

## Embedded Systems Problem Set B

### Problem G1: Reactive Synthesis

a)

From time to time, she realizes that she is late:

$$\mathbf{GF} \text{ realize\_late}$$

She will eventually start speeding, but never twice in a row:

$$(\mathbf{F} \text{ speeding}) \wedge \mathbf{G}(\text{speeding} \rightarrow \mathbf{X}\neg\text{speeding})$$

When speeding in front of a police car, she has to go to jail some time in the future:

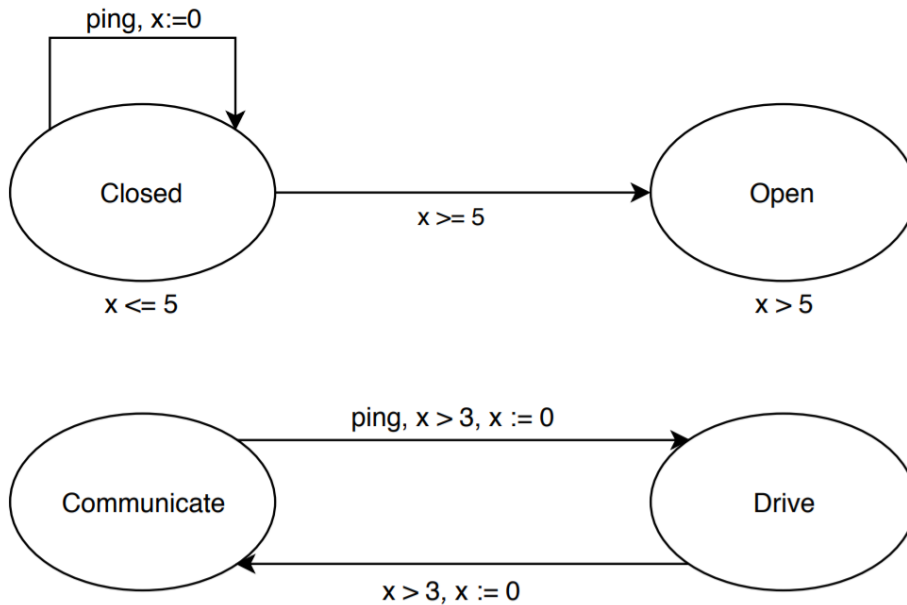
$$\mathbf{G}(\text{speeding} \wedge \text{near\_police\_car}) \rightarrow (\mathbf{F} \text{ in\_jail})$$

We get the complete requirements by conjoining the above:

$$(\mathbf{GF} r) \wedge ((\mathbf{F} s) \wedge \mathbf{G}(s \rightarrow \mathbf{X}\neg s)) \wedge (\mathbf{G}(s \wedge p) \rightarrow (\mathbf{F} j))$$

### Problem G2: Timed Synthesis

a)



b)

The train can switch modes every three seconds. A ping is send on a mode switch from communication to drive. The gate will open once it does not receive a ping for 5 seconds.

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This means that the train will be able to send one ping but has to wait for 6 seconds before it can send the next ping. By that time, the gate will have opened. Therefore, there is no winning strategy for the train.

**Problem G3: Parity Games**

**Player 0 wins from every state:**

We can see that in every infinite play, the color 4 will be seen infinitely often. From states 1 and 3, Player 0 will always go back to 4. Also Player 0 can (and should) always prevent a game from going through the 5 in the middle. Therefore, any  $x$  and  $y < 5$  will be winning plays for Player 0 from every state because the highest color will be 4 which is even.

For  $x = 5 \wedge y \leq 5$ , Player 0 cannot prevent plays from going through 5 because from the upper 4, Player 1 can go to  $x = 5$  and from the lower 4, Player 0 will go to  $y$  from where Player 0 either has to go to the 5 in the middle or to the upper 4 from where Player 1 will again choose to go to  $x = 5$ . In any case,  $x = 5$  will be visited infinitely often. Again, Player 0 will win from every state. The same holds for  $y = 5 \wedge x \leq 5$ .

The winning valuations of  $x$  and  $y$  are indicated with an ✓

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	0	1	2	3	4	5
0	✓	✓	✓	✓	✓	x
1	✓	✓	✓	✓	✓	x
2	✓	✓	✓	✓	✓	x
3	✓	✓	✓	✓	✓	x
4	✓	✓	✓	✓	✓	x
5	x	x	x	x	x	x

**Player 1 wins from every state:**

Obviously, valuation of  $x$  and  $y$  where Player 0 wins from every state, Player 1 cannot also win from every state. Therefore for every  $x < 5 \wedge y < 5$  Player 1 cannot win from every state. The question remains whether Player 1 can win from  $x < 5 \vee y < 5$ . In fact this is the case because Player 0 cannot prevent the play from going into one of the states with color 4 as explained above.

So, for every valuation of  $x$  and  $y$  one of the players wins from every state. The table for valuations where Player 1 wins from every state is the inverted table from above.

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$\begin{smallmatrix} x \\ y \end{smallmatrix}$	0	1	2	3	4	5
0	x	x	x	x	x	✓
1	x	x	x	x	x	✓
2	x	x	x	x	x	✓
3	x	x	x	x	x	✓
4	x	x	x	x	x	✓
5	✓	✓	✓	✓	✓	✓