Epistemic modals: statics and dynamics

Semantics II

February 5 & 8

Alternate title

An opinionated guide to the opinionated guide to epistemic modality

Epistemic modals

Intensional semantics (von Fintel & Heim 2011)

The possible types of linguistic meanings:

$$\tau ::= e \mid t \mid \underbrace{s \to \tau}_{\text{Adding intensions, accessed via } [\![\cdot]\!]^{g}_{\text{¢}}} \mid \tau \to \tau$$

Basic ways for meanings to compose (extended to allow application to an ***intension****):*

$$[\![A\ B]\!]^{w,g} = \begin{cases} [\![A]\!]^{w,g}\,[\![B]\!]^{w,g} \quad \text{or} \quad [\![B]\!]^{w,g}\,[\![A]\!]^{w,g} \quad \text{or} & \textit{FA, BA} \\ [\![A]\!]^{w,g}\,[\![B]\!]^{g}_{\text{¢}} \quad \text{or} \quad [\![B]\!]^{w,g}\,[\![A]\!]^{g}_{\text{¢}} \quad \text{or} & \textit{IFA, IBA} \\ \lambda x.\,[\![A]\!]^{w,g}\,x \wedge [\![B]\!]^{w,g}\,x & \textit{PM}\text{, whichever's defined} \end{cases}$$

Predicate abstraction for variable binding (unchanged):

$$[\![n\ A]\!]^{w,g} = \lambda x.\,[\![A]\!]^{w,g[n \to x]}$$

Modals

Modals can be handled very similarly to attitude verbs. The principal difference is that modals are *context-sensitive*:

$$[\![\text{may}]\!]^{w,g} = \mathbf{may}_w = \underbrace{\lambda B.\lambda p.\,\exists v \in B\,w : p\,v}_{(\mathrm{s}\to\{\mathrm{s}\})\to(\mathrm{s}\to\mathrm{t})\to\mathrm{t}}$$

The accessibility relation *B* is filled by a variable in the logical form. This variable gets its value from the assignment (i.e., from the context). This allows different kinds of worlds to be quantified over, depending on what's relevant:

- John [may B] eat one cookie.
- John [may B] be in the garden.

Exercise

Our semantics for modals pipes an evaluation world to the accessibility relation B:

$$[\![\text{may}]\!]^{w,g} = \mathbf{may}_w = \underbrace{\lambda B.\lambda p.\,\exists v \in B\,w : p\,v}_{(s\to\{s\})\to(s\to t)\to t}$$

That's a little weird, right? Usually the evaluation world gets automatically passed to whatever needs it in virtue of $[\![\cdot]\!]^{w,g}$. What's different about this case?

More on context-sensitivity for epistemics

For epistemic modals (*John may be in the garden*), it's natural to treat *B* as a contextually determined **body of information**. von Fintel & Gillies (2007: 34):

1. a. *As far as Bill knows*, John **might** be the thief.
   b. *Given what we knew at the time*, John **might** have been the thief.
   c. *Given the results of the DNA tests*, John might be the thief. But *if we take the eyewitness seriously*, John **can't** have been the thief.
2. John **has to** be in New York.

The B-variable in logical forms is assignment-dependent. Ergo, its value is:

- Contextually determined when it is free (like a free pronoun).
- Bindable in principle, as in the examples in (1).

So the context can determine more than the flavor of modality.

Limits?

English *might* has only epistemic uses. Can this be captured by assuming that the background is fixed by a variable B in the logical form?

We might try treating $\textbf{might}_w$ as defined only for certain kinds of backgrounds:

$$\textbf{Dom}\,\textbf{might}_w \subseteq \{B \mid \textbf{Epistemic}\,B\}$$

. . . And then using properties of acquaintance relations like reflexivity, transitivity, symmetry, and so on to characterize **Epistemic**. Maybe this could work.[1]

[1] See Hacquard (2009) for more on these kinds of puzzles.

Evidentiality, not weakness

Strong epistemic modals like *must* can signal that the speaker is certain of the prejacent, but that their evidence for the truth of the prejacent is somehow indirect (though indirect evidence is not ipso facto *weak* evidence).

3. It must be raining.
4. The marble is in *A* or *B*. It isn't in *A*. So it must be in *B*.

If *must* were weaker than certainty, you'd expect (4) to implicate uncertainty about the prejacent (that the ball's in *B*), analogously to (5). It definitely does not.

5. I ate some of the cookies $\rightsquigarrow$ I didn't eat all of the cookies

Non-truth-conditionality

There's an oft-expressed intuition that the meanings of epistemically modalized sentences are not precisely truth-conditional.

6. Why isn't Louise coming to our meetings these days?
   She might/must be too busy with her dissertation.
7. There might be two reds.
   That's right. There might be.
   That's right. There are.

That is, data like this may suggest that the content of an epistemically modalized sentence is not a simple possible-worlds proposition, e.g.:

$$\{w \mid \exists v \in Bw : \mathbf{two.reds}_v\}$$

On the other hand

Despite the previous data, epistemically modalized sentences semantically embed in the way you'd expect if they expressed possible-worlds propositions:

8. If the slides might have a typo, you'll need to read them carefully.
   $\rightsquigarrow$ $\textbf{if}\,(\lambda w.\,\exists v \in Bw : \textbf{typo}_v)\,(\lambda w.\,\textbf{read.carefully}_w)$
9. The slides can't have any typos. $\rightsquigarrow \lambda w.\,\neg\exists v \in Bw : \textbf{typo}_v$

Moreover, I'm not sure exactly how to interpret the diagnostics on the last slide:

10. Why isn't Louise coming to our meetings these days?
    John told me that she's very busy writing her dissertation.
11. There aren't two.
    That's right. There aren't.
    That would have been weird.

Anyway

Whether you like the non-truth-conditional data or not (I'm a bit suspect), it's a nice excuse to look at an alternative, non-truth-conditional picture of epistemic modals.

We'll develop a simple dynamic system today, on which epistemically modalized sentences don't express possible worlds propositions. As we'll see, there's a bit of independent motivation for this kind of system anyway.

A dynamic picture

Stalnaker (1978)

Stalnaker develops the notion of a **common ground**: the set of worlds that the participants in a conversation take to be live possibilities. Simplifying a lot:

$$\mathbf{CG} = \{w \mid \text{the discourse participants treat } w \text{ as possible}\}$$

Then communication can be modeled as successive update of the common ground. When a speaker asserts ϕ, the proposition expressed by ϕ is added to **CG**:

$$\mathbf{CG} \cap [\![\phi]\!]^{g}_{¢}$$

Think of this rule as a pragmatic default, not hard-coded in the semantics.

A simple language

We start by defining a simple propositional language:

$$\phi ::= \underbrace{\mathtt{Atom}}_{p,q,r,\ldots} \mid \phi \wedge \phi \mid \neg\phi$$

Up from statics

A static interpretation ($\times$ and $-$ are the usual operations on numbers):

$$\begin{aligned}
[\![p]\!]^w &= w\,p \\
[\![\phi \wedge \psi]\!]^w &= [\![\phi]\!]^w \times [\![\psi]\!]^w \\
[\![\neg\phi]\!]^w &= 1 - [\![\phi]\!]^w
\end{aligned}$$

And a pragmatic rule for updating the common ground:

$$\mathbf{Assert}(\phi, \mathbf{CG}) = \mathbf{CG} \cap \underbrace{[\![\phi]\!]_{¢}}_{\lambda w.[\![\phi]\!]^w}$$

You can think of this as a quite pared-down version of our official intensional semantics, for a simple propositional language.

A dynamic interpretation

A dynamic interpretation for this language treats sentence meanings as functions from input states s into output states. Generally, these functions are written using a more iconic *postfix* notation: $s[\phi] := [\phi]\, s$.

$$
\begin{aligned}
s[p] &= \{w \in s : w\,p\} \\
s[\phi \wedge \psi] &= s[\phi][\psi] \\
s[\neg\phi] &= s \setminus s[\phi]
\end{aligned}
$$

(Thus, for example, $[p] = \lambda s.\{w \in s : w\,p\}$.)

Conjunction amounts to function composition:

$$
\begin{aligned}
[\phi \wedge \psi] &= \lambda s.[\psi]([\phi]\, s) \\
&= [\psi] \circ [\phi] \\
&= [\phi]\, ;\, [\psi]
\end{aligned}
$$

Idle dynamicity

We have a simple semantics that uses some dynamic notions. Is the dynamic-ness playing an essential role in the system, though?

It turns out it isn't. Specifically, we observe the following property. For any s, ϕ:

$$s[\phi] = s \cap \underbrace{[\![\phi]\!]_{¢}}_{\text{the static intension of } \phi} = \textbf{Assert}(\phi, s)$$

In other words, for any ϕ, the dynamic interpretation $[\phi]$ isn't accomplishing anything beyond what we could get with $[\![\phi]\!]_{¢}$ and a Stalnakerian pragmatics.[2]

[2]We can prove this via structural induction, but the intuitive justification is beyond reproach: $s[\phi]$ always has the effect of knocking the not-ϕ worlds out of s, just like intersection with $[\![\phi]\!]_{¢}$ would.

Adding modals

We extend our propositional language with a clause for weak epistemic modals:

$$\phi ::= \texttt{Atom} \mid \phi \wedge \phi \mid \neg\phi \mid \Diamond\phi$$

(Then we can treat $\Box\phi$ as abbreviating $\neg\Diamond\neg\phi$.)

We add a new interpretation clause for modalized sentences. See if ϕ's *compatible with s*. If so, nothing happens. If no, we end up in a bad place:

$$s[\Diamond\phi] = \begin{cases} s \text{ if } s[\phi] \neq \emptyset \\ \emptyset \text{ otherwise} \end{cases}$$

In a way, then, we're still existentially quantifying over a body of information.

An exercise

What kind of update does $\Box\phi$ denote?

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&=
\end{aligned}
$$

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$$\begin{aligned} s[\Box\phi] &= s[\neg\Diamond\neg\phi] \\ &= s \setminus s[\Diamond\neg\phi] \end{aligned}$$

Let's think about the two kinds of s we might have:

- Suppose there's a not-ϕ world in s. Then

An exercise

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$$\begin{aligned} s[\Box\phi] &= s[\neg\Diamond\neg\phi] \\ &= s \setminus s[\Diamond\neg\phi] \end{aligned}$$

Let's think about the two kinds of s we might have:

- Suppose there's a not-ϕ world in s. Then $s[\Diamond\neg\phi] = s$, and $s \setminus s = \emptyset$.
- Suppose there's only ϕ worlds in s. Then

An exercise

What kind of update does $\Box\phi$ denote?

$$\begin{aligned} s[\Box\phi] &= s[\neg\Diamond\neg\phi] \\ &= s \setminus s[\Diamond\neg\phi] \end{aligned}$$

Let's think about the two kinds of s we might have:

- Suppose there's a not-ϕ world in s. Then $s[\Diamond\neg\phi] = s$, and $s \setminus s = \emptyset$.
- Suppose there's only ϕ worlds in s. Then $s[\Diamond\neg\phi] = \emptyset$, and $s \setminus \emptyset = s$.

In other words, $s[\Box\phi] = s$ if every world in s is a ϕ-world, and $\emptyset$ otherwise.

In a way, then, we're still universally quantifying over a body of information.

Distributivity

A dynamic formula ϕ may or may not be **distributive**:

$$s[\phi] = \bigcup_{w \in s} \{w\}[\phi]$$

That is, updating an info state s is the same as updating *point-wise* with each world in s, and collecting the results.

True dynamicity

In our dynamic system, formulas with $\Diamond$ aren't in general distributive. Suppose ϕ is true in u but not in v, and that s is $\{u, v\}$. Then we observe the following:

$$s[\Diamond\phi] = s = \{u, v\} \qquad \bigcup_{w \in s}\{w\}[\Diamond\phi] = \{u\} \cup \emptyset = \{u\}$$

However, if ϕ contains no epistemic modals, then it *will* be distributive: $\Diamond$ is the only part of our language that inspects global properties of s.

A similar failure of distributivity is observed for $\Box\phi$ in this model:

$$s[\Box\phi] = \emptyset \qquad \bigcup_{w \in s}\{w\}[\Box\phi] = \{u\} \cup \emptyset = \{u\}$$

True dynamicity, more generally

A dynamic system is *irreducibly* dynamic (i.e., not a reformulation of a Stalnakerian system) when it's non-distributive, or non-eliminative. Here's eliminativity:

$$s[\phi] \subseteq s$$

The dynamic system we have given isn't distributive (though it is eliminative). Later in the course, we'll have a chance to meet dynamic systems that aren't eliminative (though they will turn out to be distributive).[3]

Think about it thisly: if you never inspect global properties of s, and you never add points to s, you're doing something we could characterize as intersecting s with a set of points. In other words, you're propositional/static/non-dynamic.

[3] van Benthem (1989), Groenendijk & Stokhof (1991), Rothschild & Yalcin (2015), Charlow (2016).

Epistemic "contradictions"

One cannot really sensibly assert either of the following:

12. #It's not raining, but it might be.
13. #It might be raining, but it isn't.

On the other hand, we can imagine coherent agents who behave as follows:

14. It might be raining. . . It isn't raining.

But you can't imagine a coherent, consistent agent behaving as follows:

15. #It isn't raining. . . It might be raining.

Epistemic contradictions motivate dynamics

The truth-conditional picture predicts epistemic contradictions are ok:

$$\neg\mathbf{rain}_w \wedge \exists v \in Bw : \mathbf{rain}_v$$

That is, there are worlds w where it's not raining, but where rain is nevertheless consistent with the contextually determined body of information (see Yalcin 2007).

In contrast, the dynamic picture correctly predicts that $[\neg\mathbf{rain} \wedge \Diamond\mathbf{rain}]$ always fails:

$$s[\neg\mathbf{rain} \wedge \Diamond\mathbf{rain}] = s[\neg\mathbf{rain}][\Diamond\mathbf{rain}] = \emptyset$$

This correctly rules out (12) and (15). What about the difference b/w (13) and (14)?

Assertability

If s contains a **rain**-world, then we have the following for both (13) and (14):

$$s[\Diamond\textbf{rain} \wedge \neg\textbf{rain}] = s[\Diamond\textbf{rain}][\neg\textbf{rain}]$$
$$= \{w \in s \mid w \notin [\![\textbf{rain}]\!]_{\text{¢}}\}$$

Nevertheless, we might imagine a constraint on when ϕ is assertable by a speaker x (cf. the knowledge norm of assertion, von Fintel & Gillies's Cohesiveness):

$$\phi \text{ is } \textbf{assertable} \text{ by } x \iff s_x[\phi] = s_x$$

(Where s_x is x's information state.)

Then, $\Diamond$**rain** $\wedge$ $\neg$**rain** would never be assertable by any x, since no s_x could both contain a **rain**-world and be entirely made up of $\neg$**rain**-worlds. Nevertheless, successive utterances of $\Diamond$**rain** and $\neg$**rain** could *each* be assertable, *if* (and only if) s_x changes on the way from the assertion of $\Diamond$**rain** to the assertion of $\neg$**rain**.

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