Assignments

You do not have to type your assignment (unless you don't think we will be able to understand your handwriting).

Starting with assignment 2

- Please put the name of your GSI on the first page of the assignment.
- Staple all pages and put your name and uniqname on all pages.

Assignment Pairs

You are split into two groups: “early” and “late” team work.

- Early group: Can work in assigned pairs on assignments 2-7
- Late group: Can work in assigned pairs on assignments 8-13

If you work with your assigned partner, you turn in **only one** assignment solution.

Which group are you in?

On Canvas, check your “score” on "Early/Late Team Work Assignment”:

- Score of 1 → Early group
- Score of 2 → Late group

Assignment Pairs

Who is your partner?

Pairs will be assigned every week, so your partner can change.

How will the pairs be communicated?

Elliott Brannon will explain.

Last Time

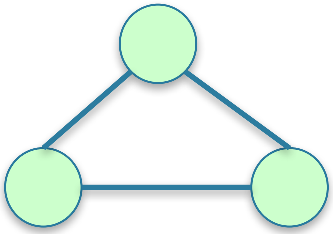
- Milgram Small World Experiment
- Average distance in instant message and Facebook
- The strength of weak ties: people found jobs through weak personal connections
- Triadic Closure
- Clustering Coefficient
  - Local Clustering Coefficient
  - Global Clustering Coefficient
    - » Transitivity
    - » Mean LCC

Clustering Coefficient

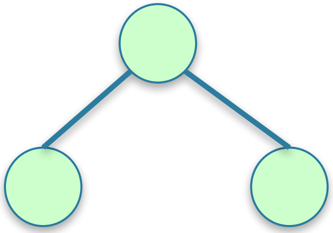
Global clustering coefficient (GCC) of a network:

Definition 1: $$\text{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

Closed triads:



Open triads:

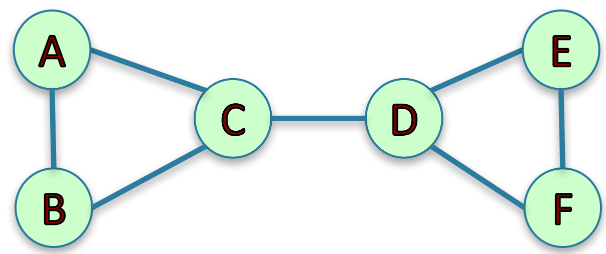


Clustering Coefficient

Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

Compute the transitivity of the network:



Clustering Coefficient

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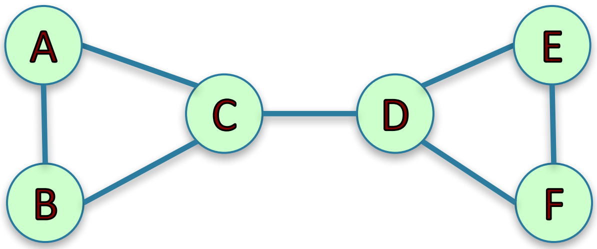
Compute the transitivity of the network:

Number of closed triads:



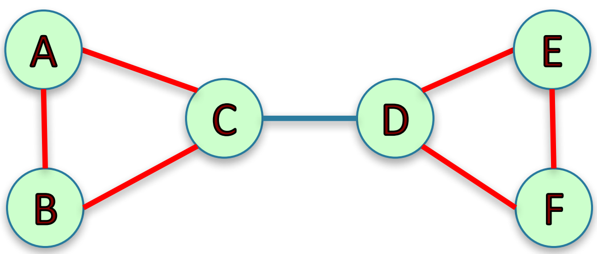
Clustering Coefficient

Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3{*}\text{Number of closed triads}}{\text{Number of open triads}}$$

Compute the transitivity of the network:

Number of closed triads: 2



Clustering Coefficient

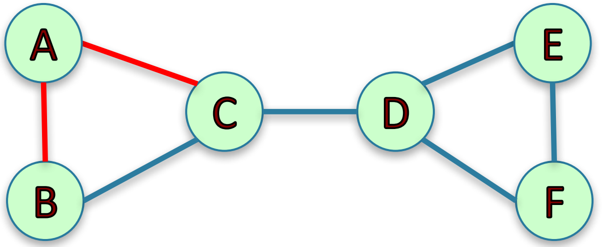
Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

Compute the transitivity of the network:

Number of closed triads: 2

Number of open triads: 1



Clustering Coefficient

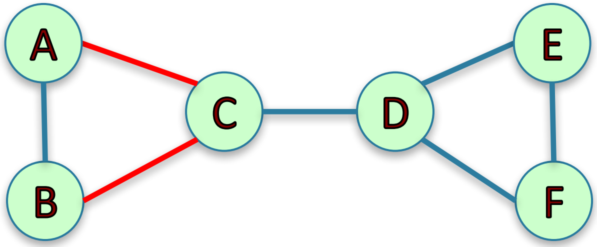
Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

Compute the transitivity of the network:

Number of closed triads: 2

Number of open triads: 2



Clustering Coefficient

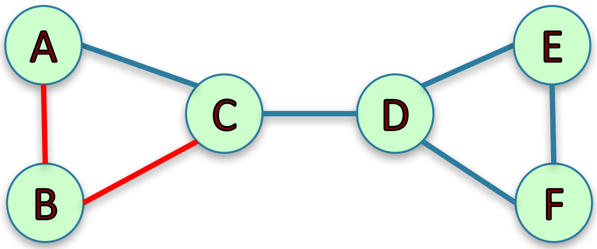
Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

Compute the transitivity of the network:

Number of closed triads: 2

Number of open triads: 3



Clustering Coefficient

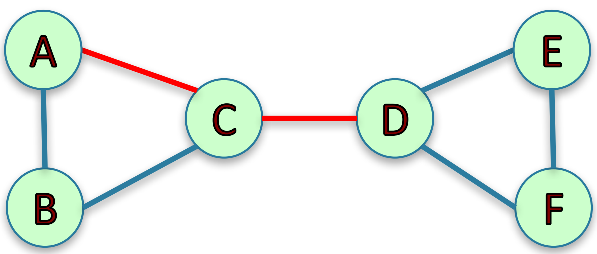
Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

Compute the transitivity of the network:

Number of closed triads: 2

Number of open triads: 4



Clustering Coefficient

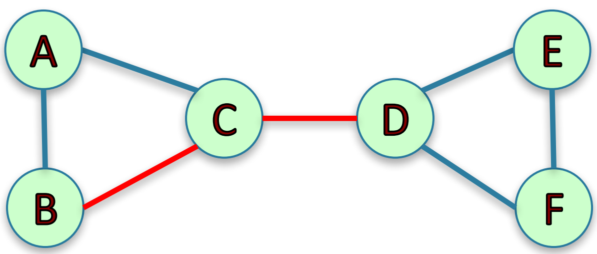
Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3*\text{Number of closed triads}}{\text{Number of open triads}}$$

Compute the transitivity of the network:

Number of closed triads: 2

Number of open triads: 5



Clustering Coefficient

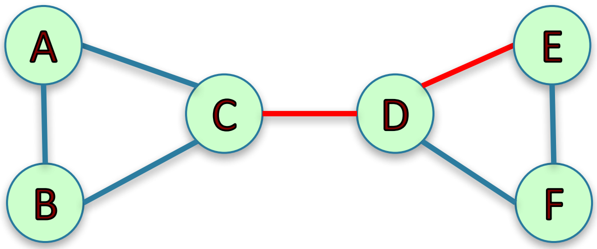
Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

Compute the transitivity of the network:

Number of closed triads: 2

Number of open triads: 6



Clustering Coefficient

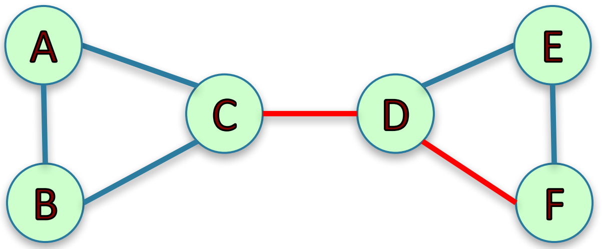
Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

Compute the transitivity of the network:

Number of closed triads: 2

Number of open triads: 7



Clustering Coefficient

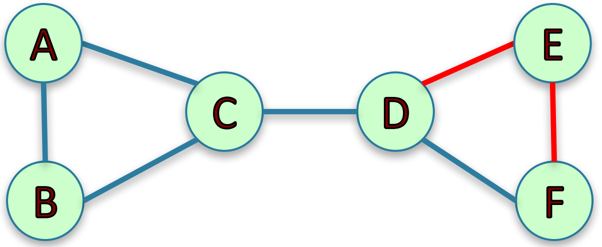
Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

Compute the transitivity of the network:

Number of closed triads: 2

Number of open triads: 8



Clustering Coefficient

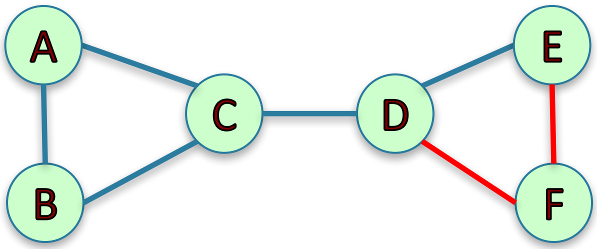
Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

Compute the transitivity of the network:

Number of closed triads: 2

Number of open triads: 9



Clustering Coefficient

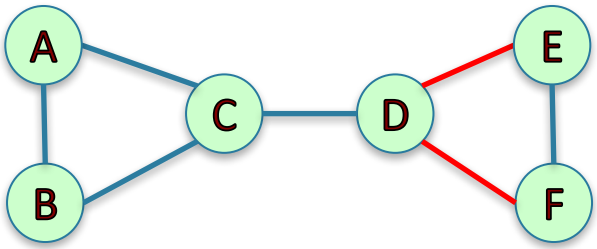
Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

Compute the transitivity of the network:

Number of closed triads: 2

Number of open triads: 10



$$\text{Transitivity} = \frac{3 * 2}{10} = \frac{3}{5}$$

Clustering Coefficient

Global clustering coefficient (GCC) of a network:

Definition 2: Mean Local Clustering Coefficient (MLCC) over all nodes

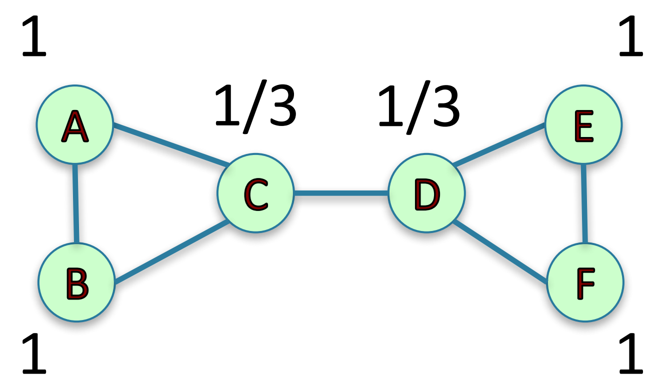
Compute the MLCC of the network:

First, compute LCC of all nodes

Now take the mean

MLCC = Mean of {1, 1, 1/3, 1/3, 1, 1}

$$\text{MLCC} = \frac{7}{9}$$



Clustering Coefficient

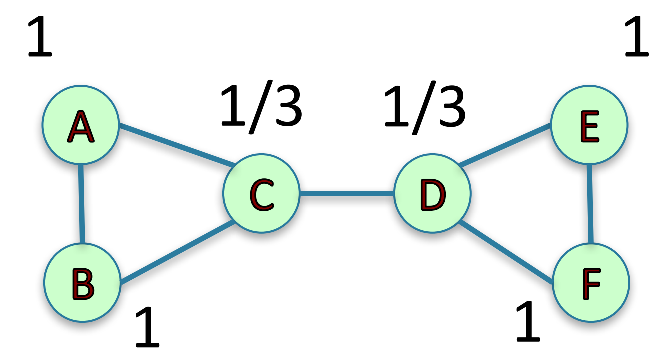
Global clustering coefficient (GCC) of a network:

$$\textbf{Transitivity} = \frac{3 * \text{Number of closed triads}}{\text{Number of open triads}}$$

MLCC = Mean Local Clustering Coefficient (MLCC) over all nodes.

$\text{Transitivity} = 3/5 = 0.6$

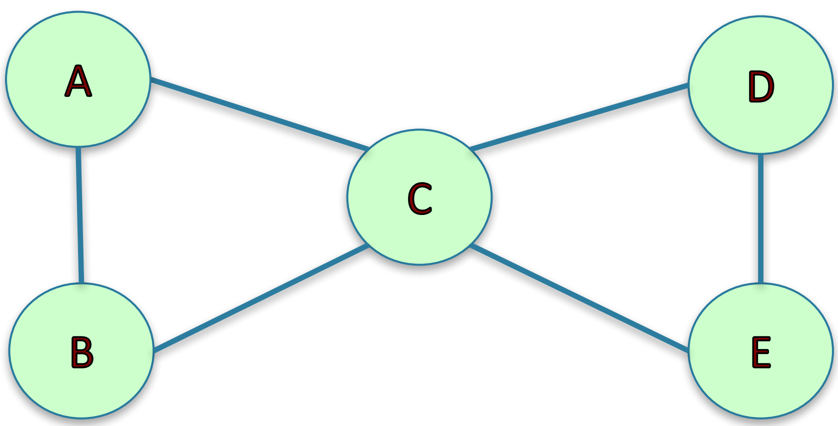
$\text{MLCC} = 7/9 = 0.77$



- Transitivity and MLCC can be very different on the same graph.
- MLCC weights every node equally
- Transitivity gives higher weight to nodes with many connections

Clustering Coefficient

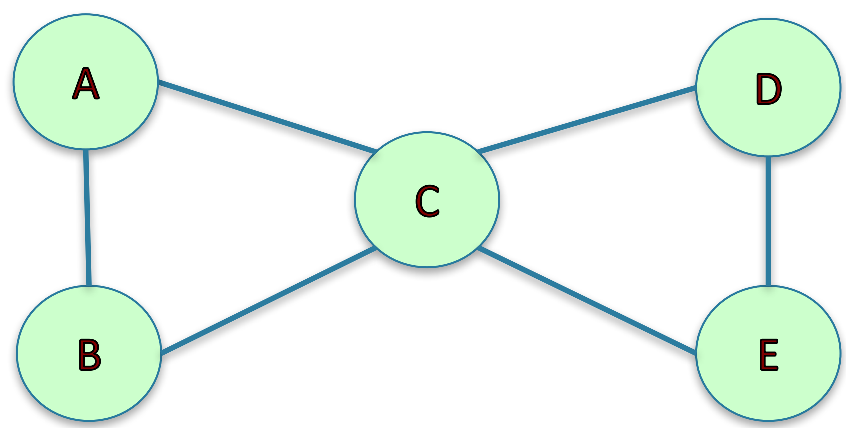
1. Compute the LCC of each node.
2. Compute the transitivity of the network
3. Compute the MLCC of the network.



Clustering Coefficient

What is the LCC of node A?

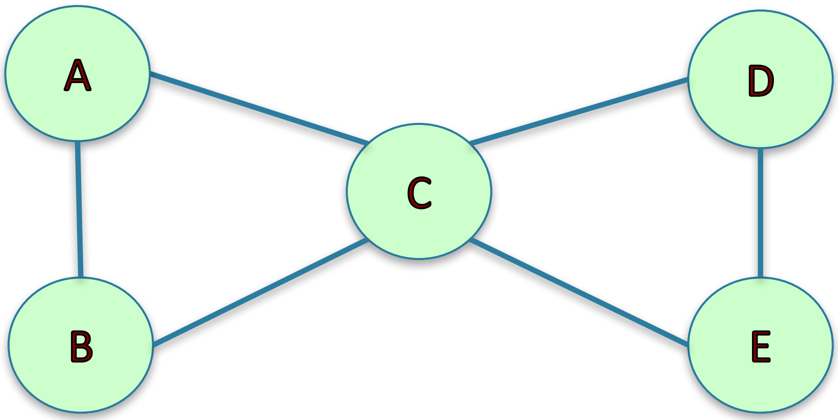
A. 0
B. 1
C. 1/2
D. 1/3



Clustering Coefficient

What is the LCC of node A?

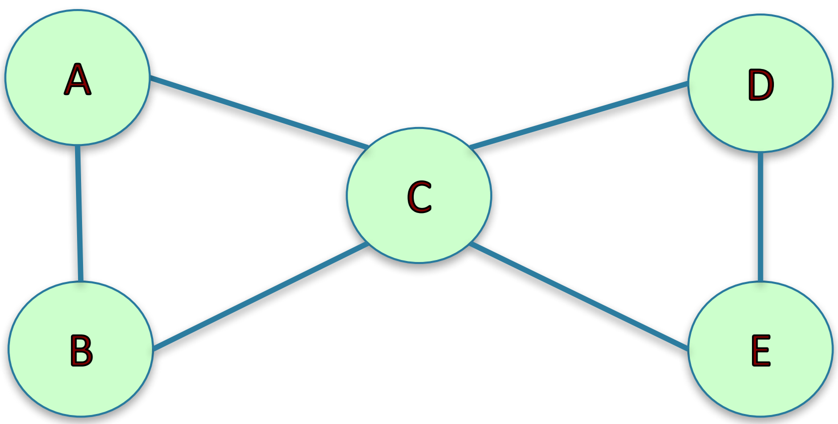
A. 0
B. 1
C. 1/2
D. 1/3



Clustering Coefficient

What is the LCC of node C?

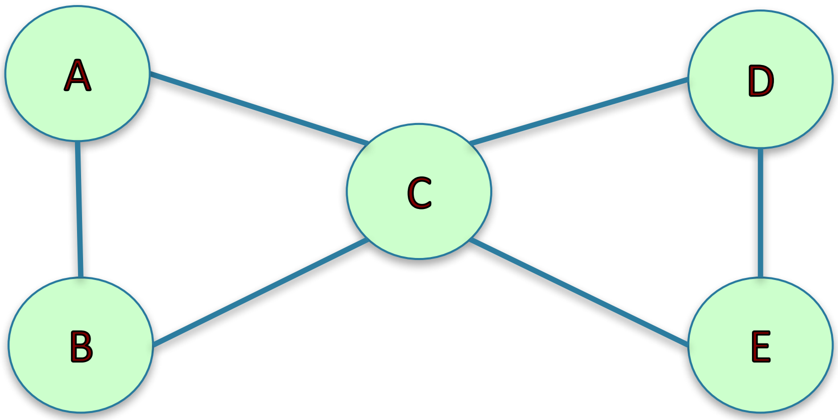
A. 0
B. 1
C. 1/2
D. 1/3



Clustering Coefficient

What is the LCC of node C?

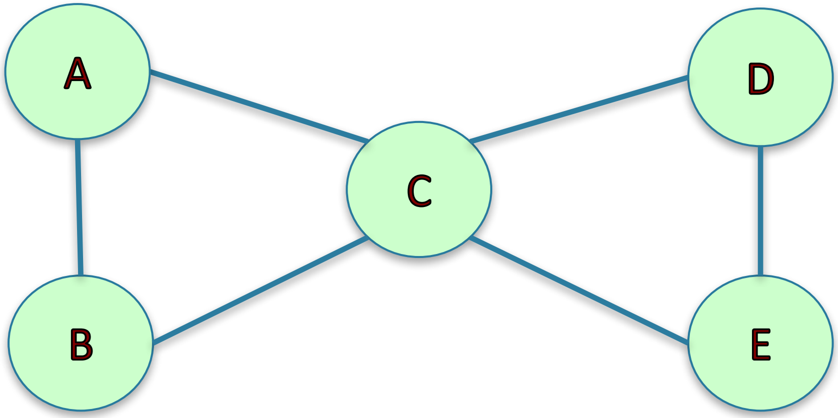
A. 0
B. 1
C. 1/2
D. 1/3



Clustering Coefficient

What is the transitivity of the network?

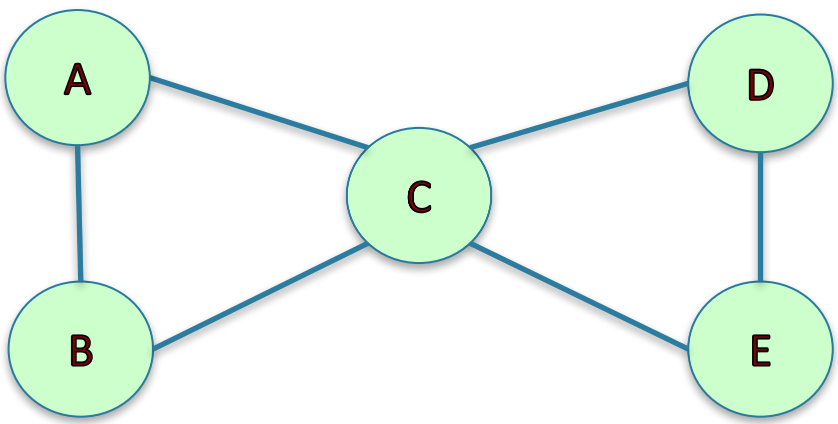
A. 2
B. 1
C. 1/3
D. 3/5



Clustering Coefficient

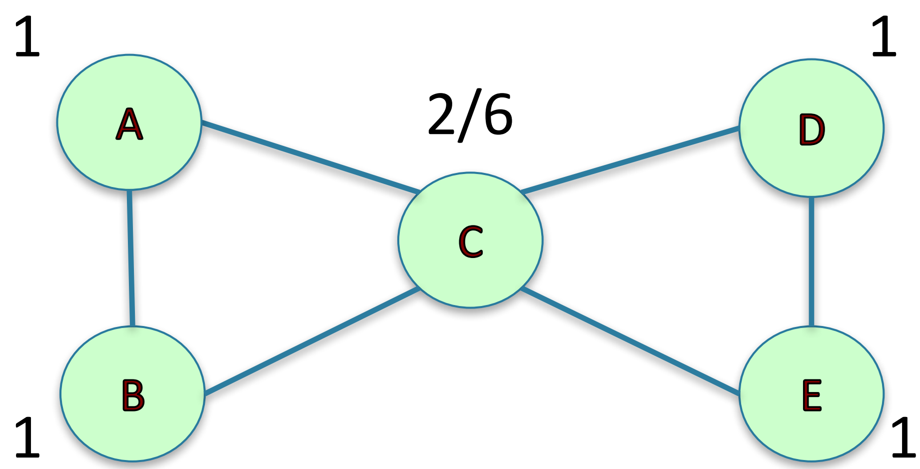
What is the transitivity of the network?

A. 2
B. 1
C. 1/3
D. 3/5



Clustering Coefficient

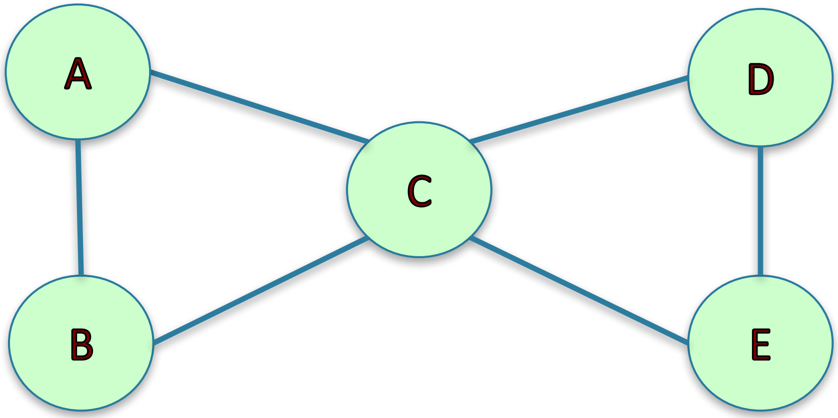
Local Clustering Coefficient



MLCC = Mean of [1,1,1,1,1/3] = 13/15

Clustering Coefficient

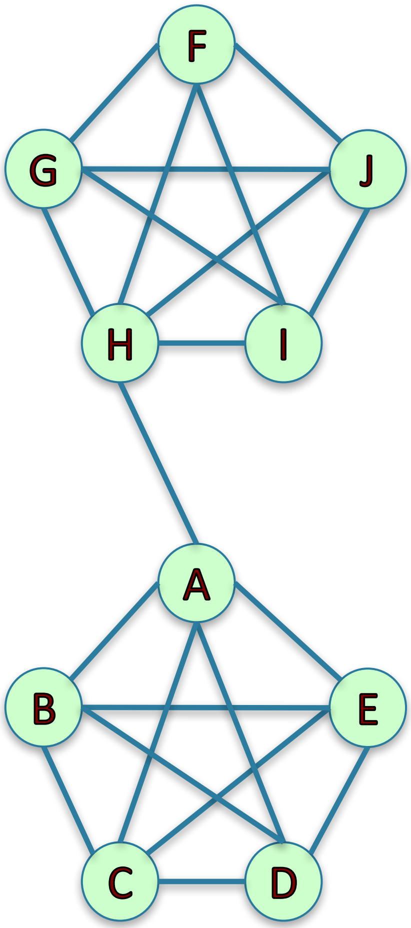
Transitivity



Number of closed triads = 2
Number of open triads = 10
Transitivity = 3*2/10 = 3/5

Bridges

What role does does the edge H-A play in this network?



Information flowing between the groups {A,B,C,D,E} and {F,G,H,I,J} necessarily passes through H-A.

Without H-A, the two groups become disconnected.

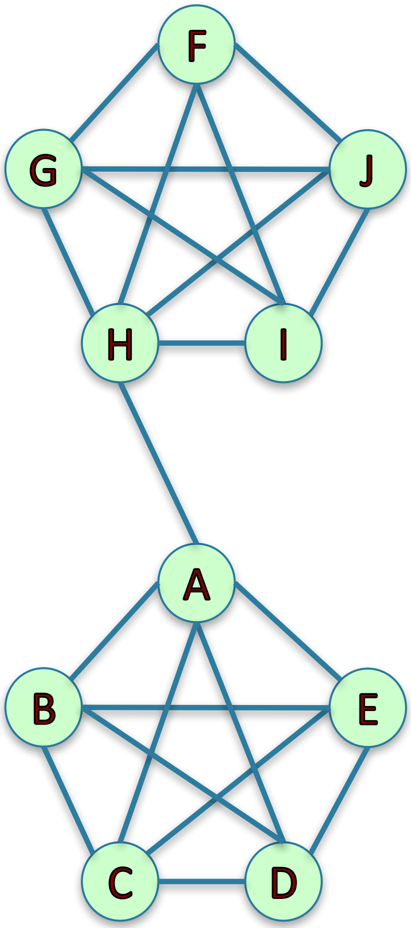
Since H and A belong to different communities, one would expect that their relationship is not very strong.

Bridges

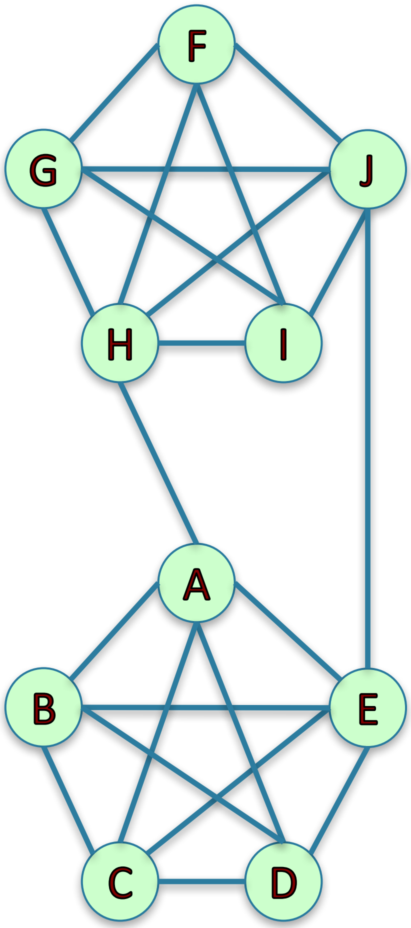
Bridge: An edge connecting two nodes A and B is a bridge if removing it would make A and B lie in different components.

Is the edge H-A a bridge?

Yes, if we remove H-A, then there is no path between H and A, so they lie in different components.



Bridges



Bridge: An edge connecting two nodes A and B is a bridge if removing it would make A and B lie in different components.

Is the edge H-A still a bridge?

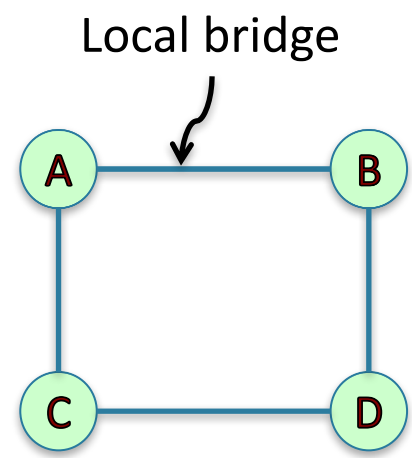
No, if we remove the edge H-A, there still a path between H and A, for example H – I – J – E – A. So H an A are still in the same component.

Are bridges common in social networks?

Probably not. There are usually **multiple** paths between any two people.

Bridges: A more useful definition

Local bridge: An edge connecting two nodes A and B is a local bridge if A and B have no friends in common.



Note:

1. Deleting a local bridge A-B increases the distance between A and B to at least 3.
2. Local bridges and triadic closure are opposite concepts. In networks with very high triadic closure (clustering coefficient), local bridges are rare.

Bridges: A more useful definition

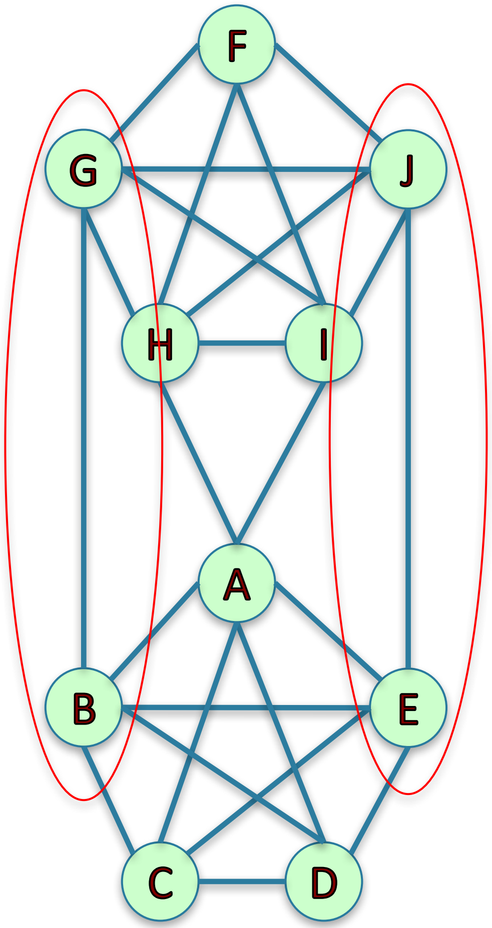
Local bridge: An edge connecting two nodes A and B is a local bridge if A and B have no friends in common.

Is H-A a local bridge?

No, H and A are both friends of I.

Find all local bridges

G-B and J-E

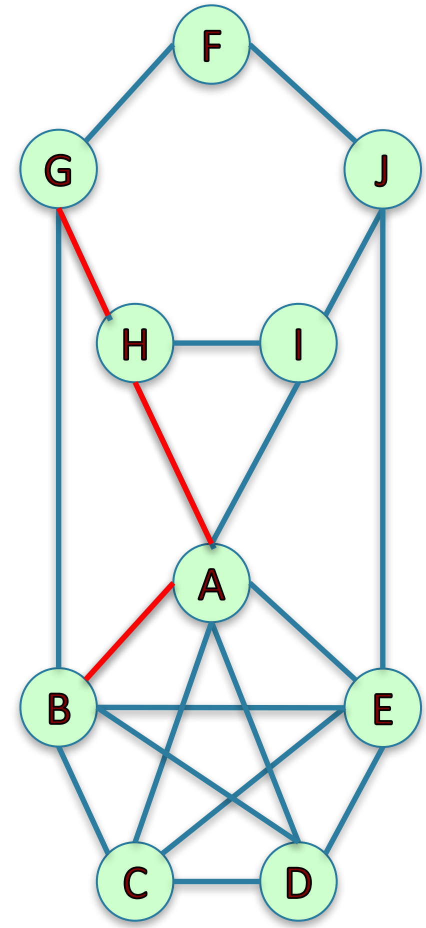


Bridges: A more useful definition

The **span of a local bridge** is the distance between its endpoint after it is removed form the network.

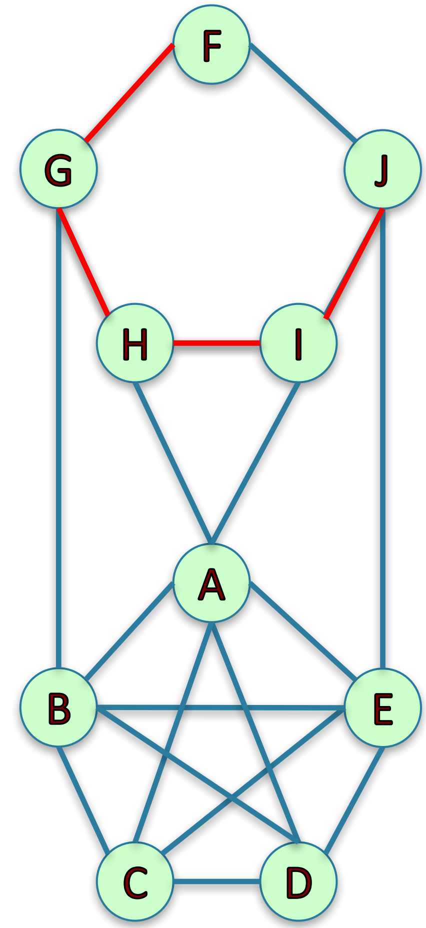
What is the span of the local bridge G-B?

3. The shortest path between G and B is 3 if we remove edge G-B.



Bridges: A more useful definition

The **span of a local bridge** is the distance between its endpoint after it is removed form the network.



What is the span of the local bridge F-J?

4. The shortest path between F and J is 4 if we remove edge F-J.

Strong and Weak Ties

In social networks, some relationships are stronger than others.

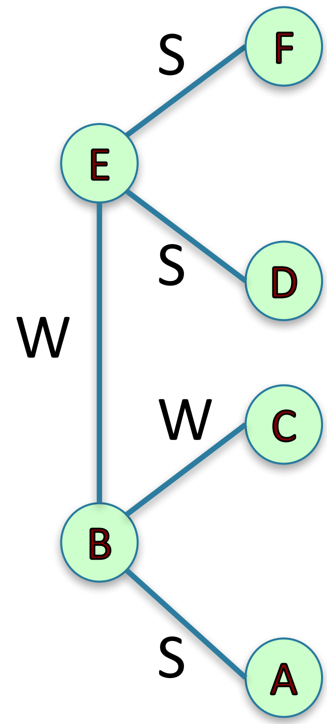
Relationships can have various degrees of strength.

Simplifying assumption: We will assume that edges in a social network can be strong (friends) or weak (acquaintance).

Strong and Weak Ties

Which edge is more likely to form, C-A or F-D?

F-D is more likely to form because both F and D have a **strong** connection with E.
C and A are both connected to B, but C is only **weakly** connected to B.

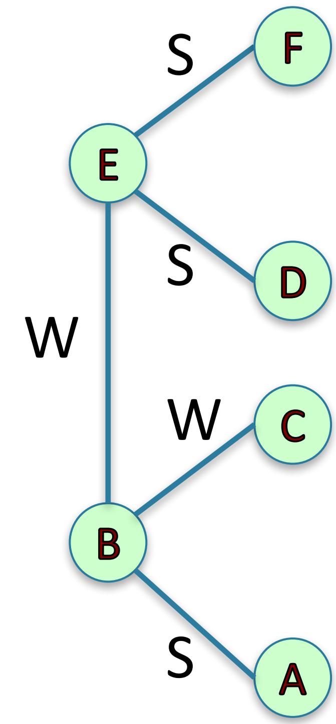


Strong Triadic Closure

Strong Triadic Closure property:

For every node A with strong ties to nodes C and D, there is an edge (weak or strong) between C and D.

Note that node E violates the strong triadic closure property, but B does not.

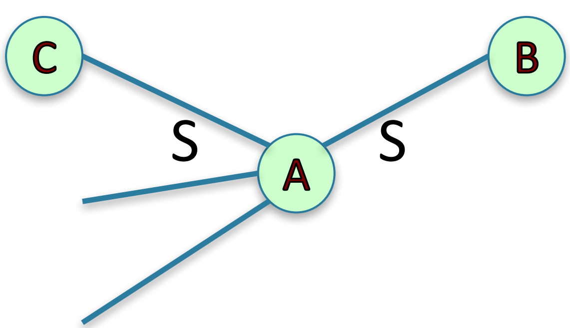


Local Bridges Are (Usually) Weak Ties

Claim: If a node A
1. Is involved in at least two strong ties
2. Satisfies the Strong Triadic Closure (STC) property

Then any local bridge A is involved in must be a weak tie.

Proof by contradiction:
Assume that a node A has a least two strong ties B, C, and possibly others



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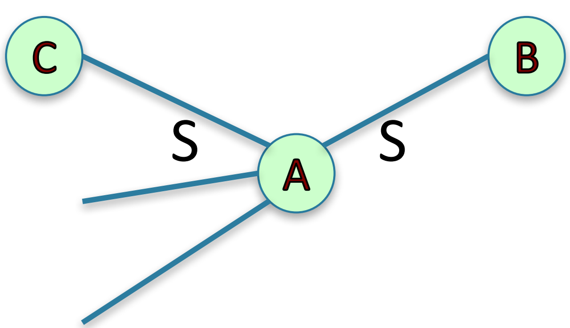
2. Satisfies the Strong Triadic Closure (STC) property

Then any local bridge A is involved in must be a weak tie.

Proof by contradiction:

Assume that a node A has a least two strong ties B, C, and possibly others, satisfies the STC property



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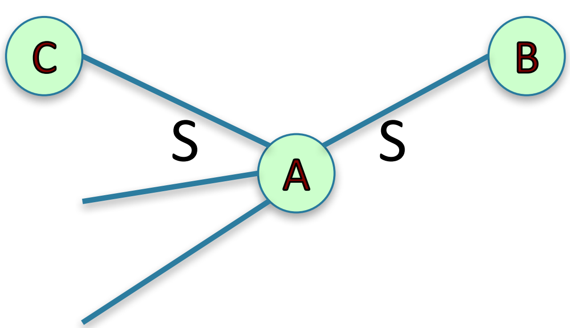
Then any local bridge A is involved in must be a weak tie.

Proof by contradiction:

Assume that a node A has a least two strong ties B, C, and possibly others, satisfies the STC property, and one A's strong ties, say A-B, is a bridge.



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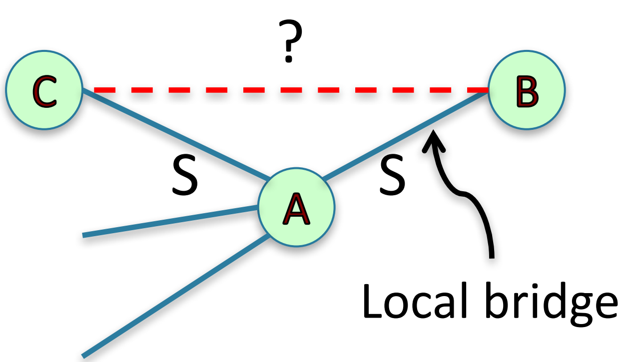


Does edge C-B exist?

Yes, because A satisfies STC

No, Because A-B is a local bridge

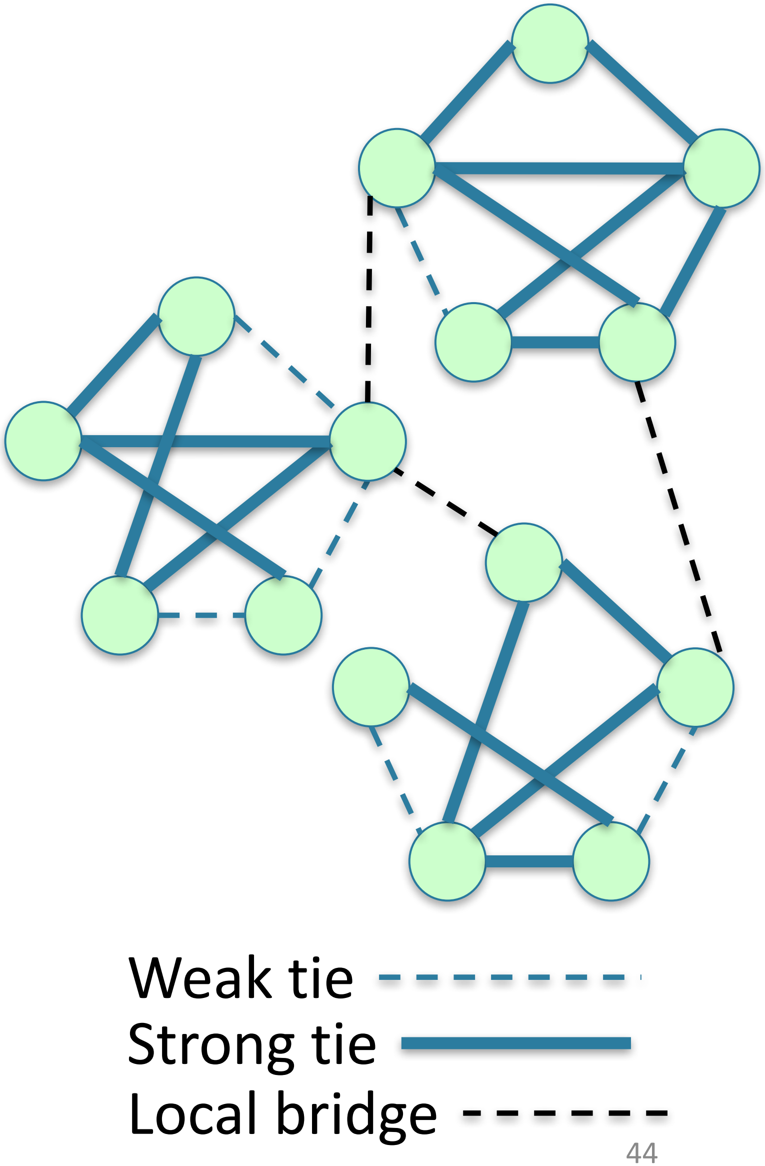
Contradiction!

The Strength of Weak Ties

We have created a model that helps us understand Granovetter's findings:

1. Strong Triadic Closure produces highly clustered groups of people.
2. Local bridges allow new and useful information to flow across these clusters.
3. These local bridges tend to be weak ties.

Aha! People were probably finding out about jobs through local bridges



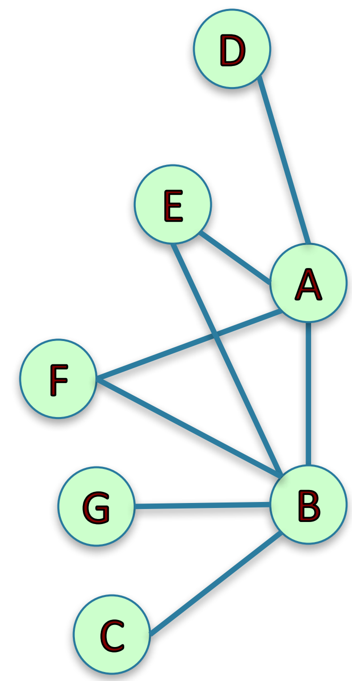
Relaxed Measure of Local Bridge

Is A—B a local bridge?

No, A and B have friends in common.

How close is A—B to being a local bridge?

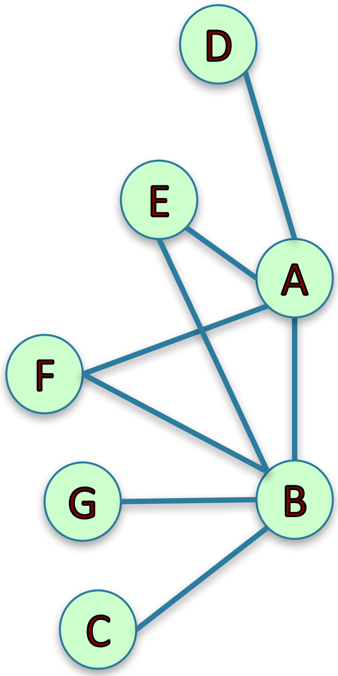
If we remove nodes E and F, then A—B would be a local bridge.



Relaxed Measure of Local Bridge

Neighborhood overlap of an edge A—B:

$$\frac{\text{Number of neighbors of both A and B}}{\text{Number of neighbors of either A or B}}$$



Relaxed Measure of Local Bridge

Neighborhood overlap of an edge A—B:

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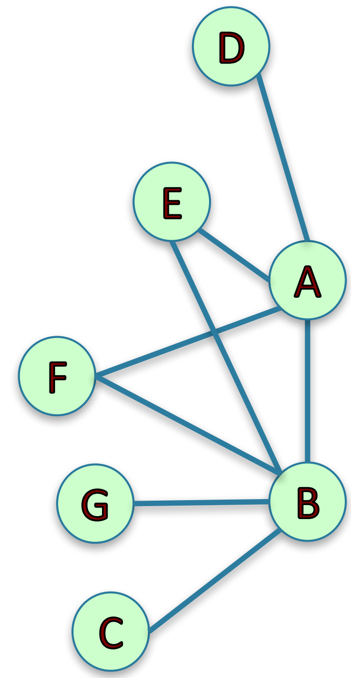
What's the neighborhood overlap of A – B?



Relaxed Measure of Local Bridge

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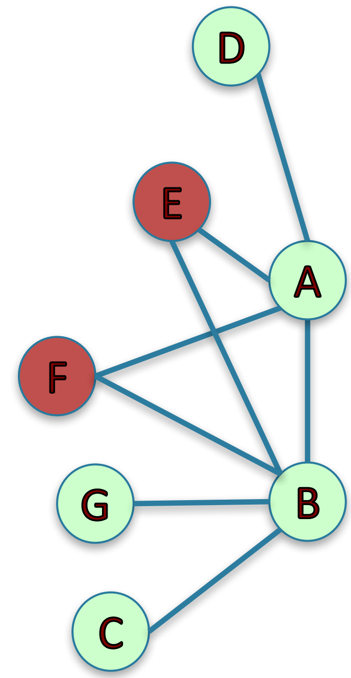
2/



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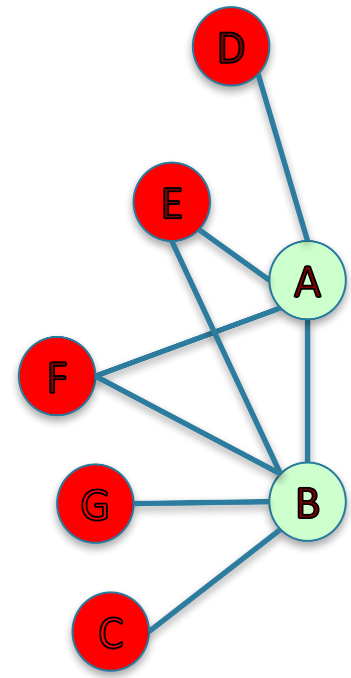
Neighborhood overlap of an edge A—B:

$$\frac{\text{Number of neighbors of both A and B}}{\text{Number of neighbors of either A or B}}$$

What's the neighborhood overlap of A – B?

2/5

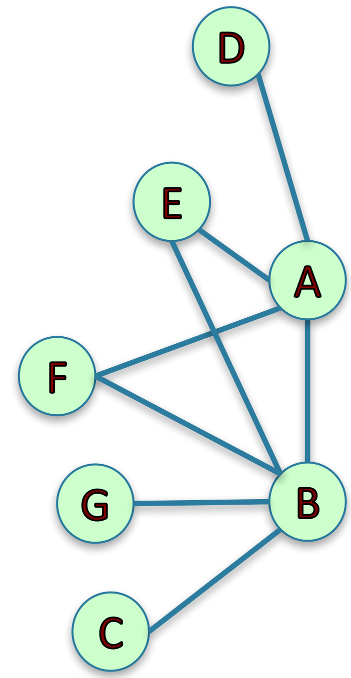


Relaxed Measure of Local Bridge

Neighborhood overlap of an edge A—B:

$$\frac{\text{Number of neighbors of both A and B}}{\text{Number of neighbors of either A or B}}$$

If an edge X – Y is a local bridge, what is its neighborhood overlap?



Relaxed Measure of Local Bridge

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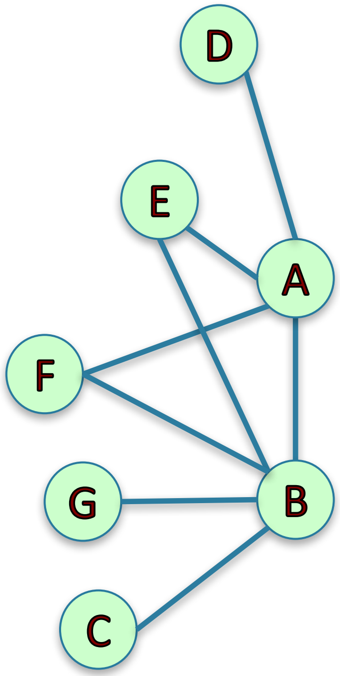


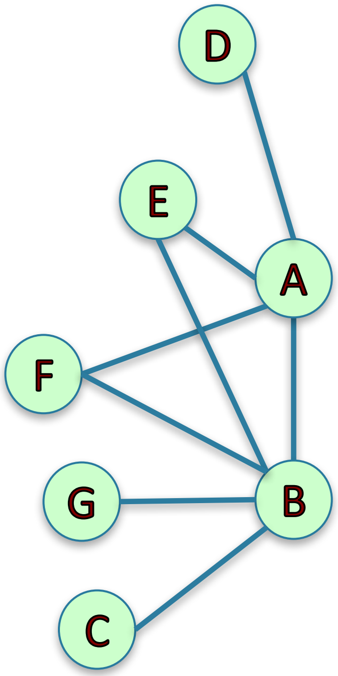
If an edge X – Y is a local bridge, what is its neighborhood overlap?

0. Since X – Y is a local bridge, then no node can be friends of X and Y.

Relaxed Measure of Local Bridge

Neighborhood overlap of an edge A—B:

$$\frac{\text{Number of neighbors of both A and B}}{\text{Number of neighbors of either A or B}}$$



Note: The larger the neighborhood overlap of an edge, the further away it is from being a local bridge.

Neighborhood over lap and local bridging are opposite concepts.

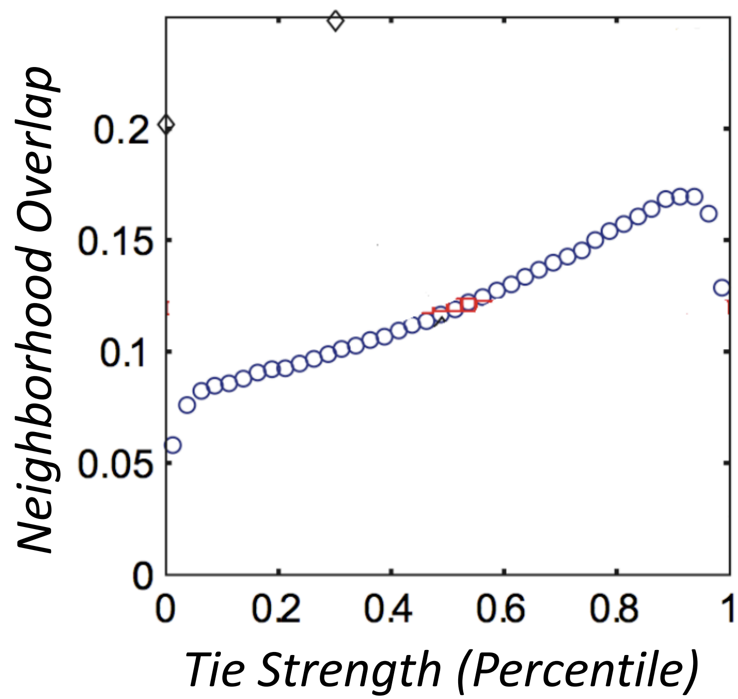
Are local bridges really weak ties?

Network:

- Nodes: people. 20% of a national population
- Edges: reciprocal cell phone calls over an 18-week period.

Measures:

- Tie strength: number of minutes spent on phone calls
- Neighborhood overlap



[Onnela et al., 2007]

Results:

Tie strength increases with neighborhood overlap.
Edges that were closer to being a local bridge were weaker.

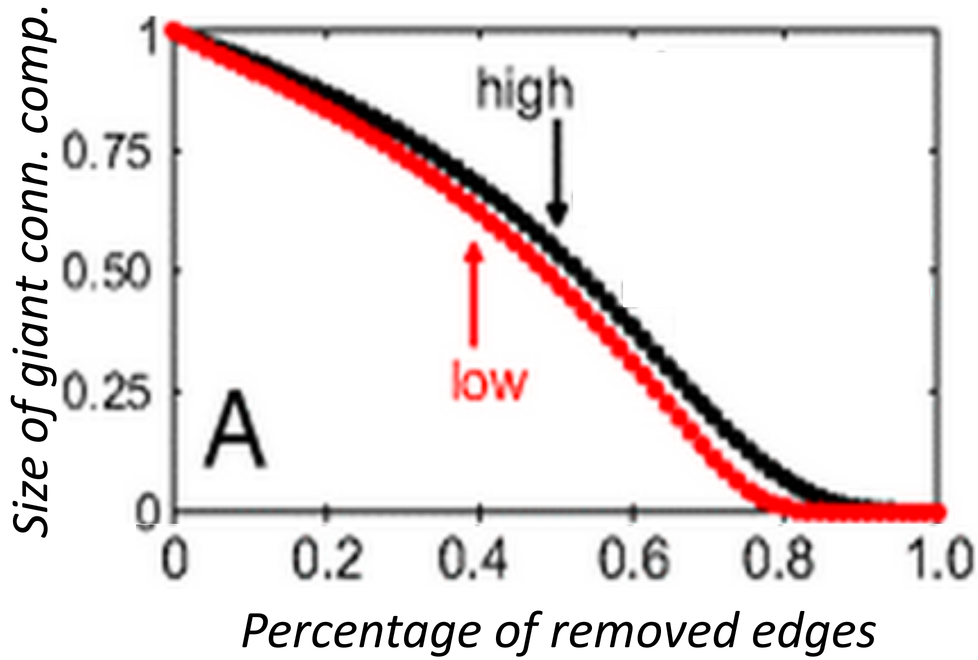
Do weak ties really hold clustered communities together?

Test:

Remove edges one by one. See what happens to the size of the giant component.

Two ways:

- Weaker ties first (low)
- Stronger ties first (high)



Results:

- When weaker ties are removed first, the size of the giant connected component decreases faster.
- Weak ties seem more crucial for connectivity. Consistent with the idea that weak ties hold communities together.

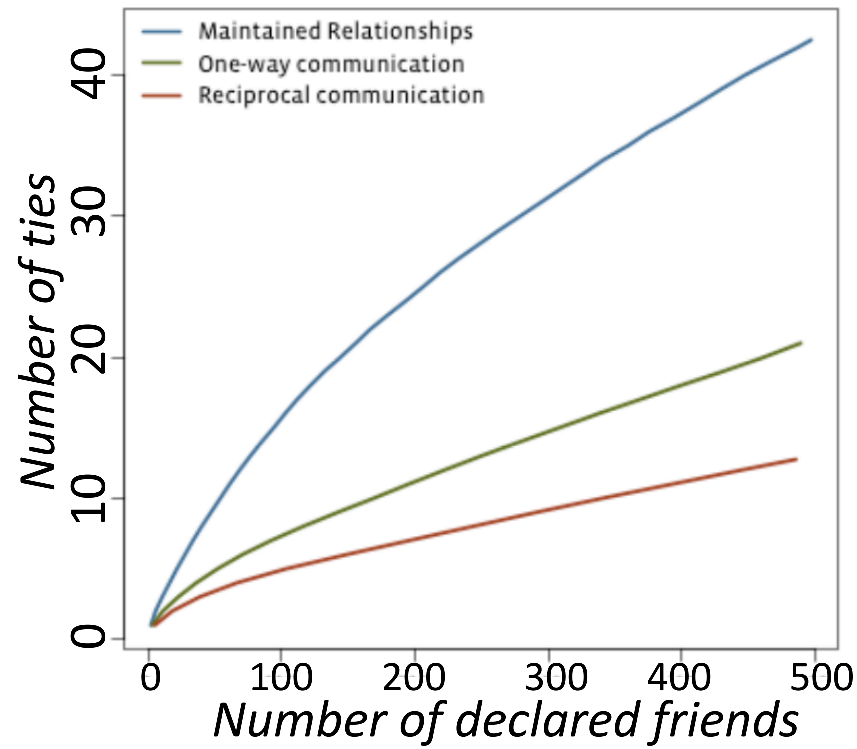
Weak and Strong Ties in Social Media

Types of ties on Facebook:

One-way communication: A sent at least one message to B.

Reciprocal communication: A sent messages to B and B replied to some.

Maintained relationship: A visited B's profile or clicked on B's content.



[Marlow et al., 2009]

Results:

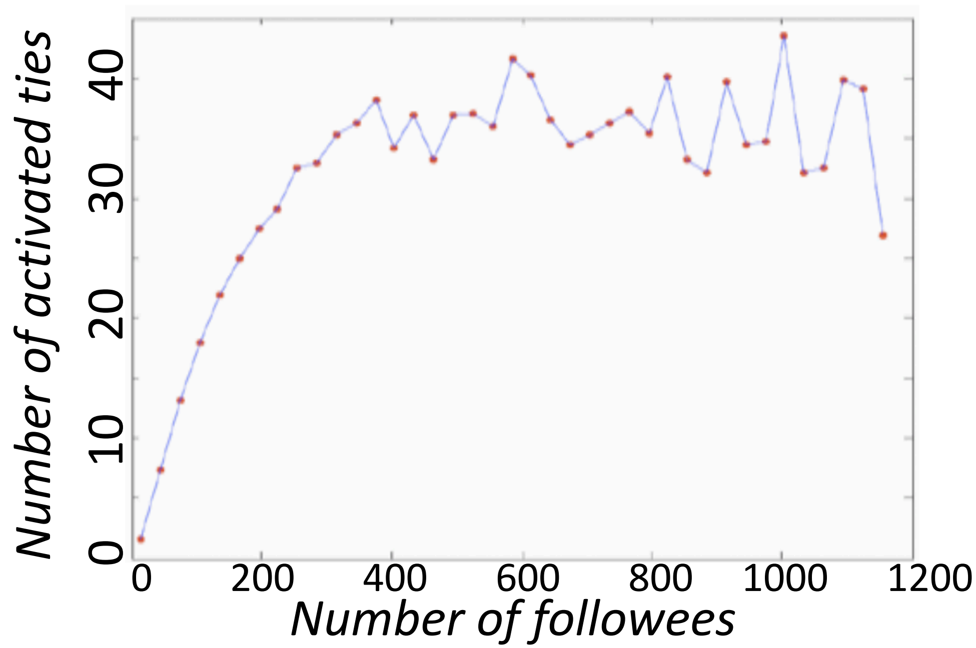
- Interaction with a very small percentage of declared friends.
- Most interactions happen *passively* through content consumption and profile visits, not communication.

Weak and Strong Ties in Social Media

Types of ties on Twitter:

Followees (weak): The set of people a user follows.

Activated (strong): The set of people a user sends at least 2 @-messages.



[Huberman - Romero - Wu, 2009]

Results:

- Weak ties can grow very large, but strong ties stay below 50.
- Strong ties require effort and time to maintain. They tend to be less common than weak ties offline as well as online.