

# Contact-Implicit Planning for Athletic, Contact-Rich Tasks

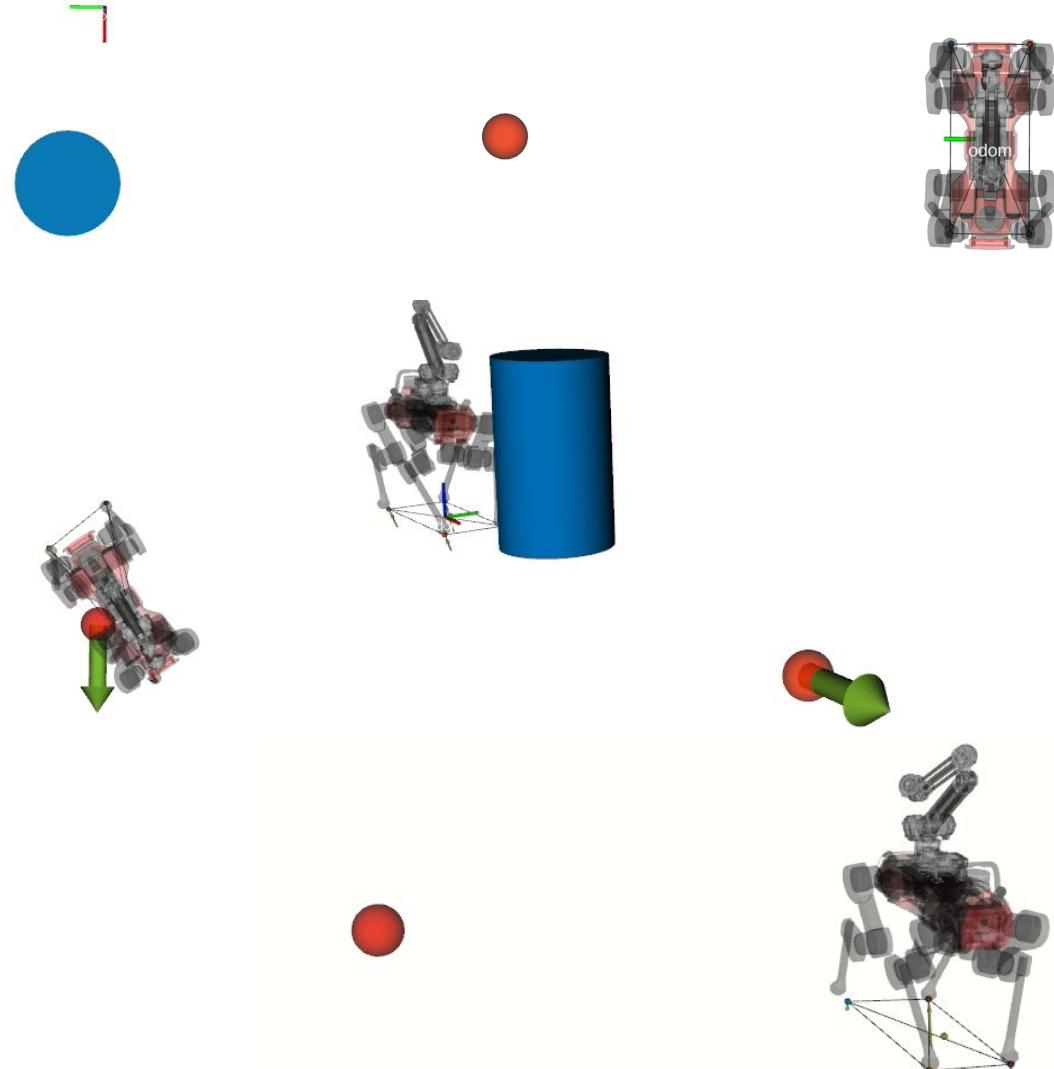
**Stefanos Charalambous**

Semester Project Final Presentation

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Committee: Mayank Mittal

24 April 2023, Zurich



# Motivation

- Physically intensive tasks require human level athleticism



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- From short reactive thinking to long horizon contact-exploiting planning



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- From short reactive thinking to long horizon contact-exploiting planning
- Spectrum of possibilities is endless

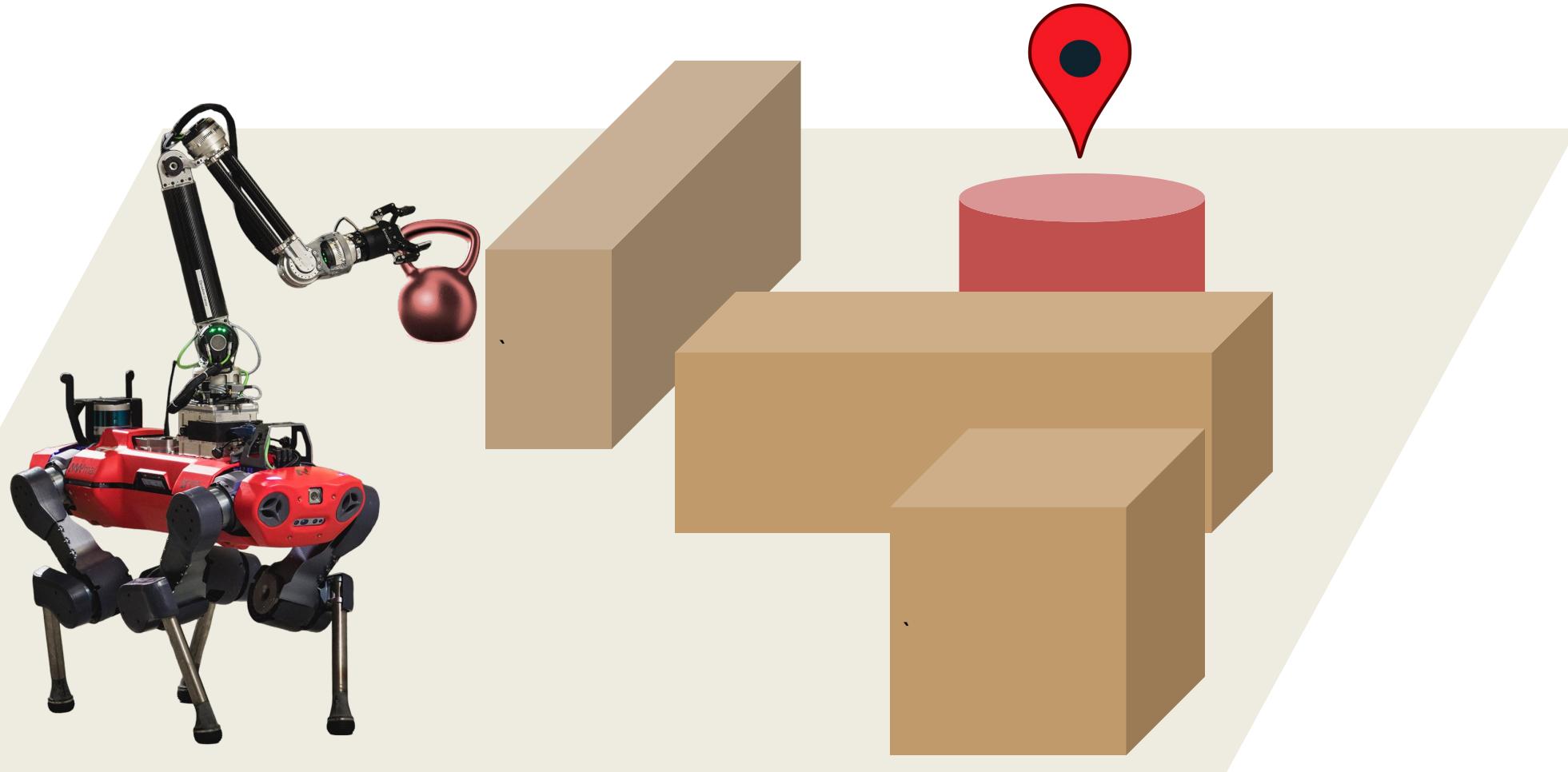


# Motivation

- Physically intensive tasks require human level athleticism
- From short reactive thinking to long horizon contact-exploiting planning
- Spectrum of possibilities is endless
- Task-agnostic approach for a platform like ANYmal or ALMA



# An example



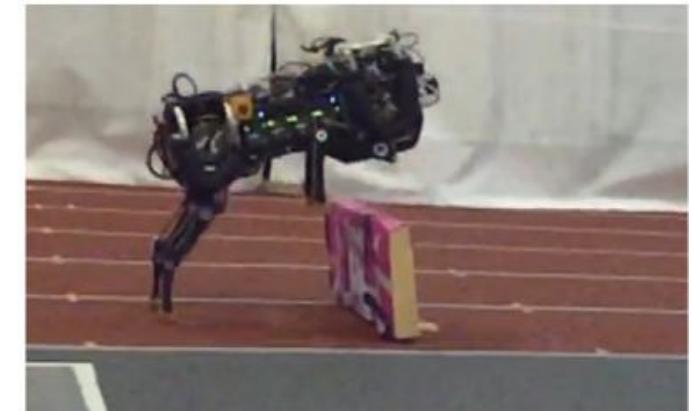
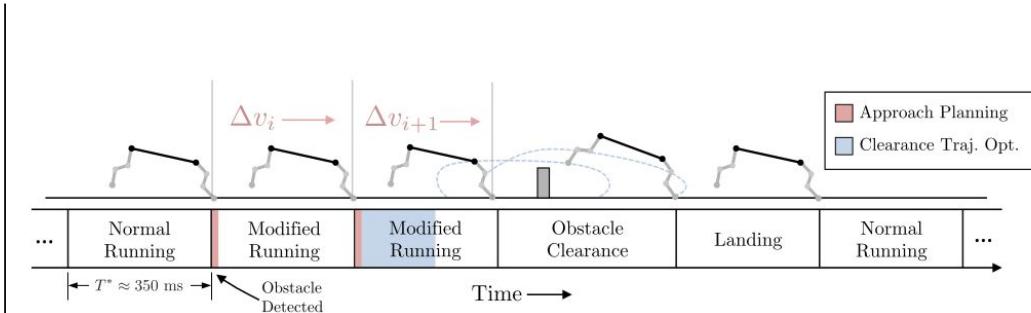
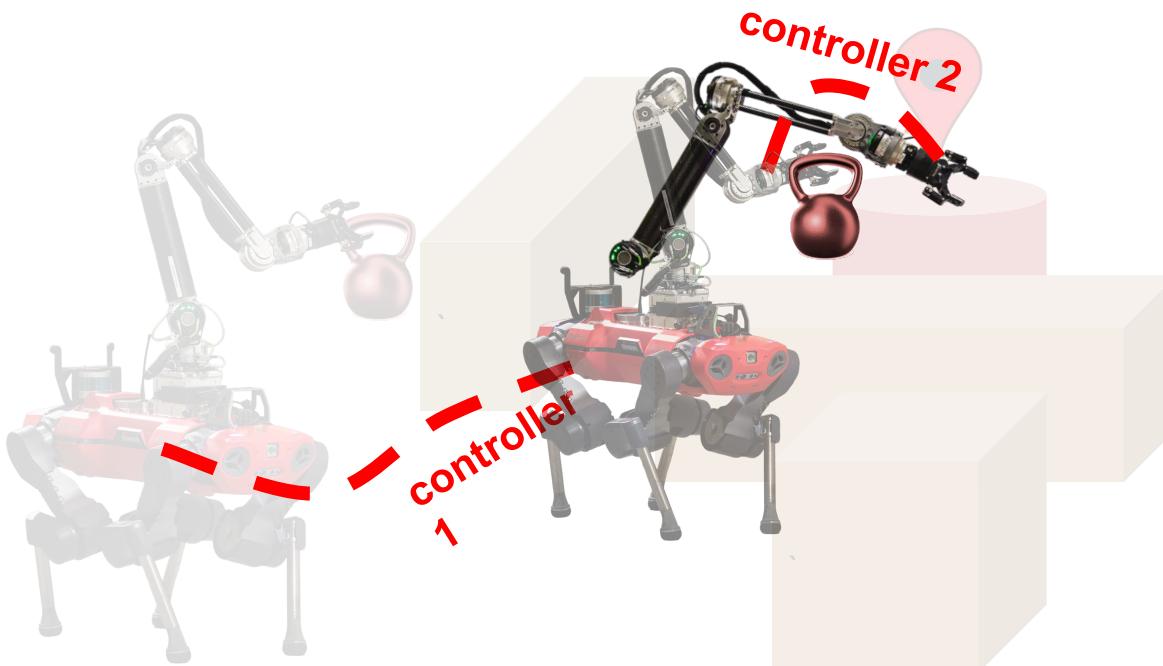
# An example



## Why is it difficult?

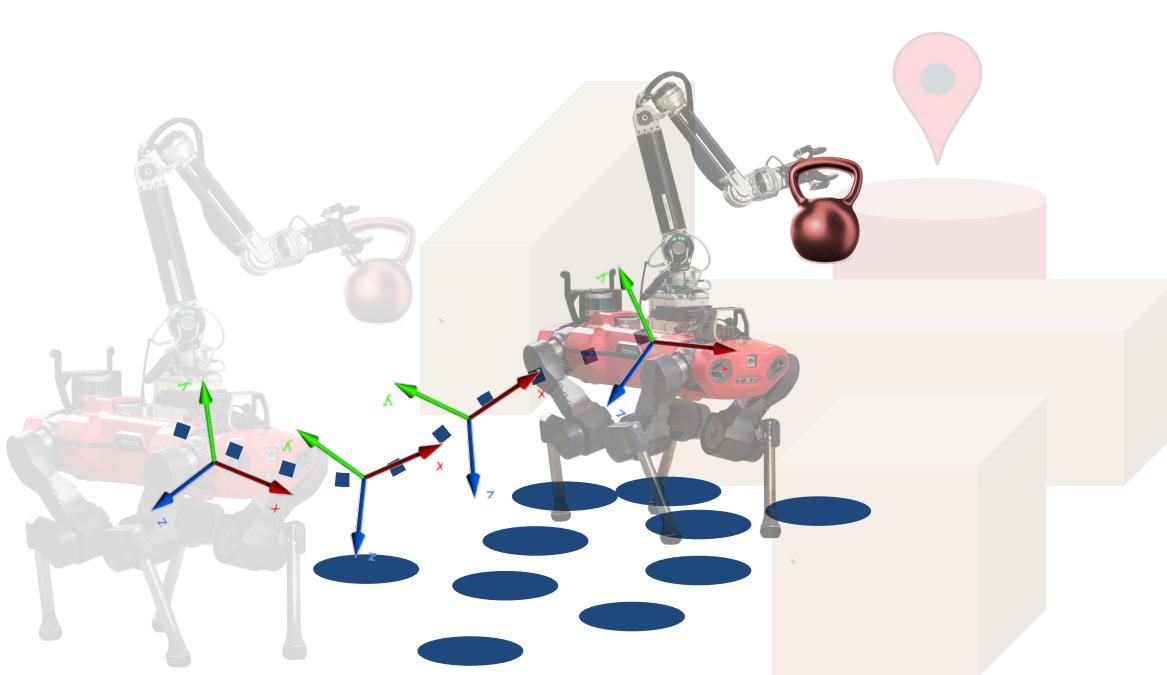
- Contact-rich
- Long horizon
- Sparse objective

# Current approaches: *Human guidance in defining behaviors*



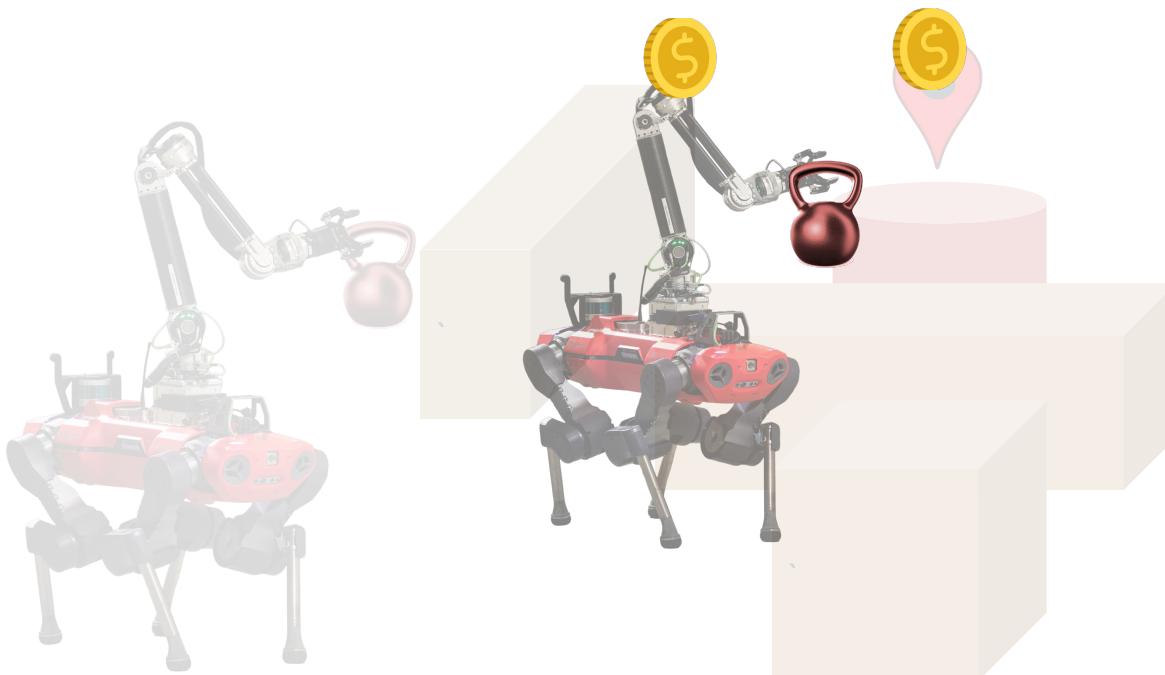
Source: Jumping over obstacles with MIT Cheetah 2

## Current approaches: *Human guidance in defining behaviors*



Source: [Atlas Gets a Grip | Boston Dynamics](#).

# Current approaches: *Human guidance in generation of behaviors*



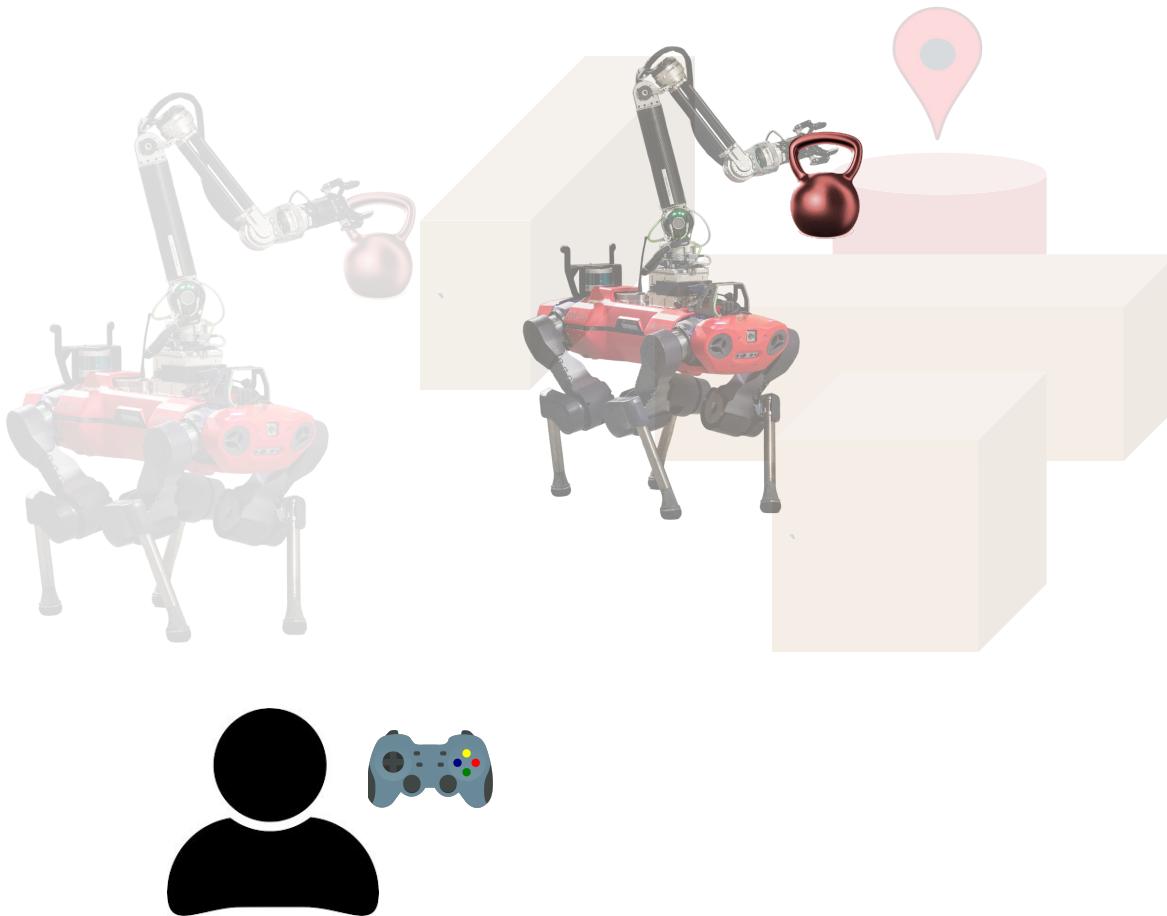
Term	Expression
Projected Ball Velocity	$\exp\{-\delta_v \ \mathbf{v}_b - \mathbf{v}_{cmd}\ ^2\}$
Robot Ball Distance	$\exp\{-\delta_p \ \mathbf{p}_b - \mathbf{p}_{FRHip}\ ^2\}$
Yaw Alignment	$\exp\{-\delta_\psi (e_{rbcmd}^2 + e_{rbbase}^2)\}$
Ball Velocity Norm	$\exp\{-\delta_n (\ \mathbf{v}_{cmd}\  - \ \mathbf{v}_b\ )^2\}$
Ball Velocity Angle	$1 - (\psi_b - \psi_{cmd})^2 / \pi^2$
Swing Phase Schedule	$[1 - \mathbf{C}_{foot}^{cmd}(\boldsymbol{\theta}^{cmd}, t)] \exp\{-\delta_{cf} \ \mathbf{f}_{foot}\ ^2\}$
Stance Phase Schedule	$[\mathbf{C}_{foot}^{cmd}(\boldsymbol{\theta}^{cmd}, t)] \exp\{-\delta_{cv} \ \mathbf{v}_{xy}^{foot}\ ^2\}$

“Reward terms for the soccer dribbling task.”



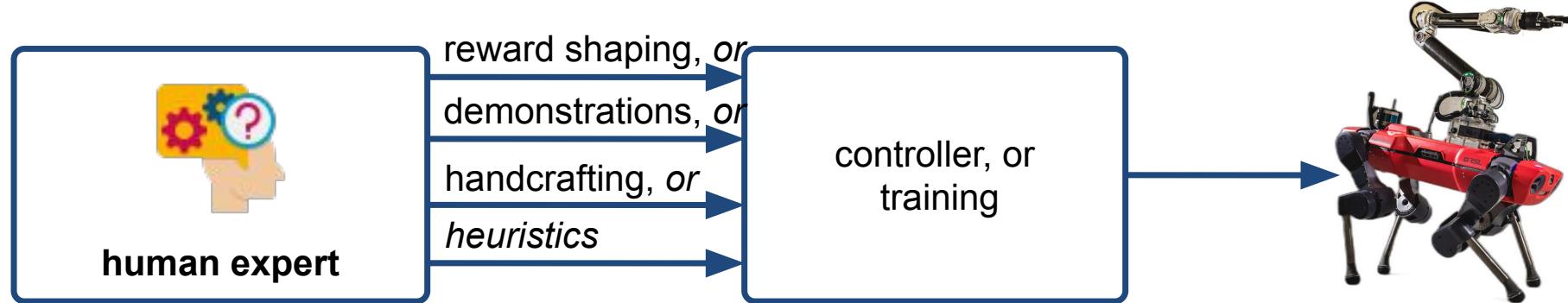
Source: Reinforcement Learning for Quadrupedal Dribbling in the Wild

## Current approaches: *Human guidance in generation of behaviors*



Source: Learning Model Predictive Controllers with Real-Time Attention for Real-World Navigation

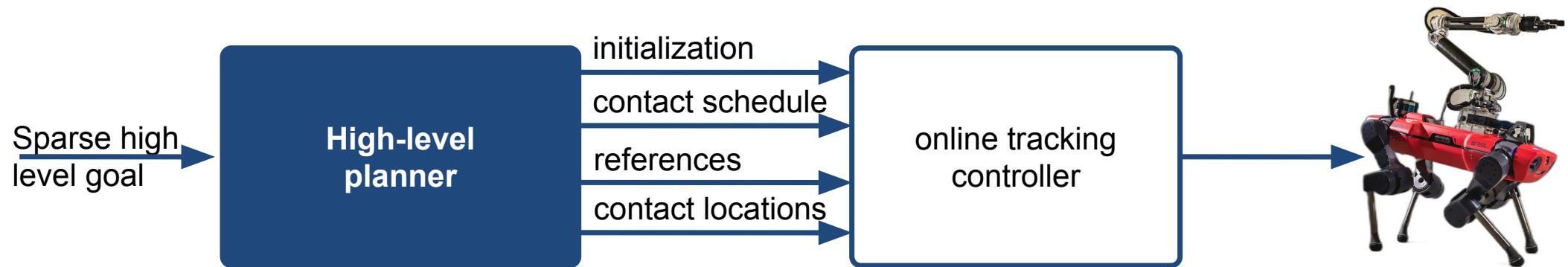
# What's currently done



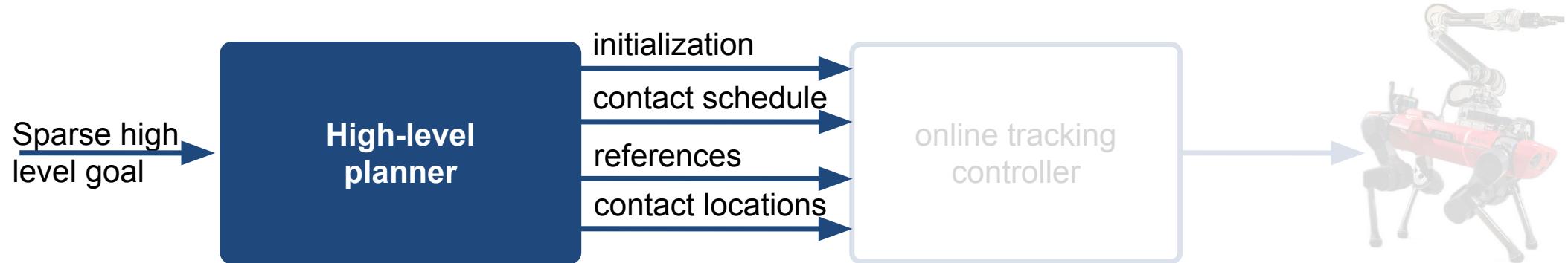
What's currently done

task-specific, costly and compromises performance.

# What we need



# What we need



**Goal:**

Automate the generation of the motion through **contact-implicit** trajectory optimization (CITO) with **minimal, high-level guidance**

# Method selection

## Framework requirements

- Minimal guidance/ task-agnostic
- Contact-implicit
- Physical feasibility
- Realizable on hardware
- Numerically stable
- Scales to multi-contact, long horizon tasks

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## Existing contact-implicit methods

- Combinatorial approaches <sup>1</sup>
- Hybrid dynamics methods
- Smooth contact
- Hard contact (constrained optimization)

1. For example, mixed-integer formulations [5] presented promising results but require symmetry and periodicity restrictions and only tractable in 2D

# Method selection

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- Hybrid dynamics methods physically incorrect
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# Method selection

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## Existing contact-implicit methods

- ~~Combinatorial approaches~~
- ~~Hybrid dynamics methods~~
- ~~Smooth contact~~
- **Hard contact (constrained optimization)**

## Background: *Complementarity Constraints*

- Contact physics as constraints on the decision variables

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## Background: Complementarity Constraints

- Contact physics as constraints on the decision variables

- **Separation**

$$0 \leq f_i^N(\mathbf{u}(k)) \perp \phi_i(\mathbf{x}(k+1)) \geq 0$$

Unilateral force constraint                      Non-penetration constraint  
No force at a distance

## Background: *Complementarity Constraints*

- Contact physics as constraints on the decision variables
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$$0 \leq f_i^N(\mathbf{u}(k)) \perp \phi_i(\mathbf{x}(k+1)) \geq 0$$

- **Sliding**

$$0 \leq f_i^N(\mathbf{u}(k)) \perp \mathbf{v}_i^T(\mathbf{x}(k+1))$$

## Background: Complementarity Constraints

- Contact physics as constraints on the decision variables
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$$0 \leq f_i^N(\mathbf{u}(k)) \perp \phi_i(\mathbf{x}(k+1)) \geq 0$$

- Sliding

$$0 \leq \underbrace{f_i^N(\mathbf{u}(k))}_{\text{No force when non-zero tangential velocity}} \perp \mathbf{v}_i^T(\mathbf{x}(k+1))$$

No force when non-zero tangential velocity

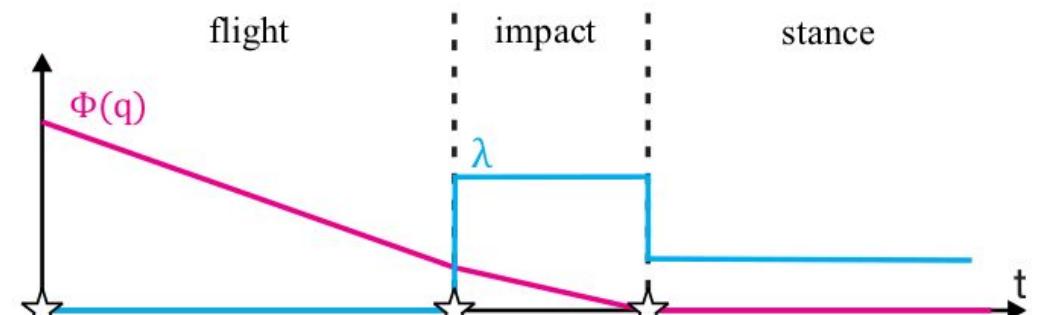
## Background: Time-stepping integration scheme <sup>1</sup>

- Note the **indexing**:

$$0 \leq f_i^N(\mathbf{u}(k)) \perp \phi_i(\mathbf{x}(k+1)) \geq 0$$

$$0 \leq f_i^N(\mathbf{u}(k)) \perp \mathbf{v}_i^T(\mathbf{x}(k+1))$$

- Contact impulse can be non-zero if and only if there is contact at the end of the interval <sup>2</sup>
- Contact impulses appear as average contact forces over time interval



Source: Contact-Implicit Trajectory Optimization using Orthogonal Collocation

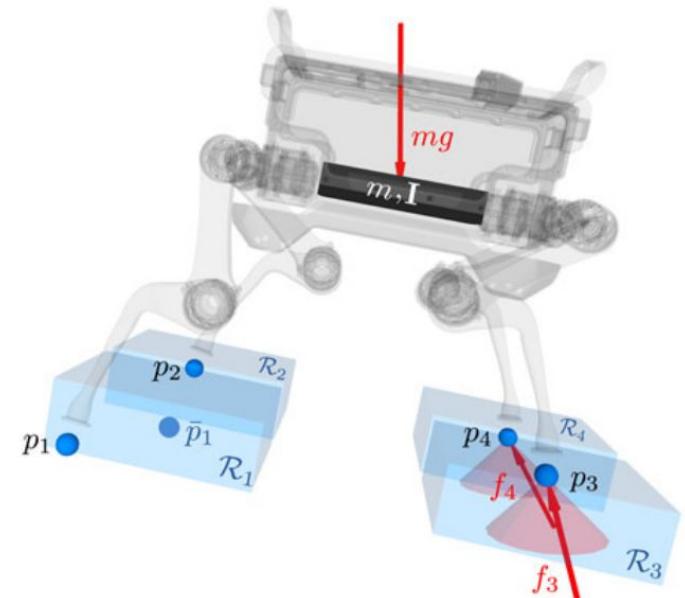
1. D. Stewart, J. C. Trinkle. "An implicit time-stepping scheme for rigid body dynamics with coulomb friction."
2. Posa, Michael Michael Antonio. Optimization for control and planning of multi-contact dynamic motion. Diss. Massachusetts Institute of Technology, 2017.

# Single Rigid Body (SRB) model

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{c} \\ \theta_{yaw} \\ \theta_{pitch} \\ \theta_{roll} \\ \dot{\boldsymbol{c}} \\ \boldsymbol{\omega} \\ \boldsymbol{r}_1 \\ \vdots \\ \boldsymbol{r}_{n_c} \\ \dot{\boldsymbol{r}}_1 \\ \vdots \\ \dot{\boldsymbol{r}}_{n_c} \end{bmatrix} \quad \boldsymbol{u} = \begin{bmatrix} \boldsymbol{f}_1 \\ \vdots \\ \boldsymbol{f}_{n_c} \\ \ddot{\boldsymbol{r}}_1 \\ \vdots \\ \ddot{\boldsymbol{r}}_{n_c} \end{bmatrix}$$

$\dot{\boldsymbol{\theta}} = \boldsymbol{T}(\boldsymbol{\theta}) \boldsymbol{\omega}$

$i = 1 \dots n_c \quad k = 0 \dots N$



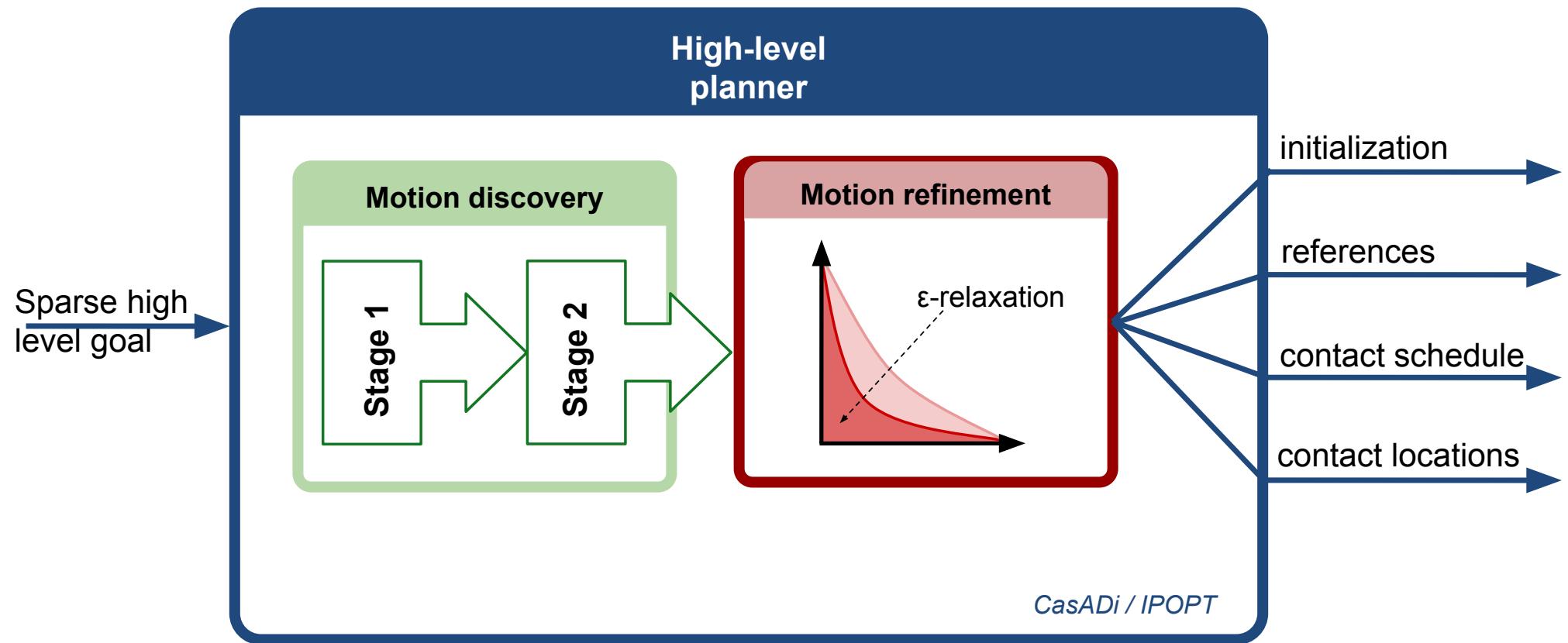
Source: Gait and Trajectory Optimization for Legged Systems Through Phase-Based End-Effector Parameterization [8]

$$\dot{\boldsymbol{\omega}} = \boldsymbol{I}_{\mathcal{W}}^{-1}(\boldsymbol{\theta}) (\sum_{i=1}^{n_c} \boldsymbol{f}_i \times (\boldsymbol{r}_i - \boldsymbol{c}) - \boldsymbol{\omega} \times \boldsymbol{I}_{\mathcal{W}}(\boldsymbol{\theta}) \boldsymbol{\omega})$$

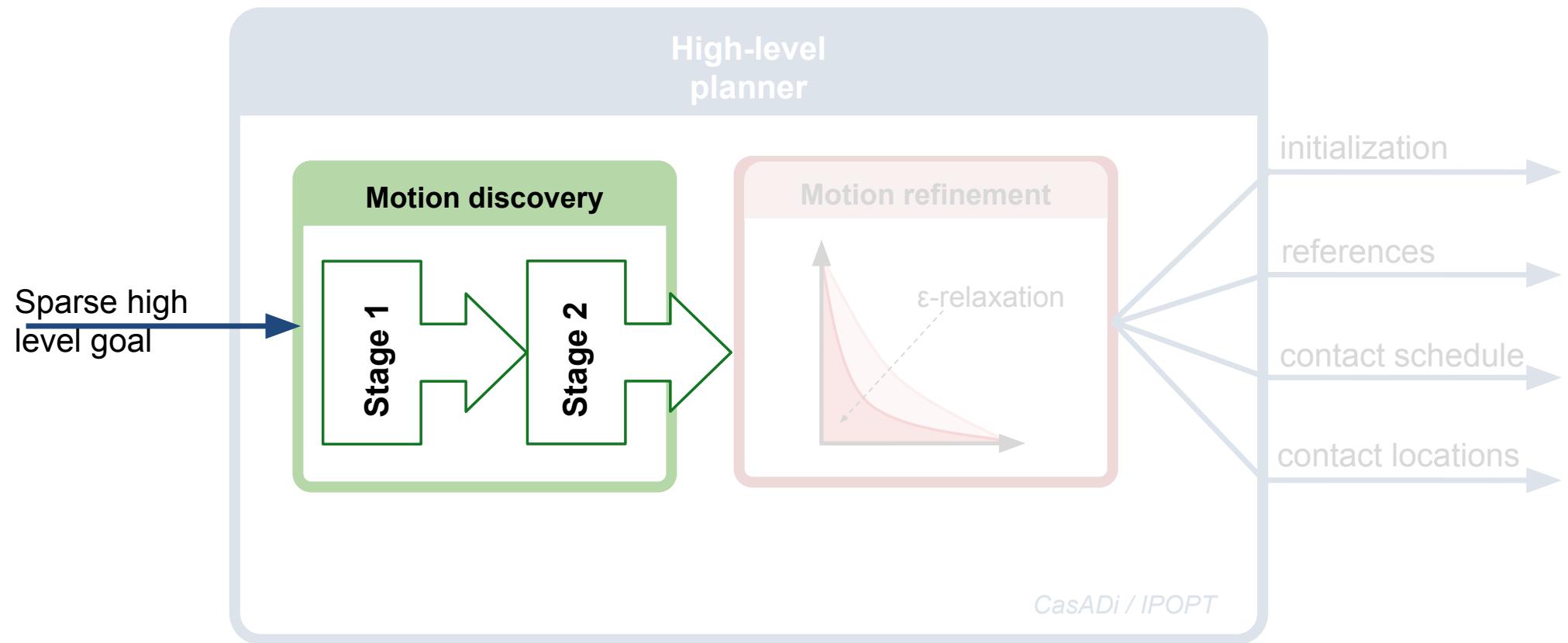
$$\ddot{\boldsymbol{c}} = \sum_{i=1}^{n_c} \frac{\boldsymbol{f}_i}{m} + \boldsymbol{g}$$

$$\text{where } \boldsymbol{I}_{\mathcal{W}}(\boldsymbol{\theta}) = {}_{\mathcal{W}}\boldsymbol{R}_{\mathcal{B}}(\boldsymbol{\theta}) \boldsymbol{I}_{\mathcal{B}} {}_{\mathcal{W}}\boldsymbol{R}_{\mathcal{B}}(\boldsymbol{\theta})^T$$

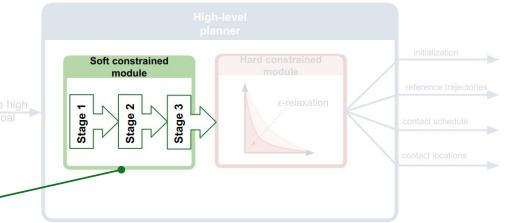
# Framework



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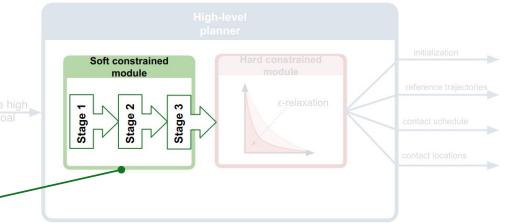
# Motion discovery



$$\begin{aligned}
 \min_{X,U} \quad & \| \mathbf{x}(N) - \mathbf{x}_{ref}(N) \|_{Q_f}^2 + \sum_{k=0}^{N-1} \| \mathbf{x}(k) - \mathbf{x}_{ref}(k) \|_Q^2 + \| \mathbf{u}(k) \|_R^2 \\
 & + w_{CIO} \sum_{i=1}^{n_c} g(f_i^N(\mathbf{u}(k))) (\phi_i(\mathbf{x}(k+1))^2 + \| \mathbf{v}_i^T(\mathbf{x}(k+1)) \|_2^2) \\
 \text{s.t.} \quad & \mathbf{x}(k+1) = f(\mathbf{x}(k), \mathbf{u}(k)) \\
 & x(0) = x_0 \\
 & \mathbf{x}(k) \in \Psi(\mathbf{x}(k)) \\
 & \| \mathbf{f}_i^T(\mathbf{u}(k)) \|_\infty \leq \mu f_i^N(\mathbf{u}(k)) \\
 & 0 \leq f_i^N(\mathbf{u}(k)) \leq f_{N,limit} \\
 & 0 \leq \phi_i(\mathbf{x}(k)) \\
 & \mathbf{A}_{x_N} \mathbf{x}(N) = \mathbf{b}_{x_N}
 \end{aligned}$$

Analogous to work by I. Mordatch et al [3]

# Motion discovery



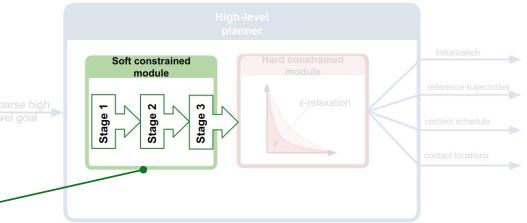
$$\begin{aligned}
 \min_{X,U} \quad & \|x(N) - x_{ref}(N)\|_{Q_f}^2 + \sum_{k=0}^{N-1} \|x(k) - x_{ref}(k)\|_Q^2 + \|\mathbf{u}(k)\|_R^2 \\
 & + w_{CIO} \sum_{i=1}^{n_c} g(f_i^N(\mathbf{u}(k))) (\phi_i(x(k+1))^2 + \|v_i^T(x(k+1))\|_2^2) \\
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 \end{aligned}$$

Quadratic state input cost

Dynamics  
Kinematic (box) constraints  
Friction pyramid  
Initial condition  
Non-penetration

Analogous to work by I. Mordatch et al [3]

# Motion discovery



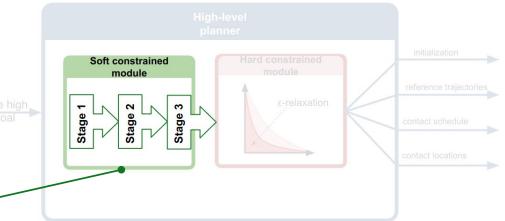
$$\begin{aligned} \min_{X,U} \quad & \|x(N) - x_{ref}(N)\|_{Q_f}^2 + \sum_{k=0}^{N-1} \|x(k) - x_{ref}(k)\|_Q^2 + \|u(k)\|_R^2 \\ & + w_{CIO} \sum_{i=1}^{n_c} g(f_i^N(u(k))) (\phi_i(x(k+1))^2 + \|\mathbf{v}_i^T(x(k+1))\|_2^2) \end{aligned}$$

$$\begin{aligned} \text{s.t.} \quad & x(k+1) = f(x(k), u(k)) \\ & x(0) = x_0 \\ & x(k) \in \Psi(x(k)) \\ & \|f_i^T(u(k))\|_\infty \leq \mu f_i^N(u(k)) \\ & 0 \leq f_i^N(u(k)) \leq f_{N,limit} \\ & 0 \leq \phi_i(x(k)) \\ & A_{x_N} x(N) = b_{x_N} \end{aligned}$$

>Contact physics as a cost

Analogous to work by I. Mordatch et al [3]

# Motion discovery



$$\begin{aligned} \min_{X,U} \quad & \|x(N) - x_{ref}(N)\|_{Q_f}^2 + \sum_{k=0}^{N-1} \|x(k) - x_{ref}(k)\|_Q^2 + \|u(k)\|_R^2 \\ & + w_{CIO} \sum_{i=1}^{n_c} g(f_i^N(u(k))) (\phi_i(x(k+1))^2 + \|v_i^T(x(k+1))\|_2^2) \end{aligned}$$

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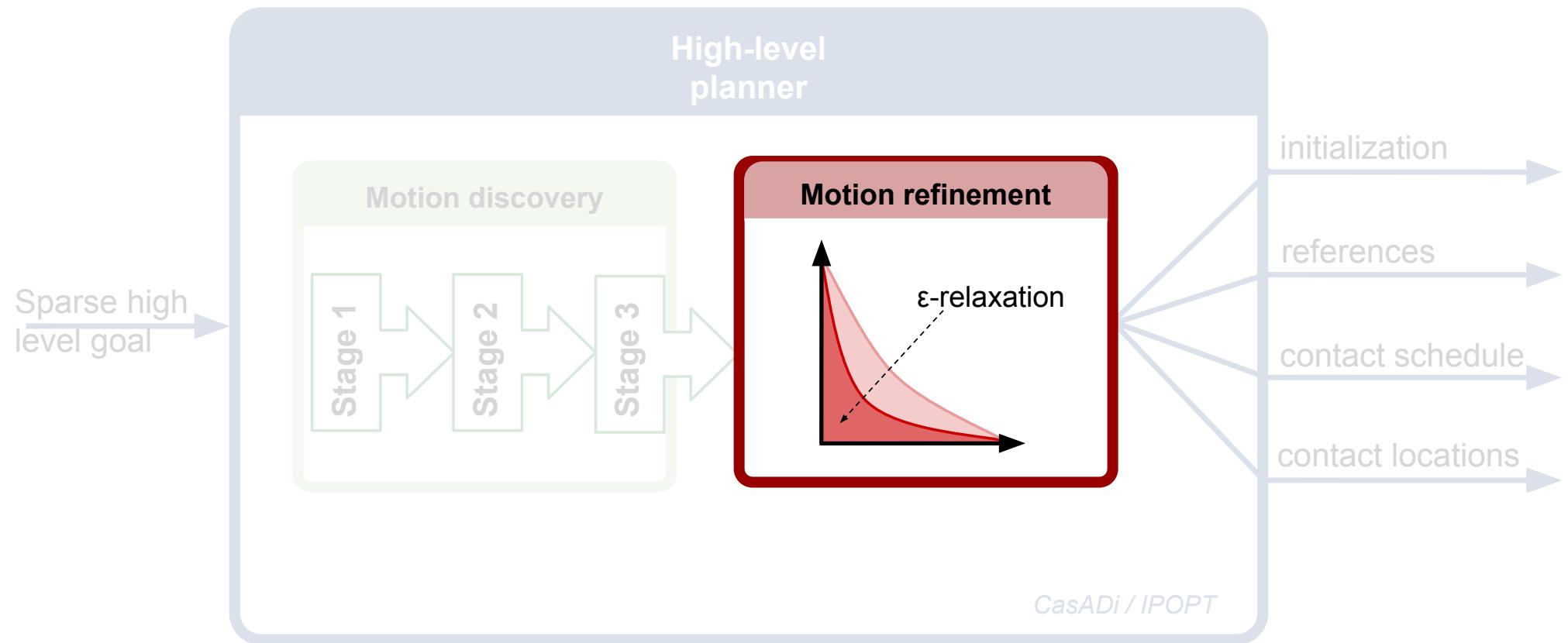
$$0 \leq \phi_i(x(k))$$

$$A_{x_N} x(N) = b_{x_N}$$

Terminal constraint to encode task

Analogous to work by I. Mordatch et al [3]

# Framework



# Motion refinement

$$\min_{X,U,Z} \quad ||\mathbf{x}(N) - \mathbf{x}_{ref}(N)||_{Q_f}^2 + \sum_{k=0}^{N-1} ||\mathbf{x}(k) - \mathbf{x}_{ref}(k)||_Q^2 + ||\mathbf{u}(k)||_R^2$$

s.t.       $\mathbf{x}(k+1) = f(\mathbf{x}(k), \mathbf{u}(k))$

$$x(0) = x_0$$

$$\mathbf{x}(k) \in \Psi(\mathbf{x}(k))$$

$$||\mathbf{f}_i^T(\mathbf{u}(k))||_\infty \leq \mu f_i^N(\mathbf{u}(k))$$

$$0 \leq f_i^N(\mathbf{u}(k)) \leq M_f \mathbf{z}_i(k)$$

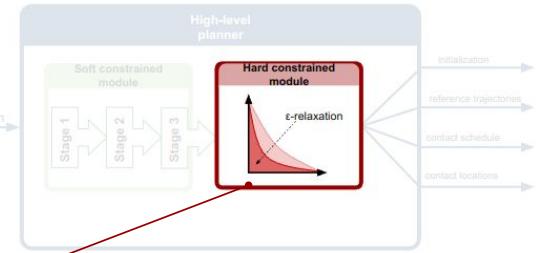
$$0 \leq \phi_i(\mathbf{x}(k+1)) \leq M_\phi (1 - \mathbf{z}_i(k))$$

$$||\mathbf{v}_i^T(\mathbf{x}(k+1))||_\infty \leq M_v (1 - \mathbf{z}_i(k))$$

$$\mathbf{z}_i(k)^T \cdot \mathbf{z}_i(k) \leq \varepsilon$$

$$\mathbf{z}(k) \in [0, 1]$$

$$\mathbf{A}_{x_N} \mathbf{x}(N) = \mathbf{b}_{x_N}$$



Analogous to work by M. Posa et al [1], [2]

# Motion refinement

$$\min_{X, U, Z} \quad \|x(N) - x_{ref}(N)\|_{Q_f}^2 + \sum_{k=0}^{N-1} \|x(k) - x_{ref}(k)\|_Q^2 + \|u(k)\|_R^2$$

$$\text{s.t.} \quad \begin{aligned} x(k+1) &= f(x(k), u(k)) \\ x(0) &= x_0 \\ x(k) &\in \Psi(x(k)) \\ \|f_i^T(u(k))\|_\infty &\leq \mu f_i^N(u(k)) \end{aligned}$$

$$0 \leq f_i^N(u(k)) \leq M_f z_i(k)$$

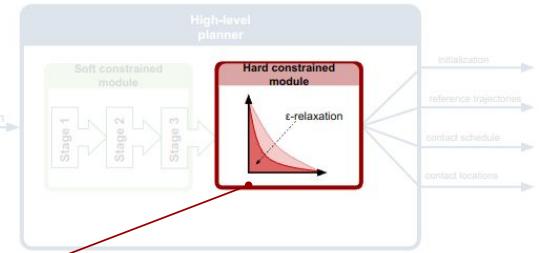
$$0 \leq \phi_i(x(k+1)) \leq M_\phi (1 - z_i(k))$$

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Quadratic state input cost  
 Dynamics  
 Initial condition  
 Kinematic (box) constraints  
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Analogous to work by M. Posa et al [1], [2]

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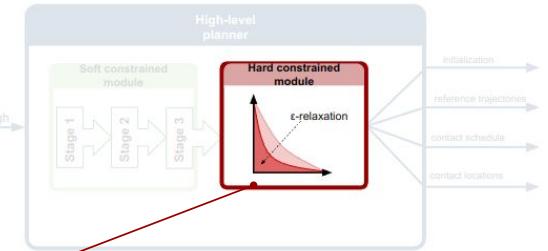
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Big-M formulation for complementarity constraints

# Motion refinement

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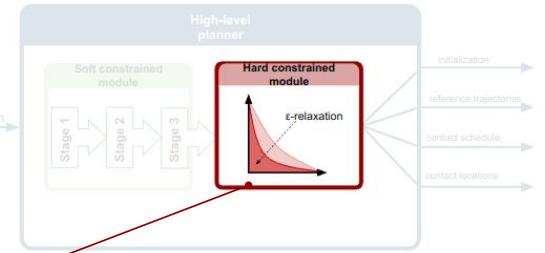
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$$z_i(k)^T \cdot z_i(k) \leq \varepsilon$$

$$z(k) \in [0, 1]$$

$$A_{x_N} x(N) = b_{x_N}$$



Relaxed orthogonality constraint on relaxed integer variables

Analogous to work by M. Posa et al [1], [2]

# Motion refinement

$$\min_{X, U, Z} \quad \|x(N) - x_{ref}(N)\|_{Q_f}^2 + \sum_{k=0}^{N-1} \|x(k) - x_{ref}(k)\|_Q^2 + \|u(k)\|_R^2$$

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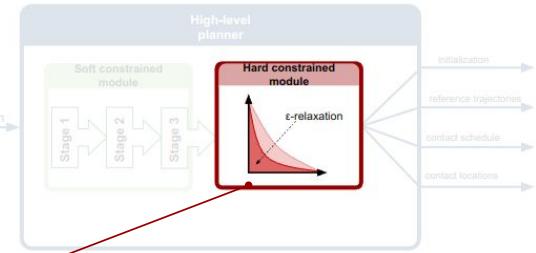
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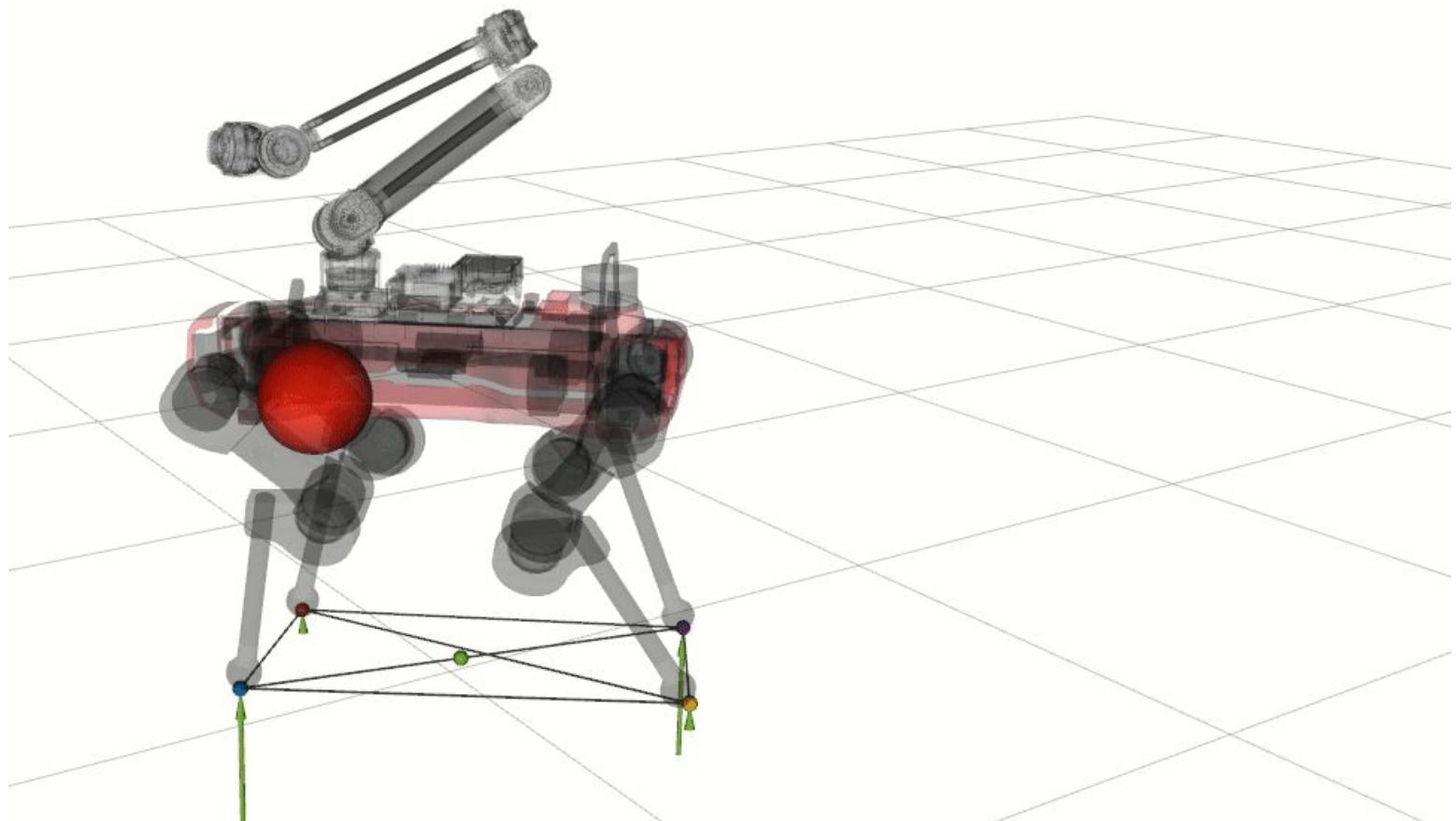
$$z(k) \in [0, 1]$$

$$A_{x_N} x(N) = b_{x_N}$$



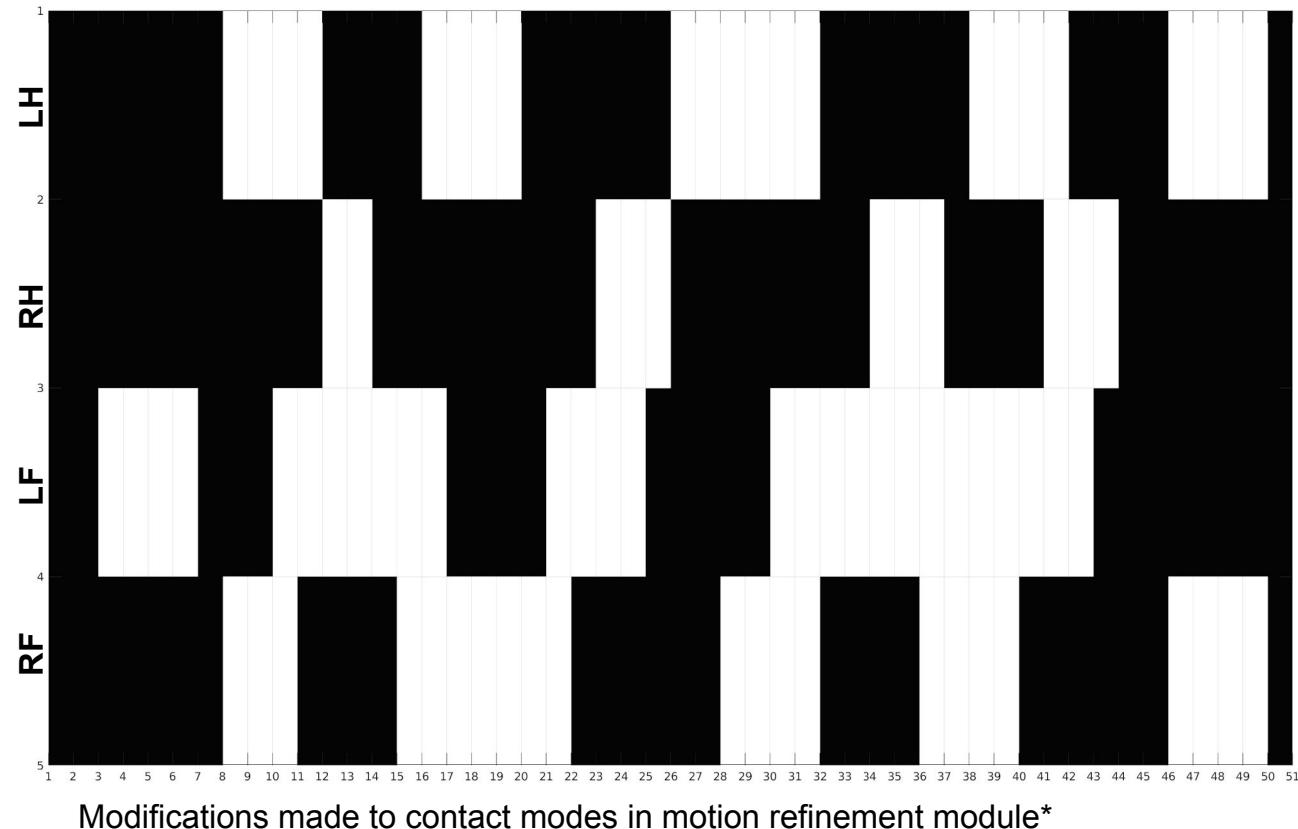
Terminal constraint  
(encodes task)

# Combined local navigation and locomotion



# Motion refinement:

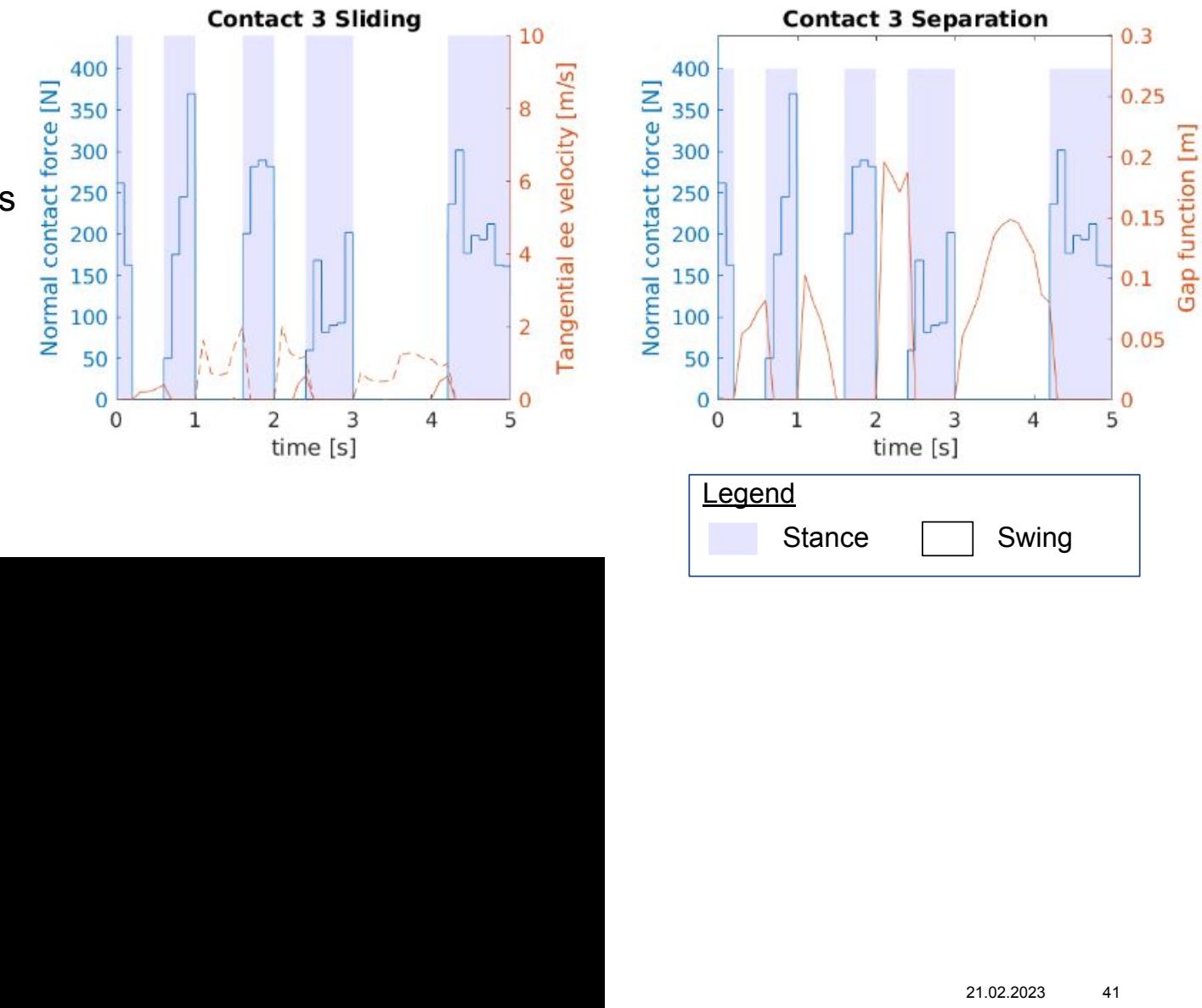
- **Direct handle on contact modes**
- For example:
  - $\geq 2$  feet in contact
  - Penalize fast contact switches
  - Initial and final modes



\* For visualization purposes. The contact modes are not explicitly encoded in the solution of the motion discovery module, but obtained through a projection operation.

# Physical feasibility

- Strict feasibility of contact physics
- Validated results in Gazebo



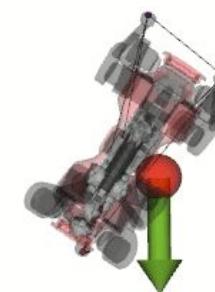
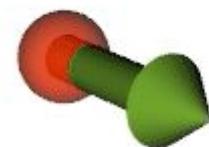
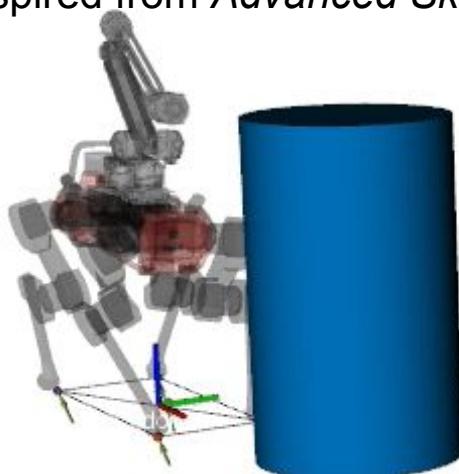
# Physical feasibility

- Online tracking controller after polishing
- Validated results on hardware



# Local navigation and locomotion

- Given only a x/y/yaw goal. No references nor initialization around obstacle
- Task inspired from *Advanced Skills by Learning Locomotion and Local Navigation End-to-End* [9]



# Limitations

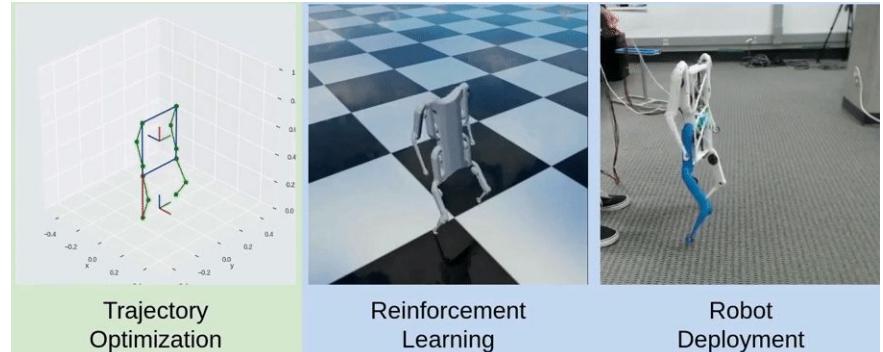
- Fast contact switches still possible to occur
- Computational time
- **Failure** in presence of **fast contact switches and dynamism** in generated references
  - Replace model-based tracking controller with **learning based controller**

# Key takeaways

- **Smooth contact models work well for motion discovery**
  - Do not act directly based on that solution: continuation method/stagewise training/ or references
- **Splines** drastically reduce search space and improve computational efficiency
  - Do not use them directly for variables that exhibit discontinuity
- **Right level, right space**
  - High level contact-implicit planner: Cartesian space  $\square$  SRBD model
  - Low level: joint space

# Future work

- **Imitation of generated references using RL**
- Extend to **loco-manipulation tasks**
  - Dynamic throw of heavy object
- Contact-implicit MPC
  - Learn mode scheduler offline (supervised learning)
- Reduce computational cost
  - C++ implementation or code generation



Source: OPT-Mimic: Imitation of Optimized Trajectories for Dynamic Quadruped Behaviors [10]



# References

- [1] Posa, Michael, Cecilia Cantu, and Russ Tedrake. "A direct method for trajectory optimization of rigid bodies through contact."
- [2] Stewart, David, and Jeffrey C. Trinkle. "An implicit time-stepping scheme for rigid body dynamics with coulomb friction."
- [3] Mordatch, Igor, Emanuel Todorov, and Zoran Popović. "Discovery of complex behaviors through contact-invariant optimization."
- [4] Aydinoglu, Alp, and Michael Posa. "Real-time multi-contact model predictive control via admm."
- [5] Valenzuela, Andrés Klee. Mixed-integer convex optimization for planning aggressive motions of legged robots over rough terrain.
- [6] Park, Hae-Won, Patrick M. Wensing, and Sangbae Kim. "Jumping over obstacles with MIT Cheetah 2."
- [7] Atlas Gets a Grip | Boston Dynamics. (n.d.). Retrieved from [https://www.youtube.com/watch?v=-e1\\_QhJ1EhQ](https://www.youtube.com/watch?v=-e1_QhJ1EhQ).
- [8] Winkler, Alexander W., et al. "Gait and trajectory optimization for legged systems through phase-based end-effector parameterization."
- [9] Rudin, Nikita, et al. "Advanced Skills by Learning Locomotion and Local Navigation End-to-End."
- [10] Fuchioka, Yuni, Zhaoming Xie, and Michiel van de Panne. "OPT-Mimic: Imitation of Optimized Trajectories for Dynamic Quadruped Behaviors."