Economics 2010c: Lecture 6
Quasi-hyperbolic discounting

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Outline:

1. Quasi-hyperbolic Discounting: aka Present Bias

2. Lifecycle simulations

3. Hyperbolic Euler Equation (Section)

1 Present Bias

Read and van Leeuwen (1998): Food experiment.

- Choose for Next Week: Fruit (74%) or Chocolate (26%)

- Choose for Today: Fruit (30%) or Chocolate (70%).

Read, Loewenstein & Kalyanaraman (1999): Video experiment

- Choose for Next Week: Low-brow (37%) or High-brow (63%)

- Choose for Today: Low-brow (66%) or High-brow (34%).

Evidence from gyms (Della Vigna and Malmendier 2004).

- Average cost of gym membership: \$75 per month.

- Average number of visits per month: 4.

- Average cost per visit: \$19.

- Cost of “pay-per-visit:” \$10.

- Present bias (Phelps and Pollak 1968, Laibson 1997):
  1, $\beta\delta$, $\beta\delta^2$, $\beta\delta^3$, ...

$$U_t = u(c_t) + \beta\delta u(c_{t+1}) + \beta\delta^2 u(c_{t+2}) + \beta\delta^3 u(c_{t+3}) + ...$$

- For exponentials: $\beta = 1$

$$U_t = u(c_t) + \delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \delta^3 u(c_{t+3}) + ...$$

- For “quasi-hyperbolics”: $\beta < 1$

$$U_t = u(c_t) + \beta \left[\delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \delta^3 u(c_{t+3}) + ...\right]$$

- To build intution, assume that $\beta \simeq \frac{1}{2}$ and $\delta \simeq 1$

$$\{1, \beta\delta, \beta\delta^2, \beta\delta^3, ...\} = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, ...\}$$

$$U_t = u(c_t) + \frac{1}{2}\left[u(c_{t+1}) + u(c_{t+2}) + u(c_{t+3}) + ...\right]$$

- Relative to the current period, all future periods are worth less (weight $\frac{1}{2}$).

- Most (for this example, *all*) of the discounting takes place between the current period and the immediate future.

- There is little (for this example, *no*) additional discounting between future periods.

$$U_t = u(c_t) + \frac{1}{2}\left[u(c_{t+1}) + u(c_{t+2}) + u(c_{t+3}) + ...\right]$$

- Preferences are dynamically inconsistent.
- At date t we prefer to be patient between $t+1$ and $t+2$.
- At date $t+1$ we want immediate gratification at $t+1$.

$$U_{t+1} = u(c_{t+1}) + \frac{1}{2}\left[u(c_{t+2}) + u(c_{t+3}) + u(c_{t+4}) + ...\right]$$

Akerlof (1992), O'Donoghue and Rabin (1999), Carroll et al (2009) on procrastination:

- Assume $\beta = \frac{1}{2}$ and $\delta = 1$.

- Suppose exercise (cost 6) generates delayed benefits (value 8).

- Exercise Today? $\quad -6 + \frac{1}{2}[8] = -2 < 0$

- Exercise Tomorrow? $\quad 0 + \frac{1}{2}[-6 + 8] = 1 > 0$

- Agent would like to exercise tomorrow.

Predictions:

- Procrastination when costs precede benefits (Della Vigna and Malmendier 2004, 2006; Ariely and Wertenbroch 2002; Augenblick, Niederle, and Sprenger 2013)

- Downward sloping consumption paths within pay-cycle (e.g., Shapiro 2005, Mastrobuoni and Weinberg 2009, Hastings and Washington 2010)

- Willingness to use commitment: savings (Ashraf, Karlan, and Yin, 2006; Beshears et al 2013), student productivity (Ariely and Wertenbroch, 2002; Houser et al., 2010, Chow 2011; Augenblick, Niederle, and Sprenger, 2013), cigarette smoking (Gine, Karlan, and Zinman, 2010), workplace productivity (Kaur, Kremer, and Mullainathan, 2010), and exercise (Milkman, Minson, and Volpp, 2012; Royer, Stehr, and Sydnor, 2012).

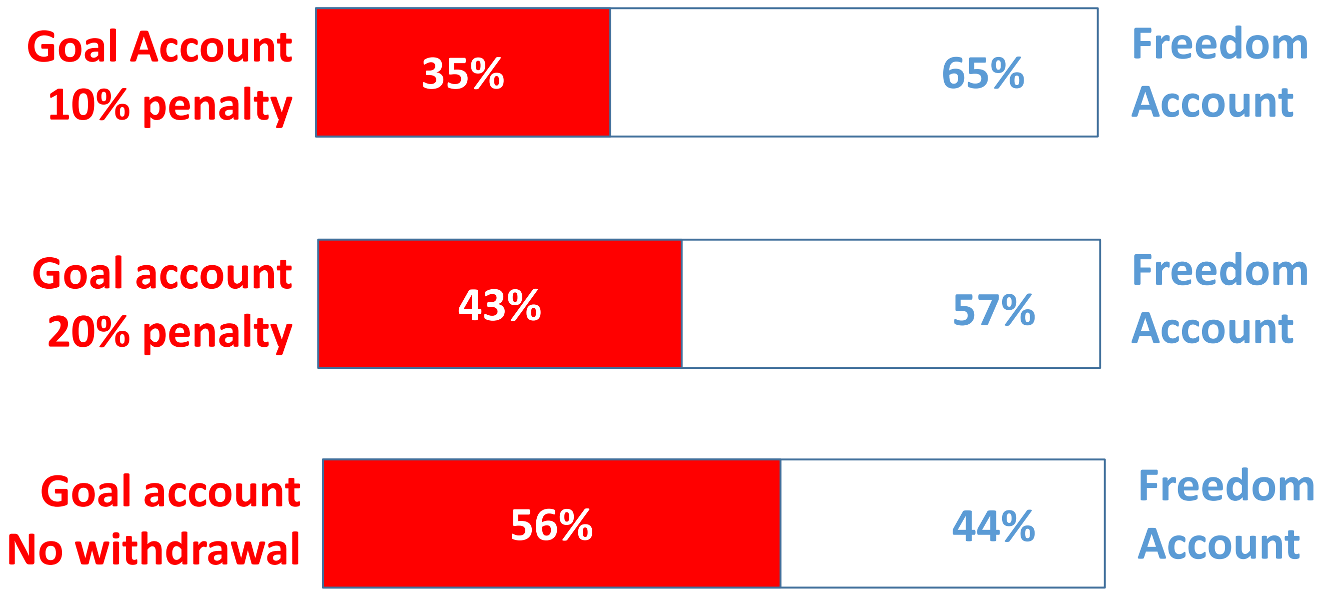
How to design a commitment contract

Participants divide $100 between:

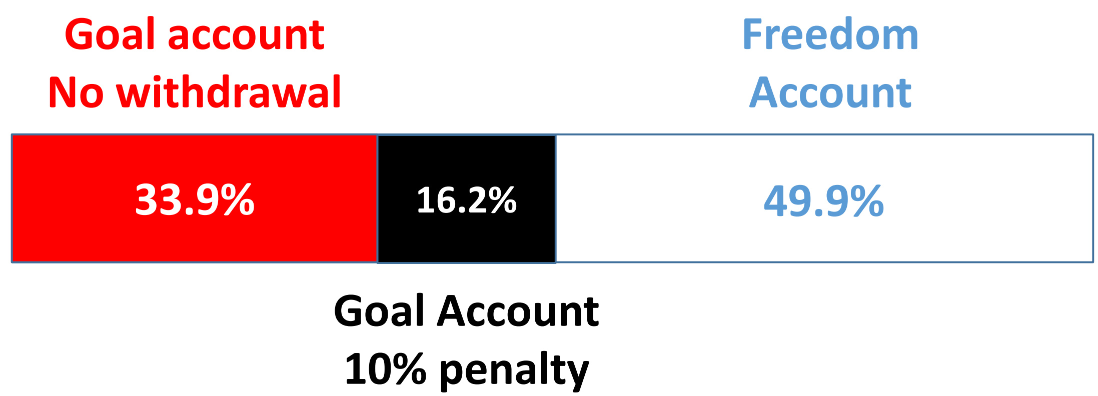
- Freedom account (22% interest)
- Goal account (22% interest)
  with a withdrawal restriction

Beshears, Choi, Madrian, Laibson, Sakong (2020)

Initial investment (dollar weighted) in the two accounts



When three accounts are offered



Beshears, Choi, Laibson, Madrian, Sakong (~~2019~~ 2020)

2 Lifecycle simulations

Laibson, Lee, Maxted, Repetto, and Tobacman (2021)

Use Method of Simulated Moments to Estimate β, δ, and $\gamma = CRRA$.

2.1 Demographic Assumptions

- Mortality

- Lifetime income profile (PSID)

- Stochastic labor income (PSID)

- Dependents (Census)

- Three educational groups: NHS, HS, COLL

2.2 Dynamic Budget Constraints

- Credit limit: $(.55)(\bar{Y}_n)$. (Calibrate from SCF.)

- Real after-tax rate of return: 2.79% (Municipal bond rate)

- Real rate of return on (partially) illiquid investment: 5.00%

- Real credit card interest rate: 11.52%

- State variables: age, liquid wealth, partially illiquid wealth, autocorrelated income.

- Choice variables: liquid wealth investment, illiquid wealth investment.

2.3 Preferences

- Instantaneous utility function: CRRA $= \gamma$.

- Quasi-hyperbolic discounting: 1, $\beta\delta$, $\beta\delta^2$, $\beta\delta^3$, ...

- Naive beliefs

2.4 Estimation (MSM)

- Use the Method of Simulated Moments (Pakes and Pollard 1989).

- Pick parameter values to minimize the gap between simulated data and empirical data.

  - Substantial retirement wealth accumulation (SCF)

  - Extensive credit card borrowing (SCF, Fed, Gross and Souleles 2000, Laibson, Repetto, and Tobacman 2000)

2.5 Estimator

Estimate parameter vector θ and evaluate models wrt data.

- $m_e = N$ empirical moments
- $m_s(\theta) =$ analogous simulated moments
- $q(\theta) \equiv (m_s(\theta) - m_e) W^{-1} (m_s(\theta) - m_e)'$, a (scalar) loss function
- Minimize loss function: $\hat{\theta} = \arg\min_{\theta} q(\theta)$

- $\hat{\theta}$ is the MSM estimator.

- Pakes and Pollard (1989) prove asymptotic consistency and normality.

- Specification tests: $q(\hat{\theta}) \sim \chi^2(N - \#parameters)$

	Moments	Age buckets
% households with credit card debt	0.81	21-30
	0.78	31-40
	0.75	41-50
	0.66	51-60
$\frac{\text{average credit card debt}}{\text{average income}}$	0.14	21-30
	0.14	31-40
	0.19	41-50
	0.20	51-60
households with credit card debt: $\frac{\text{average net worth}}{\text{average income}}$	1.19	21-30
	1.78	31-40
	2.94	41-50
	4.24	51-60
households w/o credit card debt: $\frac{\text{average net worth}}{\text{average income}}$	1.91	21-30
	2.66	31-40
	4.89	41-50
	8.10	51-60

	β flexible	β=1
(short run discount factor) β	0.50	1.00
(long-run, exponential discount factor) δ	0.99	0.96
relative risk aversion	1.32	1.42

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relative risk aversion	1.32	1.42		
	Simulated Moments		Moments	Age buckets
% households with credit card debt	0.58	0.28	0.81	21-30
	0.59	0.24	0.78	31-40
	0.57	0.26	0.75	41-50
	0.55	0.26	0.66	51-60
average credit card debt / average income	0.11	0.03	0.14	21-30
	0.14	0.03	0.14	31-40
	0.17	0.04	0.19	41-50
	0.20	0.05	0.20	51-60
households with credit card debt: average net worth / average income	1.11	0.96	1.19	21-30
	1.48	0.98	1.78	31-40
	2.45	2.02	2.94	41-50
	4.34	4.14	4.24	51-60
households w/o credit card debt: average net worth / average income	1.96	1.94	1.91	21-30
	3.04	2.88	2.66	31-40
	4.56	4.24	4.89	41-50
	7.45	6.28	8.10	51-60

The following definitions that will help you interpret the figures that follow.

- Liquid assets (x_t) is cash-on-hand *after* income has been realized in the current period. Liquid assets does not include the illiquid asset.

- If liquid savings, $(x_{t-1} - c_{t-1})$, is positive then

  $$x_t = 1.0279 \times (x_{t-1} - c_{t-1}) + \tilde{y}_t$$

  where c_{t-1} is consumption from non-durables.

- If liquid savings, $(x_{t-1} - c_{t-1})$, is negative then

  $$x_t = 1.1152 \times (x_{t-1} - c_{t-1}) + \tilde{y}_t$$

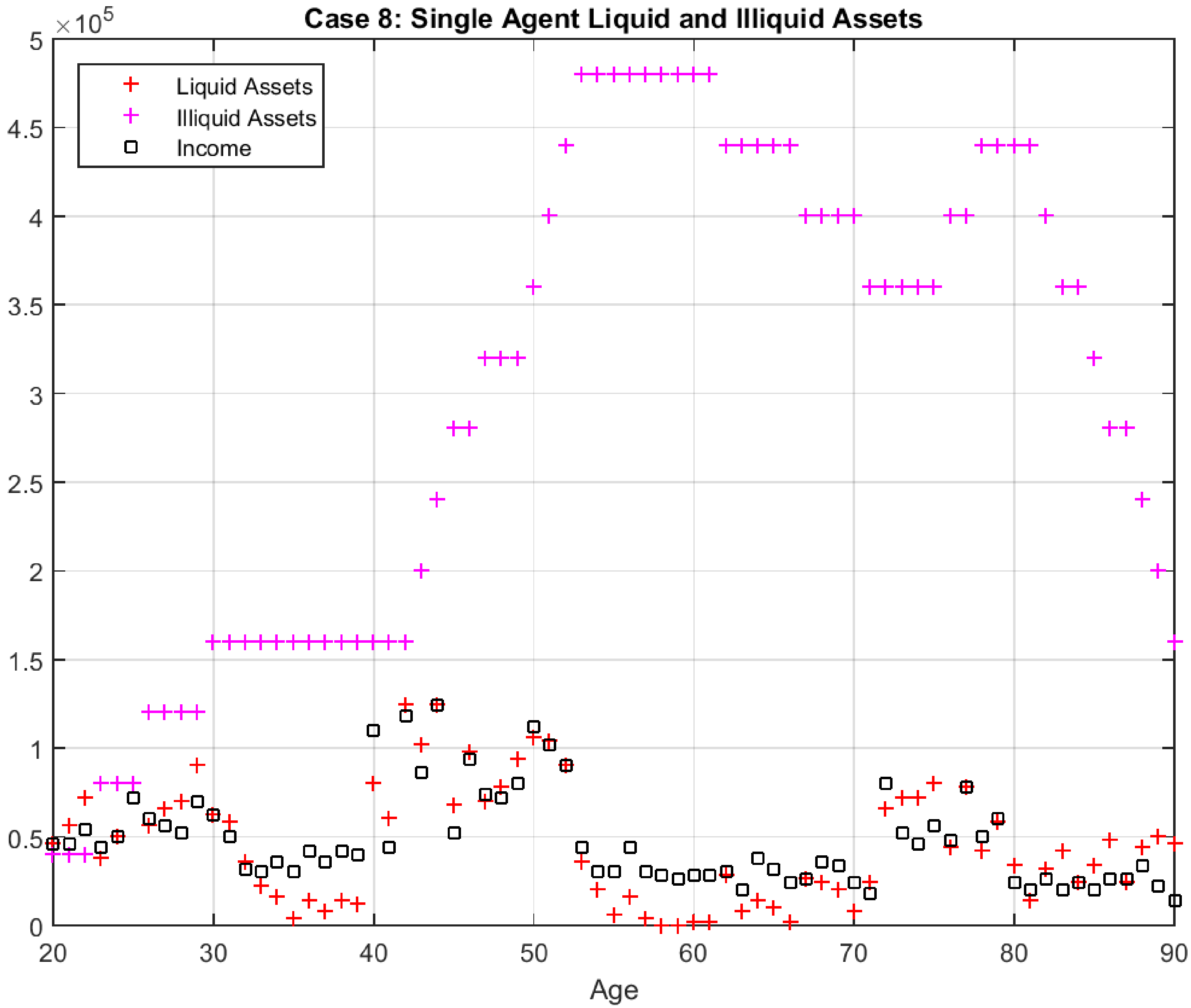
  where c_{t-1} is consumption from non-durables.

- When current period income exceeds cash-on-hand, i.e.,

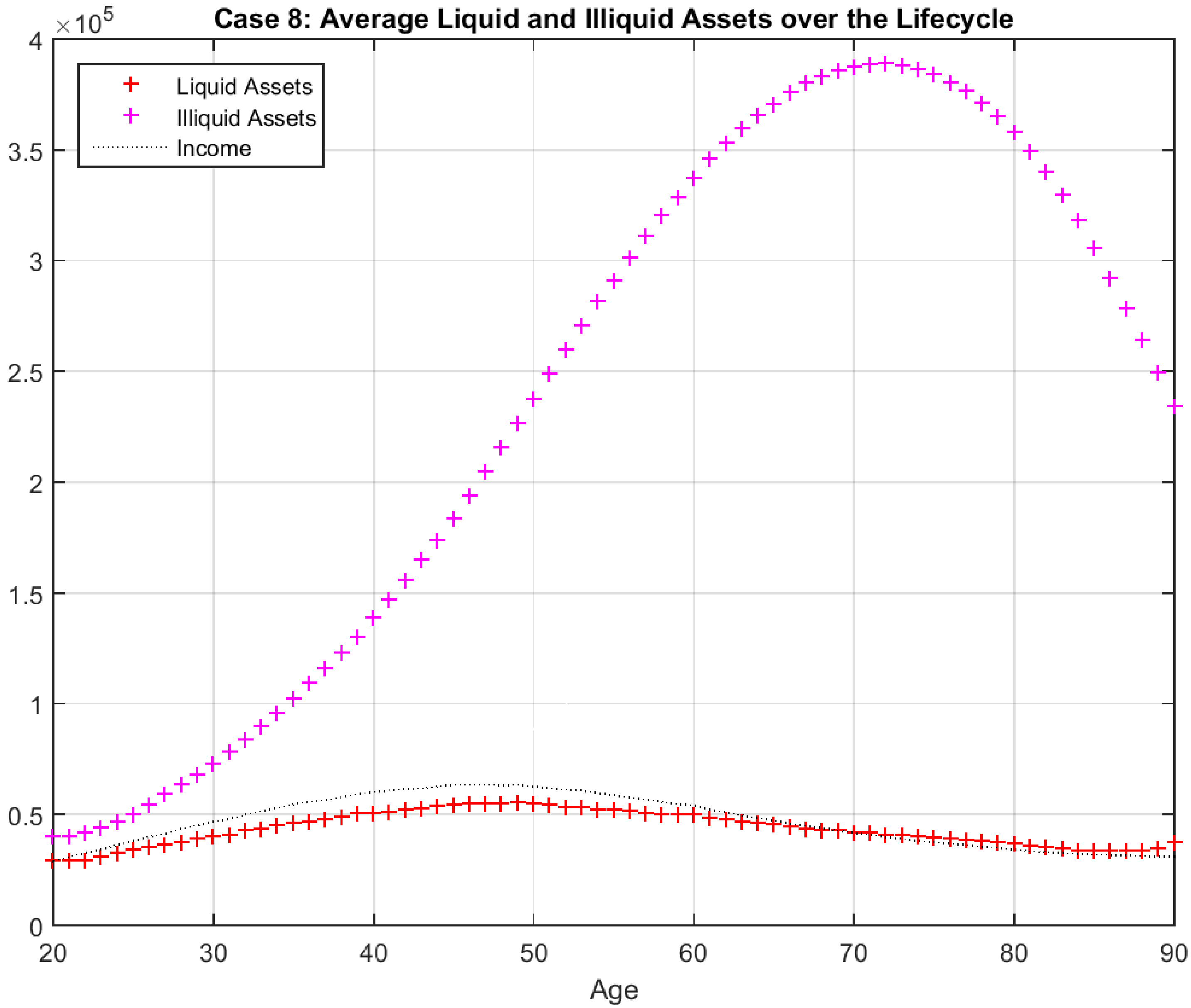
$$\tilde{y}_t > x_t$$

then liquid savings is negative: $(x_{t-1} - c_{t-1}) < 0$.

Case 8: Single Agent Income and Consumption
×10^4
14
12
10
8
6
4
2
0
20
30
40
50
60
70
80
90
Age
Income
Consumption
Consumption + 0.05*(Illiquid Assets)



Case 8: Single Agent Liquid and Illiquid Assets
×10^5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
20
30
40
50
60
70
80
90
Age
Liquid Assets
Illiquid Assets
Income



Case 8: Average Liquid and Illiquid Assets over the Lifecycle
×10^5
Liquid Assets
Illiquid Assets
Income
4
3.5
3
2.5
2
1.5
1
0.5
0
20
30
40
50
60
70
80
90
Age

3 Hyperbolic Euler Equation

- Let c represent consumption.
- Let x represent cash-on-hand.
- Let $\tilde{y}$ represent iid stochastic income.
- Let R represent gross interest rate.
- So $x_{t+1} = R(x_t - c_t) + \tilde{y}_{t+1}$.
- A (Markov) strategy is a map from state x to control c.

- Let V be the continuation-value function, W be the current-value function and C be the consumption function. Then:

$$\begin{aligned} V(x) &= U(C(x)) + \delta\, \mathsf{E}[V(R(x - C(x)) + y)] \\ W(x) &= U(C(x)) + \beta\delta\, \mathsf{E}[V(R(x - C(x)) + y)] \\ C(x) &= \operatorname*{argmax}_{c}\, U(c) + \beta\delta\, \mathsf{E}[V(R(x - c) + y)] \end{aligned}$$

- Note that V accumulates utils exponentially.

- Note that W accumulates utils quasi-hyperbolically.

- Envelope Theorem.

$$W'(x) = U'(C(x))$$

- First-order-condition.

$$U'(C(x)) = R\beta\delta\, \mathsf{E}\Big[V'(R(x - C(x)) + y)\Big]$$

- Identity linking V and W.

$$\beta V(x) = W(x) - (1 - \beta)U(C(x))$$

3.1 Problem is recursive

- Start with V.

- Find C:

$$C(x) = \operatorname*{argmax}_{c} U(c) + \beta\delta\, \mathsf{E}[V(R(x-c)+y)]\,.$$

- Find $\hat{V}$:

$$\hat{V}(x) = U(C(x)) + \delta\, \mathsf{E}[V(R(x - C(x)) + y)]$$

- In this way, generate an operator $T : V \mapsto \hat{V}$.

3.2 Can also derive an Euler Equation

We have

$$\begin{aligned} u'(c_t) &= R\beta\delta\,\mathsf{E}_t\Big[V'(x_{t+1})\Big] \\ &= R\delta\,\mathsf{E}_t\left[W'(x_{t+1}) - (1-\beta)u'(c_{t+1})\frac{dC_{t+1}}{dx_{t+1}}\right] \\ &= R\delta\,\mathsf{E}_t\left[u'(c_{t+1}) - (1-\beta)u'(c_{t+1})\frac{dC_{t+1}}{dx_{t+1}}\right]. \end{aligned}$$

So,

$$u'(c_t) = R\,\mathsf{E}_t\left[\beta\delta\left(\frac{dC_{t+1}}{dX_{t+1}}\right) + \delta\left(1 - \frac{dC_{t+1}}{dX_{t+1}}\right)\right]u'(c_{t+1}).$$

See Harris and Laibson (2003).