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4.1 k-Means Clustering

a)

b)

c)

d)

4.2 Maximum Likelihood Estimation

1. There are 6 independent variables with poisson distribution.

$$L(\lambda) = P(\{X_1 = 2\} \cap \{X_2 = 3\} \cap \{X_3 = 0\} \cap \{X_4 = 2\} \cap \{X_5 = 1\} \cap \{X_6 = 5\})$$

$$\Rightarrow^{independency} L(\lambda) = P(X_1 = 2) * P(X_2 = 3) * P(X_3 = 0) * P(X_4 = 2) * P(X_5 = 1) * P(X_6 = 5)$$

Remember: $P(X = x) = \frac{\lambda^x}{x!} * e^{-\lambda}$

$$L(\lambda) = \frac{\lambda^2}{2!} * e^{-\lambda} * \frac{\lambda^3}{3!} * e^{-\lambda} * \frac{\lambda^0}{0!} * e^{-\lambda} * \frac{\lambda^2}{2!} * e^{-\lambda} * \frac{\lambda^1}{1!} * e^{-\lambda} * \frac{\lambda^5}{5!} * e^{-\lambda}$$

$$\Leftrightarrow L(\lambda) = \frac{1}{2! * 3! * 0! * 2! * 1! * 5!} * \lambda^{13} * e^{-\lambda}$$

$$\frac{\delta L(\lambda)}{\delta \lambda} = \frac{1}{2880} * (13\lambda^{12} * e^{-8\lambda} - 8\lambda^{13} * e^{-8\lambda}) = 0$$

$$\Leftrightarrow -\frac{e^{-8\lambda} * \lambda^{12} * (8\lambda - 13)}{2280} = 0$$

$$\Rightarrow \hat{\lambda} = 0 \vee \hat{\lambda} = \frac{13}{8}$$

The likelihood function has a max at $\hat{\lambda} = \frac{13}{8}$, therefore the desired λ is $\lambda = \hat{\lambda}$.

2.
$$P(X_7 = 2) = \frac{(\frac{13}{8})^2}{2!} * e^{-\frac{13}{8}} \approx 0.2599985$$

$P(X_7 = 2)$ is at about 26%.

4.3 Composite functions

4.4 Classification