Spiking Neural Networks for Control

Max Schaufelberger
February 8, 2024 —– KTH Royal Institute of Technology

Table of Contents

Introduction

Simulation

Control

Learning

Combined Learning

BackupSlides

Control

Learning

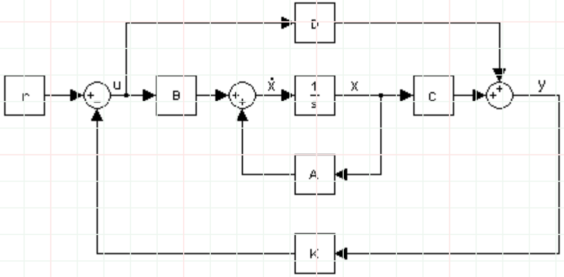
Learned Control

Conclusion

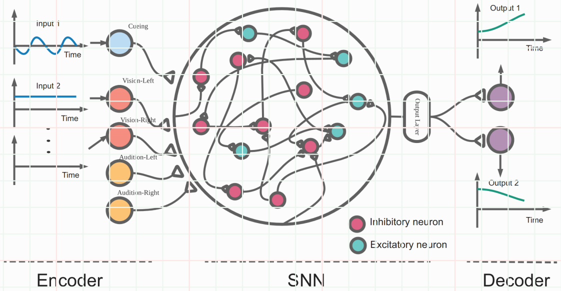
Introduction

What are we talking about

Control a Linear system



Use Spiking neural networks



What are we talking about

Control a Linear system

- Tracking of reference trajectory

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

Only stable systems

Use Spiking neural networks

- Third Generation of NN
- Working with discrete spikes
- Inherently fit for temporal data

Goal / Motivation

Artificial SNN can already solve various cognitive task such as

- Memorization
- Basic Logic
- Simulation of Dynamic Systems
- Control

Although with varying levels of biologic plausibility. We set out to build a controlled dynamic system based on SNN using learning and biologic plausibility

- Allow for black-box deployment without manual parameter tuning
- "Limit ourselves to use the brains capabilities to design a controller"

Method

1. Simulate

Use a spiking network to simulate a dynamic system

2. Control

Devise a control scheme to control the network output

3. Learn

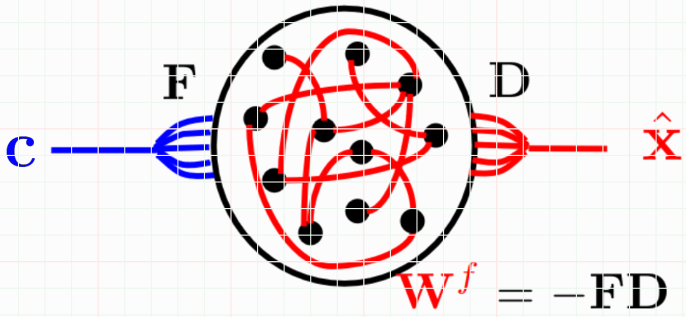
Apply biologically plausible learning rules to our network

4. Combine

Integrate all three steps into a single controller

Simulation

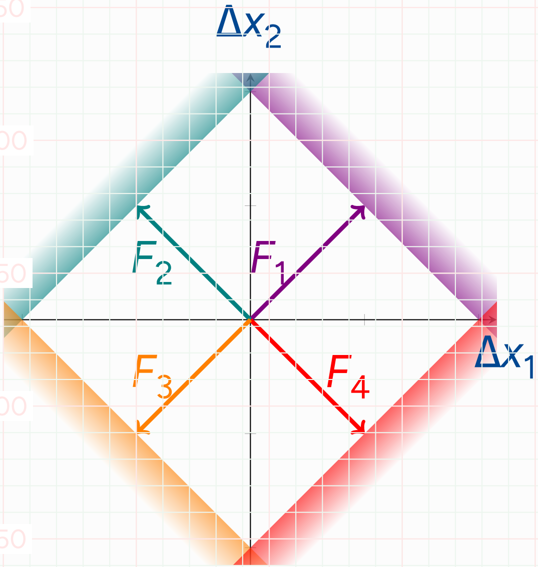
Simulation of Linear systems

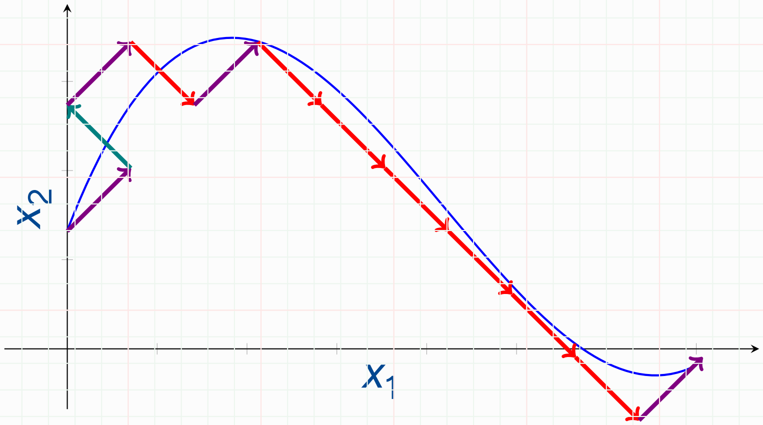


- Build NN that outputs $\hat{x}$ from the system $\dot{x} = Ax + c$ given c
- Group of LIF neurons with with intrinsic Voltage, tracking the projected error $V_i = F(x - \hat{x}) + \mu r_i$
- Network decoding $\hat{x} = F^T r$

$$\dot{V} = -\lambda_V V + Fc + W^f o(t) + W^s r(t) + \sigma_V \eta(t)$$

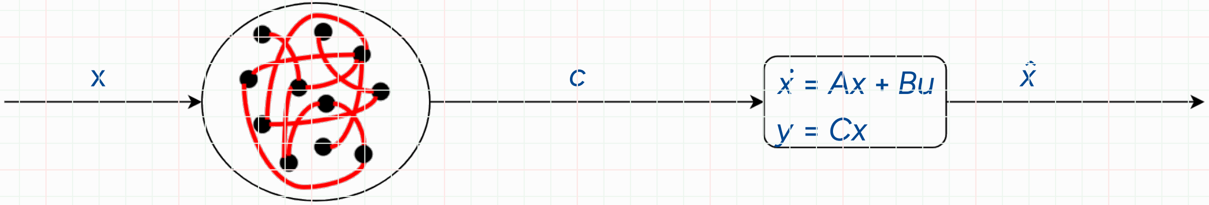
Geometric





Control

Control Concept



[HC19]

(Almost) identical network architecture

- Network output is external input into (previous) simulating network $\longleftrightarrow$ Network state contains control signal
- Governed by PD-control as $c = \dot{x} - Ax$
- In presence of output matrix $C \neq \mathbf{I} \leftrightarrow \text{rank}(B^T C^T) = rank(B^T)$

fix the layouting of this page

Examples

Insert Picture

Insert Picture

Learning

$V_i = F_i(x - \hat{x}) - \mu r_i$

Learning rules

Slow Learning rule $W^s = F(A + \lambda_d \mathbb{I})F^T$

- Online Learning of Student teacher dynamics $\dot{\hat{x}} = M\hat{x} + c$
- Error Feedback Ke during Training
- $\delta M \propto e\hat{x}^T \longrightarrow \delta W^s \propto F(e\hat{x}^T)F^T \approx Fer^T$
- Error alignment?
- Supervised Learning rule

Fast Learning rule $W^f = FF^T + \mu \mathbb{I}$

- Voltage measures system error
- Minimize average Voltage outside of Neuron Threshold
- Biologically plausible pre×post locally
- Unsupervised Learning Rule

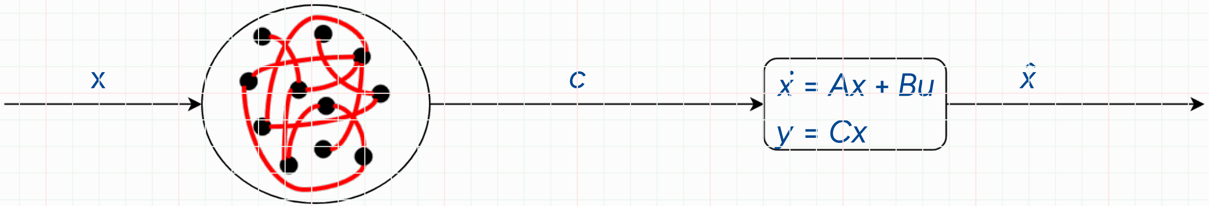
Examples

Insert Picture

Insert Picture

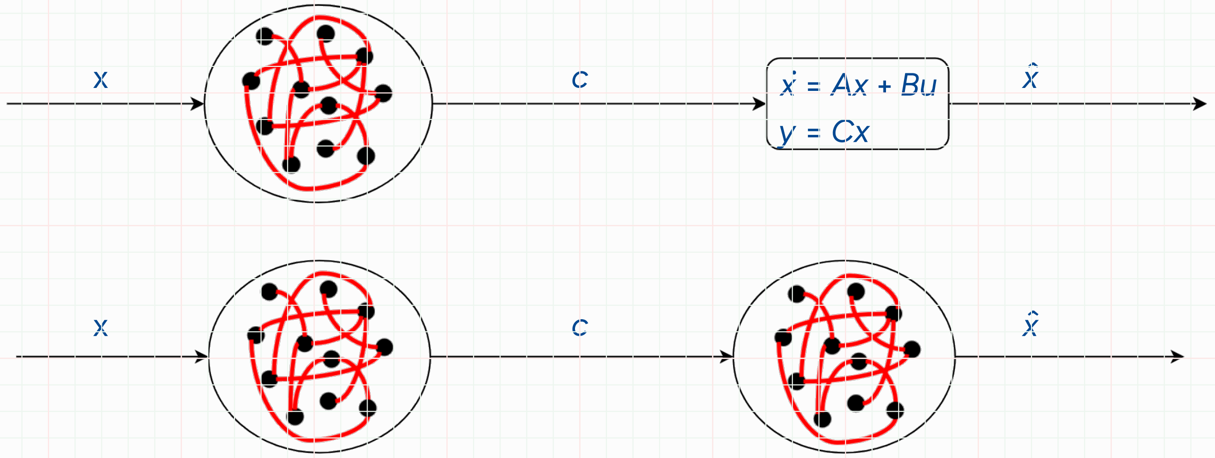
Combined Learning

Control Concept



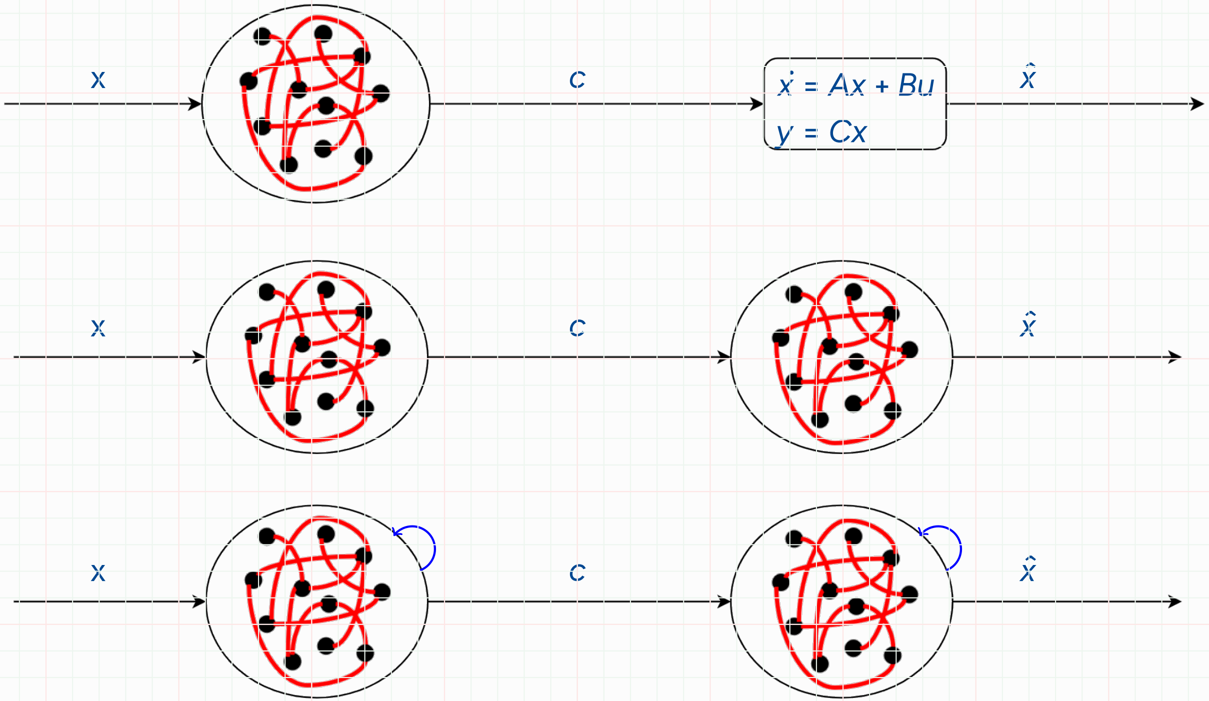
[HC19]

Control Concept



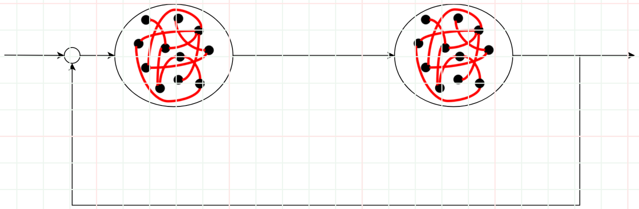
[HC19]

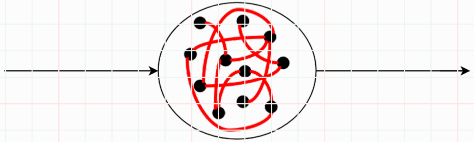
Control Concept

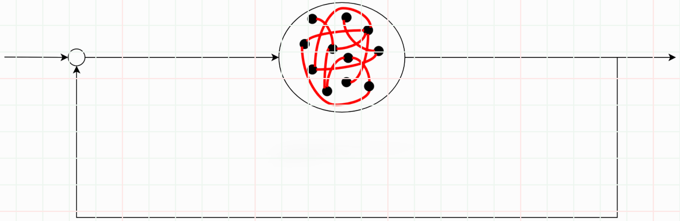


[HC19]

Control Concept II



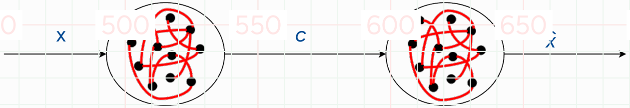




Problems

In conjunction, problems can arise:

- Divergence in Learning
- Control with Noise
- Reliance on analytic results
- Biologically implausible Learning



Dual network approach I

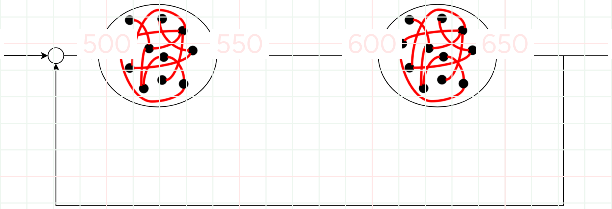
No Learning rule for control network available

Open loop control

Incapable of noise detection or correction

No compensation of training error

Highly dependent on governing dynamics from $c_{\text{cont}} = \dot{x} - Ax$



Dual network approach II

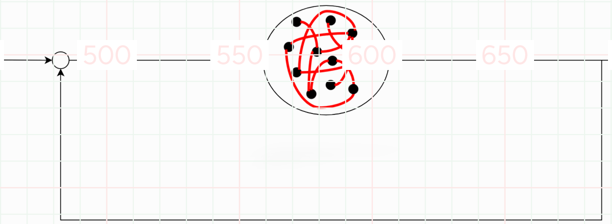
No Learning rule for control network available

~~Open loop controller~~

~~Incapable of noise detection or correction~~

~~No compensation of training error~~

Highly dependent on governing dynamics from $c_{contr} = \dot{x}_{ref} - Ax_{ref}$



Single network approach I

~~No Learning rule for control network available~~

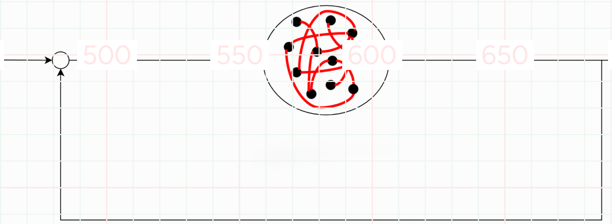
Open loop controller

Incapable of noise detection or correction

No compensation of training error

Highly dependent on governing dynamics from $c_{\text{contr}} = \dot{x}_{\text{ref}} - Ax_{\text{ref}}$

Orthonormality restriction on Input Matrix $B | B^T B = \mathbb{I}$



Single network approach II

~~No Learning rule for control network available~~

Open loop controller

Incapable of noise detection or correction

No compensation of training error

Highly dependent on governing dynamics from $c_{contr} = \dot{x}_{ref} - Ax_{ref}$

Orthonormality restriction on Input Matrix $B \in \mathbb{B} := \{M \mid M^T M = \mathbb{I}\}$ 'Write this with such that I dont have 5 versions i need to keep trakc of pt 100 150 200 250 300 350 400 450 500 550 600 650 700

100

150

200

250

300

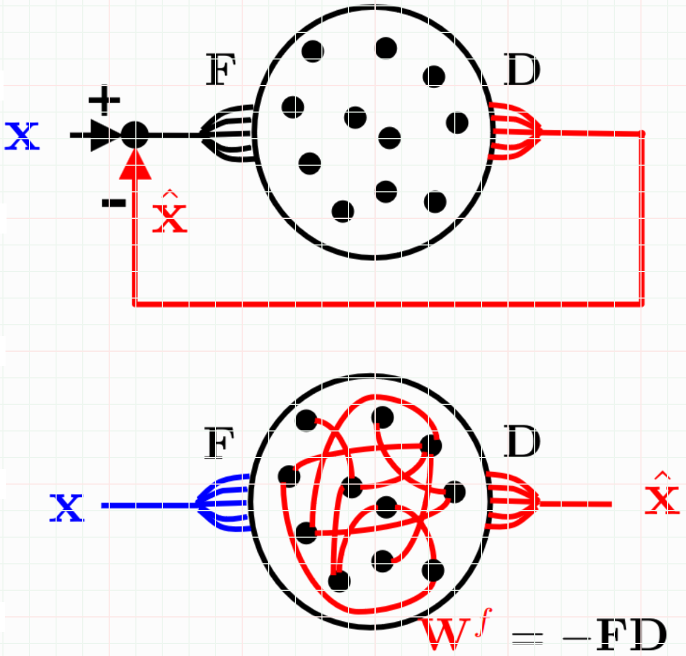
350

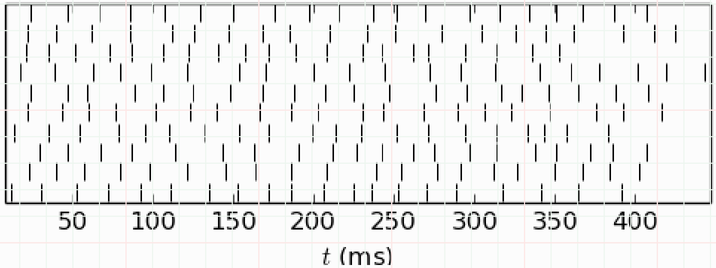
400

Jetzt hier meine versuche mit 2 netzten, 2 netze mit feedback, 1 netz, 1 netz mit feedback etc und was/warum es nicht geklappt hat

BackupSlides

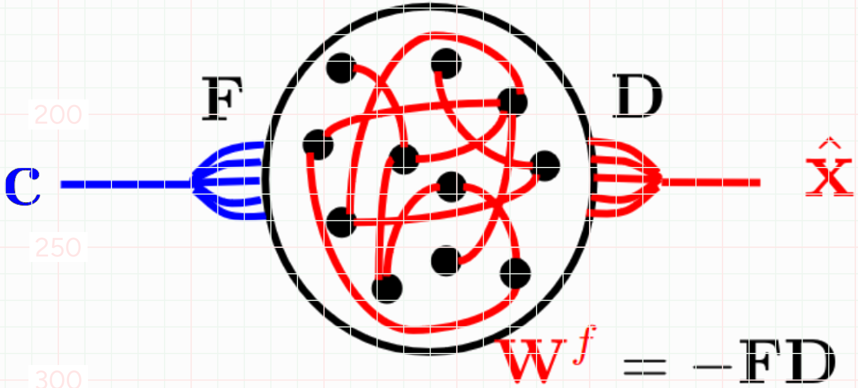
Autoencoder





$$\hat{x} = Do(t)$$
$$\dot{r} = -\lambda r + o(t) \tag{2}$$

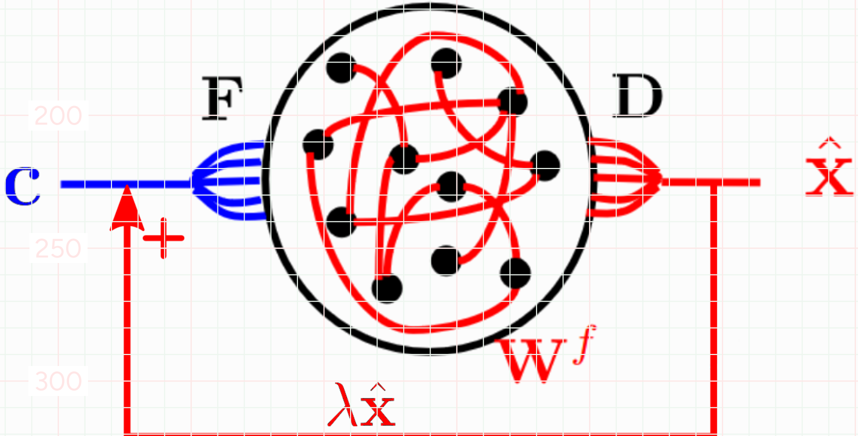
Autoencoder II

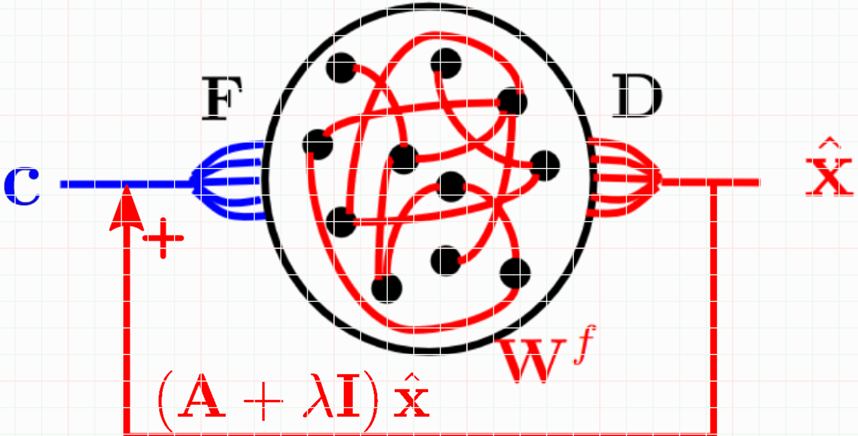


$$
\begin{aligned}
\dot{x} &= -\lambda x + c \\
\hat{x} &= Dr
\end{aligned}
\tag{3}
$$

$$\dot{r} = -\lambda r + o(t)$$
$$x = Dr$$

Autoencoder III

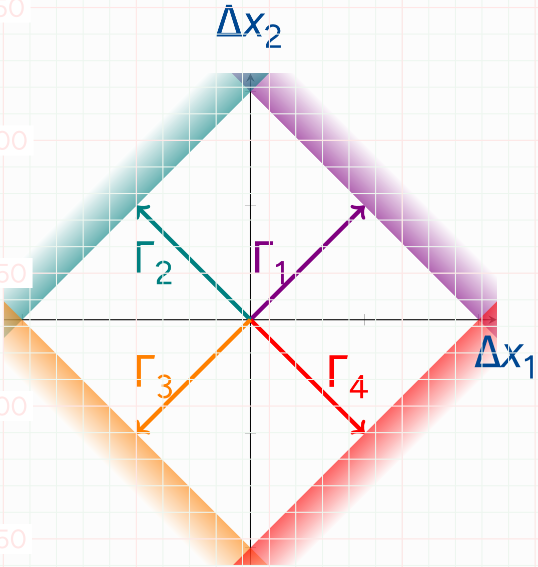


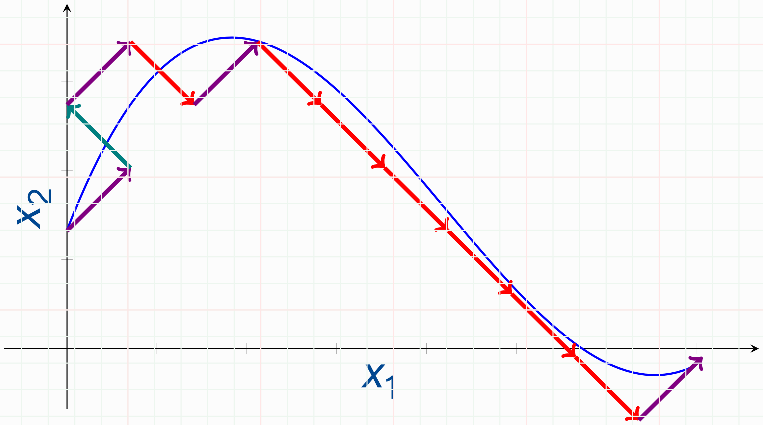


$$\dot{x} = c \tag{4}$$

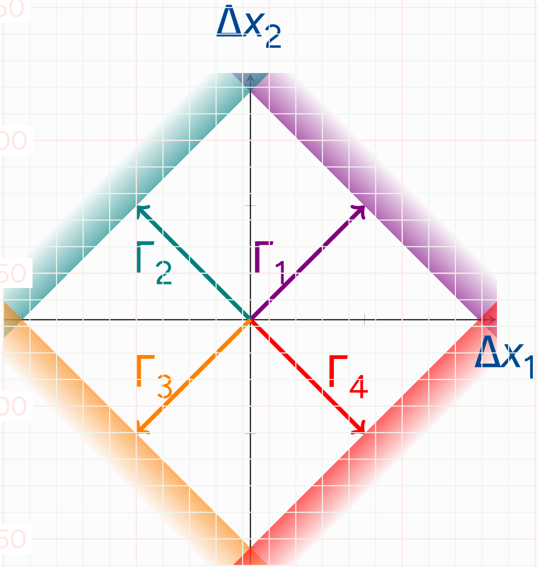
$$\dot{x} = Ax + c \tag{5}$$

Geometric





Geometric



Minimize the cost J (Greedy)

$$J = \int_0^T \|x - \hat{x}\|_2^2 + C(r)\, dt \tag{6}$$

$$\begin{aligned} V_i &= \Gamma_i^T (x - \hat{x}) - \mu r_i \\ \dot{V}_i &= -\lambda_V V_i + \Gamma^T c(t) \\ &\quad + W^f o(t) + W^s r(t) + \sigma_V \eta(t) \end{aligned} \tag{7}$$

$$W^f = \Gamma^T \Gamma + \mu I$$

$$W^s = \Gamma^T (A + \lambda_d I)\, \Gamma$$

Example Simple

content...

Example Big

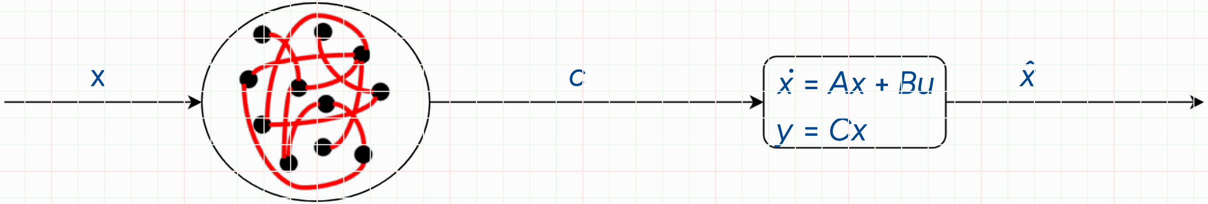
content...

Conclusion

content...

Control

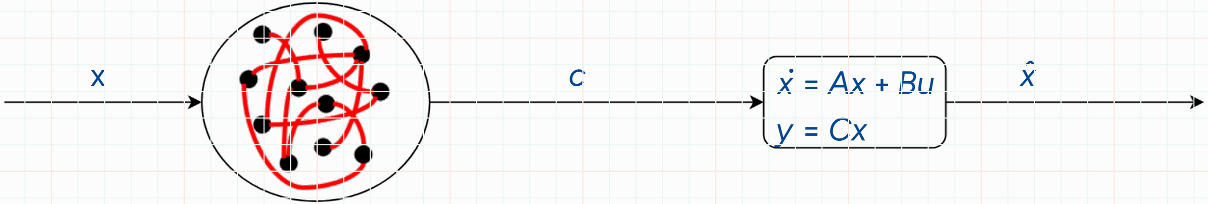
Control Concept



[HC19]

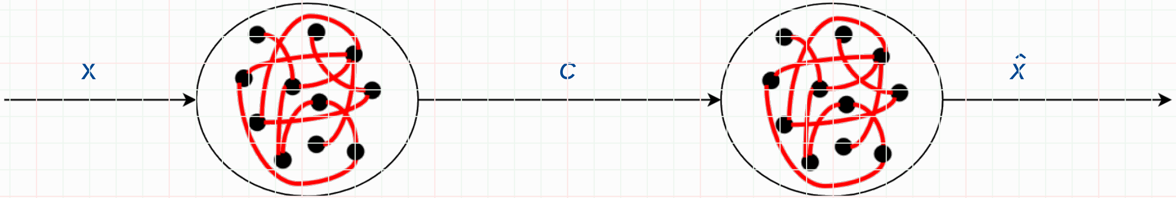
Add a separator here

Control Concept



[HC19]

Add a separator here



Control with SNN

$$u = \Gamma r + \Omega o(t) \tag{8}$$

Slow and Instantaneous decoding

$$\dot{V}(t) = -\lambda_V V(t) + \Omega^T B^T A e(t) + \Omega^T B^T c(t) + W^s r(t) + W^f o(t) + \sigma_V \eta(t) \tag{9}$$

Requires full state information on x and $\hat{x}$

$$c = \dot{x} - Ax \tag{10}$$

It is necessary on $B \in \mathbb{R}^{nxp}$

$$\text{rank}\,(B^T C^T) = p \tag{11}$$

Example in Ideal Conditions

works fine+ add plot

Example with 2 networks

works bad+ add plot

Conclusion

Conclusion

- Acceptable results in ideal conditions

Conclusion

- Acceptable results in ideal conditions
- Rank condition is limiting factor

Conclusion

- Acceptable results in ideal conditions
- Rank condition is limiting factor
- Network noise is invisible to the control

Conclusion

- Acceptable results in ideal conditions
- Rank condition is limiting factor
- Network noise is invisible to the control
- Simple open loop controller in the definition of c

Learning

Fast Learning rule

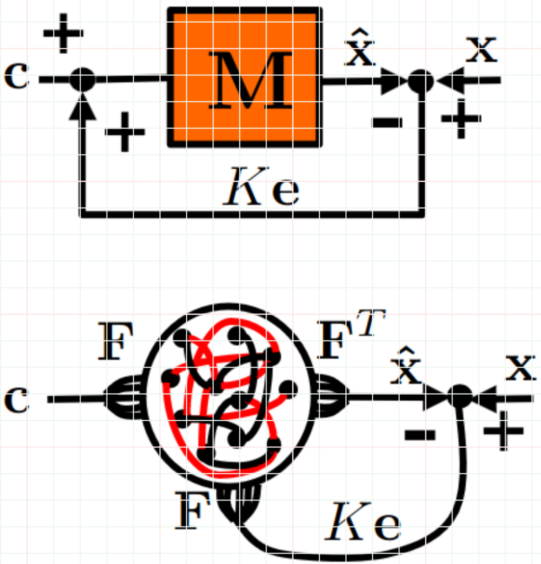
Slow Learning rule

Online Teacher-Student Scheme

Fast Learning rule

content...

Slow Learning rule



[BD15]

Online Teacher-Student Scheme for M under $\dot{x} = Mx + c$

Matrix update under squared loss

$$\delta M \propto e\hat{x}^T \longrightarrow \delta W^s \propto \Gamma \left(e\hat{x}^T \right) \Gamma^T \approx \Gamma e r \tag{12}$$

replace the F with Γ in the picture!

Learned Control

Conclusion

Conclusion

- Very limited applicability
- Open loop + rank condition limiting factor
- Too inaccurate learning of slow weights W^s
- Too dependent on initial conditions in learning

- In ideal conditions useable results achievable
- Only of theoretical interest
- Impressive accuracy
- Results are somewhat translatable to NEF and LSMs

Future Work

Future Work

- Enable non-linear dynamics

Future Work

- Enable non-linear dynamics
- Obey Dale's Law for neuron excitation and inhibition

Future Work

- Enable non-linear dynamics
- Obey Dale's Law for neuron excitation and inhibition
- Optimize Control

Future Work

- Enable non-linear dynamics
- Obey Dale's Law for neuron excitation and inhibition
- Optimize Control
- Learning of En- and Decoder Γ

Future Work

- Enable non-linear dynamics
- Obey Dale's Law for neuron excitation and inhibition
- Optimize Control
- Learning of En- and Decoder Γ
- Allow for synaptic delays

Frame title

Block

Lorem ipsum!

Bibliography

[BD15] Ralph Bourdoukan and Sophie Denève. “Enforcing balance allows local supervised learning in spiking recurrent networks”. In: **Advances in Neural Information Processing Systems**. Ed. by C. Cortes, N. Lawrence, D. Lee, M. Sugiyama, and R. Garnett. Vol. 28. Curran Associates, Inc., 2015. URL: `https://proceedings.neurips.cc/paper_files/paper/2015/file/3871bd64012152bfb53fdf04b401193f-Paper.pdf`.

[HC19] Fuqiang Huang and ShiNung Ching. “Spiking networks as efficient distributed controllers”. In: **Biological Cybernetics** 113.1 (Apr. 2019), pp. 179–190. ISSN: 0340-1200, 1432-0770. DOI: `10.1007/s00422-018-0769-7`. URL: `http://link.springer.com/10.1007/s00422-018-0769-7` (visited on 10/23/2022).

KTH
VETENSKAP
OCH KONST