

$$\boxed{7.2} \text{ e) } f(x, y) = 3x^2 - y^2$$

$$\varepsilon = 0,01$$

$$\frac{\partial^2 f}{\partial x^2} = 6$$

$$\frac{\partial^2 f}{\partial y^2} = -2$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 0$$

$$\frac{\partial f}{\partial x} = 6x$$

$$\frac{\partial f}{\partial y} = -2y$$

$$x = 5 \quad y = -1$$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \varepsilon \cdot \begin{pmatrix} \frac{\partial^2 f}{\partial x^2}(x, y) & \frac{\partial^2 f}{\partial x \partial y}(x, y) \\ \frac{\partial^2 f}{\partial y \partial x}(x, y) & \frac{\partial^2 f}{\partial y^2}(x, y) \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial f}{\partial x}(x, y) \\ \frac{\partial f}{\partial y}(x, y) \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} - 0,01 \cdot \begin{pmatrix} 6 & 0 \\ 0 & -2 \end{pmatrix} \cdot \begin{pmatrix} 30 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} - 0,01 \cdot \begin{pmatrix} 180 \\ 4 \end{pmatrix} = \begin{pmatrix} 3,2 \\ -1,04 \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 3,2 \\ -1,04 \end{pmatrix} - 0,01 \cdot \begin{pmatrix} 6 & 0 \\ 0 & -2 \end{pmatrix} \cdot \begin{pmatrix} 6 \cdot 3,2 \\ -2 \cdot (-1,04) \end{pmatrix} = \begin{pmatrix} 2,048 \\ -0,9984 \end{pmatrix}$$

following steps are analogue using Wolfram Alpha:

$$\begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1,31072 \\ -0,959464 \end{pmatrix} \quad \begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \begin{pmatrix} 0,834861 \\ -0,920125 \end{pmatrix} \quad \begin{pmatrix} x_5 \\ y_5 \end{pmatrix} = \begin{pmatrix} 0,536877 \\ -0,88332 \end{pmatrix}$$

$$f(x_4, y_4) \approx 1,26$$

$$f(x_5, y_5) \approx 0,08$$