M2: Lecture 8

Theory of the Consumption Function (I)

Andreas Schaab

Outline of today's lecture

1. Canonical income fluctuations problem in discrete time
2. Permanent income hypothesis
3. Certainty equivalence and consumption as a martingale
4. Linearization of Euler equation
5. Euler equation empirics

1. Canonical model of consumption

- The standard model is known as the **consumption-savings** or **income-fluctuations** problem

$$V(a_0) = \max_{\{c_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$a_{t+1} = R_{t+1}(a_t - c_t) + y_t$$

- a_t is wealth, $\{R_{t+1}\}$ is deterministic (gross) interest rate process, and y_t is iid. income risk
- Assume $u(\cdot)$ concave ($u' > 0$ and $u'' < 0$ for all c) and $\lim_{c \to 0} u'(c) = \infty$
- What should we assume about borrowing capacity?
  - Natural borrowing limit: $a_t \geq a^n$
  - Ad-hoc borrowing limit: $a_t \geq \underline{a}$

- Bellman equation:

$$V_t(a) = \max_{c,a'} \left\{ u(c) + \beta \mathbb{E}_t V_{t+1}(a') \right\} \;\; s.t. \;\; a' = R_{t+1}(a-c) + y$$

- Bellman equation may not be stationary or time-independent: R_t

- If income process $\{y_t\}$ persistent, would need y as second state variable

- First-order conditions (with ad-hoc borrowing limit $\underline{a}$):

$$\begin{aligned} u'(c_t(a)) &= \beta R_{t+1} \mathbb{E}_t \frac{\partial V_{t+1}(a')}{\partial a'} \quad \text{if } a' > \underline{a} \\ u'(c_t(a)) &\geq \beta R_{t+1} \mathbb{E}_t \frac{\partial V_{t+1}(a')}{\partial a'} \quad \text{if } a' = \underline{a} \end{aligned}$$

- Envelope theorem: $\frac{\partial V_t(a)}{\partial a} = u'(c_t(a))$
- So we again get a consumption Euler equation:

$$
\begin{aligned}
u'(c_t) &= \beta R_{t+1} \mathbb{E}_t u'(c_{t+1}) \quad \text{if } a' > \underline{a} \\
u'(c_t) &\geq \beta R_{t+1} \mathbb{E}_t u'(c_{t+1}) \quad \text{if } a' = \underline{a}
\end{aligned}
$$

Perturbation intuition:

- What is the cost of consuming ϵ dollars less today?

$$\text{Utility loss today} = \epsilon \cdot u'(c_t)$$

- What is the expected, discounted gain of consuming $\epsilon \cdot R_{t+1}$ dollars more tomorrow?

$$\text{Utility gain tomorrow} = \beta(\epsilon \cdot R_{t+1}) \mathbb{E}_t u'(c_{t+1})$$