TECHNISCHE UNIVERSITÄT DRESDEN

Fakultät für Elektro- und Informationstechnik, Professur für Grundlagen der Elektrotechnik und Elektronik

FliK Modul 2020

Neural Networks

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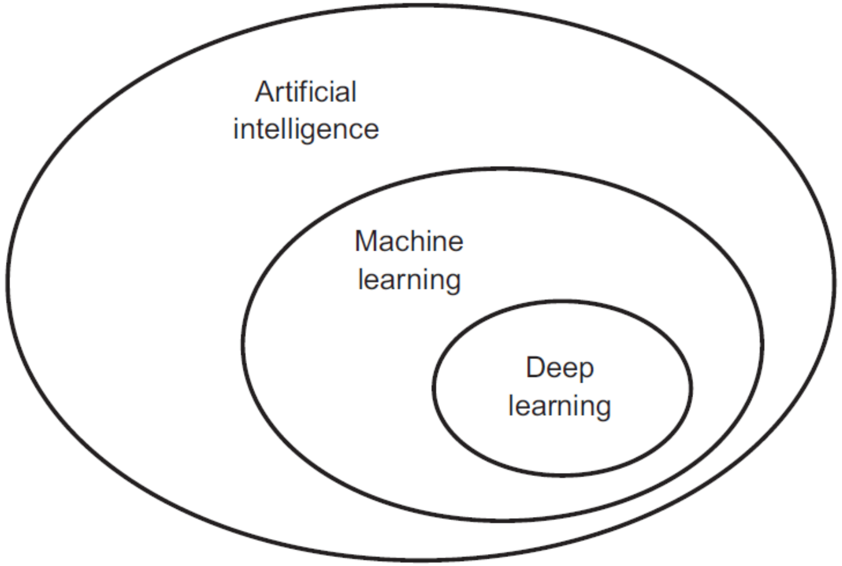
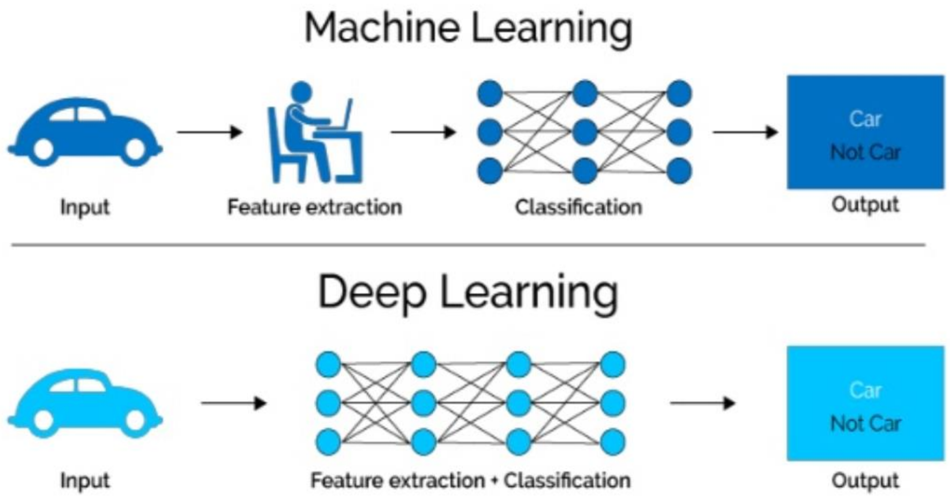
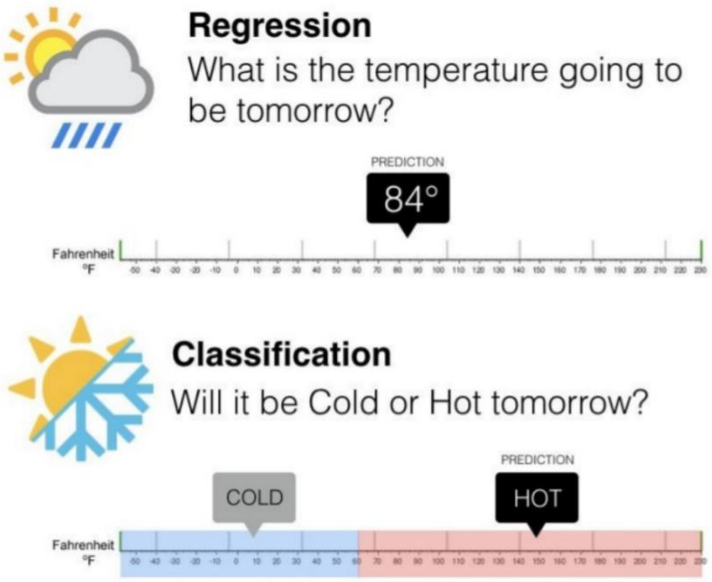
Prof. Ronald Tetzlaff

Dresden, 19-23.10.

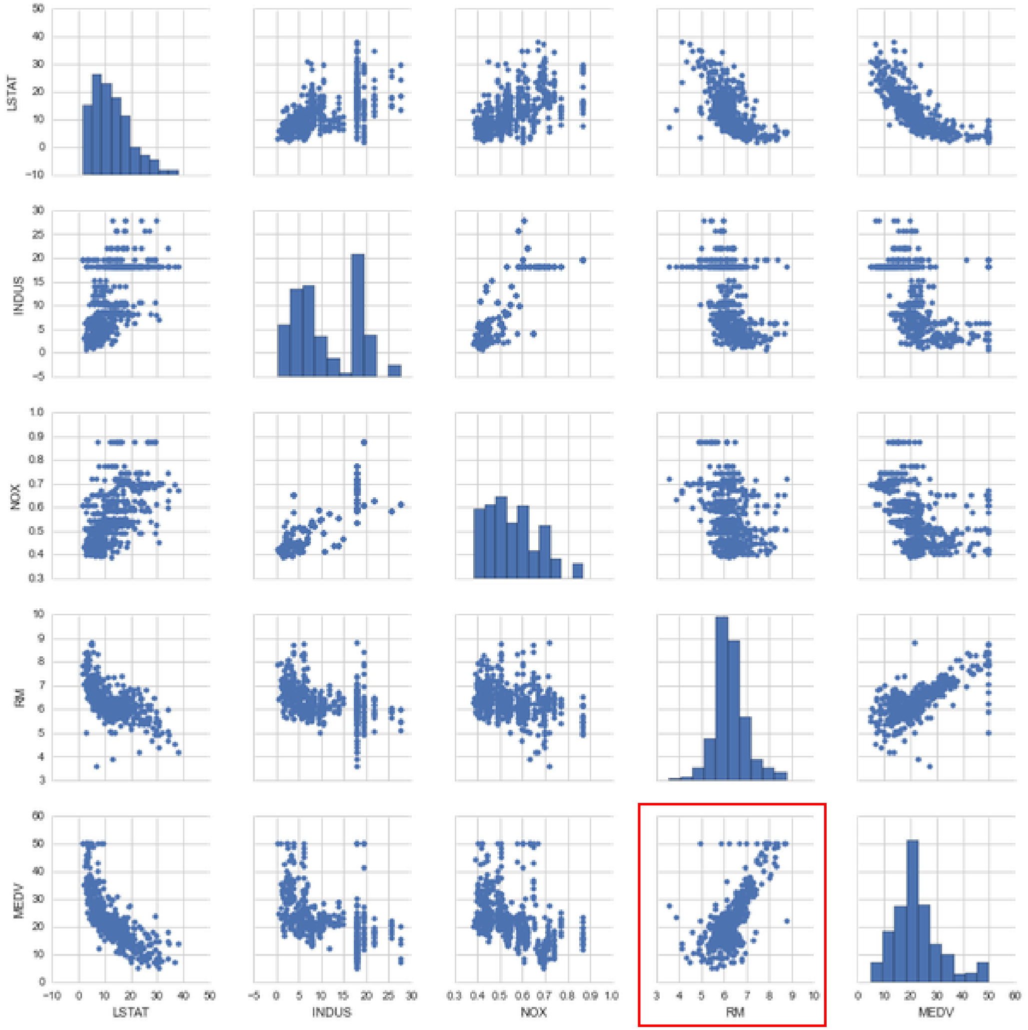
Neural Networks & Deep Learning



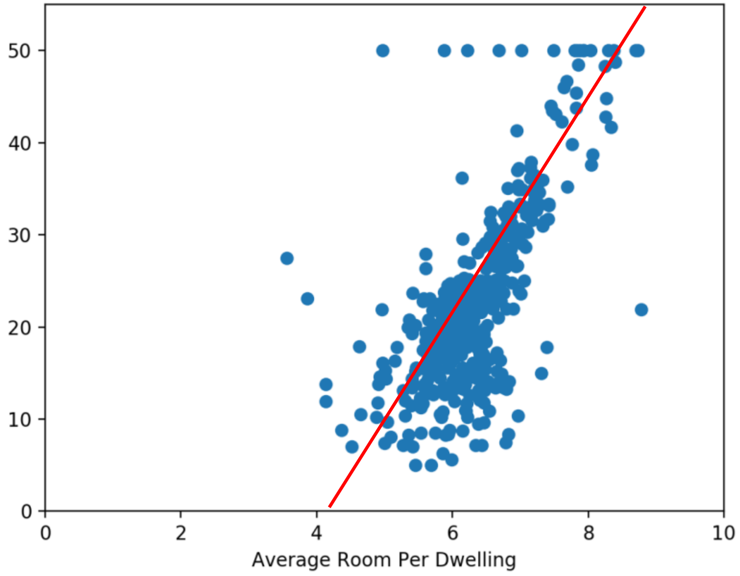




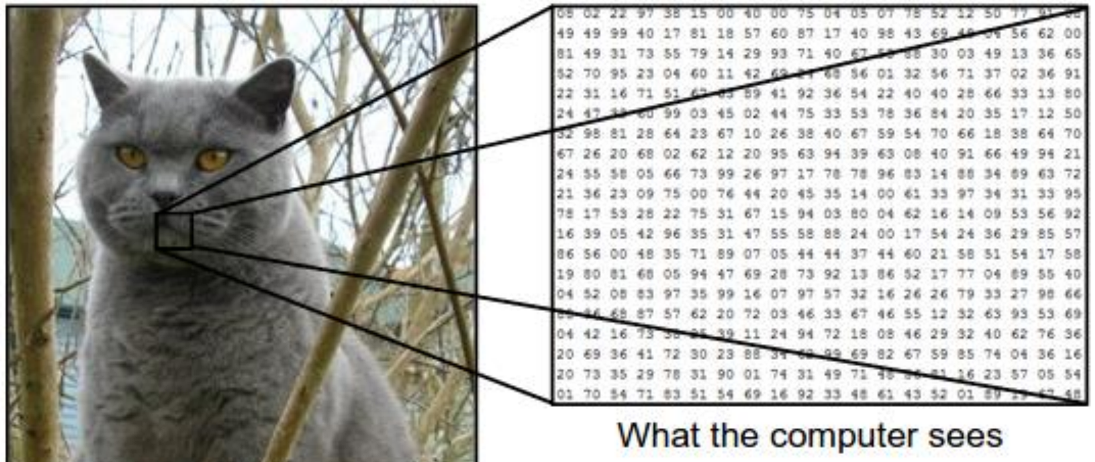
Regression: Boston Housing Dataset

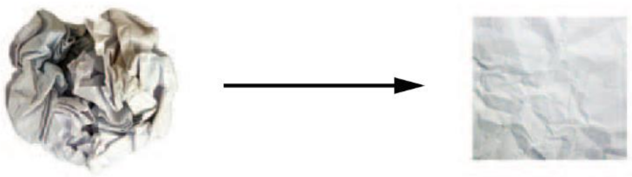


CRIM	Per capita crime rate by town
ZN	Proportion of residential land zoned for lots over 25,000 ft^2
INDUS	Proportion of nonretail business acres per town
CHAS	Charles River dummy variable (= 1 if tract bounds river; = 0 otherwise)
NOX	Nitric oxide concentration (parts per 10 million)
RM	Average number of rooms per dwelling
AGE	Proportion of owner-occupied units built prior to 1940
DIS	Weighted distances to five Boston employment centers
RAD	Index of accessibility to radial highways
TAX	Full-value property-tax rate per \$10,000
PTRATIO	Pupil/teacher ratio by town
B	$1000(Bk - 0.63)^2$ where Bk is the proportion of blacks by town
LSTAT	% Lower status of the population
MEDV	Median value of owner-occupied homes in \$1000s



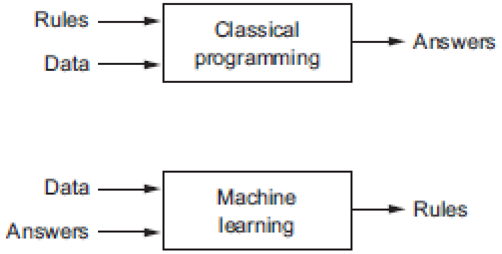
Classification:

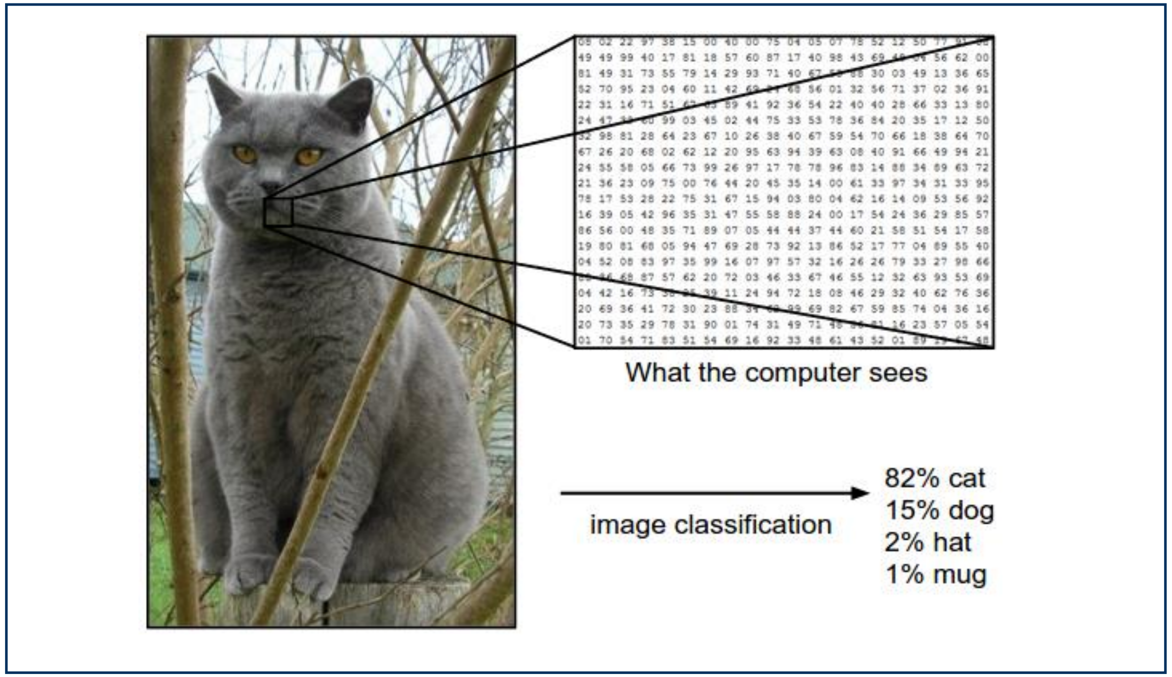




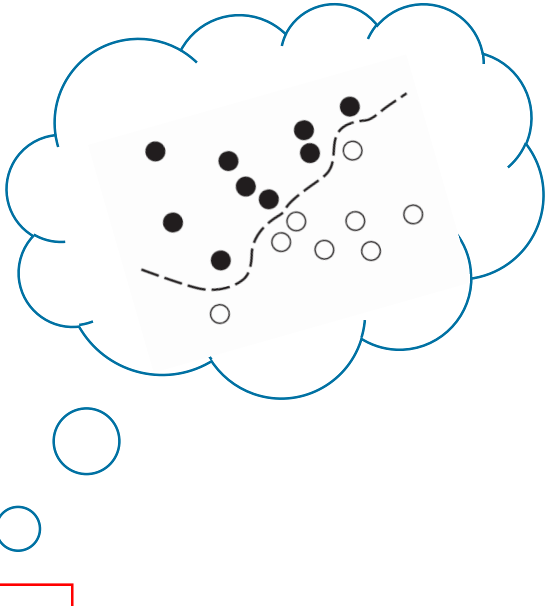
- Cat detection is a complex task!

- Neural Networks try to unfold the complexity

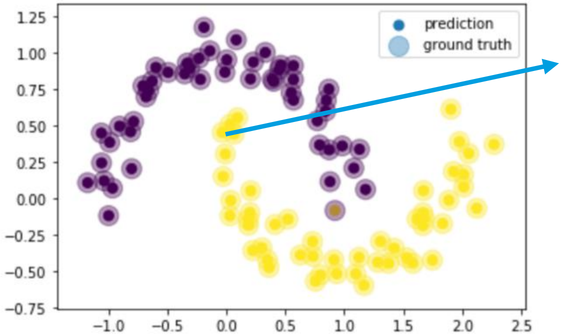




Decision Boundary



Data



Vektor/Matrix

```
[ 0.0660451 ,  0.4392075 ],
[ 0.73663111, -0.39896339],
[-1.05692838,  0.2424558 ],
[-0.80216162,  0.20271838],
[-0.70641308,  0.77076218],
[ 0.2053884 ,  0.81875305],
[ 1.37804958, -0.44658032],
```

Network

Class decision

```
1, 1, 0, 0, 0, 0, 1,
```

Multilayer Perceptron
Loss functions

A loss functions is a grade **(Metric)** how good we have been doing our task.
Mathematicaly speaking, a metric is a **measure of „Distance“.**

Network Suggestion

1,1,0,1,0

?

```
[ 0.0660451 ,  0.4392075 ],
[ 0.73663111, -0.39896339],
[-1.05692838,  0.2424558 ],
[-0.80216162,  0.20271838],
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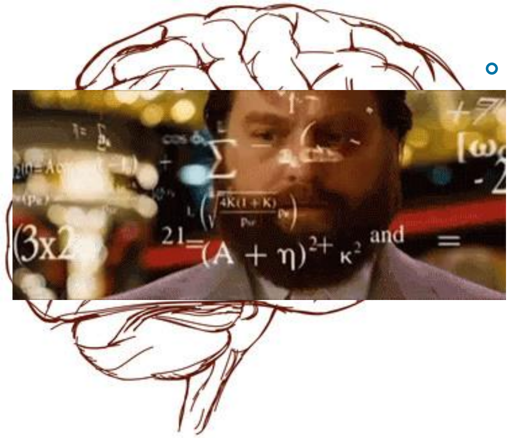


Compare

1, 1, 0, 0, 0

Ground Truth

Multilayer Perceptron
Loss functions

MSE (Naive approach)

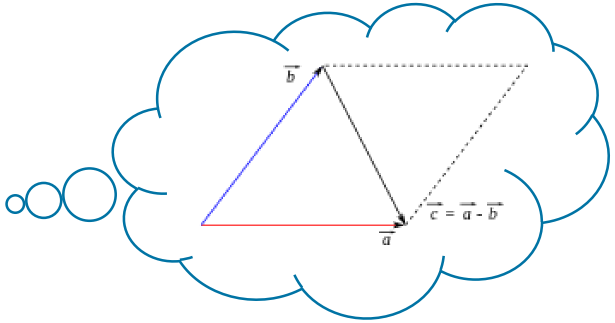
Computes the mean square Error which can defined as the **distance** of **two points** in a vector space.

Ground truth

$$\mathrm{MSE} = \boxed{\frac{1}{N}\sum_{i=1}^{N}}\,(\boxed{y_i} - \boxed{a})^2$$

Mean over all (possible) classes

Suggestion



Crossentropy

The (mean) cross entropy is a measure of **difference** between **two discrete** probability **distributions**. (created by Claude Shannon)

$$C = -\boxed{\frac{1}{N}\sum_{i=1}^{N}}\left[y_i \ln a + (1 - \boxed{y_i}) \ln(1 - \boxed{a})\right]$$

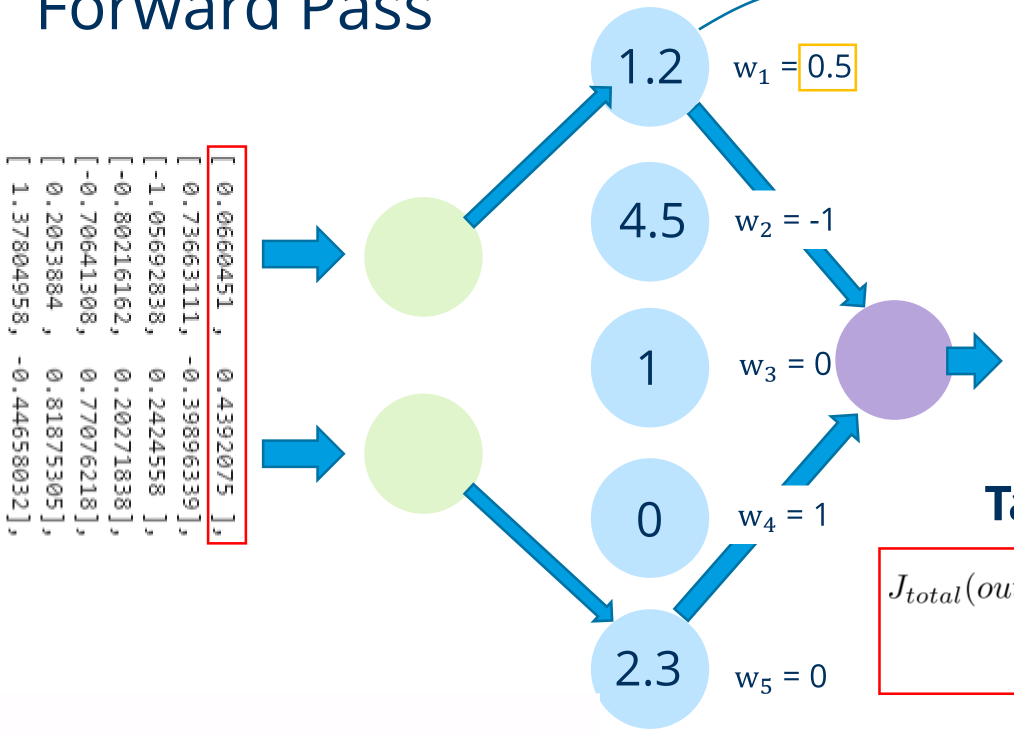
Mean over all (possible) classes

Ground truth

Suggestion

The naive approach of applying **MSE** for regression is ok. But (later) we will discuss a reason why classification **crossentropy** is the way to go in **nearly every** Situation!

Multilayer Perceptron
Forward Pass



$$output = \boxed{\sigma}\left(\underbrace{\sum_{j=1}\omega_j x_j + b}_{z}\right)$$

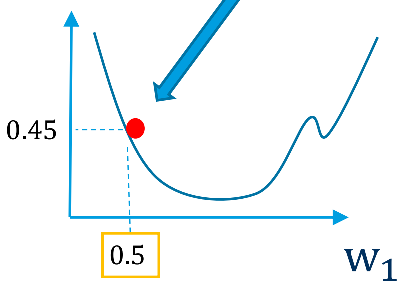
$b = 0$

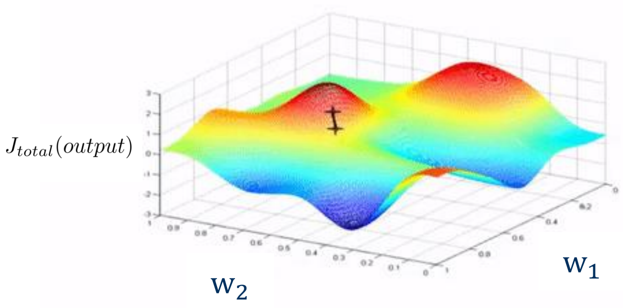
$$output = \sigma(1.2 \times 0.5 - 4.5 \times 1)$$
$$= \mathbf{0.05}$$

Target: 1, 1, 0, 0

$$J_{total}(output) = \frac{1}{2}(target - output)^2$$
$$= \frac{1}{2}(1 - 0.05)^2 = \mathbf{0.45125}$$

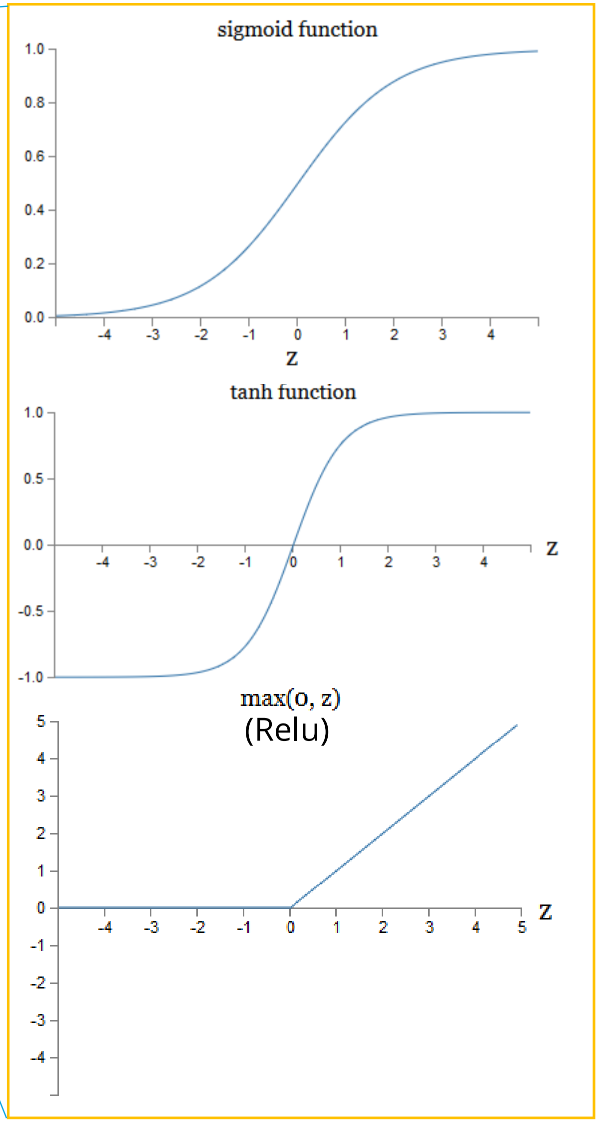
(MSE loss)





For all weights:
Sum over all training data

Activation functions

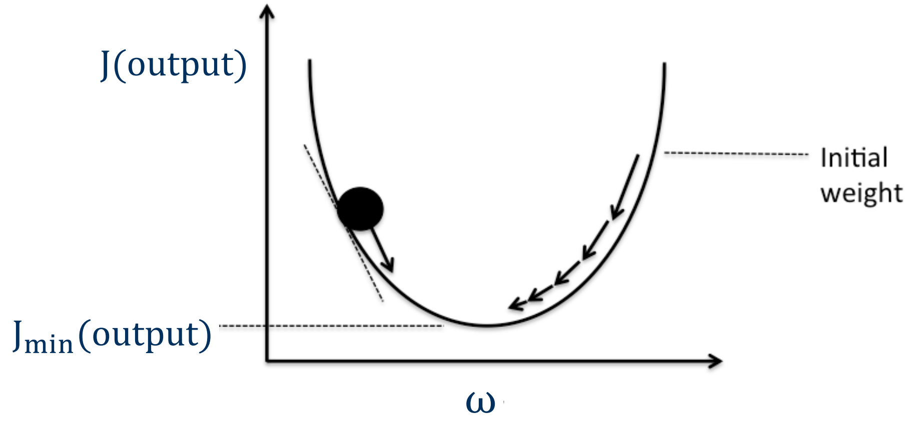


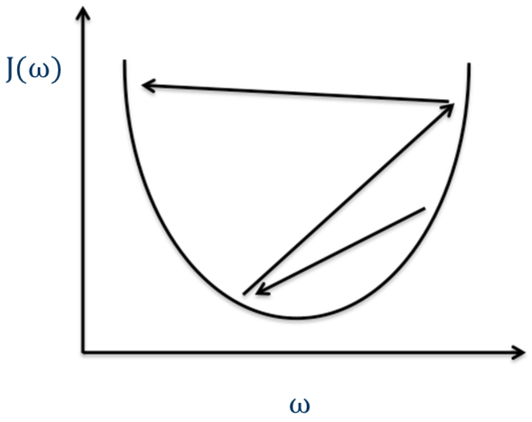
Multilayer Perceptron
Backward Pass

Gradien Descent update

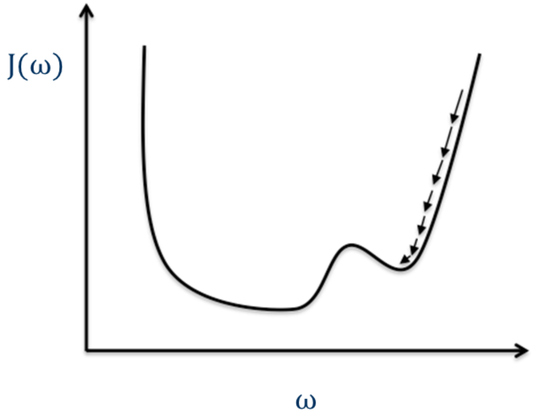
Update rule: $$w_1(new) = w_1(old) - \alpha \cdot \frac{\partial J_{total}(output)}{\partial w_1}$$

Learning rate





Large learning rate: Overshooting



Small learning rate: Many iterations until convergence and trapping in local minima

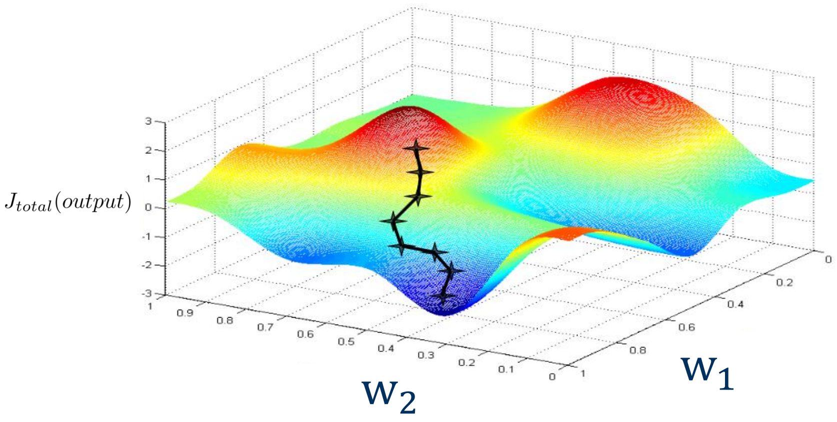
By chain rule we know for the weights that:

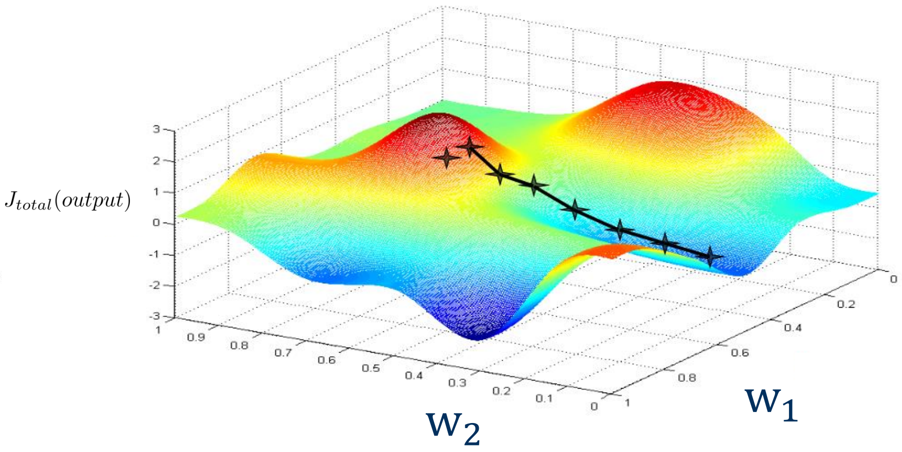
$$\frac{\partial input_{o1,1}}{\partial w_1} \cdot \frac{\partial output}{\partial input_{o1,1}} \cdot \frac{\partial J_{total}(output)}{\partial output} = \frac{\partial J_{total}(output)}{\partial w_1}$$

$$J_{total}(output) = \frac{1}{2}(target - output)^2$$ (MMSE Loss)

$$output = \frac{1}{1+e^{-input_{o1,1}}}$$ (Sigmoid)

$$input_{o1,1} = w_1 \cdot output_{h1,1} + w_2 \cdot output_{h1,2} + \ldots$$





Initialisation matters!

"Glorot" init:

$$\sigma(w) = 1/n$$
$$\mu(w) = 0$$

"He" init (Relu):

$$\sigma(w) = 2/n$$
$$\mu(w) = 0$$

Bias is set to zero.

n … Number of Neurons

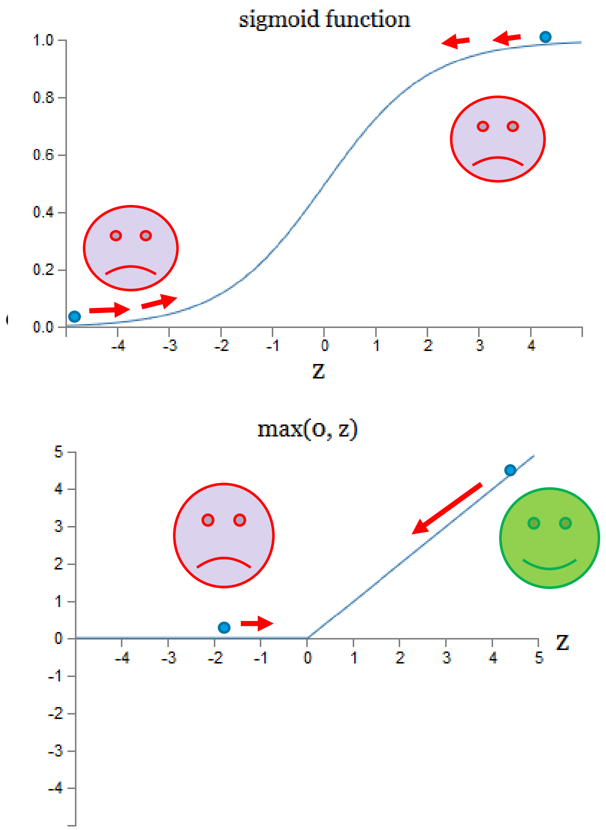
Neuron Saturation

By chain rule we know our knowledge update is:

$$\frac{\partial input_{o_{1,1}}}{\partial w_1} \cdot \boxed{\frac{\partial output}{\partial input_{o_{1,1}}}} \cdot \frac{\partial J_{total}(output)}{\partial output} = \frac{\partial J_{total}(output)}{\partial w_1}$$

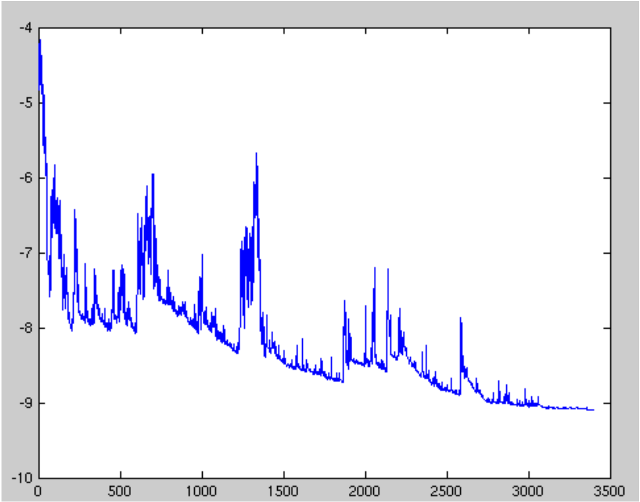
$$\boxed{output} = \frac{1}{1+e^{-input_{o_{1,1}}}} \qquad \text{(Sigmoid)}$$

So if the input of our neuron get's very large (Sigmoid & ReLU) or very low (Sigmoid) our **gradient will vanish**. This is sometimes refered as a **„dying neuron".**

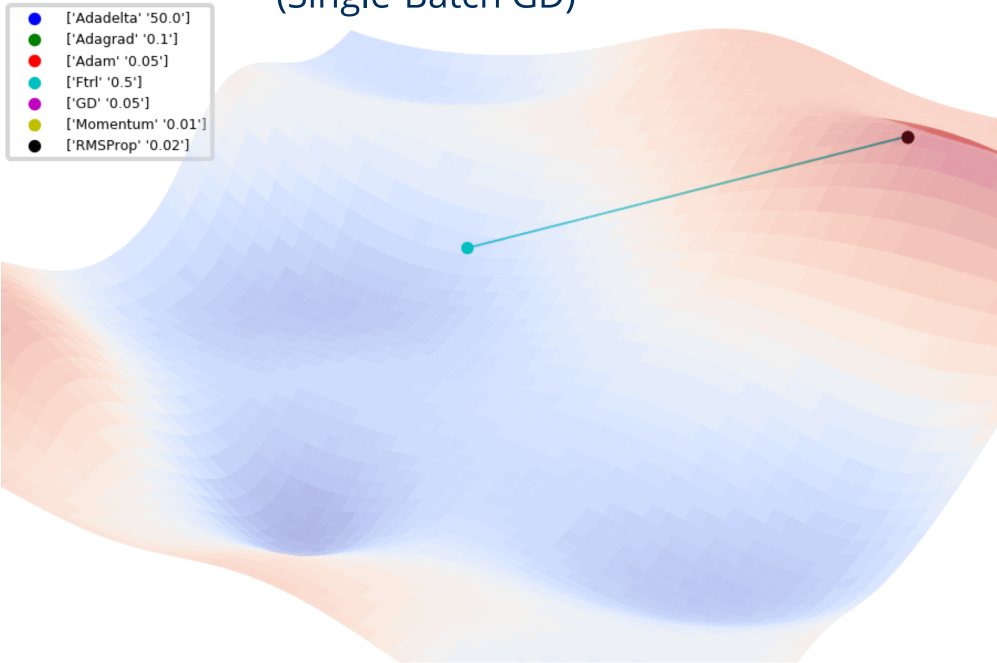


Optimizer – The “learning” Backbone
Update rules

$$\boxed{w_1(new) = w_1(old) - \alpha \cdot \frac{\partial J_{total}(output)}{\partial w_1}}$$

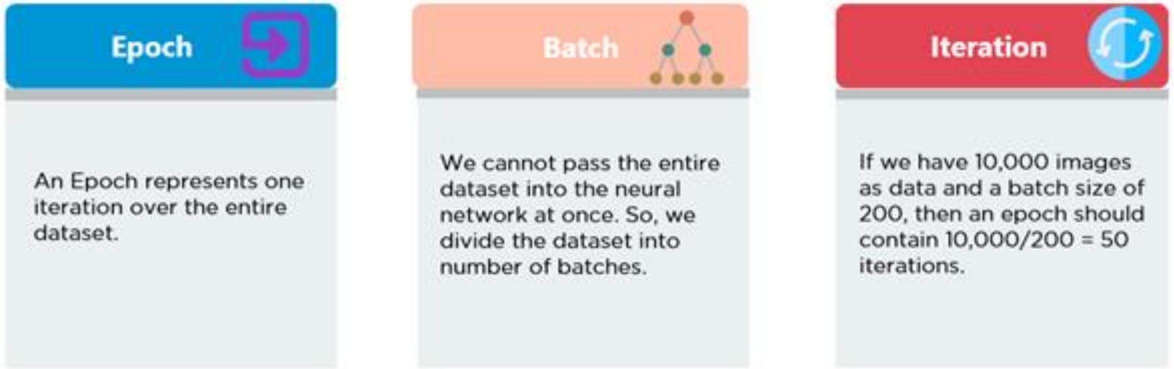


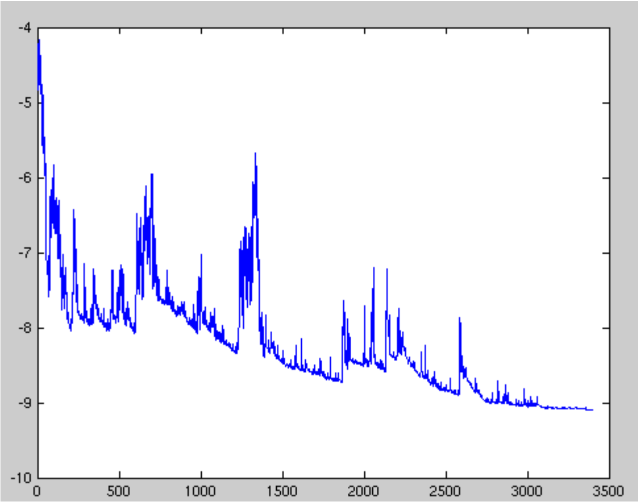
Stochastic Gradient Descent
(Single-Batch GD)



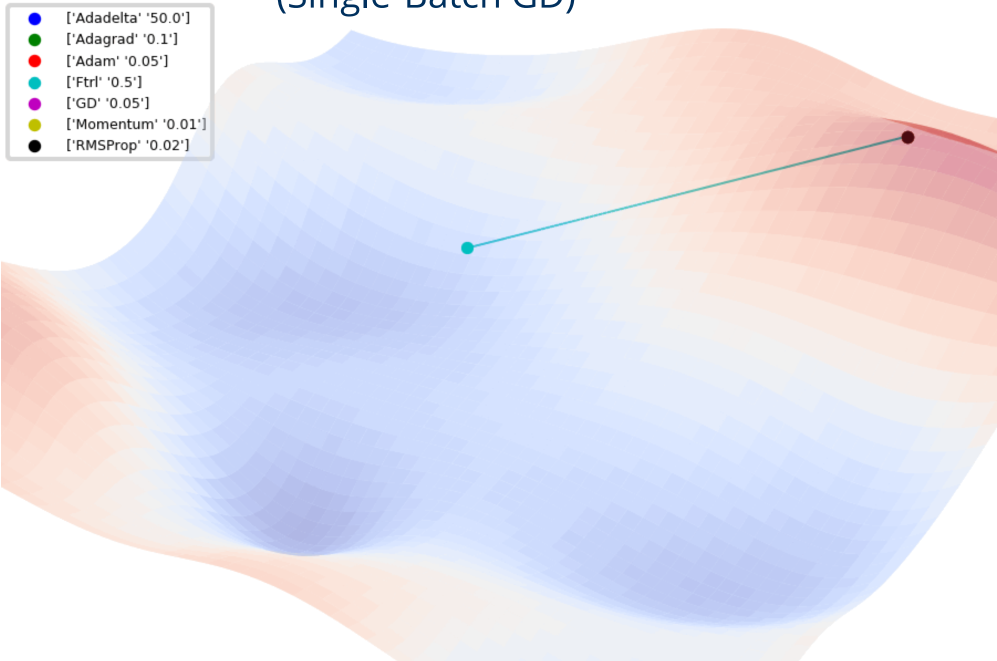
Optimizer – The "learning" Backbone
Update rules

$$w_1(new) = w_1(old) - \alpha \cdot \frac{\partial J_{total}(output)}{\partial w_1}$$





Stochastic Gradient Descent (Single-Batch GD)



Multilayer Perceptron
Decision Layers

Data → Feature Extraction → Decision Layer

How to decide which output to print?
There are two possible solutions here.

[0.0660451 , 0.4392075] →

Network output	Class
0.25	Cat
10	Horse
2	Car
4.02	House

→ Horse!

Argmax(x)

(Trained) Cross-entropie loss „x“	Corresbonding Class	Argmax(x)
0.25	Cat	0
10	Horse	10
2	Car	0
4.02	House	0

Naive approach. Just take the maximum of the output vector. → Downside: **No interpretability**

Softmax(x)

(Trained) Log-Loss „x“	Class	Softmax(x)	Argmax(Softmax(x))
0.25	Cat	0.0105	
10	Horse	0.8	→ Horse!
2	Car	0.085	
4.02	House	0.1	

Better: Interpret output as „pseudo“ probability first. The output sums up to 1 now. Then argmax! Typically used together with a Log-loss or Crossentropy

3. Exercise

Let‘s train our first classifier from scratch!