



Spiking Neural Networks for Control

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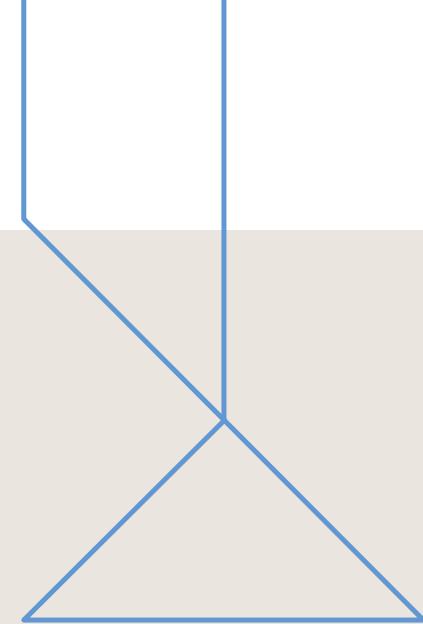
Simulation

Control

Learning

Combined Learning

Conclusion



Introduction

Goal / Motivation

Artificial SNN can already solve various cognitive task such as

- Memorization
- Basic Logic
- Simulation of Dynamic Systems
- Control

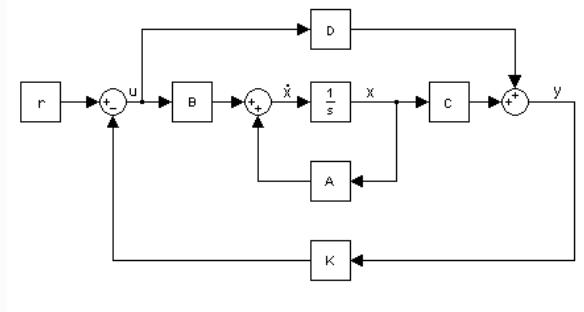
Although with varying levels of biologic plausibility. We set out to build a controlled dynamic system based on SNN using learning and biologic plausibility

- Allow for black-box deployment without manual parameter tuning
- “Limit ourselves to use the brains capabilities to design a controller”

What are we talking about

Control a Linear system

- Tracking of reference trajectory

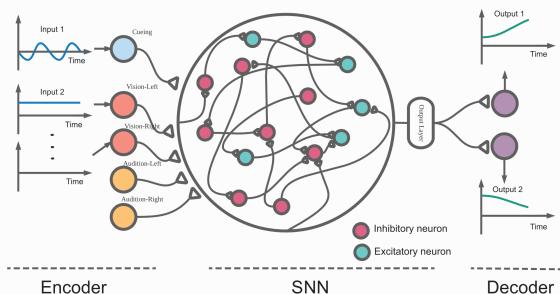


$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}\tag{1}$$

- Only stable systems

What are we talking about

Use Spiking neural networks



- Third Generation of NN
- Working with discrete spikes
- Inherently fit for temporal data

[Xue2022]

Method

1. Simulate

Use a spiking network to simulate a dynamic system

2. Control

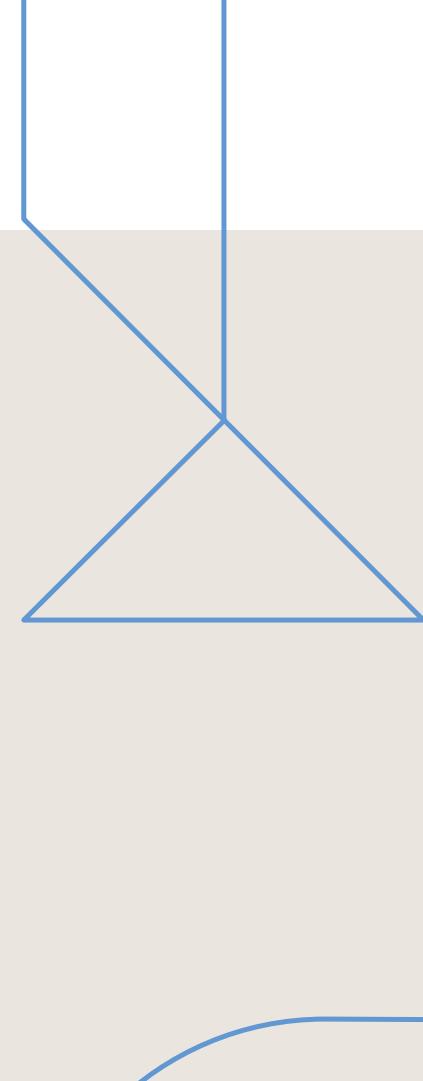
Devise a control scheme to control the network output

3. Learn

Apply biologically plausible learning rules to our network

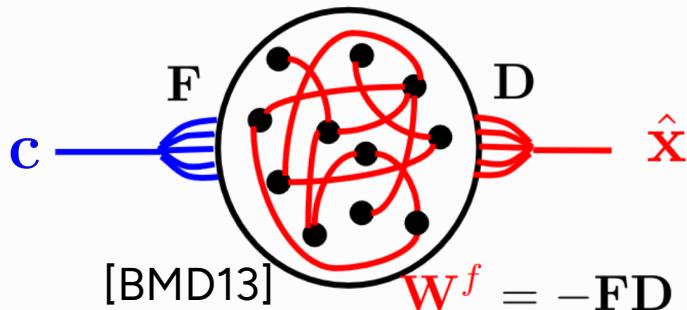
4. Combine

Integrate all three steps into a single controller



Simulation

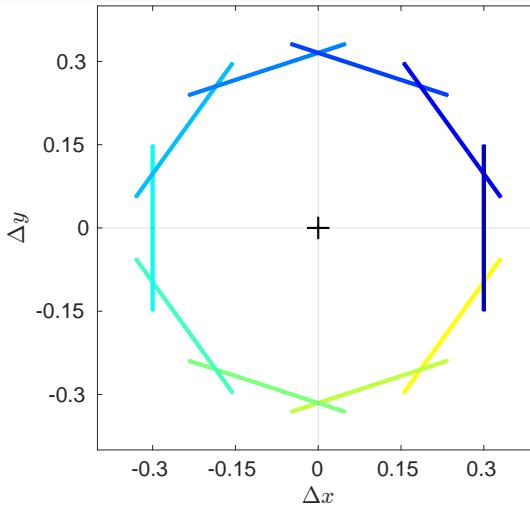
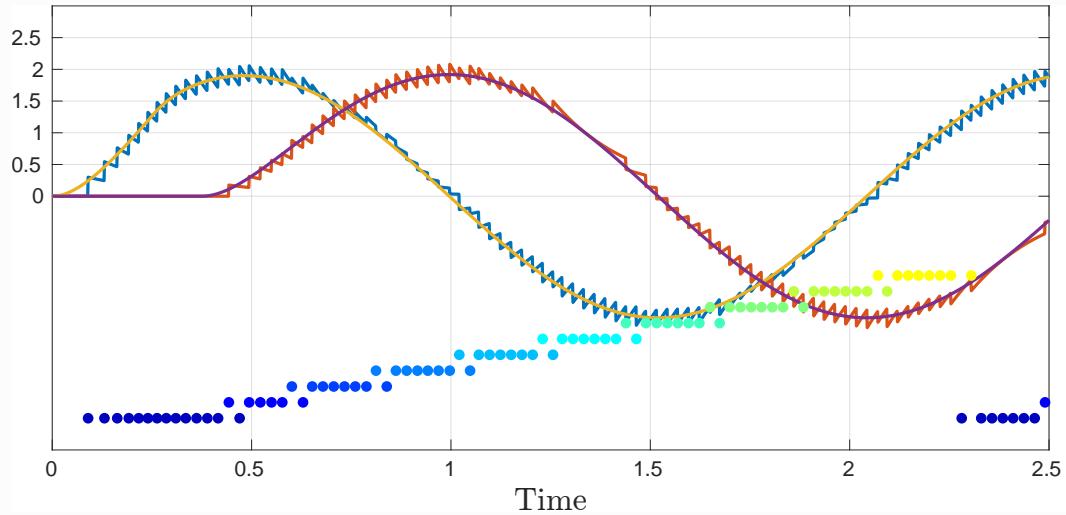
Simulation of Linear systems

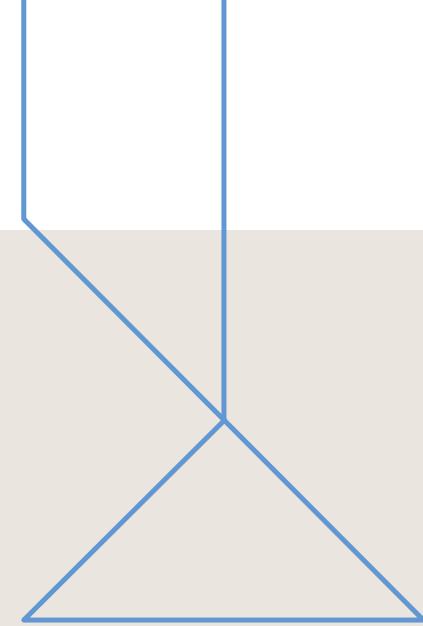


- Build NN that outputs \hat{x} from the system $\dot{x} = Ax + c$ given c
- Group of LIF neurons with intrinsic Voltage, tracking the projected error $V_i = F_i(x - \hat{x}) + \mu r_i$
- Network decoding $\hat{x} = F^T r$

$$\dot{V} = -\lambda_V V + Fc + W^f o(t) + W^s r(t) + \sigma_V \eta(t)$$

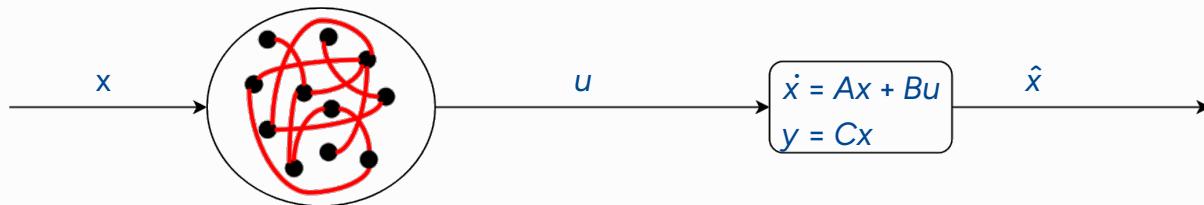
Example Simulation





Control

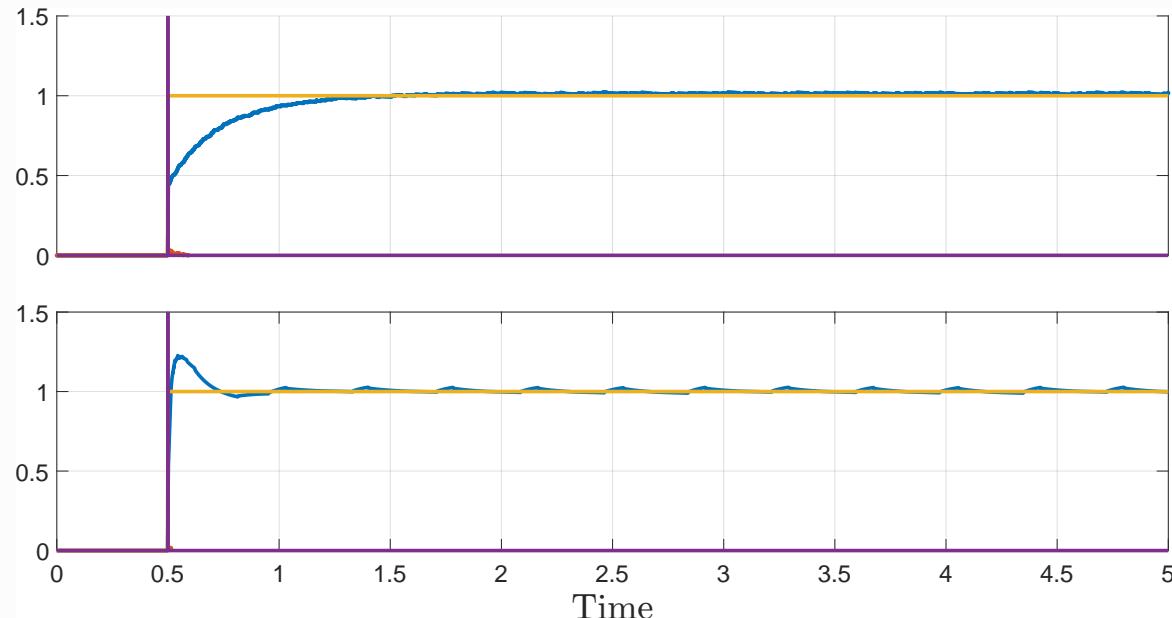
Control Concept

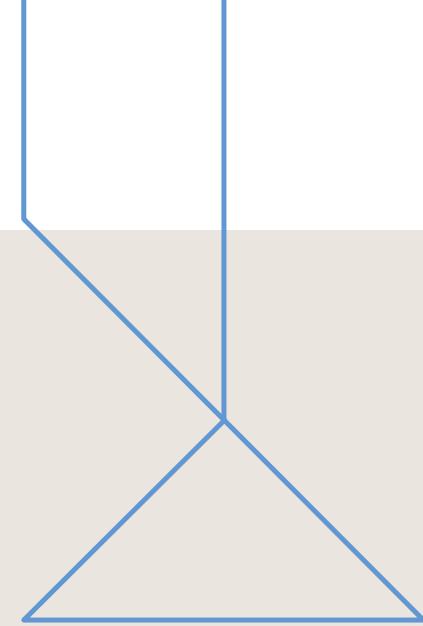


[HC19]

- (Almost) identical network architecture
- Network output is external input into (previous) simulating network \leftrightarrow Network state contains control signal
- Governed by PD-control as $c = \dot{x} - Ax$
- In presence of output matrix $C \neq I \leftrightarrow \text{rank}(B^T C^T) = \text{rank}(B^T)$

Example





Learning

Learning rules [BD15]

Slow Learning rule $W^s = F(A + \lambda_d \mathbf{I})F^T$

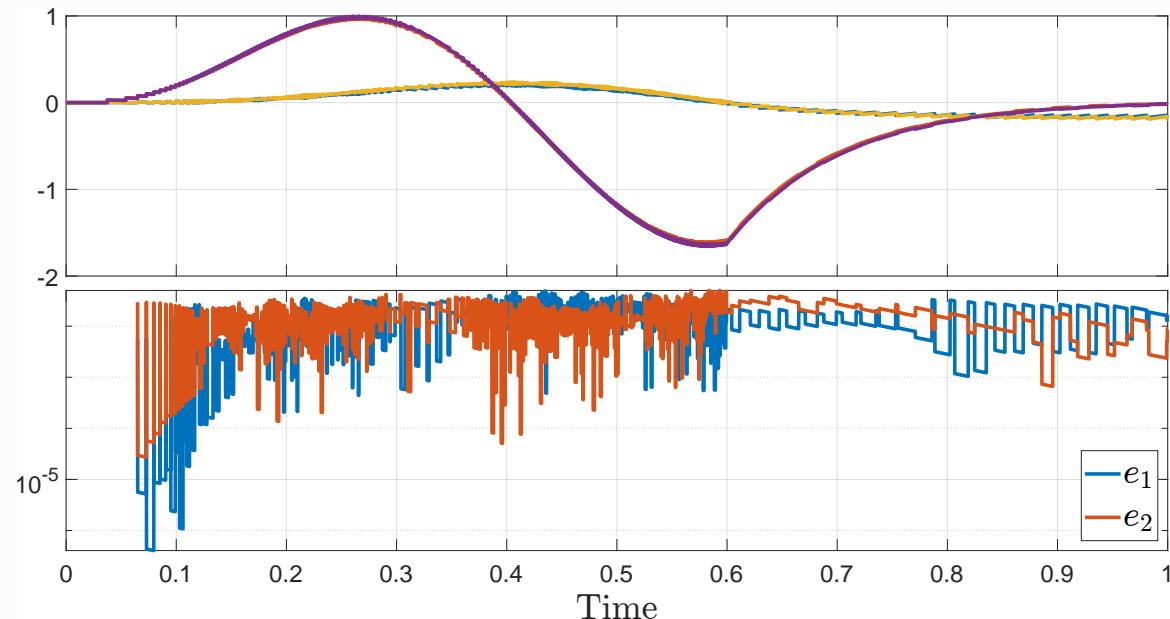
- Online Learning of Student teacher dynamics $\dot{\hat{x}} = M\hat{x} + c$
- Error Feedback e during Training
- $\delta M \propto e\hat{x}^T \rightarrow \delta W^s \propto F(e\hat{x}^T)F^T \approx Fer^T$
- Supervised Learning rule

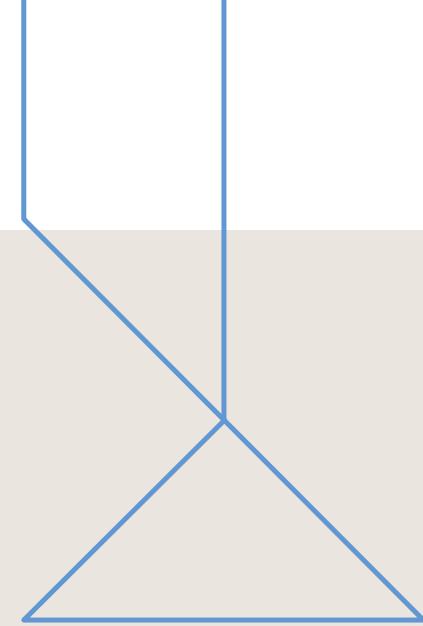
$$V_i = F_i(x - \hat{x}) - \mu r_i$$

Fast Learning rule $W^f = FF^T + \mu \mathbf{I}$

- Voltage measures system error
- Minimize average Voltage outside of Neuron Threshold
- Biologically plausible pre \times post locally
- Unsupervised Learning Rule

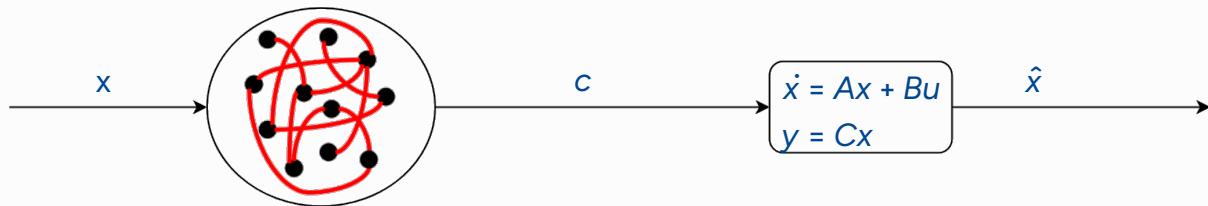
Example





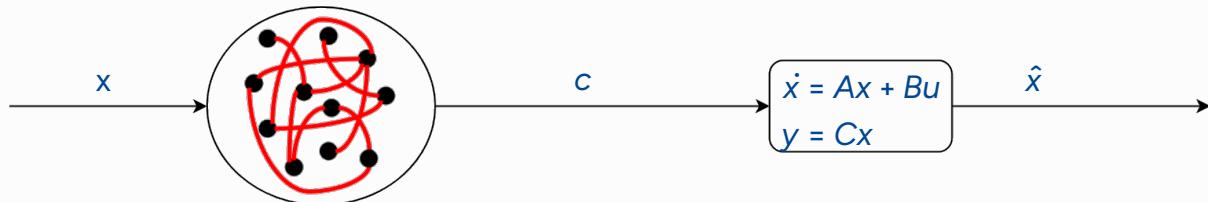
Combined Learning

Control Concept

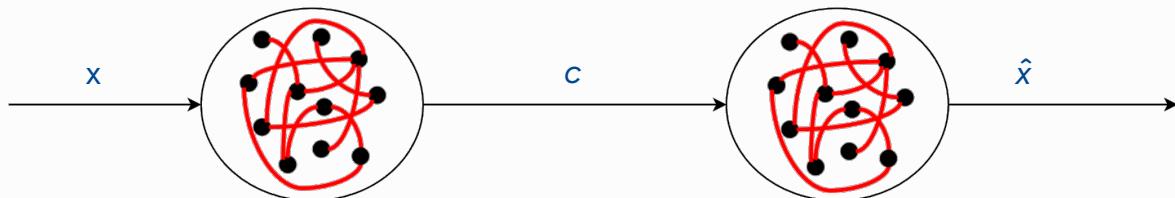


[HC19]

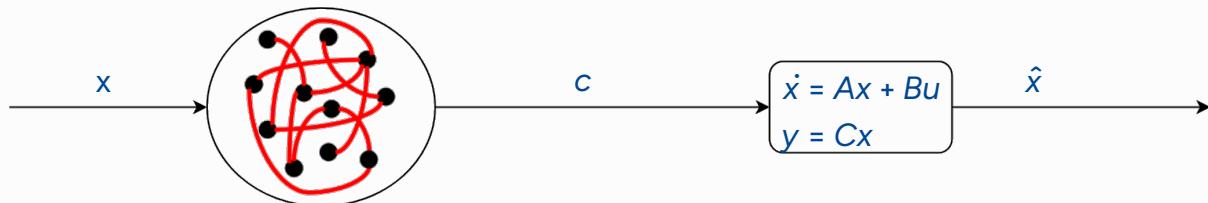
Control Concept



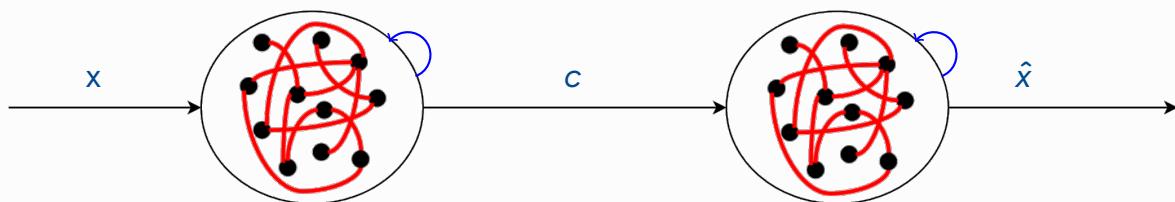
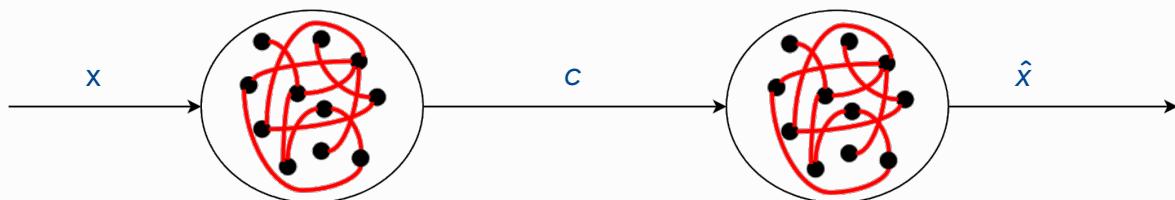
[HC19]



Control Concept



[HC19]

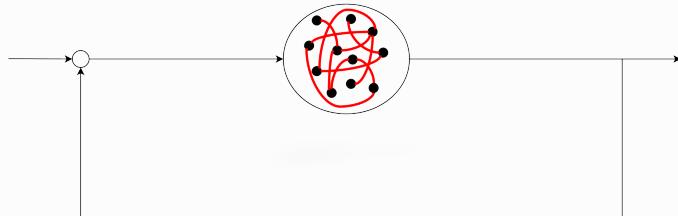
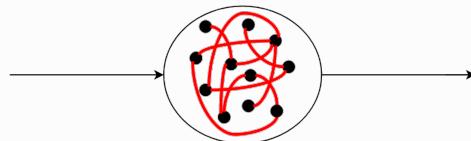
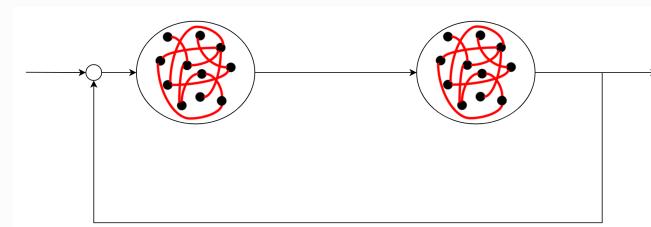


Problems

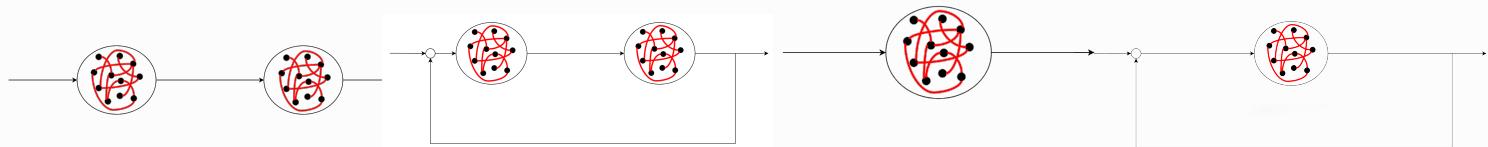
In conjunction, problems can arise:

- Divergence in Learning
- Open loop control
 - Noise detection or correction
 - Compensation of Training errors
- High dependence on governing dynamics $c_{ref} = \dot{x}_{ref} - Ax_{ref}$
- Orthonormality restriction on Input Matrix $B \in \mathbb{B} := \{M \mid MM^T = I\}$
- No biologically plausible Learning rule for control network available

Control Concept II

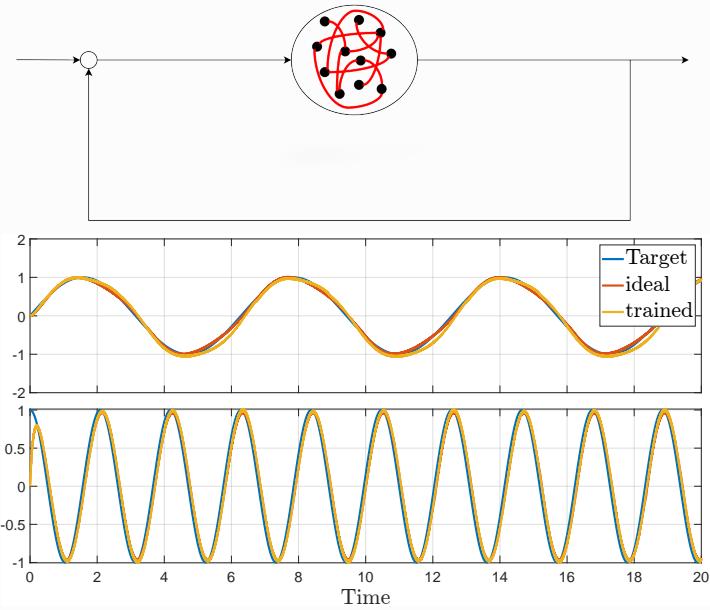
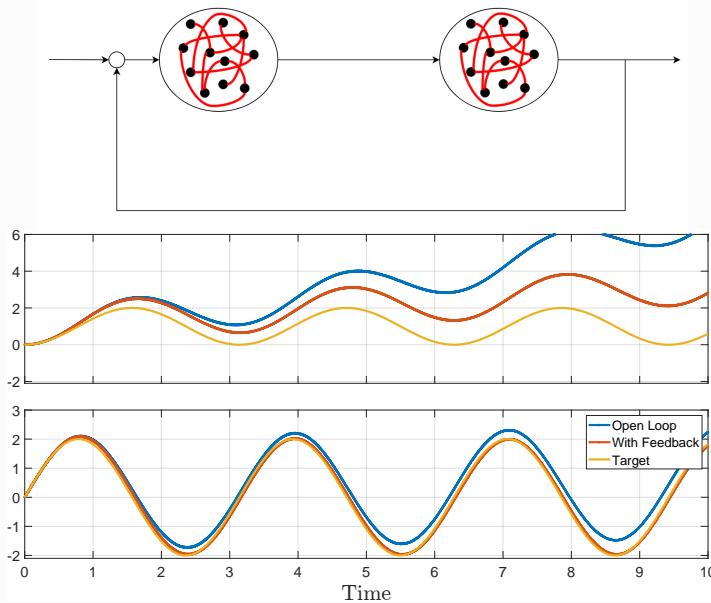


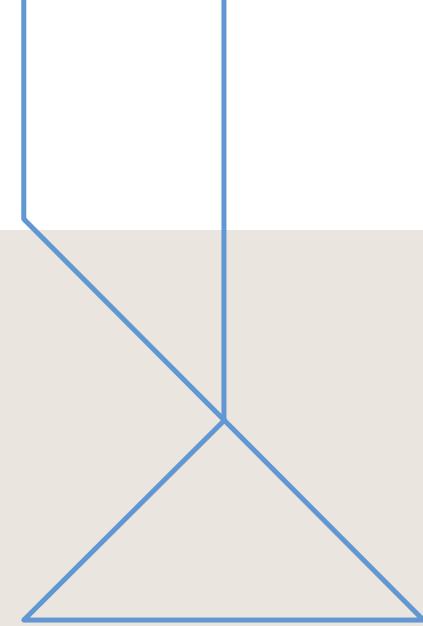
Summary



High c_{ref} dependence	Yes	No	Yes	No
Open loop control	Yes	No	Yes	No
Implausible B learning rule	Yes	Yes	No	No
Orthonormality restriction	No	No	Yes	Yes

Examples





Conclusion

Conclusion

- Open loop and inaccurate learning of slow weights W^s need to be addressed.
- Highly dependent on initial conditions in learning
- Impressive accuracy
- In ideal conditions useable results achievable
- Limited Applicability → Only of theoretical Interest
- Results are somewhat translatable to NEF and LSMs

Choice between biologic plausibility or and Input Matrix Restriction for accurate results

Future Work

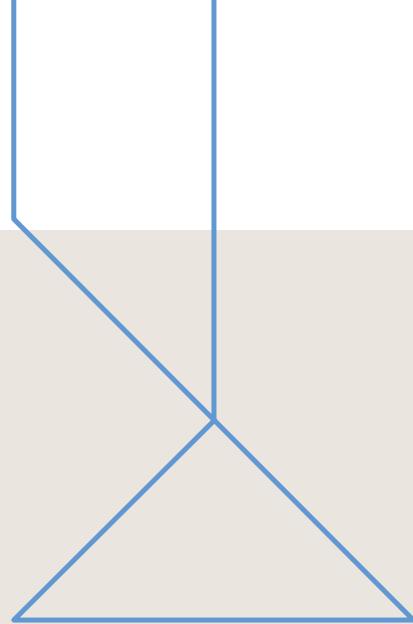
- Enable non-linear dynamics
- Obey Dale's Law for neuron excitation and inhibition
- Learning of En- and Decoder F
- Allow for synaptic delays

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- [Bre+20] Wieland Brendel, Ralph Bourdoukan, Pietro Vertechi, Christian K. Machens, and Sophie Denève. "Learning to represent signals spike by spike". In: **PLOS Computational Biology** 16.3 (Mar. 16, 2020). Publisher: Public Library of Science. DOI: [10.1371/journal.pcbi.1007692](https://doi.org/10.1371/journal.pcbi.1007692). (Visited on 09/20/2022).

Bibliography II

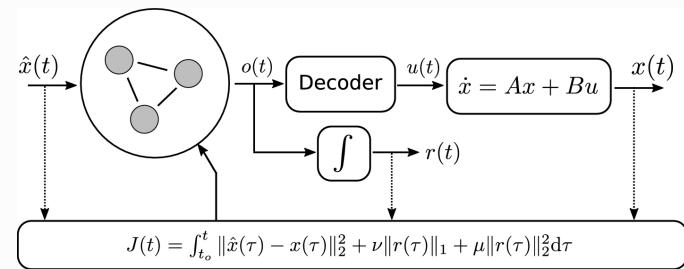
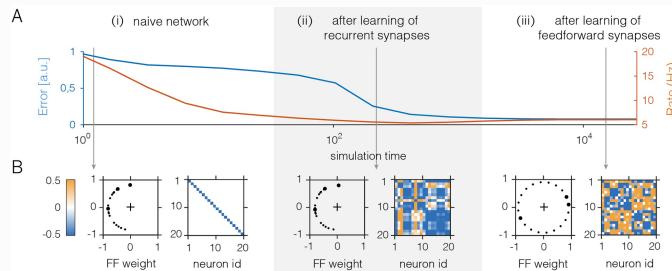
- [HC19] Fuqiang Huang and ShiNung Ching. "Spiking networks as efficient distributed controllers". In: **Biological Cybernetics** 113.1 (Apr. 2019), pp. 179–190. DOI: [10.1007/s00422-018-0769-7](https://doi.org/10.1007/s00422-018-0769-7). (Visited on 10/23/2022).



BackupSlides

Takeaways

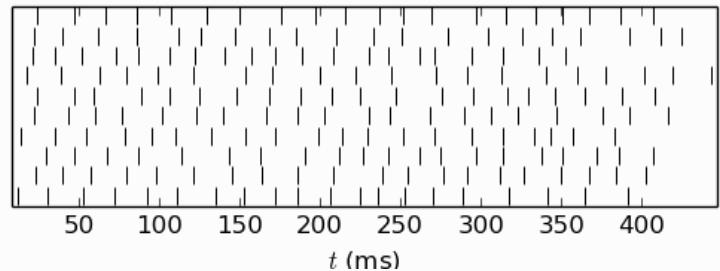
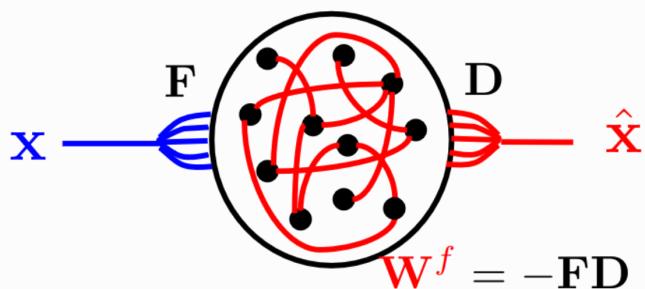
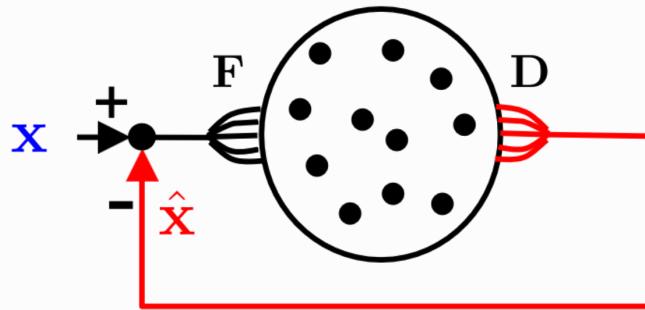
Do not trust the plots



Not the learned decoder! [Bre+20]

No Control Loop! [HC19]

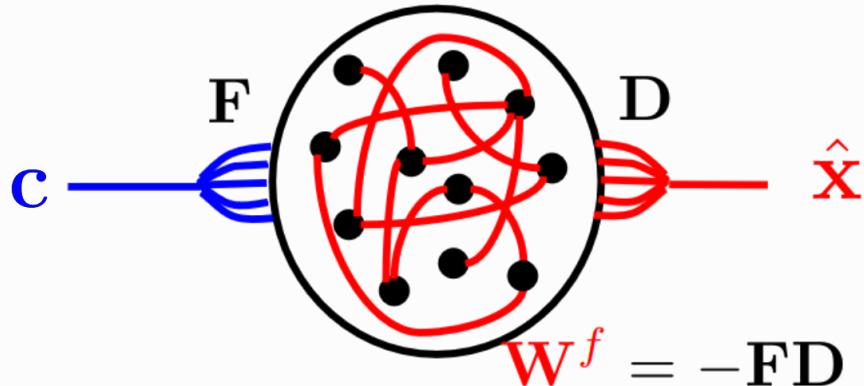
Autoencoder



$$\begin{aligned}\hat{x} &= Do(t) \\ \dot{r} &= -\lambda r + o(t)\end{aligned}\quad (2)$$

$$\dot{r} = -\lambda r + \sigma(t)$$

Autoencoder II

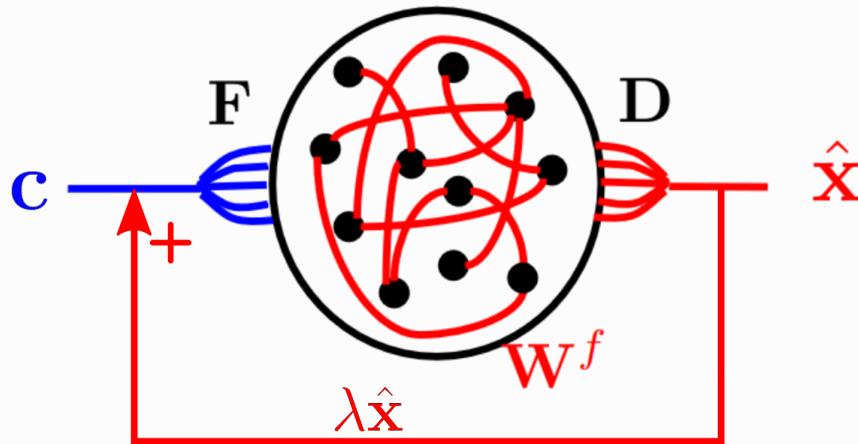


$$\begin{aligned}\dot{x} &= -\lambda x + c \\ \hat{x} &= Dr\end{aligned}\tag{3}$$

$$\dot{r} = -\lambda r + o(t)$$

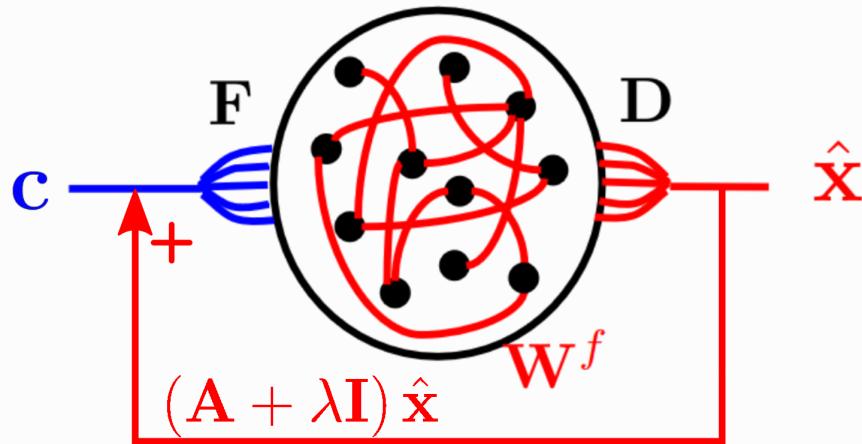
$$\hat{x} = Dr$$

Autoencoder III



$$\dot{x} = c$$

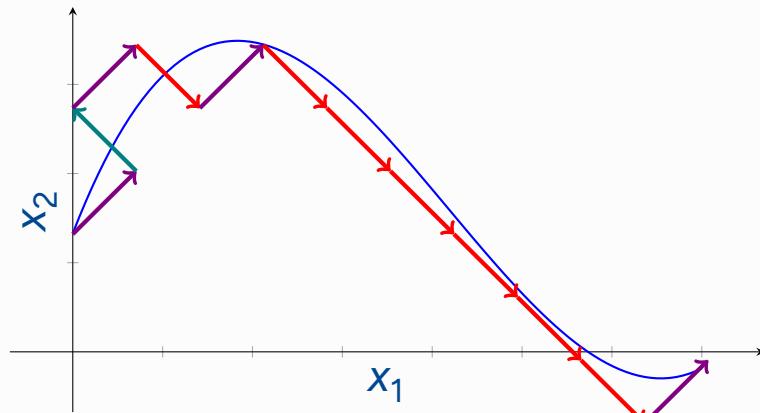
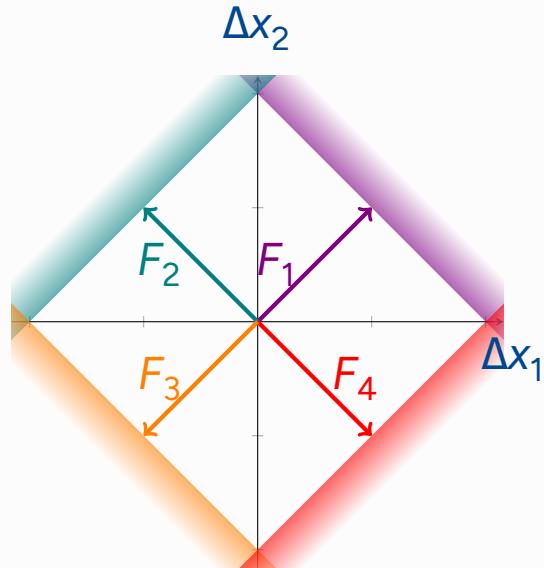
(4)



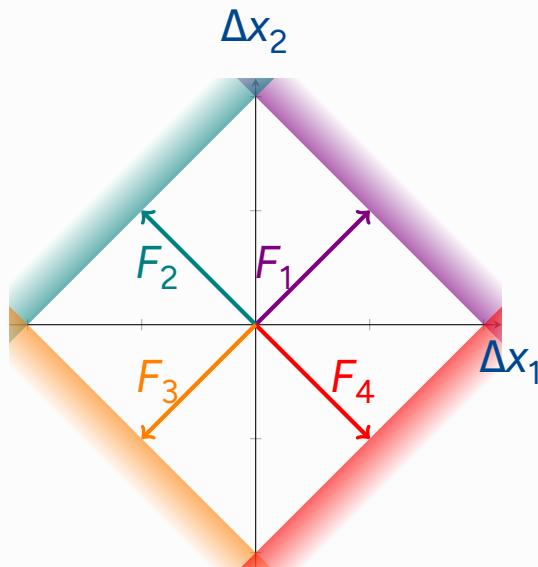
$$\dot{x} = Ax + c$$

(5)

Geometric



Geometric



Minimize the cost J (Greedy)

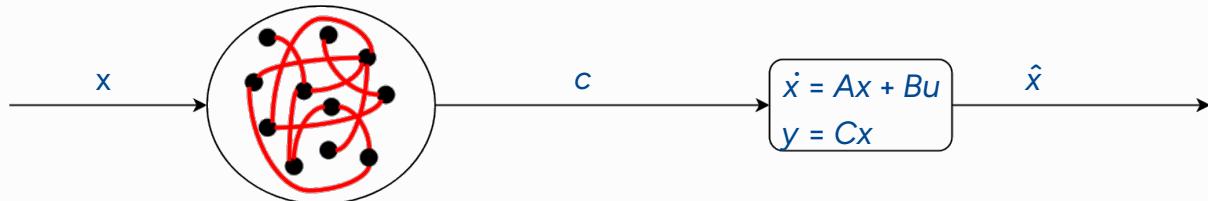
$$J = \int_0^T \|x - \hat{x}\|_2^2 + C(r) dt \quad (6)$$

$$\begin{aligned} V_i &= F_i(x - \hat{x}) - \mu r_i \\ \dot{V}_i &= -\lambda_V V_i + F_i c(t) \\ &\quad + W^f o(t) + W^s r(t) + \sigma_V \eta(t) \end{aligned} \quad (7)$$

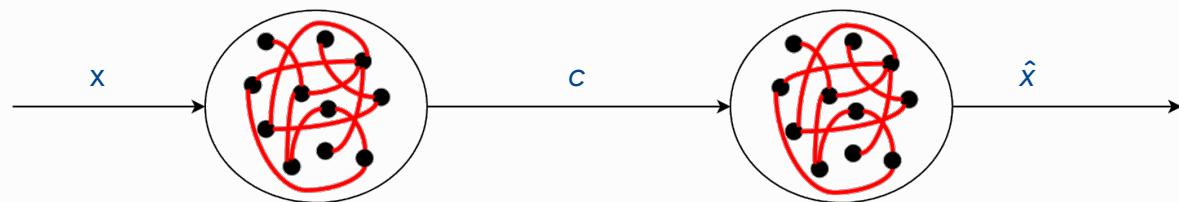
$$W^f = FF^T + \mu I$$

$$W^s = F(A + \lambda_d I)F^T$$

Control Concept



[HC19]



Control with SNN

$$u = F^T r + \Omega o(t) \quad (8)$$

Slow and Instantaneous decoding

It is necessary on $B \in \mathbb{R}^{n \times p}$

$$\text{rank}(B^T C^T) = p \quad (11)$$

$$\begin{aligned} \dot{V}(t) = & -\lambda_V V(t) + \Omega^T B^T A e(t) + \Omega^T B^T c(t) \\ & + W^s r(t) + W^f o(t) + \sigma_v \eta(t) \end{aligned} \quad (9)$$

Requires full state information on x
and \hat{x}

$$c = \dot{x} - Ax \quad (10)$$

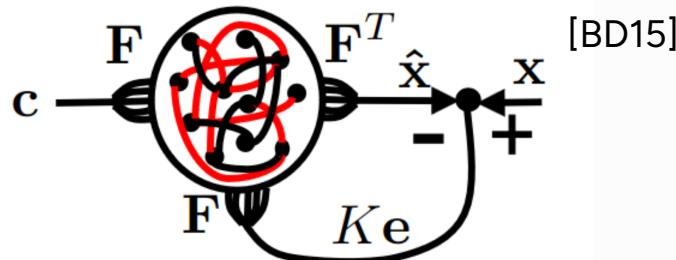
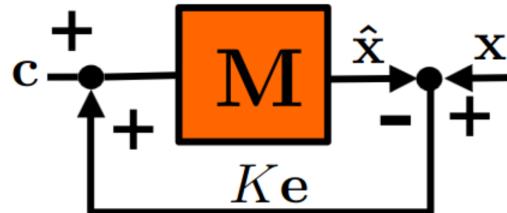
Conclusion

- Acceptable results in ideal conditions
- Rank condition is limiting factor
- Network noise is invisible to the control
- Simple open loop controller in the definition of c

$$\dot{\hat{x}} = (M - K\mathbf{I})\hat{x} + c + Kx$$

$$W^s = F(A + \lambda_d \mathbf{I})F^T$$

Slow Learning rule



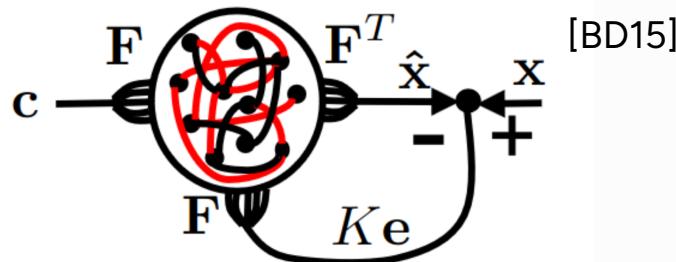
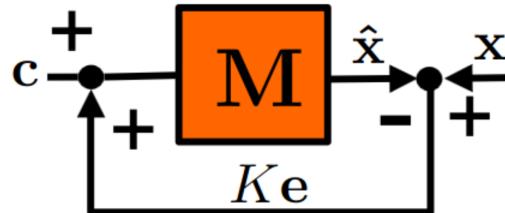
Online Teacher-Student Scheme for
 M under $\dot{x} = Mx + c$
 Matrix update under squared loss

$$\delta M \propto e\hat{x}^T \rightarrow \delta W^s \propto F^T (e\hat{x}^T)F \approx F^T e r \quad (12)$$

$$\dot{\hat{x}} = (M - K\mathbf{I})\hat{x} + c + Kx$$

$$W^s = F(A + \lambda_d \mathbf{I})F^T$$

Fast Learning rule



Online Teacher-Student Scheme for
 M under $\dot{x} = Mx + c$
 Matrix update under squared loss

$$\delta M \propto e\hat{x}^T \rightarrow \delta W^s \propto F^T (e\hat{x}^T)F \approx F^T e r \quad (13)$$

Conclusion

- Very limited applicability
- Open loop + rank condition limiting factor
- Too inaccurate learning of slow weights W^s
- Too dependent on initial conditions in learning
- In ideal conditions useable results achievable
- Only of theoretical interest
- Impressive accuracy
- Results are somewhat translatable to NEF and LSMs

