EXERCISE 3 - CONVEX OPTIMIZATION PROBLEMS

① **QCQP TO SOCP** CONVERT THE FOLLOWING QCQP INTO SOCP:

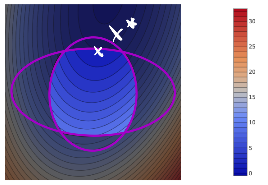
QUADRATICALLY CONSTRAINED QUADRATIC PROGRAM

SECOND-ORDER CONE PROGRAM

$$\begin{aligned} \text{MINIMIZE} \quad & x_1^2 + 4x_1x_2 + 4x_2^2 \\ \text{SUBJECT TO} \quad & 9x_1^2 + 16x_2^2 \leq 25 \\ & x_1 - x_2 = 1 \end{aligned} \implies \begin{aligned} & x^2 + 4xy + 4y^2 \quad ① \\ & 9x^2 + 16y^2 \leq 25 \quad ② \\ & x - y = 1 \quad ③ \end{aligned}$$

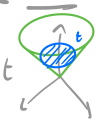
EXAMPLES: I QCQP:

$$\begin{aligned} & 3x^2 + 2(y-2)^2 - xy \\ & 2x^2 + 1.2y^2 - 2 \leq 0 \\ & x^2 + 4y^2 - 4 \leq 0 \end{aligned}$$

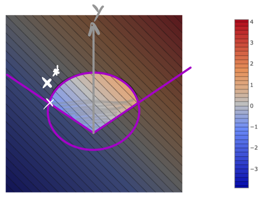


II SOCP:

$\|x\|_2 \leq t$

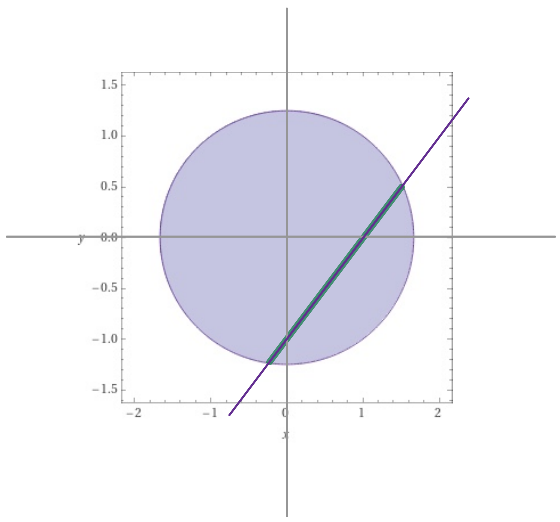


$$\begin{aligned} & x + y \\ & \sqrt{x^2} \leq 1.5y \\ & x^2 + 2y^2 \leq 1 \end{aligned}$$



② $9x^2 \leq -16y^2 + 25 \iff 9x^2 \leq -(4y-5)(4y+5)$

③ $-y = 1 - x \iff y = x - 1$



TODO FIND x_1 & x_2 WITH SUBJECT

IS THAT ENOUGH?

OR JUST SAY QCQP ⊂ SOCP

② LINEAR PROGRAMMING TRANSFORM :

FOR THE FOLLOWING OPTIMIZATION PROBLEM:

MINIMIZE $\|(2x_1 + 3x_2 , -3x_1)^T\|_\infty$ $\left\|\begin{pmatrix} 2x+3y \\ -3x \end{pmatrix}\right\|_\infty$

SUBJECT TO $|x_1 - 2x_2| \leq 3$ $|x-2y| \leq 3$

1. Express the problem as a a linear program.

TODO DO MINIMIZE FUNCTION

$\|x\|_2 = \sqrt{\sum x_i^2} = (\sum x_i^2)^{1/2}$

$\|x\|_\infty = (\sum x_i^\infty)^{1/\infty}$?

SUBJECT TO
$$x_1 - 2x_2 \leq 3$$
$$2x_2 - x_1 \leq 3$$

2. Convert the LP so that all variables are in R+ and there is no other inequality constraints than ... ≥ 0.

SUBJECT TO ?
$$2x_2 - x_1 \geq -3 \iff 2x_2 - x_1 + 3 \geq 0$$
$$x_1 - 2x_2 \geq -3 \iff x_1 - 2x_2 + 3 \geq 0$$

③ TRANSFORM GENERAL LP TO STANDARD FORM

A GENERAL LINEAR PROGRAM HAS THE FORM

MINIMIZE $c^T x + d$

SUBJECT TO $Gx \preceq h$
$Ax = b$

WHERE $G \in \mathbb{R}^{m \times n}$ AND $A \in \mathbb{R}^{p \times n}$. TRANSFORM THE GENERAL LP TO ITS STANDARD FORM:

MINIMIZE $p^T x'$

SUBJECT TO $Bx' = e$
$x' \succeq 0$

Explain in detail the relation between the feasible sets, the optimal solutions, and the optimal values of the standard form LP and the original LP.