

AOPT Exercise 7: L-BFGS and Gauss Newton Method

SCHALLER ALAIN, FONTANA JONAS, CARREL VINCENT

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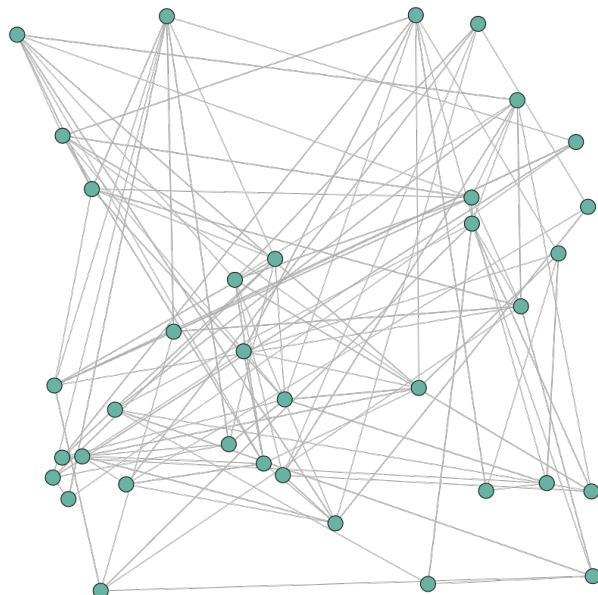


Figure 1: **Gauss Newton:** 0: f without length & Constrained spring scenario: 2: Corners – (As Gauss Newton executable uses the Newton Methods without hessian modification, and in this case, the LLT factorization fails, which makes marks the end of the method like all other **Gauss Newton without length**)

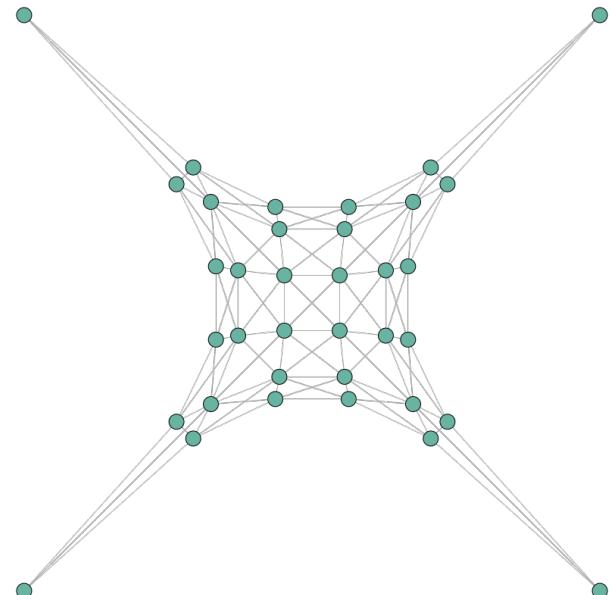


Figure 2: **Standard Newton:** 0: f without length & Constrained spring scenario: 2: Corners

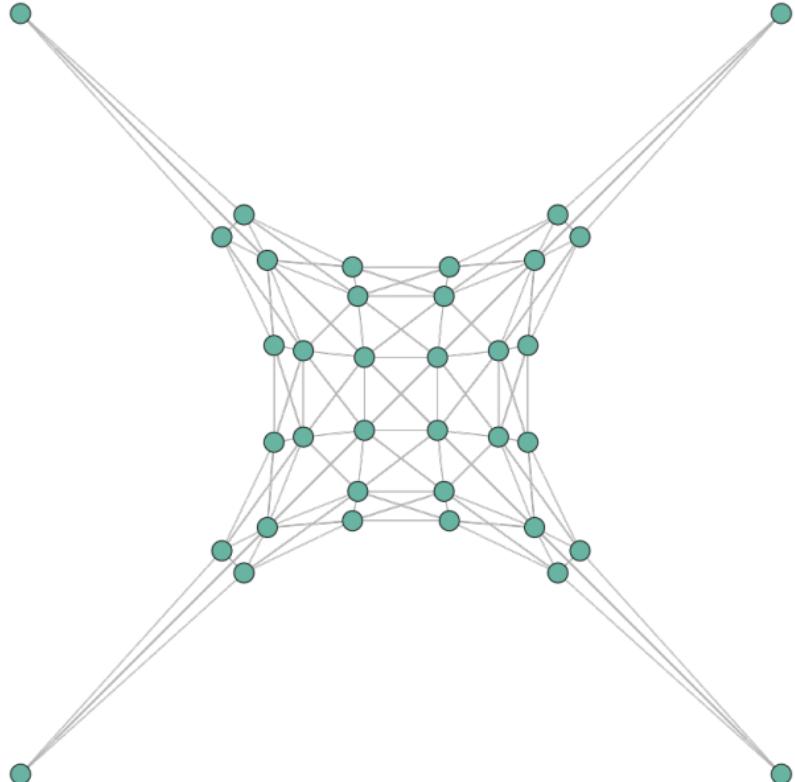


Figure 3: **Gradient Descent**: 0: f without length & Constrained spring scenario: 1: Sides

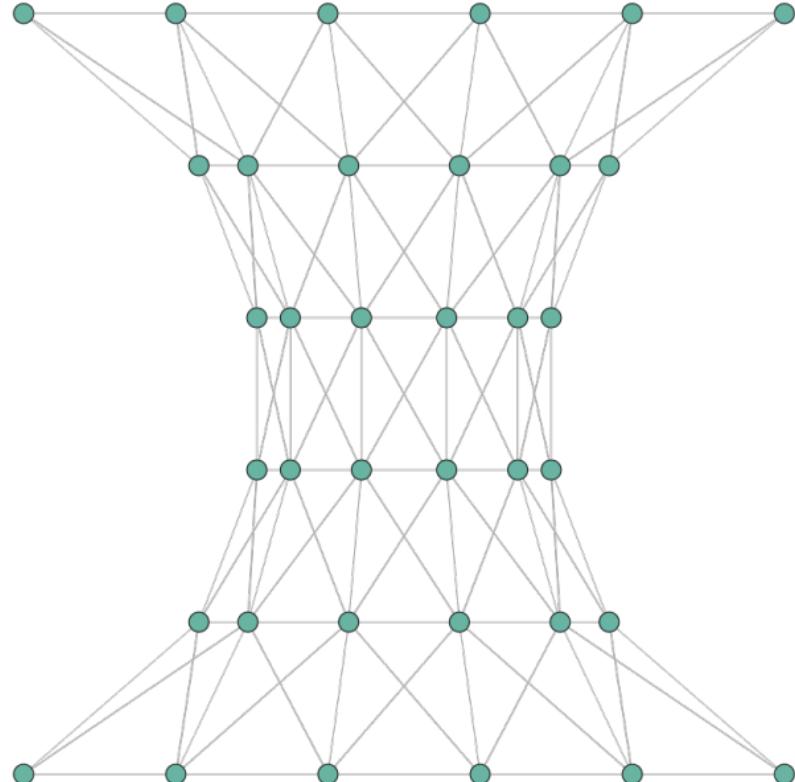


Figure 4: **Gradient Descent**: 0: f without length & Constrained spring scenario: 1: Sides

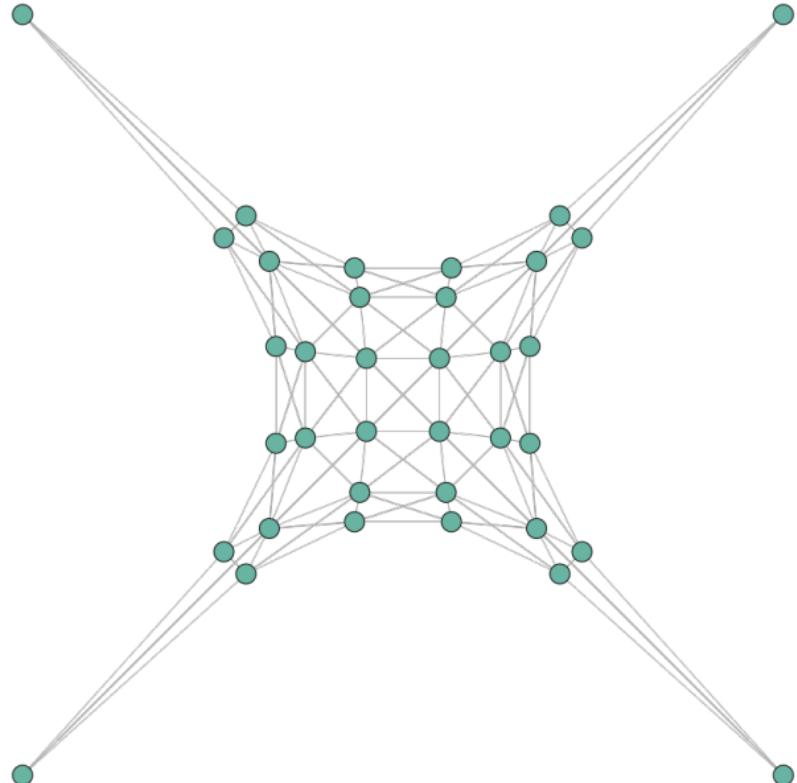


Figure 5: **L-BFGS**: 0: f without length &
Constrained spring scenario: 2: Corners

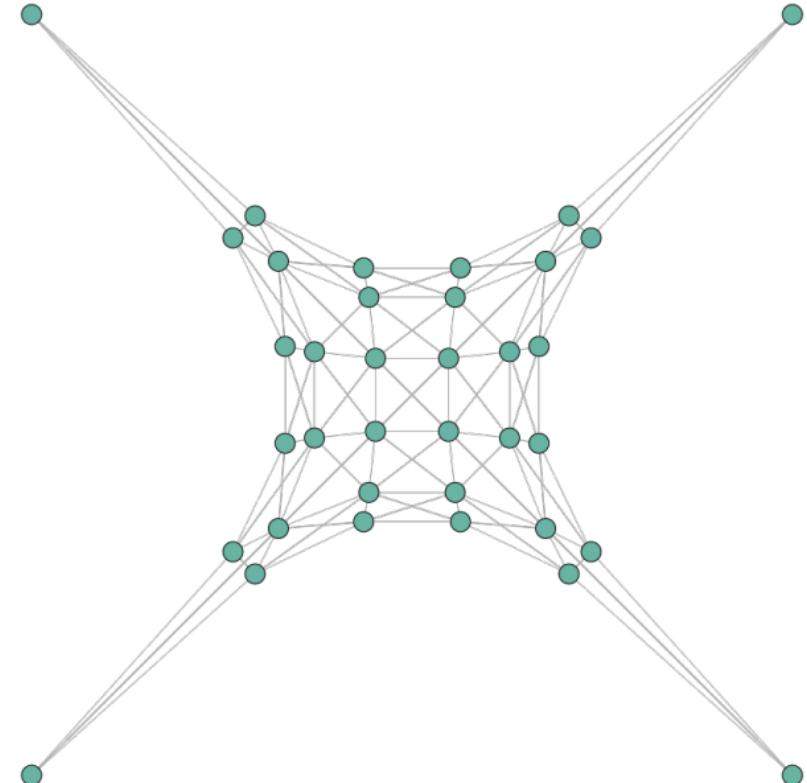


Figure 6: **L-BFGS**: 0: f without length &
Constrained spring scenario: 2: Corners

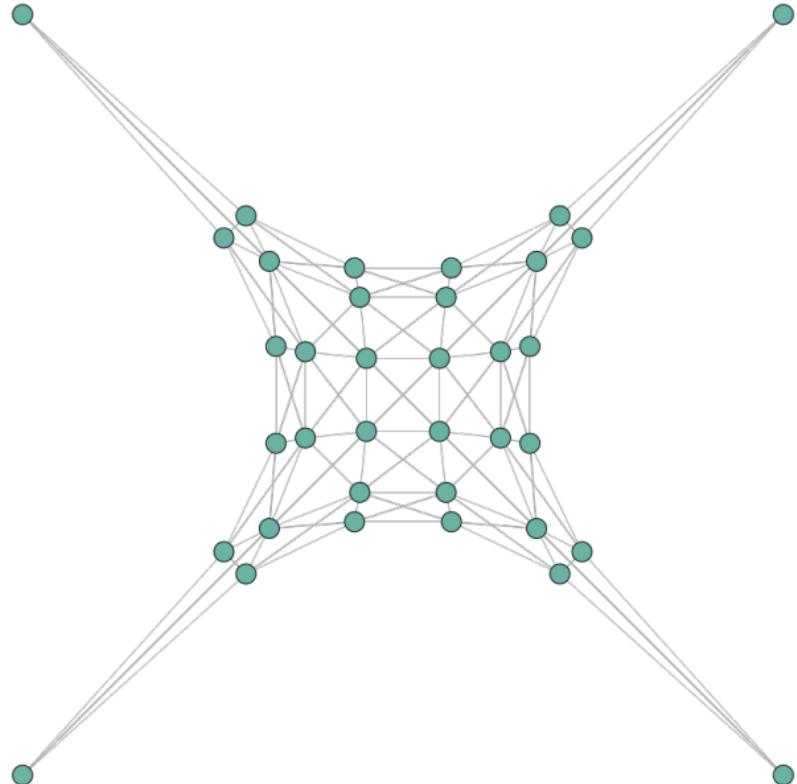


Figure 7: **L-BFGS**: 0: f without length &
Constrained spring scenario: 2: Corners

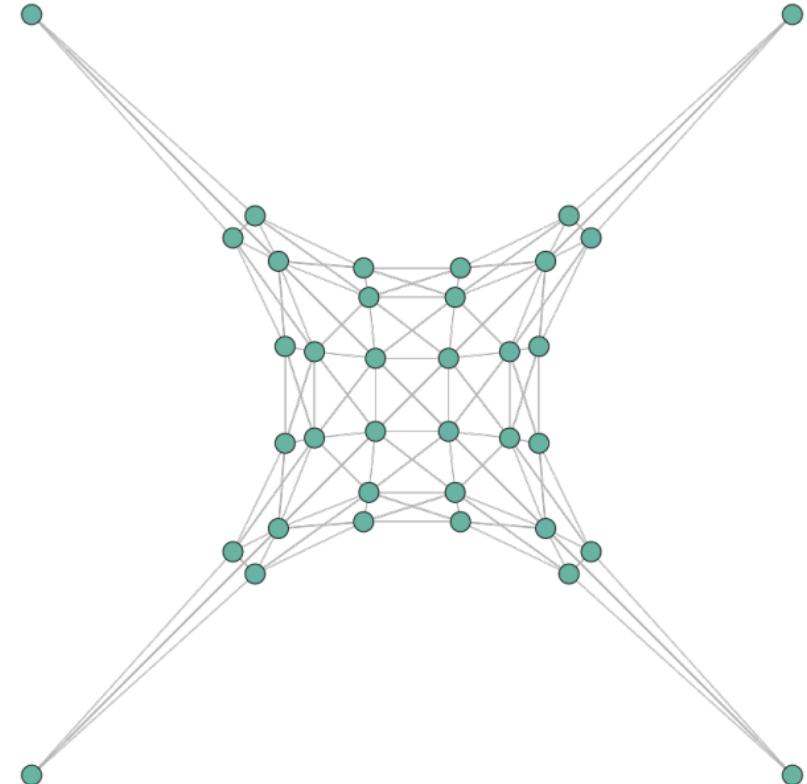


Figure 8: **L-BFGS**: 0: f without length &
Constrained spring scenario: 2: Corners

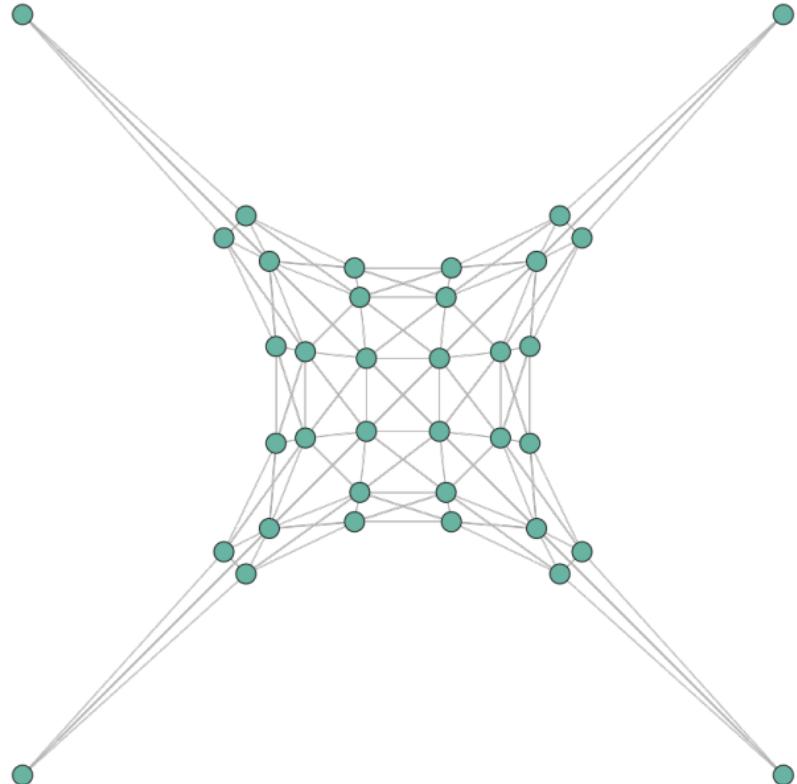


Figure 9: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

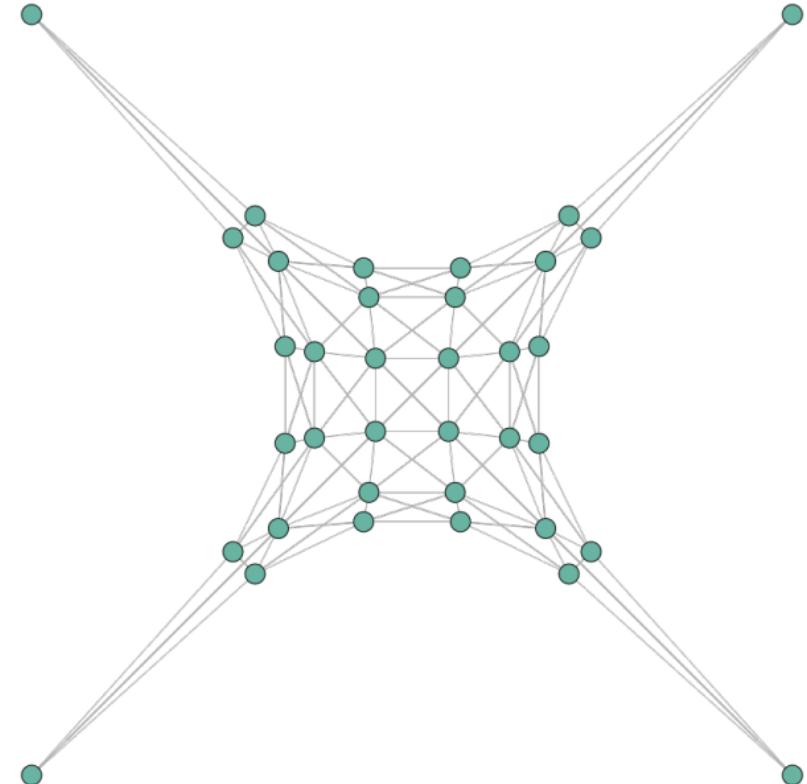


Figure 10: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

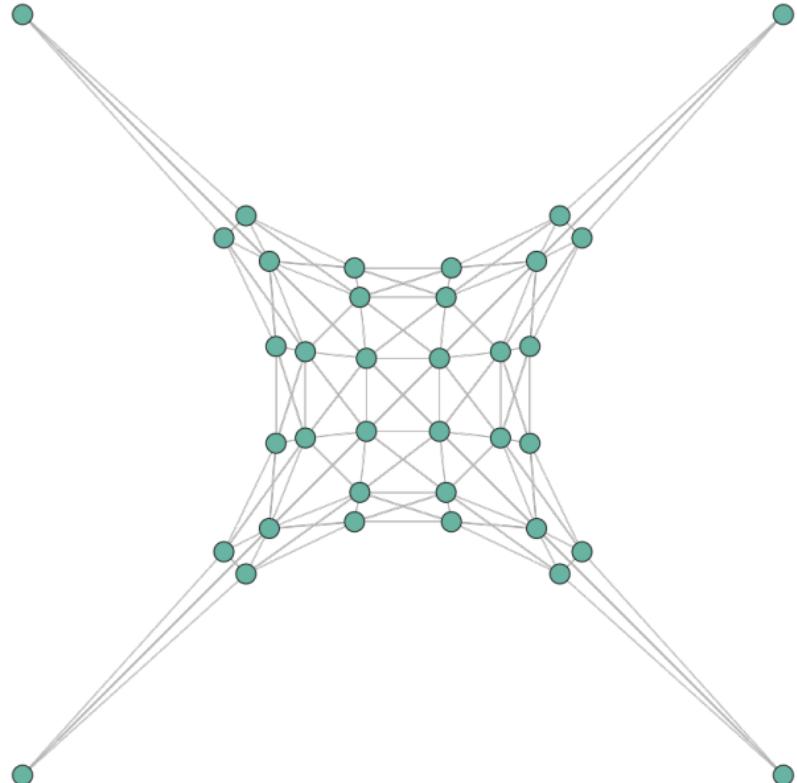


Figure 11: **L-BFGS**: 0: f without length &
Constrained spring scenario: 2: Corners

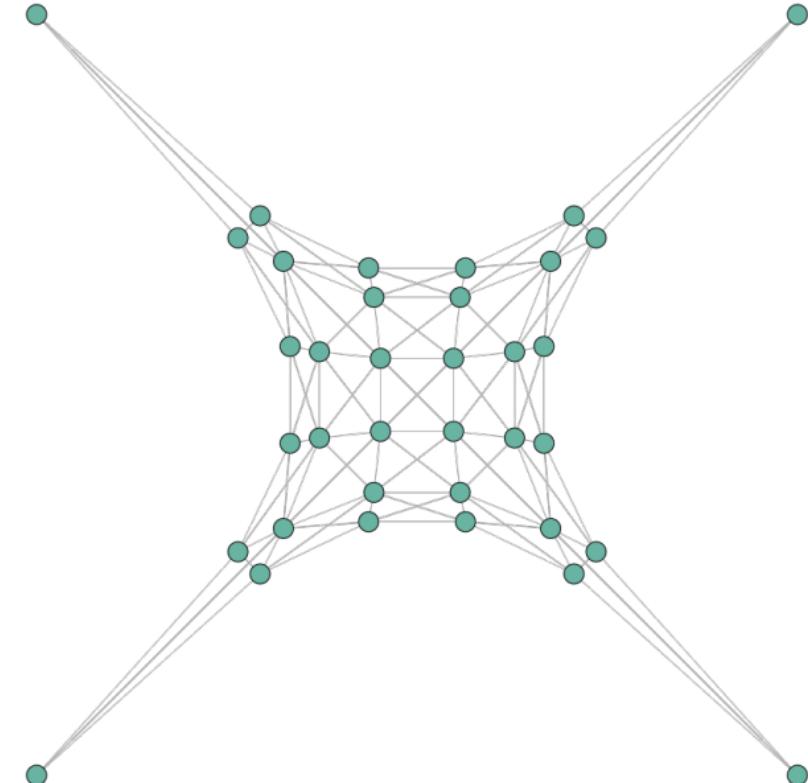


Figure 12: **L-BFGS**: 0: f without length &
Constrained spring scenario: 2: Corners

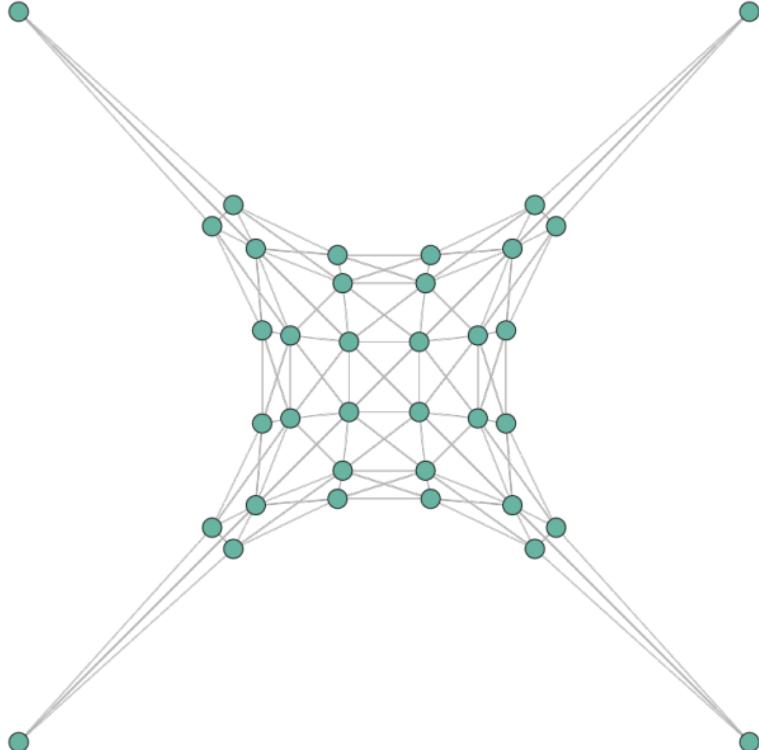


Figure 13: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

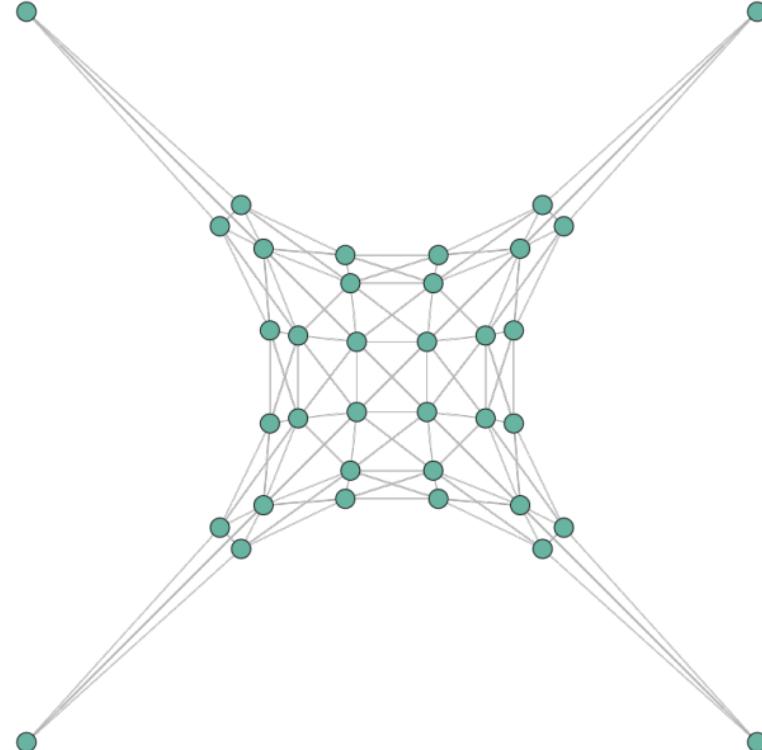


Figure 14: **Newton with projected Hessian**: 0: f without length & Constrained spring scenario: 2: Corners

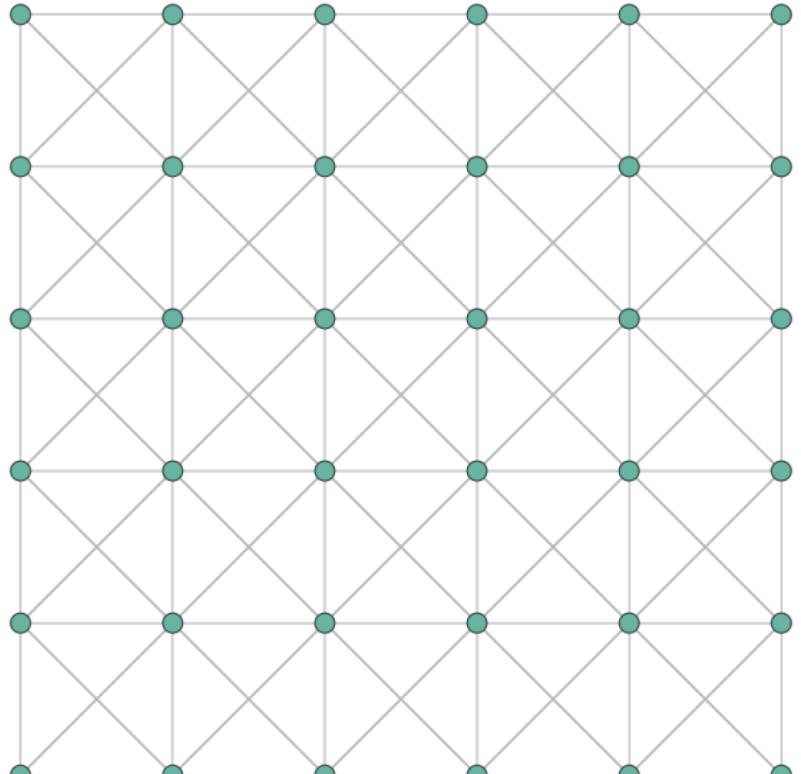


Figure 15: **Gauss Newton**: 1: f with length & Constrained spring scenario: 2: Corners

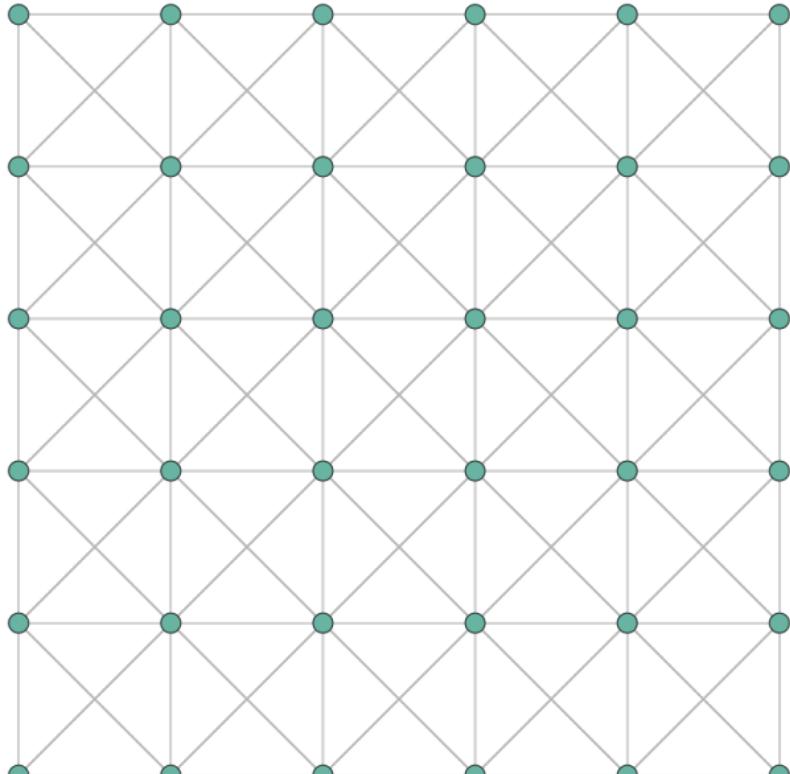


Figure 16: **Gradient Descent**: 1: f with length & Constrained spring scenario: 2: Corners

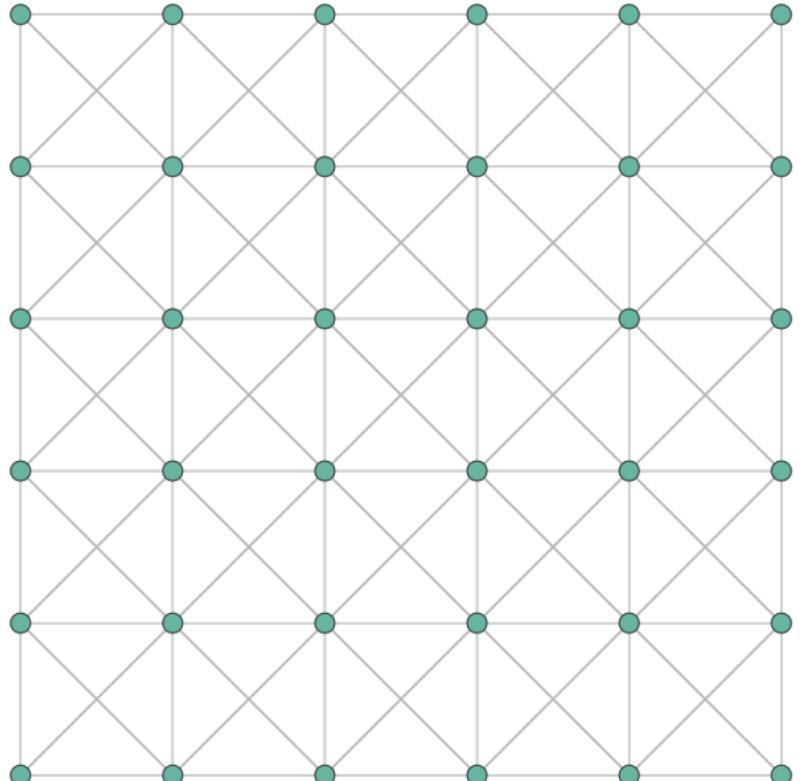


Figure 17: **Gradient Descent**: 1: f with length & Constrained spring scenario: 2: Corners

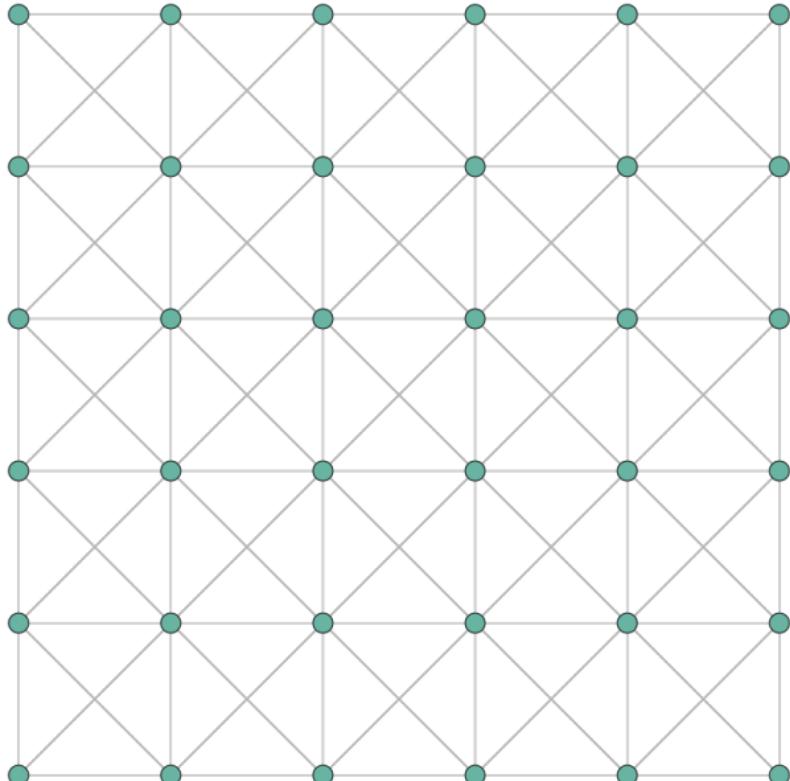


Figure 18: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

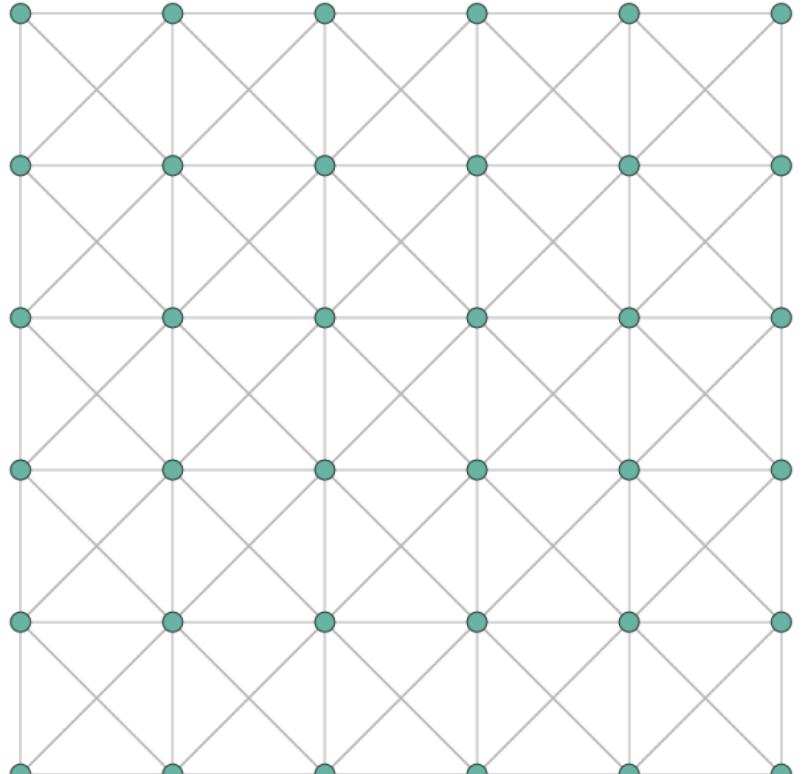


Figure 19: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

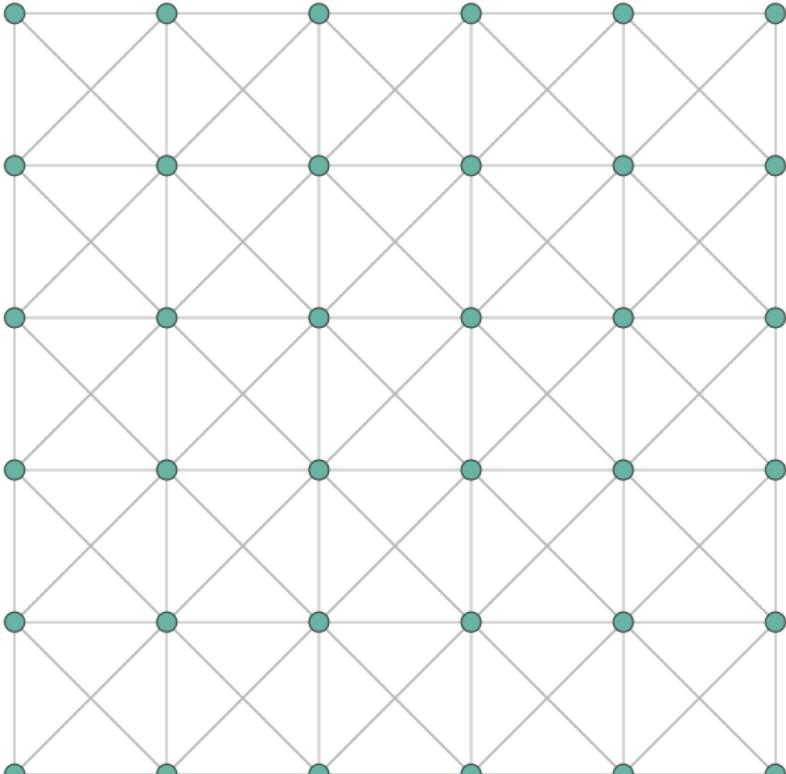


Figure 20: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

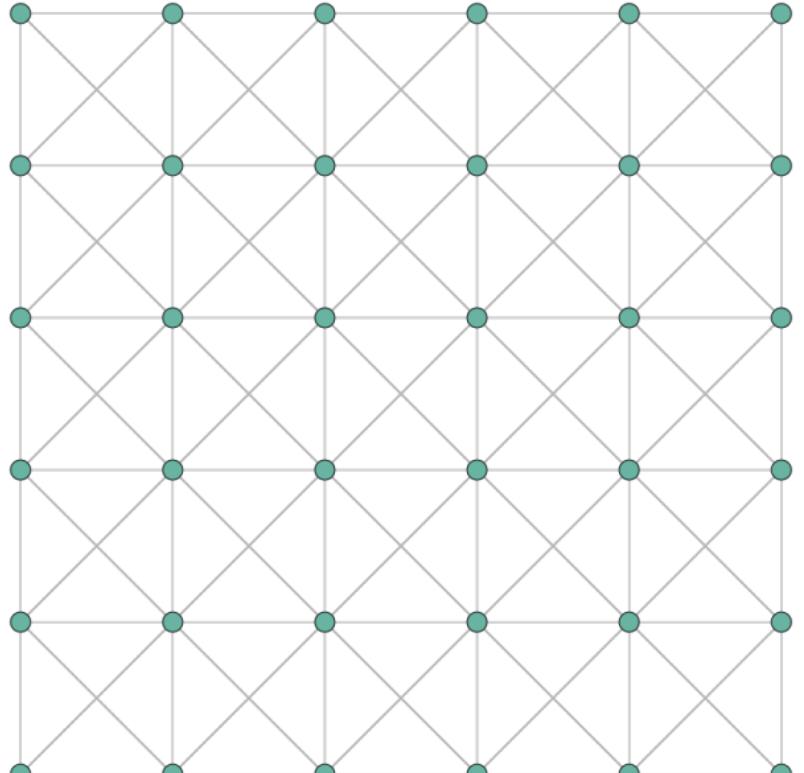


Figure 21: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

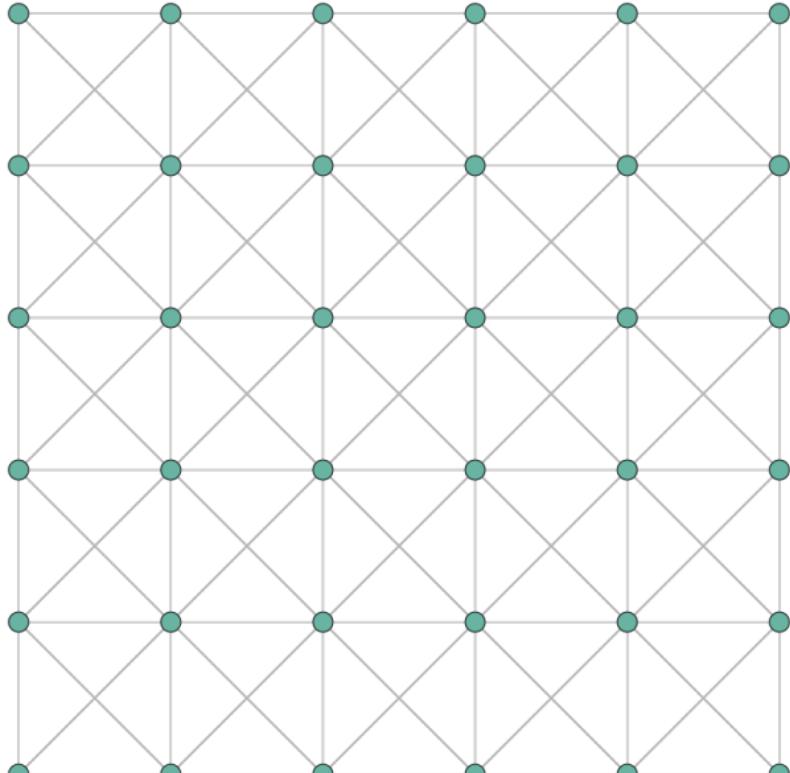


Figure 22: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

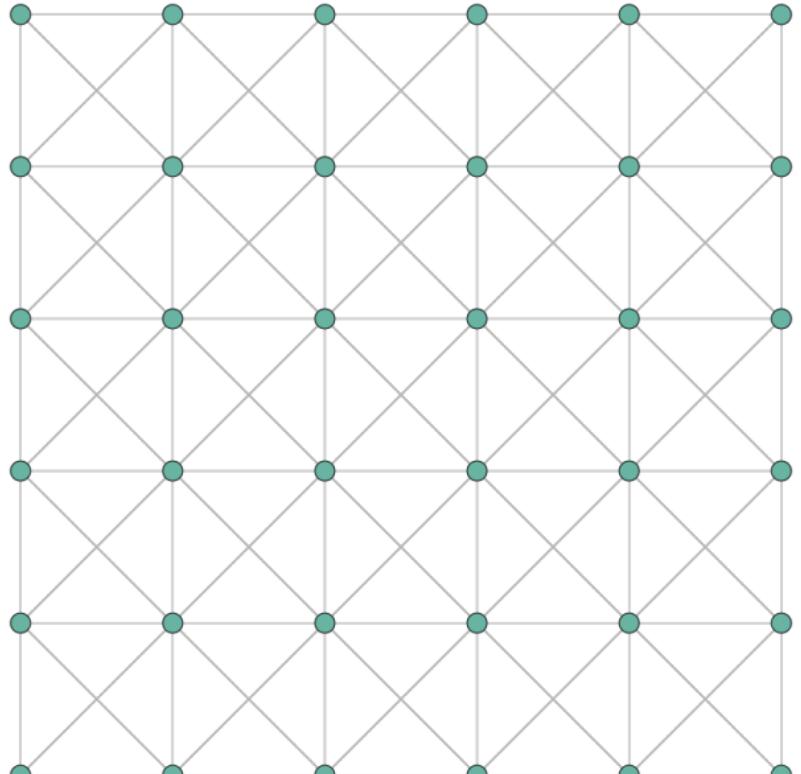


Figure 23: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

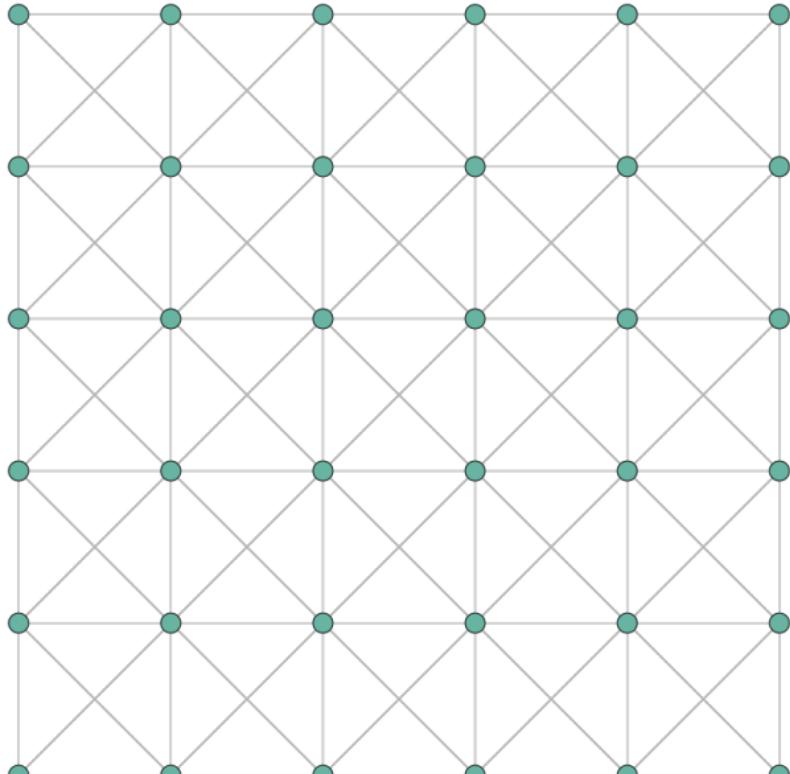


Figure 24: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

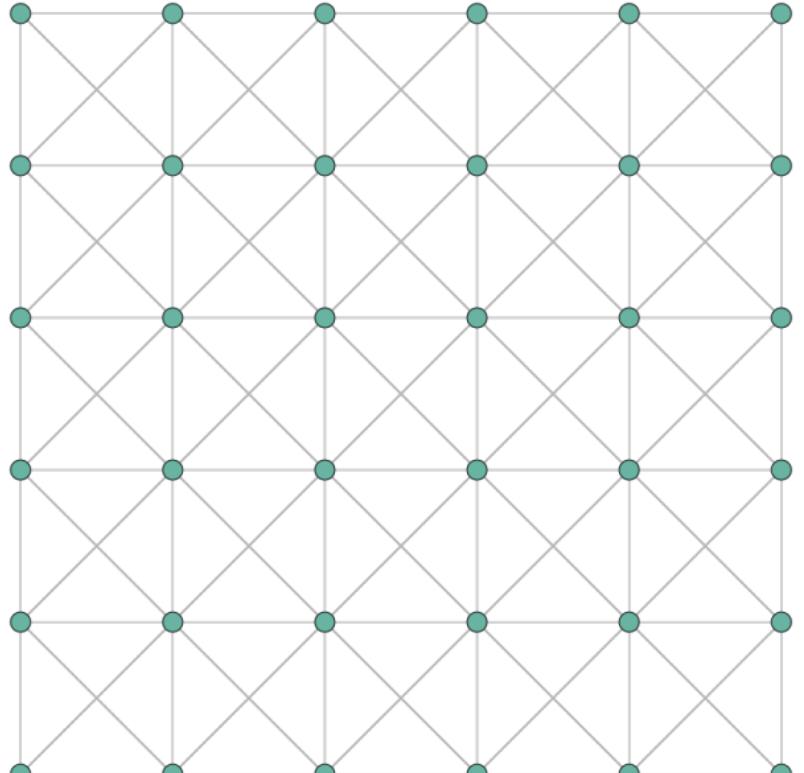


Figure 25: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

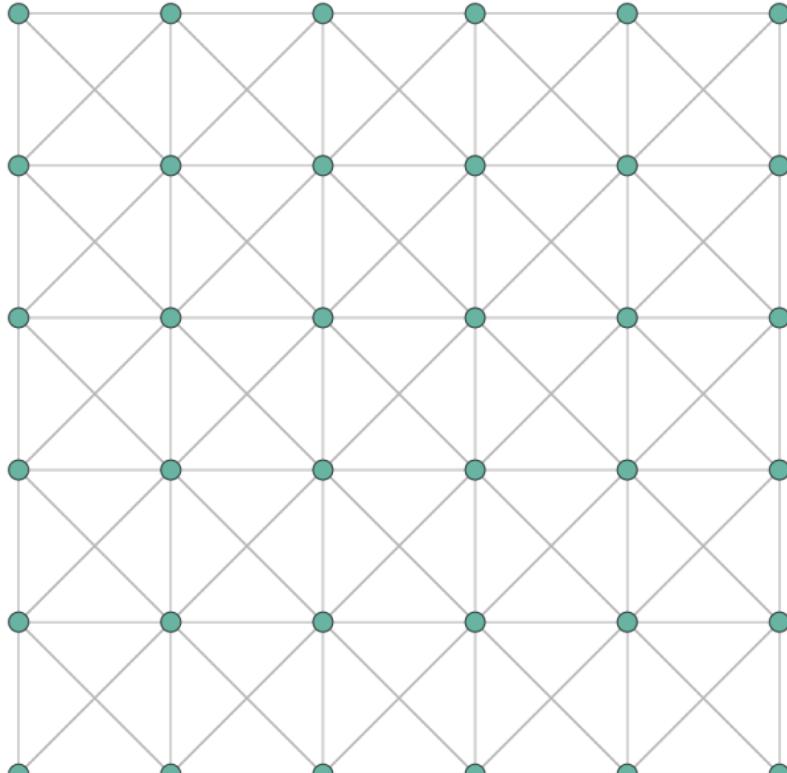


Figure 26: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

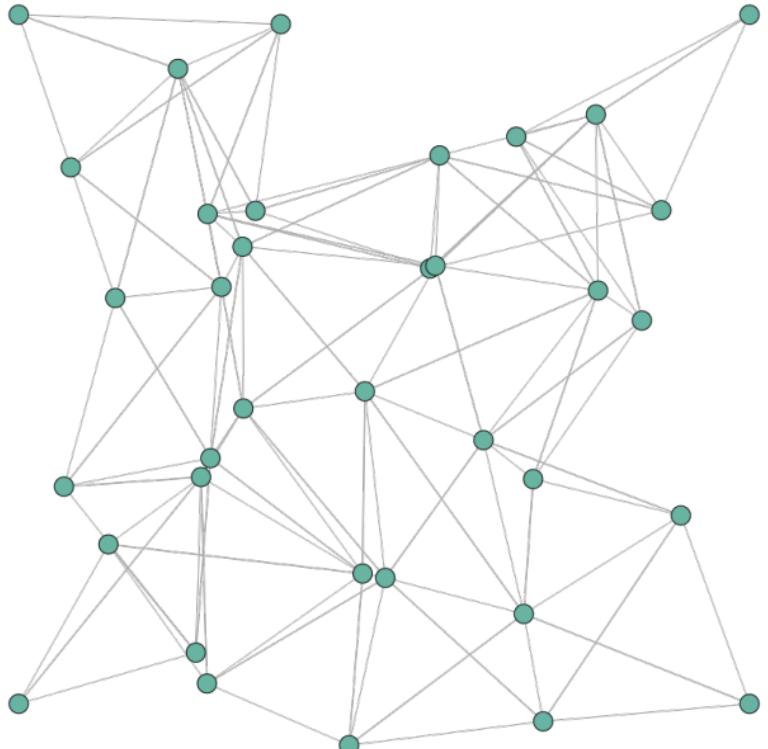


Figure 27: **Newton with projected Hessian:**
1: f with length & Constrained spring scenario:
2: Corners

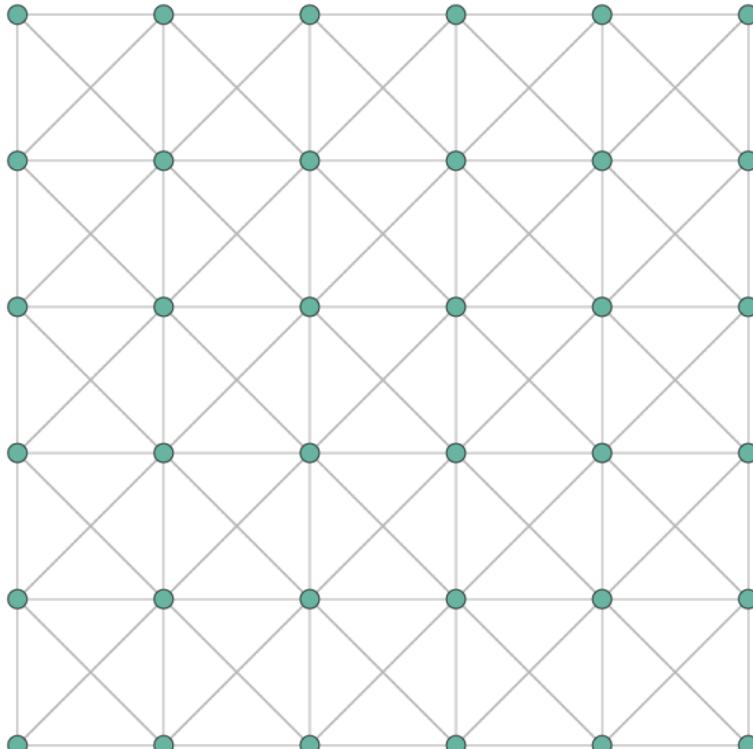


Figure 28: **Standard Newton:** 2: f with length
with positive local hessian & Constrained spring
scenario: 2: Corners

Grid size: 10

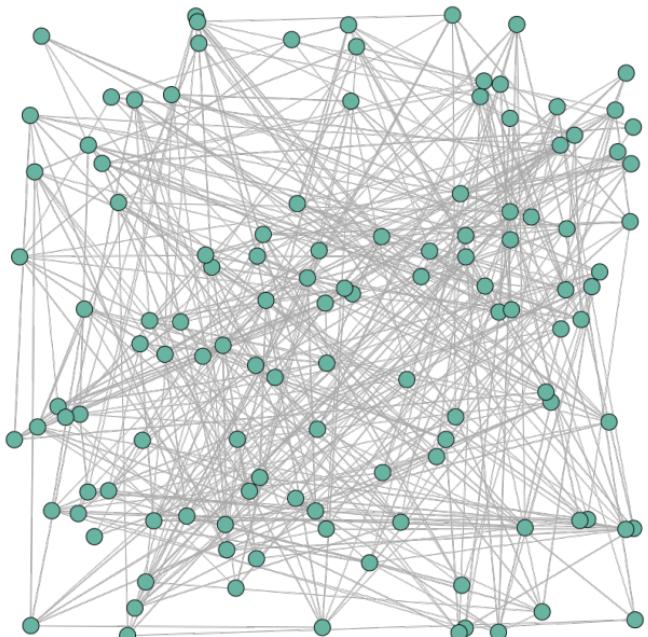


Figure 29: **Gauss Newton:** 0: f without length & Constrained spring scenario: 2: Corners – (As Gauss Newton executable uses the Newton Methods without hessian modification, and in this case, the LLT factorization fails, which makes marks the end of the method like all other **Gauss Newton without length**)

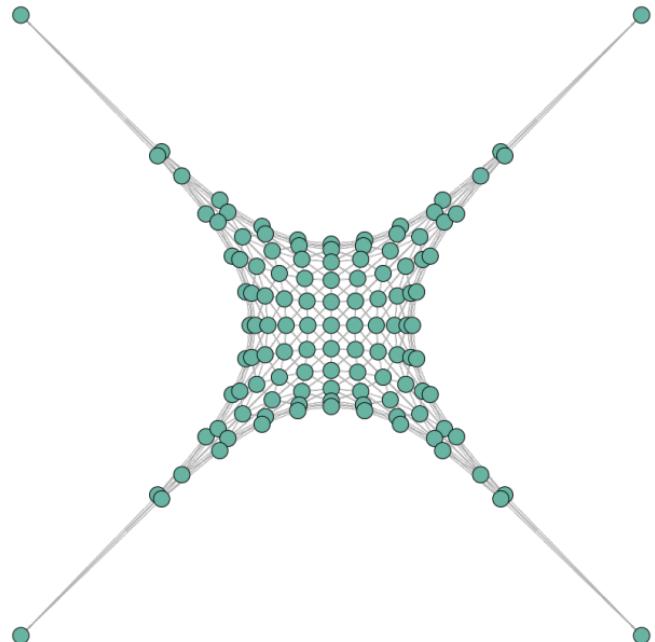


Figure 30: **Standard Newton:** 0: f without length & Constrained spring scenario: 2: Corners

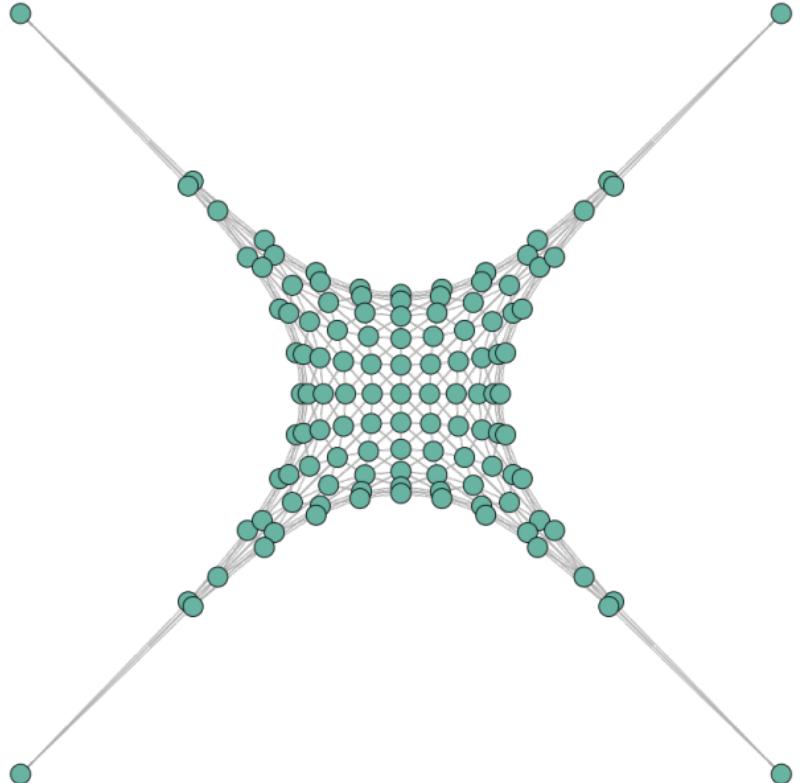


Figure 31: **Gradient Descent**: 0: f without length & Constrained spring scenario: 1: Sides

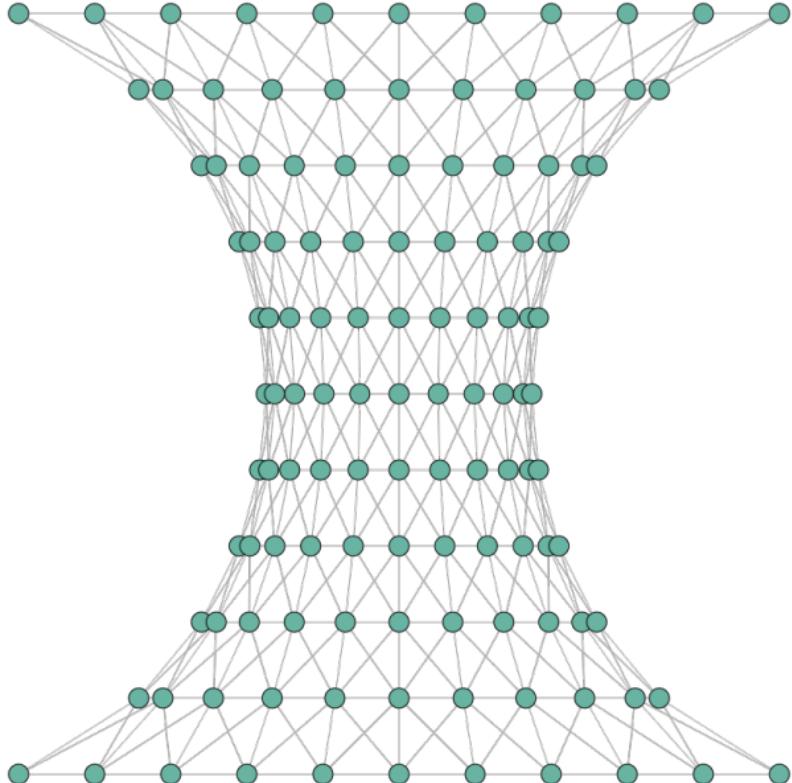


Figure 32: **Gradient Descent**: 0: f without length & Constrained spring scenario: 1: Sides

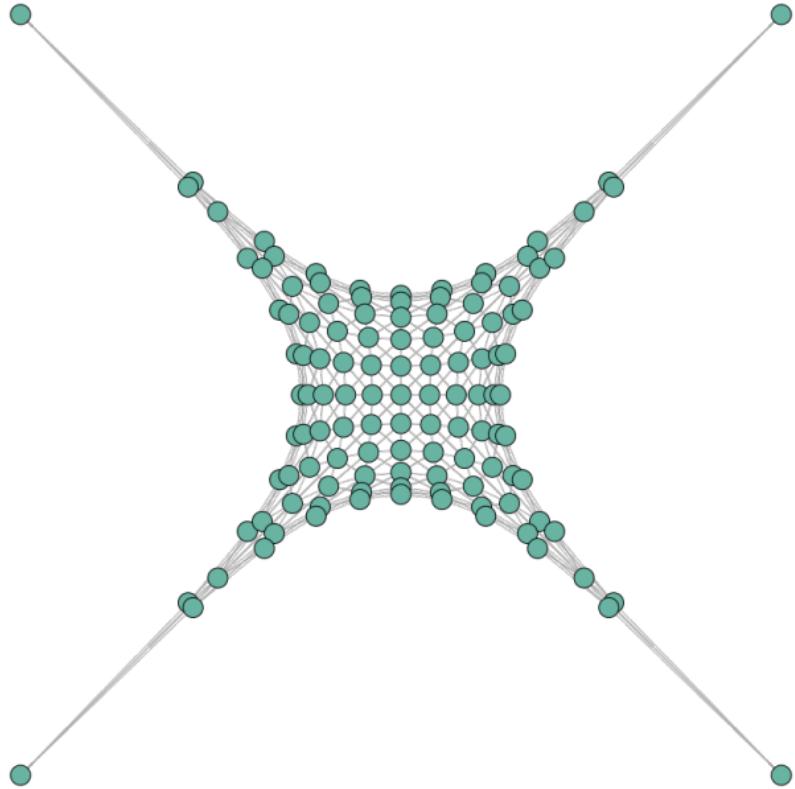


Figure 33: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

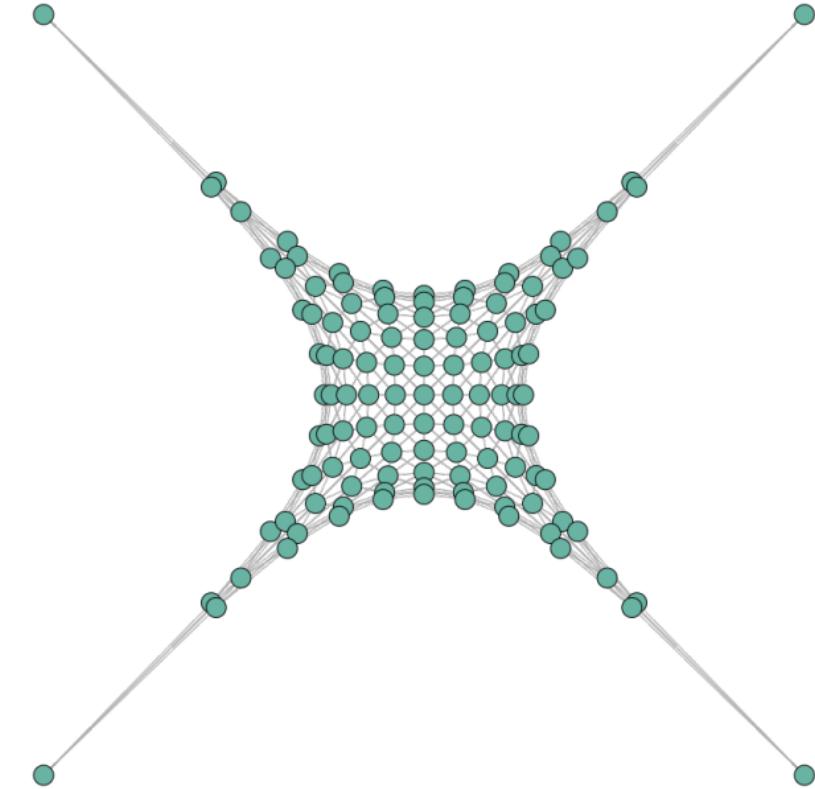


Figure 34: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

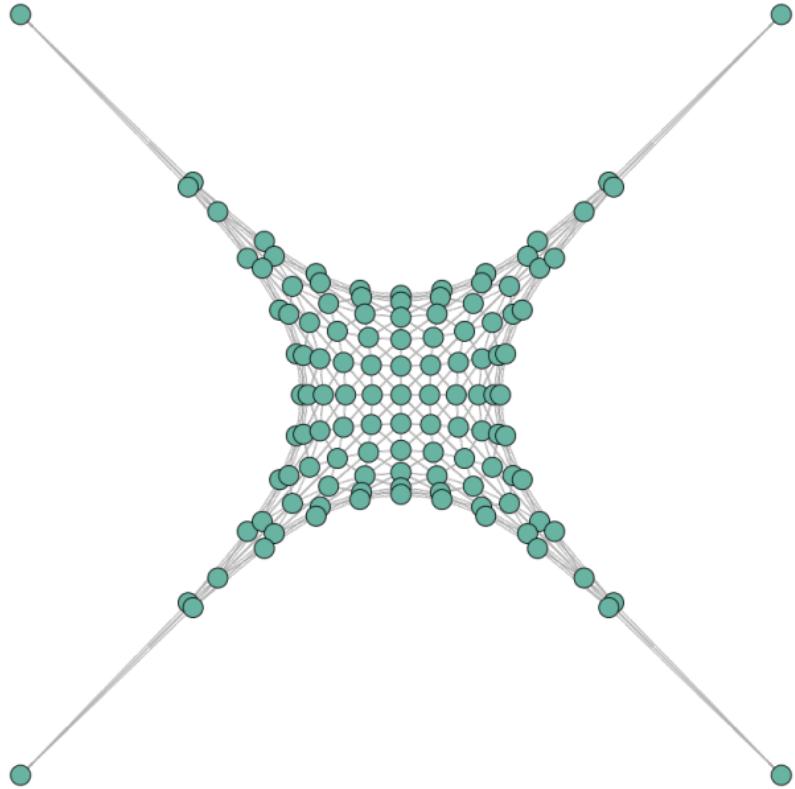


Figure 35: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

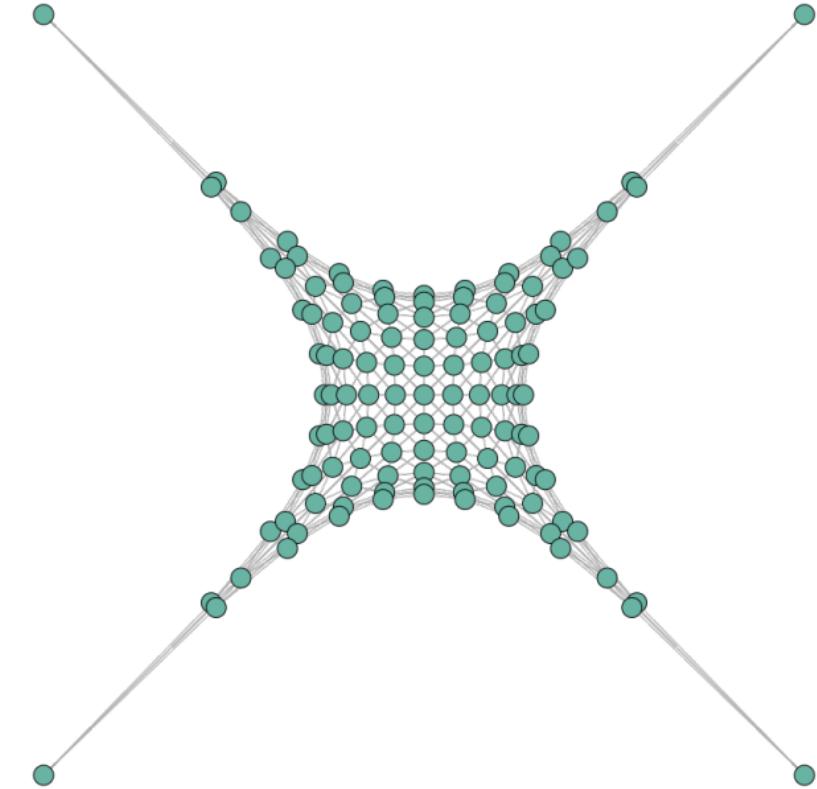


Figure 36: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

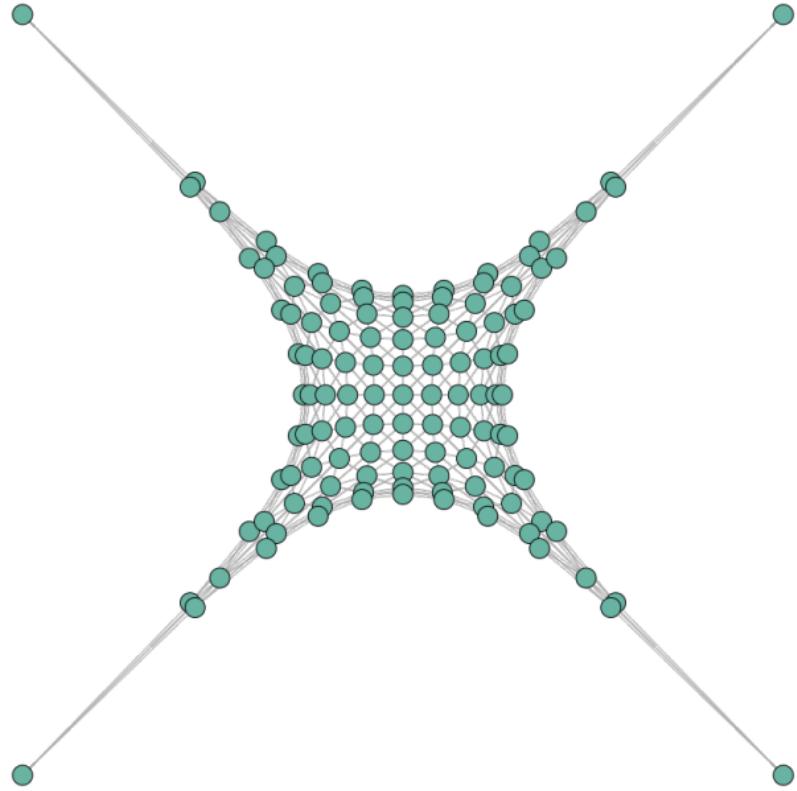


Figure 37: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

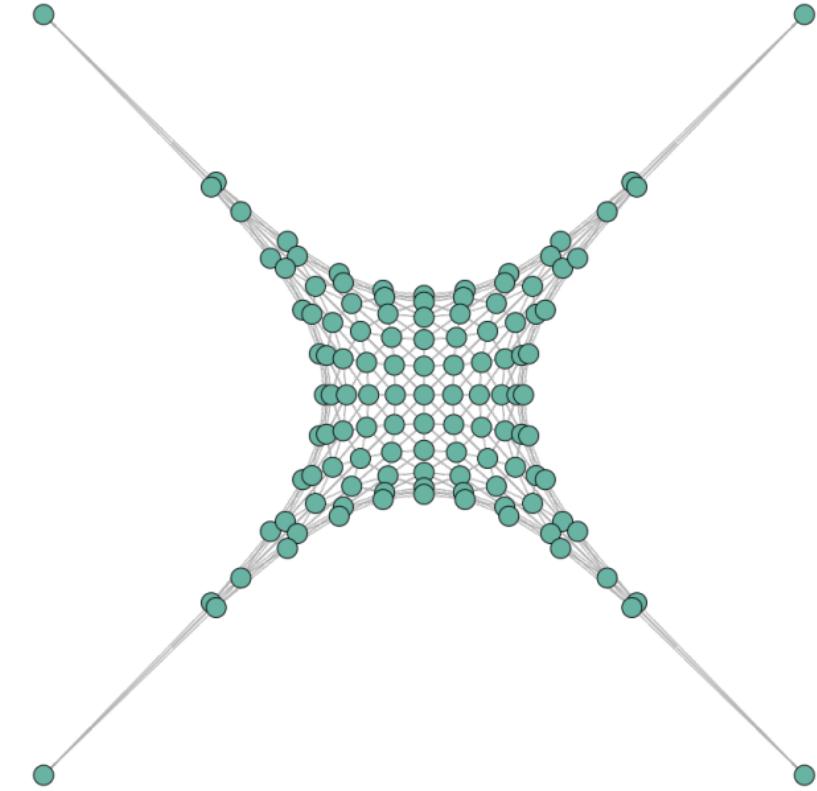


Figure 38: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

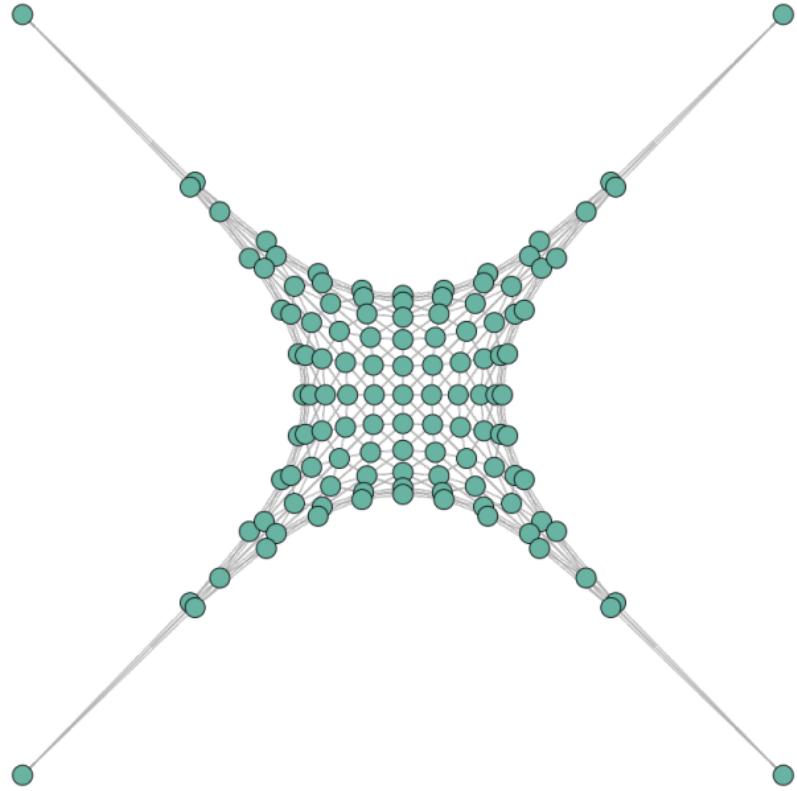


Figure 39: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

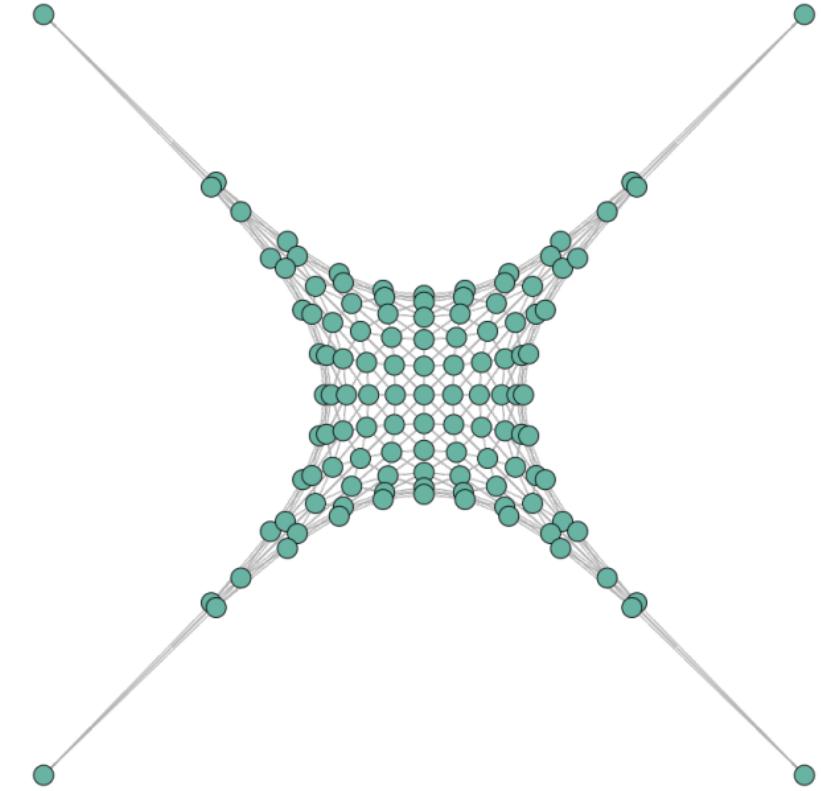


Figure 40: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

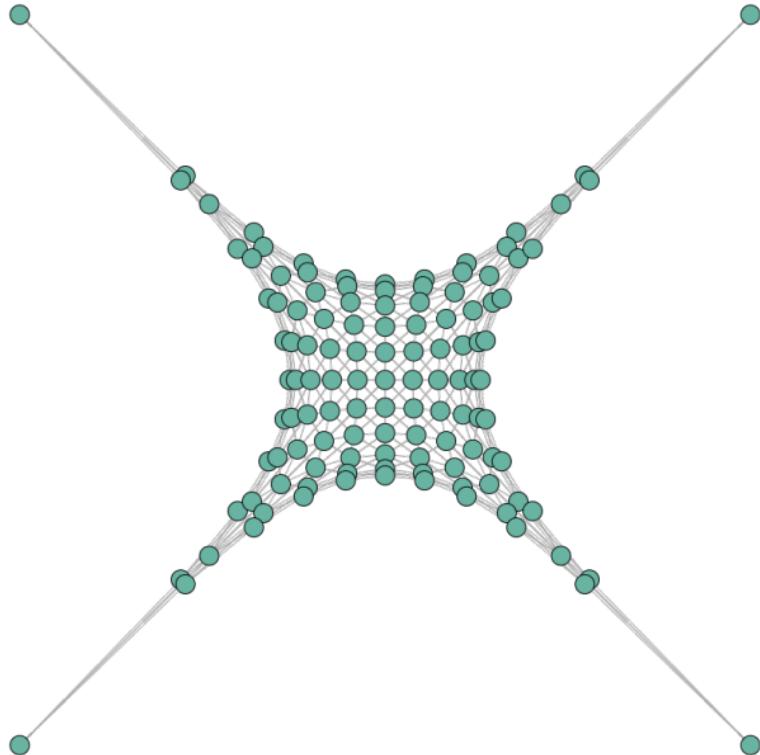


Figure 41: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

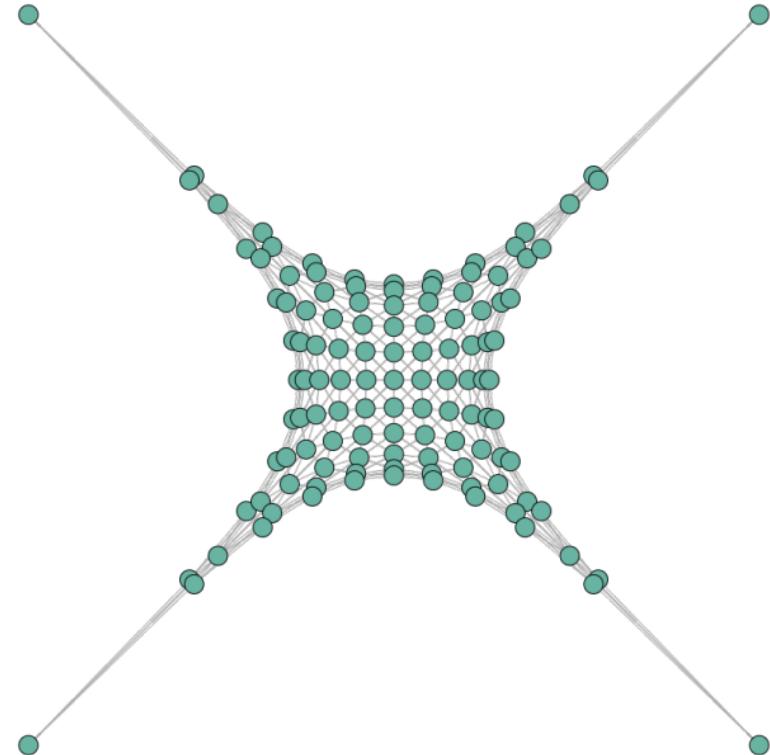


Figure 42: **Newton with projected Hessian**: 0: f without length & Constrained spring scenario: 2: Corners

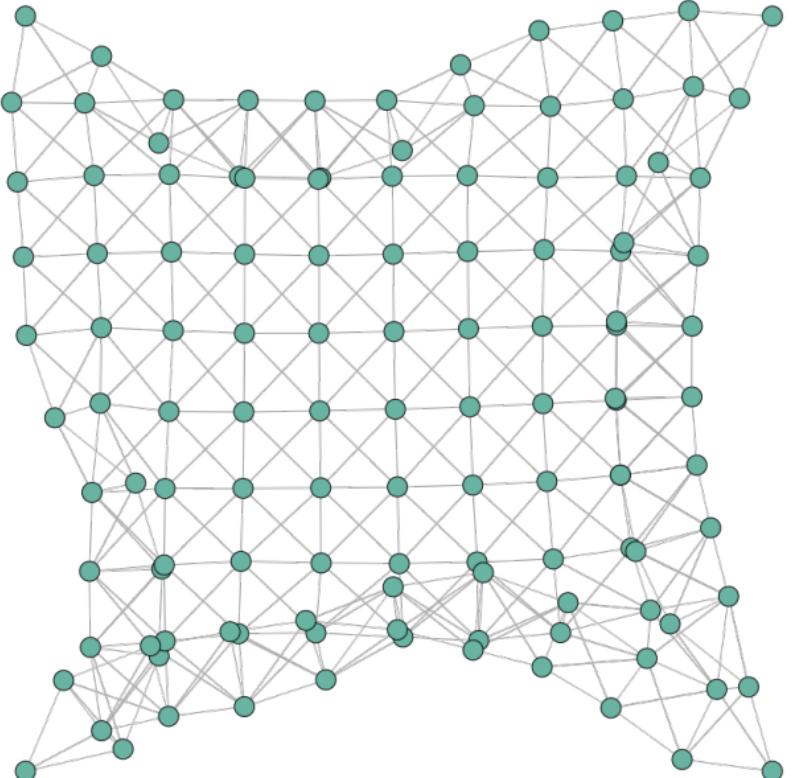


Figure 43: **Gauss Newton**: 1: f with length & Constrained spring scenario: 2: Corners

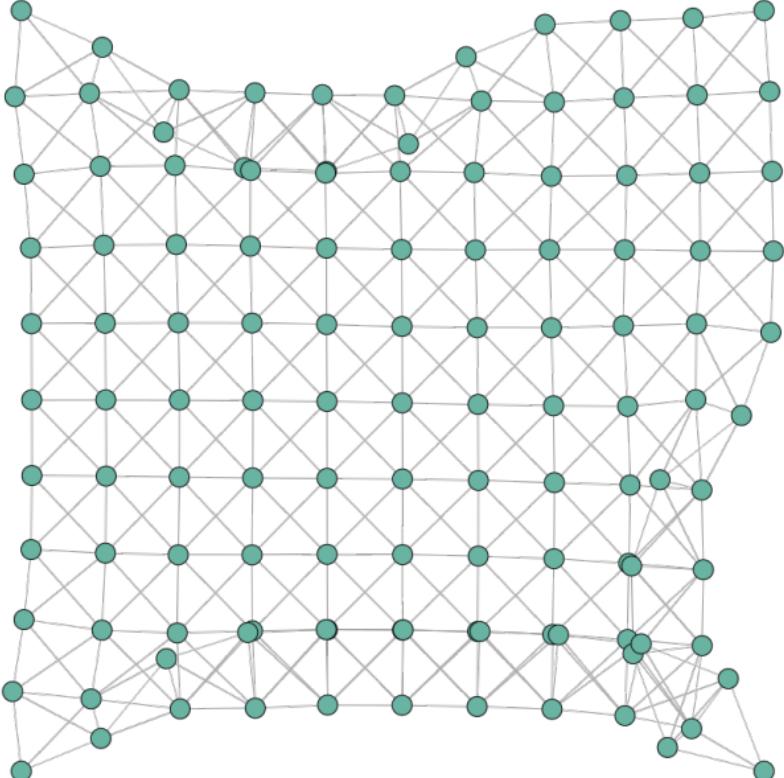


Figure 44: **Gradient Descent**: 1: f with length & Constrained spring scenario: 2: Corners

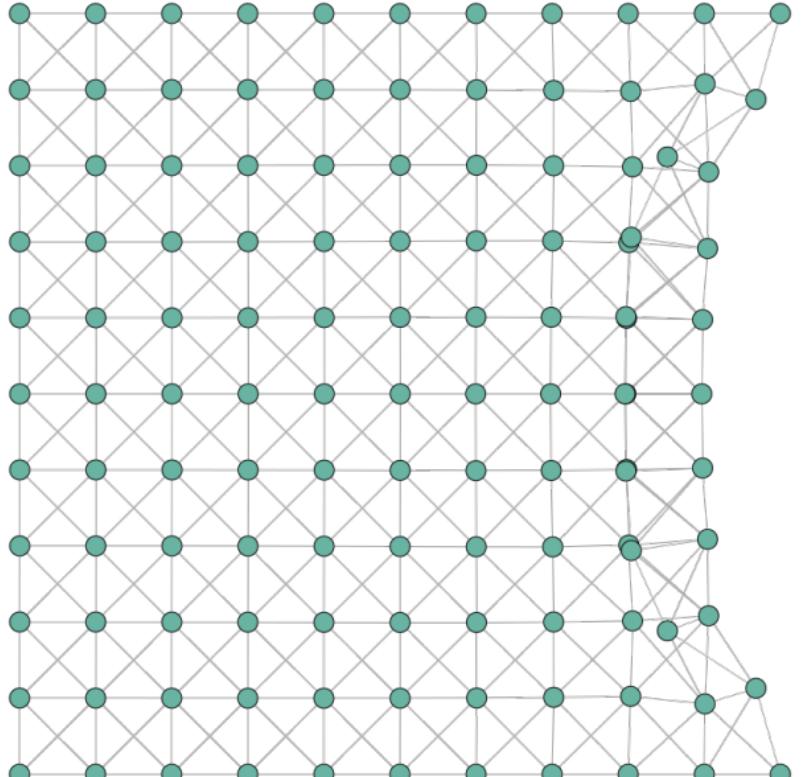


Figure 45: **Gradient Descent**: 1: f with length & Constrained spring scenario: 2: Corners

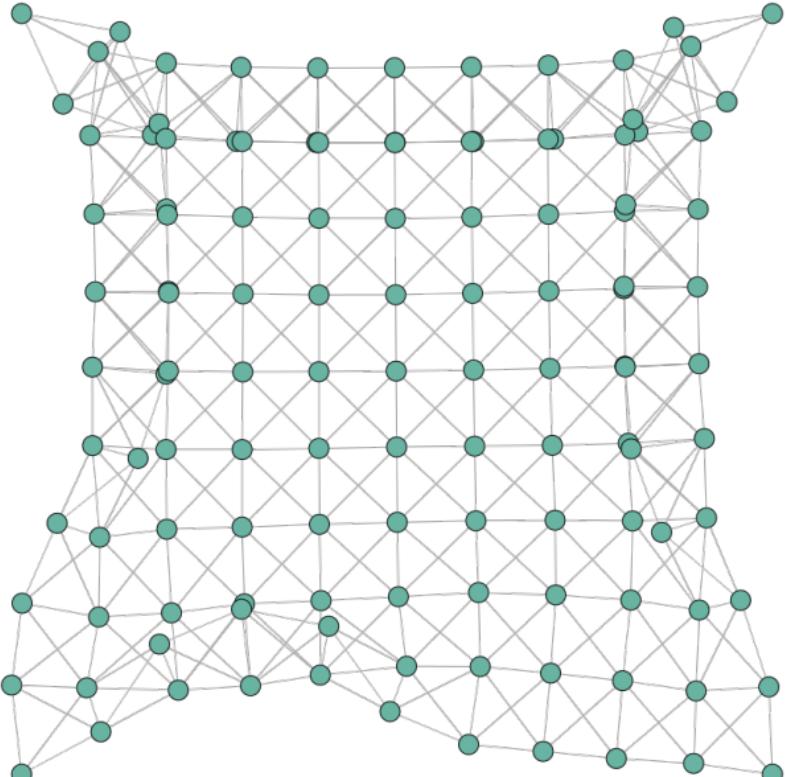


Figure 46: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

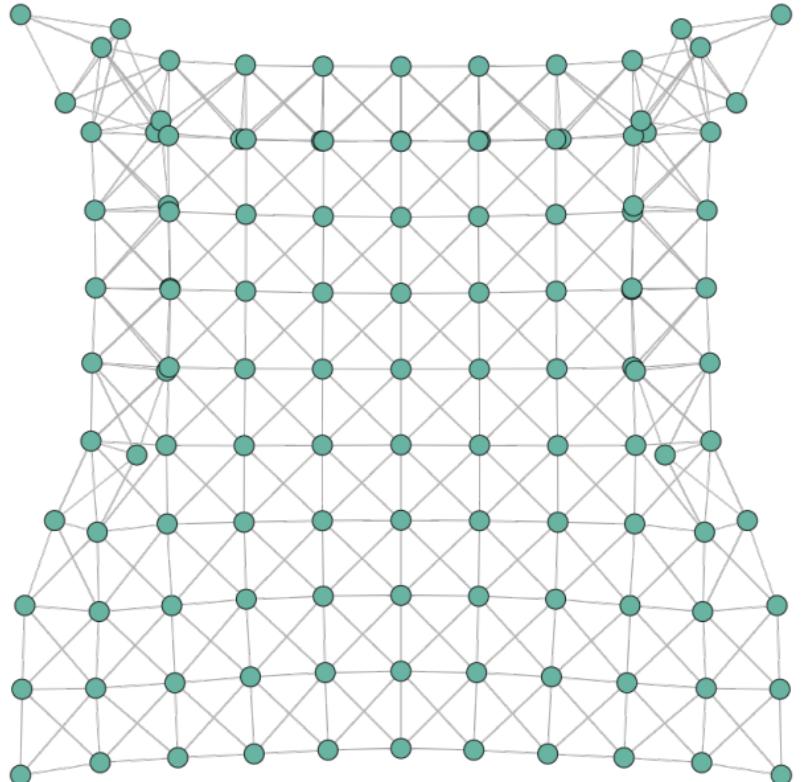


Figure 47: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

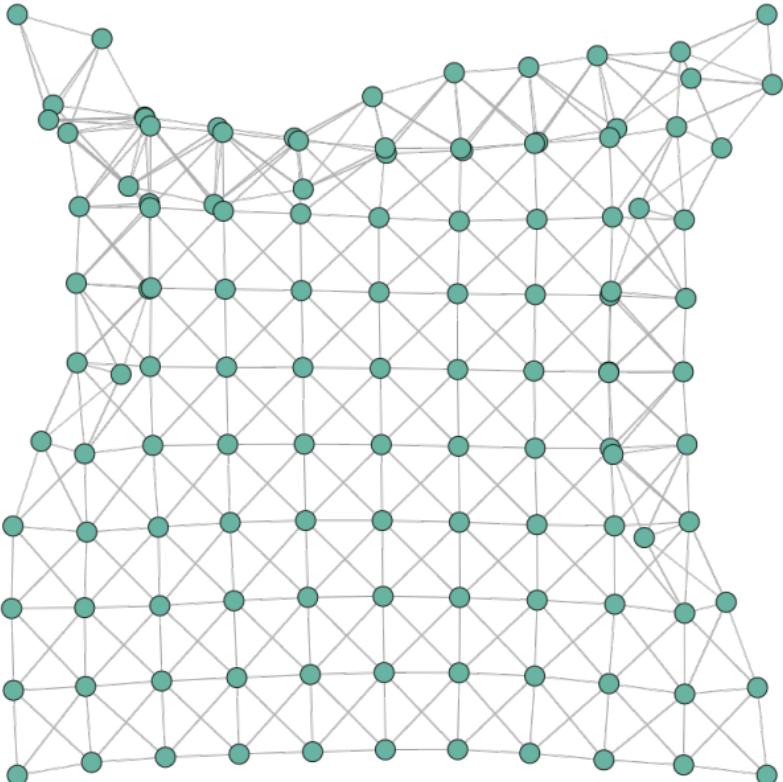


Figure 48: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

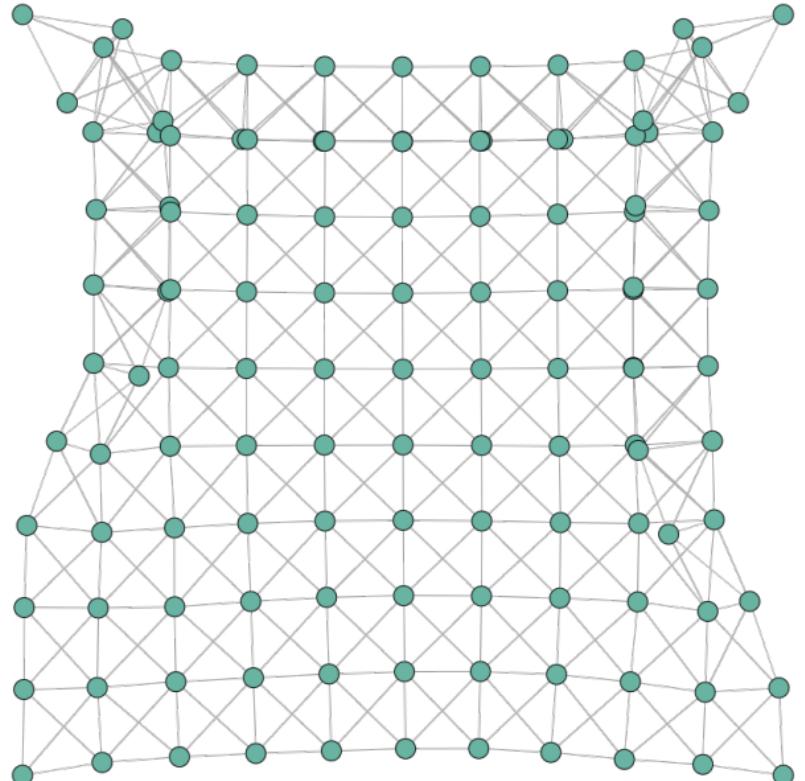


Figure 49: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

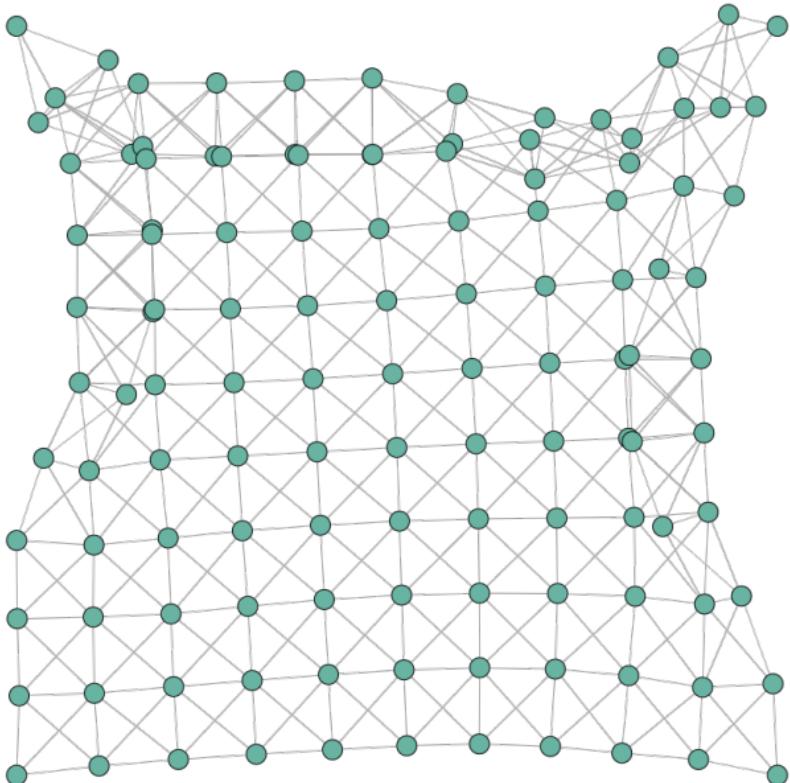


Figure 50: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

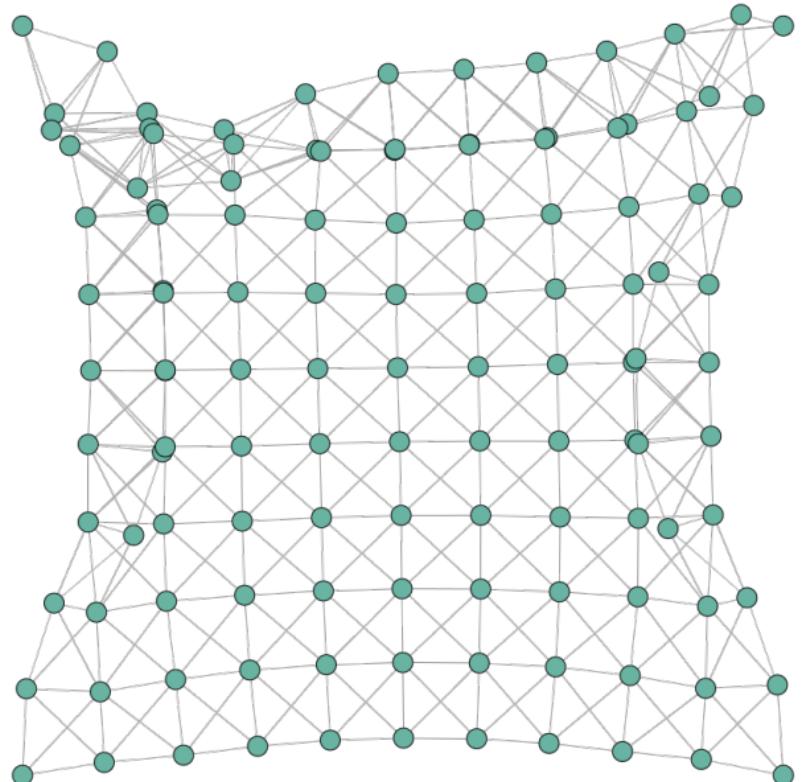


Figure 51: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

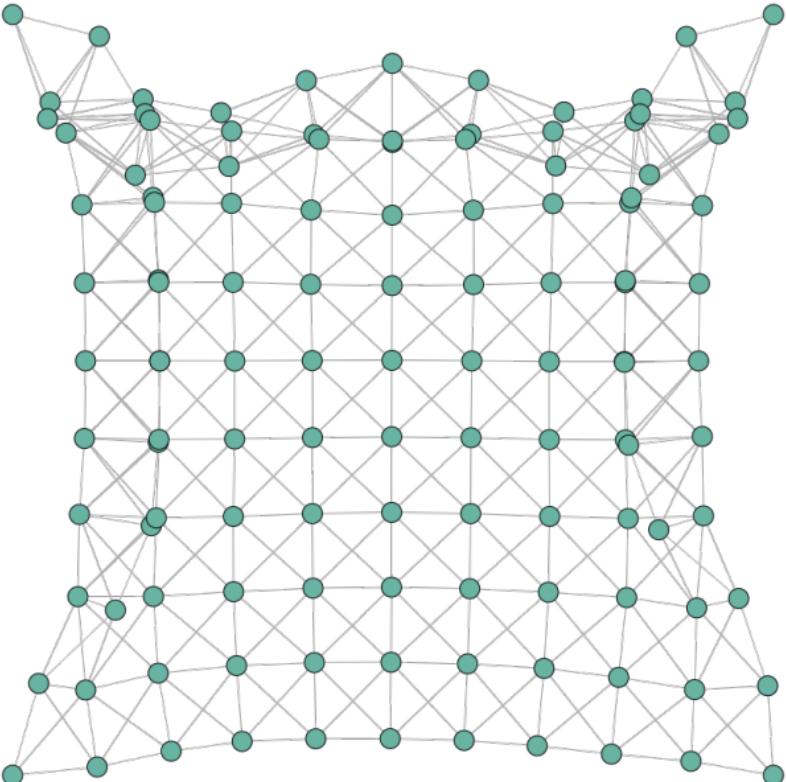


Figure 52: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

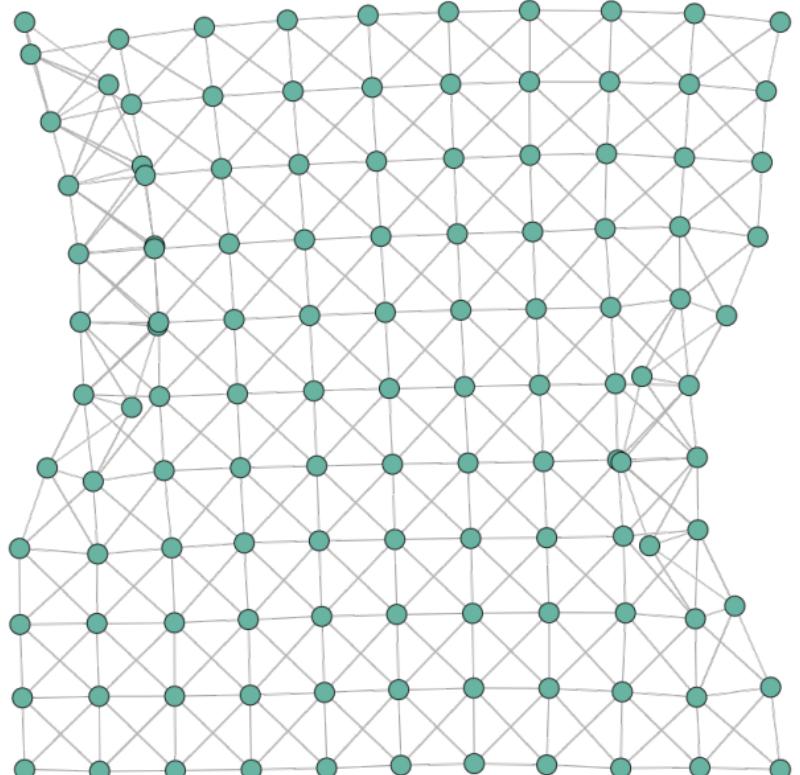


Figure 53: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

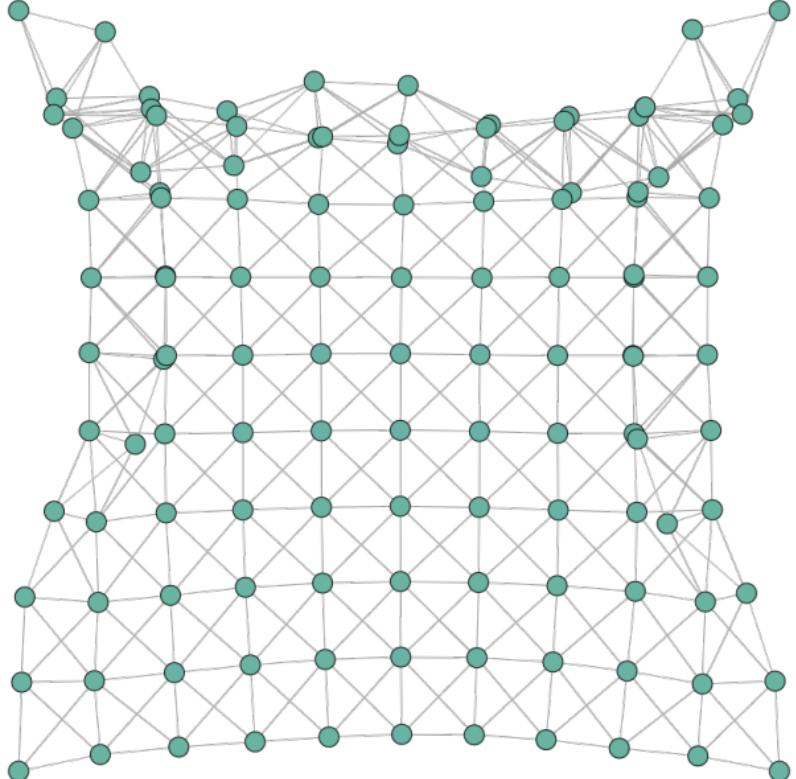


Figure 54: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners



Figure 55: **Newton with projected Hessian:**
1: f with length & Constrained spring scenario:
2: Corners

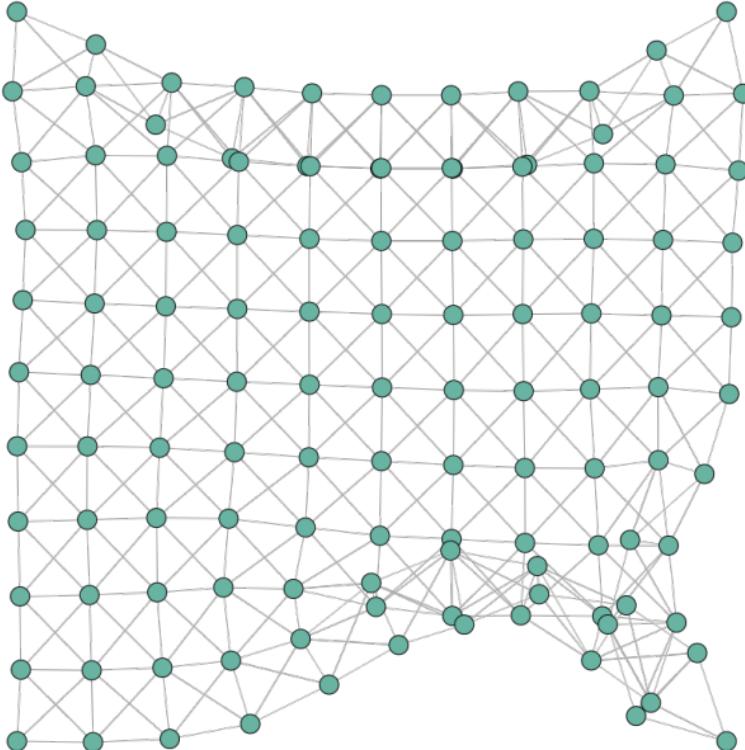


Figure 56: **Standard Newton:** 2: f with length
with positive local hessian & Constrained spring
scenario: 2: Corners

Grid size: 20

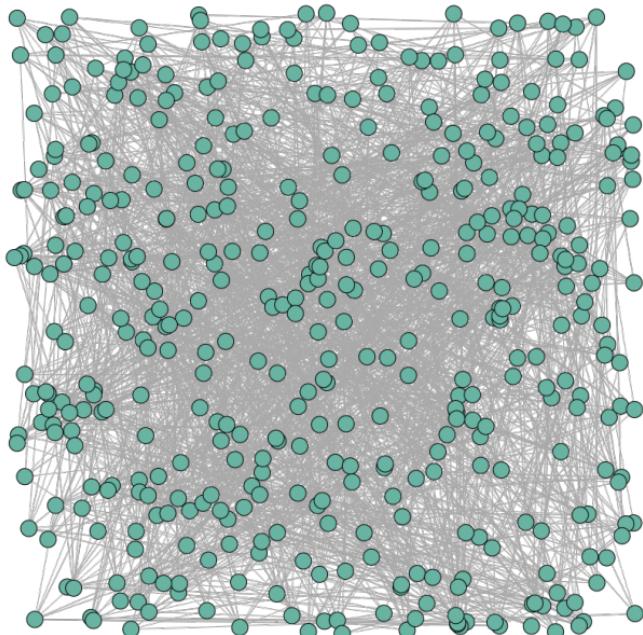


Figure 57: **Gauss Newton:** 0: f without length & Constrained spring scenario: 2: Corners – (As Gauss Newton executable uses the Newton Methods without hessian modification, and in this case, the LLT factorization fails, which makes marks the end of the method like all other **Gauss Newton without length**)

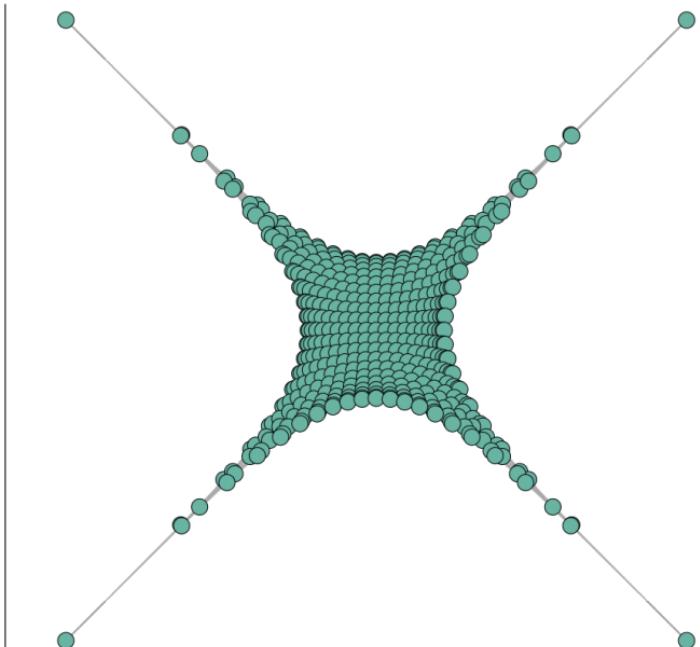


Figure 58: **Standard Newton:** 0: f without length & Constrained spring scenario: 2: Corners

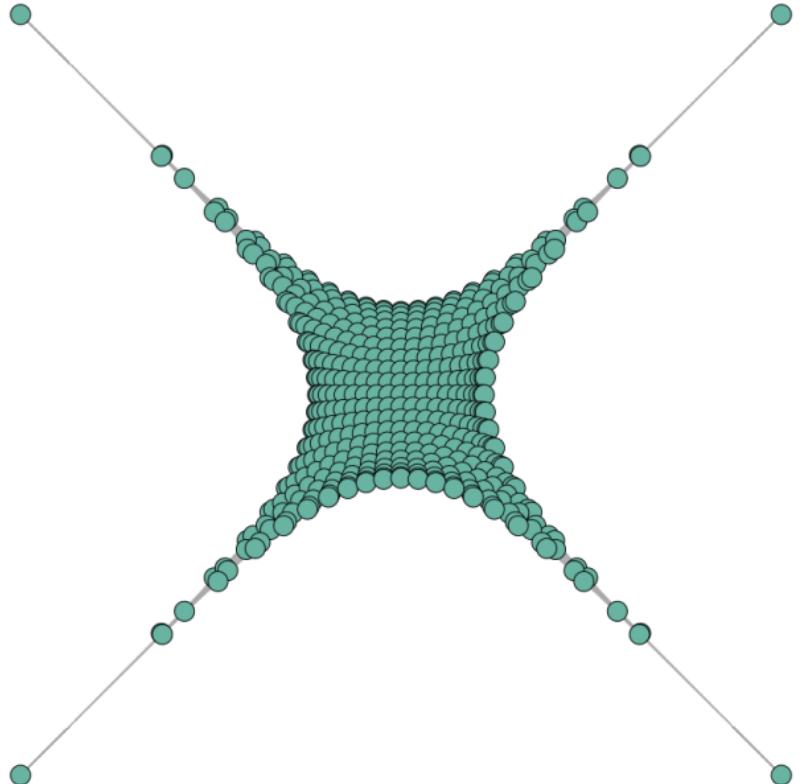


Figure 59: **Gradient Descent**: 0: f without length & Constrained spring scenario: 1: Sides

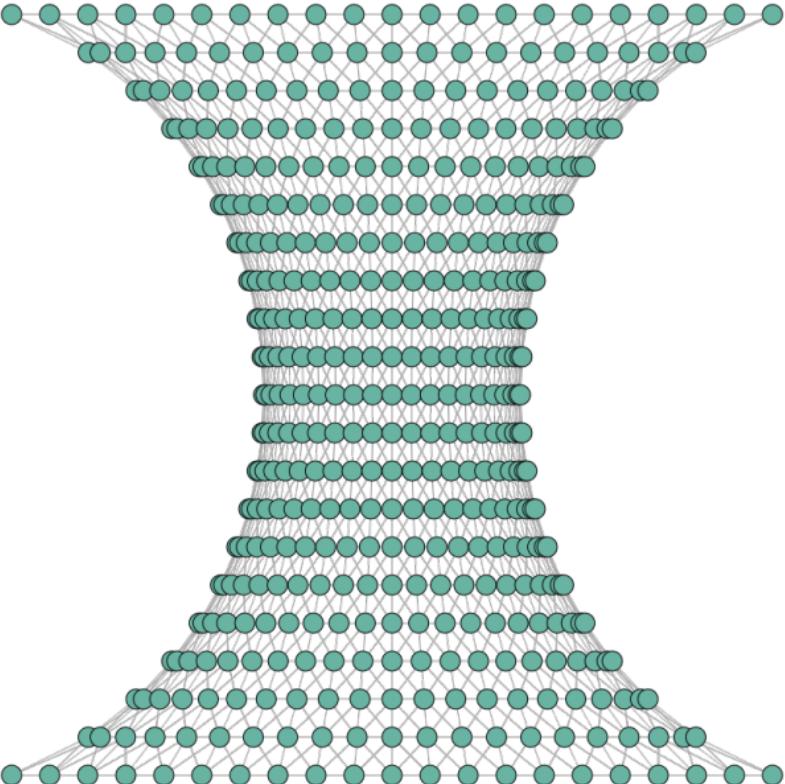


Figure 60: **Gradient Descent**: 0: f without length & Constrained spring scenario: 1: Sides

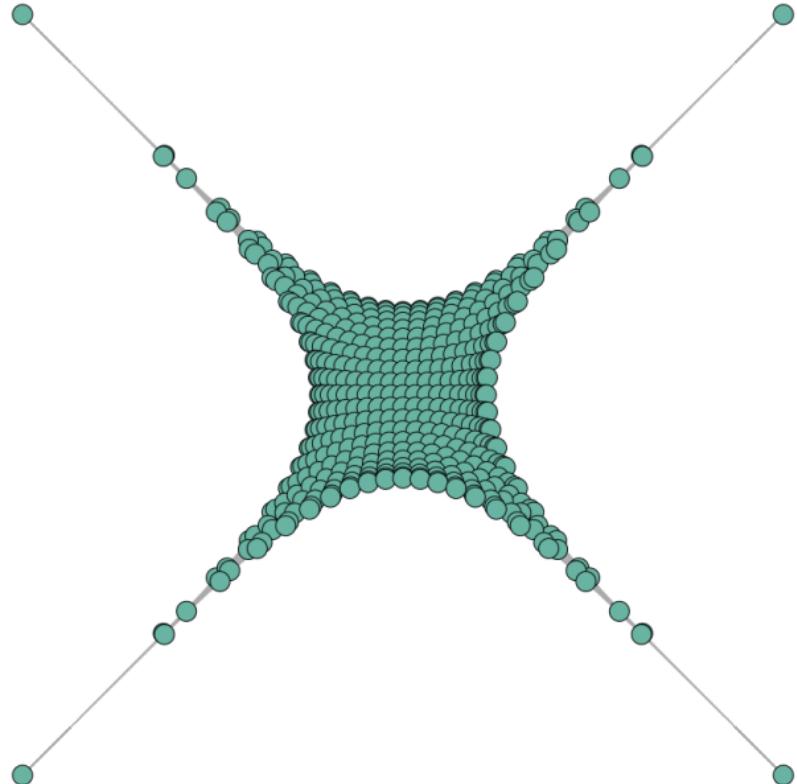


Figure 61: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

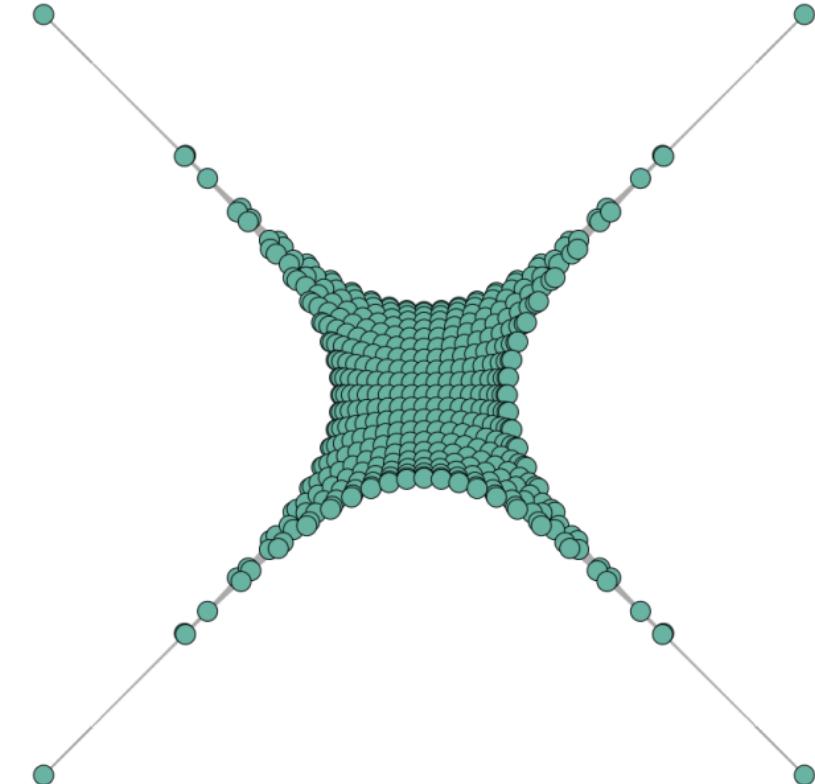


Figure 62: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

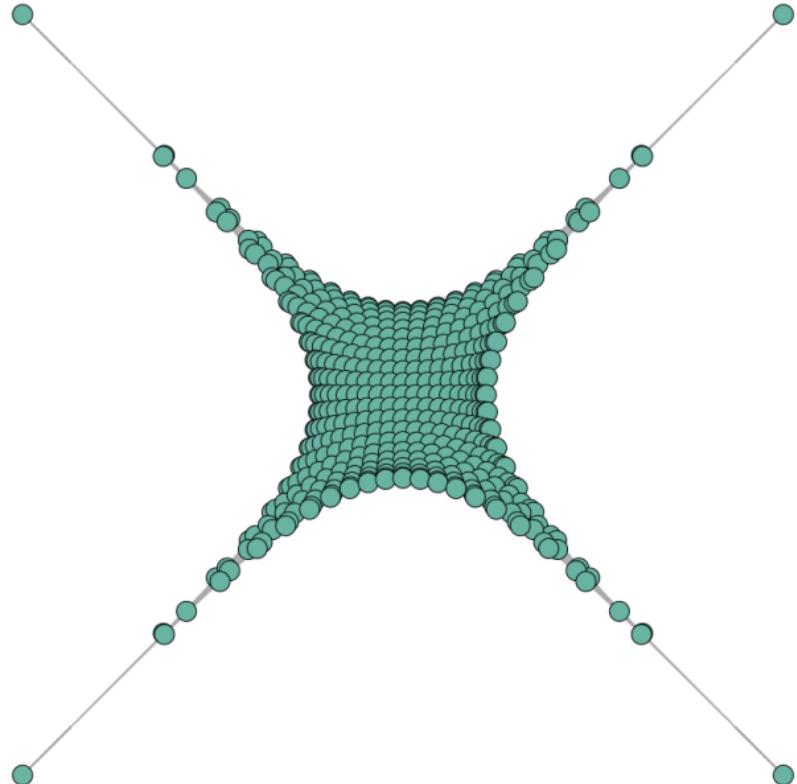


Figure 63: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

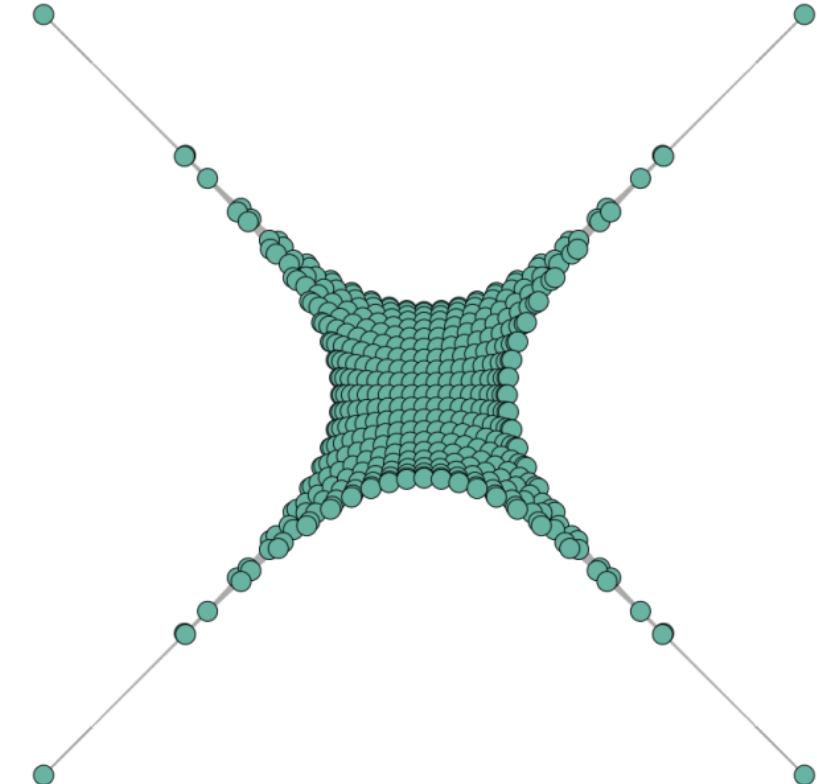


Figure 64: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

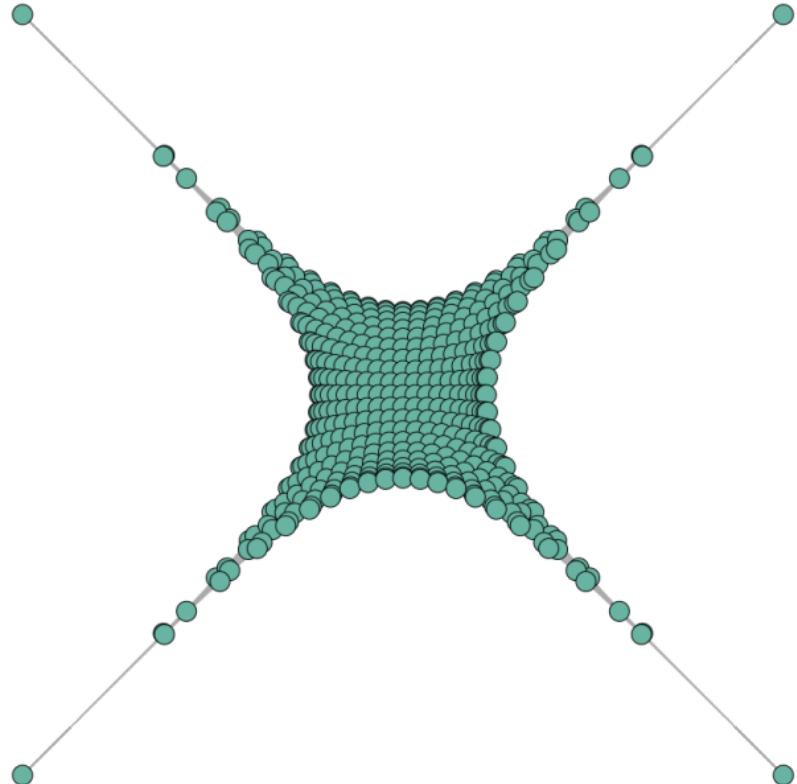


Figure 65: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

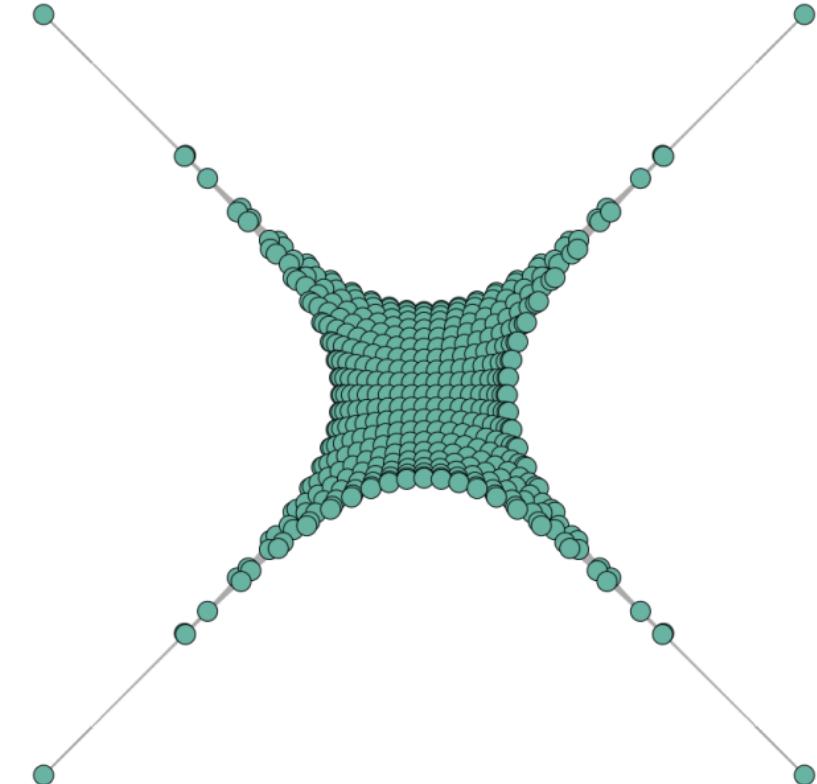


Figure 66: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

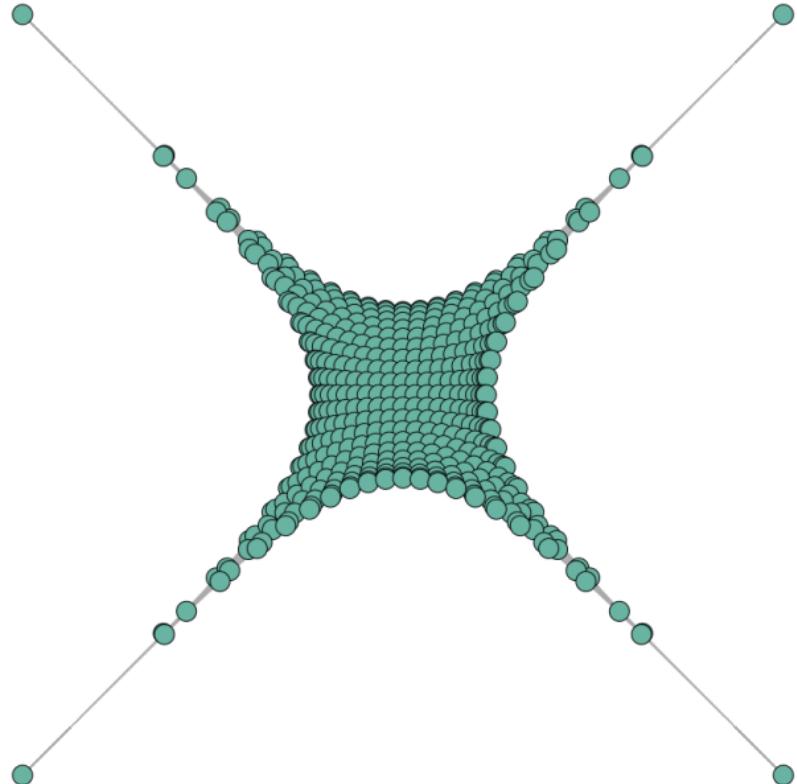


Figure 67: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

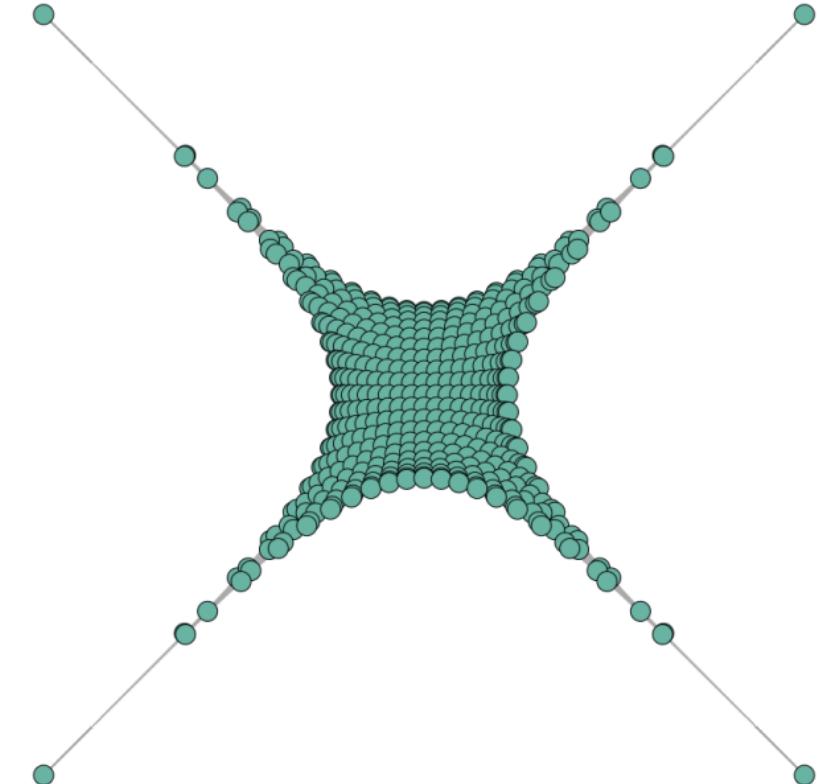


Figure 68: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

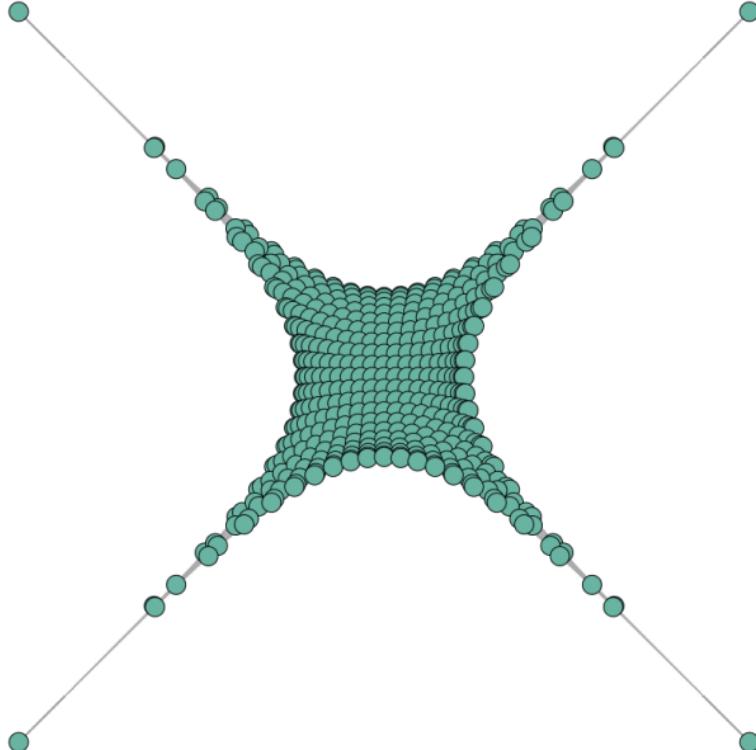


Figure 69: **L-BFGS**: 0: f without length & Constrained spring scenario: 2: Corners

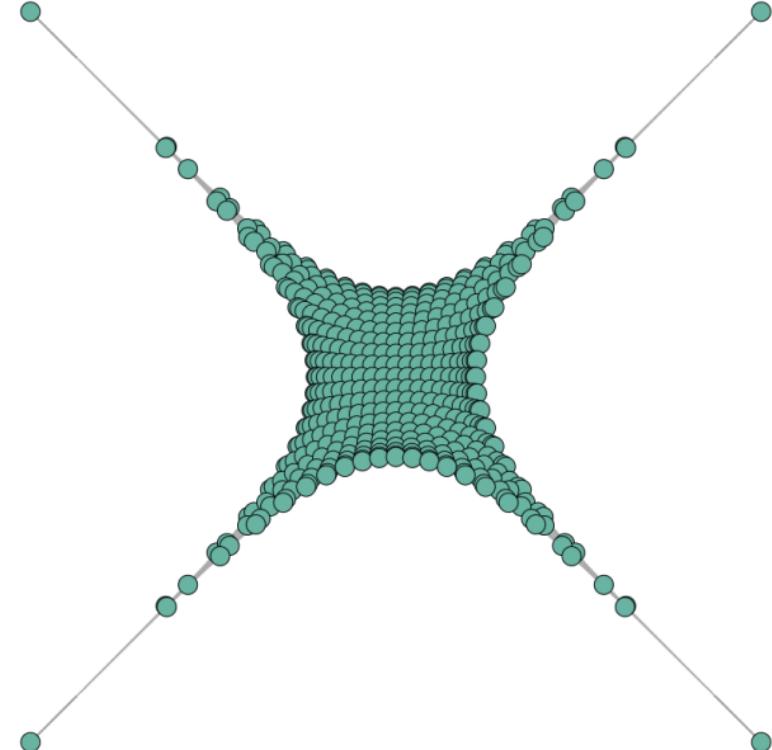


Figure 70: **Newton with projected Hessian**: 0: f without length & Constrained spring scenario: 2: Corners

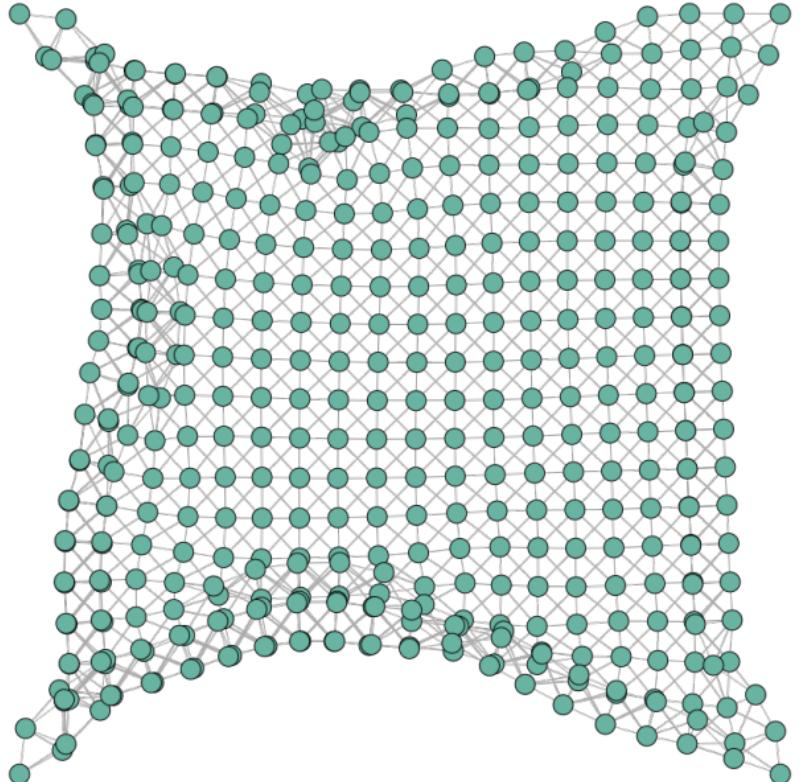


Figure 71: **Gauss Newton**: 1: f with length & Constrained spring scenario: 2: Corners

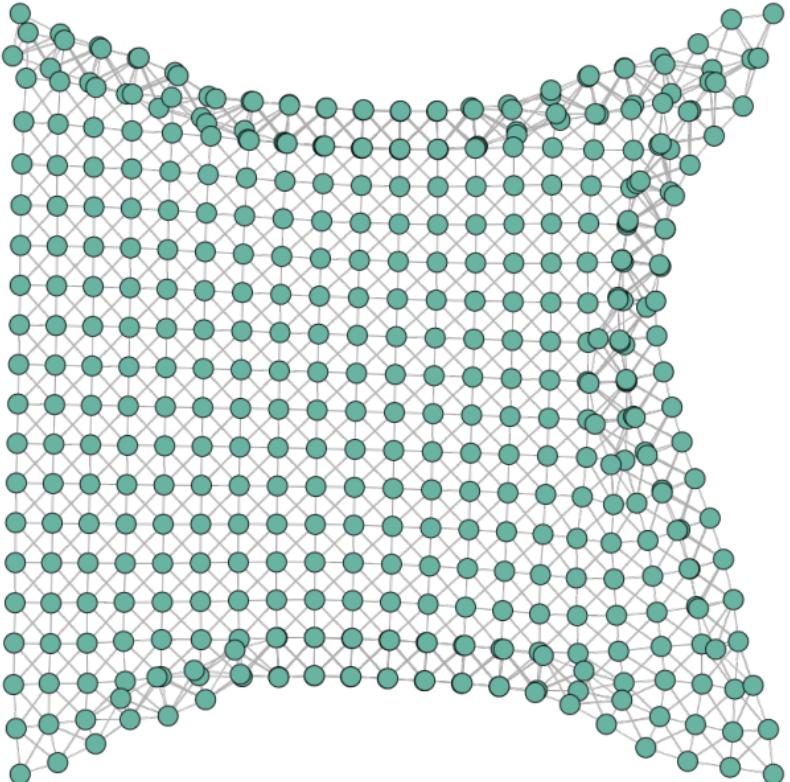


Figure 72: **Gradient Descent**: 1: f with length & Constrained spring scenario: 2: Corners

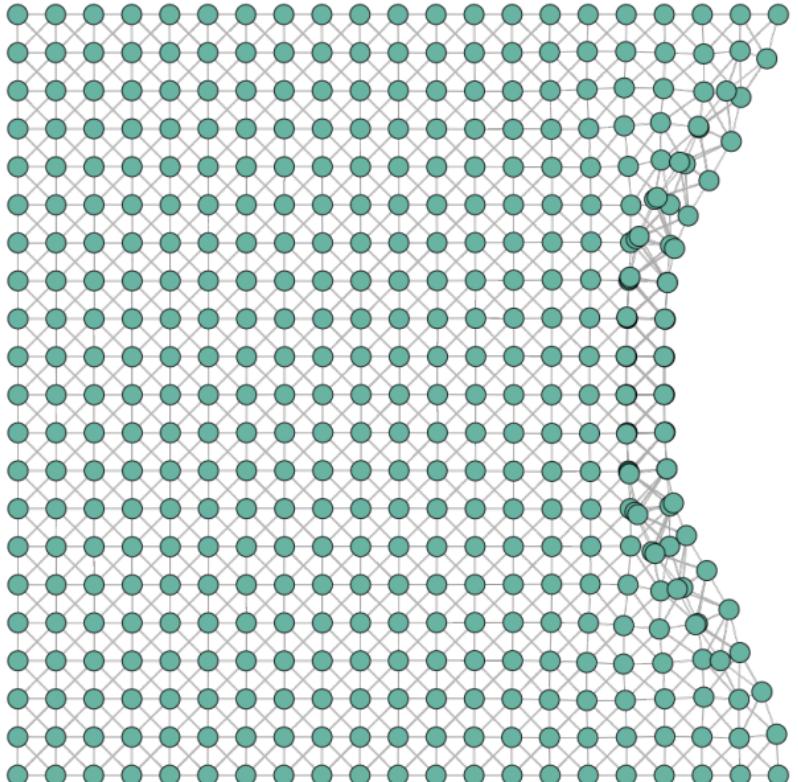


Figure 73: **Gradient Descent**: 1: f with length & Constrained spring scenario: 2: Corners

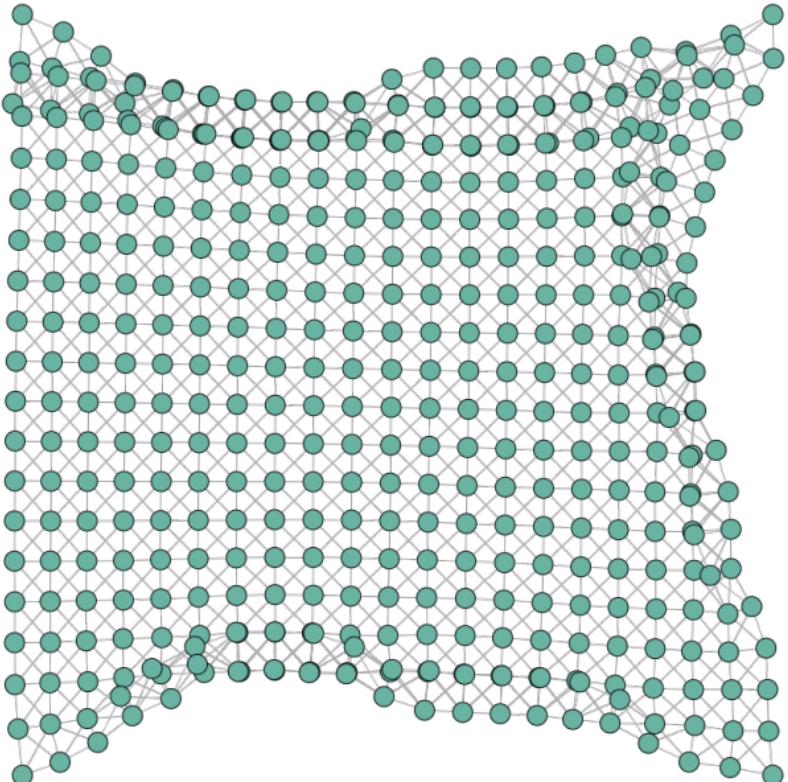


Figure 74: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

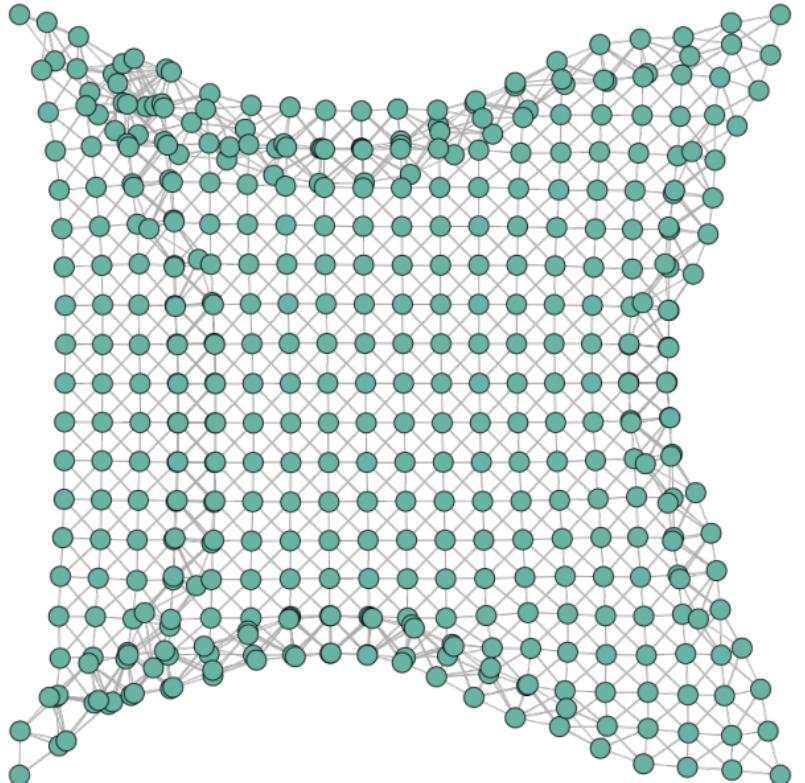


Figure 75: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

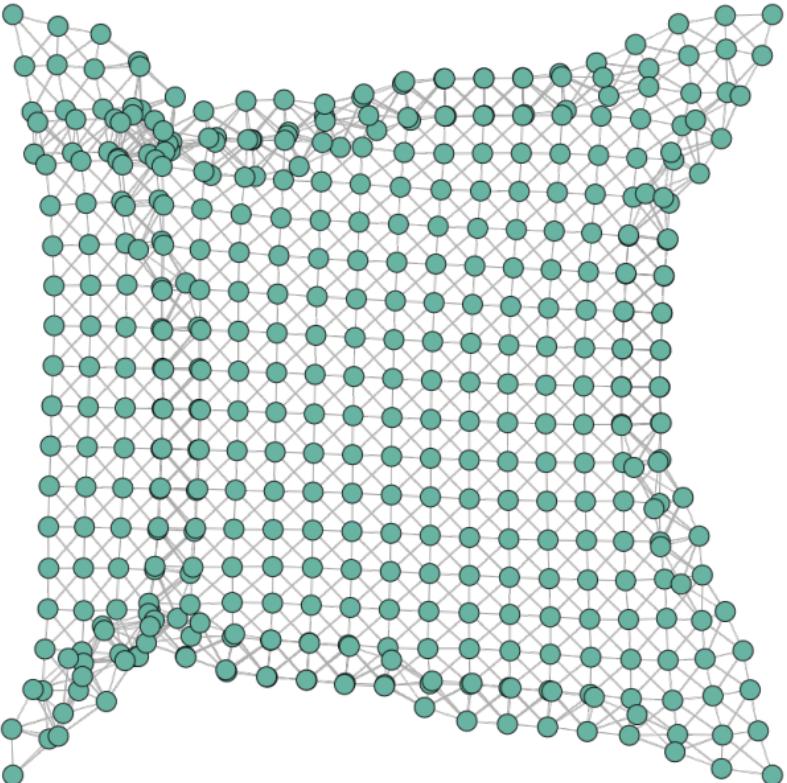


Figure 76: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

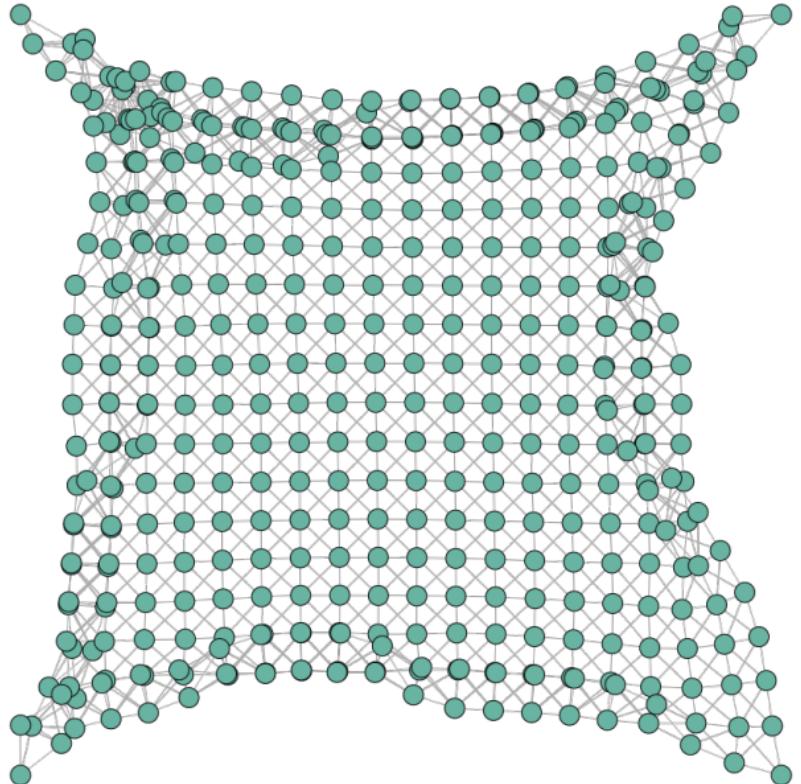


Figure 77: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

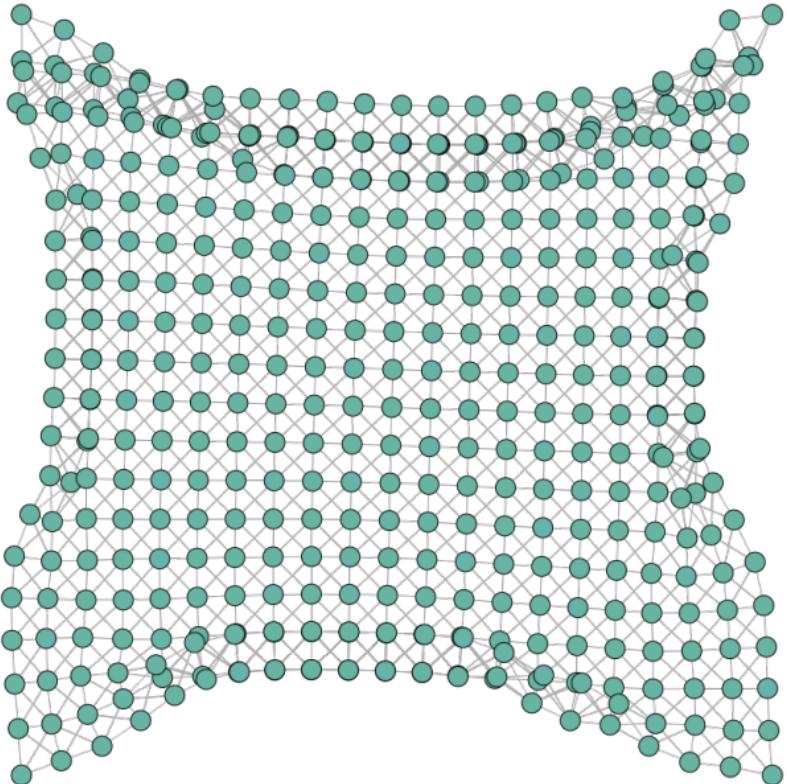


Figure 78: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

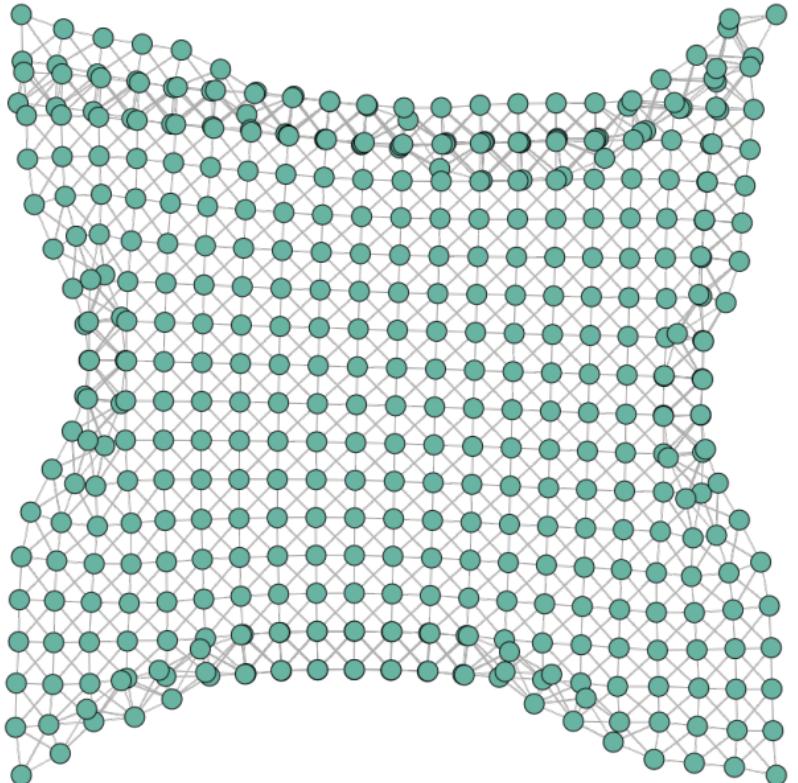


Figure 79: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

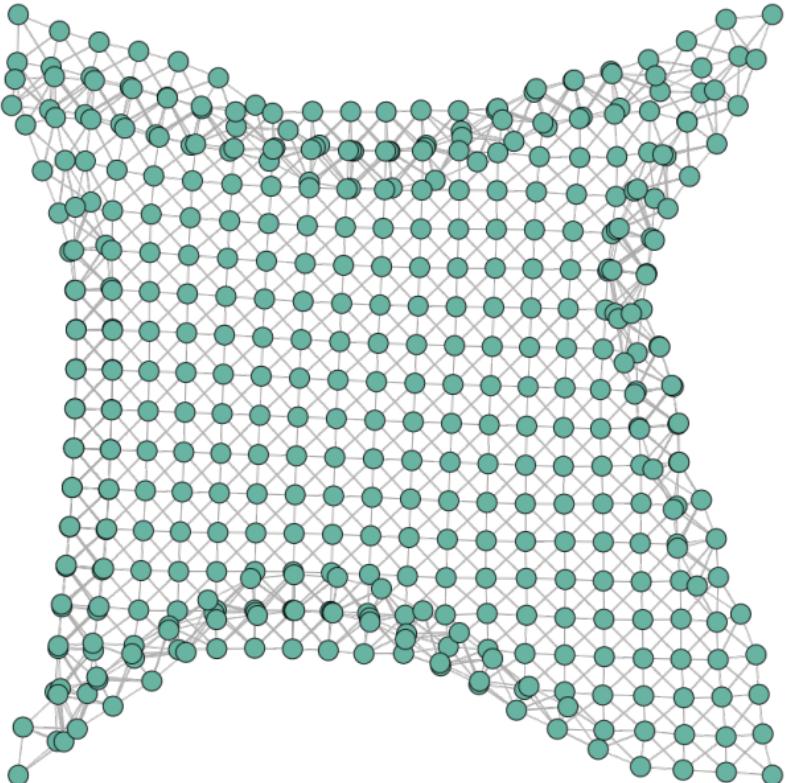


Figure 80: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

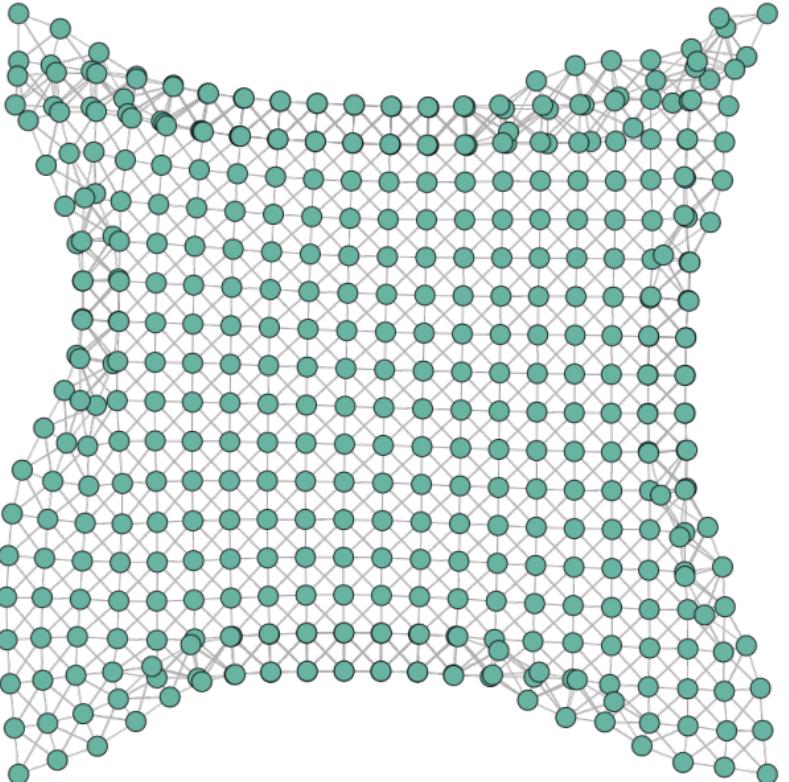


Figure 81: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

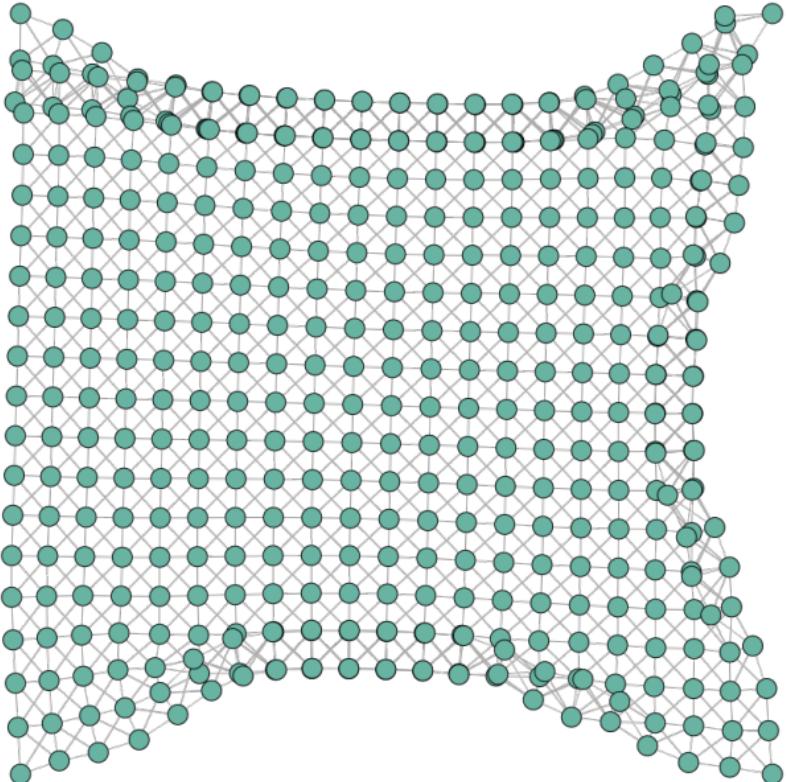


Figure 82: **L-BFGS**: 1: f with length & Constrained spring scenario: 2: Corners

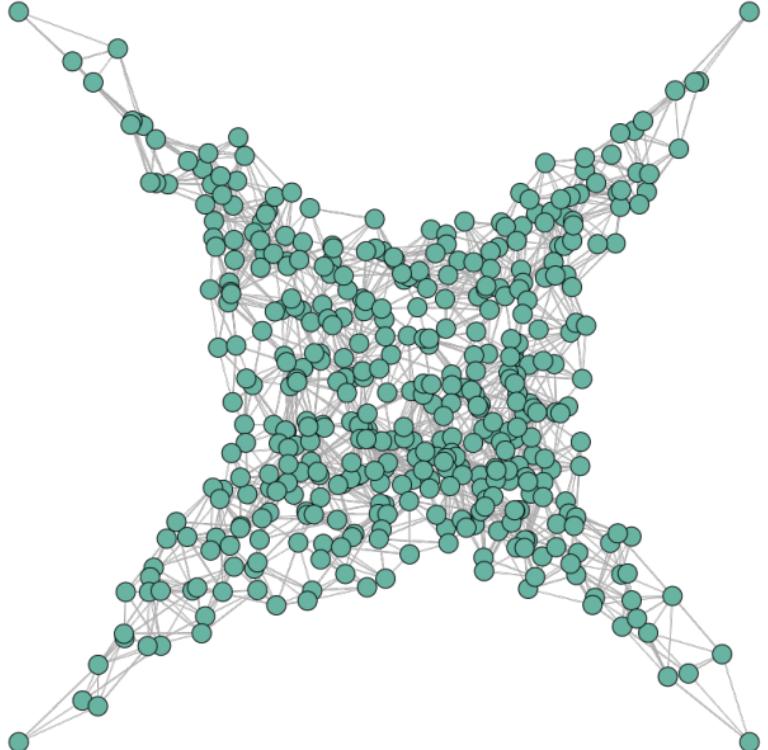


Figure 83: **Newton with projected Hessian:**
1: f with length & Constrained spring scenario:
2: Corners

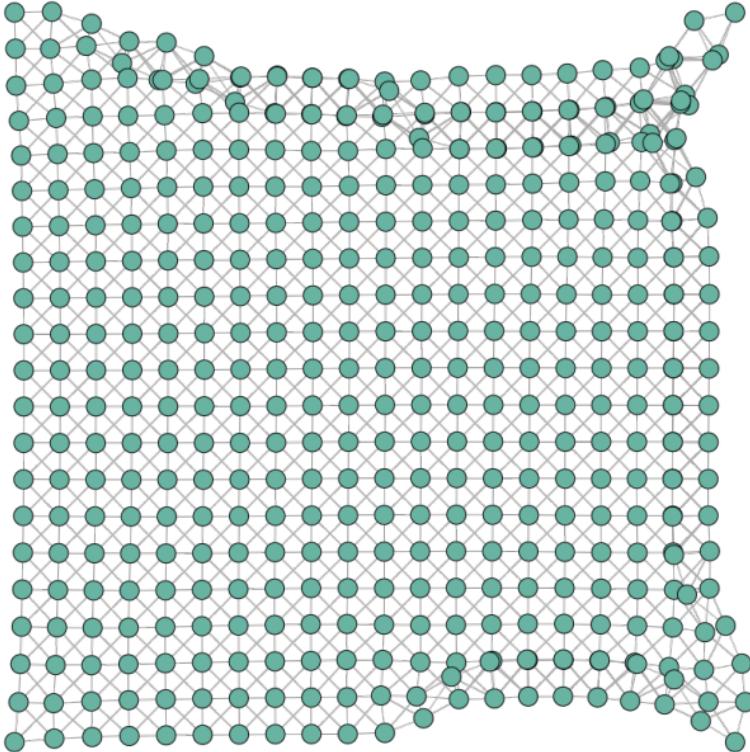


Figure 84: **Standard Newton:** 2: f with length
with positive local hessian & Constrained spring
scenario: 2: Corners

| Executable | Arg1 | Arg2 | Function index description | Newton method | Spring constraint scenario | History (m) | Grid side | Iterations | Total time [s] | Total time evaluation [s] | Total time evaluation percentage [%] | eval_f time [s] | eval_f evaluations | eval_f avg [s] | eval_g time [s] | eval_g evaluations | eval_g avg [s] | eval_g factor | eval_h time [s] | eval_h evaluations | eval_h avg [s] | eval_h factor | |
|-----------------|------|------|--|-------------------------------|----------------------------|-------------|-----------|------------|----------------|---------------------------|--------------------------------------|-----------------|--------------------|----------------|-----------------|--------------------|----------------|---------------|-----------------|--------------------|----------------|---------------|-----------|
| GaussNewton | 0 | - | 0: f without length | Standard Newton | Corners | - | 5 | 1 | 0.000747 | 0.000119 | 15.9304 | 0.00000 | 0 | nan | 0.00003 | 1 | 0.00003 | nan | 0.00009 | 1 | 0.00009 | nan | |
| NewtonMethods | 0 | 0 | 0: f without length | Standard Newton | Corners | - | 5 | 2 | 0.000399 | 0.00017 | 42.6065 | 0.00001 | 4 | 0.00000 | 0.00000 | 2 | 0.00000 | 1.33333 | 0.00016 | 2 | 0.00008 | 53.33333 | |
| GradientDescent | 0 | 1 | 0: f without length | Standard Newton | Sides | - | 5 | 12694 | 0.710848 | 0.509329 | 71.6509 | 0.49530 | 452808 | 0.00000 | 0.01403 | 12694 | 0.00000 | 1.01013 | 0.00000 | 0 | nan | nan | |
| GradientDescent | 0 | 2 | 0: f without length | Standard Newton | Sides | - | 5 | 4481 | 0.270623 | 0.180096 | 66.5487 | 0.17502 | 159083 | 0.00000 | 0.00507 | 4481 | 0.00000 | 1.02901 | 0.00000 | 0 | nan | nan | |
| LBFGS | 0 | 8 | 0: f without length | Standard Newton | Corners | 8 | 5 | 799 | 0.010789 | 0.004768 | 44.1932 | 0.00376 | 3054 | 0.00000 | 0.0101 | 800 | 0.00000 | 1.02084 | 0.00000 | 0 | nan | nan | |
| LBFGS | 0 | 9 | 0: f without length | Standard Newton | Corners | 9 | 5 | 1228 | 0.015922 | 0.005962 | 37.445 | 0.00471 | 4610 | 0.00000 | 0.00126 | 1229 | 0.00000 | 1.00112 | 0.00000 | 0 | nan | nan | |
| LBFGS | 0 | 10 | 0: f without length | Standard Newton | Corners | 10 | 5 | 859 | 0.011162 | 0.004125 | 36.9557 | 0.00326 | 3220 | 0.00000 | 0.00086 | 860 | 0.00000 | 0.98767 | 0.00000 | 0 | nan | nan | |
| LBFGS | 0 | 11 | 0: f without length | Standard Newton | Corners | 11 | 5 | 1013 | 0.012681 | 0.004915 | 38.7588 | 0.00381 | 3754 | 0.00000 | 0.00110 | 1014 | 0.00000 | 1.06872 | 0.00000 | 0 | nan | nan | |
| LBFGS | 0 | 12 | 0: f without length | Standard Newton | Corners | 12 | 5 | 719 | 0.008649 | 0.003729 | 43.1148 | 0.00289 | 2597 | 0.00000 | 0.00084 | 720 | 0.00000 | 1.04231 | 0.00000 | 0 | nan | nan | |
| LBFGS | 0 | 13 | 0: f without length | Standard Newton | Corners | 13 | 5 | 952 | 0.012662 | 0.004656 | 36.7714 | 0.00368 | 3522 | 0.00000 | 0.00098 | 953 | 0.00000 | 0.98016 | 0.00000 | 0 | nan | nan | |
| LBFGS | 0 | 14 | 0: f without length | Standard Newton | Corners | 14 | 5 | 852 | 0.011712 | 0.004078 | 34.819 | 0.00316 | 3113 | 0.00000 | 0.00092 | 853 | 0.00000 | 1.05722 | 0.00000 | 0 | nan | nan | |
| LBFGS | 0 | 15 | 0: f without length | Standard Newton | Corners | 15 | 5 | 974 | 0.013623 | 0.004793 | 35.1831 | 0.00372 | 3615 | 0.00000 | 0.00107 | 975 | 0.00000 | 1.06432 | 0.00000 | 0 | nan | nan | |
| LBFGS | 0 | 16 | 0: f without length | Standard Newton | Corners | 16 | 5 | 885 | 0.012294 | 0.004205 | 34.2037 | 0.00330 | 3193 | 0.00000 | 0.00091 | 886 | 0.00000 | 0.98972 | 0.00000 | 0 | nan | nan | |
| NewtonMethods | 2 | 0 | 0: f without length | Newton with projected Hessian | Corners | - | 5 | 2 | 0.000262 | 0.0001 | 38.1679 | 0.00000 | 4 | 0.00000 | 0.00000 | 2 | 0.00000 | 1.50000 | 0.00009 | 2 | 0.00005 | 46.50000 | |
| GaussNewton | 1 | - | 1: f with length | | Standard Newton | Corners | - | 5 | 64 | 0.00656 | 0.002652 | 40.4268 | 0.00042 | 206 | 0.00000 | 0.00069 | 64 | 0.00001 | 5.31827 | 0.00155 | 64 | 0.00002 | 11.93330 |
| GradientDescent | 1 | 1 | 1: f with length | | Corners | - | 5 | 6269 | 0.39135 | 0.247737 | 63.3032 | 0.24112 | 221556 | 0.00000 | 0.00662 | 6269 | 0.00000 | 0.96987 | 0.00000 | 0 | nan | nan | |
| GradientDescent | 1 | 2 | 1: f with length | | Corners | - | 5 | 2157 | 0.143302 | 0.089039 | 62.1338 | 0.08669 | 76999 | 0.00000 | 0.00235 | 2157 | 0.00000 | 0.96600 | 0.00000 | 0 | nan | nan | |
| LBFGS | 1 | 8 | 1: f with length | | Corners | 8 | 5 | 1651 | 0.023795 | 0.010094 | 42.4207 | 0.00804 | 6153 | 0.00000 | 0.00205 | 1652 | 0.00000 | 0.94920 | 0.00000 | 0 | nan | nan | |
| LBFGS | 1 | 9 | 1: f with length | | Corners | 9 | 5 | 1875 | 0.022669 | 0.009164 | 40.4253 | 0.00726 | 7092 | 0.00000 | 0.00190 | 1876 | 0.00000 | 0.99144 | 0.00000 | 0 | nan | nan | |
| LBFGS | 1 | 10 | 1: f with length | | Corners | 10 | 5 | 1652 | 0.021032 | 0.007913 | 37.6236 | 0.00620 | 6124 | 0.00000 | 0.00172 | 1653 | 0.00000 | 1.02665 | 0.00000 | 0 | nan | nan | |
| LBFGS | 1 | 11 | 1: f with length | | Corners | 11 | 5 | 1791 | 0.023258 | 0.009073 | 39.0102 | 0.00710 | 6519 | 0.00000 | 0.00198 | 1792 | 0.00000 | 1.01287 | 0.00000 | 0 | nan | nan | |
| LBFGS | 1 | 12 | 1: f with length | | Corners | 12 | 5 | 1836 | 0.023967 | 0.009197 | 38.3736 | 0.00729 | 6665 | 0.00000 | 0.00191 | 1837 | 0.00000 | 0.94973 | 0.00000 | 0 | nan | nan | |
| LBFGS | 1 | 13 | 1: f with length | | Corners | 13 | 5 | 1839 | 0.023386 | 0.009075 | 38.8053 | 0.00709 | 6626 | 0.00000 | 0.00198 | 1840 | 0.00000 | 1.00625 | 0.00000 | 0 | nan | nan | |
| LBFGS | 1 | 14 | 1: f with length | | Corners | 14 | 5 | 1963 | 0.025993 | 0.009633 | 37.06 | 0.00751 | 7079 | 0.00000 | 0.00213 | 1964 | 0.00000 | 1.02077 | 0.00000 | 0 | nan | nan | |
| LBFGS | 1 | 15 | 1: f with length | | Corners | 15 | 5 | 1805 | 0.026782 | 0.010276 | 38.3691 | 0.00801 | 6493 | 0.00000 | 0.00227 | 1806 | 0.00000 | 1.01823 | 0.00000 | 0 | nan | nan | |
| LBFGS | 1 | 16 | 1: f with length | | Corners | 16 | 5 | 1848 | 0.02579 | 0.008677 | 33.6448 | 0.00683 | 6755 | 0.00000 | 0.00185 | 1849 | 0.00000 | 0.99067 | 0.00000 | 0 | nan | nan | |
| NewtonMethods | 2 | 1 | 1: f with length | Newton with projected Hessian | Standard Newton | Corners | - | 5 | 8 | 0.000695 | 0.000219 | 31.5108 | 0.00002 | 21 | 0.00000 | 0.00001 | 8 | 0.00000 | 1.02717 | 0.00019 | 8 | 0.00002 | 21.34239 |
| NewtonMethods | 0 | 2 | 2: f with length with positive local hessian | | Standard Newton | Corners | - | 5 | 61 | 0.019804 | 0.015783 | 79.696 | 0.00000 | 186 | 0.00000 | 0.00006 | 61 | 0.00000 | 0.97382 | 0.01553 | 61 | 0.00025 | 247.94146 |
| GaussNewton | 0 | - | 0: f without length | Standard Newton | Corners | - | 10 | 1 | 0.001136 | 0.000193 | 16.9894 | 0.00000 | 0 | nan | 0.00007 | 1 | 0.00007 | nan | 0.00012 | 1 | 0.00012 | nan | |
| NewtonMethods | 0 | 0 | 0: f without length | Standard Newton | Corners | - | 10 | 3 | 0.001221 | 0.000397 | 32.5143 | 0.00004 | 8 | 0.00001 | 0.00002 | 3 | 0.00001 | 1.13333 | 0.00034 | 3 | 0.00011 | 22.66667 | |
| GradientDescent | 0 | 1 | 0: f without length | Standard Newton | Sides | - | 10 | 48455 | 8.41591 | 7.73977 | 91.9659 | 7.52374 | 1729685 | 0.00000 | 0.21603 | 48455 | 0.00000 | 1.02497 | 0.00000 | 0 | nan | nan | |
| GradientDescent | 0 | 2 | 0: f without length | Standard Newton | Sides | - | 10 | 15012 | 3.03963 | 2.66133 | 87.5546 | 2.57982 | 536060 | 0.00000 | 0.08151 | 15012 | 0.00001 | 1.12825 | 0.00000 | 0 | nan | nan | |
| LBFGS | 0 | 8 | 0: f without length | Standard Newton | Corners | 8 | 10 | 2543 | 0.15415 | 0.057438 | 37.2611 | 0.04503 | 9470 | 0.00000 | 0.01241 | 2544 | 0.00000 | 1.02615 | 0.00000 | 0 | nan | nan | |