A map is bijective iff at every element of Y ends exactly one arrow. By inverting the arrows we obtain the *inverse map* $f^{-1}\colon Y \to X$, which has the properties $f^{-1}(f(x)) = x$ for all $x \in X$ and $f(f^{-1}(y)) = y$ for all $y \in Y$.
If f is not bijective, then there is no inverse map f^{-1}. However, by abuse of notation one uses $f^{-1}(y)$ to denote the *preimage* of y:

$$f^{-1}(y) = \{x \in X \mid f(x) = y\}.$$

Similarly one can define the preimage $f^{-1}(B)$ of any subset $B \subset Y$.
Observe that

- f injective $\Leftrightarrow |f^{-1}(y)| \leq 1$ for all y;
- f surjective $\Leftrightarrow f^{-1}(y) \neq \varnothing$ for all y.

We can now formulate the quotient rule in the mathematical language.

> *If a map $f\colon X \to Y$ satisfies $|f^{-1}(y)| = k$ for all $y \in Y$, then $|Y| = \frac{|X|}{k}$.*

A special case of this is the bijection principle:

> *If a map $f\colon X \to Y$ is a bijection, then $|X| = |Y|$.*