Proof. Let $|X| = n$. With every subset $A \subset X$ we associate a map $\mathbf{1}_A \colon X \to \{0, 1\}$ (the *indicator function* of A) defined as

$$\mathbf{1}_A(x) = \begin{cases} 1, & \text{if } x \in A, \\ 0, & \text{if } x \notin A. \end{cases}$$

One can show that $A \mapsto \mathbf{1}_A$ is a bijection between the set of all subsets and the set of all maps $X \to \{0, 1\}$: different subsets define different maps and every map f is the indicator function of the subset $f^{-1}(1) \subset X$.
The number of all maps $X \to \{0, 1\}$ is $2^{|X|}$ by Theorem 2.4. By the bijection principle, the number of all subsets of X is the same. $\square$

Remark 2.4. The set of all maps $X \to Y$ is sometimes denoted by Y^X, so that Theorem 2.4 can be formulated as $\left|Y^X\right| = |Y|^{|X|}$. This is not the only reason for the notation Y^X. One can show that $Z^{X \cup Y} = Z^X \times Z^Y$ for disjoint X and Y, and $Z^{X \times Y} = (Z^X)^Y$.
Also, the Cartesian power

$$X^n = \underbrace{X \times \cdots \times X}_{n} = \{(x_1, \ldots, x_n) \mid x_i \in X \text{ for all } i\}$$

can be viewed as the set $X^{\{1,\ldots,n\}}$: a sequence $(x_1, \ldots, x_n)$ corresponds to a map $f\colon \{1, \ldots, n\} \to X$, $f(i) = x_i$.