Performance Analysis of Multiuser Diversity in Cooperative Multi-Relay Networks under Rayleigh-Fading Channels

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***Abstract*—In multiuser cooperative relay networks, cooperative diversity can be obtained with the help of relays, while multiuser diversity is an inherent diversity in multiuser systems. In this letter, the performance analysis of multiuser diversity in cooperative multi-relay networks is presented. Both the case of all relay participating and the case of relay selection are considered. We first derive asymptotic expressions of outage probability and symbol error probability for amplify-and-forward (AF) and decode-and-forward (DF) protocols with joint multiuser and cooperative diversity. Then, the theoretical analysis are validated by Monte Carlo simulations. Both the theoretical analysis and simulations show that a multiuser diversity order of K and a cooperative diversity order of $M+1$ can be achieved simultaneously for both AF and DF protocols (where K is the number of accessing users and M is the number of available relays). These demonstrate that the multiuser diversity can be readily combined with the cooperative diversity in multiuser cooperative relay networks.**

***Index Terms*—Multiuser diversity, cooperative communication, outage probability, symbol error probability, opportunistic scheduling.**

I. Introduction

DIVERSITY techniques are known as the effective means to cope with fading in wireless channels. Recently, cooperative communication is proposed as a promising solution to achieve spatial diversity in a distributed manner [1]-[5]. Several cooperative protocols are proposed, such as amplify-and-forward (AF) and decode-and-forward (DF) [2]. It is shown that both AF and DF can achieve full diversity [1], [5], while for simple DF cooperation with a fixed relay [2], no diversity gain can be obtained [2]. On the other hand, in multiuser systems, multiuser diversity is an important kind of diversity [6], [7], which exploits the fact that users undergo independently varying channels and at any time, there must be a user whose channel gain is near the peak. The system can choose to serve this user at that time.

Multiuser diversity in single-antenna systems is being extensively addressed, e.g., [6], [7]. For multiple-input multiple-output (MIMO) systems, the interaction between the spatial diversity and multiuser diversity is also well studied, e.g., [8]-[10]. However, there are few papers addressing the analysis of multiuser diversity in cooperative relay networks. In [4], the authors derive the diversity-multiplexing tradeoff for various MIMO relay channels. The remarkable work in [11]-[14] and the references therein present the asymptotic capacity study of large networks. Based on the random matrix theory, scaling laws for the system capacity are developed for different network configurations. These results reveal that the number of nodes in relay systems has great impact on the system performance.

In this letter, we specifically study the impact of the number of accessing users on the diversity performance. We extend the study of multiuser diversity in MIMO [8]-[10] and single-relay networks [15] to cooperative multi-relay networks based on the analysis of outage and error performance. Different from [11]-[14], we consider more practical scenarios where finite number of accessing users and relays are considered. For multiuser diversity, the well known proportional fair scheduling (PFS) algorithm [16] is adopted to schedule the accessing users. We aim at analyzing the impact of the small scale fading on the multiuser diversity, and for PFS, if the effect of large scale fading is omitted, i.e., equal average signal-to-noise ratio (SNR) for all users, it is equal to the Max SNR algorithm, where the user with the highest instantaneous SNR is selected [16]. We further assume that M relays are available for each accessing user. If relay selection is performed [5], [17], only one relay is selected to forward the data, otherwise, all relays will participate in the second phase data transmission. For relay selection, we adopt the best relay selection [5], where the relay whose path has the maximum SNR is selected. However, the analysis can be extended to address other selection methods, e.g., best harmonic mean selection, best-worst channel selection [17]. Specifically, we first present the system model of multiuser cooperative relay networks with multiple available relays. Then, asymptotic expressions of outage probability and symbol error probability are derived for both AF and DF protocol with joint multiuser and cooperative diversity. The theoretical analysis is then verified by Monte Carlo simulations. It shows that for AF and DF, whether with relay selection or not, a multiuser diversity order of K and a cooperative diversity order of $M+1$ can be achieved simultaneously, i.e., a total diversity order of $(M+1)K$.

II. System Model

In this letter, we consider multiuser cooperative relay networks with one destination, multiple accessing users and multiple relays. Each user is equipped with only one antenna. For multiuser diversity, one user with the best channel out of K accessing users is selected for transmission at a time. If relay

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selection is performed, only one "best" relay among the user's available M relays is selected to forward the data, otherwise, all relay will participate in the second hop transmission. The transmission is accomplished in two phases: in the first phase, one of the accessing user transmits while the destination and the relay listen; In the second phase, the relay(s) forward the data to the destination, adopting either AF or DF protocol. We focus on the time division multiple access (TDMA) scheme to fulfill the orthogonality requirement and assume channel state information (CSI) can be obtained at the destination via feedback channels.

1) All Relay Participating: The received SNR of the k-th user when all relay are participating can be denoted as [5]

$$\gamma_k^{AF} = \frac{E_k|h_{k,d}|^2}{N_0} + \sum_m^M \frac{\frac{E_k|h_{k,m}|^2}{N_0}\frac{E_m^k|h_{m,d}^k|^2}{N_0}}{\frac{E_k|h_{k,m}|^2}{N_0} + \frac{E_m^k|h_{m,d}^k|^2}{N_0} + 1}, \quad (1)$$

where $h_{k,d}$, $h_{k,m}$ and $h_{m,d}^k$ denote the channel coefficients of the channel from the user k to the destination, user k to the m-th relay and the m-th relay to the destination when relaying the k-th user's signal, respectively; E_k and E_m^k are the average transmission energy at the user k and at the m-th relay when relaying the k-th user's signal; N_0 is the additive white Gaussian noise (AWGN) power. We assume that all channels are independent Rayleigh fading channels. Define $a_k = E_k|h_{k,d}|^2$, $b_{k,m} = E_k|h_{k,m}|^2$, $c_{k,m} = E_m^k|h_{m,d}^k|^2$ as the received signal power at the destination from the k-th user, at the m-th relay from the k-th user and at the destination from the m-th relay, respectively, which are exponentially distributed with parameter $\overline{a}_k$, $\overline{b}_{k,m}$ and $\overline{c}_{k,m}$. Then, the k-th user's received SNR at the destination is

$$\gamma_k^{AF} = \overline{\gamma}a_k + \sum_m^M \frac{\overline{\gamma}^2 b_{k,m}c_{k,m}}{\overline{\gamma}b_{k,m} + \overline{\gamma}c_{k,m} + 1}, \quad (2)$$

where $\overline{\gamma} = 1/N_0$ is proportional to all the transmitted SNRs and the received SNR. Thus, it can be used as a measure of the average system SNR.

Similarly, for repetition-coded DF protocol [1], the received SNR of the k-th user at the destination can be expressed as

$$\gamma_k^{DF} = \overline{\gamma}a_k + \sum_{m\in\mathcal{D}(k)} \overline{\gamma}c_{k,m}, \quad (3)$$

where $\mathcal{D}(k)$ denotes the decoding set of user k, which is the set of relays that can fully decode the message of user k.

2) With Relay Selection: If relay selection is performed, for AF protocol, the one with the best relaying SNR is selected to forward the data as in [5], while for DF, among the decoding set of user k, the one with the highest relay-destination SNR is selected to forward the data. The direct transmission and the relay transmission are combined at the destination using the maximum ratio combining. Then, the receive SNR at the destination for AF and DF protocol with best relay selection can be obtained as follows, respectively,

$$\gamma_k^{SAF} = \overline{\gamma}a_k + \max_m \frac{\overline{\gamma}^2 b_{k,m}c_{k,m}}{\overline{\gamma}b_{k,m} + \overline{\gamma}c_{k,m} + 1}, \quad (4)$$

$$\gamma_k^{SDF} = \overline{\gamma}a_k + \max_{m\in\mathcal{D}(k)} \overline{\gamma}c_{k,m}. \quad (5)$$

III. Performance Analysis

When focusing on the analysis of the effect of the small scale fading, the PFS scheduler will select the accessing user who can achieve the largest SNR. Therefore, the achieved system SNR is $\gamma = \max_k \gamma_k$. We want to study the performance of multiuser diversity based on the analysis of the outage probability and symbol error probability. First, we should obtain the cumulative density function (CDF) of the achieved system SNR γ (γ^{AF}, γ^{DF}, γ^{SAF} and γ^{SDF}). Although the exact distribution can be obtained based on the convolution integral or using Laplace transformation, it is not tractable for the diversity analysis. However, in this paper, we only need the distribution in large average values to make the diversity performance analysis (in fact, the diversity order is defined as $d = -\lim_{\gamma\to\infty}\frac{P_{out}(\gamma)}{\log(\gamma)}$, which is essential a performance metric at high SNR). *Lemma 1* and *Lemma 2* provides us the CDFs of the achieved system SNRs in large average SNR regime for cooperation with all relay participating and for that with relay selection, respectively.

Lemma 1: In large average SNR regime ($\overline{\gamma} \gg 1$), the CDFs of the achieved system SNRs with multiuser diversity and all relay participating can be approximated as follows, respectively,

$$F_{\gamma^{AF}}(\gamma) \approx \frac{1}{M+1}\left(\frac{\gamma}{\overline{\gamma}}\right)^{K(M+1)} \prod_{k=1}^K \overline{a}_k \prod_{m=1}^M (\overline{b}_{k,m} + \overline{c}_{k,m}), \quad (6)$$

$$F_{\gamma^{DF}}(\gamma) \approx \left(\frac{\gamma}{\overline{\gamma}}\right)^{K(M+1)} \prod_k \overline{a}_k \sum_{\mathcal{D}(k)} \frac{1}{(|\mathcal{D}(k)|+1)!} \prod_{m\in\mathcal{D}(k)} \overline{c}_{k,m} \prod_{m\notin\mathcal{D}(k)} \overline{c}_{k,m}, \quad (7)$$

where $|\mathcal{D}(k)|$ denotes the cardinality of the random set $\mathcal{D}(k)$.

Proof: According to order statistics, for independently distributed random variables γ_k, the CDF of $\gamma = \max_k \gamma_k$ can be calculated as $F_\gamma(\gamma) = Pr[\gamma \le \gamma] = \prod_{k=1}^K Pr[\gamma_k \le \gamma]$, then with *Theorem 1* in [5] and eq. (12) in [1], we can obtain (6) and (7), respectively. ■

Lemma 2: In large average SNR regime ($\overline{\gamma} \gg 1$), the CDFs of the achieved system SNRs with multiuser diversity and relay selection can be approximated as follows, respectively,

$$F_{\gamma^{SAF}}(\gamma) \approx \frac{1}{(M+1)^K}\left(\frac{\gamma}{\overline{\gamma}}\right)^{K(M+1)} \prod_{k=1}^K \overline{a}_k \prod_{m=1}^M (\overline{b}_{k,m} + \overline{c}_{k,m}), \quad (8)$$

$$F_{\gamma^{SDF}}(\gamma) \approx \left(\frac{\gamma}{\overline{\gamma}}\right)^{K(M+1)} \prod_k \overline{a}_k \sum_{\mathcal{D}(k)} \frac{1}{(|\mathcal{D}(k)|+1)} \prod_{m\in\mathcal{D}(k)} \overline{c}_{k,m} \prod_{m\notin\mathcal{D}(k)} \overline{c}_{k,m}. \quad (9)$$

Proof: Please refer to the Appendix. ■

A. Outage Probability

The outage is defined as the channel capacity falling below a specified transmission rate R. The probability of such outage event is called outage probability. Thus, in cooperative relay networks with multiuser diversity, the outage probabilities for the cooperation with all relay participating and for that with relay selection are give by as follows,

$$P_{out}^{all} = Pr[I < R] = Pr[\gamma < 2^{(M+1)R} - 1] = F_\gamma(2^{(M+1)R} - 1), \quad (10)$$

$$P_{out}^{selection} = Pr[I < R] = Pr[\gamma < 2^{2R} - 1] = F_\gamma(2^{2R} - 1). \quad (11)$$

Based on the approximate expressions of CDF of γ in high SNR regime (6)-(9), the outage probabilities for AF and DF based multiuser cooperative relay networks can be immediately obtained. For the convenience of comparison, we also derive the outage probabilities for cooperative relay networks without multiuser diversity, which are shown as

$$P_{out}^{w/o} \approx \left(\frac{2^{2R}-1}{\overline{\gamma}}\right)^2 \frac{\overline{a}(\overline{b}+\overline{c})}{2} \quad \text{and} \quad P_{out}^{w/o} \approx \frac{2^{2R}-1}{\overline{\gamma}}\overline{b}. \qquad (12)$$

It can be seen that the asymptotic outage probabilities are proportional to $(1/\overline{\gamma})^{K(M+1)}$ for AF and DF (both for all relay participating and for relay selection). Thus a multiuser diversity order of K and a cooperative diversity order of $M+1$ can be achieved simultaneously, i.e., a total diversity order of $(M+1)K$ can be obtained for both AF and DF protocol (both for the cooperation with all relay participating and with relay selection), while for that without multiuser diversity (12), the outage probabilities are irrespective of the user number. The reason is that for multiuser diversity, the probability of choosing the "best" user increases when there are more users in the system. It is shown that the multiuser diversity and the cooperative diversity can be readily combined. Furthermore, these results can be justified by extending the results in [4], assuming a K-antenna source communicates with a single antenna destination with the help of K relays, each equipped with M antennas.

B. Symbol Error Probability

Generally, the symbol error probability (SEP) at certain SNR γ has a uniform expression for commonly used signal constellations, such as M-ary quadrature amplitude modulation (M-QAM), M-ary phase shift keying (M-PSK), etc.

$$P_s(\gamma) = \alpha Q(\sqrt{\beta\gamma}), \qquad (13)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp(\frac{-t^2}{2})dt$ is the Gaussian Q function; Parameter α and β are determined by specific constellations, for example, for Binary-PSK (BPSK) modulation, $\alpha = 1$ and $\beta = 2$; for M-PSK, $\alpha = 1$ and $\beta = 2\sin^2(\pi/M)$, and for M-QAM, $\alpha = 4$, $\beta = 3/(M-1)$.

For a certain system SNR γ, the average system SEP is therefore the symbol error probability in (13) over the SNR distribution,

$$\begin{aligned} P_s &= \mathbb{E}_\gamma\left\{\alpha Q\left(\sqrt{\beta\gamma}\right)\right\} = \alpha\mathbb{E}_\gamma\left\{Pr\left[X > \sqrt{\beta\gamma}\right]\right\} \\ &= \alpha\mathbb{E}_X\left\{Pr\left[\gamma < \frac{X^2}{\beta}\right]\right\} = \alpha\mathbb{E}_X\left\{F_\gamma\left(\frac{X^2}{\beta}\right)\right\}, \end{aligned} \qquad (14)$$

where $\mathbb{E}_y\{\cdot\}$ denotes the expectation over the distribution of y. Random variable X follows the standard normal distribution.

1) All Relay Participating: By *Lemma 1* (6) and (7), using the equation [eq. (3.461.2), 18], the SEP of AF, P_s^{AF}, can be obtained as

$$\begin{aligned} P_s^{AF} \approx & \frac{\alpha}{(2\beta\overline{\gamma})^{K(M+1)}}\frac{1}{M+1}\frac{[2K(M+1)-1]!}{[K(M+1)-1]!} \\ & \times \prod_{k=1}^{K}\overline{a}_k\prod_{m=1}^{M}\left(\overline{b}_{k,m}+\overline{c}_{k,m}\right). \end{aligned} \qquad (15)$$

Similarly, we can obtain SEP expression for DF protocol,

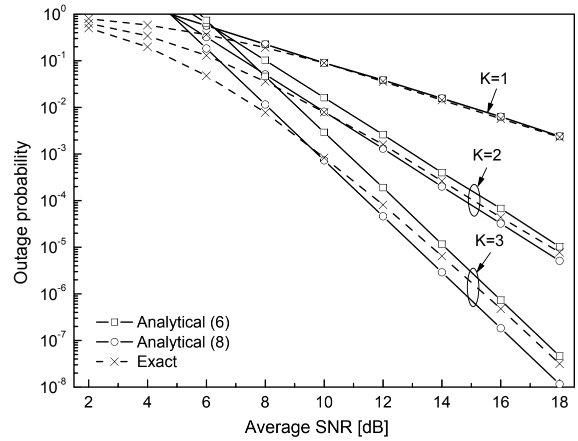

Fig. 1. Outage probability of amplify-and-forward with different numbers of accessing users ($M = 1$).

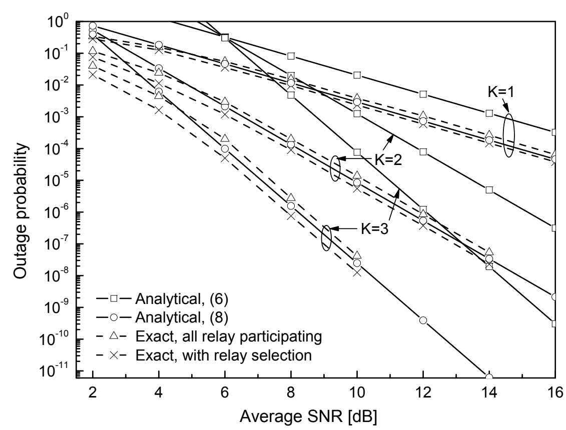

Fig. 2. Outage probability of amplify-and-forward with different numbers of accessing users ($M = 2$).

which is given by

$$\begin{aligned} P_s^{DF} \approx & \frac{\alpha}{(2\beta\overline{\gamma})^{(M+1)K}}\frac{[2K(M+1)-1]!}{[K(M+1)-1]!}\prod_k\overline{a}_k\sum_{\mathcal{D}(k)}\frac{1}{(|\mathcal{D}(k)|+1)!} \\ & \times\prod_{m\in\mathcal{D}(k)}\overline{c}_{k,m}\prod_{m\notin\mathcal{D}(k)}\overline{c}_{k,m}. \end{aligned} \qquad (16)$$

2) With Relay Selection: With *Lemma 2*, we can also obtain the SEP when relay selection is performed, which are give by (17) and (18) for AF and DF protocols, respectively,

$$\begin{aligned} P_s^{SAF} \approx & \frac{\alpha}{(2\beta\overline{\gamma})^{K(M+1)}}\frac{1}{(M+1)^K}\frac{[2K(M+1)-1]!}{[K(M+1)-1]!} \\ & \times\prod_{k=1}^{K}\overline{a}_k\prod_{m=1}^{M}\left(\overline{b}_{k,m}+\overline{c}_{k,m}\right), \end{aligned} \qquad (17)$$

$$\begin{aligned} P_s^{SDF} \approx & \frac{\alpha}{(2\beta\overline{\gamma})^{(M+1)K}}\frac{[2K(M+1)-1]!}{[K(M+1)-1]!}\prod_k\overline{a}_k\sum_{\mathcal{D}(k)}\frac{1}{(|\mathcal{D}(k)|+1)} \\ & \times\prod_{m\in\mathcal{D}(k)}\overline{c}_{k,m}\prod_{m\notin\mathcal{D}(k)}\overline{c}_{k,m}. \end{aligned} \qquad (18)$$

From these asymptotic expressions of SEP, it can be seen that a total diversity order equalling to $(M+1)K$ can be obtained for AF (DF) protocol both for the cooperation with all relay participating and for that with relay selection.

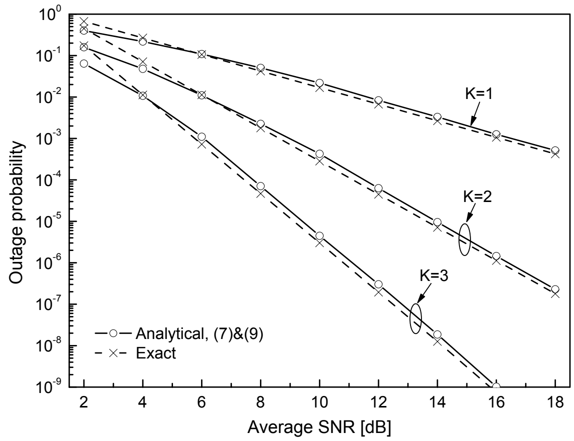

Fig. 3. Outage probability of decode-and-forward with different numbers of accessing users ($M = 1$).

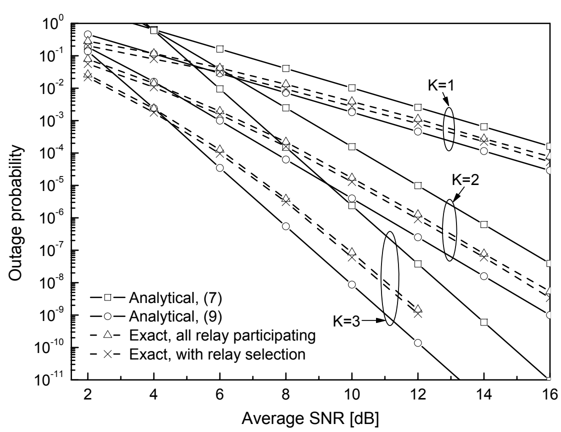

Fig. 4. Outage probability of decode-and-forward with different numbers of accessing users ($M = 2$).

IV. Numerical Results

In this section, we provide the numerical results regarding the outage probability and the symbol error probability performance. In all figures, both the exact analysis (obtained from computer simulations) and the theoretical approximate results obtained in Section III are presented.

Fig. 1-Fig. 4 give the simulation and approximate analytical results of outage probability for AF protocol and DF protocol with different numbers of accessing users K and available relays M. It can be seen that the theoretical approximation is valid. Although for $M = 2$, the analytical results are not so tight compared to that for $M = 1$, all analytical results and the simulation results decrease at the same degree, i.e., a degree of $(M + 1)K$, with the increase of SNR. These well demonstrate the benefit of multiuser diversity, which are inherent in multiuser cooperative relay networks. It indicates that we should efficiently utilize the multiuser diversity in the cooperative relay system design.

Fig. 5 and Fig. 6 show the symbol error probability results of QPSK modulation for AF and DF protocol with different numbers of accessing user K. It is shown that approximate expressions of SEP (15)-(18) are in accordance with simulation results. A diversity order of $(M + 1)K$ can also be seen, which indicates that both the number of accessing users and

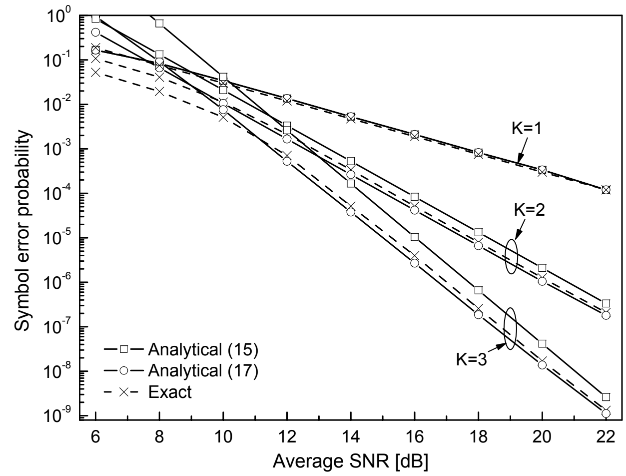

Fig. 5. Symbol error probability of amplify-and-forward with different numbers of accessing users ($M = 1$).

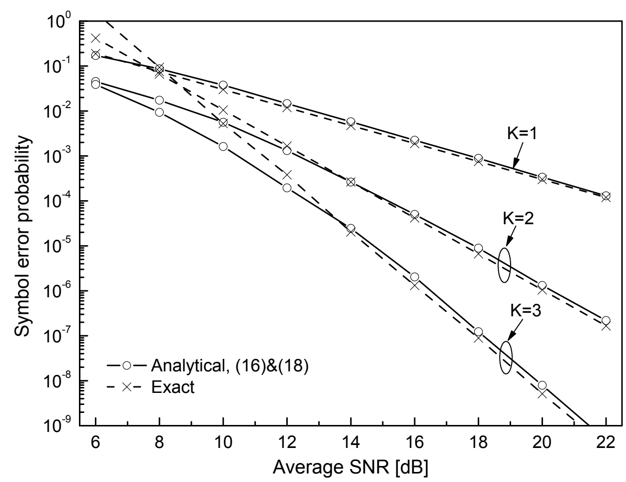

Fig. 6. Symbol error probability of decode-and-forward with different numbers of accessing users ($M = 1$).

available relays have great impact on the SEP performance.

V. Conclusion

This letter presents the analysis of the performance of multiuser diversity in cooperative relay networks with multiple available relays. Through theoretical analysis assuming full CSI at the destination, we derive asymptotic expressions of outage probability and symbol error probability for amplify-and-forward (AF) and decode-and-forward (DF) protocol based cooperative relay networks from a multiuser system perspective. Both the case of relay selection and all relay-participating are considered. All the theoretical analysis are validated by the Mento Carlo simulations. And both the theoretical and simulation analysis show that a multiuser diversity order of K and a cooperative diversity order of $(M+1)$ can be achieved simultaneously for both AF and DF protocol based cooperative relay networks with K accessing users and M available relays.

Appendix

1) Proof of Lemma 2 (8)*:* According to order statistics, for independently distributed random variables γ_k^{SAF}, the CDF of

γ^{SAF} can be calculated as

$$F_{\gamma^{SAF}}(\gamma) = Pr\left[\gamma^{SAF} \le \gamma\right] = \prod_{k=1}^{K} Pr[\gamma_k^{SAF} \le \gamma]$$
$$= \prod_{k=1}^{K} Pr\left\{\overline{\gamma}a_k + \max_m \frac{\overline{\gamma}^2 b_{k,m}c_{k,m}}{\overline{\gamma}b_{k,m} + \overline{\gamma}c_{k,m} + 1} \le \gamma\right\}$$
$$= \left(\frac{\gamma}{\overline{\gamma}}\right)^{K(M+1)} \prod_{k=1}^{K} \frac{Pr\left[a_k + \max_m \frac{b_{k,m}c_{k,m}}{b_{k,m}+c_{k,m}+\frac{1}{\overline{\gamma}}} \le \frac{\gamma}{\overline{\gamma}}\right]}{(\frac{\gamma}{\overline{\gamma}})^{M+1}}.$$

By *Theorem 4* in [5], the CDF of γ^{SAF} can be approximated as

$$F_{\gamma^{SAF}}(\gamma) \approx (\frac{\gamma}{\overline{\gamma}})^{K(M+1)} \prod_{k=1}^{K}\left[\frac{1}{(M+1)}\overline{a}_k \prod_{m=1}^{M}(\overline{b}_{k,m} + \overline{c}_{k,m})\right]$$
$$= \frac{1}{(M+1)^K}(\frac{\gamma}{\overline{\gamma}})^{K(M+1)} \prod_{k=1}^{K}\overline{a}_k \prod_{m=1}^{M}(\overline{b}_{k,m} + \overline{c}_{k,m}).$$

2) Proof of Lemma 2 (9): First, the approximate conditional CDF of γ_k^{SDF} can be calculated as

$$Pr(\gamma_k^{SDF} < \gamma|\mathcal{D}(k)) = \int_0^{\frac{\gamma}{\overline{\gamma}}} \prod_{m\in\mathcal{D}(k)} Pr\left[c_{k,m} \le \frac{\gamma}{\overline{\gamma}} - a_k\right] p_{a_k}(a_k)da_k$$
$$= \int_0^{\frac{\gamma}{\overline{\gamma}}} \prod_{m\in\mathcal{D}(k)} (1-e^{-\overline{c}_{k,m}(\frac{\gamma}{\overline{\gamma}}-a_k)})p_{a_k}(a_k)da_k,$$

where $p_{a_k}(a_k)$ denotes the PDF of a_k. Let $a_k' = \gamma/\overline{\gamma} - a_k$ and use the approximation $e^x \approx 1 + x$ when $x \to 0$, after some manipulation, the above equation can be further simplified as

$$F_{\gamma_k^{SDF}}(\gamma) \approx (\frac{\gamma}{\overline{\gamma}})^{(|\mathcal{D}(k)|+1)}\overline{a}_k\prod_{m\in\mathcal{D}(k)}\overline{c}_{k,m}\left[\frac{1}{|\mathcal{D}(k)|+1} - \frac{\overline{a}_k\frac{\gamma}{\overline{\gamma}}}{(|\mathcal{D}(k)|+1)(|\mathcal{D}(k)|+2)}\right]$$
$$\approx (\frac{\gamma}{\overline{\gamma}})^{(|\mathcal{D}(k)|+1)}\frac{1}{|\mathcal{D}(k)|+1}\overline{a}_k\prod_{m\in\mathcal{D}(k)}\overline{c}_{k,m}.$$

Then, according to the total probability law, with the probability of $Pr(\mathcal{D}(k))$ approximated as $Pr(\mathcal{D}(k)) = (\frac{\gamma}{\overline{\gamma}})^{M-|\mathcal{D}(k)|}\prod_{m\notin\mathcal{D}(k)}\overline{c}_{k,m}$ [1], the distribution of γ_k^{SDF} can be obtained as

$$Pr(\gamma_k^{SDF} < \gamma) = \sum_{\mathcal{D}(k)} Pr(\gamma_k^{SDF} < \gamma|\mathcal{D}(k))Pr(\mathcal{D}(k))$$
$$\approx (\frac{\gamma}{\overline{\gamma}})^{M+1}\overline{a}_k \sum_{|\mathcal{D}(k)|}\frac{1}{|\mathcal{D}(k)|+1}\prod_{m\in\mathcal{D}(k)}\overline{c}_{k,m}\prod_{m\notin\mathcal{D}(k)}\overline{c}_{k,m}.$$

For independently distributed γ_k^{SDF}, we can get the CDF of γ^{SDF} as

$$F_{\gamma^{SDF}}(\gamma) \approx (\frac{\gamma}{\overline{\gamma}})^{K(M+1)}\prod_k \overline{a}_k \sum_{|\mathcal{D}(k)|}\frac{1}{(|\mathcal{D}(k)|+1)}\prod_{m\in\mathcal{D}(k)}\overline{c}_{k,m}\prod_{m\notin\mathcal{D}(k)}\overline{c}_{k,m}.$$

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