

HW5 : Backtracking and Greedy Algorithms

Sources:

Problem 2

1. <https://stackoverflow.com/questions/48759175/what-is-the-space-complexity-of-the-python-sort>
2. <https://en.wikipedia.org/wiki/Timsort>

1. Backtracking

a. description

A target number n and a combination length k are passed to the function `combination()`. As a recursive approach is used for this backtracking problem, the `combination()` function makes use of a helper function `combination_helper()`. Before calling the helper function, an array, `n_arr`, of integers from 1 to $\max(n, 9)$ is created. We compare the max of n and 9 because k can be 1-9, but if $n < 9$, then the elements which make up it's sum cannot be greater than 9. Next a function call is made to `combination_helper(n, k, 0, [], n, [])` to initialize the backtracking algorithm.

In `combination_helper()` we iterate through each element in the `n_arr` with a for loop that begins at 0 and goes to $n-k$. If the length of the combination exceeds k elements or if the sum (held in *remainder*) of the combination is greater than the target n , then we backtrack to the last selection and continue trying the next possible choice. If the sum (held in *remainder*) of the elements is equal to the target n , then there is a comparison if the length of the combination is equal to k . If the combination is not equal to k , then it is too short, and we backtrack to the previous selection. However, if the combination is equal to k , then the combination meets the requirements and is appended to the *result* array.

b. pseudocode

```
def combination_helper(n, k, start, result, remainder, combination):
```

```
    if len(combination) = k+1:
        return # combination length exceeded k
    if remainder = 0:
        if len(combination) = k:
            append combination to result
            return # if len != k, then combination will be too short so only return
        else if remainder < 0:
            return # sum exceeded the target

    for i in range(start, len(n)-k):
        append n[i] to combination
        combination_helper(n, k, i+1, result, remainder-n[i], combination)
        combination.pop() # backtrack
```

```
def combination(n, k):
```

```
    result = []

    # turn n into an array of integers 1, 2, ..., max(n, 9)
    n_arr = [x for x in range(1, max(n, 9))]

    combination_helper(n_arr, k, 0, result, n, [])

    print(result)
```

2. Greedy algorithm

a. description

The `feedDog()` function takes two arrays: `hunger_level[1...n]` and `biscuit_size[1...m]`. First we sort both arrays in ascending order with the built-in Python sort method. Next we declare a variable `num_dogs` to hold the number of dogs that have been fed, and this is initialized to 0. We also declare the variables `i` and `j` to represent the current dog and the current biscuit respectively. We initialize both `i` and `j` to 0. Then we use a while loop to iterate through each dog, represented in the `hunger_level` array. If the `hunger_level[i]` of the dog is \leq to the `biscuit_size[j]`, then the dog can be fed, so `num_dogs` is incremented. `i` and `j` are also incremented because we can move on to the next dog and the next biscuit. However, if the `hunger_level[i]` of the dog is $>$ the `biscuit_size[j]`, then the dog cannot be fed with the current biscuit because its `hunger_level[i]` will not be satisfied, so we increment `j`, the current biscuit, and see if the next biscuit will satisfy the `hunger_level[i]` of the current dog. After the while loop is complete because there are no more dogs or biscuits, then `num_dogs` is returned.

b. time complexity

The function `feedDog(hunger_level[1...n], biscuit_size[1...m])` sorts two arrays: `hunger_level[1...n]` and `biscuit_size[1...m]`.

To sort both arrays I used the Python sort method, which is Timsort, so $2 * O(n \log n) \Rightarrow O(n \log n)$.