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4.1 k-Means Clustering

a)

b)

c)

d)

4.2 Maximum Likelihood Estimation

1. There are 6 independent variables with poisson distribution.

$$L(\lambda) = P(\{X_1 = 2\} \cap \{X_2 = 3\} \cap \{X_3 = 0\} \cap \{X_4 = 2\} \cap \{X_5 = 1\} \cap \{X_6 = 5\})$$

$$\Rightarrow^{independency} L(\lambda) = P(X_1 = 2) * P(X_2 = 3) * P(X_3 = 0) * P(X_4 = 2) * P(X_5 = 1) * P(X_6 = 5)$$

Remember: $P(X = x) = \frac{\lambda^x}{x!} * e^{-\lambda}$

$$L(\lambda) = \frac{\lambda^2}{2!} * e^{-\lambda} * \frac{\lambda^3}{3!} * e^{-\lambda} * \frac{\lambda^0}{0!} * e^{-\lambda} * \frac{\lambda^2}{2!} * e^{-\lambda} * \frac{\lambda^1}{1!} * e^{-\lambda} * \frac{\lambda^5}{5!} * e^{-\lambda}$$

$$\Leftrightarrow L(\lambda) = \frac{1}{2! * 3! * 0! * 2! * 1! * 5!} * \lambda^{13} * e^{-\lambda}$$

$$\frac{\delta L(\lambda)}{\delta \lambda} = \frac{1}{2880} * (13\lambda^{12} * e^{-8\lambda} - 8\lambda^{13} * e^{-8\lambda}) = 0$$

$$\Leftrightarrow -\frac{e^{-8\lambda} * \lambda^{12} * (8\lambda - 13)}{2280} = 0$$

$$\Rightarrow \hat{\lambda} = 0 \vee \hat{\lambda} = \frac{13}{8}$$

The likelihood function has a max at $\hat{\lambda} = \frac{13}{8}$, therefore the desired λ is $\lambda = \hat{\lambda}$.

2. $$P(X_7 = 2) = \frac{(\frac{13}{8})^2}{2!} * e^{-\frac{13}{8}} \approx 0.2599985$$

$P(X_7 = 2)$ is at about 26%.

4.3 Composite functions

$$f(x,y) = log(sin(xy))$$

Idea:
First order derivation:
Use chain rule with $f(x) = log(x)$ and $u = sin(xy)$

1. First order derivation:

   (a)

$$\frac{\delta}{\delta x}f(x,y) =^{Chain\ rule} \frac{\delta}{\delta u}(log(u)) * \frac{\delta}{\delta x}sin(xy) = \frac{\delta}{\delta u}\frac{ln(u)}{ln(10)} * \frac{\delta}{\delta x}sin(xy)$$

$$= \frac{1}{u * ln(10)} * \frac{\delta}{\delta x}sin(xy)) = \frac{1}{u * ln(10)} * y * cos(xy) = \frac{y * cos(xy)}{u * ln(10)}$$

   replace u with $sin(xy)$:

$$\frac{\delta}{\delta x}f(x,y) = \frac{y * cos(xy)}{sin(xy) * ln(10)}$$

   (b) $\frac{\delta}{\delta y}f(x,y)$ analogously

$$\Rightarrow \frac{\delta}{\delta y}f(x,y) = \frac{x * cos(xy)}{sin(xy) * ln(10)}$$

2. Second order derivation:

   (a)

$$\frac{\delta}{\delta xx}f(x,y) = \frac{\delta}{\delta x}\frac{y * cos(xy)}{sin(xy) * ln(10)}$$

$$= \frac{(\frac{\delta}{\delta x}y * cos(xy)) * sin(xy) * ln(10) - (\frac{\delta}{\delta x}sin(xy) * ln(10)) * y * cos(xy)}{sin^2(xy) * ln(10)^2}$$

$$= \frac{y^2 * (-sin(xy)) * sin(xy) * ln(10) - y^2 * cos(xy) * ln(10) * cos(xy)}{sin^2(xy) * ln(10)^2}$$

$$= \frac{y^2 * ln(10) * (-sin^2(xy) - cos^2(xy))}{sin^2(xy) * ln(10)^2} = -\frac{y^2}{sin^2(xy) * ln(10)}$$

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(b)

$$\frac{\delta}{\delta xy} f(x,y) = \frac{\delta}{\delta y} \frac{y * cos(xy)}{sin(xy) * ln(10)}$$

$$= \frac{(\frac{\delta}{\delta y} y * cos(xy)) * sin(xy) * ln(10) - (\frac{\delta}{\delta y} sin(xy) * ln(10)) * y * cos(xy)}{sin^2(xy) * ln(10)^2}$$

$$= \frac{cos(xy) * sin(xy) * ln(10) + xy * (-sin(xy)) * sin(xy) * ln(10) - xy * ln(10) * cos^2(xy)}{sin^2(xy) * ln(10)^2}$$

$$= \frac{cos(xy) * sin(xy) - xy * sin^2(xy) - xy * cos^2(xy)}{sin^2(xy) * ln(10)}$$

(c) $\frac{\delta}{\delta yy} f(x,y)$ analogously to $\frac{\delta}{\delta xy} f(x,y)$

$$\frac{\delta}{\delta yy} f(x,y) = \frac{-x^2}{sin^2(xy) * ln(10)}$$

(d) $\frac{\delta}{\delta yx} f(x,y)$ analogously to $\frac{\delta}{\delta xy} f(x,y)$

$$\frac{\delta}{\delta yx} f(x,y) = \frac{cos(xy) * sin(xy) - xy * sin^2(xy) - xy * cos^2(xy)}{sin^2(xy) * ln(10)}$$

4.4 Classification