# FFAEngine

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# Abstract

This paper is intended to highlight the sensitivities observed when interacting with a custom factor-based model allocation engine, called the **FFAEngine**. This engine leverages the *Fama French 3 Factor* model to perform a long/short allocation strategy in order to optimize weights within a rigid set of exchange traded funds (12 funds). Furthermore, historical data is bifurcated into several key time periods to bring light to the nature of sudden shifts in the market such as the 2008 financial crisis and how that plays a role in the allocation strategy.

# 1 Mathematical Background

## 1.1 The Fama French 3 Factor Model

The *Fama French 3 Factor* model is a factor-based asset pricing model built atop the *Capital Asset Pricing Model*, or *CAPM*. Both models are intended to describe the attributes that comprise the price of a single asset. The following is the original CAPM.

$$\rho_i = \rho_0 + \beta_i^s(\rho_m - \rho_0) \tag{1}$$

where

$$\beta_i^s = \frac{cov(R_i, R_m)}{\sigma_m^2} \tag{2}$$

and $\rho_i$ is the expected return of the asset, $\rho_0$ is the risk free rate (derived from stable securities such as government bonds), and $\rho_m$ is the expected return on the market. The key aspect of the CAPM is $\beta_i^s$ which can be described as the sensitivity between the asset and market's expected excess returns. The Fama French 3 factor model expands upon this assumption from the CAPM and presents the following.

$$\rho_i = \rho_0 + \beta_i^s(\rho_m - \rho_0) + \beta_i^{smb}\rho_{smb} + \beta_i^{hml}\rho_{hml} + \alpha_i \tag{3}$$

where $\beta_i^{smb}, \beta_i^{hml}$ are the coefficients corresponding to the sensitivities of the excess returns of small market cap to big market cap, and value to growth assets, respectively. In turn, each $\beta$ coefficient will ultimately be calculated using linear regression.

## 1.2 The Portfolio Optimization Goal

With the given Fama French 3 factor allocation model to build each asset price upon, the goal of the engine will be to examine each asset's price within a portfolio and optimize the best possible weights so as to maximize return while minimizing risk. As such, the following is the optimization problem the engine is built to solve.

$$\begin{aligned} \max_{w\in\mathbb{R}^n} \quad & \rho^T\omega - \lambda(\omega - \omega_p)^T\Sigma(\omega - \omega_p) \\ \text{s.t.} \quad & \sum_{i=1}^n \beta_i^m \omega_i = \beta_T^m \\ & \sum_{i=1}^n \omega_i = 1 \\ & -2 \le \omega_i \le 2 \end{aligned} \tag{4}$$

where

$$\beta_i^s = \frac{cov(R_i, R_m)}{\sigma_m^2} \tag{5}$$

$$\beta_p^m = \sum_{i=1}^n \beta_i^m \omega_i = \beta_T^m \tag{6}$$

in which $\beta_i^s$ is the same as the CAPM, $\beta_p^m$ is the combined beta of the portfolio equaling a target beta $\beta_T^m$, $\rho^T\omega$ describes the expected return of a portfolio, $(\omega - \omega_p)^T\Sigma(\omega - \omega_p)$ describes the portfolio's risk, $\Sigma$ is the covariance matrix between each asset's expected returns, and $\lambda$ is a hyperparameter. $\omega_p$ represents the portfolio weights from a previous iteration, and subtracting it from the current weights signifies the assurance that the weights between iterations will not deviate too far from each other. As a result, $\lambda$ can synonymously represent the penalty tuning parameter for the optimization problem representing the significance of the distance between both iteration's weights. Similarly the target $\beta_T^m$ can also be synonymous with a hyperparameter indicating how the engine would perform against the market. A target beta of one or higher will flag the engine to be a risk taker, while a low beta of zero or even less than zero will make the engine be risk-averse or even de-correlated to the market.

## 1.3 Portfolio Sensitivity Tuning

### 1.3.1 Time Periods

The engine will run against a fixed time period:

March 1 2007 - June 30 2021

And will be bifurcated into the following subperiods based on the 2008 sub-prime mortgage crisis:

**Pre-Crisis**

March 1 2007 - December 31 2007

**In-Crisis**

January 1 2008 - June 30 2009

**Post-Crisis**

July 1 2009 - June 30 2021

Using each sub-period's historical data to predict and evaluate the portfolio will bring light to the quality and quantity of the data the engine uses.

### 1.4 Look-Back Parameters

Because the engine relies on the returns of each asset, it is imperative that the engine be capable of tuning the timeline of historical data as it can change the sensitivity of the weights. As such, two look-back features, one for the covariance matrix and one for the expected returns, are added to the engine to tune how far back historically it can aggregate data from to perform its optimization. These look-back features, along with the target beta, can be grouped together to form the following term structure.

$$S_{\rho_L}^{\Sigma_L}\left(\beta_T^m\right) \tag{7}$$

For example $S_{60}^{90}\left(0.5\right)$ would mean tuning the engine such that it uses 90 days of historical data for the covariance matrix computation, 60 days for the expected returns vector computation, and sets a target sensitivity of 0.5.

## 2 Running The Engine

Running the engine yields a tearsheet of portfolio summary statistics as well as plots. It provides a series of portfolio performance signals from PnL to VaR and is used alongside the plots to gain a better understanding of how the portfolio performs given the circumstances.

### 2.1 Impact From The Target Beta

Based on the mathematical structure of the engine, the following can be summarized regarding the choice of target beta

| Choice of $\beta_T$ | Summary |
|---|---|
| $< 0$ | Shorting the market; if the market does poorly, the portfolio performs well |
| $0$ | De-correlated from the market; the market has no bearing on its returns |
| $> 0$ | Ride the market; if the market does well, so will the portfolio |

### 2.2 When $\beta_T < 0$

When the target beta is negative, the engine will try its best to short the market, which worked quite well given that the 2008 crisis had occurred quite early on. The following is the performance of the portfolio under the following condition

$$S_{90}^{120}\left(-0.5\right)$$

| Performance | Pre-Crisis |
|---|---|
| PnL | $72.1052 |
| Daily Mean | 0.217849 |
| Volatility | 0.0766214 |
| Sharpe Ratio | 0.0113728 |
| VaR 95% | -0.115074 |

### 2.3 When $\beta_T = 0$

When the target beta is 0, the engine's return will be de-correlated from the market and will have no bearing on it's performance. This worked similarly to when the target beta was negative as it decided to rebalance its portfolio quite similarly to the way the previous portfolio performed. The only difference is that it maintained a better performing reallocation method towards the end, resulting in a higher PnL. The following is the performance of the portfolio under the following condition

$$S_{90}^{120}\left(0\right)$$

| Performance | Pre-Crisis |
|---|---|
| PnL | $101.899 |
| Daily Mean | 0.23737 |
| Volatility | 0.0774037 |
| Sharpe Ratio | 0.0122666 |
| VaR 95% | -0.116931 |

### 2.4 When $\beta_T > 0$

When the target beta is greater than 0, the engine will attempt to ride the scale of the market in the hopes of mirroring its returns to match that of the market. This, when paired with the financial crisis, results in the severe performance of the portfolio as it dropped significantly in value and never recovered. The following is the performance of the portfolio under the following condition

$S_{90}^{120}\,(1)$

| Performance | Pre-Crisis |
|---|---|
| PnL | -\$197.539 |
| Daily Mean | -0.775056 |
| Volatility | 0.0720079 |
| Sharpe Ratio | -0.043054 |
| VaR 95% | -0.116965 |

### 2.5 Impact From The Term Structure

The term structure determines how far back historically the regression model needs to look back from to determine its model coefficients. This in turn plays a key role in the optimization problem (4) as it determine how each security betas will effect the target weights when attempting to meet the target beta value

### 2.6 When the term structure is $S_{40}^{200}\,(0.5)$

The following is the performance of the portfolio when using 200 historical days for the calculation of the covariance matrix and 40 days for the expected returns.

$S_{40}^{200}\,(0.5)$

| Performance | Pre-Crisis |
|---|---|
| PnL | -\$8.37556 |
| Daily Mean | 0.0216064 |
| Volatility | 0.0766831 |
| Sharpe Ratio | 0.00112705 |
| VaR 95% | -0.12165 |

### 2.7 When the term structure is $S_{200}^{40}\,(0.5)$

The following is the performance of the portfolio when using 40 historical days for the calculation of the covariance matrix and 200 days for the expected returns.

$S_{200}^{40}\,(0.5)$

| Performance | Pre-Crisis |
|---|---|
| PnL | \$897.99 |
| Daily Mean | 0.274116 |
| Volatility | 0.0640025 |
| Sharpe Ratio | 0.0171316 |
| VaR 95% | -0.0995051 |

## 3 Final Remarks

After several iterations of different model parameters and tuning the following conclusions were met

- $\beta_T < 0.5$ worked best
- $\beta_T < 0$ is a very speculative condition; a risky investor must have assumed the market would go down in order to short it.
- Term structures with large windows for expected returns provided better calculations of beta coefficients during the linear regression.
- Overall, portfolio managers should leverage this engine to find conditions where the portfolio weights are not rebalanced too frequently.

# Appendix: Plots


Figure 2.3.1: The engine's cumulative returns given $S_{90}^{120}(0)$

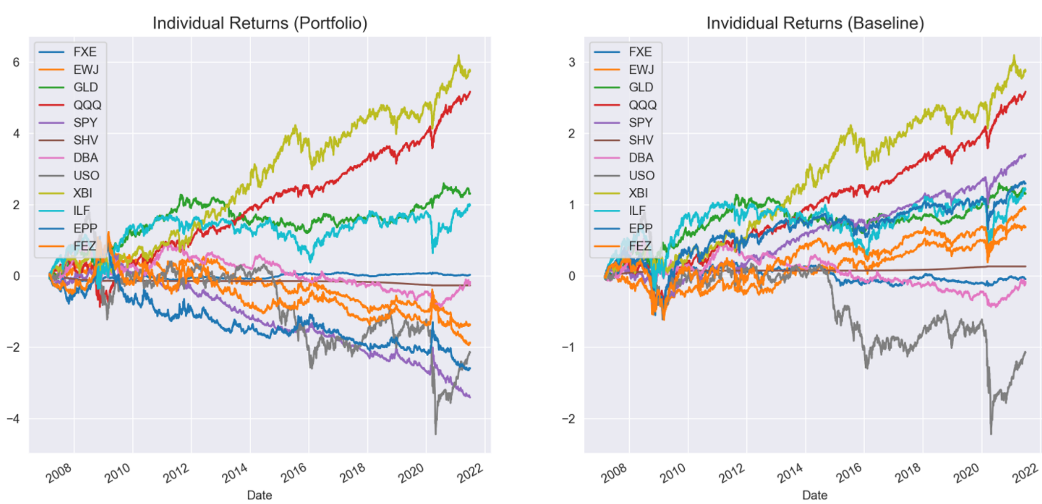


Figure 2.3.2: The engine's bifurcated returns by individual asset given $S_{90}^{120}(0)$ (Left). The baseline consists of returns when holding single assets respectively (Right).

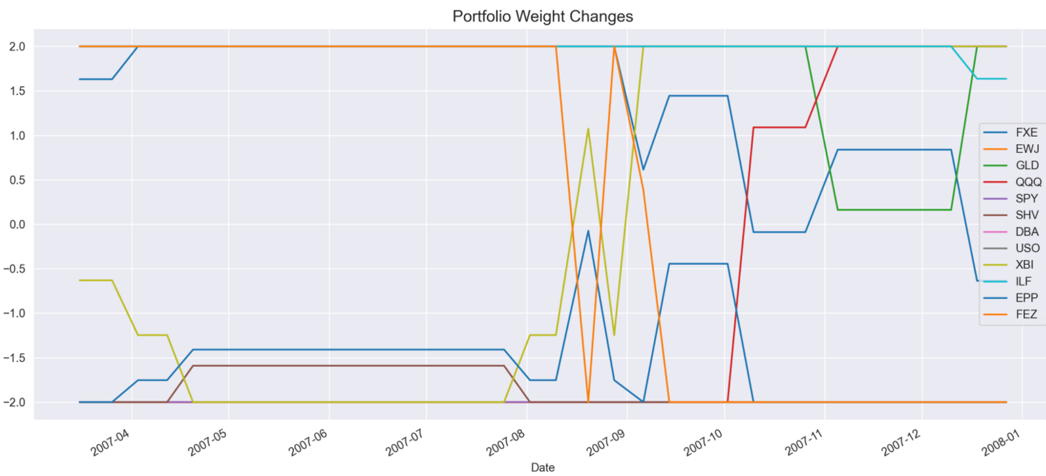


Figure 2.3.3: The engine's decisions to reallocate weights depending on conditions during a rebalance day.


Figure 2.4.1: The engine's cumulative returns given $S_{90}^{120}$ (1)

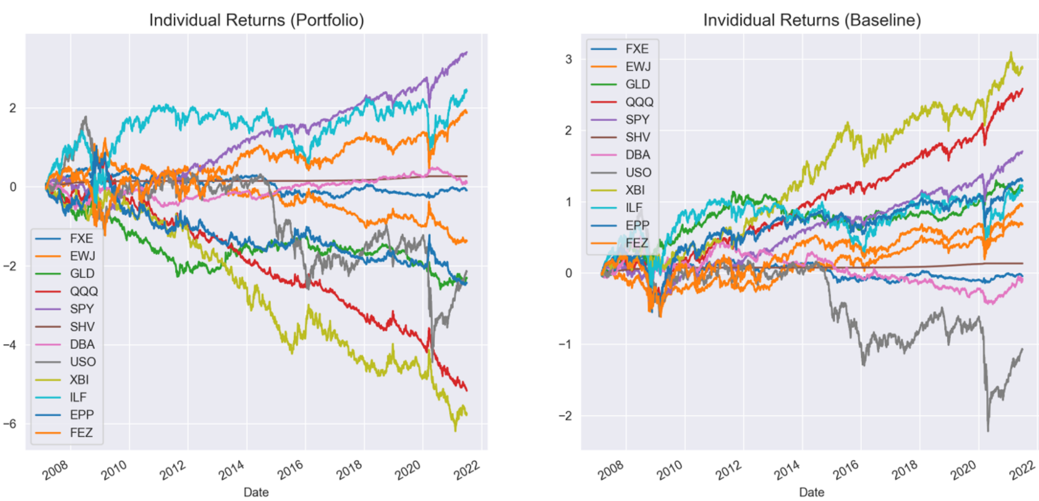


Figure 2.4.2: The engine's bifurcated returns by individual asset given $S_{90}^{120}(1)$ (Left). The baseline consists of returns when holding single assets respectively (Right).

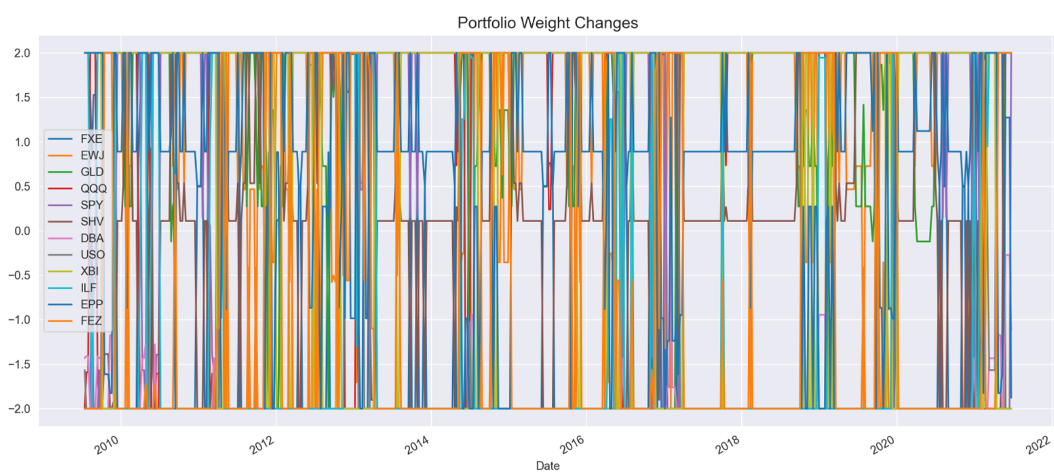


Figure 2.4.3: The engine's decisions to reallocate weights depending on conditions during a rebalance day.


Figure 3.2.1: The engine's cumulative returns given $S_{40}^{200}\,(0.5)$

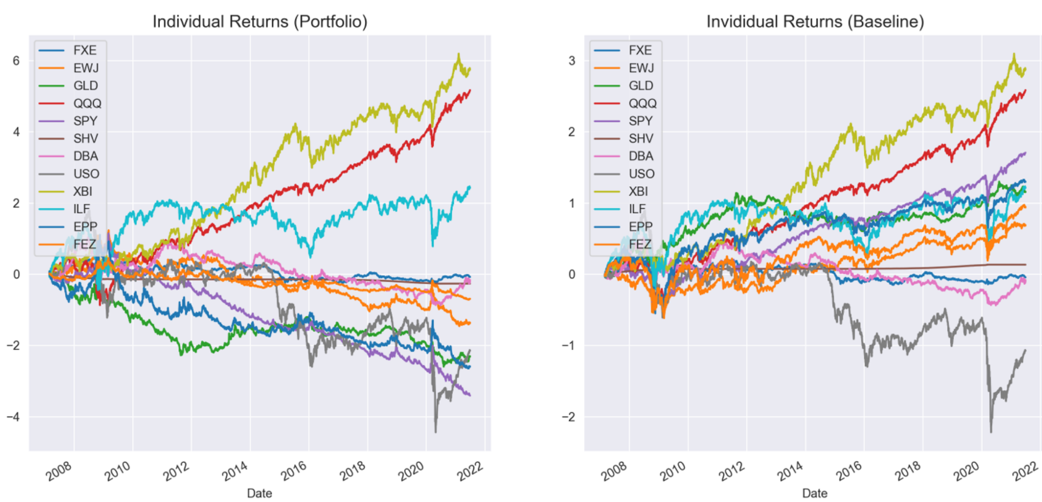


Figure 3.2.2: The engine's bifurcated returns by individual asset given $S_{40}^{200}\,(0.5)$ (Left). The baseline consists of returns when holding single assets respectively (Right).

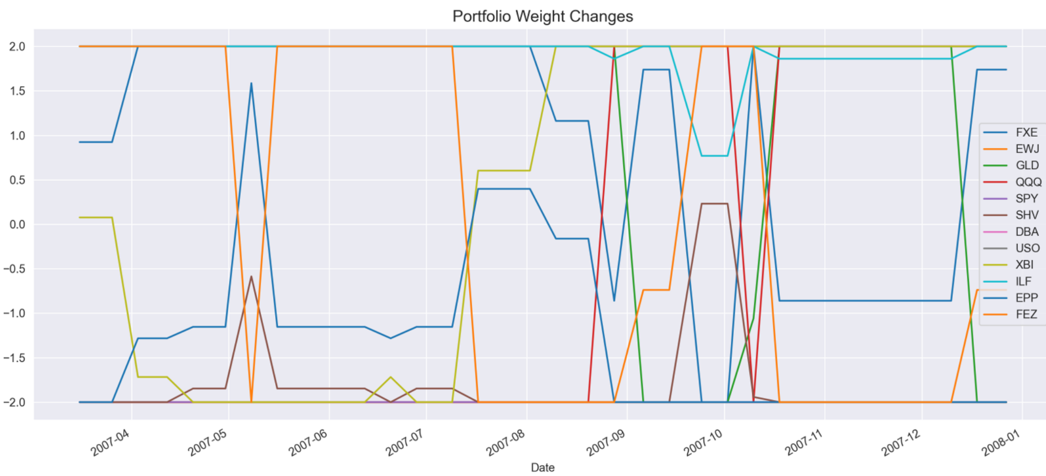


Figure 3.2.3: The engine's decisions to reallocate weights depending on conditions during a rebalance day.

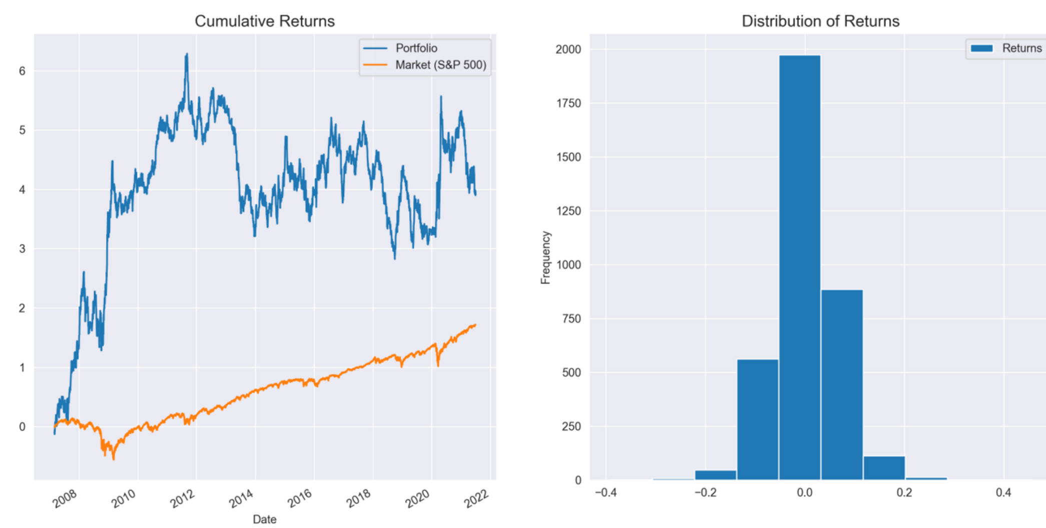


Figure 3.3.1: The engine's cumulative returns given $S_{20}^{40}(0.5)$

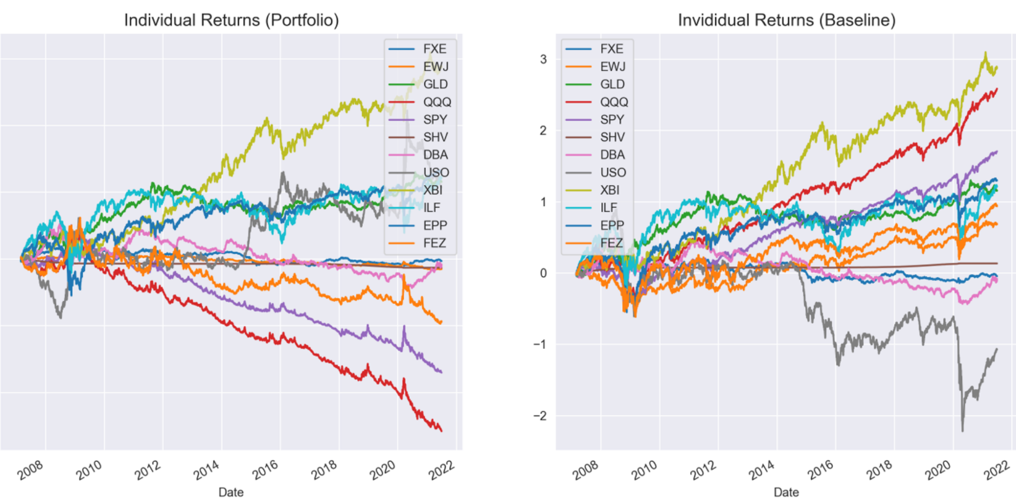


Figure 3.3.2: The engine's bifurcated returns by individual asset given $S_{20}^{40}(0.5)$ (Left). The baseline consists of returns when holding single assets respectively (Right).

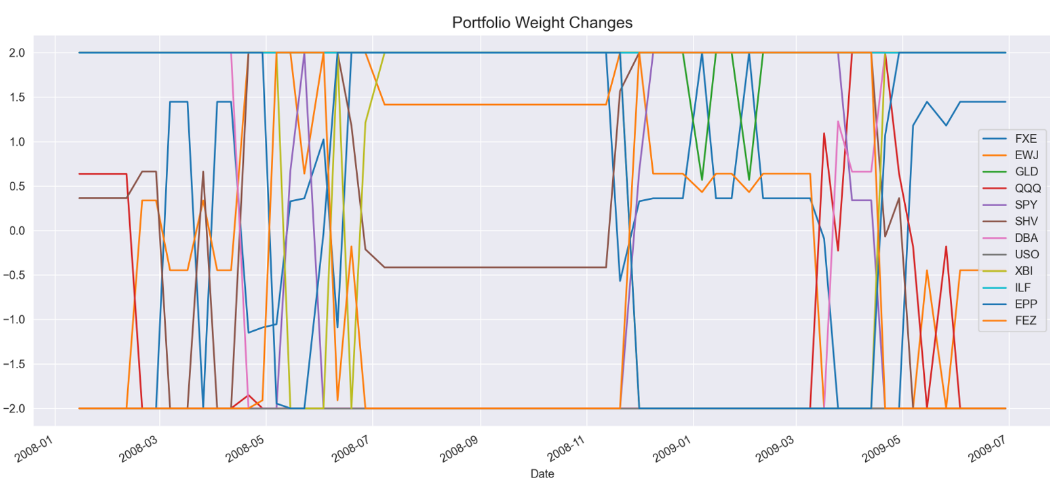


Figure 3.3.3: The engine's decisions to reallocate weights depending on conditions during a rebalance day.