

This is our original definition:  $\binom{n}{k}$  is the number of unordered choices of  $k$  elements out of  $n$ . In a more abstract language, this is the number of  $k$ -element subsets of an  $n$ -element set.

We have proved that

$$\binom{n}{k} = \frac{n(n-1) \cdot \dots \cdot (n-k+1)}{k!} = \frac{n!}{k!(n-k)!}. \quad (1)$$