Spiking Neural Networks for Control

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Introduction

Goal / Motivation

Artificial SNN can already solve various cognitive task such as

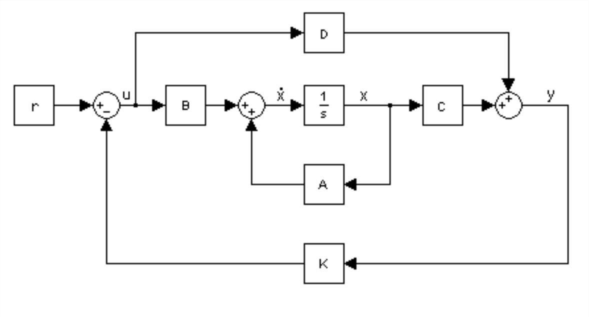
- Memorization
- Basic Logic
- Simulation of Dynamic Systems
- Control

Although with varying levels of biologic plausibility. We set out to build a controlled dynamic system based on SNN using learning and biologic plausibility

- Allow for black-box deployment without manual parameter tuning
- ”Limit ourselves to use the brains capabilities to design a controller”

What are we talking about

Control a Linear system



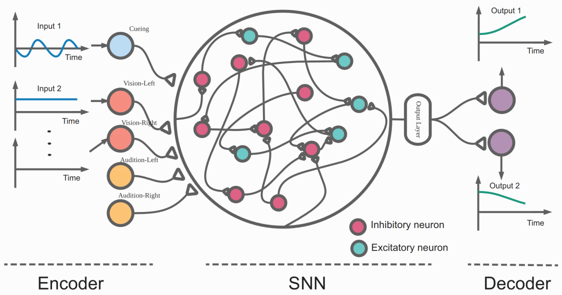
- Tracking of reference trajectory

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

- Only stable systems

What are we talking about

Use Spiking neural networks



[Xue+22]

- Third Generation of NN
- Working with discrete spikes
- Inherently fit for temporal data

Method

1. Simulate

Use a spiking network to simulate a dynamic system

2. Control

Devise a control scheme to control the network output

3. Learn

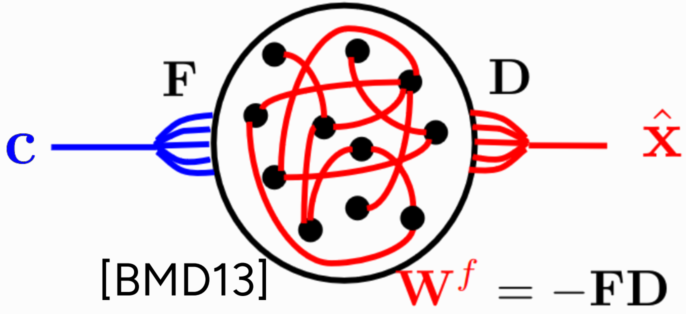
Apply biologically plausible learning rules to our network

4. Combine

Integrate all three steps into a single controller

Simulation

Simulation of Linear systems

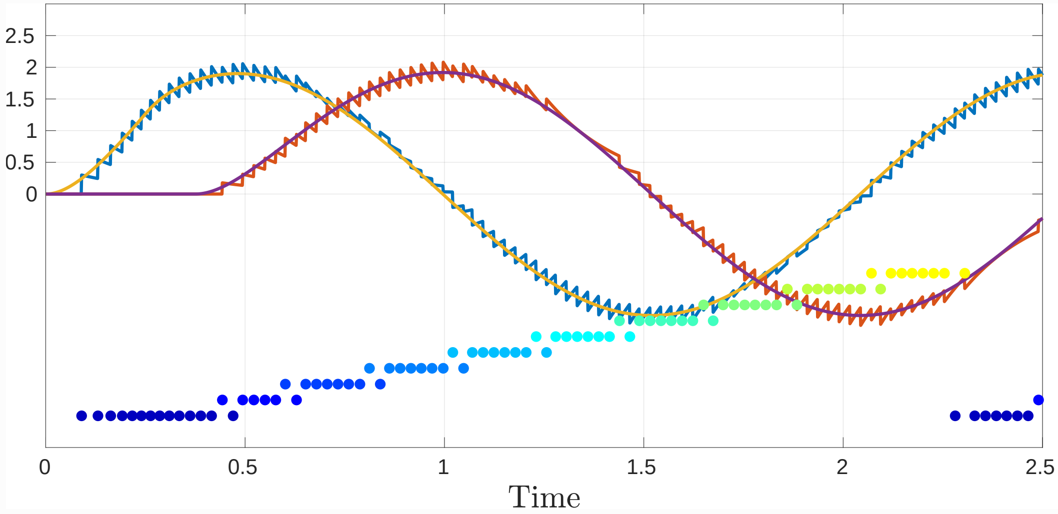


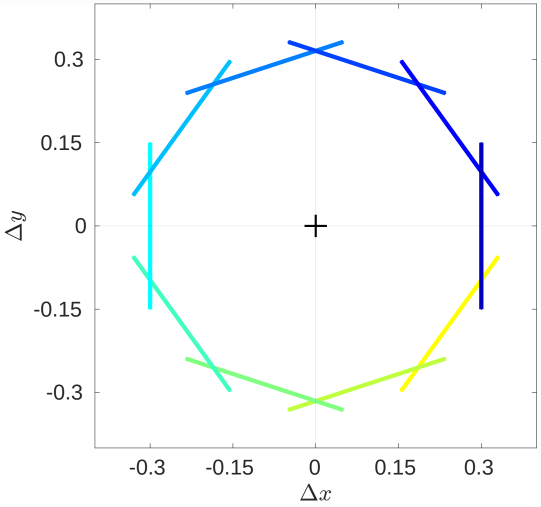
[BMD13]

- Build NN that outputs $\hat{x}$ from the system $\dot{x} = Ax + c$ given c
- Group of LIF neurons with with intrinsic Voltage, tracking the projected error $V_i = F_i(x - \hat{x}) + \mu r_i$
- Network decoding $\hat{x} = F^T r$

$$\dot{V} = -\lambda_V V + Fc + W^f o(t) + W^s r(t) + \sigma_V \eta(t)$$

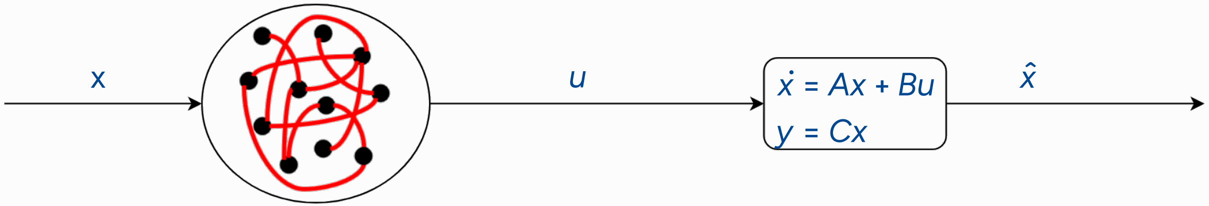
Example Simulation





Control

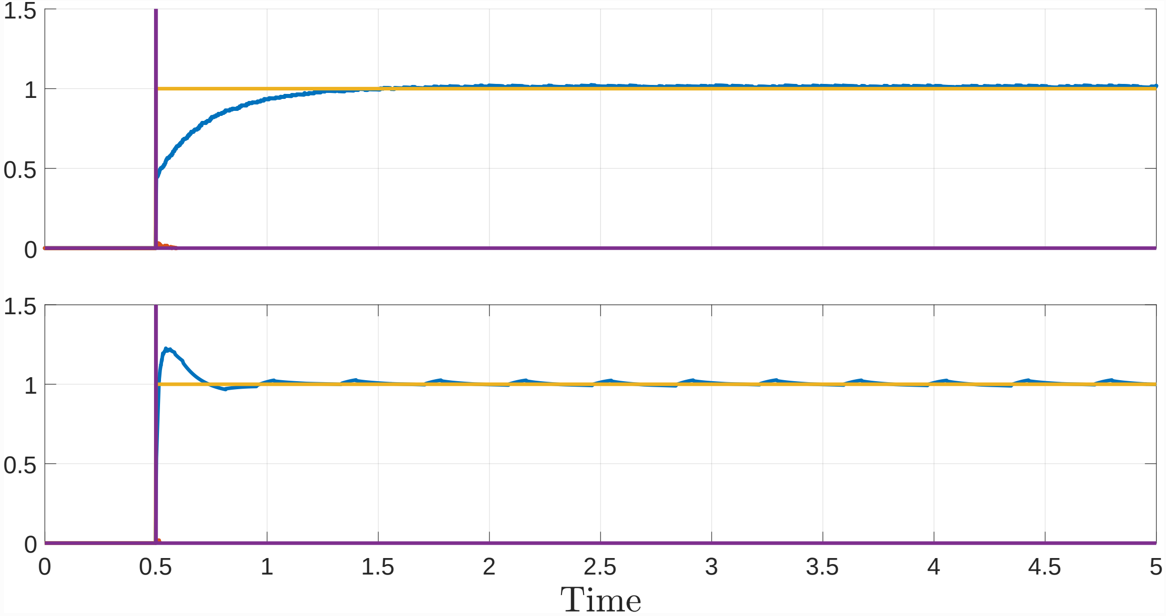
Control Concept



[HC19]

- (Almost) identical network architecture
- Network output is external input into (previous) simulating network $\longleftrightarrow$ Network state contains control signal
- Governed by PD-control as $c = \dot{x} - Ax$
- In presence of output matrix $C \neq \mathbf{I} \leftrightarrow \mathrm{rank}(B^T C^T) = rank(B^T)$

Example



Learning

$V_i = F_i(x - \hat{x}) - \mu r_i$

Learning rules [BD15]

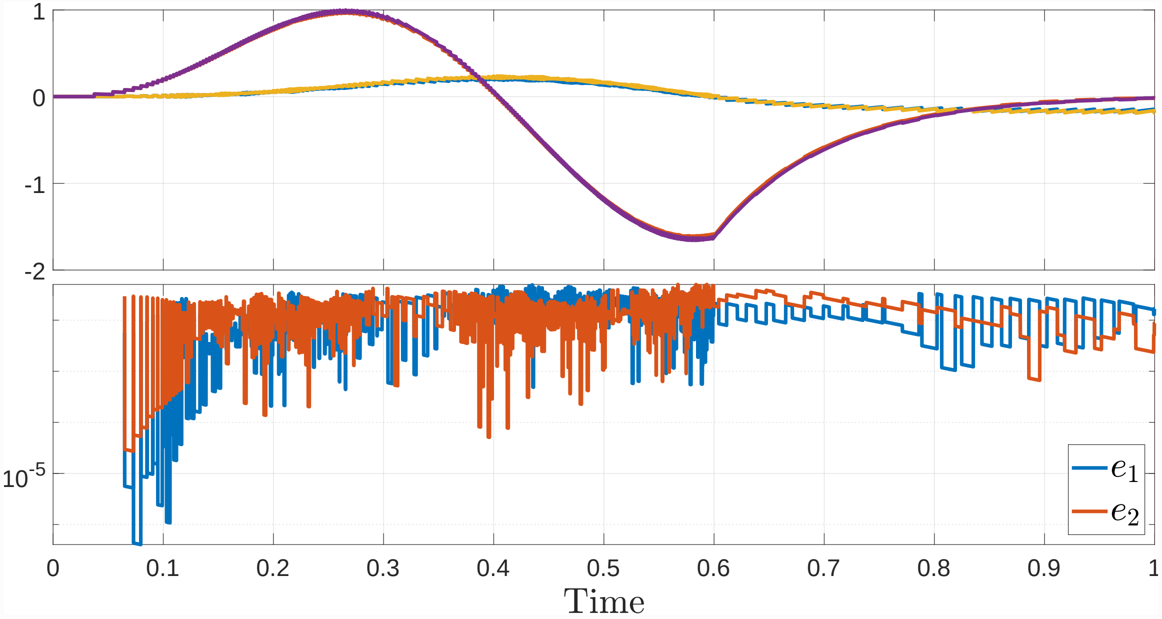
Slow Learning rule $W^s = F(A + \lambda_d \mathbf{I})F^T$

- Online Learning of Student teacher dynamics $\dot{\hat{x}} = M\hat{x} + c$
- Error Feedback e during Training
- $\delta M \propto e\hat{x}^T \longrightarrow \delta W^s \propto F(e\hat{x}^T)F^T \approx Fer^T$
- Supervised Learning rule

Fast Learning rule $W^f = FF^T + \mu \mathbf{I}$

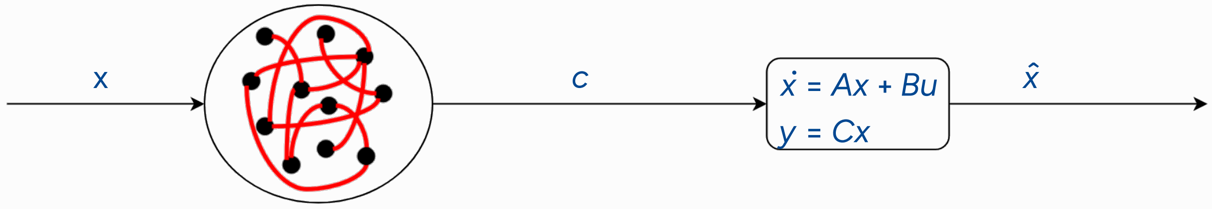
- Voltage measures system error
- Minimize average Voltage outside of Neuron Threshold
- Biologically plausible pre × post locally
- Unsupervised Learning Rule

Example



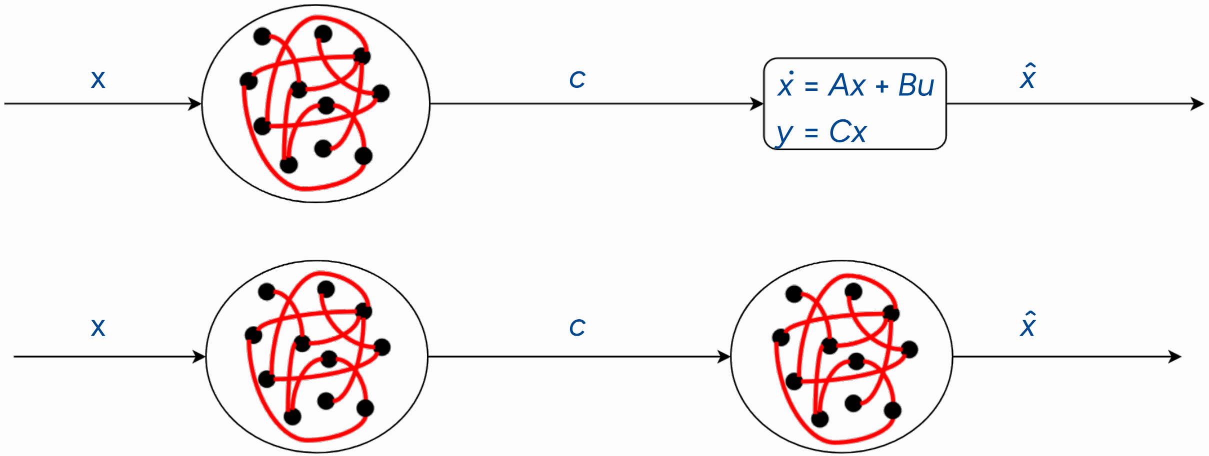
Combined Learning

Control Concept



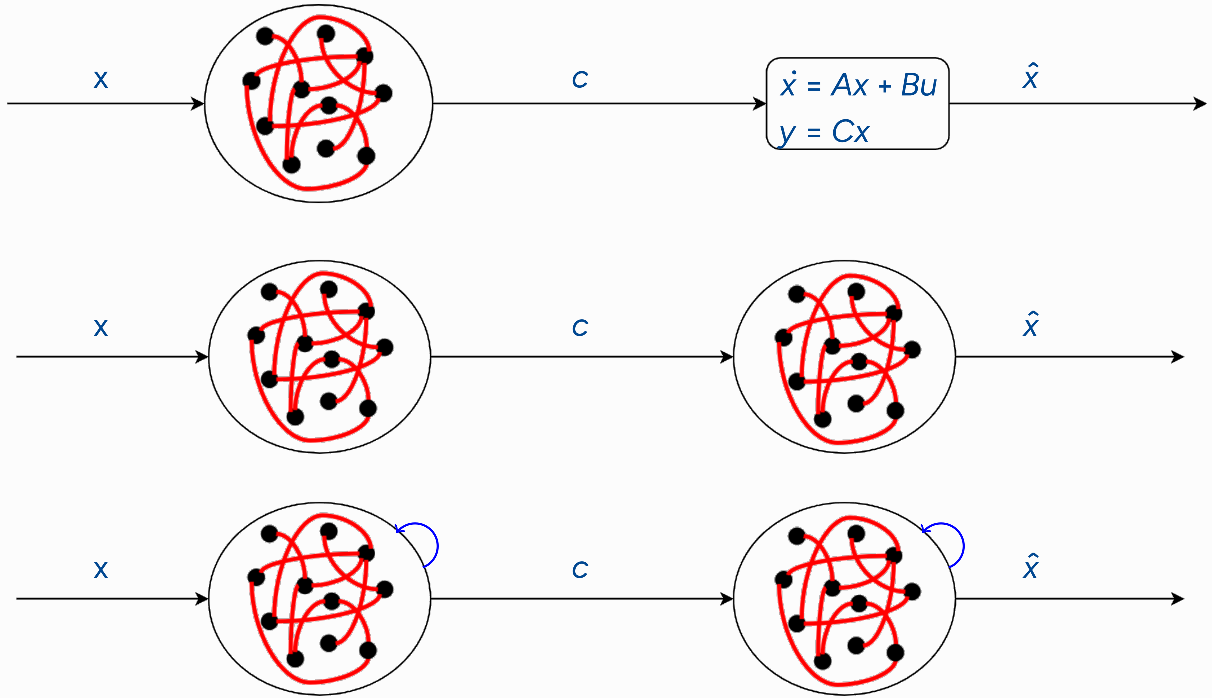
[HC19]

Control Concept



[HC19]

Control Concept



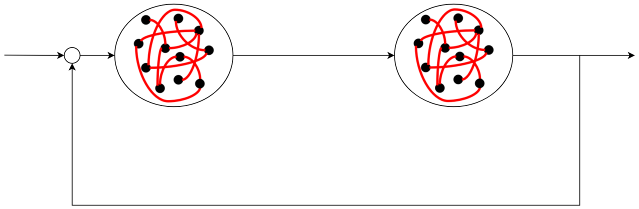
[HC19]

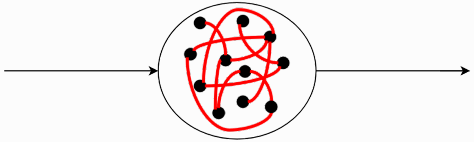
Problems

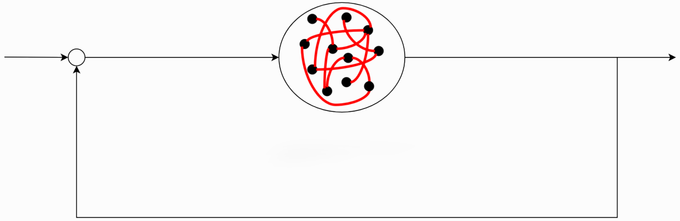
In conjunction, problems can arise:

- Divergence in Learning
- Control with Noise
- Open loop control
  - incapable of noise detection or correction
  - No compensation of Training errors
- No biologically plausible Learning rule for control network available

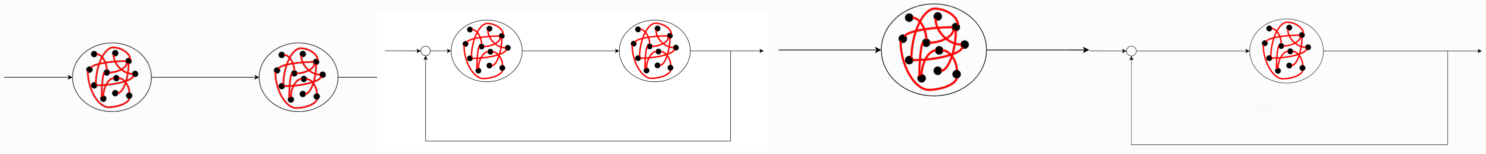
Control Concept II



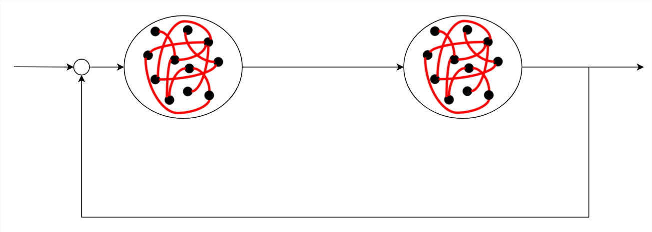




Summary



High c_{ref} dependence	Yes	No	Yes	No
Open loop control	Yes	No	Yes	No
Implausible B learning rule	Yes	Yes	No	No
Orthonormality restriction	No	No	Yes	Yes



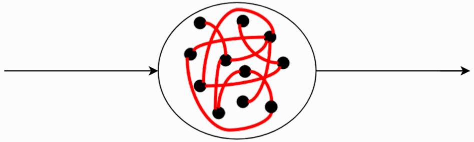
Dual Network with Feedback

No Learning rule for control network available

~~Open loop Control:~~

- ~~Incapable of noise detection or correction~~
- ~~No Compensation of Training error~~

~~Highly dependent on governing dynamics from~~ $\cancel{c_{\text{contr}} = \dot{x}_{\text{ref}} - Ax_{\text{ref}}}$



Single Network

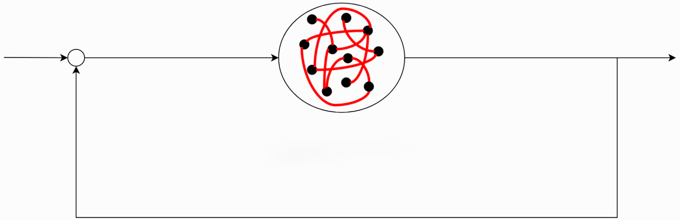
~~No Learning rule for control network available~~

Open Loop Control:

- Incapable of noise detection or correction
- No Compensation of Training error

Highly dependent on governing dynamics from $c_{\text{contr}} = \dot{x}_{\text{ref}} - Ax_{\text{ref}}$

Orthonormality restriction on Input Matrix $B \in \mathbb{B} := \left\{ M \mid MM^T = \mathbf{I} \right\}$



Single Network with Feedback

~~No Learning rule for control network available~~

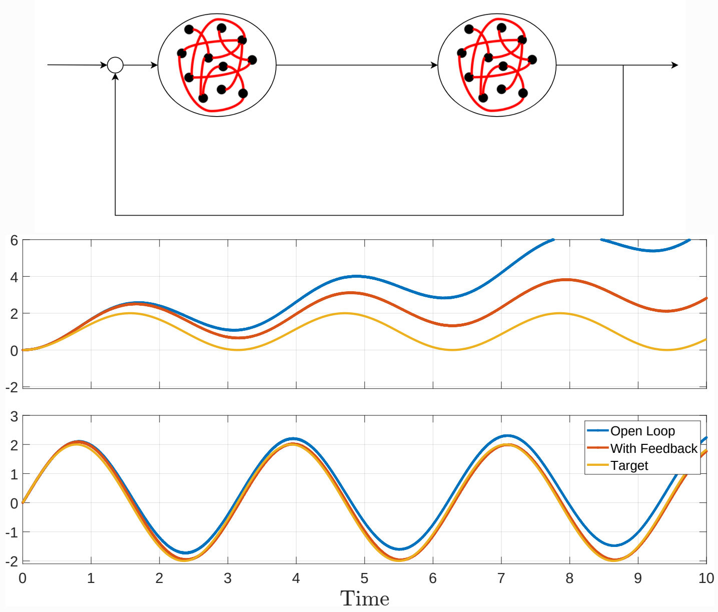
~~Open loop Control:~~

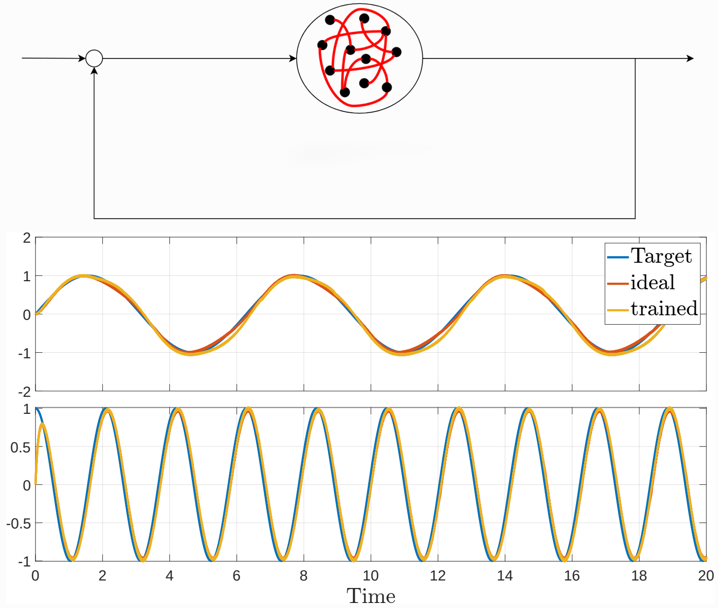
- ~~Incapable of noise detection or correction~~
- ~~No Compensation of Training error~~

~~Highly dependent on governing dynamics from~~ $\cancel{c_{\text{contr}} = \dot{x}_{\text{ref}} - Ax_{\text{ref}}}$

Orthonormality restriction on Input Matrix $B \in \mathbb{B} := \{M \mid MM^T = \mathbf{I}\}$

Examples


Open Loop
With Feedback
Target
Time


Target
ideal
trained
Time

Conclusion

Conclusion

- Open loop and inaccurate learning of slow weights W^s need to be addressed.
- Highly dependent on initial conditions in learning
- Impressive accuracy

- In ideal conditions useable results achievable
- Limited Applicability $\rightarrow$ Only of theoretical Interest
- Results are somewhat translatable to NEF and LSMs

Choice between biologic plausibility or and Input Matrix Restriction for accurate results

Future Work

- Enable non-linear dynamics
- Obey Dale's Law for neuron excitation and inhibition
- Learning of En- and Decoder F
- Allow for synaptic delays

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[BD15] Ralph Bourdoukan and Sophie Denève. “Enforcing balance allows local supervised learning in spiking recurrent networks”. In: **Advances in Neural Information Processing Systems**. Ed. by C. Cortes, N. Lawrence, D. Lee, M. Sugiyama, and R. Garnett. Vol. 28. Curran Associates, Inc., 2015. URL: https://proceedings.neurips.cc/paper_files/paper/2015/file/3871bd64012152bfb53fdf04b401193f-Paper.pdf.

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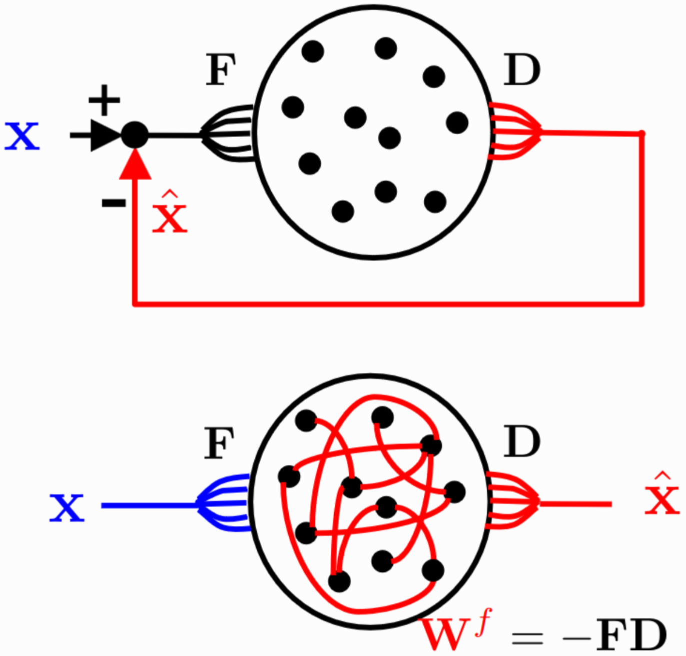
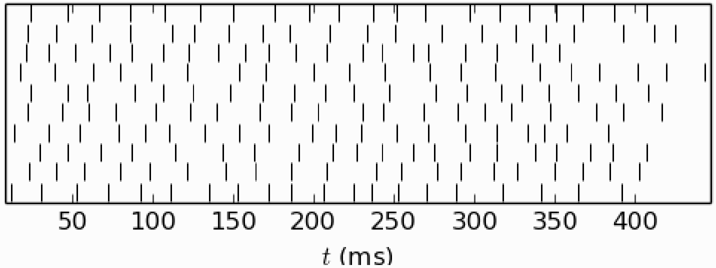
BackupSlides

Autoencoder

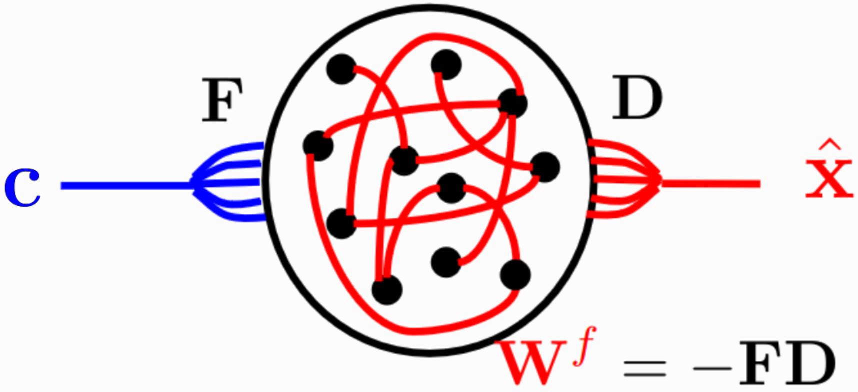




$$\begin{aligned} \hat{x} &= Do(t) \\ \dot{r} &= -\lambda r + o(t) \end{aligned} \tag{2}$$

$$\dot{r} = -\lambda r + \sigma(t)$$

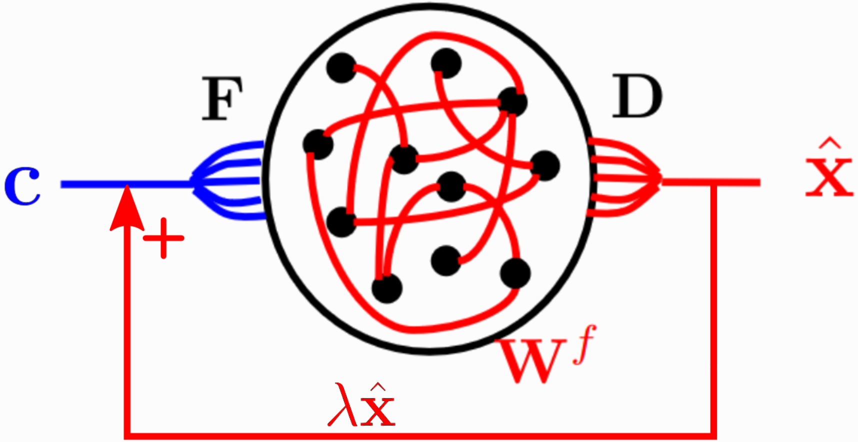
Autoencoder II

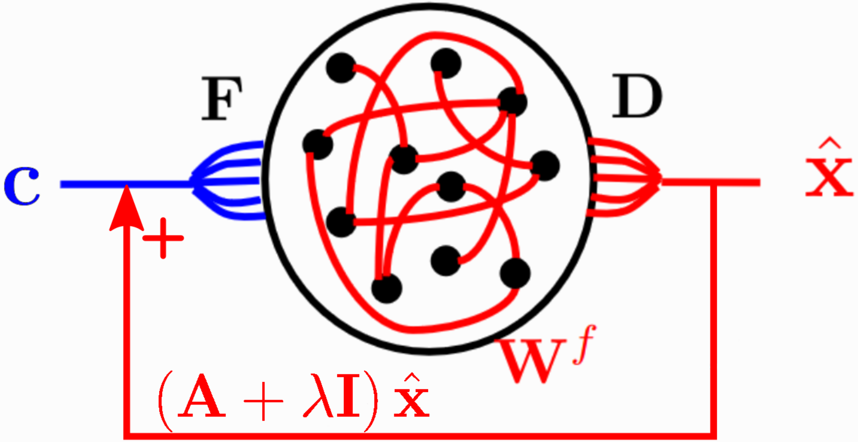


$$\begin{aligned} \dot{x} &= -\lambda x + c \\ \hat{x} &= Dr \end{aligned} \tag{3}$$

$$\dot{r} = -\lambda r + o(t)$$
$$\hat{x} = Dr$$

Autoencoder III

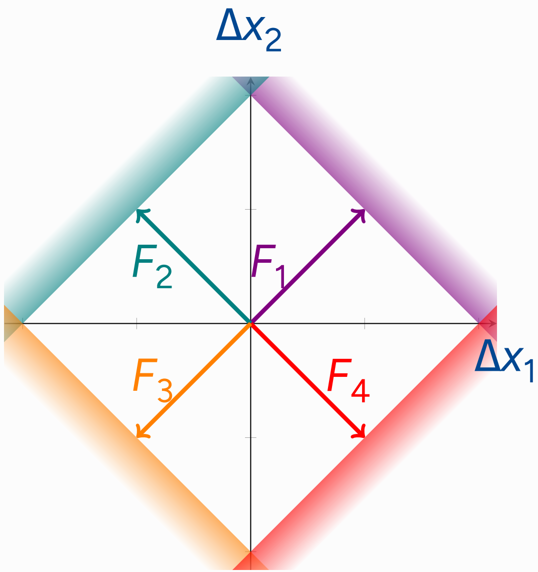


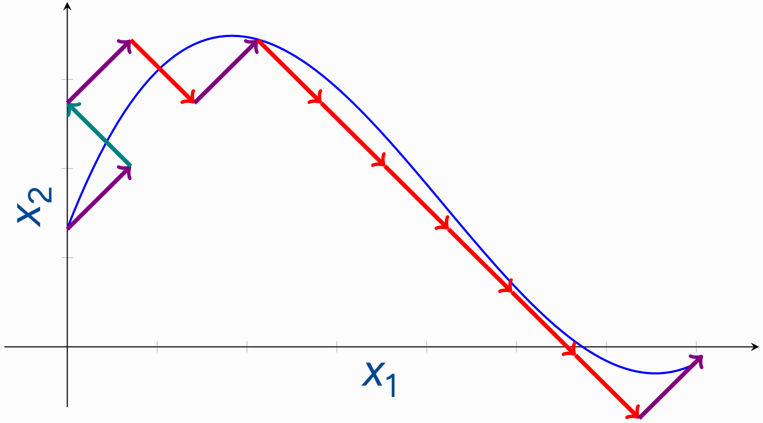


$$\dot{x} = c \tag{4}$$

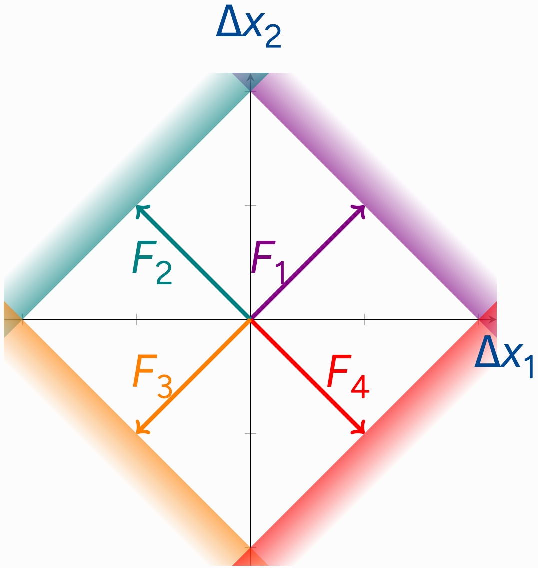
$$\dot{x} = Ax + c \tag{5}$$

Geometric





Geometric

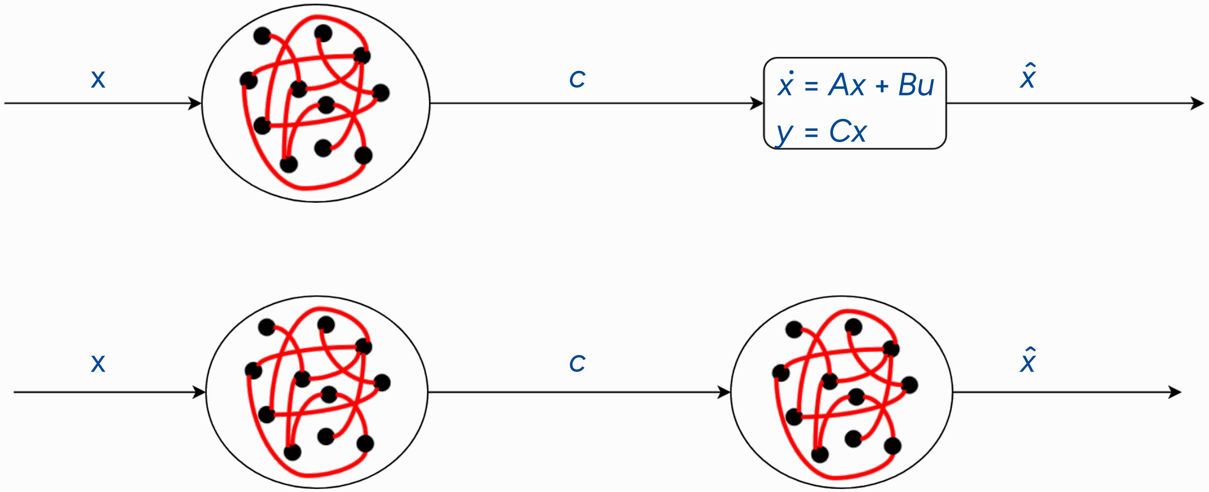


Minimize the cost J (Greedy)

$$J = \int_0^T \|x - \hat{x}\|_2^2 + C(r)\, dt \tag{6}$$

$$\begin{aligned} V_i &= F_i(x - \hat{x}) - \mu r_i \\ \dot{V}_i &= -\lambda_V V_i + F_i c(t) \\ &\quad + W^f o(t) + W^s r(t) + \sigma_V \eta(t) \\ W^f &= FF^T + \mu I \\ W^s &= F\,(A + \lambda_d I)\,F^T \end{aligned} \tag{7}$$

Control Concept



[HC19]

Control with SNN

$$u = F^T r + \Omega o(t) \tag{8}$$

Slow and Instantaneous decoding

$$\dot{V}(t) = -\lambda_V V(t) + \Omega^{\mathrm{T}} B^T A e(t) + \Omega^{\mathrm{T}} B^T c(t) + W^s r(t) + W^f o(t) + \sigma_V \eta(t) \tag{9}$$

Requires full state information on x and $\hat{x}$

$$c = \dot{x} - Ax \tag{10}$$

It is necessary on $B \in \mathbb{R}^{nxp}$

$$\text{rank}\,(B^T C^T) = p \tag{11}$$

Conclusion

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- Acceptable results in ideal conditions

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- Rank condition is limiting factor

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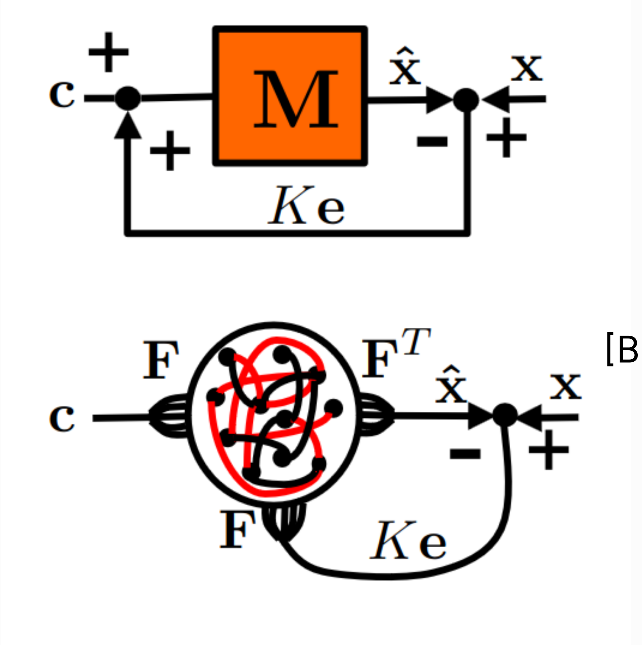
- Rank condition is limiting factor
- Network noise is invisible to the control

Conclusion

- Acceptable results in ideal conditions
- Rank condition is limiting factor
- Network noise is invisible to the control
- Simple open loop controller in the definition of c

$$\dot{\hat{x}} = (M - K\mathbf{I})\hat{x} + c + Kx$$
$$W^s = F\left(A + \lambda_d \mathbf{I}\right) F^T$$

Slow Learning rule



[BD15]

Online Teacher-Student Scheme for M under $\dot{x} = Mx + c$

Matrix update under squared loss

$$\delta M \propto e\hat{x}^T \longrightarrow \delta W^s \propto F^T \left(e\hat{x}^T\right) F \approx F^T er \tag{12}$$

Conclusion

- Very limited applicability
- Open loop + rank condition limiting factor
- Too inaccurate learning of slow weights W^s
- Too dependent on initial conditions in learning

- In ideal conditions useable results achievable
- Only of theoretical interest
- Impressive accuracy
- Results are somewhat translatable to NEF and LSMs