

Proof. Without loss of generality, let $X = \{1, 2, \dots, m\}$. In order to define a map $f: X \rightarrow Y$ we need to make m choices, each time from n possibilities: $f(1)$ can take n different values, so can $f(2)$, and so on up to $f(m)$. Thus there are

$$\underbrace{n \cdots n}_m = n^m$$

different maps $X \rightarrow Y$.

