

Proof. Without loss of generality, $X = \{1, \dots, k\}$. We have n possibilities to choose $f(1)$. After this, for $f(2)$ there remain $n - 1$ possibilities, then $n - 2$ possibilities for $f(3)$, and so on up to $f(k)$, for which $n - (k - 1) = n - k + 1$ possibilities remain.

The above argument works for $k \leq n$, but the formula is true for $k > n$ as well: there are no injective maps in this case, and the product $n(n - 1) \cdots (n - k + 1)$ vanishes because it contains a zero factor. \square