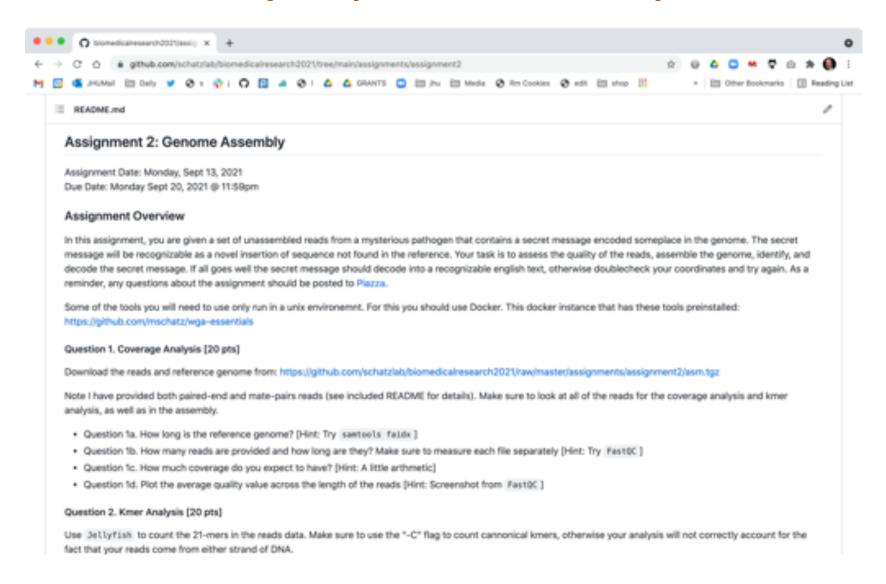
Read Mapping Michael Schatz

September 22, 2021 Lecture 7. Computational Biomedical Research



Assignment 2: Genome Assembly Due Monday Sept 20 @ 11:59pm

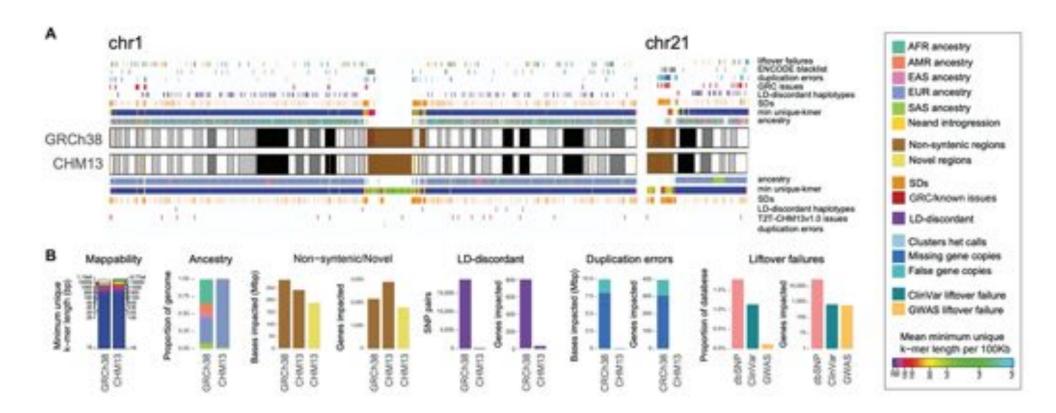


Assignment 3: Variant Calling

Postponed for I week!



The complete sequence of a human genome



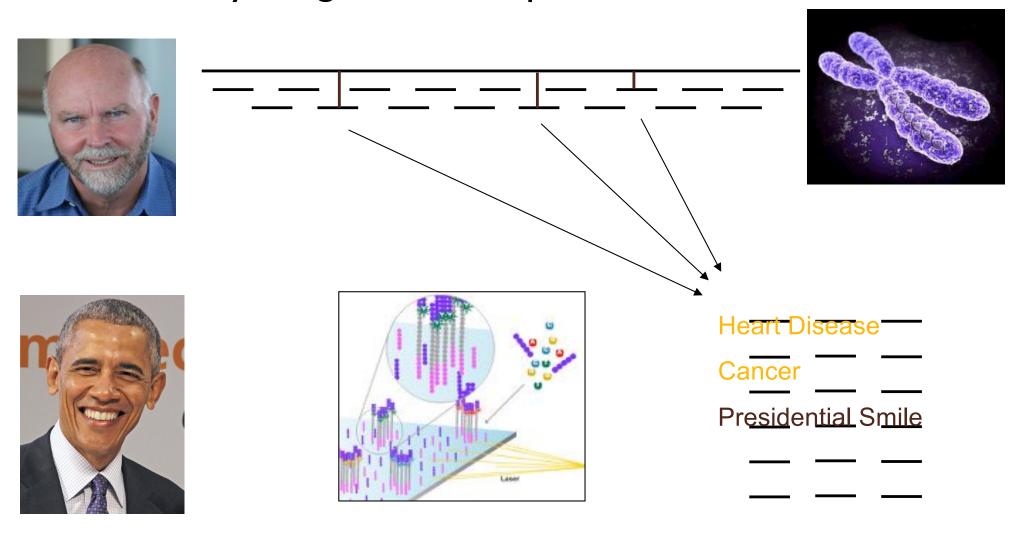
CHM13v1.1 genome size is **3.057 Gbp with zero Ns**Every chromosome is telomere-to-telomere, quality estimated >Q70
~190 Mbp (3–6%) of new sequence vs. GRCh38, fixes thousands of errors

A complete reference genome improves analysis of human genetic variation

Aganezov, S*, Yan, SM*, Soto, DC*, Kirsche, M*, Zarate, S*, et al. (2021) bioRxiv. doi: https://doi.org/10.1101/2021.07.12.452063

Personal Genomics

How does your genome compare to the reference?



- Where is GATTACA in the human genome?
- Strategy I: Brute Force

I	2	3	4	5	6	7	8	9	10	П	12	13	14	15	•••
Т	G	Α	Т	Т	Α	С	Α	G	Α	Т	Т	Α	С	С	•••
G	Α	Т	Т	A	С	Α									

No match at offset I

- Where is GATTACA in the human genome?
- Strategy I: Brute Force

I	2	3	4	5	6	7	8	9	10	П	12	13	14	15	•••
Т	G	Α	Т	Т	Α	С	Α	G	Α	Т	Т	Α	С	С	•••
	G	Α	Т	Т	Α	С	Α								

Match at offset 2

- Where is GATTACA in the human genome?
- Strategy I: Brute Force

I	2	3	4	5	6	7	8	9	10	П	12	13	14	15	•••
Т	G	Α	Т	Т	Α	С	Α	G	Α	Т	Т	Α	С	С	•••
		G	Α	Т	Т	Α	С	Α	•••						

No match at offset 3...

- Where is GATTACA in the human genome?
- Strategy I: Brute Force

I	2	3	4	5	6	7	8	9	10	П	12	13	14	15	•••
Т	G	Α	Т	Т	Α	С	Α	G	Α	Т	Т	Α	С	С	•••
								G	Α	Т	Т	Α	С	Α	

No match at offset 9 <- Checking each possible position takes time

Brute Force Analysis

- Brute Force:
 - At every possible offset in the genome:
 - Do all of the characters of the query match?
- Analysis
 - Simple, easy to understand

— Genome length = n	[3B]
– Query length = m	[7]
Comparisons: (n-m+1) * m	[21B]

Overall runtime: O(nm)

[How long would it take if we double the genome size, read length?] [How long would it take if we double both?]

Brute Force Reflections

Why check every position?

GATTACA can't possibly start at position 15

[WHY?]

1	2	3	4	5	6	7	8	9	10	П	12	13	14	15	•••
Т	G	Α	Т	Т	Α	С	Α	G	Α	Т	Т	Α	С	С	• • •
								G	Α	Т	Т	Α	С	Α	

- Improve runtime to O(n + m)

[3B + 7]

- If we double both, it just takes twice as long
- Knuth-Morris-Pratt, 1977
- Boyer-Moyer, 1977, 1991
- For one-off scans, this is the best we can do (optimal performance)
 - We have to read every character of the genome, and every character of the query
 - For short queries, runtime is dominated by the length of the genome

How can we make this go faster?

Lets play the hi/lo game

Im thinking of a secret number ...

Call out a number, and I'll tell you if you guessed it, or are too hi or too low

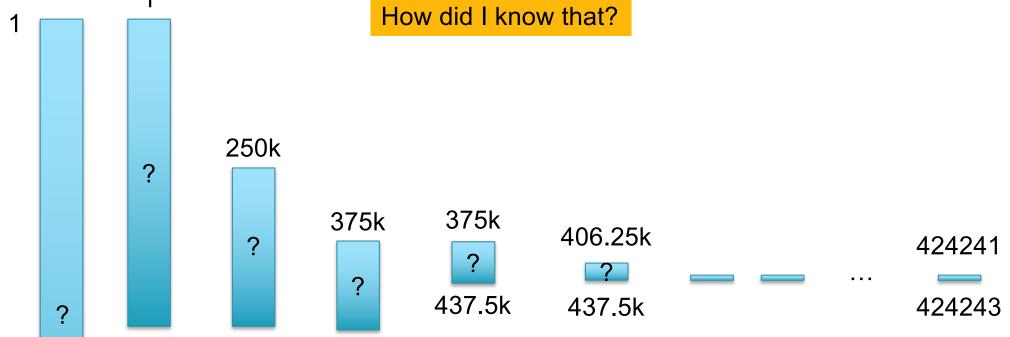
Just like grade school

Except my secret number is between 1 and 1,000,000 ©

Possibilities: 1 Results??? 2 3 999,998 999,999

1,000,000

<20 guesses to find 424242

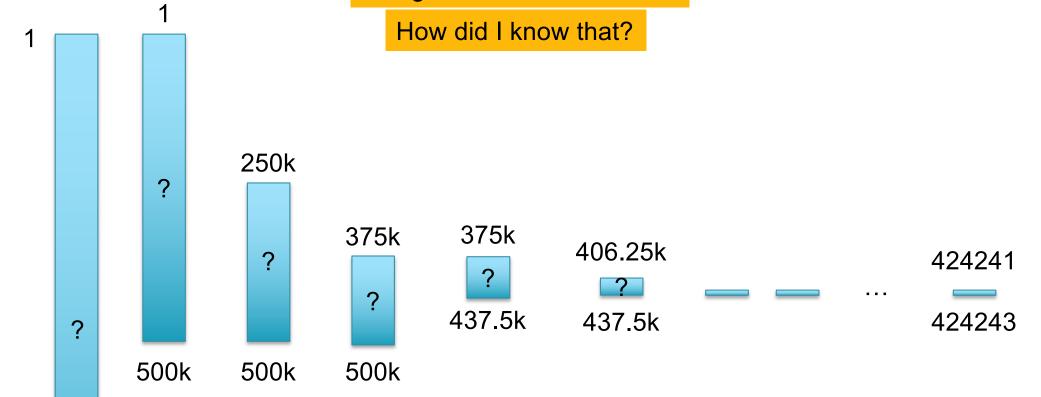


500k

500k

500k

<20 guesses to find 424242



How many times can I cut 1,000,000 in half?

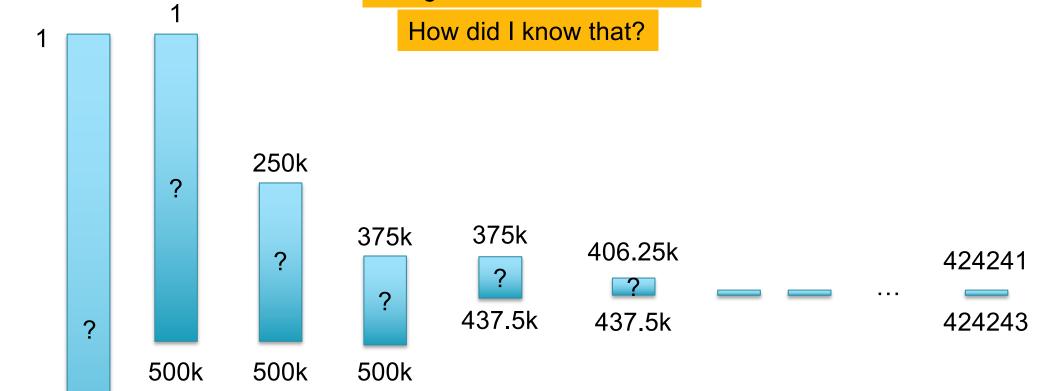
Find smallest x such that: $1000000/(2^x) \le 1$

 $1,000,000 \le 2^{x}$

 $x = log_2(1,000,000)$

x = 19.93

<20 guesses to find 424242



How many times can I cut 1,000,000 in half? x = 19.93 = 20

How many times can I cut 2,000,000 in half? x = 20.93 = 21

How many times can I cut 1,000,000,000 in half? x = 29.89 = > 30

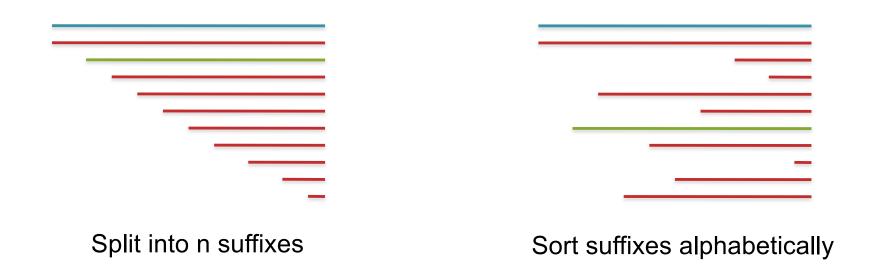
How many times can I cut 3,000,000,000 in half? x = 31.48 => 32

Searching the Phone Book

- What if we need to check many queries?
 - We don't need to check every page of the phone book to find 'Schatz'
 - Sorting alphabetically lets us immediately skip 96% (25/26) of the book without any loss in accuracy

Searching the Phone Book

- What if we need to check many queries?
 - We don't need to check every page of the phone book to find 'Schatz'
 - Sorting alphabetically lets us immediately skip 96% (25/26) of the book without any loss in accuracy
- Sorting the genome: Suffix Array (Manber & Myers, 1991)
 - Sort every suffix of the genome



[Challenge Question: How else could we split the genome?]

Suffix Arrays: Searching the Phone Book

GATTACA All suffixes Suffix array **GATTACA** Α ACA ATTACA TTACA ATTACA TACA CA ACA **GATTACA**

TACA

TTACA

CA

Suffix Arrays: Searching the Phone Book

GAT	TACA
All suffixes	Suffix array
	# p
GATTACA	17 A
ATTACA	2 5 ACA
TTACA	3 3 ATTACA
TACA	4 6 CA
ACA	5 1 GATTACA
CA	6 4 TACA
A	7 3 TTACA

Suffix Arrays: Searching the Phone Book

GAT	TACA
All suffixes	Suffix array
	# p
GATTACA	1 7
ATTACA	2 5
TTACA	3 3
TACA	4 6
ACA	5 1
CA	6 4
A	7 3

- Strategy 2: Binary search
 - Compare to the middle, refine as higher or lower
- Searching for GATTACA
 - Lo = I; Hi = 15;



#	Sequence	Pos
Ι	ACAGATTACC	6
2	ACC	13
3	AGATTACC	8
4	ATTACAGATTACC	3
5	ATTACC	10
6	C	15
7	CAGATTACC	7
8	CC	14
9	GATTACAGATTACC	2
10	GATTACC	9
П	TACAGATTACC	5
12	TACC	12
13	TGATTACAGATTACC	I
14	TTACAGATTACC	4
15	TTACC	П



- Strategy 2: Binary search
 - Compare to the middle, refine as higher or lower
- Searching for GATTACA
 - Lo = I; Hi = 15; Mid = (1+15)/2 = 8
 - Middle = Suffix[8] = CC



#	Sequence	Pos
I	ACAGATTACC	6
2	ACC	13
3	AGATTACC	8
4	ATTACAGATTACC	3
5	ATTACC	10
6	C	15
7	CAGATTACC	7
8	CC	14
9	GATTACAGATTACC	2
10	GATTACC	9
П	TACAGATTACC	5
12	TACC	12
13	TGATTACAGATTACC	I
14	TTACAGATTACC	4
15	TTACC	11



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 - Lo = I; Hi = 15; Mid = (1+15)/2 = 8
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I	ACAGATTACC	6
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5	ATTACC	10
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7	CAGATTACC	7
8	CC	14
9	GATTACAGATTACC	2
10	GATTACC	9
П	TACAGATTACC	5
12	TACC	12
13	TGATTACAGATTACC	I
14	TTACAGATTACC	4
15	TTACC	П



- Strategy 2: Binary search
 - Compare to the middle, refine as higher or lower
- Searching for GATTACA
 - Lo = I; Hi = 15; Mid = (1+15)/2 = 8
 - Middle = Suffix[8] = CC=> Higher: Lo = Mid + I
 - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
 - Middle = Suffix[12] = TACC

#	Sequence	Pos
I	ACAGATTACC	6
2	ACC	13
3	AGATTACC	8
4	ATTACAGATTACC	3
5	ATTACC	10
6	C	15
7	CAGATTACC	7
8	CC	14
9	GATTACAGATTACC	2
10	GATTACC	9
Ш	TACAGATTACC	5
12	TACC	12
13	TGATTACAGATTACC	I
14	TTACAGATTACC	4
15	TTACC	П



Lo

- Strategy 2: Binary search
 - Compare to the middle, refine as higher or lower
- Searching for GATTACA
 - Lo = I; Hi = 15; Mid = (1+15)/2 = 8
 - Middle = Suffix[8] = CC=> Higher: Lo = Mid + I
 - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
 - Middle = Suffix[12] = TACC=> Lower: Hi = Mid I
 - Lo = 9; Hi = 11;





#	Sequence	Pos
I	ACAGATTACC	6
2	ACC	13
3	AGATTACC	8
4	ATTACAGATTACC	3
5	ATTACC	10
6	C	15
7	CAGATTACC	7
8	CC	14
9	GATTACAGATTACC	2
10	GATTACC	9
П	TACAGATTACC	5
12	TACC	12
13	TGATTACAGATTACC	T
14	TTACAGATTACC	4
15	TTACC	П

- Strategy 2: Binary search
 - Compare to the middle, refine as higher or lower
- Searching for GATTACA
 - Lo = I; Hi = 15; Mid = (1+15)/2 = 8
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 - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
 - Middle = Suffix[12] = TACC=> Lower: Hi = Mid I
 - Lo = 9; Hi = 11; Mid = (9+11)/2 = 10
 - Middle = Suffix[10] = GATTACC

#	Sequence	Pos
I	ACAGATTACC	6
2	ACC	13
3	AGATTACC	8
4	ATTACAGATTACC	3
5	ATTACC	10
6	C	15
7	CAGATTACC	7
8	CC	14
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10	GATTACC	9
П	TACAGATTACC	5
12	TACC	12
13	TGATTACAGATTACC	I
14	TTACAGATTACC	4
15	TTACC	Ш

Lo →

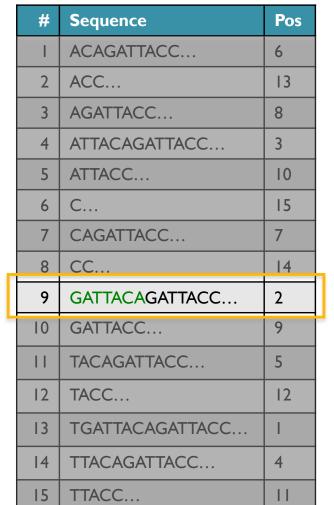
Ηį

- Strategy 2: Binary search
 - Compare to the middle, refine as higher or lower
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 - Lo = I; Hi = 15; Mid = (1+15)/2 = 8
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 - Middle = Suffix[12] = TACC=> Lower: Hi = Mid I
 - Lo = 9; Hi = 11; Mid = (9+11)/2 = 10
 - Middle = Suffix[10] = GATTACC=> Lower: Hi = Mid I
 - Lo = 9; Hi = 9;



#	Sequence	Pos
I	ACAGATTACC	6
2	ACC	13
3	AGATTACC	8
4	ATTACAGATTACC	3
5	ATTACC	10
6	C	15
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 - Middle = Suffix[12] = TACC=> Lower: Hi = Mid I
 - Lo = 9; Hi = 11; Mid = (9+11)/2 = 10
 - Middle = Suffix[10] = GATTACC=> Lower: Hi = Mid I
 - Lo = 9; Hi = 9; Mid = (9+9)/2 = 9
 - Middle = Suffix[9] = GATTACA...=> Match at position 2!





Binary Search Analysis

Binary Search

```
Initialize search range to entire list

mid = (hi+lo)/2; middle = suffix[mid]

if query matches middle: done

else if query < middle: pick low range

else if query > middle: pick hi range

Repeat until done or empty range
```

[WHEN?]

- Analysis
 - More complicated method
 - How many times do we repeat?
 - How many times can it cut the range in half?
 - Find smallest x such that: $n/(2^x) \le 1$; $x = \lg_2(n)$

[32]

- Total Runtime: O(m lg n)
 - More complicated, but much faster!
 - Looking up a query loops 32 times instead of 3B





Suffix Array Construction

How can we store the suffix array?
 [How many characters are in all suffixes combined?]

$$S = 1 + 2 + 3 + \dots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} = O(n^2)$$

- Hopeless to explicitly store 4.5 billion billion characters
- Instead use implicit representation
 - Keep I copy of the genome, and a list of sorted offsets
 - Storing 3 billion offsets fits on a server (12GB)
- Searching the array is very fast, but it takes time to construct
 - This time will be amortized over many, many searches
 - Run it once "overnight" and save it away for all future queries



5		

	•	_	
9	Q		

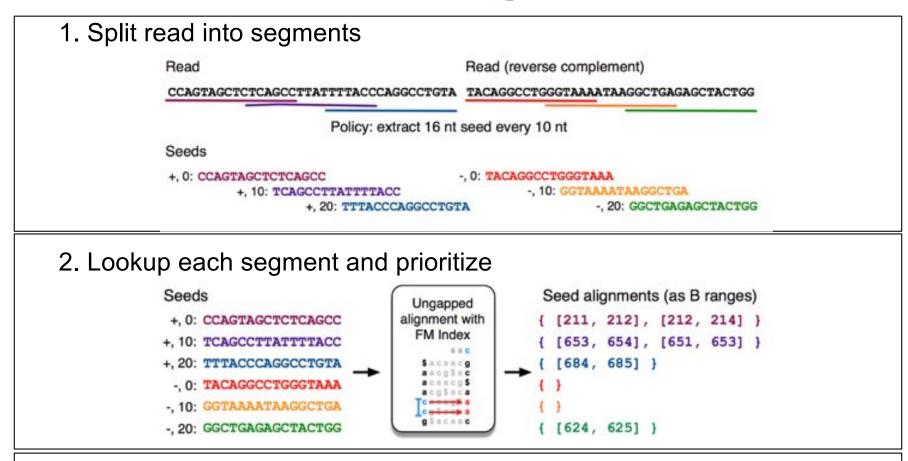
3	
J	

I	0	
ı	5	

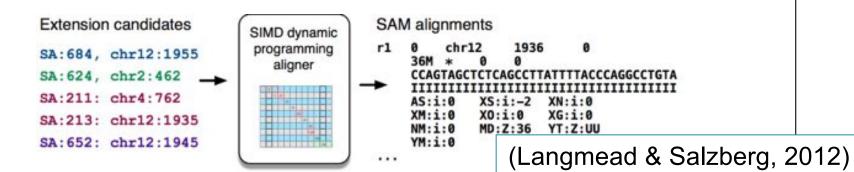
_	
7	
•	

14
17

Bowtie2 Algorithm



3. Evaluate end-to-end match



Sapling: Accelerating Suffix Array Queries with Learned Data Models





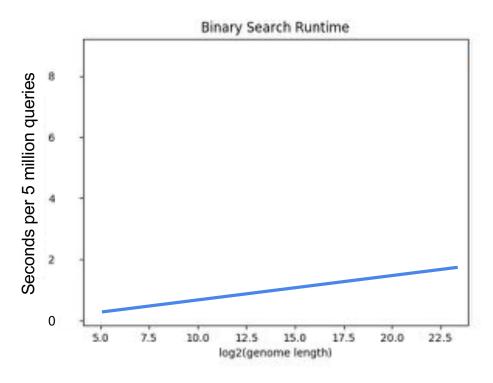


Melanie Kirsche

Arun Das

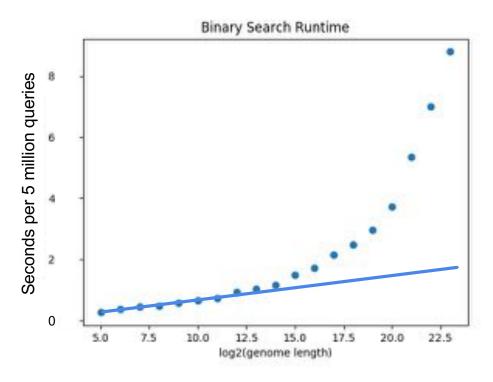
https://github.com/mkirsche/sapling

Caching and Binary Search



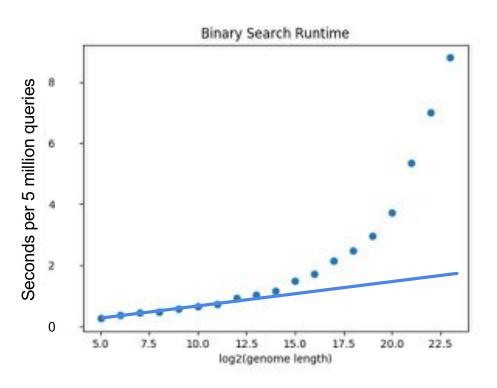
In theory, searching should scale linearly with log2 of the genome size

Caching and Binary Search



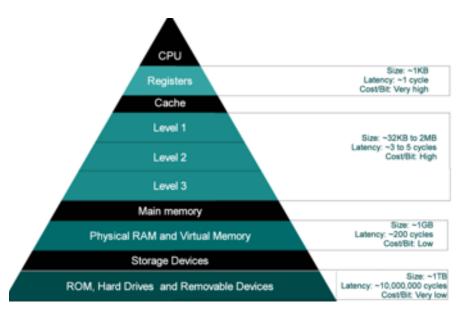
In practice, searching is much slower for large genome sizes

Caching and Binary Search



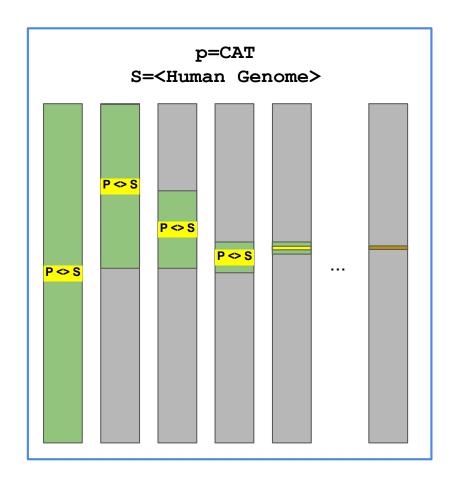
In practice, searching is much slower for large genome sizes

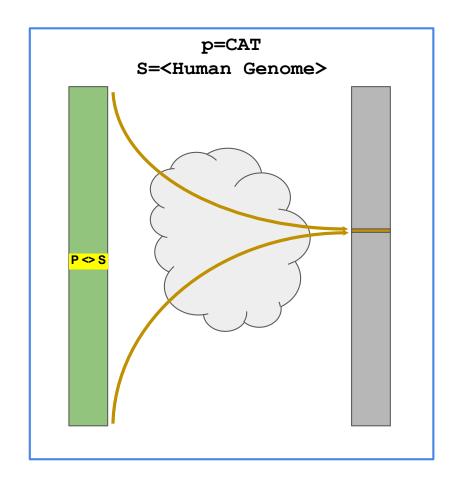




Binary search suffers from poor locality causing many lookups in main memory

Suffix Array Prediction





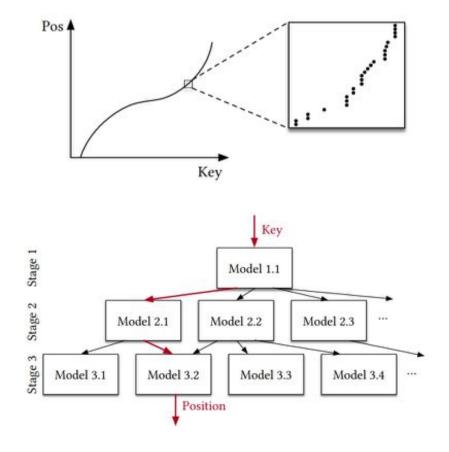
What if instead of a slow algorithmic approach to find the correct rows, we could somehow quickly guess/predict the correct rows?

Learned Index Structures

Researchers at Google using neural networks to replace classical data structures such as B-Trees, HashMaps, and Bloom Filters

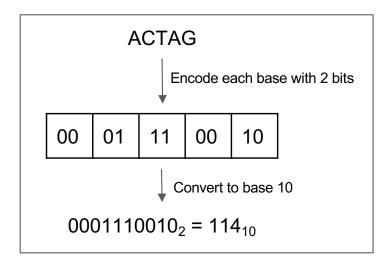
Train network to predict position of a data point in the structure given its value

Compute the maximum error E = |predicted position - actual position| among all points in data structure. Then, narrow search to within E of predicted value.

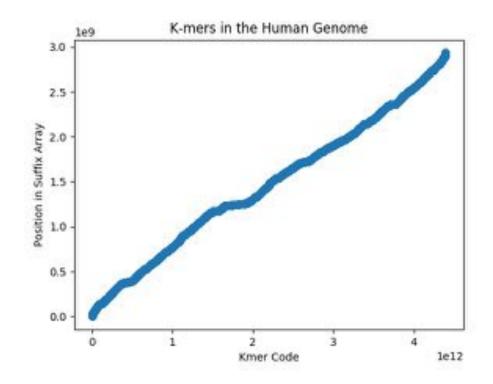


Kraska et. al. "The Case for Learned Index Structures". SIGMOD 2018.

Suffix Array Search as a Prediction Task

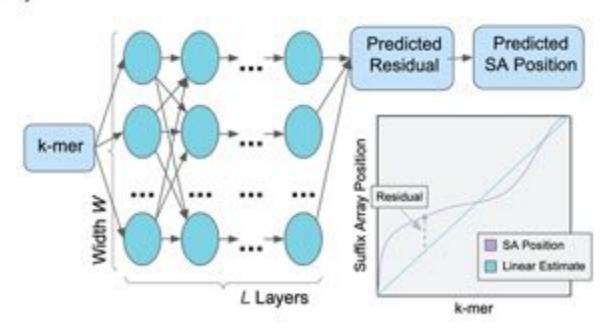


Goal: Given a query string and a suffix array, predict the suffix array position where the suffix begins with that query string

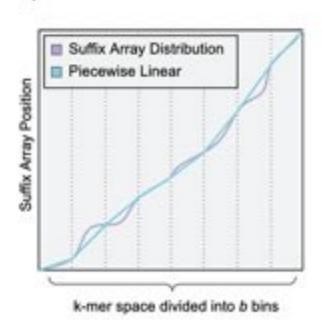


Prediction Schemes

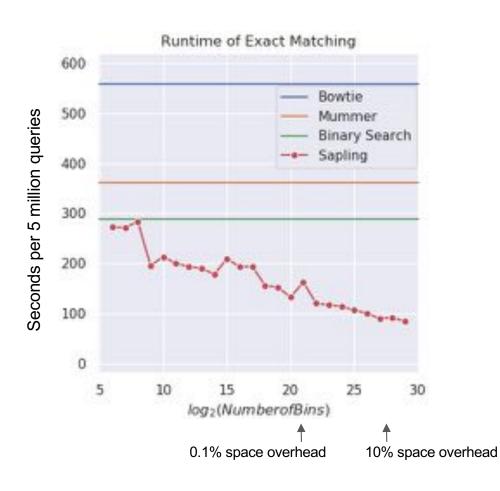
a) ANN Architecture

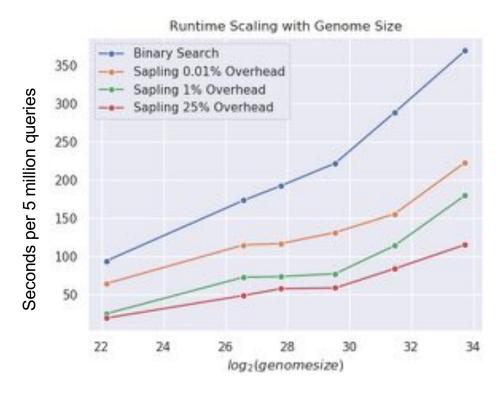


b) Piecewise Linear Architecture



Performance Results

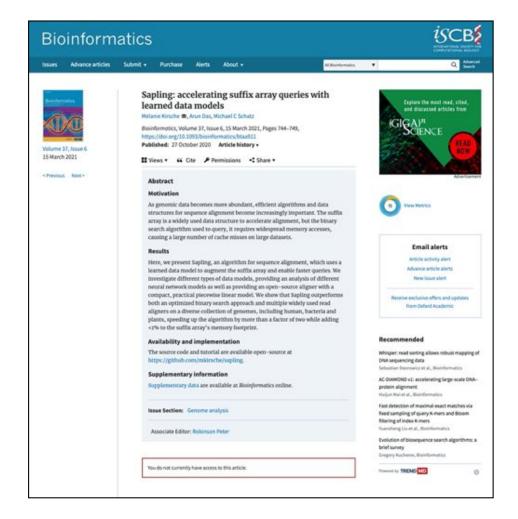




Conclusion

SAPLING allows faster string searching than existing approaches

- Scales well with genome length and could be used for searching large collections of genomes, e.g. metagenomics search
- Technique of treating data structure lookups as predictive function evaluation has the potential to speed up many other genomic data structures



https://github.com/mkirsche/sapling