Ex:

$\dot{s}_1 = -s_1 + 2s_2$

$\dot{s}_2 = s_2$

$s(t) = e^{At}$

$A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

Closed Form

Eigenvalue: $\begin{vmatrix} -1-\lambda & 2 \\ 0 & 1-\lambda \end{vmatrix} = (1-\lambda)\cdot(1-\lambda) - 0\cdot 2 = \lambda^2 - 1$

$\lambda = 1 \lor \lambda = -1$

diagonal $D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (as also called J in the Lecture)

Eigenvectors: $\underline{\lambda = 1}$ $\begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{matrix} 1 \\ 1 \end{matrix}$

$\underline{\lambda = -1}$ $\begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

transformation Matrix $P = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$P \cdot D \cdot P^{-1} = A$

$s(t) = P \cdot e^{Dt} \cdot P^{-1} \cdot s_0$

$e^{D} = \begin{bmatrix} e & 0 \\ 0 & \frac{1}{e} \end{bmatrix}$

Unique Property of sinc:

- 1 at 0 and 0 at every other frequency point.
- Sum of all sincs at frequency points = constant 1.

Problem B2: Exam Task ?

a) ...

b) use of Controllability Matrix
   + compute Rank of it (number of rows which are linearly independent)

c) Consider Eigenvalues of $A - BF$

$\dot{s} = (A - BF)s + BFR$ closed loop Controller

→ calculate using Matlab / Wolfram A etc.

$$F = \begin{bmatrix} \frac{1}{6} & \frac{13}{3} \end{bmatrix}$$

Eigenvalues $-1+i$, $-1-i$

$e^{i\varphi} = \cos(\varphi) + i \cdot \sin(\varphi)$

$e^{-1+i} = e^{-1} \cdot e^{i}$

$\boxed{B3}$

$\bar{O}(s) = Cs + D\underline{I}$ (only for systems in which the output does not depend on the input?)

$\bar{O}(\alpha s_0 + \beta s_1) = C(\alpha s_0 + \beta s_0)$