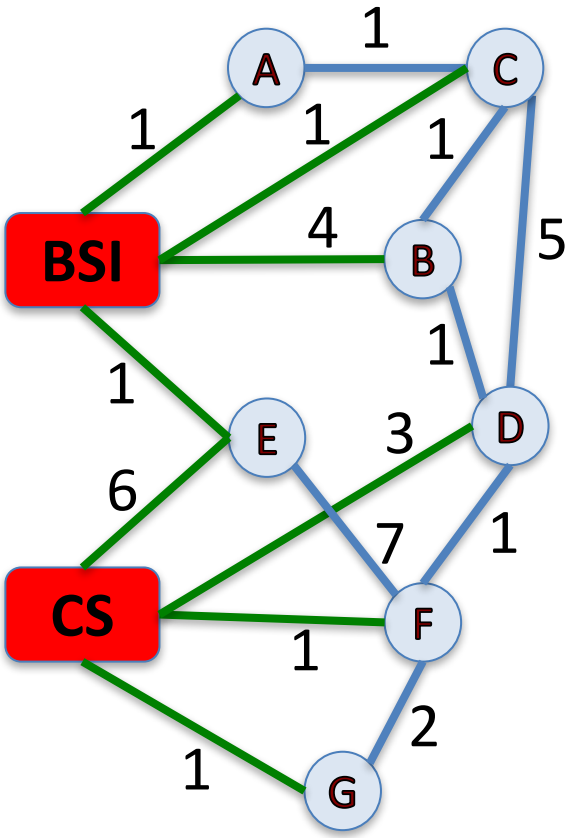
Last Time

- Affiliation network
- Social affiliation network
- Closure in social affiliation network
  - Triadic closure
  - Group closure
  - Membership closure
- Signed networks
  - Balanced and unbalanced complete networks

Warm Up

Consider the shown social-affiliation network. Every edge has a number next to it. These numbers represent the order in which the edges appeared in the network.

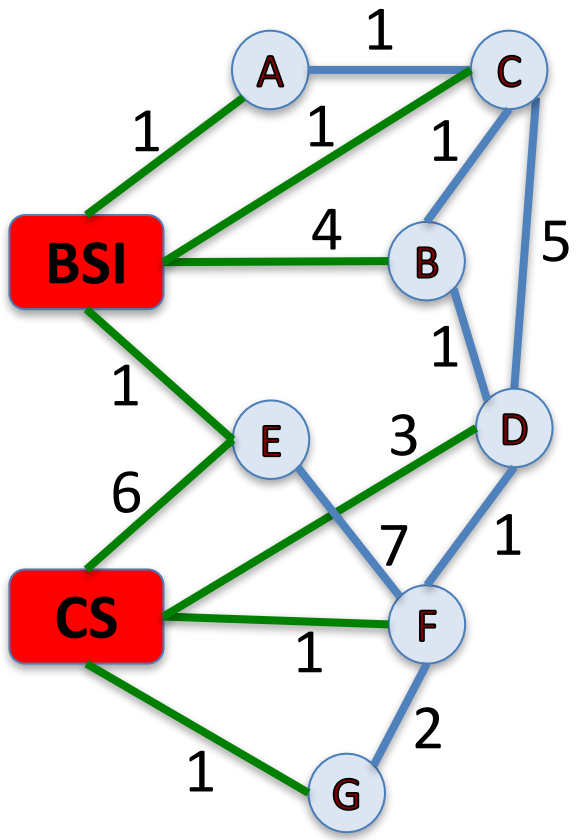
For each edge that arrived after time 1, say whether it is an example of *(i) triadic closure, (ii) group or focal closure, or (iii) membership closure*.



Warm Up

1. Edge G—F is an example of:

   A. Triadic closure
   B. Group closure
   C. Membership closure
   D. Friendship closure
   E. None of the above



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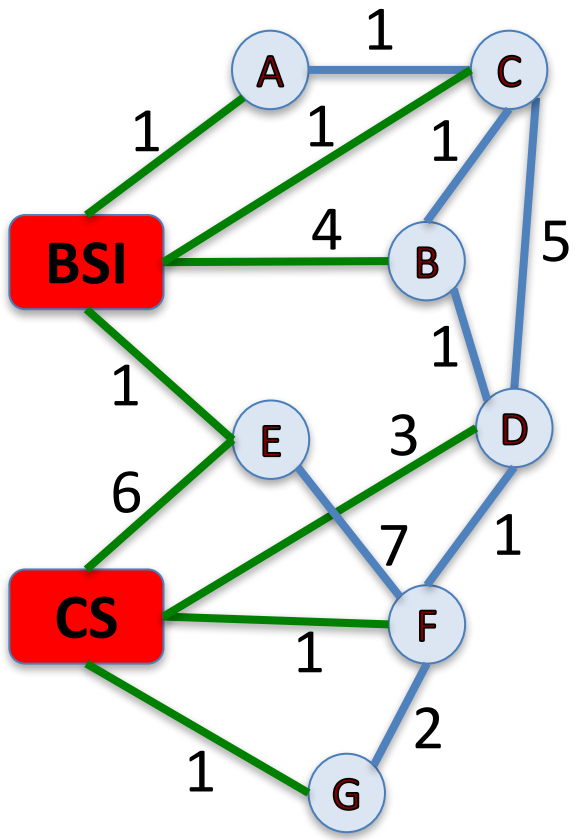
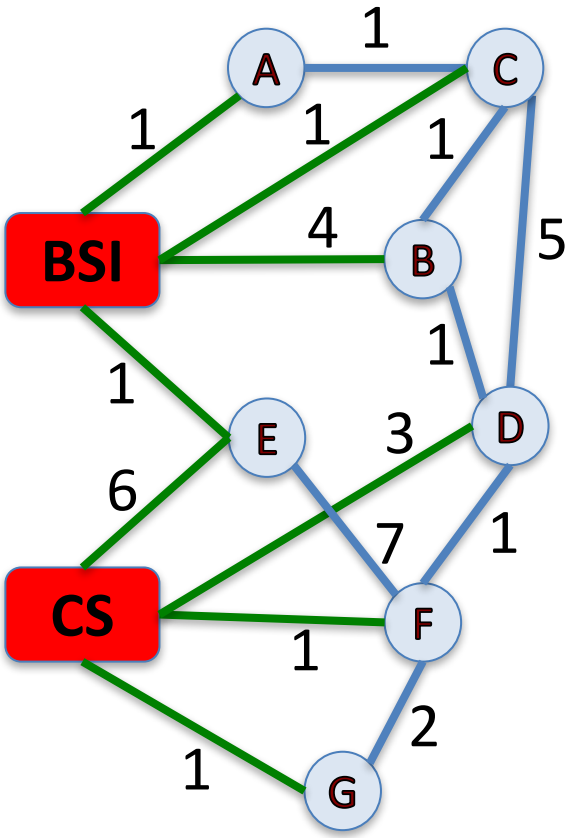


Warm Up

2. Edge C—D is an example of:

   A. Triadic closure
   B. Group closure
   C. Membership closure
   D. Friendship closure
   E. None of the above



Warm Up

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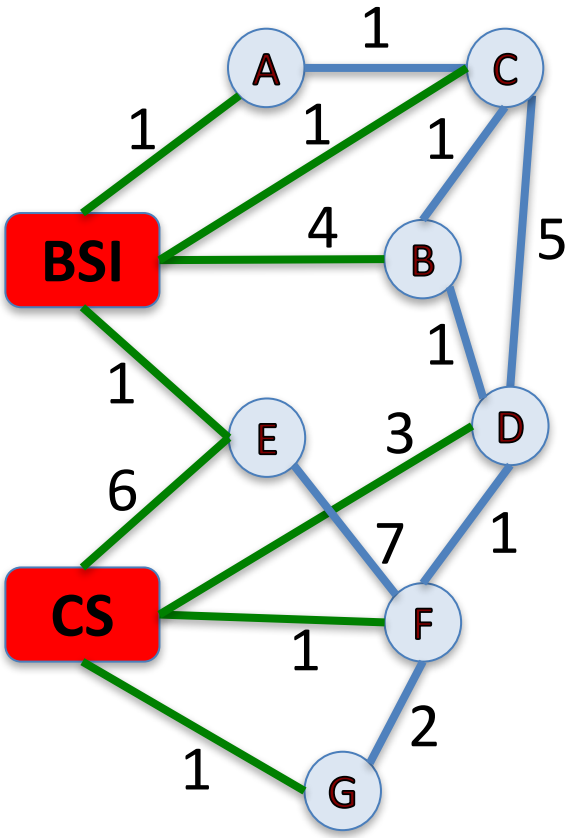
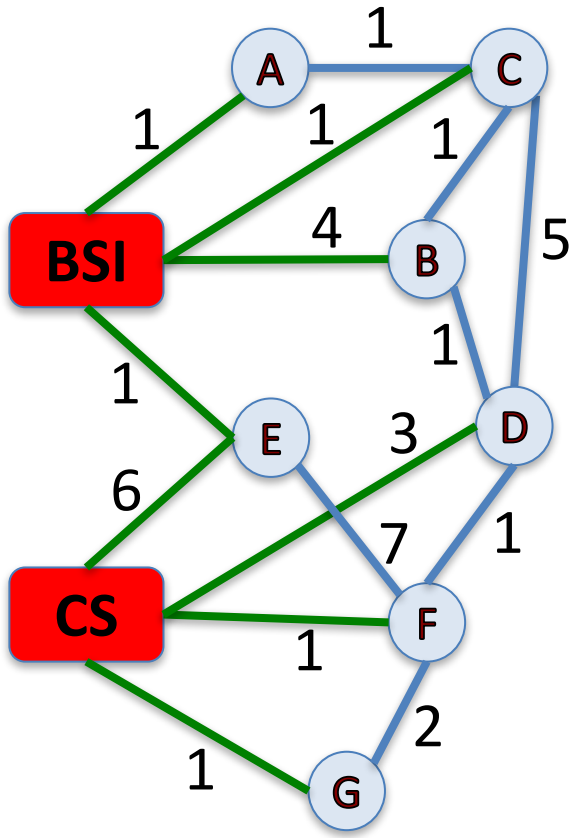
   E. None of the above



Warm Up

3. Edge BSI—B is an example of:

   A. Triadic closure
   B. Group closure
   C. Membership closure
   D. Friendship closure
   E. None of the above



Warm Up

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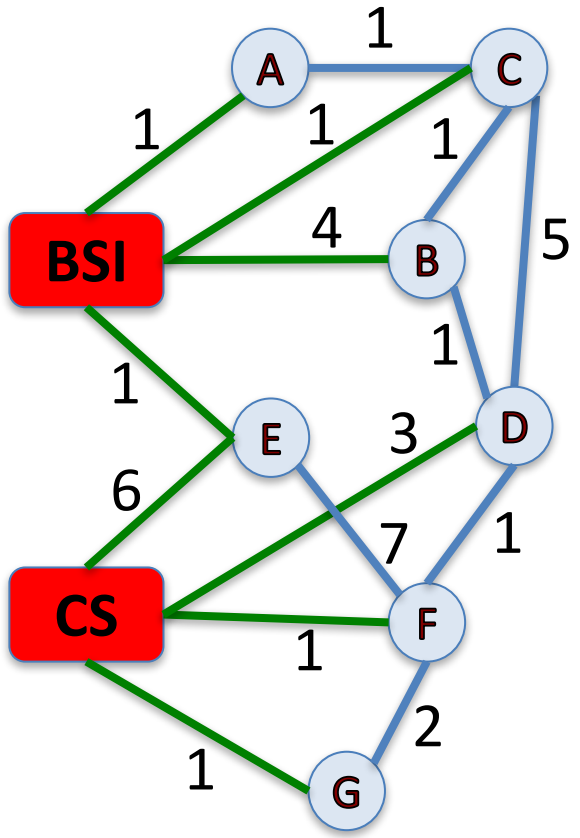
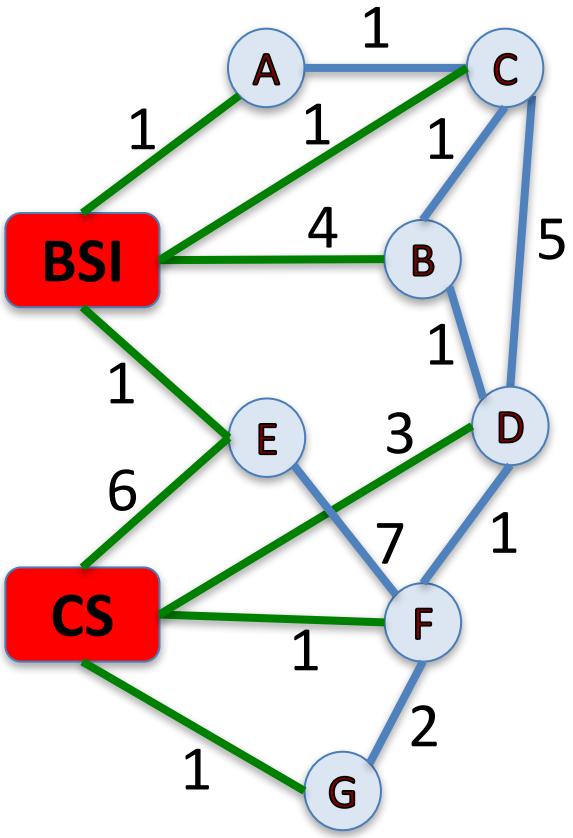


Warm Up

4. Edge CS—E is an example of:

   A. Triadic closure
   B. Group closure
   C. Membership closure
   D. Friendship closure
   E. None of the above



Warm Up

4. Edge CS—E is an example of:

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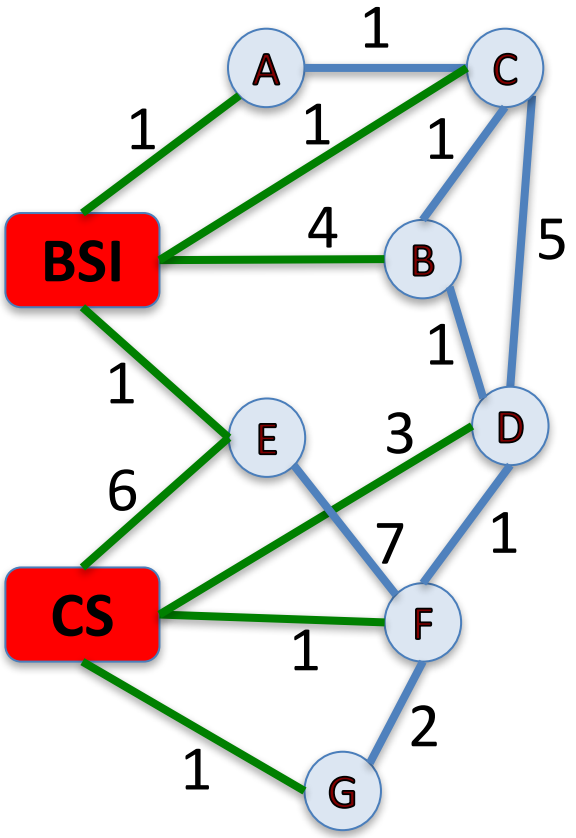


Structural Balance

Balance Theorem:

A labeled complete graph is balanced, if and only if

1. either all pairs of nodes are friends,
2. or all the nodes can be divided into two groups, X and Y, such that all pairs of nodes in X are friends, all pairs of nodes in Y are friends, and every node in X is the enemy of every node in Y.

Structural Balance

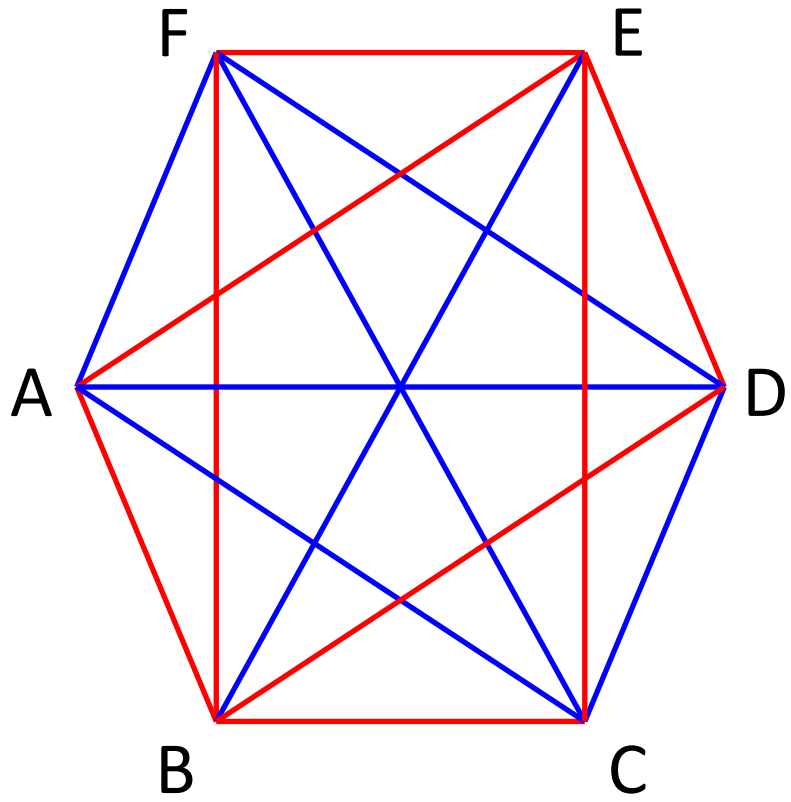
Friendship: Blue, Antagonism: Red

Is this network balanced?

Rather than checking each triangle, try to find groups with positive edges within groups and negative edges across.

Group 1: {F, A, C, D}
Group 2: {B, E}

Yes, the network is balanced



Structural Balance

Balance Theorem (Hard direction):
If a labeled complete graph is balanced, then

1. either all pairs of nodes are friends,
2. or all the nodes can be divided into two groups, X and Y, such that all pairs of nodes in X are friends, all pairs of nodes in Y are friends, and every node in X is the enemy of every node in Y.

Proof:

Proof of Balance Theorem

For any complete graph, one of the following must be true:

1. There are no negative edges.
2. At least one negative edge is present.

If there are no negative edges, then all nodes are friends and there is nothing to prove.

Proof of Balance Theorem

If at least one negative edge is present, we want to *construct* two groups of nodes that satisfy the property stated in the theorem.

How to construct the two groups?

Proof of Balance Theorem

- Pick any node with a negative edge (let's call it A) in the balanced network. Put A in X.

- Go through every other node. If it is A's friend, put it in X. If it is A's enemy, put it in Y.

- Now we need to verify:
  1. Nodes in X are mutual friends.
  2. Nodes in Y are mutual friends.
  3. Nodes across two groups are enemies.

Proof of Balance Theorem

- Pick any node with a negative edge (let's call it A) in the balanced network. Put A in X.
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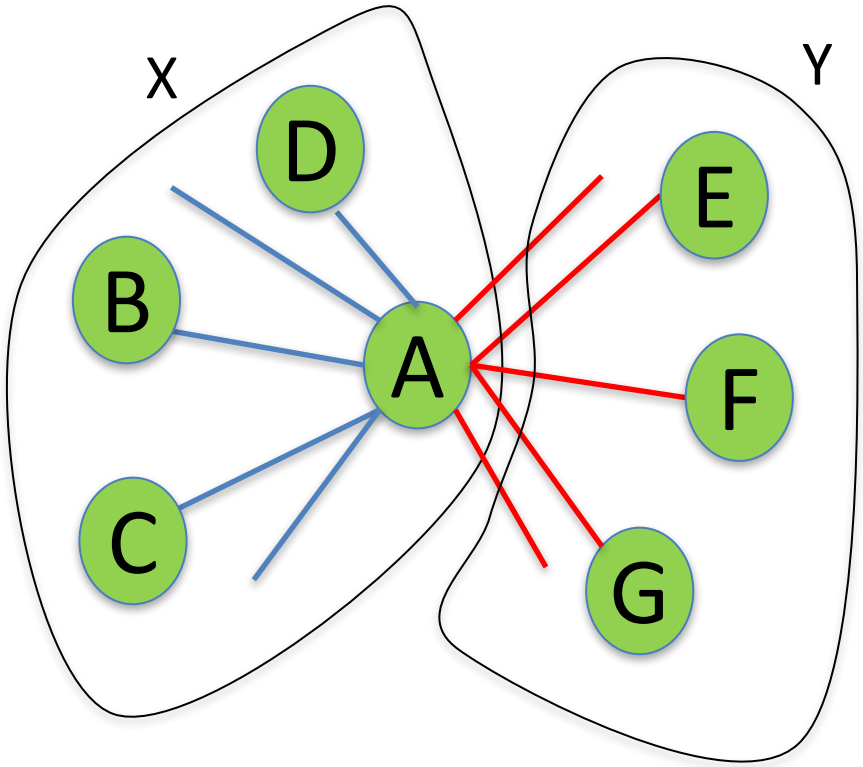

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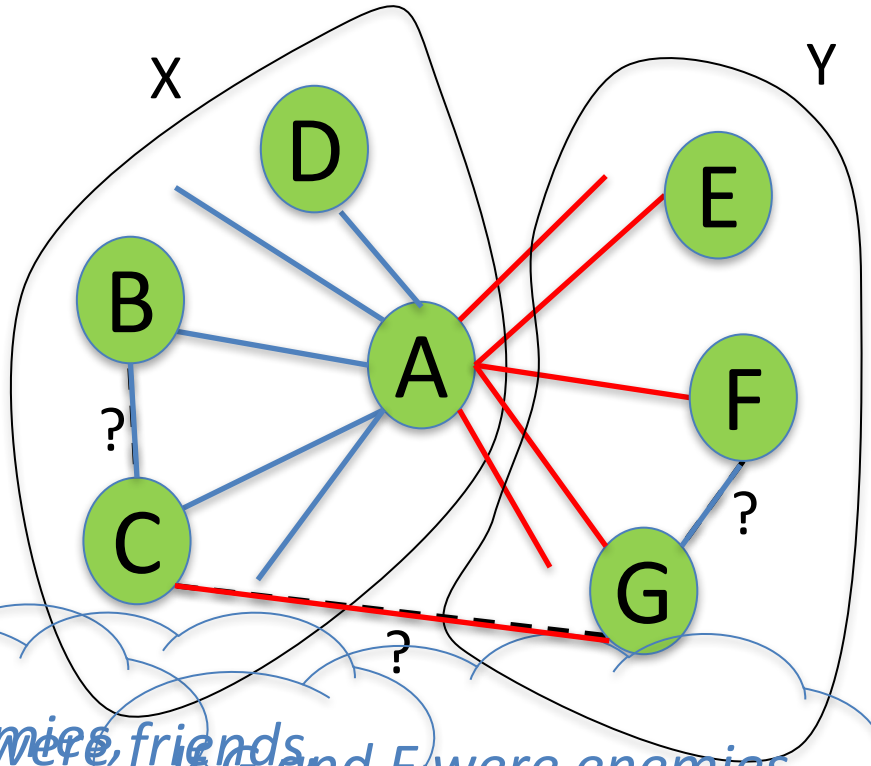


If B and C were friends, triangle A—B—C would be unbalanced

If C and G were friends, triangle C—A—G would be unbalanced

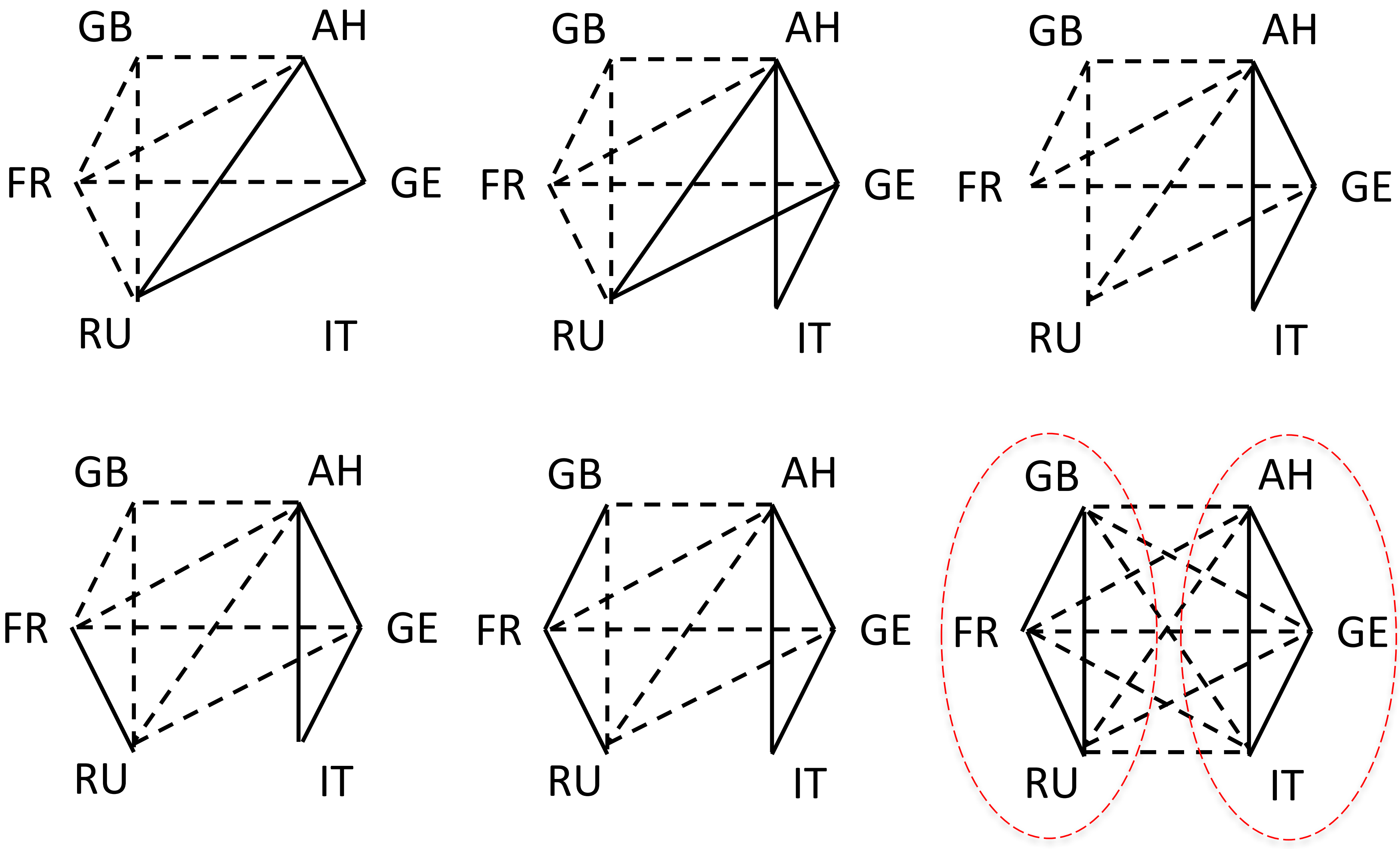
If G and F were enemies, triangle A—F—G would be unbalanced

Applications

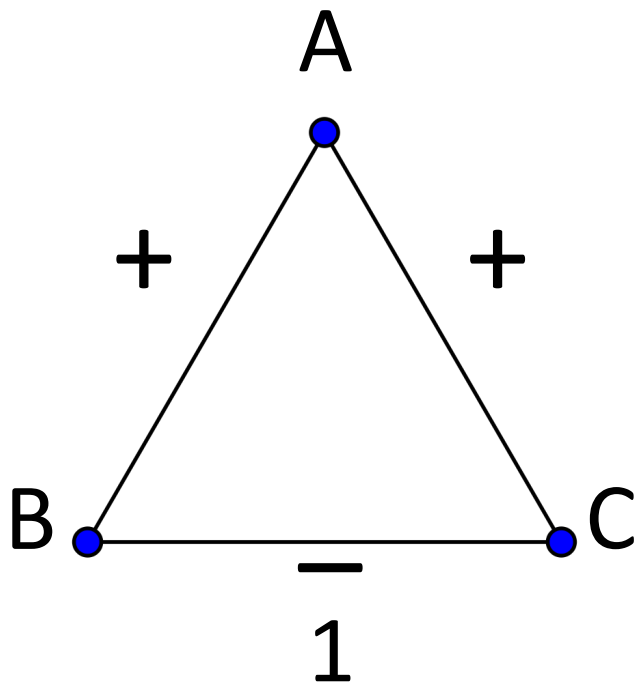
- Our discussion treats structural balance as a static property.
- However, people commonly reassess their friends and enemies and the structural balance changes dynamically.
- International relations

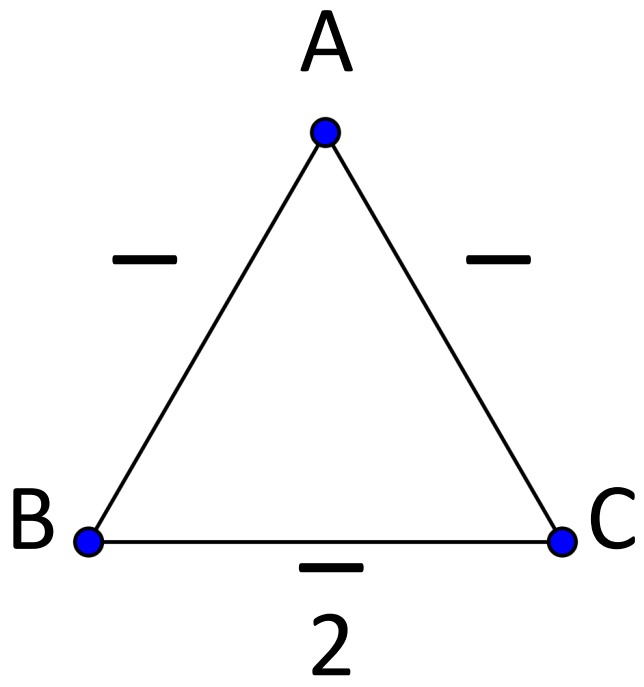

Europe Pre-World War I
NORWAY
Oslo
SWEDEN
Stockholm
St. Petersburg
DENMARK
Copenhagen
North Sea
Baltic Sea
RUSSIA
GREAT BRITAIN
London
NETH.
Amsterdam
Berlin
GERMANY
Brussels
BELGIUM
LUX.
Paris
ATLANTIC OCEAN
FRANCE
Bern
SWITZ.
Vienna
AUSTRIA-HUNGARY
ROMANIA
Bucharest
Belgrade
SERBIA
Sofia
BULGARIA
Black Sea
Constantinople (Istanbul)
PORTUGAL
Madrid
SPAIN
ANDORRA
ITALY
MONTENEGRO
Rome
Tirane
ALBANIA
OTTOMAN EMPIRE
GREECE
Athens
Mediterranean Sea
N
0
400 Miles
0
800 Kilometers
8°W
0°
8°E
16°E
24°E
32°E
40°E
48°E
50°N
42°N
34°N

International Relations



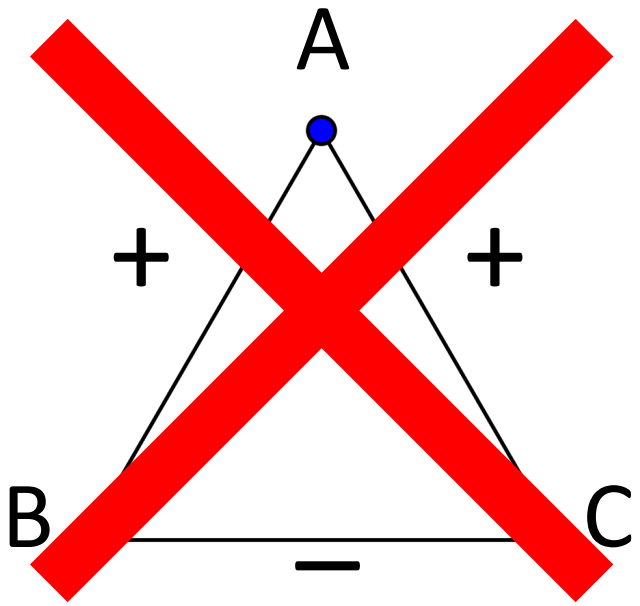
Weak Structural Balance





Weak Structural Balance

Weak Structural Balance Property:
There is *no* set of three nodes such that the edges among them consist of exactly two positive edges and one negative edge.



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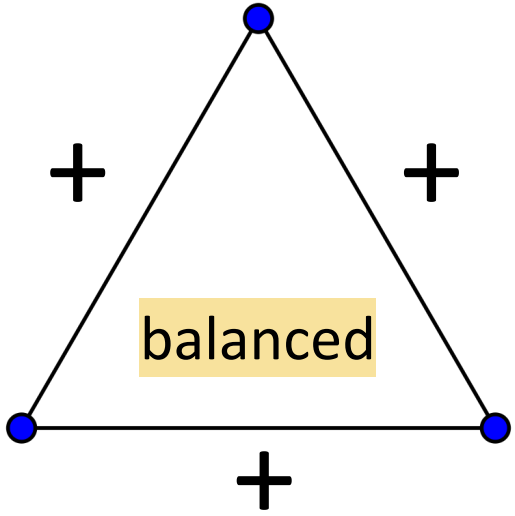
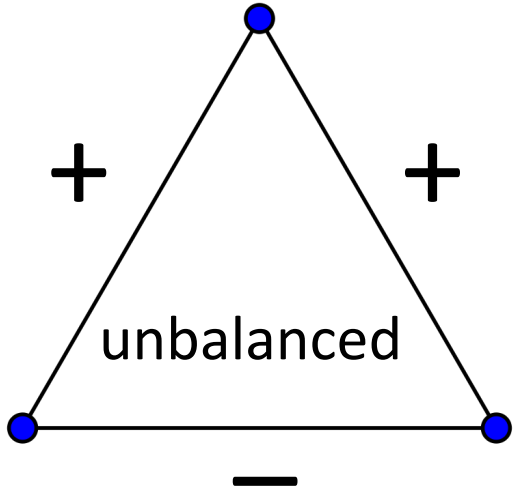
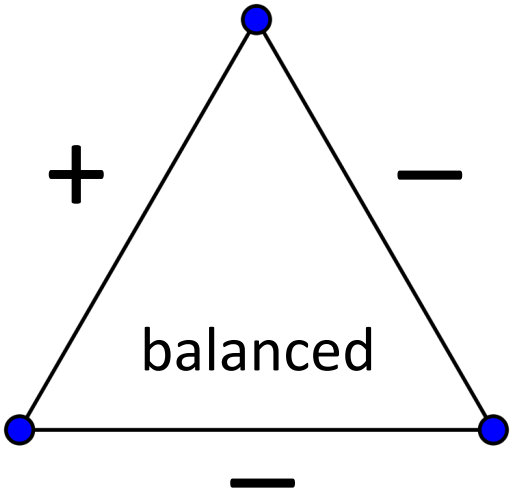
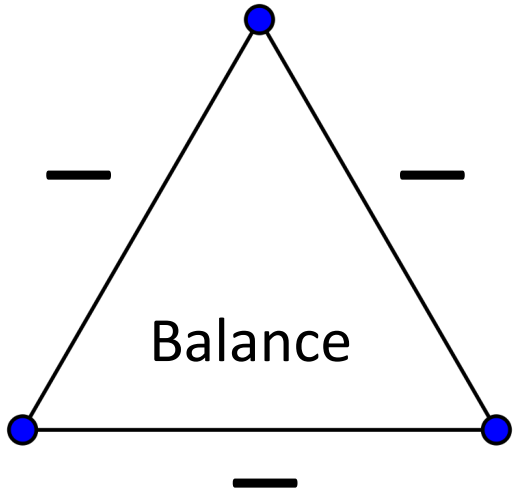


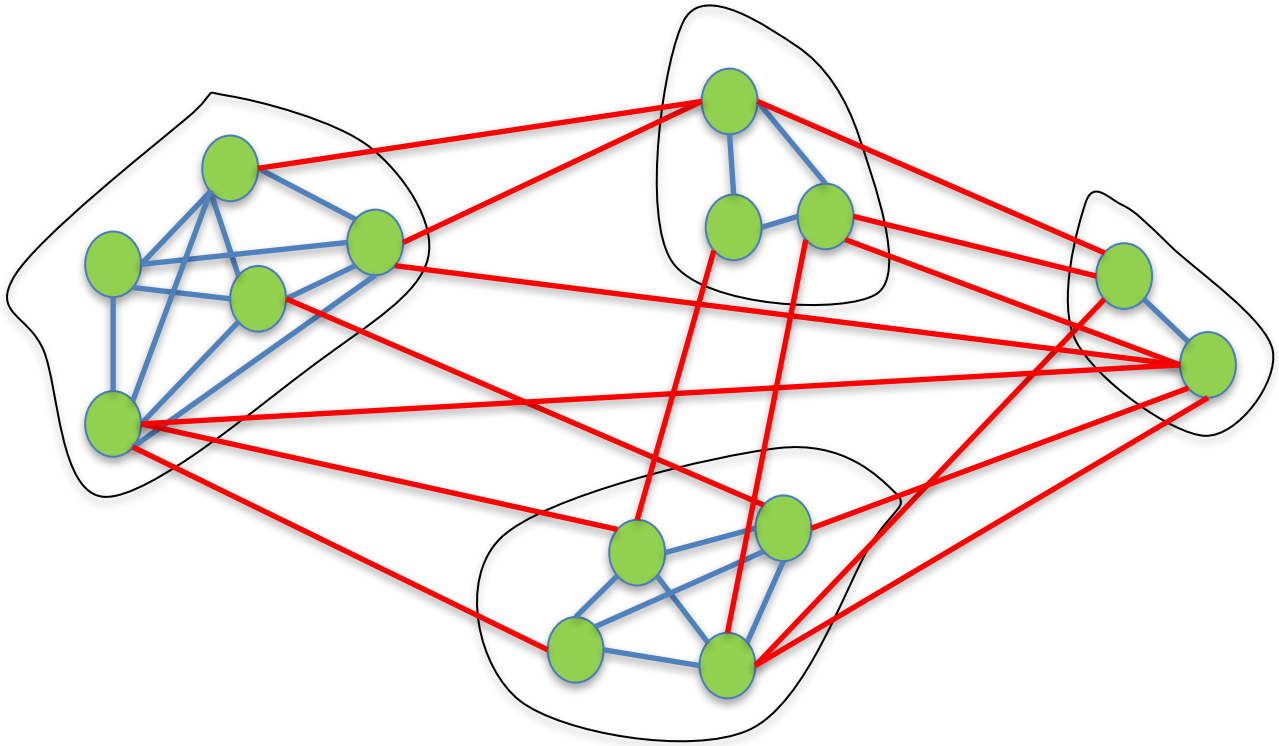






Weaker Structural Balance

Weak Balance Theorem:
If a labeled complete graph is weakly balanced, its nodes can be divided into groups in such a way that every two nodes belonging to the same group are friends, and every two nodes belonging to different groups are enemies.



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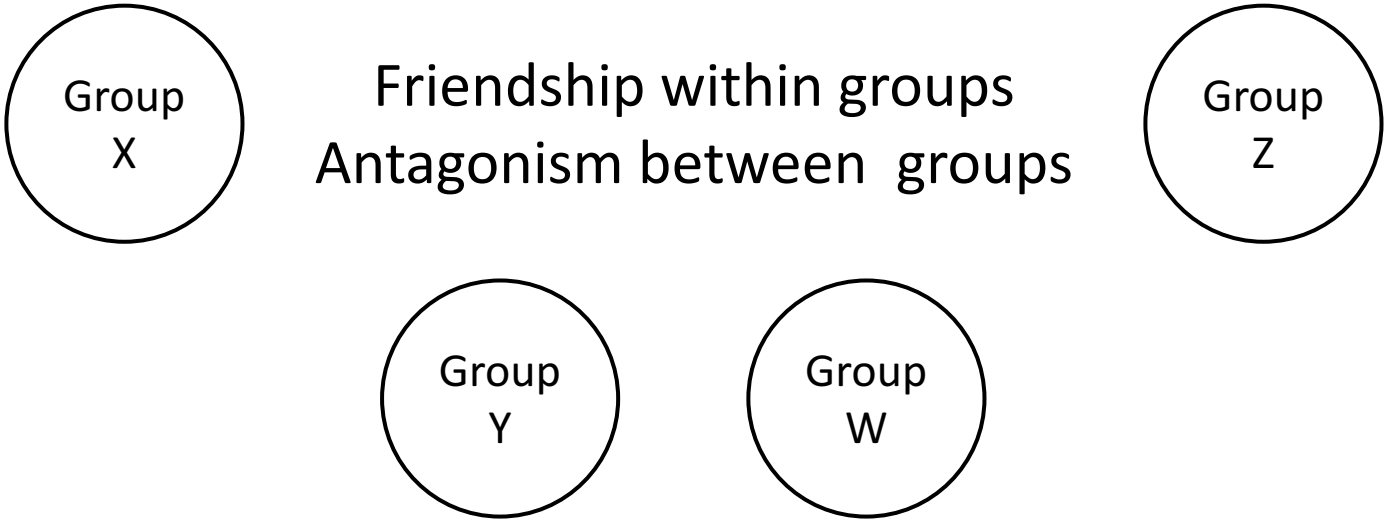
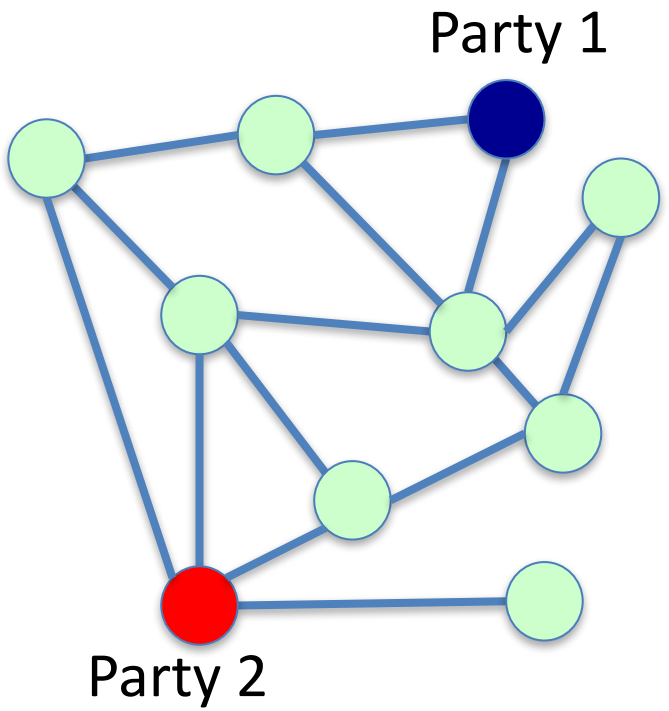


See proof in section 5.4 of textbook

Game Theory

Game Theory

Each person wants to go to the party where most of her friends are going.



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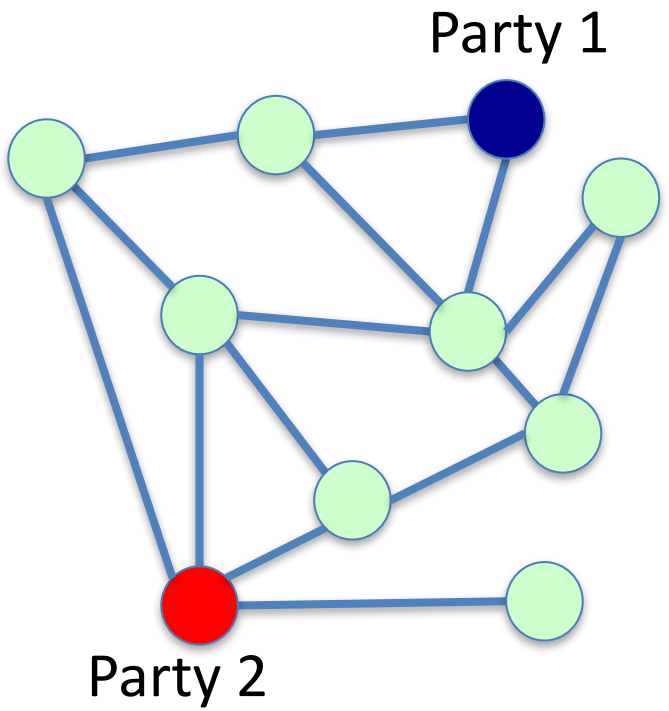
Each person decides which party to go to simultaneously – no coordination possible.



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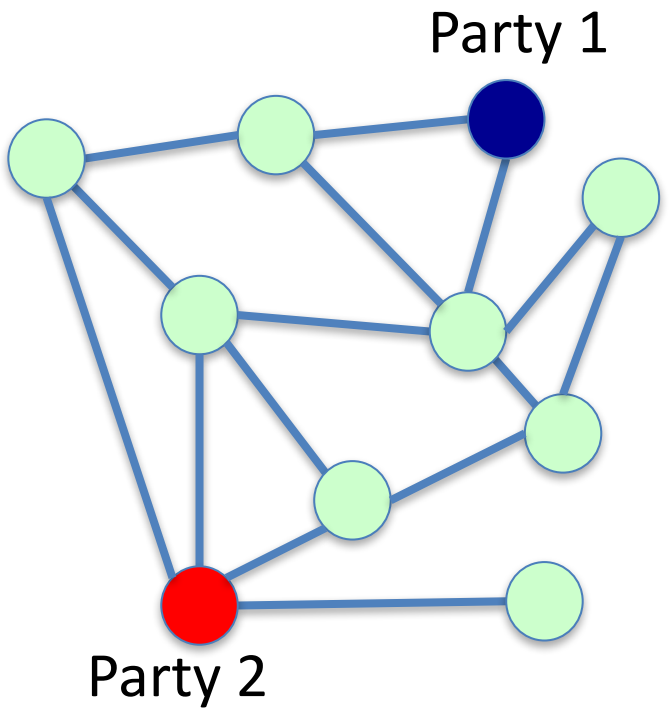
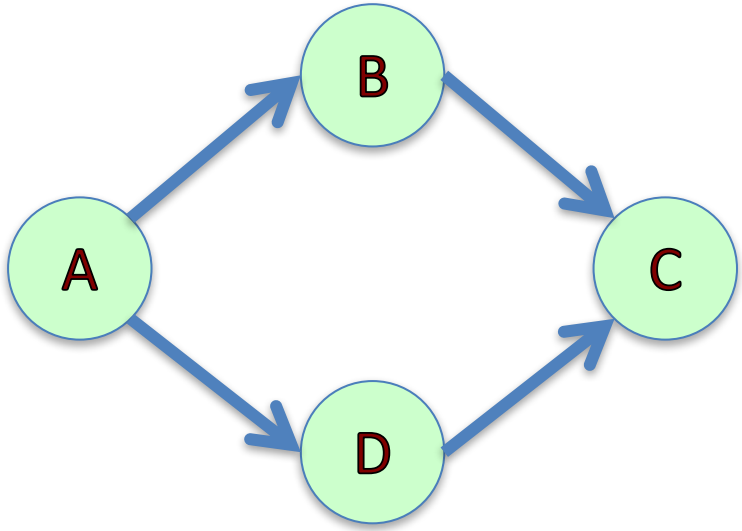


Game Theory

People want to travel from A to C.

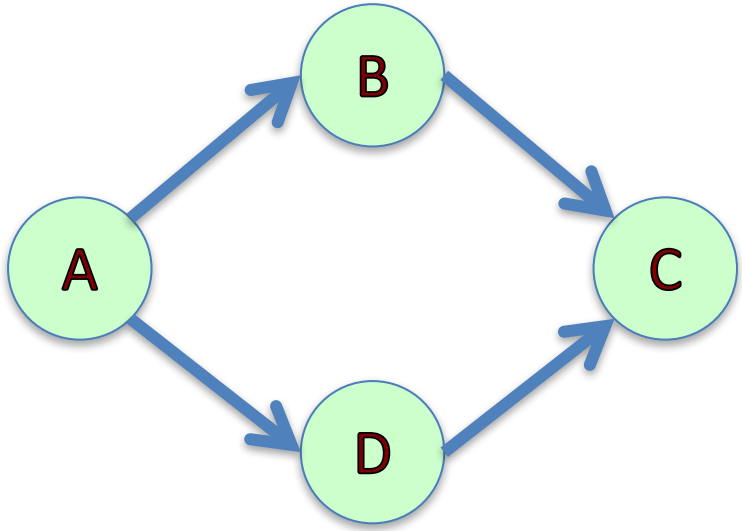


Highway network

Game Theory

People want to travel from A to C.

Each person must decide between taking route A-B-C or A-D-C. Simultaneous decision – no coordination



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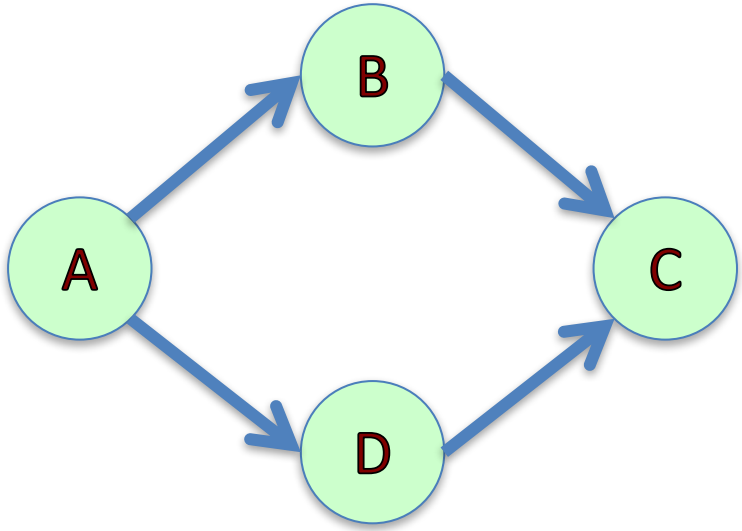
The route chosen by the smallest number of people is the fastest.



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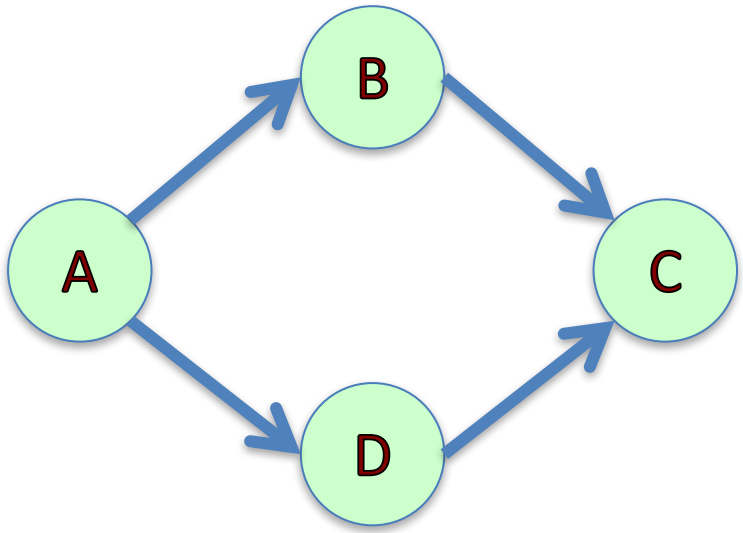

Highway network

Presentation vs. Exam

- Presentation with a partner and exam tomorrow.

Presentation vs. Exam

- Presentation with a partner and exam tomorrow.
- You and your partner only have time to prepare for one.

Presentation vs. Exam

- Presentation with a partner and exam tomorrow.
- You and your partner only have time to prepare for one.
- Exam:
  - If you study for the exam you will get a 92
  - If you don’t study you will get an 80.

Presentation vs. Exam

- Presentation with a partner and exam tomorrow.
- You and your partner only have time to prepare for one.
- Exam:
  - If you study for the exam you will get a 92
  - If you don’t study you will get an 80.
- Presentation:
  - If both you and your partner prepare you get 100
  - If only one of you prepares you get 92
  - If neither of you prepare you get 84

Presentation vs. Exam

- Presentation with a partner and exam tomorrow.
- You and your partner only have time to prepare for one.
- Exam:
  - If you study for the exam you will get a 92
  - If you don’t study you will get an 80.
- Presentation:
  - If both you and your partner prepare you get 100
  - If only one of you prepares you get 92
  - If neither of you prepare you get 84
- You cannot coordinate with your partner

Presentation vs. Exam

You study for	Your Partner studies for	Your exam grade	Your presentation grade	Your average
Exam	Exam			
Exam	Presentation			
Presentation	Exam			
Presentation	Presentation			

Presentation vs. Exam

You study for	Your Partner studies for	Your exam grade	Your presentation grade	Your average
Exam	Exam	92	84	
Exam	Presentation			
Presentation	Exam			
Presentation	Presentation			

Presentation vs. Exam

You study for	Your Partner studies for	Your exam grade	Your presentation grade	Your average
Exam	Exam	92	84	88
Exam	Presentation			
Presentation	Exam			
Presentation	Presentation			

Presentation vs. Exam

You study for	Your Partner studies for	Your exam grade	Your presentation grade	Your average
Exam	Exam	92	84	88
Exam	Presentation	92	92	92
Presentation	Exam			
Presentation	Presentation			

Presentation vs. Exam

You study for	Your Partner studies for	Your exam grade	Your presentation grade	Your average
Exam	Exam	92	84	88
Exam	Presentation	92	92	92
Presentation	Exam	80	92	86
Presentation	Presentation			

Presentation vs. Exam

You study for	Your Partner studies for	Your exam grade	Your presentation grade	Your average
Exam	Exam	92	84	88
Exam	Presentation	92	92	92
Presentation	Exam	80	92	86
Presentation	Presentation	80	100	90

Presentation vs. Exam

You study for	Your Partner studies for	Your exam grade	Your presentation grade	Your average
Exam	Exam	92	84	88
Exam	Presentation	92	92	92
Presentation	Exam	80	92	86
Presentation	Presentation	80	100	90

Presentation vs. Exam

The game:

		Your Partner	
		Presentation	**Exam**
You	**Presentation**	90, 90	86, 92
	Exam	92, 86	88, 88

Presentation vs. Exam

The game:

- **Players**: You and Your Partner

		Your Partner	
		Presentation	**Exam**
You	**Presentation**	90, 90	86, 92
	Exam	92, 86	88, 88

Presentation vs. Exam

The game:

- **Players**: You and Your Partner
- **Strategies**: Prepare for “Presentation” and “Exam”

		Your Partner	
		Presentation	**Exam**
You	**Presentation**	90, 90	86, 92
	Exam	92, 86	88, 88

Presentation vs. Exam

The game:

- **Players**: You and Your Partner
- **Strategies**: Prepare for “Presentation” and “Exam”
- **Payoffs**: The average grade of presentation and exam

Your Partner

You		**Presentation**	**Exam**
	Presentation	90, 90	86, 92
	Exam	92, 86	88, 88

Presentation vs. Exam

The game:

- **Players**: You and Your Partner
- **Strategies**: Prepare for “Presentation” and “Exam”
- **Payoffs**: The average grade of presentation and exam

Your Partner’s payoff

Your payoff

		Your Partner	
		Presentation	**Exam**
You	**Presentation**	90, 90	86, 92
	Exam	92, 86	88, 88

Presentation vs. Exam

The game:

- **Players**: You and Your Partner
- **Strategies**: Prepare for “Presentation” and “Exam”
- **Payoffs**: The average grade of presentation and exam
- **Payoff Matrix**: Summarizes the game.

		Your Partner	
		Presentation	**Exam**
You	**Presentation**	90, 90	86, 92
	Exam	92, 86	88, 88

Game Assumptions

- Players only care about the payoffs in the game.

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- Each player has complete information about the game:
  - Strategies available to all players.
  - Payoffs for each choice of strategies for all players.

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Game Assumptions

- Players only care about the payoffs in the game.

- Each player has complete information about the game:
  - Strategies available to all players.
  - Payoffs for each choice of strategies for all players.

- All players are *rational*:
  - Players want to get the highest payoff possible
  - Players will choose strategies that optimize their payoff, if they can.

How would you play this game?

		Your Partner: Presentation	Your Partner: Exam
You	**Presentation**	90, 90	86, 92
	Exam	92, 86	88, 88

How would you play this game?

- If your partners studies for the exam →

		Your Partner	
		Presentation	**Exam**
You	**Presentation**	90, 90	86, 92
	Exam	92, 86	88, 88

How would you play this game?

- If your partners studies for the exam →

		Your Partner: Presentation	Your Partner: Exam
You	Presentation	90, 90	86, 92
	Exam	92, 86	88, 88

How would you play this game?

- If your partners studies for the exam → you should study for the exam to get 88 instead of 86.

		Your Partner	
		Presentation	**Exam**
You	**Presentation**	90, 90	86, 92
	Exam	92, 86	88, 88

How would you play this game?

- If your partners studies for the exam → you should study for the exam to get 88 instead of 86.
- If your partner prepares for the presentation →

		Your Partner	
		Presentation	**Exam**
You	**Presentation**	90, 90	86, 92
	Exam	92, 86	88, 88

How would you play this game?

- If your partners studies for the exam → you should study for the exam to get 88 instead of 86.
- If your partner prepares for the presentation → you should study for the exam to get 92 instead of 90.

		Your Partner	
		Presentation	**Exam**
You	**Presentation**	90, 90	86, 92
	Exam	92, 86	88, 88