Topics in Heterogeneous Agent Macro: The New Keynesian Model

Lecture 8

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Overview

- New Keynesian model = RBC model with (i) monopolistic competition and (ii) sticky prices / wages

- References: Gali (2008), Woodford (2003), Clarida-Gali-Gertler (1999), Gali-Monacelli (2005)

- Closely follow the exposition in Gali (2008) except:
  - Continuous time, with $t \in [0, \infty)$
  - No linearization

- Abstract from aggregate uncertainty: focus on one-time, unanticipated shocks under perfect foresight (demand, supply and cost-push shocks)

- Heterogeneous Agent New Keynesian (HANK) model: replace representative household block

Model

Households

- Economy populated by a representative household with preferences over consumption C_t and hours worked N_t
- Household's lifetime utility defined as

$$V_0 = \max_{\{C_t, N_t\}_{t \geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\int_0^t \rho_s ds} U(C_t, N_t)\, dt,$$

- ρ_t denotes the time-varying discount rate
- Households face the budget constraint

$$\dot{B}_t = r_t B_t + w_t N_t + T_t - C_t$$

- Notation: B_t stock of bonds, r_t real interest rate, w_t real wage rate, T_t lump-sum rebate

Households

- Household's problem characterized by two first-order necessary conditions
- Optimal labor supply ("labor-leisure condition"):

$$w_t = -\frac{U_{N,t}}{U_{C,t}} \tag{1}$$

- Notation: $U_{x,t}$ is a shorthand for $\frac{\partial}{\partial x} U_t(\cdot)$
- Economics: households always supply labor until the real wage is equalized with the private marginal rate of substitution between consumption and labor
- Optimal consumption-savings ("Euler equation"):

$$\frac{\dot{U}_{C,t}}{U_{C,t}} = \rho_t - r_t. \tag{2}$$

Production: overview

- Two types of firms: retailer and $j \in [0, 1]$ intermediate input producers ("firms")
- Firms sell intermediate varieties to retailer who bundles them up to sell homogeneous final good to household. This is where we get a demand function for each good j
- Firms are monopolistically competitive and face dynamic pricing decision subject to adjustment costs

Production: retailer

- Retailer operates a CES aggregation technology

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon}{\epsilon - 1}}$$

- ϵ_t is potentially time-varying elasticity of substitution between varieties $Y_{j,t}$

- Retailer sells final consumption good at the consumer price index P_t and purchases intermediate varieties at prices $P_{j,t}$

- Profit maximization implies the demand functions

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\epsilon_t} Y_t \tag{3}$$

Production: (intermediate) firms

- Firms produce intermediate varieties with the linear production function

$$Y_{j,t} = A_t N_{j,t}$$

- A_t is aggregate productivity and $N_{j,t}$ firm j's labor demand
- Firm j sells at price $P_{j,t}$, profit = revenue net of operating expenses

$$\Pi_{j,t} = P_{j,t} Y_{j,t} - (1 - \tau^L) W_t N_{j,t}$$

- As in Gali (2008), we introduce employment subsidy τ^L
- Firms maximize NPV of future profit streams, discounted at nominal interest rate (representative household's SDF)

Firms' dynamic pricing problem

- Two ways to model price stickiness: Calvo and Rotemberg. Here: firms pay quadratic Rotemberg adjustment cost $\frac{\delta}{2}\pi_{j,t}^2 P_t Y_t$ to adjust nominal price

- Firm problem:

$$\max_{\{\pi_{j,t}, N_{j,t}\}_{t\geq 0}} \int_0^\infty e^{-\int_0^t i_s ds}\left(P_{j,t}Y_{j,t} - (1-\tau^L)W_t N_{j,t} - \frac{\delta}{2}\pi_{j,t}^2 P_t\right)dt, \tag{4}$$

  s.t. $\dot{P}_{j,t} = P_{j,t}\pi_{j,t}$, taking as given $\{W_t, Y_t, P_t\}_{t\geq 0}$ and initial condition $P_{j,0}$

- Any two firms j and j' with same initial price $P_{j,0} = P_{j',0}$ adopt identical inflation and production policies $\implies$ initializing economy with symmetric initial price distribution $P_{j,0} = P_{j',0}$ for all $j' \in [0,1]$ implies firms remain symmetric ex-post

- Symmetric firms is a big advantage of Rotemberg adjustment cost relative to Calvo (implies non-trivial distribution of prices and thus price dispersion as a state variable)

Derivation of Phillips curve

- Firm's problem associated with Lagrangian

$$L = \int_0^\infty e^{-\int_0^t \rho_s ds} \left(P_{j,t}^{1-\epsilon_t} P_t^{\epsilon_t} Y_t - \frac{W_t}{A_t} P_{j,t}^{-\epsilon_t} P_t^{\epsilon_t} Y_t - \frac{\delta}{2} \pi_{j,t}^2 X_t + \lambda_{j,t} \left[-\dot{P}_{j,t} + P_{j,t} \pi_{j,t} \right] \right) dt,$$

where we substituted in for the retailer's demand function

- In language of optimal control theory: $P_{j,t}$ is a state variable and $\pi_{j,t}$ a control variable
- Denote by X_t the scale of the adjustment cost ($\implies$ modeling trick to get nice solution)
- Integrating by parts:

$$\begin{aligned}\int_0^\infty e^{-\int_0^t \rho_s ds} \lambda_{j,t} \dot{P}_{j,t} dt &= e^{-\int_0^t \rho_s ds} \lambda_{j,t} P_{j,t} \Big|_0^\infty - \int_0^\infty \frac{d}{dt} \left(e^{-\int_0^t \rho_s ds} \lambda_{j,t} \right) P_{j,t} dt \\ &= -\lambda_{j,0} P_{j,0} + \int_0^\infty e^{-\int_0^t \rho_s ds} \rho_t \lambda_{j,t} P_{j,t} dt - \int_0^\infty e^{-\int_0^t \rho_s ds} \dot{\lambda}_{j,t} P_{j,t} dt\end{aligned}$$

where we assume the transversality condition $\lim_{t\to\infty} e^{-\int_0^t \rho_s ds} \lambda_t P_{j,t} = 0$

Derivation of Phillips curve

- Lagrangian therefore becomes

$$L = \int_0^\infty e^{-\int_0^t \rho_s ds} \left(P_{j,t}^{1-\epsilon_t} P_t^{\epsilon_t} Y_t - \frac{W_t}{A_t} P_{j,t}^{-\epsilon_t} P_t^{\epsilon_t} Y_t - \frac{\delta}{2} \pi_{j,t}^2 X_t + \lambda_{j,t} P_{j,t} \pi_{j,t} \right.$$
$$\left. + \dot{\lambda}_{j,t} P_{j,t} - \rho_t \lambda_{j,t} P_{j,t} \right) dt + \lambda_{j,0} P_{j,0}.$$

- Recall: firm takes as given all macroeconomic aggregates as well as initial price $P_{j,0}$
- By the fundamental lemma of the calculus of variations, the following necessary conditions must hold for any optimal policy:

$$0 = (1-\epsilon_t) P_{j,t}^{-\epsilon_t} P_t^{\epsilon_t} Y_t + \epsilon \frac{W_t}{A_t} P_{j,t}^{-\epsilon_t - 1} P_t^{\epsilon_t} Y_t + \lambda_{j,t} \pi_{j,t} + \dot{\lambda}_{j,t} - \rho_t \lambda_{j,t}$$
$$0 = -\delta \pi_{j,t} X_t + \lambda_{j,t} P_{j,t},$$

as well as the initial condition for the multiplier

$$\lambda_{j,0} = 0.$$

Derivation of Phillips curve

- Differentiate $\lambda_{j,t} P_{j,t} = \delta \pi_{j,t} X_t$ to get:

$$\dot{\lambda}_{j,t} P_{j,t} = \delta \dot{\pi}_{j,t} X_t - \lambda_{j,t} P_{j,t} \pi_{j,t} + \delta \pi_{j,t} \dot{X}_t$$

- Plug into FOC for $P_{j,t}$:

$$\begin{aligned} 0 &= (1-\epsilon_t) P_{j,t}^{1-\epsilon_t} P_t^{\epsilon_t} Y_t + \epsilon \frac{W_t}{A_t} P_{j,t}^{-\epsilon_t} P_t^{\epsilon_t} Y_t + \lambda_{j,t} P_{j,t} (\pi_{j,t} - \rho_t) + \dot{\lambda}_{j,t} P_{j,t} \\ &= (1-\epsilon_t) P_{j,t}^{1-\epsilon_t} P_t^{\epsilon_t} Y_t + \epsilon \frac{W_t}{A_t} P_{j,t}^{-\epsilon_t} P_t^{\epsilon_t} Y_t + \delta \pi_{j,t} X_t (\pi_{j,t} - \rho_t) + \delta \dot{\pi}_{j,t} X_t - \lambda_{j,t} P_{j,t} \pi_{j,t} + \delta \pi_{j,t} X \end{aligned}$$

- Restrict attention to symmetric equilibrium with $P_{j,t} = P_t$:

$$0 = (1-\epsilon_t) P_t Y_t + \epsilon \frac{W_t}{A_t} Y_t + \delta \pi_t X_t (\pi_t - \rho_t) + \delta \dot{\pi}_t X_t - \delta \pi_t^2 X_t + \delta \pi_t \dot{X}_t.$$

- Now use modeling trick, by choosing scale $X_t = P_t Y_t$:

$$0 = \frac{\epsilon_t}{\delta} \left[\frac{w_t}{A_t} - \frac{\epsilon_t - 1}{\epsilon_t} \right] + \pi_t (\pi_t - \rho_t) + \dot{\pi}_t + \pi_t \frac{\dot{Y}_t}{Y_t}$$

Summary of Production

- New Keynesian Phillips curve (NKPC) for aggregate consumer price inflation:

$$\dot{\pi}_t = \pi_t \left(i_t - \pi_t - \frac{\dot{Y}_t}{Y_t} \right) - \frac{\epsilon_t}{\delta} \left[\frac{(1-\tau^L)w_t}{A_t} - \frac{\epsilon_t - 1}{\epsilon_t} \right]$$

- Aggregate production function In a symmetric equilibrium:

$$Y_t = A_t N_t,$$

- Aggregate corporate sector profits:

$$\Pi_t = Y_t - (1-\tau^L) w_t N_t = (1 - mc_t) Y_t$$

where we define $mc_t = (1-\tau^L)\frac{w_t}{A_t}$ as real marginal cost

Government

- Fiscal authority is stylized: pays for employment subsidy with lump-sum tax
- Corporate profits rebated to household, so in equilibrium the lump-sum rebate is

$$T_t = \Pi_t - \tau^L w_t N_t$$

- Monetary policy sets path of nominal interest rates $\{i_t\}_{t \geq 0}$
- Given path of nominal rates and inflation, real interest rate follows from Fisher relation

$$r_t = i_t - \pi_t \tag{5}$$

- Closing the model requires either (i) interest rate rule or (ii) letting planner choose $\{i_t\}$ optimally

Market clearing and equilibrium

- Labor market clearing already implicit in our notation
- Goods market clearing:
$$Y_t = C_t \tag{6}$$
- Bond market clearing:
$$B_t = 0 \tag{7}$$

Definition. (Competitve Equilibrium) *Given paths of the nominal interest rates $\{i_t\}$ and shocks $\{A_t, \rho_t, \epsilon_t\}$ as well as a symmetric initial price distribution, competitive equilibrium comprises paths of prices $\{\pi_t, r_t, w_t\}$ and allocations $\{Y_t, N_t, C_t\}$ such that (i) households and firms optimize, and (ii) markets clear.*

Summary of equilibrium conditions

Assume isoelastic (CRRA) preferences with $U(C, N) = \log C_t - \frac{1}{1+\eta} N_t^{1+\eta}$:

$$
\begin{aligned}
\frac{\dot{C}_t}{C_t} &= i_t - \pi_t - \rho_t \\
Y_t &= C_t \\
Y_t &= A_t N_t \\
w_t &= N_t^{\eta} C_t \\
\dot{\pi}_t &= \pi_t \left(i_t - \pi_t - \frac{\dot{Y}_t}{Y_t} \right) - \frac{\epsilon_t}{\delta} \left[\frac{(1-\tau^L) w_t}{A_t} - \frac{\epsilon_t - 1}{\epsilon_t} \right].
\end{aligned}
$$

The 3-equation representation

Lemma. (Non-Linear Representation of NK Model) Given paths for the nominal interest rate $\{i_t\}$ and shocks $\{A_t, \rho_t, \epsilon_t\}$, and assuming a symmetric initial price distribution, output and inflation satisfy the dynamic IS equation

$$\frac{\dot{Y}_t}{Y_t} = i_t - \pi_t - \rho_t \tag{8}$$

and the NKPC

$$\dot{\pi}_t = \rho_t \pi_t - \frac{\epsilon_t}{\delta}\left[(1-\tau^L)\left(\frac{Y_t}{A_t}\right)^{1+\eta} - \frac{\epsilon_t - 1}{\epsilon_t}\right]. \tag{9}$$

- This is an exact, non-linear representation
- Standard 3-equation model adds Taylor rule

Steady state

- Consider constant shock realizations $\{A_{\text{ss}}, \rho_{\text{ss}}, \epsilon_{\text{ss}}\} = \{A, \rho, \epsilon\}$
- Given a constant long-run inflation rate π_{ss}, the associated steady state is given by

$$Y_{\text{ss}} = A\left[\frac{1}{1-\tau^L}\left(\rho\frac{\delta}{\epsilon}\pi_{\text{ss}} + \frac{\epsilon-1}{\epsilon}\right)\right]^{\frac{1}{1+\eta}}$$

$$r_{\text{ss}} = \rho$$

- Rest of the allocation is given by $C_{\text{ss}} = Y_{\text{ss}} = AN_{\text{ss}}$, and prices by $i_{\text{ss}} = \pi_{\text{ss}} + \rho$, and

$$w_{\text{ss}} = A\left[\frac{1}{1-\tau^L}\left(\rho\frac{\delta}{\epsilon}\pi_{\text{ss}} + \frac{\epsilon-1}{\epsilon}\right)\right].$$

Flexible price allocation

- Flexible price limit ($\delta \to 0$) implies
$$\frac{(1-\tau^L)w_t}{A_t} = \frac{\epsilon_t - 1}{\epsilon_t}$$
Economics: firms employ labor until wages are equal to a markdown on marginal product of labor net of the employment subsidy

- Using ˜ to denote the flexprice allocation, this implies
$$\tilde{Y}_t = A_t \left(\frac{1}{1-\tau^L} \frac{\epsilon_t - 1}{\epsilon_t} \right)^{\frac{1}{1+\eta}} \tag{10}$$

- Refer to $\tilde{Y}_t$ as *natural output*. From IS equation, naturate rate of interest:
$$\tilde{r}_t = \rho_t + \frac{\dot{\tilde{Y}}_t}{\tilde{Y}_t}. \tag{11}$$

- First-best: At a first-best allocation, the marginal rate of substitution between consumption and labor must be equalized with the marginal rate of transformation.

Wage Rigidity

Overview

- Most HANK papers use wage instead of price rigidity
- Standard NK model counter-factually predicts counter-cyclical profits
- Assumptions on the rebate of these profits across households has large quantitative implications in HANK models
- Also, standard NK model with frictionless labor supply choice predicts counter-factually large marginal propensities to earn (via income effect on labor supply)
- Sticky wages fix both of these problems

Labor market

- Households supply labor to each of $k \in [0,1]$ unions, denoted $n_{k,t}dk$
- Union k differentiates labor inputs and sells labor variety $N_{k,t}$ to labor packer at $W_{k,t}$
- The labor packer operates a CES aggregation technology,

$$N_t = \left(\int_0^1 N_{k,t}^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}}.$$

- Labor packer sells aggregate labor N_t to firms at aggregate nominal wage rate W_t
- Implies demand functions for labor varieties and nominal wage index:

$$N_{k,t} = \left(\frac{W_{k,t}}{W_t} \right)^{-\epsilon} N_t \qquad W_t = \left[\int W_{k,t}^{1-\epsilon} dk \right]^{\frac{1}{1-\epsilon}}$$

Union problem

- Unions face Rotemberg adjustment cost to adjust nominal wages
- Key assumption: unions ration labor so $N_{k,t} = n_{k,t}$ (trivial with RA)
- Union problem: choose path of wages to maximize stakeholder value

$$\max_{\pi^w_{k,t}, W_{k,t}, N_{k,t}} \int_0^\infty e^{-\rho t} \left[u(C_t) - v\left(\int_0^1 N_{k,t} dk \right) - \frac{\delta}{2} \int \left(\pi^w_{k,t} \right)^2 dk \right] dt$$

taking as given household consumption behavior and s.t.

$$\dot{W}_{k,t} = \pi^w_{k,t} W_{k,t}$$

$$N_{k,t} = \left(\frac{W_{k,t}}{W_t} \right)^{-\epsilon} N_t$$

Summary of labor market and production

- New Keynesian wage Phillips curve:

$$\dot{\pi}_t^w = \rho \pi_t^w + \frac{\epsilon}{\delta}\left[(1+\tau^L)\frac{\epsilon-1}{\epsilon} w_t u'(C_t) - v'(N_t)\right] N_t$$

- Production: representative firm produces the final consumption good

$$Y_t = A_t N_t$$

- Assume perfect competition and flexible prices:

$$w_t = \frac{W_t}{P_t} = A_t$$

Equilibrium representation

- Definition of competitive equilibrium almost identical
- Can summarize competitive equilibrium conditions as

$$\begin{aligned}
\frac{\dot{U}_{C,t}}{U_{C,t}} &= \rho_t - i_t + \pi_t^w - \frac{\dot{A}_t}{A_t} \\
\dot{\pi}_t^w &= \rho_t \pi_t^w + \frac{\epsilon_t}{\delta}\left[(1+\tau^L)\frac{\epsilon_t - 1}{\epsilon_t} A_t u'(C_t) - v'(N_t)\right] N_t \\
C_t &= A_t N_t - G_t
\end{aligned}$$

Assume isoelastic (CRRA) preferences with $u(c) = \frac{1}{1-\gamma} c^{1-\gamma}$ and $v(n) = \frac{1}{1+\eta} n^{1+\eta}$

Equilibrium representation

- Set $G_t = 0$, assume symmetric initial wage distribution
- Given paths for the nominal interest rate $\{i_t\}$ and shocks $\{A_t, \rho_t, \epsilon_t\}$, we have the IS

$$\frac{\dot{Y}_t}{Y_t} = \frac{i_t - \rho_t - \pi_t^w + \frac{\dot{A}_t}{A_t}}{\gamma} \tag{IS}$$

  and the NKPC

$$\dot{\pi}_t^w = \rho_t \pi_t^w + \frac{\epsilon_t}{\delta}\left[(1+\tau^L)\frac{\epsilon_t - 1}{\epsilon_t} Y_t^{1-\gamma} - \left(\frac{Y_t}{A_t}\right)^{1+\eta}\right]. \tag{NKPC}$$

Efficiency

- Efficiency requires:
  1. the MRS between consumption and labor must be equalized with the MRT
  2. households' MRS must be equalized across all histories

- Since households are always on their labor supply curve, this requires that for all t

$$w_t^{\text{FB}} = A_t \qquad\qquad -\frac{U_{N,t}^{\text{FB}}}{U_{C,t}^{\text{FB}}} = w_t^{\text{FB}}$$

- Alternatively, we require

$$(C_t^{\text{FB}})^\gamma (N_t^{\text{FB}})^\eta = A_t \quad \Longrightarrow \quad (Y_t^{\text{FB}})^\gamma (Y_t^{\text{FB}})^\eta = A_t^{1+\eta} \quad \Longrightarrow \quad Y_t^{\text{FB}} = A_t^{\frac{1+\eta}{\gamma+\eta}}$$

- In the absence of cost-push shocks, the flexwage allocation with employment subsidy $\tau^L = \frac{1}{\epsilon-1}$ is efficient

Keynesian Cross

Overview

- What do different Keynesian models say about AD amplification?
- Start with the simplest "old Keynesian" static IS-LM model:
  IS block:

$$
\begin{aligned}
Y &= C + I + G \\
C &= \bar{C} + \text{MPC}(Y - T) - C_r r \\
I &= \bar{I} - I_r r
\end{aligned}
$$

  LM equation:

$$
\frac{M}{P} = l_Y Y - l_r r
$$

Overview

- Assume a flat LM curve $l_Y = 0$, then:

$$Y = \bar{C} + \bar{I} + G + \text{MPC}(Y - T) + (C_r + I_r)\frac{1}{l_r}\frac{M}{P}$$

- This implies

$$\begin{aligned}(1 - \text{MPC})Y &= \bar{C} + \bar{I} + G - \text{MPC}T + (C_r + I_r)\frac{1}{l_r}\frac{M}{P} \\ Y &= \frac{1}{1 - \text{MPC}}\left(\bar{C} + \bar{I} + G - \text{MPC}T + (C_r + I_r)\frac{1}{l_r}\frac{M}{P}\right) \\ dY &= \frac{1}{1 - \text{MPC}}dG - \frac{\text{MPC}}{1 - \text{MPC}}dT\end{aligned}$$

  assuming that M does not respond

- $\implies$ (old static) Keynesian Cross

Modern Keynesian Cross

- All New Keynesian models admit *dynamic* IS-LM representations
- Going back to RANK model:

$$V_0 = \max_{\{C_t, N_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\int_0^t \rho_s ds} U(C_t, N_t)\, dt$$
$$s.t. \quad \dot{B}_t = r_t B_t + e_t - C_t$$

- Following Heathcote-Storesletten-Violante (2017), assume tax retention function

$$e_t = \tau_t \Big(w_t N_t + \Pi_t \Big)^{1-\lambda} = \tau_t Y_t^{1-\lambda} \equiv Y_t - T_t$$

- Define government's tax revenue as $T_t = Y_t - Z_t$ (total minus private income)
- This implies an *aggregate consumption function*

$$C_t = \mathcal{C}_t \left(\left\{ Y_s - T_s \right\}_{s\geq 0}, \left\{ r_s \right\}_{s\geq t} \right)$$

Modern Keynesian Cross

- Plugging into the goods market clearing condition

$$Y_t = \mathcal{C}_t(\{Y_s - T_s\}_{s\geq 0}, \{r_s\}_{s\geq t}) + G_t$$

- For simplicity: discretize time using appropriate FD scheme

- Assume central bank adopts rule $dr_t = 0$ (similar to holding $\frac{M}{P}$ fixed):

$$dY_0 = \sum_{n=1}^{\infty} \frac{\partial \mathcal{C}_0}{\partial Y_{t_n}}(dY_{t_n} - dT_{t_n}) + dG_0$$
$$d\boldsymbol{Y} = \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) + d\boldsymbol{G}$$

- If invertible:

$$d\boldsymbol{Y} = (\boldsymbol{I} - \boldsymbol{M})^{-1}d\boldsymbol{G} - (\boldsymbol{I} - \boldsymbol{M})^{-1}\boldsymbol{M}d\boldsymbol{T}$$

HANK

Overview

- One-asset HANK: useful for analytical characterizations
- But not ideal for quantitative work: cannot match joint distribution of assets and MPCs in the data (see Kaplan-Moll-Violante 2018)
- Labor market, production, government and market clearing as in RANK
- What's new: replace representative household with Huggett model

Households

Preferences: Households' private lifetime utility is

$$V_0(\cdot) = \max \mathbb{E}_0 \int_0^\infty e^{-\int_0^t \boldsymbol{\rho}_s ds} \underbrace{U_t(c_t, n_t)}_{\text{Instantaneous Utility Flow}} dt$$

Budget constraint: $\dot{a}_t = r_t a_t + e_t - c_t$

- Households trade a real bond a_t, borrowing constraint: $a_t \geq \underline{a}$
- Earnings following HSV2017: $e_t = \tau_t (w_t z_t n_t + z_t \Pi_t)^{1-\lambda}$
- Idiosyncratic labor productivity z_t: two-state Markov process

Cross-sectional distribution: denote joint density $g_t(a, z)$

- Heterogeneity in income and wealth: identify households by their states (a, z)
- Aggregation: $C_t = \iint c_t(a, z) g_t(a, z) \, da \, dz$

Labor Markets and Production

Off-the-shelf model of **nominal wage rigidity**: Erceg et al. (2000), Auclert-Rognlie-Straub (2023)

- Labor rationing: households work same hours, $n_t = N_t$
- Unions pay Rotemberg cost to adjust wages $\rightarrow$ passed to households as utility cost:

$$U_t(c_t, N_t) = u(c_t) - v(N_t) - \frac{\delta}{2}(\pi_t^w)^2$$

- New Keynesian **wage Phillips curve**:

$$\dot{\pi}_t^w = \boldsymbol{\rho}_t \pi_t^w + \underbrace{\frac{\boldsymbol{\epsilon}_t}{\delta}}_{\text{NKPC slope}} \iint N_t \left(\underbrace{\frac{\boldsymbol{\epsilon}_t - 1}{\boldsymbol{\epsilon}_t}}_{\substack{\text{Desired}\\ \text{Markup}}} \underbrace{(1+\tau^L)}_{\substack{\text{Employment}\\ \text{Subsidy}}} z\, w_t\, \underbrace{u'(c_t(a,z)) \; - \; v'(N_t)}_{\text{Individual MRS}} \right) g_t(a,z)\, da\, dz$$

Production: representative firm produces consumption good $Y_t = \boldsymbol{A}_t N_t$

- Perfect competition + flexible prices: $\frac{W_t}{P_t} = w_t = \boldsymbol{A}_t$ (wages = MRT $\neq$ MRS)

Remaining Model Details

Government

- Define government's tax revenue as: $T_t = Y_t - \iint e_t(a,z) g_t(a,z)\, da\, dz$
- Policy instrument: path of interest rates $\{i_t\}_{t \geq 0}$
- Fisher relation: $r_t = i_t - \pi_t$, where CPI inflation is $\pi_t = \pi_t^w - \frac{\dot{\boldsymbol{A}}_t}{\boldsymbol{A}_t}$

Market clearing

Goods: $$Y_t = \iint c_t(a,z) g_t(a,z)\, da\, dz + G_t$$

Bonds: $$0 = B_t = \iint a g_t(a,z)\, da\, dz$$

Equilibrium

Definition. *Given initial density* $g_0(a,z)$ *and sequences of monetary policy* $\{i_t\}_{t>0}$ *and shocks* $\{\boldsymbol{A}_t, \boldsymbol{\rho}_t, \boldsymbol{\epsilon}_t\}_{t>0}$, *an equilibrium is defined as paths for*

(i) prices $\{\pi_t^w, \pi_t, w_t, r_t\}_{t>0}$

(ii) aggregates $\{Y_t, N_t, C_t\}_{t>0}$

(iii) individual allocation rules $\{c_t(a,z)\}_{t>0}$

(iv) densities $\{g_t(a,z)\}_{t>0}$

such that households optimize, unions and firms optimize, labor is rationed, markets for goods and bonds clear, and densities $\{g_t(a,z)\}_{t>0}$ *are consistent with household behavior.*

Sources of suboptimality:

(**1**) Monopolistic competition

(**2**) Nominal rigidity

(**3**) Labor rationing

(**4**) Incomplete markets

Aggregate Consumption Function

- Total income split into post-tax private income and tax revenue: $Y_t = Z_t + T_t$
- Post-tax private income: $Z_t = \iint e_t(a,z)g_t(a,z)\,da\,dz = \tau_t Y_t^{1-\lambda} \iint z^{1-\lambda} g_t(a,z)\,da\,dz$
- Given household's post-tax income is:

$$e_t(a,z) = z^{1-\lambda}\tau_t Y_t^{1-\lambda} = \frac{z^{1-\lambda}}{\iint \tilde{z}^{1-\lambda} g_t(a,\tilde{z})\,da\,d\tilde{z}} Z_t$$

- Since $\frac{z^{1-\lambda}}{\iint \tilde{z}^{1-\lambda} g_t(a,\tilde{z})\,da\,d\tilde{z}}$ is constant: $e_t(a,z) = e(z; Z_t)$

Aggregate Consumption Function

- Household budget constraint therefore given by:

$$\dot{a}_t = r_t a_t + \frac{z^{1-\lambda}}{\iint \tilde{z}^{1-\lambda} g_t(a, \tilde{z})\, da\, d\tilde{z}} Z_t - c_t$$

- Dynamic programming implies:

$$c_t(a, z) = c\left(a, z; \left\{Z_s, r_s\right\}_{s \geq t}\right)$$

- Aggregating consumption function:

$$C_t = \iint c\left(a, z; \left\{Z_s, r_s\right\}_{s \geq t}\right) g_t(a, z)\, da\, dz = \mathcal{C}_t\left(\left\{Z_s, r_s\right\}_{s \geq 0}\right)$$

Intertemporal Keynesian Cross in HANK

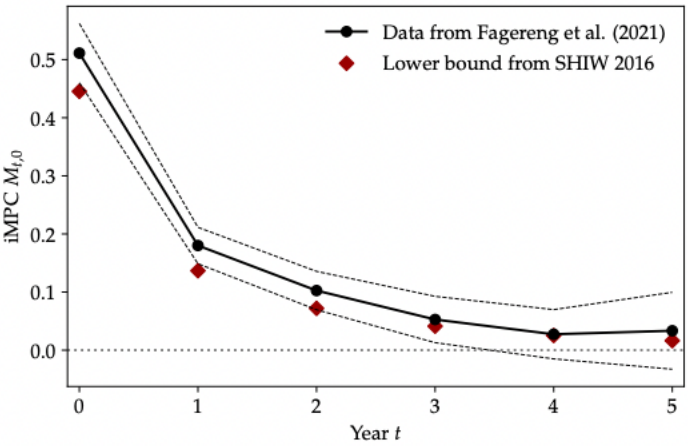
Discretize again in time for simplicity:

$$Y_0 = \mathcal{C}_0\left(\left\{Y_s - T_s, r_s\right\}_{s\geq 0}\right) + G_0$$
$$dY_0 = dG_0 + \sum_{n=1}^{\infty}\left[\frac{\partial \mathcal{C}_0}{\partial Y_{t_n}}(dY_{t_n} - dT_{t_n}) + \frac{\partial \mathcal{C}_0}{\partial r_{t_n}} dr_{t_n}\right]$$

Proposition. The IKC is given by

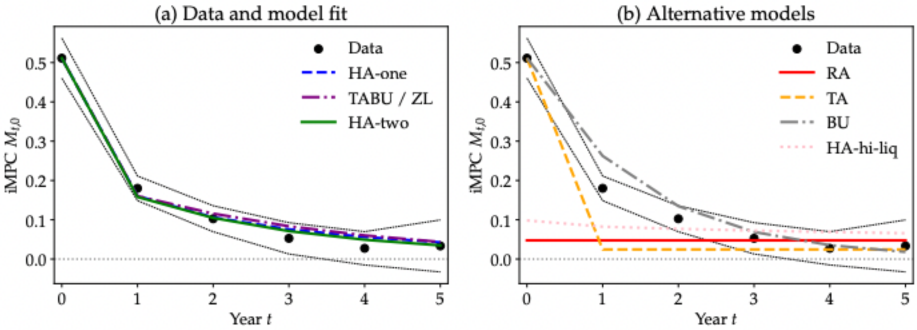
$$d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{dT} + \boldsymbol{M}\, d\boldsymbol{Y} + \boldsymbol{M}^r d\boldsymbol{r}$$

Figure 1: iMPCs in the Norwegian and Italian data



Source: Auclert-Rognlie-Straub (2023)

Figure 2: iMPCs in the Norwegian data and several models



Notes: All models are calibrated to match $r = 0.05$. RA does not have any other free parameter. The single free parameter in BU (λ), TA (μ), HA-one (A/Z) and HA-two (ν) is calibrated to match $M_{00} = 0.51$. The additional free parameter in TABU and ZL (μ) is calibrated to match $M_{10} = 0.16$ (its value in the HA-one model). The HA-two and HA-hi-liq models are calibrated to an aggregate ratio of assets to post-tax income of $A/Z = 6.29$, its value in the model with capital in section 7.

Source: Auclert-Rognlie-Straub (2023)