncratic states by$y_t \in Y$. For simplicity, both Z and Y are assumed to be finite. We let $z^t = (z_0, \ldots, z_t)$ and $y^t = (y_0, \ldots, y_t)$ denote the history of aggregate and idiosyncratic states.

[64] Stahl (1984) proves that there always exist monotonic paths of integration in a classical demand context. While a formal proof of existence of such paths for the general framework considered here is outside of the scope of this paper, there is no reason to believe this result cannot be extended to natural applications.

[65] Importantly, the index i in this section, which indexes ex-ante heterogeneity, is completely different from the index i in the body of the paper, in which i indexes individuals. Formally, s^t in the body of the paper maps to z^t in this section, while i maps to the triple $\left\{i, y_0, y^t\right\}$.

States can be exogenous, in which case we refer to them as shocks, are they can be endogenous state variables (e.g., wealth or asset holdings). We denote the unconditional probability of transitioning from state y_0 given an initial aggregate state z_0 to a state (y^t, z^t) for an individual of ex-ante type i by $\pi_t^i \left(y^t, z^t \middle| y_0, z_0\right)$. We assume that the economy starts at an initial aggregate state z_0, which a cross-sectional distribution of individuals represented by $dG(y_0, i)$, where $\iint dG(y_0, i) = 1$. Given our assumptions, at a give date t, there is a single aggregate state (of any dimension), but there are as many idiosyncratic states (of any dimension) of individuals in the economy.

The lifetime utility of an individual of type i, with initial idiosyncratic state y_0, given an aggregate state z_0, is given by

$$V_i(y_0, z_0) = \sum_{t=0}^{T} (\beta_i)^t \sum_{z^t} \sum_{y^t} \pi_t^i \left(y^t, z^t \middle| y_0, z_0\right) u_i \left(c_t^i \left(y^t, z^t\right), n_t^i \left(y^t, z^t\right)\right),$$

where, for simplicity, we assume that β_i and $u_i(\cdot)$ are not functions of y^t and z^t. It is straightforward to extend our results to environments in which β_i and $u_i(\cdot)$ can be directly functions of y^t and z^t. Hence, we can express the change in the lifetime utility of an individual i with initial idiosyncratic state at a given initial aggregate state z_0 induced by a marginal policy change as follows:

$$\frac{dV_i(y_0, z_0)}{d\theta} = \sum_{t=0}^{T} (\beta_i)^t \sum_{z^t} \sum_{y^t} \pi_t^i \left(y^t, z^t \middle| y_0, z_0\right) \frac{\partial u_i(y^t, z^t)}{\partial c_t^i} \frac{du_{i|c}(y^t, z^t)}{d\theta}.$$

In the case of a welfarist planner, the counterpart of Equation (6) is now

$$\begin{aligned}
\frac{dW^{\mathcal{W}}(z_0)}{d\theta} &= \iint \lambda_i(y_0, z_0) \frac{dV_i(y_0, z_0)}{d\theta} dG(y_0, i) \\
&= \iint \lambda_i(y_0, z_0) \sum_{t=0}^{T} (\beta_i)^t \sum_{z^t} \sum_{y^t} \pi_t \left(y^t, z^t \middle| y_0, z_0\right) \frac{\partial u_i(y^t, z^t)}{\partial c_t^i} \frac{du_{i|c}(y^t, z^t)}{d\theta} dG(y_0, i),
\end{aligned}$$

where $\lambda_i(y_0, z_0) = \frac{\partial \mathcal{W}\left(\{V_i(y_0, z_0)\}_{i, y_0}\right)}{\partial V_i}$. Hence, a desirable policy change for a DS-planner, that is, the counterpart of Definition 2, is now based on

$$\frac{dW^{DS}(z_0)}{d\theta} = \iint \sum_{t=0}^{T} \sum_{z^t} \sum_{y^t} \omega_t^i \left(y^t, z^t \middle| y_0, z_0\right) \frac{du_{i|c}(y^t, z^t)}{d\theta} dG(y_0, i),$$

where $\omega_t^i \left(y^t, z^t \middle| y_0, z_0\right)$ denotes the DS-weight assigned to an individual of type i, whose idiosyncratic state at the time of the assessment is y_0, when the aggregate state at the time of the assessments is z_0, for a date t in which the idiosyncratic state of such individual is y^t and the aggregate state is z^t.

In this case, note that it is possible to define an individual multiplicative decomposition — the counterpart of Lemma 1 — that takes the form:

$$\omega_t^{i,y_0}\left(y^t, z^t\middle| z_0\right) = \underbrace{\tilde{\omega}^{i,y_0}(z_0)}_{\text{individual}}\underbrace{\tilde{\omega}_t^{i,y_0}(z_0)}_{\text{dynamic}}\underbrace{\tilde{\omega}_t^{i,y_0}\left(y^t, z^t|z_0\right)}_{\text{stochastic}}.$$

In this case, the individual multiplicative decomposition of a normalized welfarist planner — the counterpart of Proposition 5 — takes the form:

$$\tilde{\omega}_t^{i,y_0,\mathcal{W}}\left(y^t, z^t|z_0\right) = \frac{(\beta_i)^t \pi_t^i\left(y^t, z^t\right| y_0, z_0) \frac{\partial u_i\left(y^t,z^t\right)}{\partial c_t^i}}{\sum_{z^t}\sum_{y^t}(\beta_i)^t \pi_t^i\left(y^t, z^t\right| y_0, z_0) \frac{\partial u_i\left(y^t,z^t\right)}{\partial c_t^i}} = \frac{\pi_t^i\left(y^t, z^t\right| y_0, z_0) \frac{\partial u_i\left(y^t,z^t\right)}{\partial c_t^i}}{\sum_{z^t}\sum_{y^t}\pi_t^i\left(y^t, z^t\right| y_0, z_0) \frac{\partial u_i\left(y^t,z^t\right)}{\partial c_t^i}}$$

$$\tilde{\omega}_t^{i,y_0,\mathcal{W}}(z_0) = \frac{(\beta_i)^t\sum_{z^t}\sum_{y^t}\pi_t^i\left(y^t, z^t\right| y_0, z_0) \frac{\partial u_i\left(y^t,z^t\right)}{\partial c_t^i}}{\sum_{t=0}^T\sum_{z^t}\sum_{y^t}(\beta_i)^t \pi_t^i\left(y^t, z^t\right| y_0, z_0) \frac{\partial u_i\left(y^t,z^t\right)}{\partial c_t^i}}$$

$$\tilde{\omega}^{i,y_0,\mathcal{W}}(z_0) = \frac{\lambda_i(y_0,z_0)\sum_{t=0}^T\sum_{z^t}\sum_{y^t}(\beta_i)^t \pi_t^i\left(y^t, z^t\right| y_0, z_0) \frac{\partial u_i\left(y^t,z^t\right)}{\partial c_t^i}}{\iint\lambda_i(y_0,z_0)\sum_{t=0}^T\sum_{z^t}\sum_{y^t}(\beta_i)^t \pi_t^i\left(y^t, z^t\right| y_0, z_0) \frac{\partial u_i\left(y^t,z^t\right)}{\partial c_t^i} dG(y_0,i)}.$$

In this case, note that $\sum_{z^t}\sum_{y^t}\tilde{\omega}_t^{i,y_0,\mathcal{W}}\left(y^t, z^t\right| z_0) = 1$, $\forall i$, $\forall y_0$; $\sum_t \tilde{\omega}_t^{i,y_0,\mathcal{W}}(z_0) = 1$, $\forall i$, $\forall y_0$; and $\iint \tilde{\omega}^{i,y_0,\mathcal{W}}(z_0)\, dG(y_0,i) = 1$. Interestingly, under mild assumptions, note that there is scope to further decompose the individual and stochastic components as follows:

$$\tilde{\omega}^{i,y_0}(z_0) = \underbrace{\tilde{\omega}^{i}(z_0)}_{\text{ex-ante}}\ \underbrace{\tilde{\omega}^{y_0|i}(z_0)}_{\text{state variable}} \qquad \text{(individual)} \tag{OA25}$$

$$\tilde{\omega}_t^{i,y_0}\left(y^t, z^t|z_0\right) = \underbrace{\tilde{\omega}_t^{i,y_0}\left(z^t|z_0\right)}_{\text{aggregate}}\underbrace{\tilde{\omega}_t^{i,y_0}\left(y^t|z^t, z_0\right)}_{\text{idiosyncratic}}. \qquad \text{(stochastic)} \tag{OA26}$$

The two sub-components of the individual component capture redistribution towards immutable ex-ante heterogeneity (indexed by i) and initial idiosyncratic state-variable heterogeneity (indexed by y_0). The two sub-components of the stochastic component will allow us to decompose the risk-sharing component into pure risk-sharing of idiosyncratic and risk-transfer of aggregate risk.

Proposition 19. *(Welfare assessments: aggregate additive decomposition) The aggregate welfare assessment of a DS-planner, $\frac{dW^{DS}(z_0)}{d\theta}$, can be decomposed into four components: i) an aggregate efficiency component, ii) a risk-sharing component, iii) an intertemporal-sharing component, and iv) a redistribution component, as follows:*

$$\frac{dW^{DS}(s_0)}{d\theta} = \underbrace{\sum_{t=0}^{T} \mathbb{E}_{i,y_0}\left[\tilde{\omega}_t^{i,y_0}(z_0)\right] \sum_{z^t} \mathbb{E}_{i,y^0,y^t}\left[\frac{\tilde{\omega}_t^{i,y_0}(y^t,z^t|z_0)}{G_t(y^t,z^t|y_0,z_0,i)}\right] \mathbb{E}_{i,y^0,y^t}\left[\frac{du_{i|c}(y^t,z^t)}{d\theta}\right]}_{=\Xi_{AE}\ \textit{(Aggregate Efficiency)}}$$
$$+ \underbrace{\sum_{t=0}^{T} \mathbb{E}_{i,y_0}\left[\tilde{\omega}_t^{i,y_0}(z_0)\right] \sum_{z^t} \mathbb{C}ov_{i,y^0,y^t}\left[\frac{\tilde{\omega}_t^{i,y_0}(y^t,z^t|z_0)}{G_t(y^t,z^t|y_0,z_0,i)}, \frac{du_{i|c}(y^t,z^t)}{d\theta}\right]}_{=\Xi_{RS}\ \textit{(Risk-sharing)}}$$
$$+ \underbrace{\sum_{t=0}^{T} \mathbb{C}ov_{i,y_0}\left[\tilde{\omega}_t^{i,y_0}(z_0), \sum_{z^t}\sum_{y^t} \tilde{\omega}_t^{i,y_0}\left(y^t,z^t|z_0\right) \frac{du_{i|c}(y^t,z^t)}{d\theta}\right]}_{=\Xi_{IS}\ \textit{(Intertemporal-sharing)}}$$
$$+ \underbrace{\mathbb{C}ov_{i,y_0}\left[\tilde{\omega}^{i,y_0}(z_0), \sum_{t=0}^{T} (\beta_i)^t \sum_{z^t}\sum_{y^t} \pi_t^i\left(y^t,z^t\middle| y_0,z_0\right) \frac{\partial u_i(y^t,z^t)}{\partial c_t^i} \frac{du_{i|c}(y^t,z^t)}{d\theta}\right]}_{=\Xi_{RD}\ \textit{(Redistribution)}},$$

where we denote by $G_t(y^t,z^t|y_0,z_0,i)$ the transition likelihood with which an individual i that starts at states y_0 and z_0 transitions to histories y^t and z^t at date t.

Typically, in applications, $G_t(y^t,z^t|y_0,z_0,i)$ will equal $\pi_t^i(y^t,z^t|\,y_0,z_0)$, but not always, for instance when agents have heterogeneous beliefs. The definition of intertemporal-sharing and redistribution are exactly identical to those in Proposition 1. The definitions of risk-sharing and aggregate efficiency, which crucially hinge on taking cross-sectional average and covariances conditional on the values of idiosyncratic states, need to be slightly adjusted to account for the fact that agents transition between different states.

Finally, note that by combining Equation (OA25) with the definition of Ξ_{RD}, it is possible to provide a subdecomposition of the redistribution term into three terms:

$$\Xi_{RD} = \mathbb{C}ov_{i,y_0}\left[\tilde{\omega}^i(z_0)\,\tilde{\omega}^{y_0|i}(z_0), \frac{dV_i^{DS}(y_0,z_0)}{d\theta}\right]$$
$$= \underbrace{\mathbb{E}_{i,y_0}\left[\tilde{\omega}^i(z_0)\right] \mathbb{C}ov_{i,y_0}\left[\tilde{\omega}^i(z_0), \frac{dV_i^{DS}(y_0,z_0)}{d\theta}\right]}_{\text{ex-ante redistribution}} + \underbrace{\mathbb{E}_{i,y_0}\left[\tilde{\omega}^{y_0|i}(z_0)\right] \mathbb{C}ov_{i,y_0}\left[\tilde{\omega}^{y_0|i}(z_0), \frac{dV_i^{DS}(y_0,z_0)}{d\theta}\right]}_{\text{state-variable redistribution}}$$
$$+ \underbrace{\mathbb{E}_{i,y_0}\left[\left(\tilde{\omega}^i(z_0) - \mathbb{E}_{i,y_0}\left[\tilde{\omega}^i(z_0)\right]\right)\left(\tilde{\omega}^{y_0|i}(z_0) - \mathbb{E}_{i,y_0}\left[\tilde{\omega}^{y_0|i}(z_0)\right]\right)\left(\frac{dV_i^{DS}(y_0,z_0)}{d\theta} - \mathbb{E}_{i,y_0}\left[\frac{dV_i^{DS}(y_0,z_0)}{d\theta}\right]\right)\right]}_{\text{ex-ante/state-variable coskewness redistribution}}.$$

A similar subdecomposition emerges combining Equation (OA26) with the definition of Ξ_{RS}. In this case

$$
\begin{aligned}
\Xi_{RS} &= \sum_{t=0}^{T} \mathbb{E}_{i,y_0}\left[\tilde{\omega}_t^{i,y_0}\left(z_0\right)\right] \sum_{z^t} \mathbb{C}ov_{i,y^0,y^t}\left[\frac{\tilde{\omega}_t^{i,y_0}\left(y^t, z^t|z_0\right)}{G_t\left(y^t, z^t|y_0, z_0, i\right)}, \frac{du_{i|c}\left(y^t, z^t\right)}{d\theta}\right] \\
&= \underbrace{\sum_{t=0}^{T} \mathbb{E}_{i,y_0}\left[\tilde{\omega}_t^{i,y_0}\left(z_0\right)\right] \sum_{z^t} \mathbb{C}ov_{i,y^0,y^t}\left[\frac{\tilde{\omega}_t^{i,y_0}\left(z^t|z_0\right)}{G_t\left(z^t|z_0\right)}, \frac{du_{i|c}\left(y^t, z^t\right)}{d\theta}\right]}_{\text{risk transfer}} \\
&+ \underbrace{\sum_{t=0}^{T} \mathbb{E}_{i,y_0}\left[\tilde{\omega}_t^{i,y_0}\left(z_0\right)\right] \sum_{z^t} \mathbb{C}ov_{i,y^0,y^t}\left[\frac{\tilde{\omega}_t^{i,y_0}\left(y^t|z^t, z_0\right)}{G_t\left(y^t|z^t, z_0, y_0, i\right)}, \frac{du_{i|c}\left(y^t, z^t\right)}{d\theta}\right]}_{\text{idiosyncratic risk sharing}} \\
&+ \underbrace{\sum_{t=0}^{T} \mathbb{E}_{i,y_0}\left[\tilde{\omega}_t^{i,y_0}\left(z_0\right)\right] \mathbb{E}_{i,y^0,y^t}\left[\begin{array}{c} \left(\frac{\tilde{\omega}_t^{i,y_0}\left(z^t|z_0\right)}{G_t(z^t|z_0)} - \mathbb{E}_{i,y^0,y^t}\left[\frac{\tilde{\omega}_t^{i,y_0}\left(z^t|z_0\right)}{G_t(z^t|z_0)}\right]\right) \times \left(\frac{\tilde{\omega}_t^{i,y_0}\left(y^t|z^t,z_0\right)}{G_t(y^t|z^t,z_0,y_0,i)} - \mathbb{E}_{i,y^0,y^t}\left[\frac{\tilde{\omega}_t^{i,y_0}\left(y^t|z^t,z_0\right)}{G_t(y^t|z^t,z_0,y_0,i)}\right]\right) \\ \times \left(\frac{du_{i|c}\left(y^t,z^t\right)}{d\theta} - \mathbb{E}_{i,y^0,y^t}\left[\frac{du_{i|c}\left(y^t,z^t\right)}{d\theta}\right]\right) \end{array}\right]}_{\text{risk coskewness}}.
\end{aligned}
$$

where, under mild assumptions, we can define $G_t\left(y^t, z^t|y_0, z_0, i\right) = G_t\left(z^t|z_0\right) G_t\left(y^t|z^t, z_0, y_0, i\right)$.