Simulation of particle motion

1. **Projectile Motion with Air Resistance (Euler Methods)**
   Simulate a projectile ($m = 1\,kg$) launched from $(0, 0)$ with initial velocity $\mathbf{v}_0 = (10\,m/s, 10\,m/s)$ under gravitational force $\mathbf{F}_g = -mg\hat{\mathbf{y}}$ ($g = 9.81$) and velocity-dependent drag $\mathbf{F}_d = -k\mathbf{v}$.

   (a) Implement motion using Explicit Euler in Unity ($\Delta t = 0.01$).

   (b) Compare trajectories for $k = 0$, $k = 0.1$, and $k = 0.5$.

2. **Harmonic Oscillator with RK4**
   A particle of mass $m = 1\,kg$ moves under the force $\mathbf{F}(x) = -kx\hat{\mathbf{x}}$ in 2D, with $k = 4\,N/m$ and initial conditions $s(0) = (2, 0)\,m$ and $v(0) = (0, 0)m/s$. Implement RK4 ($\Delta t = 0.001\,s$) and visualize for $t \in [0, 5\pi]$

3. Simulate Earth's orbit around a fixed Sun using Newtonian gravity $\mathbf{F} = -\frac{GMm}{r^3}\mathbf{r}$. Use the same values in the material's course

4. **Mass-Spring System**
   A spring exerts force $\mathbf{F} = -k(\mathbf{x} - \mathbf{x}_0) - b\mathbf{v}$ (Hooke's Law), where k is stiffness, b damping, and $\mathbf{x}_0$ rest position.

   (a) Create a 1D system ($m = 1\,kg$, $k = 20\,N/m$, $x_0 = 0\,m$) with initial displacement $x(0) = 0.5\,m$, $v(0) = 0\,m/s$.

   (b) Implement motion using Verlet integration in Unity. $b = 0\,kg/s$

   (c) Add damping $b = 0.5\,\mathrm{kg/s}$

5. **Charged Particle in Magnetic Field (RK4)**
   A particle with charge q moves under $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ where $\mathbf{B} = B_0\hat{\mathbf{z}}$.

   (a) Derive the equations of motion and implement RK4 in Unity (3D).

   (b) Visualize helical motion for initial velocity $\mathbf{v}_0 = (v_0, 0, v_0)$, $q = 1, v_0 = 5$, $B_0 = 2T$ and $\Delta t = 0.01$.

6. **Pendulum Dynamics (Euler vs Verlet)**
   Simple pendulum ($l = 2\,\mathrm{m}$, $m = 1\,\mathrm{kg}$) with $\theta(0) = 45°$, $\dot{\theta}(0) = 0$ (Read and take the equation from https://en.wikipedia.org/wiki/Pendulum_(mechanics).

   (a) Implement both methods ($\Delta t = 0.01\,\mathrm{s}$ and $\Delta t = 0.1\,\mathrm{s}$)

   (b) Add air resistance $b = 0.1\,\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}$ and plot energy decay