**Directions:** This assignment has **6** questions. There are two files: HW2.hs, which is where you should put your answers, and the easier to read HW2.pdf. Your answers will either be replacing undefined pieces of code, or tracing evaluation steps (the second of these must be done in a comment, since these evaluation steps aren't themselves valid Haskell). Start by loading HW2.hs into ghci, and periodically check your work by saving your changes and using :r to refresh/update ghci with them.

## 1 Warm up

We'll start by defining a type Shape for the possible moves in Rock, Paper, Scissors, and a type Result for the possible outcomes of one game.

```
data Shape = Rock | Paper | Scissors
  deriving Show
-- The second line of this definition and the next, which you can ignore, just
-- says we'd like these values to display on the screen in the obvious way.

data Result = Win | Lose | Tie
  deriving Show
```

(1) whatItBeats is a function that takes a Shape as an input and returns the shape that it beats. play is a function taking two Shapes, which tells if the first Shape won, lost, or tied. These definitions are incomplete. Complete them by replacing each occurrence of undefined with the right value.

## 2 Practice with evaluation

(2) Using the above definitions and the evaluation rules for let and lambda, evaluate the following three expressions, one step at a time, by filling out each of the blank lines below. You can check your answers by pasting the starting expression into ghci and having it evaluate it for you! (But you will be graded on the intermediate steps too!)

```
a. let guaranteedTie = (\x → play x x) in guaranteedTie Paper

⇒

⇒

⇒
```

```
b. let unbeatable = (\y → play y (whatItBeats y)) in unbeatable Scissors

⇒

⇒

c. (\f → f (f (f Rock))) whatItBeats

⇒

⇒

⇒

⇒

⇒

⇒

⇒
```

As a reminder, here are our one-step evaluation rules for let and lambda:

```
• let v = e1 in e2 \Longrightarrow [e1/v]e2
• (v \rightarrow e2) e1 \Longrightarrow [e1/v]e2
```

In both cases, [e1/v]e2 means, "e1 replaces v in e2".

## 3 Practice with recursion

Here is the Nat type from Tuesday's lecture. It says that a Nat is either Z (zero), or the S (successor) of a Nat.

```
data Nat = Z | S Nat
  deriving Show
```

And here's the toInt function we defined then. It takes a Nat to an Int by defining a base case and a recursive step. The recursive step for n assumes that we can compute toInt (subOne n). Once we have this number, we know that toInt n has to be 1 greater than it.

```
toInt :: Nat -> Int
toInt Z = 0
toInt n = 1 + (toInt (subOne n))
subOne (S n) = n -- referenced in toInt, from the lecture slides
```

a. Define a toNat function that goes in the opposite direction, converting an Int to a Nat. I've started you off by giving the base case. Hint: this function will be very similar to toInt (technical jargon: they are inverses). You can check your answer by refreshing ghci with :r, and then calling, say, toNat 5. (Watch your parentheses: in general, a b c is grouped as (a b) c, not a (b c)!)

```
toNat :: Int -> Nat
toNat 0 = Z
toNat i = undefined
```

b. Using your definition, walk through the evaluation of toNat 2, one step at a time:

```
toNat 2

⇒

⇒

⇒

⇒

⇒
```

(4) a. Define an add function that sums two Nats. Hint: for the recursive step, you will find it useful to remember the following fact about addition: (1 + m) + n == m + (1 + n). You can check your answer by refreshing ghci with :r, and then calling, say, add (S (S Z))) (S (S Z)).

```
add :: Nat -> Nat -> Nat
add Z n = undefined
add (S m) n = undefined
```

b. Using your definition, walk through the evaluation of add (S (S Z)) (S Z).

```
\begin{array}{c} \text{add (S (S Z)) (S Z)} \\ \Longrightarrow \\ \Longrightarrow \\ \Longrightarrow \end{array}
```

## 4 Recursion on lists

Recall the type we rolled for IntLists: an IntList is either Empty, or the Cons of an Int onto an IntList:

```
data IntList = Empty | Cons Int IntList
  deriving Show
```

(5) Define concatIntList, such that concatenating Cons 1 (Cons 2 Empty) and Cons 3 (Cons 4 Empty) gives Cons 1 (Cons 2 (Cons 3 (Cons 4 Empty))). Big hint: this definition will be **extremely** close to the definition of add you gave above (at least if you followed the hint I gave there!). The reason: for the recursive case, if we know how to compute concatIntList xs ys, then concatenating the slightly longer list Cons x xs with ys is just Consing/adding x to concatIntList xs ys.

```
concatIntList :: IntList -> IntList
concatIntList Empty ys = ys
concatIntList (Cons x xs) ys = undefined
```

Now convert your definition into one that works on Haskell's native representation of lists, recalling that Empty is the IntList version of [], and Cons x xs is the IntList version of x:xs. You are reimplementing Haskell's (++).

(6) Finally, define a plus10 function that takes a list as an input and returns a list that's like the old one, but with every number increased by 10. Test your definition by entering plus10 [1..10] into ghci.

```
plus10 :: [Int] -> [Int]
plus10 [] = []
plus10 (x:xs) = undefined
```