Performance Analysis of Cooperative Diversity with Incremental-Best-Relay Technique over Rayleigh Fading Channels

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***Abstract*—In this paper, we introduce a comprehensive analysis of the incremental-best-relay cooperative diversity, in which we exploit limited feedback from the destination terminal, e.g., a single bit indicating the success or failure of the direct transmission. If the destination provides a negative acknowledgment via feedback; in this case only, the best relay among M available relays retransmits the source signal in an attempt to exploit spatial diversity by combining the signals received at the destination from the source and the best relay. Furthermore, we study the end-to-end performance of the incremental-best-relay cooperative-diversity networks using decode-and-forward and amplify-and-forward relaying over independent non-identical Rayleigh fading channels. Closed-form expressions for the bit error rate, the outage probability and average channel capacity are determined. Results show that the incremental-best-relay cooperative diversity can achieve the maximum possible diversity order, compared with the regular cooperative-diversity networks, with higher channel utilization. In particular, the incremental-best-relay technique can achieve $M+1$ diversity order at low signal-to noise ratio (SNR) and considerable virtual array gain at high SNR.**

***Index Terms*—Cooperative diversity networks, decode-and-forward, amplify-and-forward, Multiple relay networks, incremental relaying, best relay, error probability, outage probability.**

I. Introduction

COOPERATIVE-DIVERSITY networks technology is a promising solution for the high data-rate coverage required in future wireless communications systems. There are two main advantages of this technology; the low transmit RF power requirements, and the spatial diversity gain [1]–[4]. The basic idea is that in addition to the direct transmission from the transmitter to the receiver, there are other nodes, which can be used to enhance the diversity by relaying the source signal to the destination.

In regular cooperative-diversity networks, in addition to the direct link all relays participates in sending the source signal to the destination. Then, the destination combines all received signals from the indirect links and the direct link using some combining technique such as the Maximum Ratio Combining (MRC). The regular cooperative-diversity scheme was analyzed for various system and channel models. The average symbol error rate (SER) of a two-hop amplify-and-forward cooperative system is analyzed in [5]–[7] for the Rayleigh and Nakagami-*m* fading channels. In [8] and [9], the authors derived closed-form expressions for the outage probability for Rayleigh and Nakagami-*m* channels, respectively with decode-and-forward relays. Furthermore, the enhancement of spatial-diversity by applying space-time coding was investigated in [10], [11] for non-regenerative (amplify-and-forward) and distributed regenerative (decode-and-forward) relaying, respectively.

It is widely known that the advantages of the regular cooperative diversity come at a spectral efficiency cost. This is mainly because of

- the half-duplex constraint (i.e., the fact that the relays cannot transmit and receive simultaneously at the same frequency band [1], [2]), resulting in two stages transmissions from the source terminals to the destination ones.
- the relays must transmit in orthogonal channels (time or frequency or codes) to avoid the interference with each other

To increase the spectral efficiency of cooperative-diversity networks the best-relay selection scheme, where the best relay only takes part and retransmit the source signal to the destination, is proposed [12]. The authors in [12] showed that this selection scheme has the same diversity order in terms of the outage probability as the regular cooperative-diversity networks. Closed-form expressions for the error probability, outage probability and average channel capacity for the best-relay cooperative diversity networks over Rayleigh fading channels are determined in [13]. In [14], asymptotic error performance was derived for the error probability. The authors in [15], [16] determined the error probability and outage probability of the best-relay adaptive decode-and-forward cooperative-diversity networks.

In different scenario, incremental-relaying cooperative-diversity networks try to save the channels by restricting the relaying process to the necessary conditions [1]. This can be implemented by exploiting a limited feedback from the destination terminal, (e.g., a single bit indicating the success or failure of the direct transmission). If the source-destination SNR is sufficiently high, the feedback indicates success of

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the direct transmission, and the relay does nothing. If the source-destination SNR is not sufficiently high for successful direct transmission, the feedback requests that the relay resends what it received from the source. In the latter case, the destination combines the two signals using MRC technique or any other combining technique [1].

Up to the date of the literature, there is little attention found on the incremental relaying protocol. The incremental relaying protocol was studied in [17] where the problem was approached from an information theoretic perspective. In [18], the authors analyzed by simulation the incremental relaying protocol with Rate-compatible punctured convolutional codes (RCPC codes) in a two-user cooperative diversity system. This protocol can be considered as an extension of hybrid automatic repeat-request (H-ARQ) with RCPC to the cooperative diversity system. Finally, some preliminary performance analysis and proposed protocols for incremental relaying can be found in [19] and [20].

Further to increase the spectral efficiency of the cooperative-diversity networks, The authors in [21]–[23] have proposed a new algorithm called incremental-best-relay technique. The incremental-best-relay cooperative relaying networks exploits limited feedback from the destination terminal, as in the incremental relaying technique. If the destination provides a negative acknowledgment via feedback, in that case only the best relay among the M relays retransmits the source signal in an attempt to exploit spatial diversity by combining the signals that the destination receives from source and the best relay. It can be seen that in this new algorithm we have two features. First, we restrict the relaying process to the necessary conditions. Second, to achieve high diversity order without sacrificing the spectral efficiency, only the best relay (not all the relays) retransmits another copy of the source signal to the destination. Furthermore, this algorithm will not increase the hardware complexity compared to the incremental relying introduced in [1], since the same ACK/NACK can be used.

In this paper, we present a completely analytical approach in obtaining closed-form expressions for the error rate, outage probability and average channel capacity of the incremental-best-relay cooperative diversity networks equipped with decode-and-forward and amplify-and-forward relays over independent non-identical Rayleigh fading channels.

The remainder of this paper is organized as follows. Section II discusses the system model. Performance analysis is given in Section III. Section IV includes the performance results. Finally, the conclusions are given in Section V.

II. System Model

As shown in Fig. 1, a source node (S) and a destination node (D) communicate over a channel with a flat Rayleigh fading coefficient ($h_{S,D}$). A number of potential relaying nodes $R_i, (i = 1, \cdots, M)$ are available to relay the signal to provide the destination with another copy of the original signal. The channel coefficients between S and R_i (h_{S,R_i}) and between R_i and D ($h_{R_i,D}$) are also flat Rayleigh fading coefficients. In addition, $h_{S,D}$, h_{S,R_i} and $h_{R_i,D}$ are mutually-independent and non-identical. We also assume here that the additive white Gaussian noise (AWGN) terms of all links have zero mean and equal variance (N_0). All terminals are equipped with a single antenna.

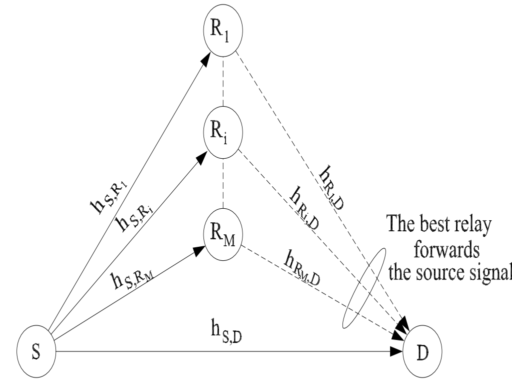

Fig. 1. Illustration of a multi branch cooperative diversity network.

Assuming time division multiplexing for simplicity[1], in the first time slot, the source sends its signal. All the M relays and the destination receive faded noisy versions of the source signal. Based on the quality of the received signal at the destination, the destination decides whether relaying is needed or not. For sufficient signal quality, all the relays do nothing (the destination performs detection using the source signal) and the source sends a new message in the second time slot. For insufficient signal quality at the destination, the best relay that gives the highest SNR at the destination forwards the received source signal to the destination. In this case, the destination combines the two signals using MRC techniques.

Mathematically speaking, the received signal from the source at the destination $(y_{S,D})$ and at the relay (y_{S,R_i}) can be written as

$$\begin{aligned} y_{S,D} &= h_{S,D}\sqrt{E_s}x + n_{S,D} \\ y_{S,R_i} &= h_{S,R_i}\sqrt{E_s}x + n_{S,R_i} \end{aligned} \tag{1}$$

where E_s is the transmitted signal energy, x is a transmitted symbol signal with unit energy and $n_{S,D}$ and n_{S,R_i} are the AWGN terms. In the second time slot, if it is necessary, the best relay processes the received signal and generates the relayed signal x_r and transmits it to the destination. The received signal at the destination from the best relay is given by

$$y_{R_{\text{best}},D} = h_{R_{\text{best}},D}\sqrt{E_s}x_r + n_{R_{\text{best}},D} \tag{2}$$

where $n_{R_{\text{best}},D}$ is the AWGN term of the $R_{\text{best}} \rightarrow D$ link.

We assume that the acknowledgment feedback transmission by the destination is based on the comparison of the received SNR from the source with SNR threshold (γ_0), which defines the minimum SNR for which the destination can detect the signal successfully without the need of the relayed signal.

While a large value of γ_0 lowers the probability of error, it reduces the bandwidth efficiency because the best relay forwards the signal more often but this, of course, increases the diversity benefit. Note that for direct transmission only, γ_0

[1]Frequency division multiplexing and code division multiplexing can also be used.

is equal to 0 while for the conventional best-relay cooperative-diversity networks in which the best relay always resend the source signal to destination, γ_0 is equal to ∞.

Finally, to capture the effect of the path-loss on the performance we use the model (which is commonly used in the literature e.g., [1]), where $\mathbf{E}\left(h^2_{S,R_i}\right) = \left(d_{S,D}/d_{S,R_i}\right)^\alpha$, $\mathbf{E}\left(h^2_{R_i,D}\right) = \left(d_{S,D}/d_{R_i,D}\right)^\alpha$ and $\mathbf{E}\left(h^2_{S,D}\right) = 1$, where $d_{i,j}$ is the distance between terminal i and j, α is the path-loss exponent and $\mathbf{E}(\bullet)$ is the statistical average operator.

III. Error Performance Analysis

In this section, we derive closed-form expressions for the error probability for the two relaying schemes, amplify-and-forward and decode-and-forward. The average unconditional error probability of the combined signal using the incremental-best-relay technique for both amplify-and-forward and decode-and-forward schemes can be written as

$$P(e) = \Pr\left(\gamma_{S,D} \le \gamma_0\right) P_{div}(e) + \left(1 - \Pr\left(\gamma_{S,D} \le \gamma_0\right)\right) P_{direct}(e) \quad (3)$$

where $\gamma_{S,D} = h^2_{S,D} E_s/N_0$ is the instantaneous SNR between S and D, $P_{div}(e)$ is the average probability that an error occurs in the combined signal (at the destination) from the source and the best relay signals. The fading parameter $h_{S,D}$ follows the Rayleigh distribution; therefore, $\gamma_{S,D}$ follows the exponential distribution. Hence, it is straightforward to show that

$$\Pr\left(\gamma_{S,D} \le \gamma_0\right) = 1 - \exp\left(-\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right) \quad (4)$$

where $\bar{\gamma}_{S,D} = \mathrm{E}\left(h^2_{S,D}\right) E_s/N_0$ is the average SNR between S and D, and $P_{direct}(e)$ is the probability of error at the destination given that the destination decides that the relay does not forward the source signal. In this case, the destination relies only on the direct signal from the source. For several Gray bit-mapped constellations employed in practical systems, the conditional error probability takes the form of $a \times \operatorname{erfc}\left(\sqrt{b\gamma_{S,D}}\right)$, with $\operatorname{erfc}(x)$ being the complementing error function defined as $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp\left(-x^2\right) dx$ and (a, b) are constants depending on the type of modulation (e.g. BPSK: $a = 0.5$ and $b = 1$, QPSK: $a = 0.5$ and $b = 0.5$). Then, the corresponding average error probability $(P_{direct}(e))$ can be written as

$$P_{direct}(e) = \int_0^\infty P_{direct}(e|\gamma) f_{\gamma_{S,D}}\left(\gamma|\gamma_{S,D} > \gamma_0\right) d\gamma \quad (5)$$

where $P_{direct}(e|\gamma)$ is the conditional error probability and $f_{\gamma_{S,D}}\left(\gamma|\gamma_{S,D} > \gamma_0\right)$ is the conditional probability density function (PDF) of $\gamma_{S,D}$ given that $\gamma_{S,D}$ is greater than the threshold value γ_0. The conditional PDF $f_{\gamma_{S,D}}\left(\gamma|\gamma_{S,D} > \gamma_0\right)$ can be found to be as

$$f_{\gamma_{S,D}}\left(\gamma|\gamma_{S,D} > \gamma_0\right) = \begin{cases} 0, & \text{if } \gamma \le \gamma_0; \\ \dfrac{\exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)}{\bar{\gamma}_{S,D}} \exp\left(\frac{-\gamma}{\bar{\gamma}_{S,D}}\right), & \text{if } \gamma > \gamma_0 \end{cases} \quad (6)$$

Substituting (6) into (5) and by solving the integration and doing some necessary manipulations, the average error probability can be written in a closed-from as

$$P_{direct}(e) = a \operatorname{erfc}\left(\sqrt{b\gamma_0}\right) - a \exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right) \sqrt{\frac{b\bar{\gamma}_{S,D}}{1 + b\bar{\gamma}_{S,D}}} \times \operatorname{erfc}\left(\sqrt{\gamma_0\left(b + 1/\bar{\gamma}_{S,D}\right)}\right) \quad (7)$$

where $\bar{\gamma}_{S,D} = \mathbf{E}\left(h^2_{S,D}\right) E_s/N_0$ is the average SNR between S and D. Note that for $\gamma_0 = 0$, $a = 0.5$ and $b = 1$, we obtain the well-known probability of error for BPSK transmission over Rayleigh fading channel [24]. Furthermore, we should note that as γ_0 increases, $P_{direct}(e)$ decreases, which plays an important role on the value of $P(e)$ for high SNR region as we will see later. If the best relay forwards another copy of the source signal, then the destination combines the signals it receives from source and relay using MRC. The expression of $P_{div}(e)$ depends on the relaying scheme that will be used at the relay as discussed in the following subsections.

A. Amplify-and-Forward

In the amplify-and-forward scheme, the relayed signal (x_r) is an amplified version of $y_{S,R_{\text{best}}}$ and can be written as $x_r = G \times y_{S,R_{\text{best}}}$ where $G = \sqrt{E_s/\left(E_s h^2_{S,R_{\text{best}}} + N_0\right)}$ is the gain factor. We choose the amplifier gain to depend upon the fading coefficient $h_{S,R_{\text{best}}}$, which the best relay can estimate with high accuracy. It is widely known that the exact and upper-bound of the total SNR at the destination can be written as

$$\begin{aligned} \gamma_{AF} &= \gamma_{S,D} + \max_{i \in M}\left[\frac{\gamma_{S,R_i}\gamma_{R_i,D}}{\gamma_{S,R_i} + \gamma_{R_i,D} + 1}\right] \\ &\le \gamma_{S,D} + \max_{i \in M}\left[\min\left(\gamma_{S,R_i}\gamma_{R_i,D}\right)\right] \\ &= \gamma_{S,D} + \max_{i \in M}(\gamma_i) = \gamma_{S,D} + \gamma_b = \gamma_{ub} \end{aligned} \quad (8)$$

where $\gamma_i = \min\left(\gamma_{S,R_i}\gamma_{R_i,D}\right)$ and $\gamma_b = \max_{i \in M}(\gamma_i)$. Our subsequent analysis exclusively relies on γ_{ub}, since this upper bound is shown to be quite accurate [7]. Finally, The PDF of γ_i can be expressed in terms of the average SNR $\bar{\gamma}_{S,R_i} = \mathbf{E}\left(h^2_{S,R_i}\right) E_s/N_0$ and $\bar{\gamma}_{R_i,D} = \mathbf{E}\left(h^2_{R_i,D}\right) E_s/N_0$ as $f_{\gamma_i}(\gamma) = \left(1/\bar{\gamma}_i\right)\exp\left(-\gamma/\bar{\gamma}_i\right)$, where $\bar{\gamma}_i = \bar{\gamma}_{S,R_i}\bar{\gamma}_{R_i,D}/\left(\bar{\gamma}_{S,R_i} + \bar{\gamma}_{R_i,D}\right)$.

The lower bound average error probability $P_{div}(e)$ can be written as

$$P_{div}(e) = a \int_0^\infty f_{\gamma_{ub}}\left(\gamma_{ub}|\gamma_{S,D} \le \gamma_0\right) \operatorname{erfc}\left(\sqrt{b\gamma_{ub}}\right) d\gamma_{ub} \quad (9)$$

To find the conditional PDF, $f_{\gamma_{ub}}\left(\gamma_{ub}|\gamma_{S,D} \le \gamma_0\right)$, we need to know the PDF of $\gamma_{S,D}$ and γ_b. It is known that the PDF of $\gamma_{S,D}$ is given by $f_{\gamma_{S,D}}(\gamma) = \frac{1}{\bar{\gamma}_{S,D}} \exp\left(\frac{-\gamma}{\bar{\gamma}_{S,D}}\right)$. The PDF of γ_b can be written as [13]

$$f_{\gamma_b}(\gamma) = \sum_{n=1}^{M} (-1)^{n+1} \sum_{k_1=1}^{M-n+1} \sum_{k_2=k_1+1}^{M-n+2} \cdots \sum_{k_n=k_{n-1}+1}^{M} \prod_{j=1}^{n} \exp\left(-\frac{\gamma}{\bar{\gamma}_{k_j}}\right) \sum_{j=1}^{n} \frac{1}{\bar{\gamma}_{k_j}} \quad (10)$$

$$f_{\gamma_{ub}}\left(\gamma|\gamma_{S,D}\leq\gamma_0\right)=\frac{1}{1-\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)}\begin{cases}\sum_{n=1}^{M}(-1)^{n+1}\sum_{k_1=1}^{M-n+1}\sum_{k_2=k_1+1}^{M-n+2}\cdots\sum_{k_n=k_{n-1}+1}^{M}\\ \times\left(\frac{1}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}-\bar{\gamma}_{S,D}}\exp\left(\sum_{j=1}^{n}\frac{-\gamma}{\bar{\gamma}_{k_j}}\right)-\frac{1}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}-\bar{\gamma}_{S,D}}\exp\left(\frac{-\gamma}{\bar{\gamma}_{S,D}}\right)\right), & \text{if } \gamma\leq\gamma_0;\\ \sum_{n=1}^{M}(-1)^{n+1}\sum_{k_1=1}^{M-n+1}\sum_{k_2=k_1+1}^{M-n+2}\cdots\sum_{k_n=k_{n-1}+1}^{M}\\ \times\left(\frac{1-\exp\left(-\gamma_0\left(\frac{1}{\bar{\gamma}_{S,D}}-\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}\right)\right)}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}-\bar{\gamma}_{S,D}}\exp\left(-\gamma\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}\right)\right), & \text{if } \gamma>\gamma_0\end{cases}\tag{11}$$

$$\begin{aligned}P_{div}(e)=&\sum_{n=1}^{M}(-1)^{n+1}\sum_{k_1=1}^{M-n+1}\sum_{k_2=k_1+1}^{M-n+2}\cdots\sum_{k_n=k_{n-1}+1}^{M}\frac{a}{(\omega-\bar{\gamma}_{S,D})\left(1-\exp\left(-\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\right)}\left\{\bar{\gamma}_{S,D}\sqrt{\frac{b\bar{\gamma}_{S,D}}{1+b\bar{\gamma}_{S,D}}}\operatorname{erf}\left(\sqrt{\lambda}\right)\right.\\&-\omega\sqrt{\frac{b\omega}{1+b\omega}}\operatorname{erf}\left(\sqrt{\zeta}\right)+\exp\left(-\gamma_0\left(\frac{1}{\bar{\gamma}_{S,D}}+\frac{1}{\omega}\right)\right)\left(\bar{\gamma}_{S,D}\exp\left(\frac{\gamma_0}{\omega}\right)-\omega\exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\right)\operatorname{erfc}\left(\sqrt{b\gamma_0}\right)-(\bar{\gamma}_{S,D}-\omega)\\&\left.+\left(1-\exp\left(-\gamma_0\left(\frac{1}{\bar{\gamma}_{S,D}}-\frac{1}{\omega}\right)\right)\right)\left[\omega\exp\left(\frac{-\gamma_0}{\omega}\right)\operatorname{erfc}\left(\sqrt{b\gamma_0}\right)-\omega\sqrt{\frac{b\omega}{1+b\omega}}\operatorname{erfc}\left(\sqrt{\zeta}\right)\right]\right\}\end{aligned}\tag{12}$$

After doing some necessary manipulations, the PDF, $f_{\gamma_{ub}}(\gamma_{ub}|\gamma_{S,D}\leq\gamma_0)$, can be written as in (11) on the top of this page. Then the approximate $P_{div}(e)$ reduces to the expression given in (12) on the top of this page, where $\omega=\left(\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}\right)^{-1}$, $\lambda=\frac{\gamma_0(1+b\bar{\gamma}_{S,D})}{\bar{\gamma}_{S,D}}$ and $\zeta=\frac{\gamma_0(1+b\omega)}{\omega}$.

By substituting (4), (7) and (12) into (3) we can have a tight lower bound closed-form expression for the error probability, $P(e)$.

B. Decode-and-Forward

In decode-and-forward scheme, the source broadcasts its signal to the set of M-relay nodes and the destination node. We define the decoding set (C) as the set of relays with the ability to fully decode the source message correctly. Then, the best relay from the decoding set C decodes and forwards (retransmits) the source information to the destination (where the retransmission will happen only for insufficient signal quality between S and D). Then, the destination combines the direct and the best indirect link using MRC. Hence if the relaying is needed, the equivalent SNR at the destination can be written as

$$\gamma_{DF}=\gamma_{S,D}+\max_{i\in C}(\gamma_{R_i,D})\tag{13}$$

Actually, due to the difficulty of finding the PDF of γ_{DF} given in (13), it will be difficult to find a closed form expression for the error probability of the adaptive incremental-best-relay decode-and-forward cooperative networks especially over non-identical fading channels. To bypass this difficulty we invoke the technique described in [8] where we visualize the wireless cooperative network depicted in Fig. 1 as effectively having $M+1$ paths between the source and destination. Then, we define a random variable ξ_i representing the received instantaneous SNR at the destination on the i^{th} indirect link ($S\rightarrow R_i\rightarrow D$). Since the relay is assumed to forward the source information only if it is decoded correctly, we can write the PDF of ξ_i as

$$\begin{aligned}f_{\xi_i}(x)=&f_{\xi_i|R_i\text{ Decodes Incorrectly}}(x)\Pr(R_i\text{ Decodes Incorrectly})\\&+f_{\xi_i|R_i\text{ Decodes Correctly}}(x)\Pr(R_i\text{ Decodes Correctly})\end{aligned}\tag{14}$$

The PDF of the instantaneous SNR from S to R_i (γ_{S,R_i}) is $f_{\gamma_{S,R_i}}(x)=\frac{1}{\bar{\gamma}_{S,R_i}}\exp(-x/\bar{\gamma}_{S,R_i})$, where $\bar{\gamma}_{S,R_i}=\mathbf{E}\left(h_{S,R_i}^2\right)E_s/N_0$ is the average SNR between S and R_i. As in [8], the PDF of ξ_i can be expressed as

$$f_{\xi_i}(x)=B_i\delta(x)+(1-B_i)\frac{1}{\bar{\gamma}_{R_i,D}}\exp\left(-\frac{x}{\bar{\gamma}_{R_i,D}}\right)\tag{15}$$

where

$$\begin{aligned}B_i&=a\int_0^{\infty}\operatorname{erfc}\left(\sqrt{b\gamma_{S,R_i}}\right)f_{\gamma_{S,R_i}}(\gamma_{S,R_i})\,d\gamma_{S,R_i}\\&=a\left[1-\sqrt{\frac{b\bar{\gamma}_{S,R_i}}{b\bar{\gamma}_{S,R_i}+1}}\right]\end{aligned}\tag{16}$$

The unconditional PDF of the SNR at the destination given in (15) represents the i^{th} cascaded link ($S\rightarrow R_i\rightarrow D$) and accounts for the possible incorrect detection of the source message as well as the fading on the i^{th} relay to destination link. By using this PDF, the total SNR at the destination γ_{DF} can be rewritten as

$$\gamma_{DF}=\gamma_{S,D}+\max_{i\in M}(\xi_i)=\gamma_{S,D}+\chi\tag{17}$$

where $\chi=\max_{i\in M}(\xi_i)$. Note that the expressions of γ_{DF} in (13) and (17) are equivalent; however, the expression given in (17) is analytically more tractable than the expression given in (13). As a result, this facilitates the derivation of the total SNR statistics (cumulative distribution (CDF) function and PDF). The PDF of χ can be found as follows: The CDF of χ can be written as

$$F_\chi(x) = \Pr\left(\max_{i\in M} \xi_i \le x\right) = \prod_{i=1}^{M} \Pr(\xi_i \le x) = \prod_{i=1}^{M} F_{\xi_i}(x) \quad (18)$$

where $F_{\xi_i}(x)$ is the CDF of ξ_i, which can easily be derived from (15) as $F_{\xi_i}(x) = 1 - (1 - B_i)\exp(-x/\bar{\gamma}_{R_i,D})$. By using the CDF of ξ_i, the PDF of χ can be found by taking the derivative of (18) with respect to x and after some manipulations, $f_\chi(x)$, can be written as

$$f_\chi(x) = \delta(x)\prod_{i=1}^{M} B_i + \sum_{k=1}^{M}(-1)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M} \times \prod_{i=1}^{k}(1-B_{\lambda_i})\exp\left(\frac{-x}{\bar{\gamma}_{R_{\lambda_i},D}}\right)\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}, \quad x \ge 0 \quad (19)$$

After some necessary manipulations, $f_{\gamma_{DF}}(x|\gamma_{S,D} \le \gamma_0)$ can be written as in (20) on the top of the next page. Since $P_{div}(e) = a\int_0^\infty f_{\gamma_{DF}}(\gamma_{DF}|\gamma_{S,D} \le \gamma_0)\,\mathrm{erfc}\left(\sqrt{b\gamma_{DF}}\right)d\gamma_{DF}$, then substituting (20) and doing the integral and after some simplifications, $P_{div}(e)$ can be obtained as in (21) on the top of the next page, where $\lambda = \frac{\gamma_0(1+b\bar{\gamma}_{S,D})}{\bar{\gamma}_{S,D}}$, $\zeta = \frac{\gamma_0(1+b\nu)}{\nu}$ and $\nu = \left(\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right)^{-1}$.

By substituting (21), (7) and (4) into (3) we can have a closed-form expression for the error probability of the incremental-best-relay adaptive decode-and-forward cooperative diversity over Rayleigh flat fading channels. Finally, although the derived error probabilities for both relaying schemes have multiple summations/multiplications, all these summations/multiplications are finite and easy to handle numerically.

Although the previously derived expressions for the error probabilities for DF and AF enable numerical evaluation of the system performance and may not be computationally intensive, they do not offer insight into the effect of the system parameters. Here, we simplify the error probability expressions by focusing on high SNR. In high SNR region we can assume that $\Pr(\gamma_{S,D} \le \gamma_0)$ goes to zero and this simplifies (3) to

$$\begin{aligned} P(e) &= \underbrace{\Pr(\gamma_{S,D} \le \gamma_0)}_{\to 0} P_{div} + \underbrace{(1 - \Pr(\gamma_{S,D} \le \gamma_0))}_{\to 1} P_{direct} \\ &\approx a\int_{\gamma_0}^{\infty}\frac{1}{\bar{\gamma}_{S,D}}\exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\exp\left(-\frac{\gamma}{\bar{\gamma}_{S,D}}\right)\mathrm{erfc}\left(\sqrt{b\gamma}\right)d\gamma \\ &\approx a\,\mathrm{erfc}\left(\sqrt{b\gamma_0}\right) - a\exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\sqrt{\frac{b\bar{\gamma}_{S,D}}{1+b\bar{\gamma}_{S,D}}} \\ &\times \mathrm{erfc}\left(\sqrt{\gamma_0(b + 1/\bar{\gamma}_{S,D})}\right) \end{aligned} \quad (22)$$

From this equation we can see that at high SNR (SNR $>> \gamma_0$), the error performance for the incremental-best-relay cooperative-diversity network outperforms the direct transmission system.

Furthermore as γ_0 increases we can find from the second line of (22) that $P_{direct}(e)$ will also reduce since the region of the integration will decrease. This means, as we will see in the result sections, that for approximately SNR $< \gamma_0$ the incremental-best-relay cooperative-diversity network achieves a diversity order of $M+1$ and for high SNR (SNR $> \gamma_0$) the incremental-best-relay cooperative-diversity network achieves virtual array gain with a diversity order of 1.

IV. Outage Probability and Throughput Analysis

A. Outage Probability Analysis

In this subsection, we derive closed-form expressions for the outage probability (P_{out}). In incremental-best-relay cooperative-diversity networks, if the SNR of the direct link at the destination is less than the threshold value γ_0, the destination will need assistance from the best relay to send another copy of the source signal. In this case, the best relay sends another copy of the signal but there is still a probability that the overall SNR at the destination is less than γ_0, and in this subsection we will determine this probability.

1) Amplify-and-Forward: The outage probability can be derived as

$$\begin{aligned} P_{out} &= \Pr(\gamma_{S,D} + \gamma_b \le \gamma_0 | \gamma_{S,D} \le \gamma_0)\Pr(\gamma_{S,D} \le \gamma_0) \\ &= \Pr(\gamma_{S,D} + \gamma_b \le \gamma_0) \end{aligned} \quad (23)$$

Since the two links (direct and best indirect links) are independent, then the outage probability can be found by finding the convolution of the two PDFs to find the overall PDF and then finding the CDF. After doing some necessary manipulations, P_{out} can be derived and written in closed form as

$$\begin{aligned} P_{out} &= \sum_{n=1}^{M}(-1)^{n+1}\sum_{k_1=1}^{M-n+1}\sum_{k_2=k_1+1}^{M-n+2}\cdots\sum_{k_n=k_{n-1}+1}^{M} \\ &\times\left(1 - \frac{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}} - \bar{\gamma}_{S,D}}\exp\left(\sum_{j=1}^{n}\frac{-\gamma_0}{\bar{\gamma}_{k_j}}\right)\right. \\ &\left. + \frac{\bar{\gamma}_{S,D}}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}} - \bar{\gamma}_{S,D}}\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)\right) \end{aligned} \quad (24)$$

2) Decode-and-Forward: In the case of decode-and-forward, the outage probability can be derived as follows

$$\begin{aligned} P_{out} &= \Pr(\gamma_{S,D} + \chi \le \gamma_0 | \gamma_{S,D} \le \gamma_0)\Pr(\gamma_{S,D} \le \gamma_0) \\ &= \Pr(\gamma_{S,D} + \chi \le \gamma_0) \end{aligned} \quad (25)$$

Then P_{out} can be derived as

$$\begin{aligned} P_{out} &= \left(\prod_{i=1}^{M} B_i\right)\left(1 - \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)\right) \\ &+ \sum_{k=1}^{M}(-1)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M}\prod_{i=1}^{k}(1-B_{\lambda_i}) \\ &\times\left[1 + \frac{1}{\bar{\gamma}_{S,D} - 1/\sum_{i=1}^{k}1/\bar{\gamma}_{R_{\lambda_i},D}}\times\right. \\ &\left.\left(\frac{\prod_{i=1}^{k}\exp\left(-\gamma_0/\bar{\gamma}_{R_{\lambda_i},D}\right)}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}} - \bar{\gamma}_{S,D}\exp(-\gamma_0/\bar{\gamma}_{S,D})\right)\right] \end{aligned} \quad (26)$$

$$f_{\gamma_{DF}}\left(x|\gamma_{S,D}\leq\gamma_0\right)=\frac{1}{1-\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)}\times\begin{cases}\frac{\prod_{i=1}^{M}B_i}{\bar{\gamma}_{S,D}}\exp\left(-\frac{x}{\bar{\gamma}_{S,D}}\right)+\sum_{k=1}^{M}\left(-1\right)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M}\\ \times\frac{\prod_{i=1}^{k}\left(1-B_{\lambda_i}\right)}{\bar{\gamma}_{S,D}-1/\sum_{i=1}^{k}1/\bar{\gamma}_{R_{\lambda_i},D}}\left[\exp\left(-x/\bar{\gamma}_{S,D}\right)-\prod_{i=1}^{k}\exp\left(-x/\bar{\gamma}_{R_{\lambda_i},D}\right)\right], & \text{if } x\leq\gamma_0;\\ \sum_{k=1}^{M}\left(-1\right)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M}\\ \times\frac{\prod_{i=1}^{k}\left(1-B_{\lambda_i}\right)}{\left(1/\sum_{i=1}^{k}1/\bar{\gamma}_{R_{\lambda_i},D}\right)-\bar{\gamma}_{S,D}}\exp\left(-x\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right)\left[1-\exp\left(-\gamma_0\left(1/\bar{\gamma}_{S,D}-\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right)\right)\right], & \text{if } x>\gamma_0\end{cases}\tag{20}$$

$$\begin{aligned}P_{div}\left(e\right)=&\frac{a\prod_{i=1}^{M}B_i}{1-\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)}\left[1-\sqrt{\frac{b\bar{\gamma}_{S,D}}{1+b\bar{\gamma}_{S,D}}}\text{erf}\left(\sqrt{\frac{\gamma_0\left(1+b\bar{\gamma}_{S,D}\right)}{\bar{\gamma}_{S,D}}}\right)-\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)\text{erfc}\left(\sqrt{b\gamma_0}\right)\right]\\&+\sum_{k=1}^{M}\left(-1\right)^{k+1}\sum_{\lambda_1=1}^{M-k+1}\sum_{\lambda_2=\lambda_1+1}^{M-k+2}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{M}\frac{a}{\left(\nu-\bar{\gamma}_{S,D}\right)\left(1-\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)\right)}\\&\times\left\{\bar{\gamma}_{S,D}\sqrt{\frac{b\bar{\gamma}_{S,D}}{1+b\bar{\gamma}_{S,D}}}\text{erf}\left(\sqrt{\lambda}\right)-\nu\sqrt{\frac{b\nu}{1+b\nu}}\text{erf}\left(\sqrt{\zeta}\right)+\exp\left(-\gamma_0\left(\frac{1}{\bar{\gamma}_{S,D}}+\frac{1}{\nu}\right)\right)\right.\\&\times\left(\bar{\gamma}_{S,D}\exp\left(\frac{\gamma_0}{\nu}\right)-\nu\exp\left(\frac{\gamma_0}{\bar{\gamma}_{S,D}}\right)\right)\text{erfc}\left(\sqrt{b\gamma_0}\right)-\left(\bar{\gamma}_{S,D}-\nu\right)\\&\left.+\left[1-\exp\left(-\gamma_0\left(\frac{1}{\bar{\gamma}_{S,D}}-\frac{1}{\nu}\right)\right)\right]\left[\nu\exp\left(\frac{-\gamma_0}{\nu}\right)\text{erfc}\left(\sqrt{b\gamma_0}\right)-\nu\sqrt{\frac{b\nu}{1+b\nu}}\text{erfc}\left(\sqrt{\zeta}\right)\right]\right\}\end{aligned}\tag{21}$$

B. Average Channel Capacity

The channel capacity, in the Shannon's sense, is an important performance metric since it provides the maximum achievable transmission rate under which the errors are recoverable. The average channel capacity can be expressed as

$$C_{avg}=\Pr\left(\gamma_{S,D}\leq\gamma_0\right)C_{div}+\left(1-\Pr\left(\gamma_{S,D}\leq\gamma_0\right)\right)C_{direct}\tag{27}$$

where C_{div} is the average channel capacity in the combined diversity transmission from S and R_{best} to the D and C_{direct} is the average channel capacity of the direct link when $\gamma_{S,D}\geq\gamma_0$.

C_{direct} for both schemes, amplify-and-forward and decod-and-forward, can be written as

$$\frac{C_{direct}}{BW}=\int_0^{\infty}\log_2\left(1+\gamma_{S,D}\right)f_{\gamma_{S,D}}\left(\gamma_{S,D}|\gamma_{S,D}>\gamma_0\right)d\gamma_{S,D}\tag{28}$$

where $f_{\gamma_{S,D}}\left(\gamma_{S,D}|\gamma_{S,D}>\gamma_0\right)$ given in (6). The integration in (28) can be solved in closed form as

$$\begin{aligned}&C_{direct}\\&=\frac{BW}{\ln\left(2\right)}\left[\exp\left(\frac{1+\gamma_0}{\bar{\gamma}_{S,D}}\right)E_1\left(\frac{1+\gamma_0}{\bar{\gamma}_{S,D}}\right)+\ln\left(1+\gamma_0\right)\right]\end{aligned}\tag{29}$$

where $E_1\left(x\right)$ is the exponential integral defined as $E_1\left(x\right)=\int_x^{\infty}\frac{\exp(-t)}{t}dt$ [27, eq. 5.1.1]. Note that for $\gamma_0=0$, we obtain the well known average channel capacity over Rayleigh fading channel [26]. The expression of C_{div} depends on the relaying scheme that will be used at the relay as discussed in the following subsections.

1) Amplify-and-Forward: In the amplify-and-forward scheme, C_{div} can be written as

$$C_{div}=\frac{BW}{2}\int_0^{\infty}f_{\gamma_{ub}}\left(\gamma_{ub}|\gamma_{S,D}\leq\gamma_0\right)\log_2\left(1+\gamma_{ub}\right)d\gamma_{ub}\tag{30}$$

The reason of the $\frac{1}{2}$ factor here is that we need in this case two orthogonal channels or two time slots for transmitting the data. By subtituting the PDF derived in (11) into (30), the closed-form expression for C_{div} can be written as in (31) on the top of the next page. By substituting (31), (4) and (29) into (27) we can have a closed-form expression for the channel capacity, C_{avg}, of the amplify-and-forward best-relay incremental relaying cooperative relaying protocol over Rayleigh flat fading channels.

2) Decode-and-Forward: By following the same steps as in amplify-and-forward, C_{div} can be written as in (32) on the top of the next page. By substituting (32), (4) and (29) into (27) we can have a closed-form expression for C_{avg}.

In high SNR region we can assume that $\Pr\left(\gamma_{S,D}\leq\gamma_0\right)$ goes to zero and this simplifies (27) to

$$\begin{aligned}C_{avg}&=\underbrace{\Pr\left(\gamma_{S,D}\leq\gamma_0\right)}_{\to0}C_{div}+\underbrace{\left(1-\Pr\left(\gamma_{S,D}\leq\gamma_0\right)\right)}_{\to1}C_{direct}\\&\approx BW\int_0^{\infty}\log_2\left(1+\gamma_{S,D}\right)f_{\gamma_{S,D}}\left(\gamma_{S,D}|\gamma_{S,D}>\gamma_0\right)d\gamma_{S,D}\\&\approx\frac{BW}{\ln\left(2\right)}\left[\exp\left(\frac{1+\gamma_0}{\bar{\gamma}_{S,D}}\right)E_1\left(\frac{1+\gamma_0}{\bar{\gamma}_{S,D}}\right)+\ln\left(1+\gamma_0\right)\right]\end{aligned}\tag{33}$$

$$C_{div} = \frac{BW/2}{1-\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)} \sum_{n=1}^{M} (-1)^{n+1} \sum_{k_1=1}^{M-n+1} \sum_{k_2=k_1+1}^{M-n+2} \cdots \sum_{k_n=k_{n-1}+1}^{M} \frac{1}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}} - \bar{\gamma}_{S,D}}$$
$$\times \Bigg\{ \bar{\gamma}_{S,D} \exp\left(\frac{1}{\bar{\gamma}_{S,D}}\right) E_1\left(\frac{1+\gamma_0}{\bar{\gamma}_{S,D}}\right) - \frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}} \exp\left(\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}\right) E_1\left(\frac{1+\gamma_0}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}}\right) + \ln(1+\gamma_0)$$
$$\times \left[\bar{\gamma}_{S,D} \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right) - \frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}} \exp\left(-\gamma_0 \sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}\right)\right] - \bar{\gamma}_{S,D} \exp\left(\frac{1}{\bar{\gamma}_{S,D}}\right) E_1\left(\frac{1}{\bar{\gamma}_{S,D}}\right)$$
$$+ \frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}} \exp\left(\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}\right) E_1\left(\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}\right) + \left[1-\exp\left(-\gamma_0\left(\frac{1}{\bar{\gamma}_{S,D}} - \sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}\right)\right)\right]$$
$$\times \left[\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}} \exp\left(-\gamma_0 \sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}\right) \ln(1+\gamma_0) + \frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}} \exp\left(\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}\right) E_1\left(\frac{1+\gamma_0}{\frac{1}{\sum_{j=1}^{n}\frac{1}{\bar{\gamma}_{k_j}}}}\right)\right]\Bigg\} \tag{31}$$

$$C_{div} = \frac{\frac{BW}{2\ln(2)} \prod_{i=1}^{M} B_i}{1-\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)} \left[\exp\left(\frac{1}{\bar{\gamma}_{S,D}}\right)\left(E_1\left(\frac{1}{\bar{\gamma}_{S,D}}\right) - E_1\left(\frac{1+\gamma_0}{\bar{\gamma}_{S,D}}\right)\right) - \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)\ln(\gamma_0+1)\right]$$
$$+ \frac{BW/(2\ln(2))}{1-\exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right)} \sum_{k=1}^{M} (-1)^{n+1} \sum_{\lambda_1=1}^{M-n+1} \sum_{\lambda_2=\lambda_1+1}^{M-n+2} \cdots \sum_{\lambda_k=\lambda_{k-1}+1}^{M} \frac{\prod_{i=1}^{k}(1-B_{\lambda_i})}{\frac{1}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}} - \bar{\gamma}_{S,D}}$$
$$\times \Bigg\{ \bar{\gamma}_{S,D} \exp\left(\frac{1}{\bar{\gamma}_{S,D}}\right) E_1\left(\frac{1+\gamma_0}{\bar{\gamma}_{S,D}}\right) - \frac{\exp\left(\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right)}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}} E_1\left(\frac{1+\gamma_0}{\frac{1}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}}}\right) + \ln(1+\gamma_0)$$
$$\times \left[\bar{\gamma}_{S,D} \exp\left(\frac{-\gamma_0}{\bar{\gamma}_{S,D}}\right) - \frac{1}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}} \exp\left(-\gamma_0 \sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right)\right] - \bar{\gamma}_{S,D} \exp\left(\frac{1}{\bar{\gamma}_{S,D}}\right) E_1\left(\frac{1}{\bar{\gamma}_{S,D}}\right)$$
$$+ \frac{1}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}} \exp\left(\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right) E_1\left(\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right) + \left[1-\exp\left(-\gamma_0\left(\frac{1}{\bar{\gamma}_{S,D}} - \sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right)\right)\right]$$
$$\times \left[\frac{\exp\left(-\gamma_0 \sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right)}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}} \ln(1+\gamma_0) + \frac{1}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}} \exp\left(\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}\right) E_1\left(\frac{1+\gamma_0}{\frac{1}{\sum_{i=1}^{k}\frac{1}{\bar{\gamma}_{R_{\lambda_i},D}}}}\right)\right]\Bigg\} \tag{32}$$

From (33) we can see that the channel capacity, in low SNR region, has been improved due to diversity gain. However, at high SNR we will have approximately the channel capacity of the direct transmission only. This means that there is no reduction in the spectral efficiency of the cooperative diversity networks.

C. Throughput Analysis

Throughput is the amount of data which is successfully transmitted per unit time, that is $r/\mathbf{E}(T)$ [25], where r bits/sec/Hz is the target rate, and $\mathbf{E}(T)$ is the average delay, i.e., the expected number of transmissions (original transmission plus retransmission) is given by

$$\mathbf{E}(T) = 1 \times \Pr(T=1) + 2 \times \Pr(T=2) \tag{34}$$

where 1 represents only one time slot which is needed if there is no retransmission by the best relay and 2 represents two time slots needed if there is retransmission by the best relay. Now, consider the probability of each of the two cases. In case 1, the transmission takes only one time slot. The probability of successful transmission after the first time slot is given by $\Pr(T=1) = p_1 = \exp(-\gamma_0/\bar{\gamma}_{S,D})$, and the probability of successful transmission after the two time slots is given by $\Pr(T=2) = 1-\exp(-\gamma_0/\bar{\gamma}_{S,D}) = 1-p_1$. Finally, after simple manipulations, the throughput can be written as

$$\text{Throughput} = \frac{r}{2-p_1} \tag{35}$$

From (35) we can see as p_1 decreases the throughput decreases. For regular cooperative diversity networks, $p_1 = 0$ and the throughput is $r/2$, while for direct transmission alone,

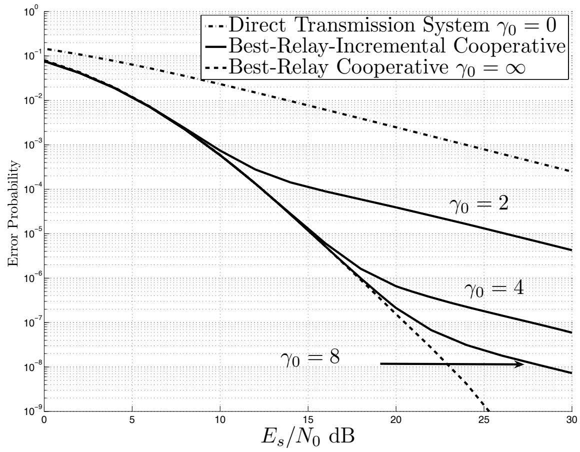

Fig. 2. Error performance for the decode-and-forward incremental-best- relay selection scheme over Rayleigh fading channels.

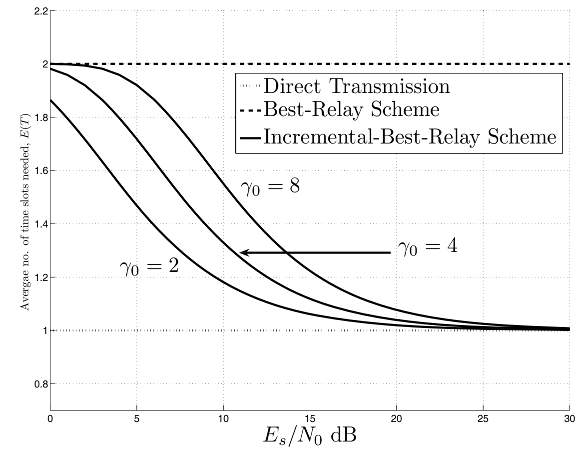

Fig. 4. Time delay performance for the incremental-best-relay selection scheme over Rayleigh fading channels.

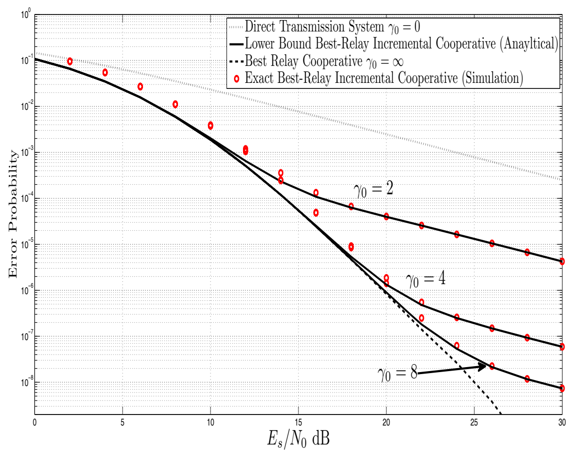

Fig. 3. Error performance for the amplify-and-forward incremental-best-relay selection scheme over Rayleigh fading channels.

$p_1 = 1$ and the throughput is r (the best value). It is clearly seen the throughput of the best-relay incremental cooperative diversity techniques lies between $r/2$ and r.

V. Results and Discussions

In this section, we show numerical results of the analytical BER for binary phase shift keying (BPSK) modulation, and outage probability. We plot the performance curves of the average BER, outage probability and average channel capacity versus the SNR of the transmitted signal (E_s/N_0 dB).

Figs. 2 and 3 compare the error performance of the incremental-best-relay cooperative diversity with that of best-relay cooperative and conventional direct system for different values of γ_0 and for $M = 3$ for amplify-and-forward and decode-and-forward, respectively. From these two figures we can conclude the following:

First, as expected, the best-relay scheme outperforms all other cooperative diversity networks and can achieve $M + 1$ diversity order for the entire SNR region. However, the incremental-best-relay cooperative diversity can achieve the same error performance of the best-relay scheme at low SNR while at high SNR (for approximately SNR $> \gamma_0$) the error performance will tend to have diversity order of 1 and virtual array gain[2]. From Figs. 2 and 3 it is apparent that the virtual array gain depends on the value of γ_0: for high values of γ_0 we have higher virtual array gain and vice versa. It should be also noted that when $\gamma_0 = \infty$, we go back to the best relay scheme. Furthermore, for high values of γ_0 the region, where the incremental-best-relay cooperative diversity has the same performance of the best-relay cooperative diversity networks, also increases.

Second, it can be seen from Figs. 2 and 3 that there is a critical SNR point where the error performance will no longer has $M + 1$ diversity order and this critical value actually depends on γ_0. Even more, it can be seen that this critical value starts earlier for adaptive decode-and-forward scheme compared to amplify-and-forward scheme. In addition, it can be seen that adaptive decode-and-forward scheme slightly outperforms the amplify-and-forward scheme. It is worthy to mention that this improvement of the adaptive decode-and-forward comes with extra complexity at the relays compared with the amplify-and-forward scheme. Finally, it should be noted that for high SNR, amplify-and-forward and decode-and-forward have exactly the same error performance because they have the same asymptotic error performance which validates equation (22).

It should be noted that the value of γ_0 is an important factor to determine the error performance of the whole system. Actually this value can be determined based on the quality of service and transmission delay that could be accepted (voice, video or data transmission). In Fig. 4, we show the average number of time slots needed per message for different values of γ_0. We should note that for the best-relay scheme we need two time slots regardless of the number of the relays and for the direct transmission we need one time slot. It is obvious that increasing γ_0 will increase the number of time

[2]If the error rate is plotted versus the SNR on a log-log scale the diversity order can be interpreted as the slope of the so-obtained curve whereas the virtual array gain corresponds to the horizontal position of the curve.

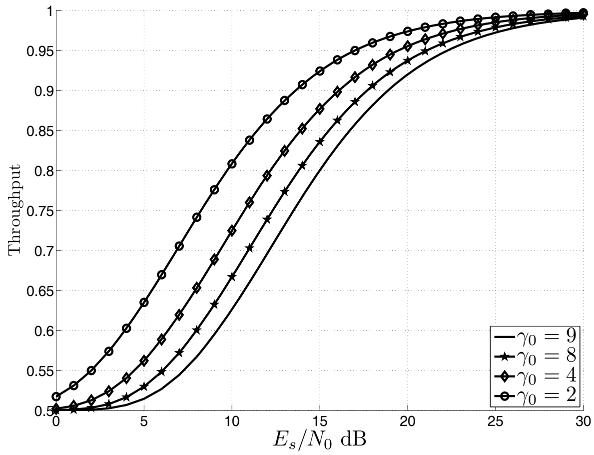

Fig. 5. Throughput of incremental relaying system for different values of γ_0.

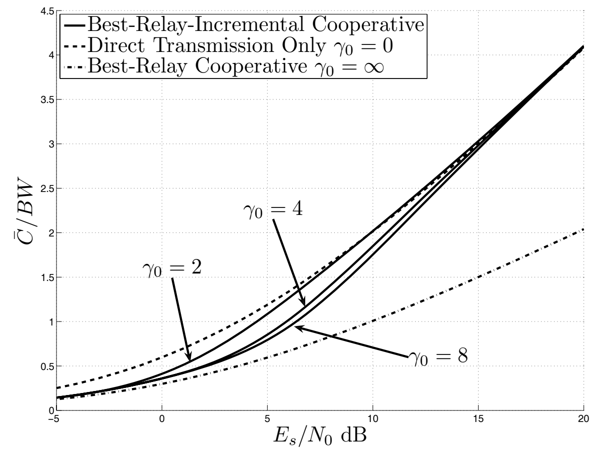

Fig. 6. Channel capacity performance for amplify-and-forward best-relay incremental relaying for different values of γ_0. Note that direct transmission equivalent to $\gamma_0 = 0$ and regular cooperative diversity equivalent to $\gamma_0 = \infty$.

slots needed to complete the transmission and vice versa. Therefore, the value of γ_0 depends on the quality of service to be offered. For example, for $\gamma_0 = 8$ at SNR = 15 dB, the average number of time slots per message is 1.2 and the bit error probability $\approx 1 \times 10^{-5}$ while for direct transmission the bit error probability is $\approx 10^{-2}$. Furthermore, for $\gamma_0 = 8$ at SNR = 20 dB, the average number of time slots per message is almost 1 (almost the same as the direct link) with bit error probability of 10^{-7} while the direct link has bit error probability of 3×10^{-3}. This big improvement (more than 3 orders of magnitude) can be used in a hidden way to increase the data rate by going further to higher modulation level.

Fig. 5 shows the throughput of incremental-best-relay relaying cooperative-diversity network for $r = 1$ bit/s. Fig. 5 shows when the SNR detection threshold (γ_0) increases, the throughput decreases. Fig. 5 also shows that incremental-best-relay cooperative-diversity network gives high throughput (very close to direct system) at medium and high SNR while incremental-best-relay technique has much better error performance. For example, for the incremental-best-relay cooperative-diversity network at 15 dB, the throughput reaches 0.85 with $P(e) = 1 \times 10^{-4}$ for amplify-and-forward, $P(e) = 1 \times 10^{-5}$ for decode-and-forward with $\gamma_0 = 8$, while the direct transmission has throughput of 1 and $P(e) = 10^{-2}$.

Figs. 6 and 7 show the channel capacity performance for both amplify-and-forward and decode-and-forward schemes, respectively. From Figs. 6 and 7, we can conclude that as γ_0 increases, the channel capacity slightly decrease. In these two figures, we can see that incremental-best-relay cooperative-diversity network significantly outperforms the best-relay cooperative diversity network. Also, we can see that there is almost no difference in channel capacity performance between the incremental-best-relay cooperative-diversity network with direct transmission. Finally, we can see that incremental-best-relay cooperative-diversity network significantly improves the error probability without sacrificing the spectral efficiency, hence this technique can be used for the applications that need high data rate.

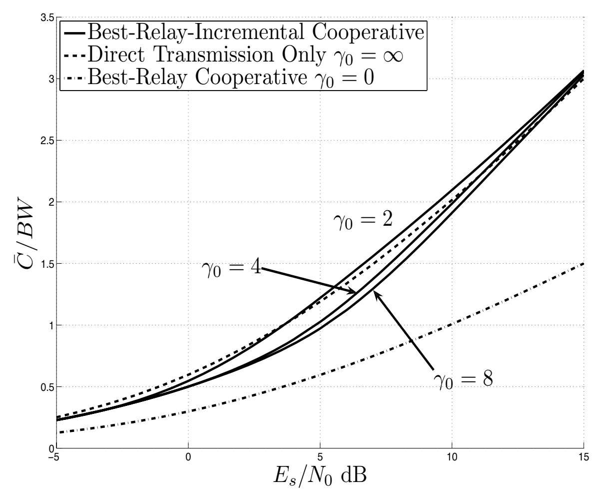

Fig. 7. Channel capacity performance for decode-and-forward best-relay incremental relaying for different values of γ_0. Note that direct transmission equivalent to $\gamma_0 = 0$ and regular cooperative diversity equivalent to $\gamma_0 = \infty$.

VI. Conclusions

Incremental-best-relay cooperative-diversity network is an efficient technique that can be used to save the channel resources and use extra channel resources only when it is necessary. We have derived closed-form expressions for the error probability, outage probability and channel capacity.

Results show that the incremental-best-relay cooperative-diversity network can achieve significant spatial diversity with a high channel capacity compared with the best-relay cooperative-diversity networks. Also, results show that the error performance is highly dependent on the SNR threshold employed at the destination. Obviously, the value of this threshold depends on the application used at the destination. Furthermore, it can also be seen that incremental-best-relay cooperative-diversity network has high channel capacity and throughput comparable to that of the direct transmission particularly at medium and high SNR.

The implementation issues for this kind of cooperative techniques is a new open area of research and it is considered to be as a future work.

REFERENCES

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, Dec. 2004.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, pp. 2415-2425, Oct. 2003.

[3] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity, part I: system description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927-1938, Nov. 2003.

[4] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity, part II: implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939-1948, Nov. 2003.

[5] M. O. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1126-1131, Nov. 2003.

[6] P. A. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a Rayleigh-fading environment," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 1416-1421, Sep. 2004.

[7] S. Ikki and M. H. Ahmed, "Performance analysis of cooperative diversity wireless networks over Nakagami-m fading channel," *IEEE Commun. Lett.*, vol. 11, no. 4, pp. 334-336, Apr. 2007.

[8] N. C. Beaulieu and J. Hu, "A closed-form expression for the outage probability of decode-and-forward relaying in dissimilar Rayleigh fading channels," *IEEE Commun. Lett.*, vol. 10, pp. 813-815, Dec. 2006.

[9] H. A. Suraweera, P. J. Smith, and J. Armstrong, "Outage probability of cooperative relay networks in Nakagami-m fading channels," *IEEE Commun. Lett.*, vol. 10, pp. 834-836, Dec. 2006.

[10] O. Canpolat, M. Uysal, and M. M. Fareed, "Analysis and design of distributed space time trellis codes with amplify-and-forward relaying," *IEEE Trans. Veh. Technol.*, vol. 56, pp. 1649-1660, July 2007.

[11] S. Yiu, R. Schober, and L. Lampe, "Distributed space-time block coding," *IEEE Trans. Commun.*, vol. 54, pp. 1195-1206, July 2006.

[12] A. Bletsas, H. Shin, M. Z. Win, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659-672 Mar. 2006.

[13] S. Ikki and M. H. Ahmed, "Performance of multiple-relay cooperative diversity systems with best relay selection over Rayleigh fading channels," *EURASIP J. Advances Signal Process.*, vol. 2008.

[14] Y. Zhao, R. S. Adve, and T. J. Lim, "Symbol error rate of selection amplify-and-forward relay systems," *IEEE Commun. Lett.*, vol. 10, no. 11, Nov. 2006.

[15] S. Ikki and M. H. Ahmed, "Performance analysis of adaptive decode-and-forward cooperative diversity networks with the best relay selection," *IEEE Trans. Commun.*, vol. 58, pp. 68-72, Jan. 2010.

[16] A. Adinoyi, Y. Fan, H. Yanikomeroglu, and V. Poor, "On the performance of selective relaying," *IEEE VTC2008 Fall*, Sep. 2008.

[17] B. Zhao and M. C. Valenti, "Practical relay networks: a generalization of hybrid-ARQ," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 1, pp. 7-18, Jan. 2005.

[18] P. Tarasak, H. Minn, and Y. H. Lee, "Analysis of incremental relaying protocol with RCPC in cooperative diversity systems," *IEEE VTC'05 (Fall)*, vol. 4, pp. 2537-2541, Sep. 2005.

[19] V. Mahinthan, H. Rutagemwa, J. W. Mark, and X. Shen, "Performance of adaptive relaying schemes in cooperative diversity systems with ARQ," in *Proc. IEEE GLOBECOM*, Nov. 2007.

[20] G. Yu, Z. Zhang, and P. Qiu, "Efficient ARQ protocols for exploiting cooperative relaying in wireless sensor networks," *Comput. Commun.*, vol. 30, pp. 2765-2773, 2007.

[21] K.-S. Hwang, Y.-C. Ko, and M.-S. Alouini, "Performance analysis of incremental opportunistic relaying over identically and non-identically distributed cooperative paths," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 1953-1961, Apr. 2009.

[22] GK.-S. Hwang, Y.-C. Ko, and M.-S. Alouini, "Performance analysis of opportunistic incremental relaying with adaptive modulation," in *Proc. IEEE International Symp. Wireless Pervasive Comput.*, 2008.

[23] J. S. Chen and J. X. Wang, "Performance analysis of opportunistic incremental relaying with adaptive modulation," *IET Commun.*, vol. 3, no. 12, pp. 1948-1957, 2009.

[24] J. G. Proakis, *Digital Communications*, 4th edition. McGraw-Hill, 2001.

[25] I. Stanojev, O. Simeone, Y. B. Ness, and C. You, "Performance of multi-relay collaborative hybrid-ARQ protocols over fading channels," *IEEE Commun. Lett.*, vol. 10, no. 7, pp. 522-524, July 2006.

[26] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*. Wiley, 2000.

[27] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th edition. Dover, 1970.

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