Particle Collisions

1. **Projectile Motion with Euler Method**
   Simulate a projectile ($m = 1\,kg$) launched from $(0, 0)$ with initial velocity $\mathbf{v}_0 = (10\,m/s, 10\,m/s)$ under gravitational force $\mathbf{F}_g = -mg\hat{\mathbf{y}}$ ($g = 9.81$).

   Implement the collision between the particle and the ground. Use different values for ε to see the different behaviors.

2. **Particle Motion Inside a Cube with Euler Method**

   Simulate a particle ($m = 1\,kg$) moving inside a cube, starting from the initial position $\mathbf{p}_0 = (1\,m, 1\,m, 1\,m)$ with an initial velocity $\mathbf{v}_0 = (5\,m/s, 5\,m/s, 5\,m/s)$.

   The cube is defined by the limits:

   $$0 \leq x \leq 5, \quad 0 \leq y \leq 5, \quad 0 \leq z \leq 5.$$

   Implement a simulation using the Euler method, updating the particle's position and velocity at each time step $\Delta t = 0.01$.

   Collision Handling: - When the particle collides with one of the cube's wall, apply the reflection rule:

   $$\mathbf{v} = \mathbf{v} - (1 + \varepsilon)(\mathbf{n} \cdot \mathbf{v})\mathbf{n}.$$

   where $\mathbf{n}$ is the normal vector of the ground. - Use different values for ε: - $\varepsilon = 1$ (Perfectly elastic collision) - $\varepsilon = 0.5$ (Partially inelastic collision) - $\varepsilon = 0$ (Perfectly inelastic collision, the particle stops moving)

   Extension:

   - Extend the simulation to handle collisions with all six walls of the cube.
   - Observe the behavior of the particle as it bounces inside the cube

   Extension 2: - For each wall put a different value of ε.

3. **Particle Motion Inside a Triangle with Euler Method**

   Simulate a particle ($m = 1\,kg$) moving freely inside a **triangular** region, starting from the initial position $\mathbf{p}_0 = (1\,m, 1\,m)$ with an initial velocity $\mathbf{v}_0 = (3\,m/s, 4\,m/s)$. The particle moves with **constant velocity**, meaning there is no external force acting on it.

   The triangular region is defined by the vertices:

   $$A = (0, 0), \quad B = (5, 0), \quad C = (2.5, 5).$$

   Implement a simulation using the Euler method, updating the particle's position at each time step Δt.

   Collision Handling: - When the particle collides with one of the triangle's edges, apply the reflection rule:

   $$\mathbf{v} = \mathbf{v} - (1 + \varepsilon)(\mathbf{n} \cdot \mathbf{v})\mathbf{n}.$$

   where $\mathbf{n}$ is the normal vector of the edge.

   - Use different values for ε: - $\varepsilon = 1$ (Perfectly elastic collision) - $\varepsilon = 0.5$ (Partially inelastic collision) - $\varepsilon = 0$ (Perfectly inelastic collision, the particle stops moving)

   Extension: - Implement a method to detect which edge the particle collides with. - Observe how the particle's trajectory evolves as it bounces inside the triangle.

4. Verify that in a mechanical system, the following equality is satisfied

   $$\Delta U = -W.$$