

*First proof.* This is immediate from (??): if we replace  $k$  by  $n - k$ , then the factors in the denominator simply interchange.  $\square$

*Second proof.* The theorem can be proved even without knowing a formula for  $\binom{n}{k}$ , by a bijective argument. Associate to every subset of an  $n$ -element set its complement:  $A \mapsto X \setminus A$ . This is a bijection, and it sends  $k$ -element subsets to  $(n - k)$ -element subsets. Thus there are as many  $k$ -element subsets as there are  $(n - k)$ -element subsets.  $\square$