Numerical Methods

1. Write a menu-driven program for finding roots of a nonlinear equation using Bisection, Regula Falsi and Newton-Raphson method.

2. Use the above program to find 3 roots of the equation xtan(x)=c where c is a user-input constant. Use both bisection method and Newton-Raphson method.

3. There are three real roots of the equation $x^3 - 2.5x^2 - 2.46x + 3.96 = 0$ in the domain [-4, +4]. Write a program to first find out the disjoint subintervals in the given domain those cover the roots. Hence find the roots by Newton-Raphson method. **[OPTIONAL]**

4. Write a menu-driven program for solving a system of linear equations using Gauss-Elimination method, Jacobi's method and Gauss Elimination with pivoting method

5. Using the above program solve the following system of equations :

   i. $x + y + z = 6$, $x + y - z = 0$, $x - y + z = 2$

   ii. $x_1 + x_2 + x_3 = 3$, $2x_1 + 3x_2 + x_3 = 6$, $x_1 - x_2 - x_3 = -3$

   iii. $2x_1 + 4x_2 + 2x_3 = 15$, $2x_1 + x_2 + 2x_3 = -5$, $4x_1 + x_2 - 2x_3 = 0$

6. Write a menu-driven program for implementing Interpolation using Lagrange's formula, Newton's forward difference formula, and Newton's backward difference formula.

7. For the following table of values:

x	1	2	3	4
f(*x*)	1	8	27	64

Find $f(2.5)$ using all three methods and comment on your answer

1. An experiment gave the following table of values for the dependent variable *y* for a set of known values of *x*. Obtain an appropriate least squares fit for the data.

x	1	2	3	4	5	6	7	8	9
y	5.5	7.0	9.6	11.5	12.6	14.4	17.6	19.5	20.5

2. For the data of the following table, obtain the regression line of *x* on *y* and of *y* on *x*. Compare the two lines. Show that both lines pass through $(\bar{x}, \bar{y})$ where $\bar{x}$ and $\bar{y}$ are means of *x* and *y* respectively. **[OPTIONAL]**

x	1.2	2.1	2.8	4.1	4.9	6.2	7.1	7.9	8.9
y	4.2	6.8	9.8	13.4	15.5	19.6	21.6	25.4	28.6

Show that in general the linear regression lines pass through the means$\left(\frac{\sum x_i}{n}, \frac{\sum y_i}{n}\right)$.

3. Find $\frac{dy}{dx}$ using forward, backward and central differencing schemes for $y = Sin\, x$ using (i) $\Delta x = 0.1$ (ii) $\Delta x = 0.01$ for $0 \leq x \leq \pi$ and determine the relative percentage error, defined as $\left| \frac{\left.\frac{dy}{dx}\right|_{numerical} - \left.\frac{dy}{dx}\right|_{exact}}{\left.\frac{dy}{dx}\right|_{exact}} \right| \times 100$ in each case with the exact derivative given by $\left.\frac{dy}{dx}\right|_{exact} = \cos(x)$. Comment on the result.

4. Consider a circle described by the equation $x^2 + y^2 = a^2$ where a is a user input constant. Determine the curvature of a point on the circle given by $(aCos\,\theta, aSin\theta)$ for $0 \leq \theta \leq \frac{\pi}{2}$. The curvature of a curve $y = f(x)$ is given by

$$\kappa = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

Determine the derivatives using central differencing scheme. Explore the sensitivity of the result to choice of Δx and compare your result at each point with 1/*a*. **[OPTIONAL]**

Repeat the problem for $y = \sin(kx)$ where $1 \leq k \leq 5$ is a user-defined constant. Comment on the appropriate choice of Δx for different values of *k*.

5. Write a function integrate to find the integration of a function *f* within the limits *a* and *b* by Trapezoidal and Simpson's 1/3rd integration method. *f*, *a* and *b* should be provided as arguments of the function integrate. Write the main function to take the name of the integrand function and the limits of integration as command line arguments.

6. Write a program to solve the following differential equations by (i) Euler method, (ii) Runge-Kutta second order method. Compare your solutions. In each method estimate the truncation error and choose an appropriate step size.

$$\frac{dy}{dx} = 2xy, y(0) = 0.5, \text{solution for } 1 \geq x \geq 0$$

$$\frac{dy}{dx} = x^2 + y^2, y(0) = 1, solution\ for\ 1 \geq x \geq 0$$

7. Write a program to solve the following second order differential equation by second order and fourth order Runge-Kutta method:

$$\frac{d^2y}{dx^2} + 0.5\frac{dy}{dx} + 4y = 5, y(0) = y'^{(0)} = 0.$$

Find the solution in the range $2 \geq x \geq 0$. **[OPTIONAL]**