



Applied Optimization

Exercise 1 - Convex Sets

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1 Convex Sets

1. Example sets

Sketch the following sets in \mathbb{R}^2

1. SPAN
2. SPAN
3. AFF
4. CONV

2. Convexity

Let $C \subseteq \mathbb{R}^n$ be a convex set, with $x_1, \dots, x_k \in C$, and let $\theta_1, \dots, \theta_k \in \mathbb{R}$ satisfy $\theta_i \geq 0, \theta_1 + \dots + \theta_k = 1$. Show that $\theta_1 x_1 + \dots + \theta_k x_k \in C$. (The definition of convexity is that this holds for $k = 2$; you must show it for arbitrary k .) Hint. Use induction on k .

3. Linear Equations

Show that the solution set of linear equations $\{x | Ax = b\}$ with $x \in \mathbb{R}^n, A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$ is an affine set.

4. Linear Inequalities

1. Show that the solution set of linear inequalities $\{x | Ax \preceq b, Cx = d\}$ with $x \in \mathbb{R}^n, A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m, C \in \mathbb{R}^{k \times n}$ and $d \in \mathbb{R}^k$ is a convex set. Here \preceq means componentwise less or equal.
2. Is it an affine set?

5. Voronoi description of halfspace

Let a and b be distinct points in R^n . Show that the set of all points that are closer (in Euclidean norm) to a than b , i.e., $\{x \mid \|x - a\|^2 \leq \|x - b\|^2\}$, is a halfspace. Describe it explicitly as an inequality of the form $c^T x \leq d$. Draw a picture.

2 Convex Illumination Problem

Show that the solution $p^* = (p_1^*, p_2^*, \dots, p_n^*)^T \in R^n$ of the non-convex illumination problem from the lecture

$$\begin{aligned} & \text{minimize} && \max_{k=1\dots m} |\log I_k - \log I_{des}| \\ & \text{subject to} && 0 \leq p_j \leq p_{max}, \quad j = 1\dots n \end{aligned}$$

with $I_k = \sum_{j=1}^n a_{kj} p_j$ for geometric constants $a_{kj} \in R$, a constant desired illumination $I_{des} \in R$ and an upper bound $p_{max} \in R$ on the lamp power, is identical to the solution of the following equivalent (convex) problem

$$\begin{aligned} & \text{minimize} && \max_{k=1\dots m} h(I_k/I_{des}) \\ & \text{subject to} && 0 \leq p_j \leq p_{max}, \quad j = 1\dots n \end{aligned}$$

with $h(u) = \max\{u, \frac{1}{u}\}$.