

Statistical Inference for Data Science

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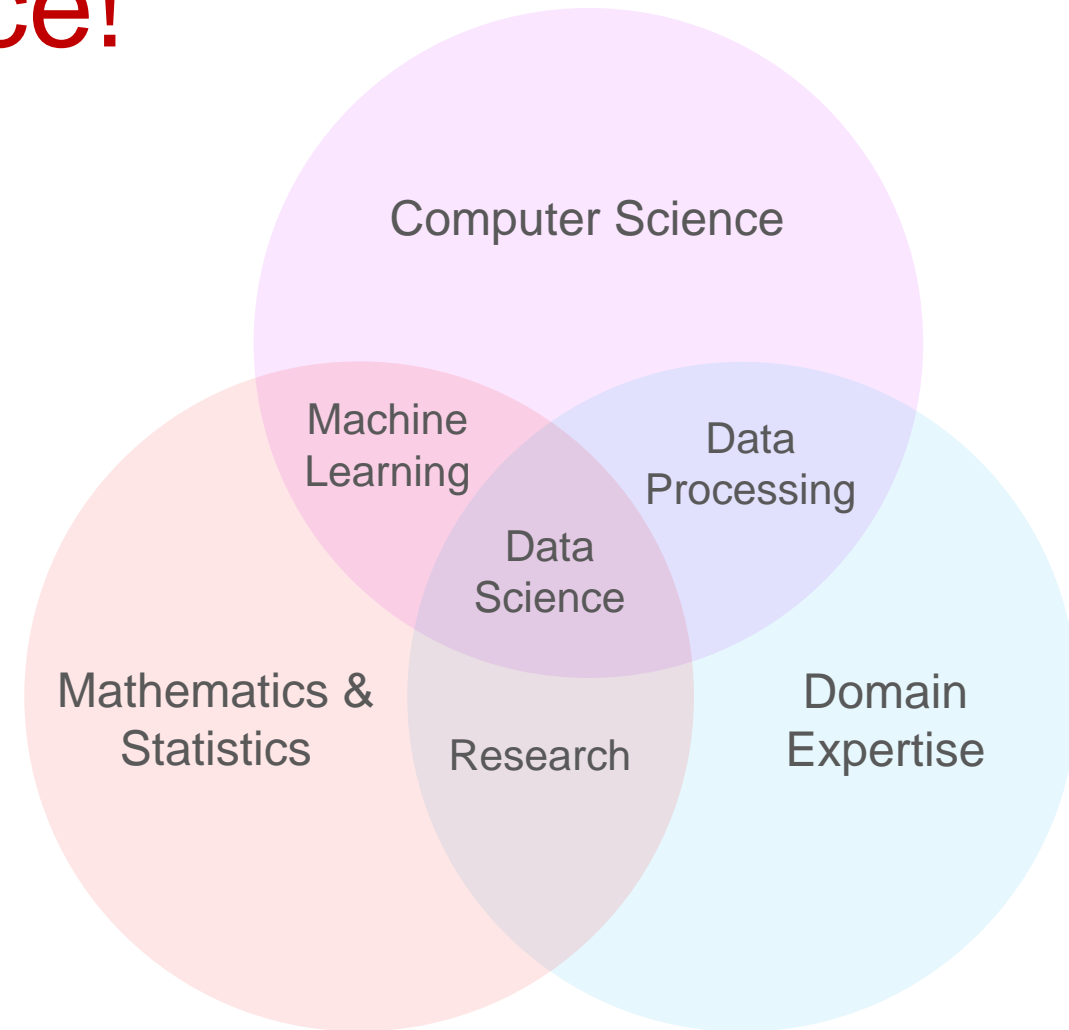


Welcome to Data Science!

Data Science uses

- Mathematics and Statistics
- Computer Science
- Domain expertise

on data to build information and extract knowledge.



Module 2

Day 1	Descriptive Statistics and Probability
Day 2	Parameter estimation
Day 3	Hypothesis testing
Day 4	Putting it all together
Project	Presentation session (date to be fixed!)



Caution

- This module aims to give a brief overview on basic statistics.
- That means in a short amount of time we'll see a lot.
- While this may be repetition for some,
- For others there may be a lot of new things.
- I'll try my best to accommodate everyone's needs.

Teaching

- Introductory lectures
- In-depth self-study of the content with notebooks
- Discussion sessions based on your questions
Please ask questions 😊
- I am open to modifications if wished for!



Project

Formal

- Group of 2-3 people
- 15min presentation, 15min discussion
- Half-day presence on presentation session

Content

- Choose your own data set
- answer research questions using statistics

Iris data set

- Due to time restrictions we use a **single** data set in this module
- **3 classes:** versicolor, setosa, virginica
- **4 characteristics**
petal: *length, width*
sepal: *length, width*



Iris Setosa



Iris Virginica

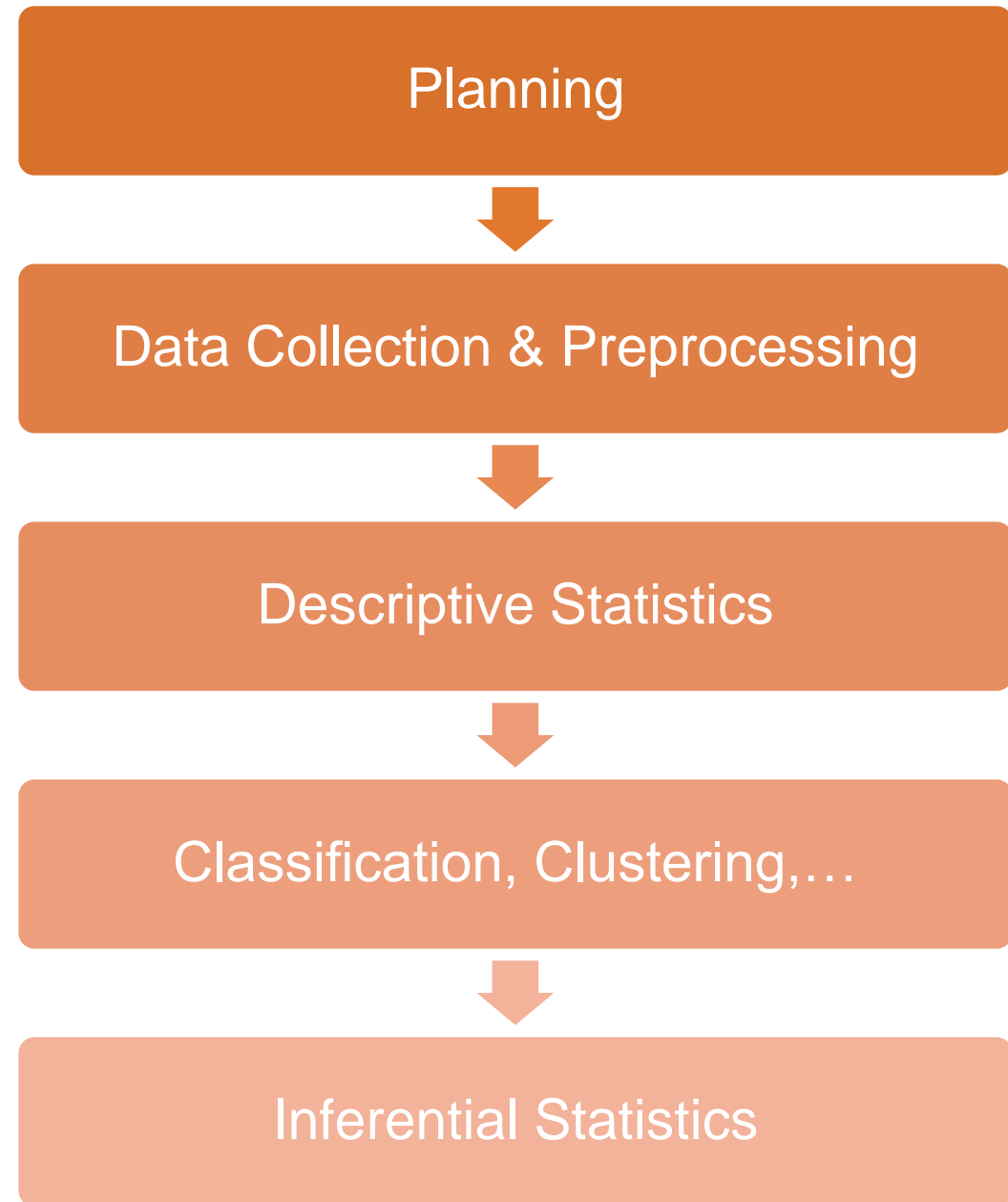


Iris Versicolor



Any questions so far?

General Procedure



Descriptive Statistics

Why?

- Get an overview of the data
- Identify Patterns
- Identify possible problems eg. outliers
- Get a feeling for the quality of the data

➡ good description is the basis for good inference

Descriptive Statistics

The two **main tasks** of descriptive statistics are

- the quantitative description and summary, and
- the graphical representation of data

What tools are suitable depends on the type of the variable we want to describe.

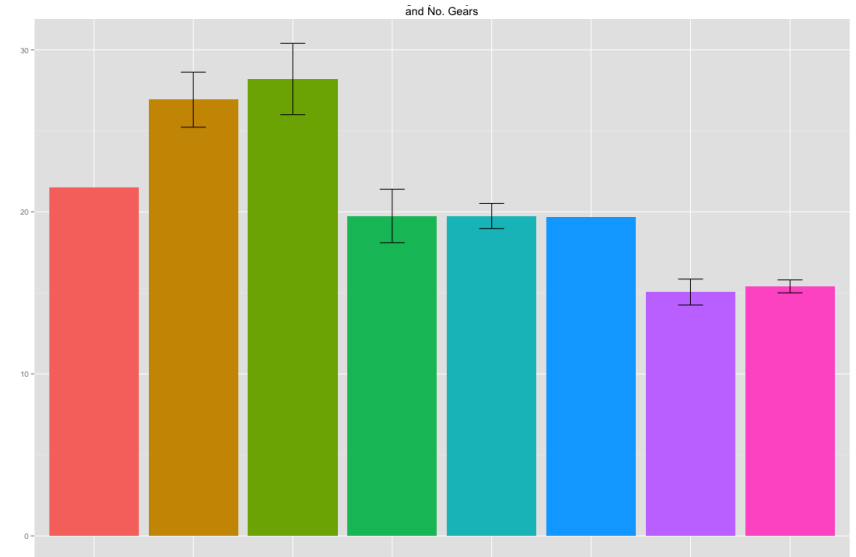
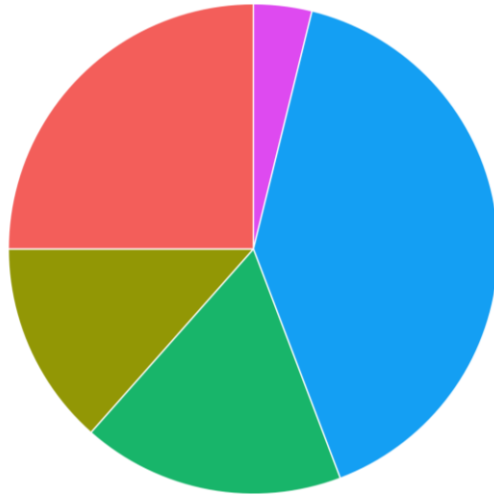
Categorical Variables

(quantitative)

- Absolute frequency (eg. number of female participants)
- Relative frequency (eq. number of female participants divided by the sample size)

Categorical Variables

(graphical)



(Either absolute or relative frequencies can be displayed)

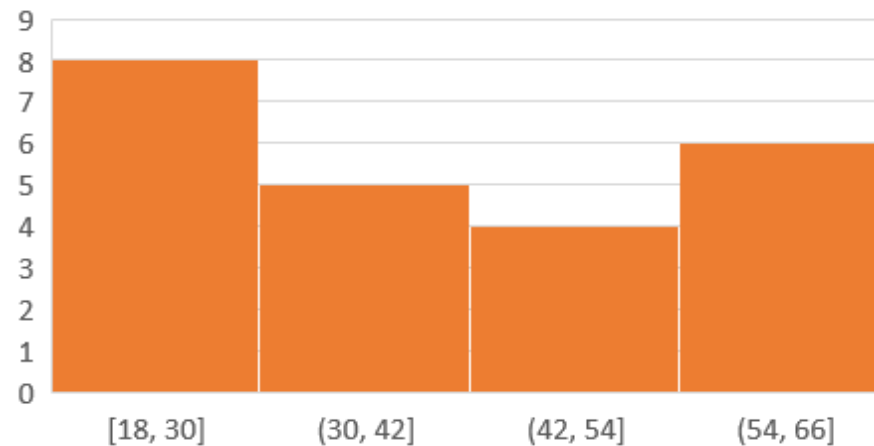
Numerical Variables

(categorization)

Summary tables

Age	Nr. of People
18-30	8
30-42	5
42-54	4
54-66	6

Histograms



Location

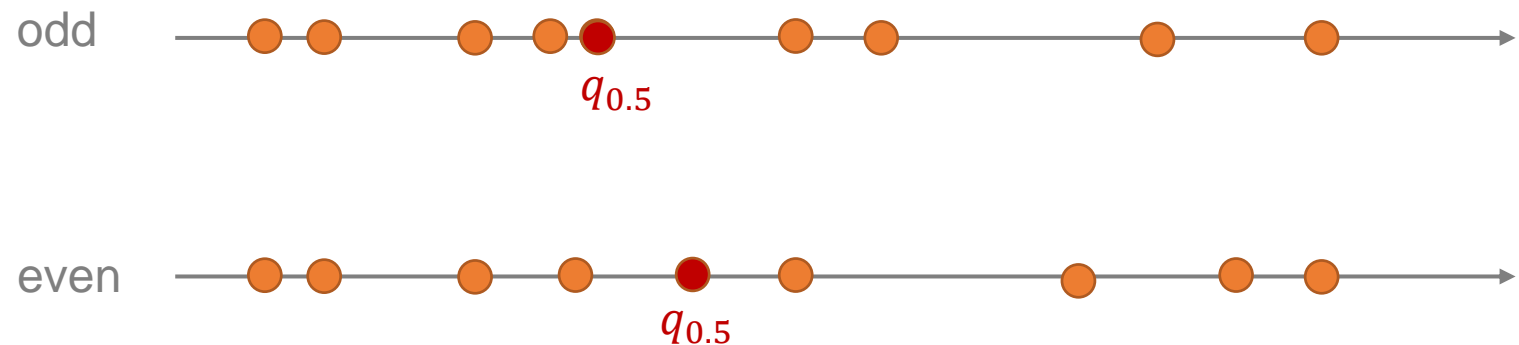
(Numerical Variables)

What are typical values for the variable X ?

- Sample Mean:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

- Sample Median: «center of the observations»



➡ median is more robust than the mean

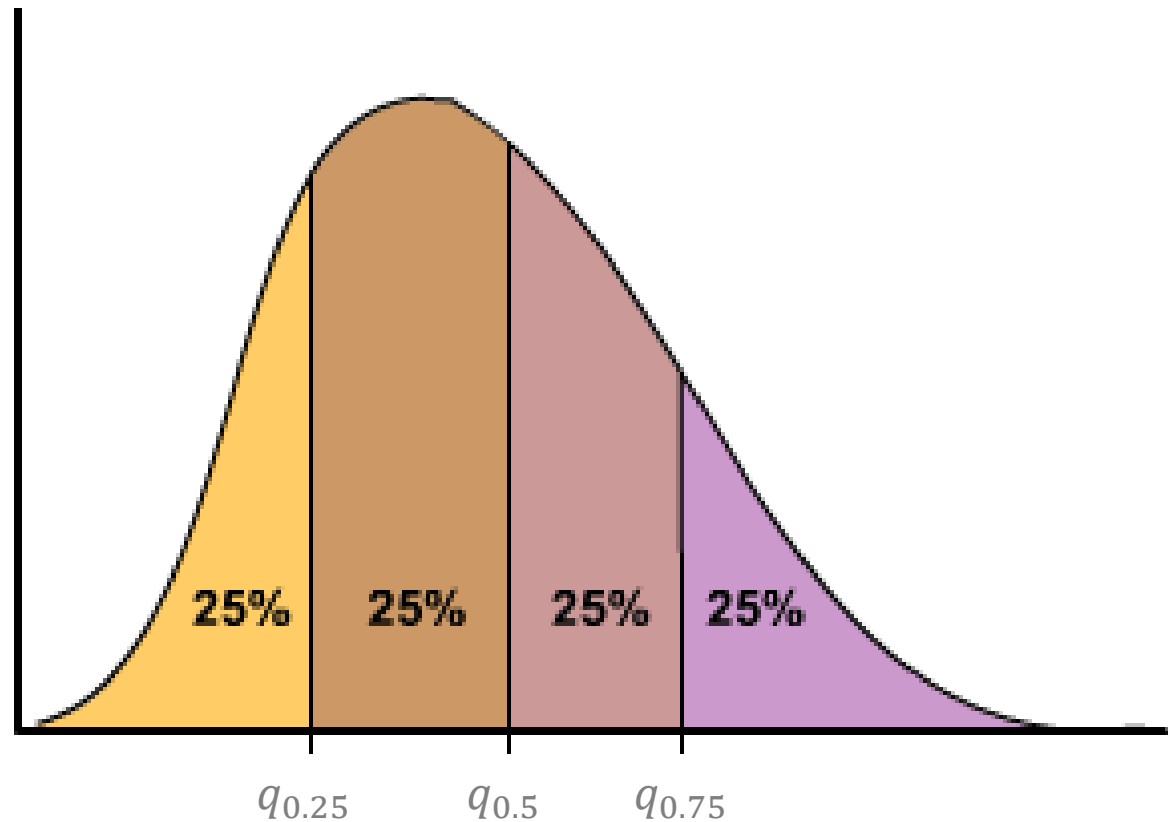
Quantiles

(Numerical Variables)

Generalizing the idea of the median to other fractions.

Typical for descriptive analyses: $q_{0.25}$, $q_{0.5}$, $q_{0.75}$

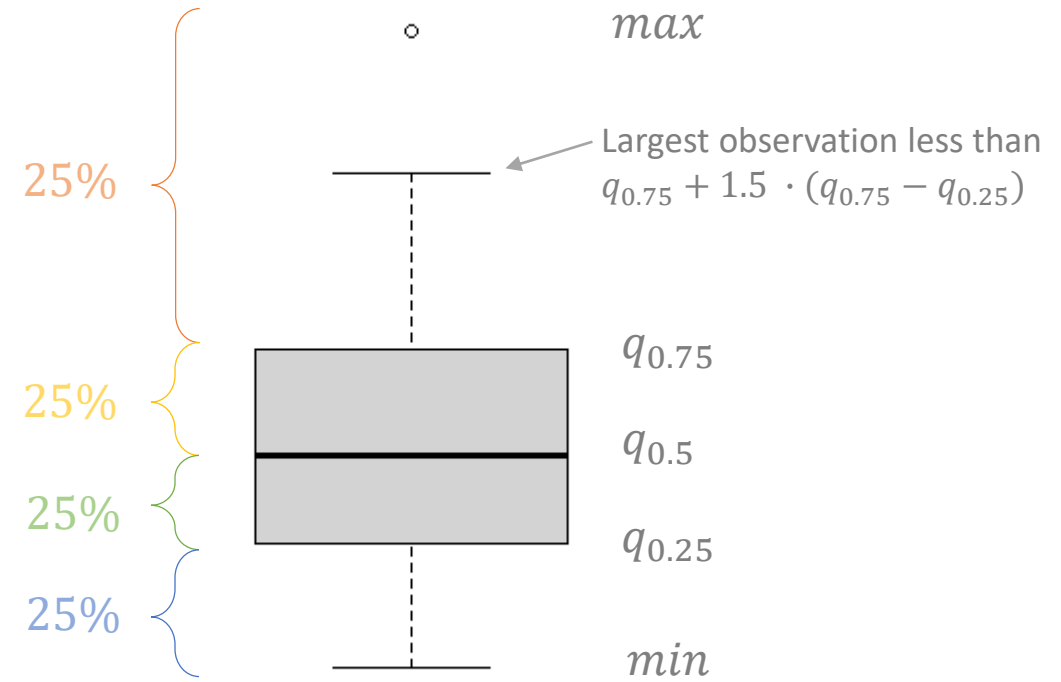
Typical for hypothesis testing: $q_{0.01}$, $q_{0.05}$, $q_{0.95}$, $q_{0.99}$



Boxplots

(Numerical Variables)

Graphical display of the quantiles



Spread

(Numerical Variables)

How strong is the deviation from the center?

- Sample standard deviation:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

- **IQR** (inter quartile range):

$$IQR = q_{0.75} - q_{0.25}$$



$$S = 1.16, IQR = 1.34$$



$$S = 4.05, IQR = 5.93$$

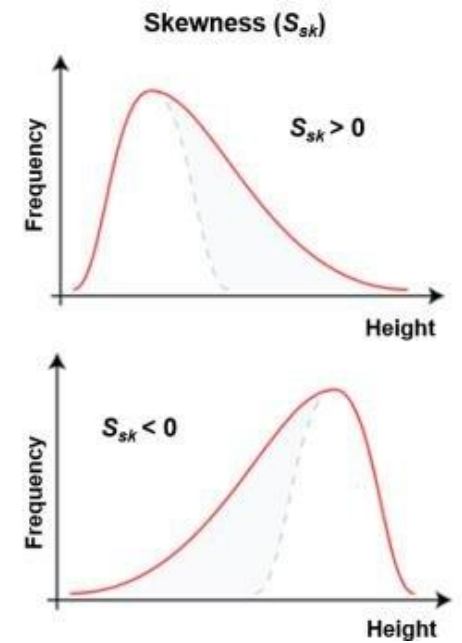
Shape

(Numerical Variables)

Is the distribution symmetric?

- Skewness:

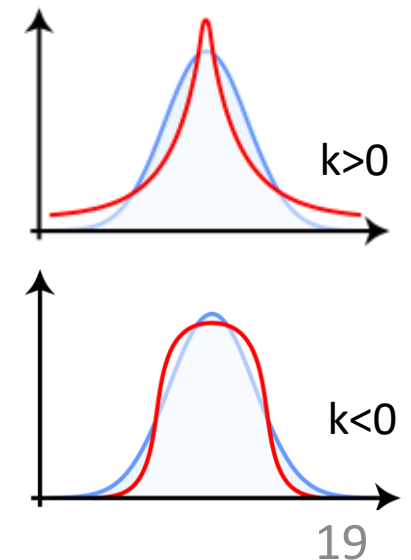
$$S_{sk} = \frac{1}{n} \sum_{i=1}^n \frac{(x_i - \bar{x})^3}{s^3}$$



Does the distribution look like a bell curve?

- Kurtosis:

$$k = \frac{1}{n} \sum_{i=1}^n \frac{(x_i - \bar{x})^4}{s^4} - 3$$



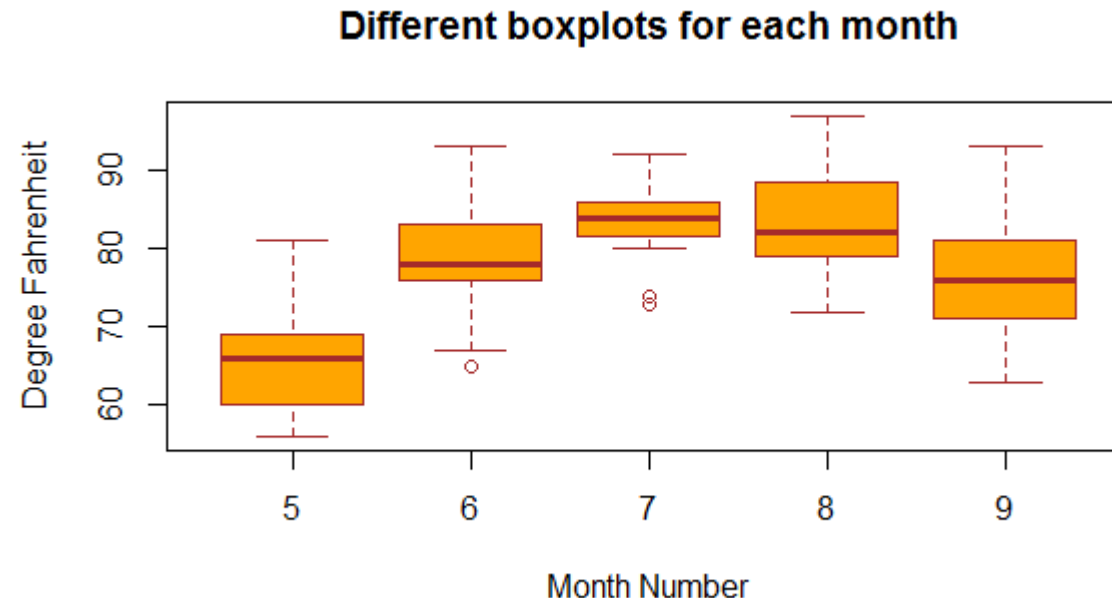
Simultaneous description

(of two features)

- Contingency table (2 categorical features)

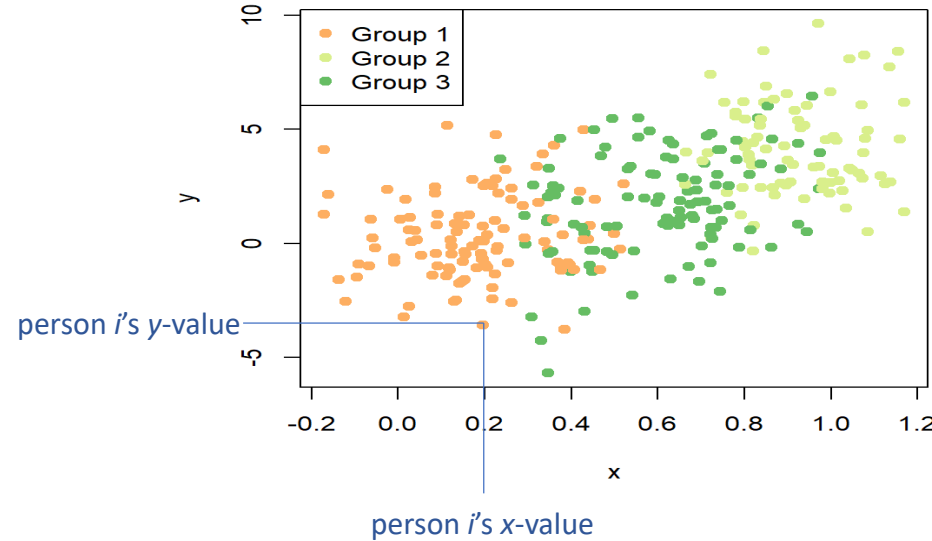
	Male	Female	Total
Blonde	4	8	12
Brunette	7	9	16
Total	11	17	28

- Boxplots (1 categorical and 1 numerical feature)



Simultaneous description (of two features)

- Scatterplot (2 numerical features)

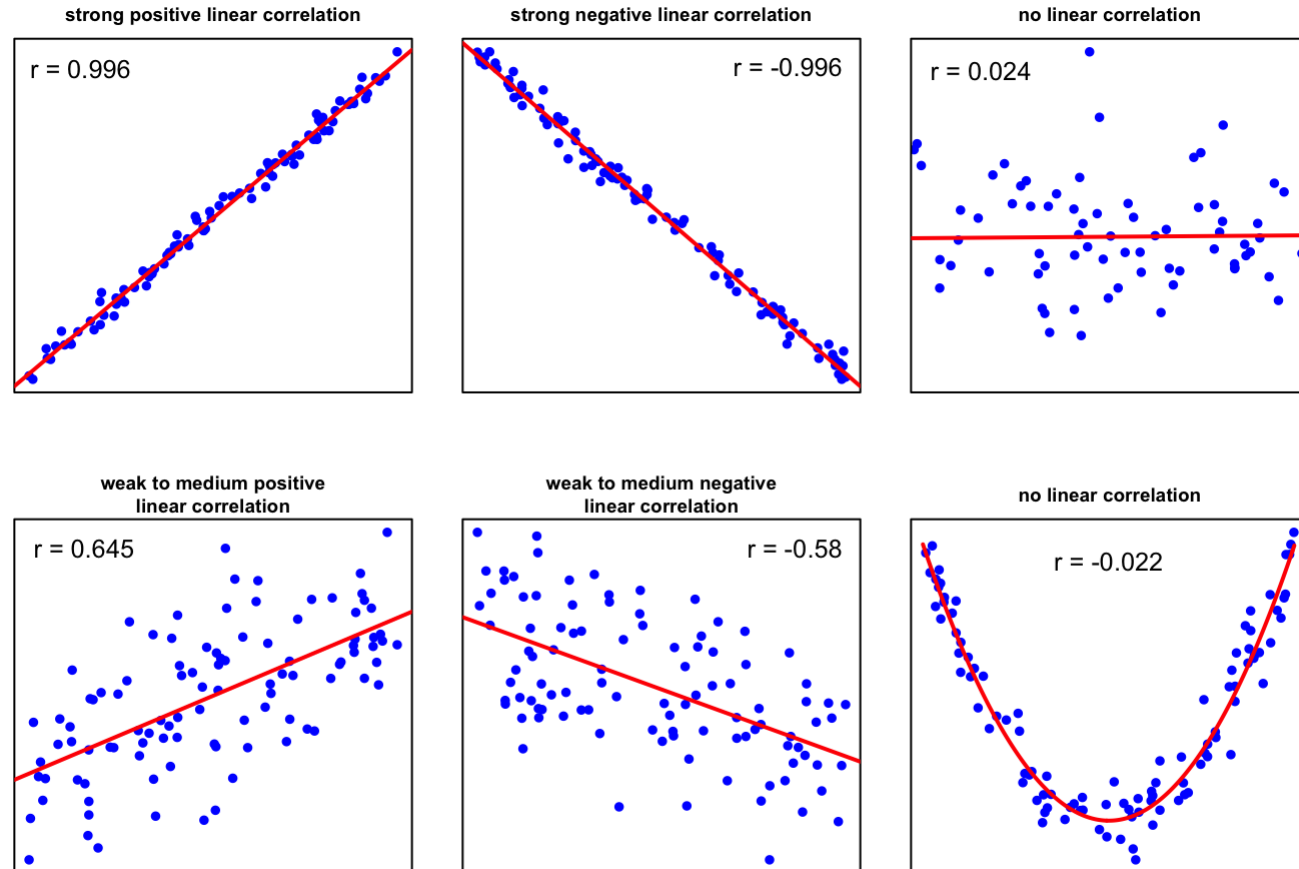


- Pearson Correlation (2 numerical features)

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

Simultaneous description (of two features)

- Pearson Correlation (2 numerical features)

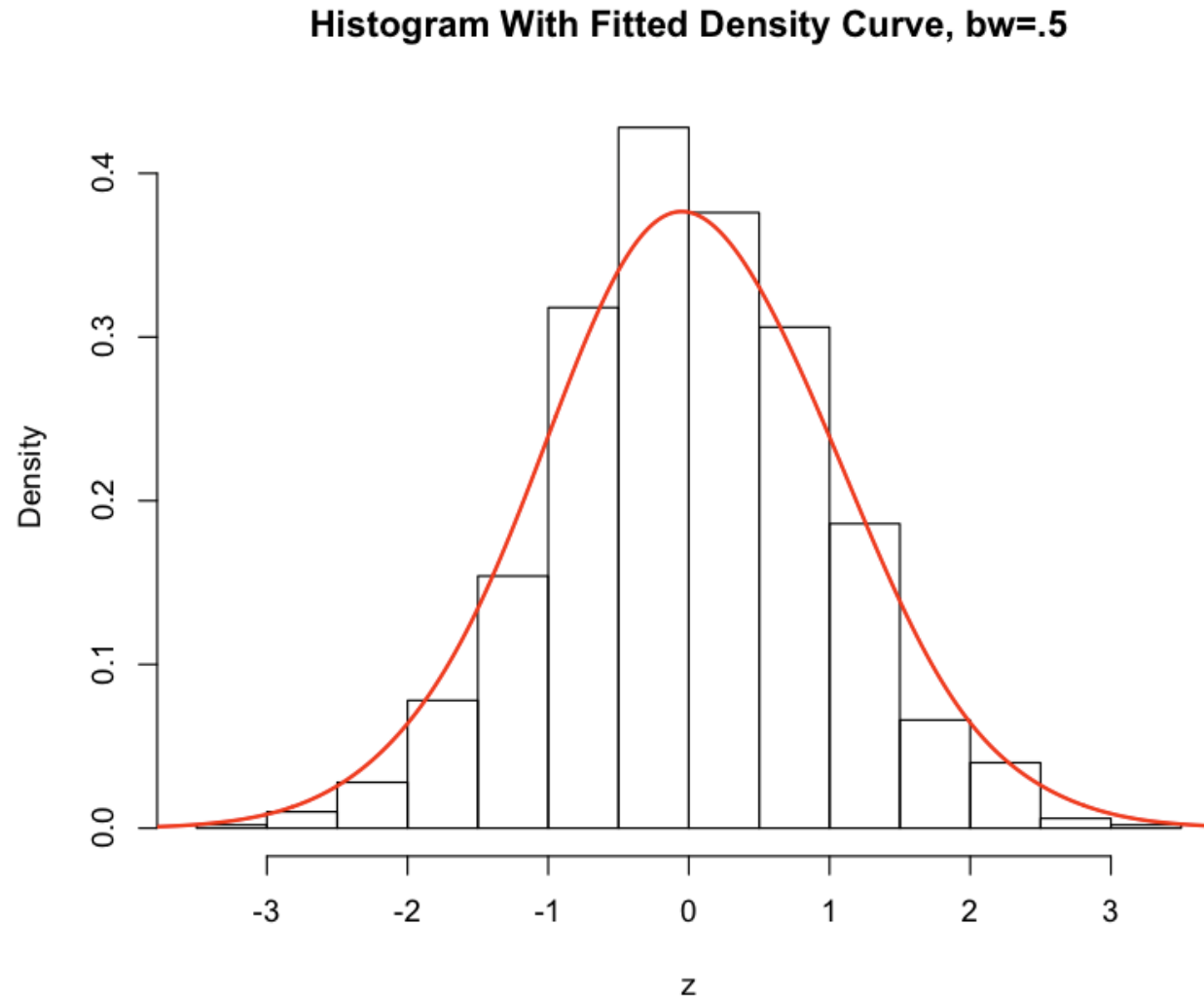


Probability

- Descriptive statistic is an important first step but does not provide us with the means we aim for eventually.
- In general, we want confirm a hypothesis on a population based on sample of said population.
- To this end, we need a mathematical framework for dealing this uncertainty.
- To quantify the uncertainty one often works with probability distributions.

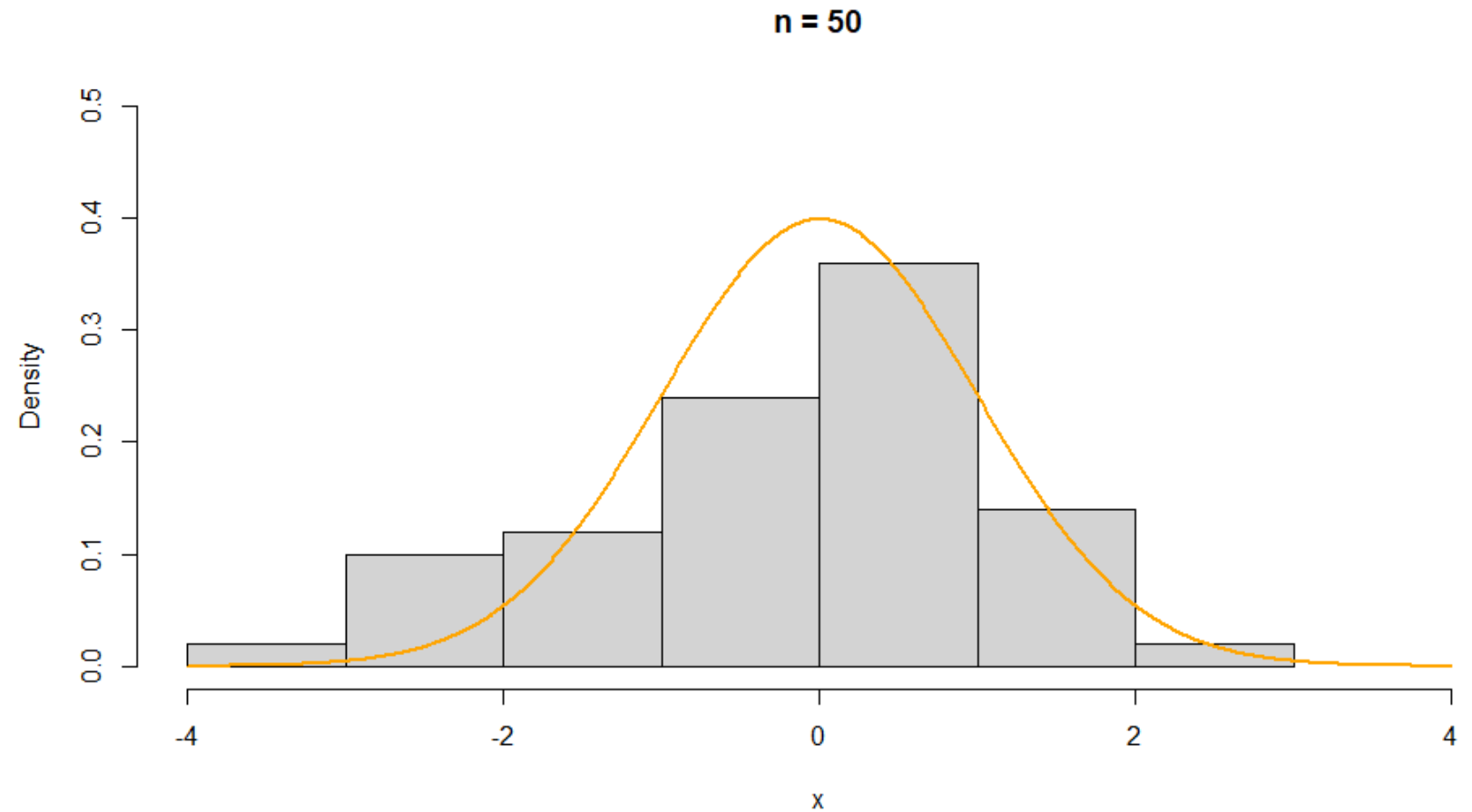
Probability

Probability density function (pdf)



Probability

Probability density function (pdf)



Sketch of idea

