1 Outline

- Continuized CCGs offer a grammar-wide generalization of scope-taking ("ubiquitous scopal pied piping") that brings quantifiers and scope-taker more generally into the compositional fold. Three combinators, **lift**, **triv**, and **scope**, form the backbone of the grammar, allowing scope-takers to interact with their linguistic context. In addition, a **bind** shifter is introduced to facilitate quantificational binding.

- Extant CCG treatments of E-type anaphora (i.e. cross-sentential and donkey anaphora): Barker & Shan 2008. Motivation for pursuing another approach. BS approach has not-so-good empirical coverage (Charlow 2010). de Groote 2006 can be combined with BS regime, but...?

- Proposal: replace **lift** and **triv** with options that countenance side effects (Shan 2005). Any side effects regime can be grafted onto a continuized CCG, by replacing **lift** and **triv** with monadic functors (Moggi 1989, Wadler 1992, 1994, 1995, Shan 2002).

- We provide a general technique for integrating a monadic approach to side effects with continuations-based approaches to scope in CCG. We relate our approach to the ContT monad transformer (Liang et al. 1995). Offers a type-theoretic way to track effects, integrate them into a well-developed CCG framework for scope-taking.

- Therefore, these results are of interest both for the categorial grammarian interested in donkey anaphora and scope-taking, as well as more generally. Any side-effects regime a semanticist thinks is motivated can be accommodated along these lines. Further, because of the inherent modularity, adding side effects necessitates neither fiddling with the basic compositional machinery, nor messing with lexical items which don't exploit side effects.

- Dynamic semantics is (Shan 2001):[1]
  - State: ability to manipulate the discourse context, i.e. create discourse referents.
  - Nondeterminism: analogizes indefinites to referential expressions. Treats indefinites as referring expressions, though ones which refer indeterminately.

- Corollary: there is no need to settle on a single ("the") grammar. Different and quite varied side effects regimes can be modularly grafted onto a simple applicative ("pure") core. Lexical entries that would seem incongruous in a flat-footed standard perspective integrate seamlessly in a single grammar.

- Monads as a natural way to extend a continuations-based grammar with tools for dynamic binding and exceptional scope. In the end: you have functional application, plus the functors from whichever monads are implicated in a given language.

- The standard continuations-based perspective of Barker 2002, Shan & Barker 2006, Barker & Shan 2014 is an instantiation of a more general perspective.

- Standard dynamic techniques (DPL, DMG) not reducible to monads.

- Broader question: how this relates to the idea that continuations can simulate any monad (Filinski 1994). I don't understand this result well enough to say anything.

2 Adding side effects to k

- Standard continuized grammar:
  - **lift**: $\lambda k.\, k\, x$
  - **triv**: $\lambda x.\, x$
  - **scope**: $\lambda k.\, m\,(\lambda f.\, n\,(\lambda x.\, k\,(f\, x)))$
- To do: insert figure with inference rules.
- Type-theoretic details here
- Adding side effects (Wadler 1994, 1995, Shan 2002): monads
- Monad laws / punting
- Relating monads to continuized grammars:
  - Replace **lift** with $\star$
  - Replace **triv** with η
  - **scope** stays the same
- Two type constructors:
  - Bipartite Cont: $\mathsf{K}\, a\, b ::= (a \to b) \to b$
  - Unary Monadic:

3 Finding the dynamic monad

- The meat of PLA (Dekker 1994): sentences are relations on stacki. Non-empty relations correspond to truth. Non-functional pairs in the relation correspond to nondeterminism introduced by indefinites (and perhaps disjunction).

$$[\![\text{a linguist}]\!] = \lambda k i. \bigcup_{x \in \mathsf{ling}} k\, x\, \widehat{i x}$$

1 NB: does not characterize all varieties of dynamic semantics. Dynamic treatments following Groenendijk & Stokhof 1990 (e.g. Zimmermann 1991, Dekker 1993, Szabolcsi 2003, de Groote 2006) provide a way for indefinites to extend their binding domain but do not treat indefinites as nondeterministic analogs of proper names.

$$\frac{\Gamma \vdash f : b/a \quad \Delta \vdash e : a}{\Gamma \cdot \Delta \vdash f\,e : b}\,/ \qquad \frac{\Delta \vdash e : a \quad \Gamma \vdash f : a\backslash b}{\Delta \cdot \Gamma \vdash f\,e : b}\,\backslash \qquad \frac{\Delta \vdash m : \mathsf{K}(b/a)r \quad \Gamma \vdash n : \mathsf{K}ar}{\Delta \cdot \Gamma \vdash \mathbf{S}_{/} m\,n : \mathsf{K}br}\,/\!\!/ \qquad \frac{\Delta \vdash m : \mathsf{K}ar \quad \Gamma \vdash n : \mathsf{K}(a\backslash b)r}{\Delta \cdot \Gamma \vdash \mathbf{S}_{\backslash} m\,n : \mathsf{K}br}\,\backslash\!\!\backslash$$

$$\frac{\Gamma \vdash e : a}{\Gamma \vdash \lambda k.\,k\,e : \mathsf{K}ar}\uparrow \qquad \frac{\Gamma \vdash m : \mathsf{K}rr}{\Gamma \vdash m\,(\lambda x.\,x) : r}\downarrow$$

Figure 1: Partial multimodal continuized grammar, no side effects.

$$\frac{\Gamma \vdash f : b/a \quad \Delta \vdash e : a}{\Gamma \cdot \Delta \vdash f\,e : b}\,/ \qquad \frac{\Delta \vdash e : a \quad \Gamma \vdash f : a\backslash b}{\Delta \cdot \Gamma \vdash f\,e : b}\,\backslash \qquad \frac{\Delta \vdash m : \mathsf{K}(b/a)r \quad \Gamma \vdash n : \mathsf{K}ar}{\Delta \cdot \Gamma \vdash \mathbf{S}_{/} m\,n : \mathsf{K}br}\,/\!\!/ \qquad \frac{\Delta \vdash m : \mathsf{K}ar \quad \Gamma \vdash n : \mathsf{K}(a\backslash b)r}{\Delta \cdot \Gamma \vdash \mathbf{S}_{\backslash} m\,n : \mathsf{K}br}\,\backslash\!\!\backslash$$

$$\frac{\Gamma \vdash e : \mathsf{M}a}{\Gamma \vdash \lambda k.\,e \star k : \mathsf{K}a\mathsf{M}r}\uparrow \qquad \frac{\Gamma \vdash m : \mathsf{K}r\mathsf{M}r}{\Gamma \vdash m\,\eta : \mathsf{M}r}\downarrow$$

Figure 2: Partial multimodal continuized grammar, with side effects.

- A different perspective on this: treating nondeterminism and state modification as side effects, within a functional programming setting for side effects.
- Monad for nondeterminism:

  Definition 1 (The Set monad).

$$\begin{aligned} \mathsf{M}a &::= a \to t \\ \eta\,x &:= \{x\} \\ m \star k &:= \bigcup_{x \in m} k\,x \end{aligned}$$

- Monad for state (generalization of monad for environment-sensitivity). Assume that γ is the type of "contexts of evaluation". For our purposes, we might think of γ as inhabited by *sequences of discourse referents*.

  Definition 2 (The State monad).

$$\begin{aligned} \mathsf{M}a &::= \gamma \to a \times \gamma \\ \eta\,x &:= \lambda i.\,\langle x, i\rangle \\ m \star k &:= \lambda i.\,k\,(m\,i)_0\,(m\,i)_1 \end{aligned}$$

- Given our identification of γ with the set of sequences of discourse referents, a natural operation to suppose as associated with dref introduction is sequence extension (cf. de Groote 2006, Unger 2012):

  Definition 3 (Sequence extension).

$$m^{\triangleright} := m \star \lambda x i.\,\langle x, \widehat{i x}\rangle$$

- An example:

$$(\eta\,\mathsf{a})^{\triangleright} \star \lambda x.\,\eta\,(\mathsf{left}\,x) = \lambda i.\,\langle \mathsf{left}\,\mathsf{a}, \widehat{i\mathsf{a}}\rangle$$

- Use StateT to stitch the two together. Given any monad $\mathcal{M} = \langle \mathsf{L}, \eta_{\mathsf{L}}, \star_{\mathsf{L}}\rangle$, StateT is a recipe for building a new monad with which adds State-type functionality to $\mathcal{M}$:[2]

  Definition 4 (The StateT monad transformer).

$$\begin{aligned} \mathsf{M}a &::= \gamma \to \mathsf{L}\,(a \times \gamma) \\ \eta\,x &:= \lambda i.\,\eta_{\mathsf{L}}\,\langle x, i\rangle \\ m \star k &:= \lambda i.\,m\,i \star_{\mathsf{L}} \lambda \pi.\,k\,\pi_0\,\pi_1 \end{aligned}$$

  Definition 5 (The State_Set monad).

$$\begin{aligned} \mathsf{M}a &::= \gamma \to (a \times \gamma) \to t \\ \eta\,x &:= \lambda i.\,\{\langle x, i\rangle\} \\ m \star k &:= \lambda i.\,\bigcup_{\pi \in m i} k\,\pi_0\,\pi_1 \end{aligned}$$

- Static lexicon, dynamic lexicon
- Modular treatment of binding.

$$\begin{aligned} \text{Previous}: &\quad \mathbf{bind}\,m := \lambda k.\,m\,(\lambda x.\,k\,x\,x) \\ \text{Proposal}: &\quad \mathbf{bind}\,m := \lambda k.\,m\,(\lambda x i.\,k\,x\,\widehat{i x}) \end{aligned}$$

- Summing up: three combinators for "order-insensitive" (i.e. continuized combination). **unit**, **run**, **bind**

	lift m	M **triv**	**bind** M
Previous	$\lambda k.\,k\,m$	$M\,(\lambda x.\,x)$	$\lambda k.\,m\,(\lambda x.\,k\,x\,x)$
Proposal	$\lambda k.\,m \star k$	$M\,\eta$	$\lambda k.\,m\,(\lambda x i.\,k\,x\,\widehat{i x})$

4 Examples

- Some upshots: no dynamic conjunction, completely standard model theory (cf. de Groote 2006). "Contexts of evaluation" are constructed on the fly.

2 Fn. about SetT

- Cross-sentential anaphora: the indefinite's side effects influence the evaluation of the second clause, even as the indefinite scopes within its clause.

$$\begin{aligned}\textbf{a.man.left} \star \lambda p.\, \textbf{he.tired} \star \lambda q.\, \eta\, (p \wedge q) \\ = \textbf{a.man} \star \lambda x.\, \eta\, (\mathsf{left}\, x \wedge \mathsf{tired}\, x)\end{aligned}$$

- Compare universals. After ending the derivation at the clause boundary, we're left with a pure computation. The universal's side effects have died on evaluation.

$$\eta\, (\forall x.\, \mathsf{ling}\, x \Rightarrow \mathsf{left}\, x)$$

- Donkey anaphora works similarly. Take the following. The restrictor c here acquires a kind of monadic scope, via $\star$, over the nuclear scope k. This means any side effects inside c influence the context of evaluation for k. However, once k is grabbed, the wide-scoping negation discharges side effects (as is standard in dynamic systems).

$$[\![\mathsf{every}]\!] \coloneqq \lambda c k.\, \textbf{not}\, (\textbf{a}\, c \star \lambda x.\, \textbf{not}\, (k\, x))$$

- Islands: a clause must denote a $\mathsf{M}t$

  de Groote 2001 Charlow 2014 Bumford to appear

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