Dynamic Optimization: Problem Set #5

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Problem 1

Credit: David Laibson (https://projects.iq.harvard.edu/econ2010c/problem-sets-david-laibson)

Please solve Problem 1 (“a simple consumption problem”) of David’s problem set (PSET) #3.

Problem 2

Credit: David Laibson (https://projects.iq.harvard.edu/econ2010c/problem-sets-david-laibson)

Please solve Problem 2 (“true / false / uncertain”) of David’s problem set (PSET) #3.

Problem 3

Credit: David Laibson (https://projects.iq.harvard.edu/econ2010c/problem-sets-david-laibson)

Please solve Problem 3 (“three period hyperbolic discounting model”) of David’s problem set (PSET) #3.

Problem 4

Credit: David Laibson (https://projects.iq.harvard.edu/econ2010c/problem-sets-david-laibson)

Please solve Problem 4 (“a procrastination problem”) of David’s problem set (PSET) #3.

Problem 5

Credit: Pablo Kurlat

Consider the following model.

- The world lasts for two periods
- There are two ty pes of households. -γ type A households. Preferences:

$$u(c_1, c_2) = \log(c_1) + \beta \log(c_2)$$

$-1-\gamma$ type B households. Consumption rule:

$$c_1 = a + b(Y_1 + sT)$$

- All households get the same income

- Y_1 is period-1 GDP
- sT is a lump-sum transfer from the government in period 1.s is a constant
- Period 2 GDP Y_2 is exogenously given - e.g. flexible prices and inelastic labor supply
- Period 1 GDP Y_1 is demand-determined
- The real interest r is fixed at zero - e.g. due to rigid prices and the zero lower bound
- The government spends G in period 1
- The government charges a lump-sum tax $\tau = sT + G$ on everyone in period 2
- Notation: - Aggregate consumption:

$$C_1 \equiv \gamma c_1^A + (1-\gamma) c_1^B$$

- Direct effect of transfers on consumption

(a) Solve for consumption of A and B households and for total consumption. Use the following notation

  - $c_1^i(T, \tau, Y_1)$ for $i = A, B$ denotes the consumption choice given transfers, taxes and period-1 GDP.
  - $c_1^i(T, G, Y_1)$ for $i = A, B$ denotes the consumption choice given transfers, government spending and period-1 GDP, once we have taken into account the government budget constraint, which implies a certain level of τ for given levels of T and G.
  - $C_1(T, \tau, Y_1) \equiv \gamma c_1^A(T, \tau, Y_1) + (1-\gamma) c_1^B(T, \tau, Y_1)$

- $C_1(T,G,Y_1) \equiv \gamma c_1^A(T,G,Y_1) + (1-\gamma)c_1^B(T,G,Y_1)$

(b) Compute the following quantities

(a) $\frac{dY_1}{dG}$

(b) $\frac{\partial C_1(T,\tau,Y_1)}{\partial T}$

(c) $\frac{\partial C_1(T,G,Y_1)}{\partial T}$

(d) $\frac{\partial C_1(T,G,Y_1)}{\partial G}$

(e) $\frac{dY_1}{dT}$

(c) Consider the following empirical experiments:

(a) A period-1 transfer T_i is assigned to household i in period 1. T_i varies randomly across the population. The government budget is balanced with a lump-sum tax paid equally by everybody in period 2 .

(b) A lump-sum transfer T is assigned equally to everyone. A random sample of the population gets in in period 1 and the rest get it in period 2. The government budget is balanced with a lump-sum tax paid equally by everybody in period 2 . In both cases a researcher estimates a regression:

$$c_{i1} = \alpha + \delta T_{i1}$$

What quantity is the researcher estimating in each case? What quantity are Parker et. al. (2013) estimating?

(d) Consider a government-spending-multiplier design where we use exogenous variation in (nationallevel) military spending to measure the contemporaneous government spending multiplier. What quantity are we measuring?

(e) Consider the case where $\gamma = 0$.

(e.1) Solve for the equilibrium level of Y_1 as a function of T and G.

(e.2) Show that:

$$\frac{dY_1}{dT} = \frac{dY_1}{dG}\frac{\partial C_1(T,\tau,Y_1)}{\partial T}$$

and also:

$$\frac{dY_1}{dT} = \frac{dY_1}{dG}\frac{\frac{\partial C_1(T,G,Y_1)}{\partial T}}{1+\frac{\partial C_1(T,G,Y_1)}{\partial G}}$$

(f) Now consider the case where $\gamma > 0$

(a) Solve for the equilibrium level of Y_1 as a function of T and G.

(b) Show that (2) does not hold but (3) does hold.

(g) Suppose that we want to measure $\frac{dY_1}{dT}$ by combining the estimates of Parker et. al. (2013) and estimates of the government spending multiplier. Will we get the right answer?