Economics 2010c: Lecture 3
The Classical Consumption Model

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Outline:

1. Consumption: Basic model and early theories

2. Linearization of the Euler Equation

3. Empirical tests without "precautionary savings effects"

1 Application: Consumption.

Sequence Problem (SP): Find $v(x)$ such that

$$v(x_0) = \sup_{\{c_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \delta^t u(c_t)$$

subject to a static budget constraint for consumption,

$$c_t \in \Gamma^C(x_t),$$

and a dynamic budget constraint for assets,

$$x_{t+1} \in \Gamma^X\left(x_t, c_t, \tilde{R}_{t+1}, \tilde{y}_{t+1}\right).$$

Here x is the vector of assets, c is consumption, R is the vector of financial asset returns, and y is the vector of income (other than capital income).

For instance, consider the case where the only asset is cash-on-hand (so x is cash-on-hand) and consumption is constrained to lie between 0 and $x + L$, where L is the borrowing limit (e.g., the household's credit line). Then,

$$c_t \in \Gamma^C(x_t) \equiv [0, x_t + L]$$

$$x_{t+1} \in \Gamma^X\left(x_t, c_t, \tilde{R}_{t+1}, \tilde{y}_{t+1}\right) \equiv \tilde{R}_{t+1}(x_t - c_t) + \tilde{y}_{t+1}$$

$$x_0 = y_0.$$

- We will assume that $\tilde{y}$ is exogenous and iid.
  - However, in the actual economy, $\tilde{y}$ is not independent of x. See analysis of means testing in Hubbard, Skinner, and Zeldes (1995).
  - For example, Medicaid, college financial aid, and SNAP all have asset tests.
- We will always assume that u is concave ($u'' < 0$ for all $c > 0$).
- We will assume $\lim_{c \downarrow 0} u'(c) = \infty$, so $c > 0$ as long as $x > 0$.
  - I'll highlight one exception to this assumption.

1.1 Bellman Equation representation

- The state variable, x, is stochastic, so it is not directly chosen (rather a distribution for x_{t+1} is chosen at time t).

- It is more convenient to think about c as the choice variable.

Bellman Equation:

$$v(x) = \sup_{c \in [0, x+L]} \{ u(c) + \delta E v(x_{+1}) \} \quad \forall x$$

$$x_{+1} = \tilde{R}_{+1}(x - c) + \tilde{y}_{+1}$$

$$x_0 = y_0.$$

1.2 Necessary Conditions

- First Order Condition:

$$\begin{array}{ll} u'(c_t) = \delta E_t \tilde{R}_{t+1} v'(x_{t+1}) & \text{if} \quad 0 < c_t < x_t + L \\ u'(c_t) \geq \delta E_t \tilde{R}_{t+1} v'(x_{t+1}) & \text{if} \quad \quad c_t = x_t + L \end{array}$$

- Envelope Theorem: $v'(x) = u'(c)$. Prove this. What if $c = x + L$?

- Euler Equation:

$$\begin{array}{ll} u'(c_t) = \delta E_t \tilde{R}_{t+1} u'(c_{t+1}) & \text{if} \quad 0 < c_t < x_t + L \\ u'(c_t) \geq \delta E_t \tilde{R}_{t+1} u'(c_{t+1}) & \text{if} \quad \quad c_t = x_t + L \end{array}$$

1.3 Perturbation intuition behind the Euler Equation:

- What is the cost of consuming ε dollars less today?

$$\text{Utility loss today} = \varepsilon \cdot u'(c_t)$$

- What is the (discounted, expected) gain of consuming $\tilde{R}_{t+1}\,\varepsilon$ dollars more tomorrow?

$$\text{Utility gain tomorrow} = \delta E_t\left[\left(\tilde{R}_{t+1}\,\varepsilon\right)\cdot u'(c_{t+1})\right]$$

Let's now rederive the Euler Equation:

1. Suppose $u'(c_t) < \delta E_t \tilde{R}_{t+1} u'(c_{t+1})$. Then cut c_t by ε and raise c_{t+1} by $\tilde{R}_{t+1}\,\varepsilon$ to generate a net utility gain:

$$\varepsilon \cdot \left[-u'(c_t) + \delta E_t \tilde{R}_{t+1} u'(c_{t+1}) \right] > 0.$$

This perturbation is always possible on the equilibrium path, so:

$$u'(c_t) \geq \delta E_t \tilde{R}_{t+1} u'(c_{t+1}).$$

2. Suppose $u'(c_t) > \delta E_t \tilde{R}_{t+1} u'(c_{t+1})$, then raise c_t by ε and cut c_{t+1} by $\tilde{R}_{t+1}\,\varepsilon$ to generate a net utility gain:

$$\varepsilon \cdot \left[u'(c_t) - \delta E_t \tilde{R}_{t+1} u'(c_{t+1}) \right] > 0.$$

This perturbation is possible on the equilibrium path as long as $c < x + L$, so:

$$u'(c_t) \leq \delta E_t \tilde{R}_{t+1} u'(c_{t+1}) \quad \text{as long as } c_t < x_t + L.$$

It follows that

$$\begin{aligned} u'(c_t) &= \delta E_t \tilde{R}_{t+1} u'(c_{t+1}) \quad \text{if} \quad c_t < x_t + L \\ u'(c_t) &\geq \delta E_t \tilde{R}_{t+1} u'(c_{t+1}) \quad \text{if} \quad c_t = x_t + L \end{aligned}$$

1.4 Important consumption models:

1.4.1 Life Cycle Hypothesis: Modigliani & Brumberg (1954)

- $\tilde{R}_t = R$, $\delta R = 1$.

- Deterministic or stochastic income, y_t for all t.

- Perfect capital markets (i.e., no moral hazard), so that future income, y_t, can be exchanged for current capital. Let's assume that your counter-party is risk neutral.

- **Bellman Equation**:

$$v(x) = \sup_{c \leq x} \{ \, u(c) + \delta v \left(x_{+1} \right) \, \} \quad \forall x$$

$$x_{+1} = R(x - c)$$

$$x_0 = E_0 \sum_{t=0}^{\infty} R^{-t} \, y_t$$

- Sometimes referred to as "eating a pie/cake problem."

- Euler Equation implies,

$$u'(c_t) = \delta R u'(c_{t+1}) = u'(c_{t+1}).$$

- Hence, consumption is constant.

Budget constraint:

$$\sum_{t=0}^{\infty} R^{-t} c_t = E_0 \sum_{t=0}^{\infty} R^{-t} y_t$$

Substitute Euler Equation, $c_0 = c_t$ to find

$$\sum_{t=0}^{\infty} R^{-t} c_0 = E_0 \sum_{t=0}^{\infty} R^{-t} y_t$$

So Euler Equation + budget constraint implies

$$c_0 = \left(1 - \frac{1}{R}\right) \left(E_0 \sum_{t=0}^{\infty} R^{-t} y_t\right) \qquad \forall t$$

Consumption is an annuity. Note that

$$\left(1 - \frac{1}{R}\right) \simeq \left(1 - \frac{1}{1}\right) + \frac{1}{R^2_{|R=1}}(R-1) = R - 1 = r$$

Remark 1.1 *What's the value of your annuity?*

Remark 1.2 *Friedman's Permanent Income Hypothesis (Friedman, 1957) is much like Modigliani's Life-Cycle Hypothesis, though Friedman was more behavioral. He didn't imagine a full lifecycle of consumption smoothing, but instead a five-year look-ahead window.*

1.4.2 Certainty Equivalence Model: Hall (1978)

- $\tilde{R}_t = R$, $\delta R = 1$

- Quadratic utility: $u(c) = \alpha c - \frac{\beta}{2}c^2$
  - This admits negative consumption.
  - And this does **not** imply $\lim_{c \downarrow 0} u'(c) = \infty$.

- Now assume you can't sell claims to labor income.

- Euler Equation

$$u'(c_t) = \delta R E_t u'(c_{t+1})$$

implies,

$$\begin{aligned} -\beta c_t &= -\beta E_t c_{t+1} \\ c_t &= E_t c_{t+1} = E_t c_{t+n}. \end{aligned}$$

So consumption is a random walk:

$$c_{t+1} = c_t + \eta_{t+1}.$$

- So Δc_{t+1} can not be predicted by any information available at time t.

Budget constraint at date t:

$$\sum_{s=0}^{\infty} R^{-s}\, c_{t+s} = x_t + \sum_{s=1}^{\infty} R^{-s}\, \widetilde{y}_{t+s}$$

$$E_t \sum_{s=0}^{\infty} R^{-s}\, c_{t+s} = x_t + E_t \sum_{s=1}^{\infty} R^{-s}\, \widetilde{y}_{t+s}$$

Substitute $c_t = E_t c_{t+s}$ to find

$$\sum_{s=0}^{\infty} R^{-s}\, c_t = x_t + E_t \sum_{s=1}^{\infty} R^{-s}\, \widetilde{y}_{t+s}$$

So Euler Equation + budget constraint implies

$$c_t = \left(1 - \frac{1}{R}\right) \left(x_t + E_t \sum_{s=1}^{\infty} R^{-s}\, \widetilde{y}_{t+s}\right) \qquad \forall t$$

2 Linearizing Euler Equation

Recall Euler Equation:

$$u'(c_t) = E_t \delta R_{t+1} u'(c_{t+1})$$

Want to transform this equation so it is more amenable to empirical analysis.

Assume that R_{t+1} is known at time t.

Assume u is an isoelastic (i.e., constant relative risk aversion) utility function,

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}.$$

(Aside: $\lim_{\gamma \to 1} \frac{c^{1-\gamma}-1}{1-\gamma} = \ln c$. Important special case.)

Note that

$$u'(c) = c^{-\gamma}.$$

We can rewrite the Euler Equation as

$$c_t^{-\gamma} = E_t \delta R_{t+1} c_{t+1}^{-\gamma}$$

$$1 = E_t \exp\left[\ln\left(\delta R_{t+1} c_{t+1}^{-\gamma} c_t^{\gamma}\right)\right]$$

$$1 = E_t \exp\left[-\rho + r_{t+1} - \gamma \ln(c_{t+1}/c_t)\right]$$

where $-\ln \delta = \rho$ and $\ln R_{t+1} = r_{t+1}$.

Since, $\ln(c_{t+1}/c_t) = \ln(c_{t+1}) - \ln(c_t)$, we write,

$$1 = E_t \exp\left[r_{t+1} - \rho - \gamma \Delta \ln c_{t+1}\right].$$

Assume that $\Delta \ln c_{t+1}$ is conditionally normally distributed. So,

$$1 = \exp\left[E_t r_{t+1} - \rho - \gamma E_t \Delta \ln c_{t+1} + \frac{1}{2}\gamma^2 V_t \Delta \ln c_{t+1}\right].$$

Taking the natural log of both sides yields,

$$E_t \Delta \ln c_{t+1} = \frac{1}{\gamma}(E_t r_{t+1} - \rho) + \frac{\gamma}{2} V_t \Delta \ln c_{t+1}.$$

3 Empirical tests without precautionary savings effects

Recall Euler Equation:

$$u'(c_t) = E_t \delta R_{t+1} u'(c_{t+1})$$

We write the linearized Euler Equation in regression form:

$$\Delta \ln c_{t+1} = \frac{1}{\gamma}(E_t r_{t+1} - \rho) + \frac{\gamma}{2} V_t \Delta \ln c_{t+1} + \varepsilon_{t+1}$$

where ε_{t+1} is orthogonal to any information known at date t.

The conditional variance term is often referred to as the "precautionary savings term," (more on this later).

We sometimes (counterfactually) assume that $V_t \Delta \ln c_{t+1}$ is constant (i.e., independent of time). So the Euler Equation reduces to:

$$\Delta \ln c_{t+1} = \text{constant} + \frac{1}{\gamma}(E_t r_{t+1} - \rho) + \varepsilon_{t+1}$$

When we replace the precautionary term with a constant, we are effectively ignorning its effect (since it is no longer separately identified from the other constant term: $\frac{\rho}{\gamma}$).

Hundreds of papers have estimated a linearized Euler Equation:

$$\Delta \ln c_{t+1} = \text{constant} + \frac{1}{\gamma} E_t r_{t+1} + \beta X_t + \varepsilon_{t+1}$$

The principal goals of these regressions are twofold:

1. Estimate $\frac{1}{\gamma}$, the elasticity of intertemporal substitution (EIS) $= \frac{\partial \Delta \ln c_{t+1}}{\partial E_t r_{t+1}}$. For example, see Hall (1988).

    - For this model, the EIS is the inverse of the CRRA.

2. Test the orthogonality restriction:

$$\{\Omega_t \equiv \text{information set at date } t\} \perp \varepsilon_{t+1}.$$

- In other words, test the restriction that information available at time t does **not** predict consumption growth in the following regression

$$\Delta \ln c_{t+1} = \text{constant} + \frac{1}{\gamma} E_t r_{t+1} + \beta X_t + \varepsilon_{t+1}.$$

- For example, does the date t expectation of income growth, $E_t \Delta \ln Y_{t+1}$, predict date $t+1$ consumption growth?

$$\Delta \ln c_{t+1} = \text{constant} + \frac{1}{\gamma} E_t r_{t+1} + \alpha E_t \Delta \ln Y_{t+1} + \varepsilon_{t+1}$$

$\hat{\alpha} \in [0.1, 0.35]$, so $E_t \Delta \ln Y_{t+1}$ covaries with $\Delta \ln c_{t+1}$ (e.g., Campbell and Mankiw 1989, Shea 1995, Shapiro 2005, Parker and Broda 2014, Gelman, Kariv, Shapiro, Silverman, and Tadelis 2015, Ganong and Noel 2018).

- Orthogonality restriction is violated: information at date t predicts consumption growth from t to $t+1$.

- In other words, the assumptions (1) the Euler Equation is true, (2) the utility function is in the CRRA class, (3) the linearization is accurate, and (4) $V_t \Delta \ln c_{t+1}$ is constant, are jointly rejected.

Related literature estimates the marginal propensity to consume non-durables (MPC) out of wealth "windfalls."

$$c_{t+1} = \text{constant} + \alpha w_{t+1} + \varepsilon_{t+1}$$

$\hat{\alpha} \in [0.1, 0.35]$, so MPC is much higher than classical models would lead us to expect (e.g., see Havranek and Sokolova 2020, Ganong, Jones, Noel, Greig, Farrell, and Wheat 2020, Parker et al 2013, Kueng 2018, Fagereng et al 2019). Note that marginal propensity for expenditure (MPX) is about three times as high as $\hat{\alpha}$ due to expenditures on durables (e.g., Laibson, Maxted, and Moll 2022).

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A note on Shea's methodology (for estimating $E_t \Delta \ln Y_{t+1}$)

1. Assign respondents to unions with national or regional bargaining
   - national: trucking, postal service, railroads
   - regional: lumber in Pacific Northwest, shipping on East Coast
2. Assign respondents to dominant local employer
   - automobile worker living in Genesee County, MI (GM's Flint plant)