# Spiking Neural Networks for Control

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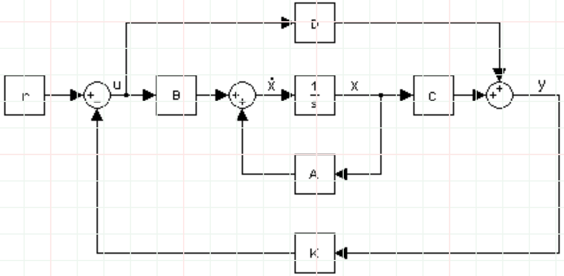
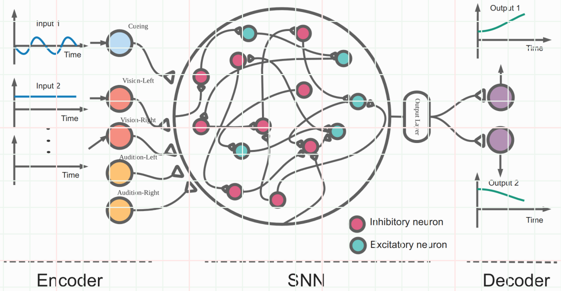
# Introduction

# What are we talking about

## Control a Linear system




## Use Spiking neural networks




[Xue+22]

# What are we talking about

## Control a Linear system

- Tracking of reference trajectory

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

Only stable systems

## Use Spiking neural networks

- Third Generation of NN
- Working with discrete spikes
- Inherently fit for temporal data

# Goal / Motivation

Artificial SNN can already solve various cognitive task such as

- Memorization
- Basic Logic
- Simulation of Dynamic Systems
- Control

Although with varying levels of biologic plausibility. We set out to build a controlled dynamic system based on SNN using learning and biologic plausibility

- Allow for black-box deployment without manual parameter tuning
- ”Limit ourselves to use the brains capabilities to design a controller”

# Method

## 1. Simulate

Use a spiking network to simulate a dynamic system

## 2. Control

Devise a control scheme to control the network output

## 3. Learn

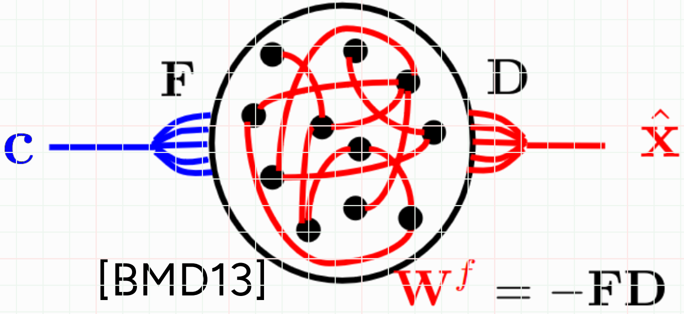
Apply biologically plausible learning rules to our network

## 4. Combine

Integrate all three steps into a single controller

# Simulation

# Simulation of Linear systems

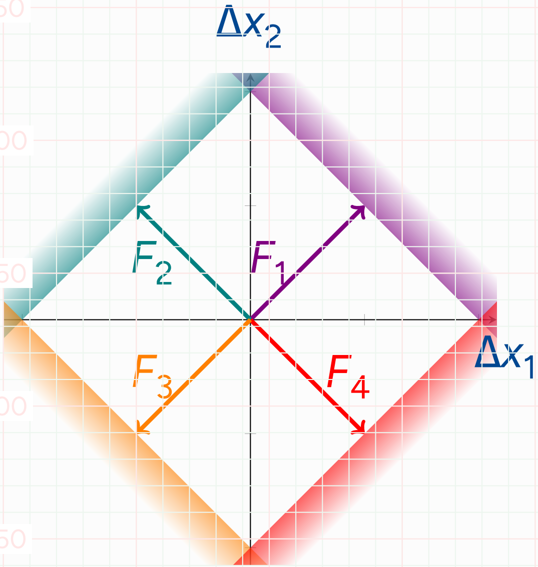


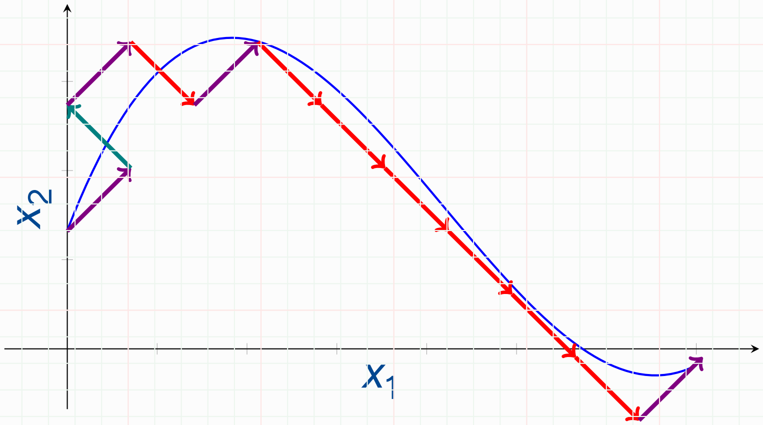

[BMD13]

- Build NN that outputs $\hat{x}$ from the system $\dot{x} = Ax + c$ given $c$
- Group of LIF neurons with with intrinsic Voltage, tracking the projected error $V_i = F(x - \hat{x}) + \mu r_i$
- Network decoding $\hat{x} = F^T r$

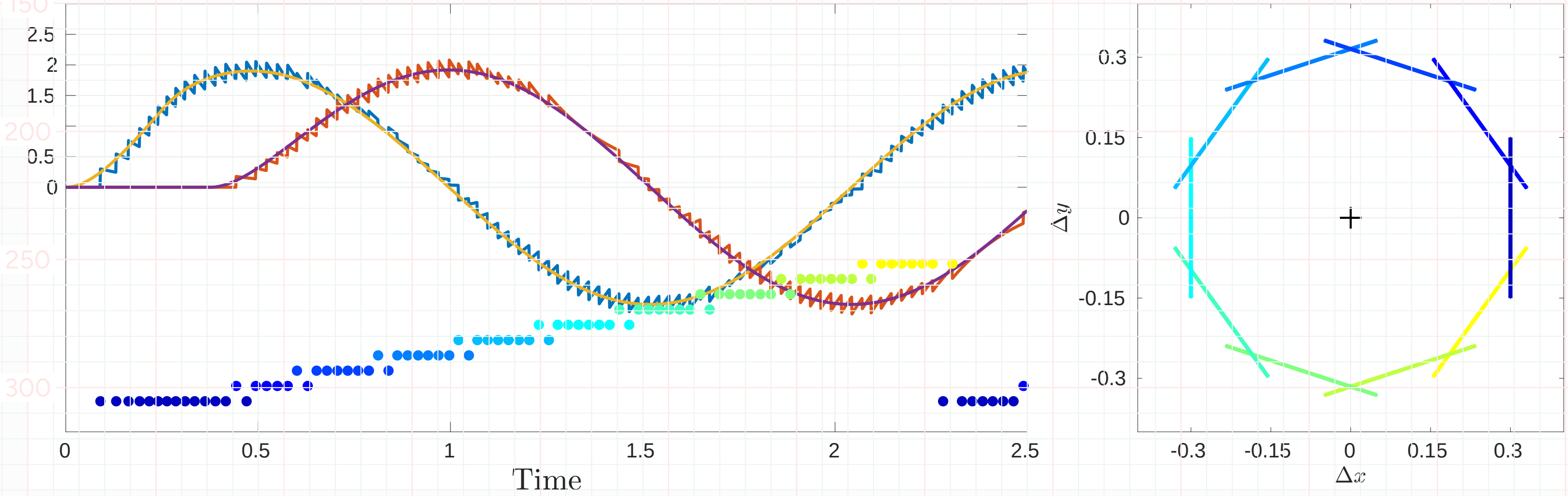
$$\dot{V} = -\lambda_V V + Fc + W^f o(t) + W^s r(t) + \sigma_V \eta(t)$$

# Geometric



# Example Simulation



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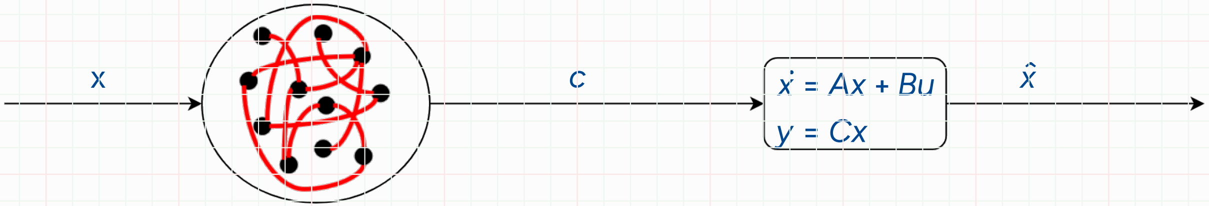
# Control

# Control Concept

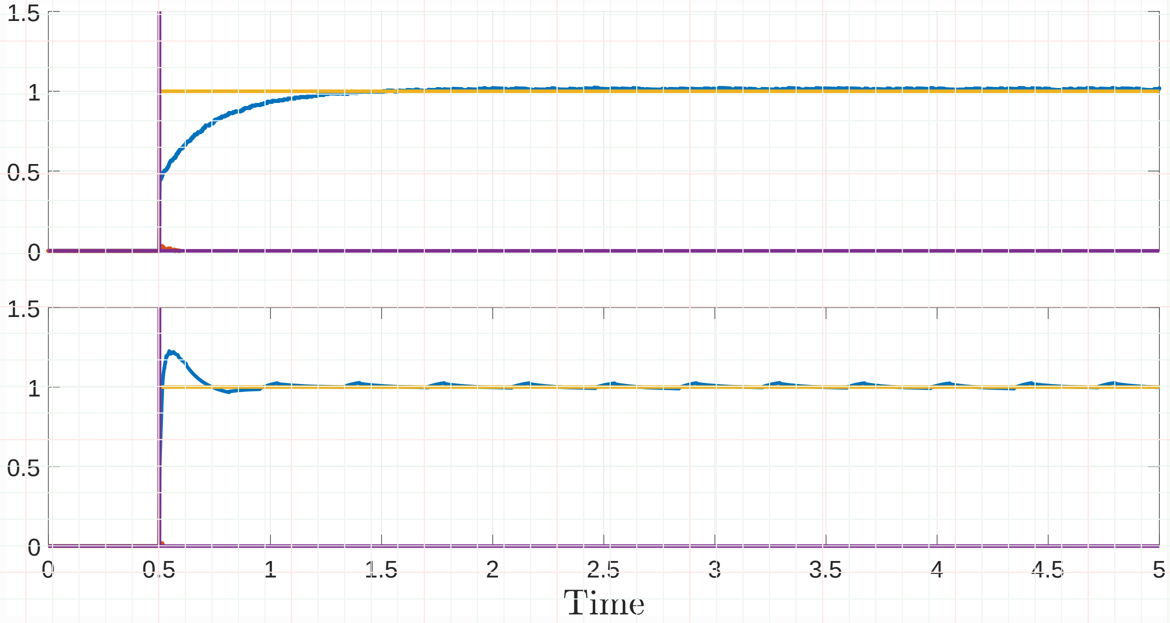



[HC19]

(Almost) identical network architecture

- Network output is external input into (previous) simulating network $\longleftrightarrow$ Network state contains control signal
- Governed by PD-control as $c = \dot{x} - Ax$
- In presence of output matrix $C \neq \mathbf{I} \leftrightarrow \text{rank}(B^T C^T) = rank(B^T)$

# Example



# Learning

$V_i = F_i(x - \hat{x}) - \mu r_i$

# Learning rules [BD15]

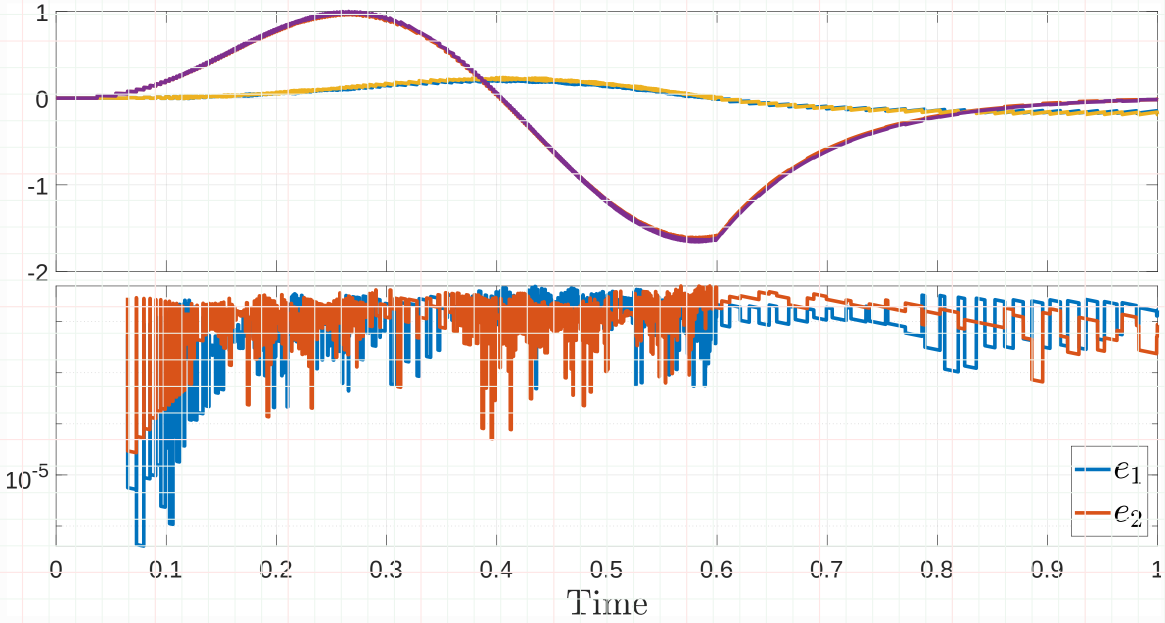
**Slow Learning rule** $W^s = F(A + \lambda_d \mathbf{I})F^T$

- Online Learning of Student teacher dynamics $\dot{\hat{x}} = M\hat{x} + c$
- Error Feedback $Ke$ during Training
- $\delta M \propto e\hat{x}^T \longrightarrow \delta W^s \propto F(e\hat{x}^T)F^T \approx Fer^T$
- Supervised Learning rule

**Fast Learning rule** $W^f = FF^T + \mu\mathbf{I}$

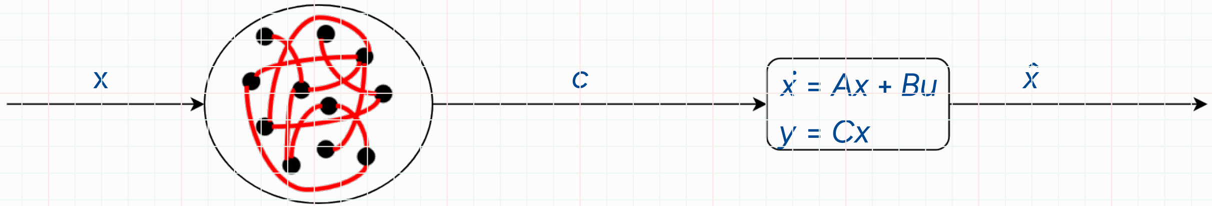
- Voltage measures system error
- Minimize average Voltage outside of Neuron Threshold
- Biologically plausible pre × post locally
- Unsupervised Learning Rule

# Example



# Combined Learning

# Control Concept




[HC19]

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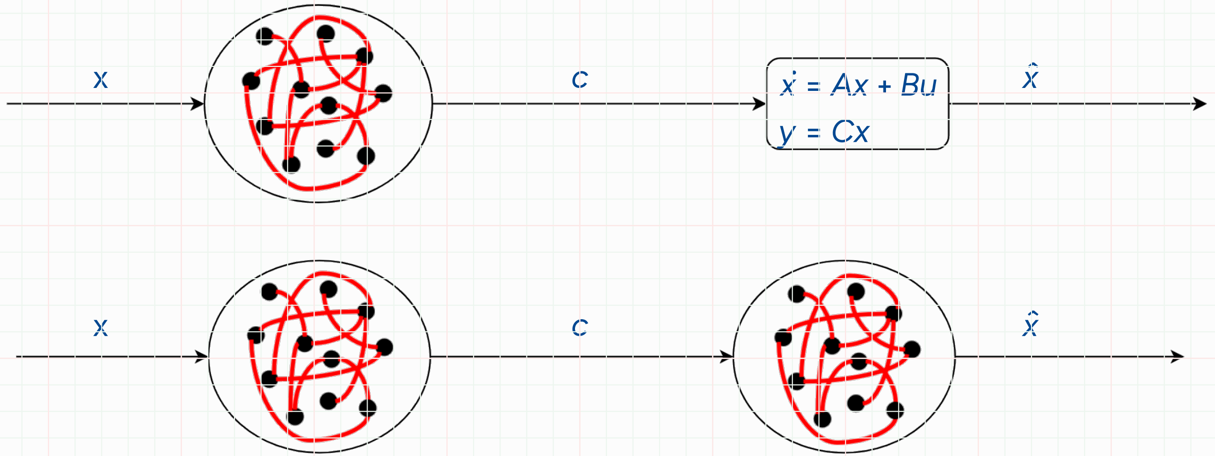


[HC19]

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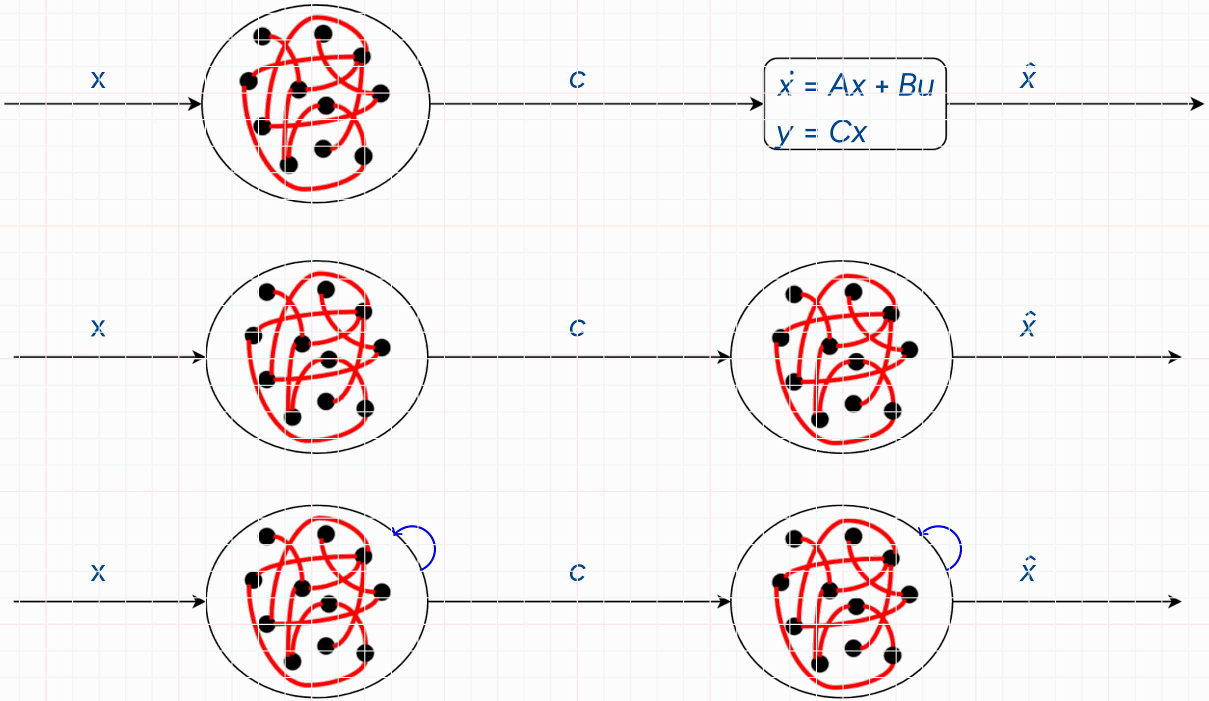
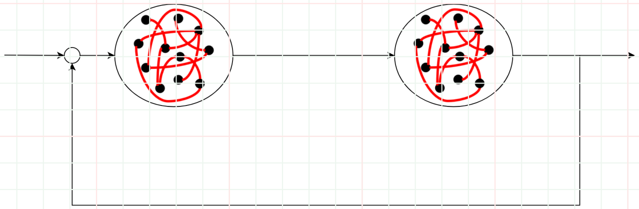
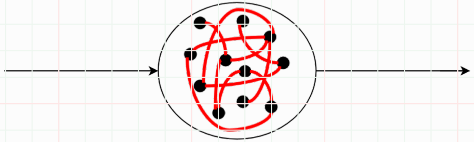
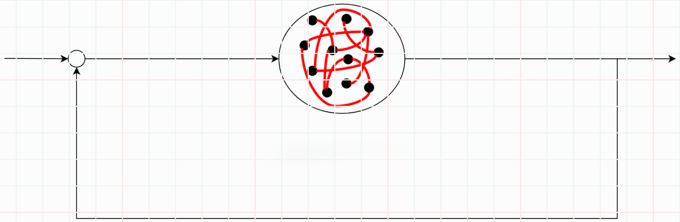



[HC19]

# Control Concept II



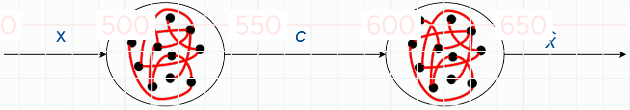




# Problems

In conjunction, problems can arise:

- Divergence in Learning
- Control with Noise
- Reliance on analytic results
- Biologically implausible Learning



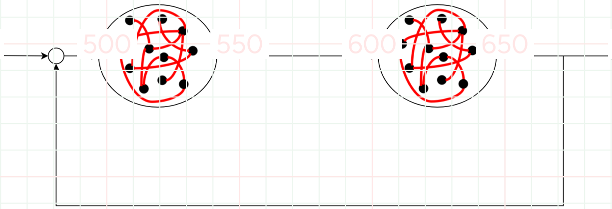

# Dual Network

No Learning rule for control network available

Open Loop Control:

- Incapable of noise detection or correction
- No Compensation of Training error

Highly dependent on governing dynamics from $c_{contr} = \dot{x}_{ref} - Ax_{ref}$



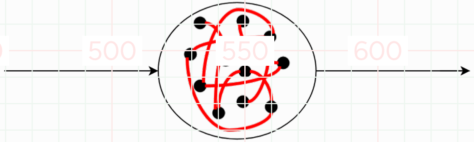
# Dual Network with Feedback

No Learning rule for control network available

~~Open loop Control:~~

- ~~Incapable of noise detection or correction~~
- ~~No Compensation of Training error~~

~~Highly dependent on governing dynamics from~~ $\cancel{e_{contr} = \dot{x}_{ref} - Ax_{ref}}$



# Single Network

~~No Learning rule for control network available~~

Open Loop Control:

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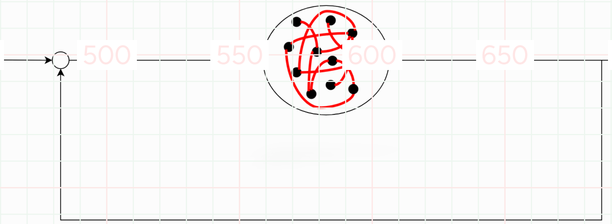
Highly dependent on governing dynamics from $c_{\text{contr}} = \dot{x}_{\text{ref}} - Ax_{\text{ref}}$

Orthonormality restriction on Input Matrix $B \in \mathbb{B} := \{M \mid MM^T = \mathbb{I}\}$



# Single Network with Feedback

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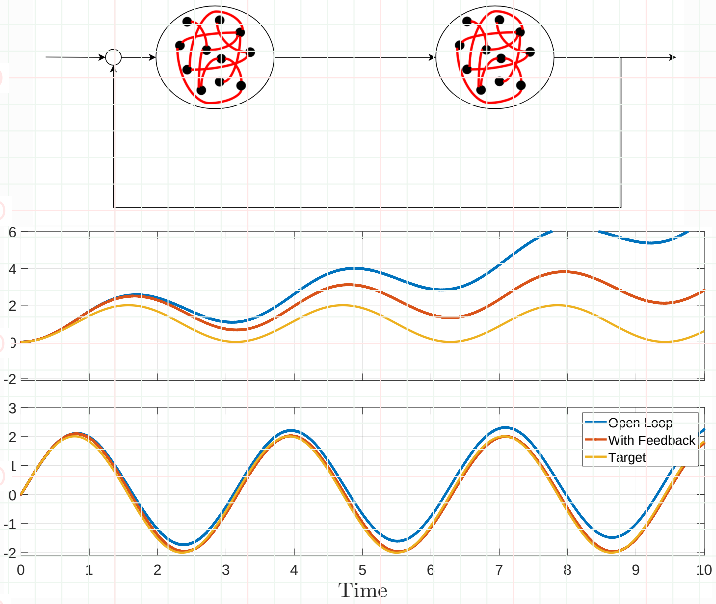
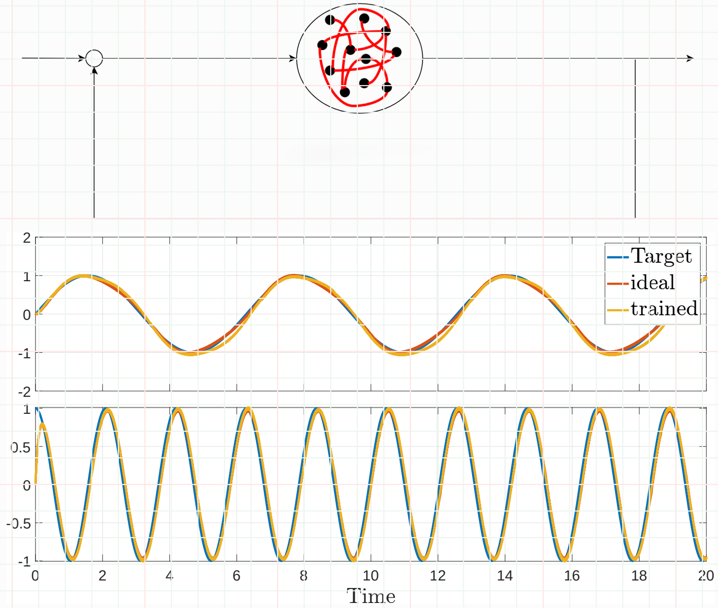
Orthonormality restriction on Input Matrix $B \in \mathbb{B} := \{M \mid MM^T = \mathbb{I}\}$

# Examples


Open Loop
With Feedback
Target
Time


Target
ideal
trained
Time

# Conclusion

# Conclusion

- Open loop and inaccurate learning of slow weights $W^s$ need to be addressed.
- Highly dependent on initial conditions in learning
- Impressive accuracy

- In ideal conditions useable results achievable
- Limited Applicability → Only of theoretical Interest
- Results are somewhat translatable to NEF and LSMs

Choice between biologic plausibility or and Input Matrix Restriction for accurate results

# Future Work

- Enable non-linear dynamics
- Obey Dale's Law for neuron excitation and inhibition
- Optimize Control
- Learning of En- and Decoder $F$
- Allow for synaptic delays

# Bibliography I


[BD15] Ralph Bourdoukan and Sophie Denève. "Enforcing balance allows local supervised learning in spiking recurrent networks". In: **Advances in Neural Information Processing Systems**. Ed. by C. Cortes, N. Lawrence, D. Lee, M. Sugiyama, and R. Garnett. Vol. 28. Curran Associates, Inc., 2015. URL: https://proceedings.neurips.cc/paper_files/paper/2015/file/3871bd64012152bfb53fdf04b401193f-Paper.pdf.

[BMD13] Martin Boerlin, Christian K. Machens, and Sophie Denève. "Predictive Coding of Dynamical Variables in Balanced Spiking Networks". In: **PLOS Computational Biology** 9.11 (Nov. 14, 2013). Publisher: Public Library of Science, e1003258. ISSN: 1553-7358. DOI: 10.1371/journal.pcbi.1003258. URL: https://journals.plos.org/ploscompbiol/article?id=10.1371/journal.pcbi.1003258 (visited on 09/20/2022).

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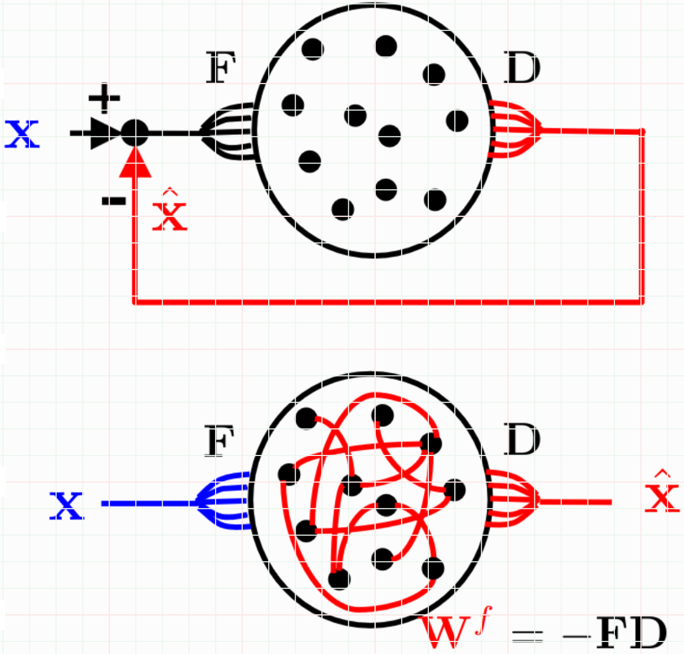
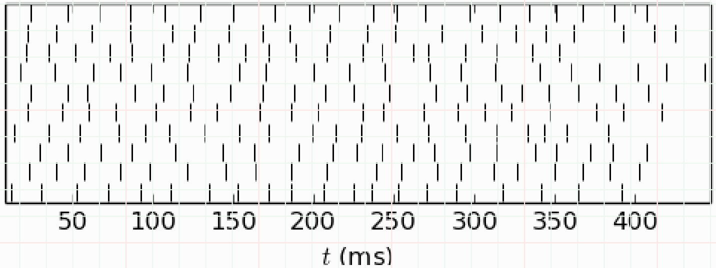
[HC19] Fuqiang Huang and ShiNung Ching. “Spiking networks as efficient distributed controllers”. In: **Biological Cybernetics** 113.1 (Apr. 2019), pp. 179–190. ISSN: 0340-1200, 1432-0770. DOI: 10.1007/s00422-018-0769-7. URL: http://link.springer.com/10.1007/s00422-018-0769-7 (visited on 10/23/2022).

[Xue+22] Xiaohe Xue, Ralf D. Wimmer, Michael M. Halassa, and Zhe Sage Chen. “Spiking Recurrent Neural Networks Represent Task-Relevant Neural Sequences in Rule-Dependent Computation”. In: **Cognitive Computation** 15.4 (Feb. 2022), pp. 1167–1189. ISSN: 1866-9964. DOI: 10.1007/s12559-022-09994-2. URL: http://dx.doi.org/10.1007/s12559-022-09994-2.

# BackupSlides

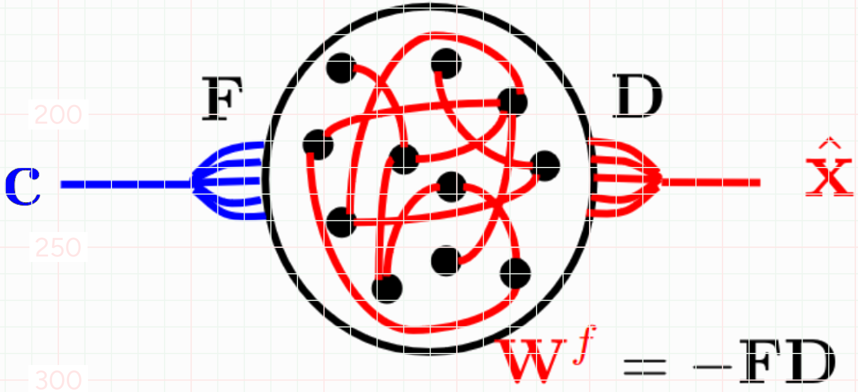
# Autoencoder



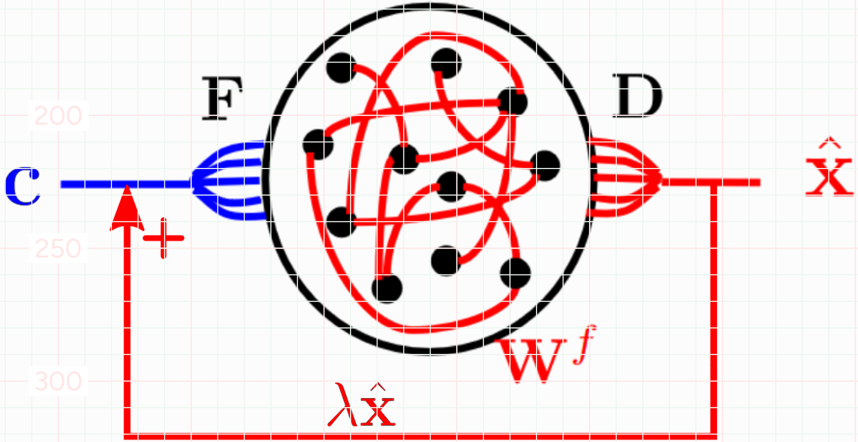

$$\hat{x} = Do(t)$$
$$\dot{r} = -\lambda r + o(t) \tag{2}$$

# Autoencoder II

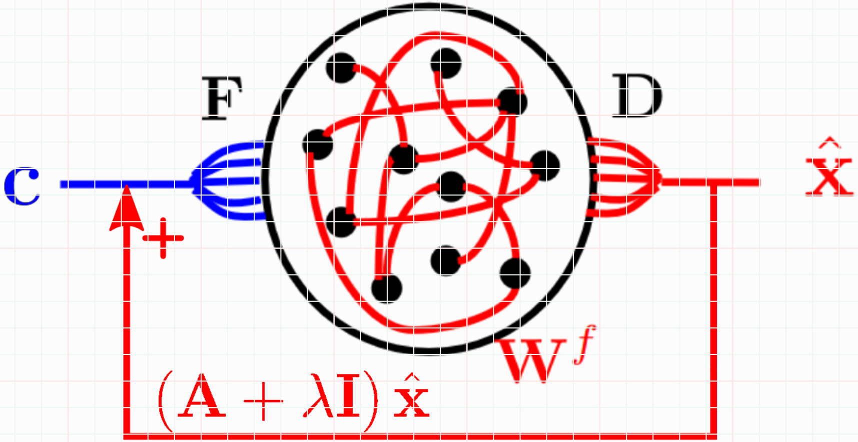



$$\begin{aligned}\dot{x} &= -\lambda x + c \\ \hat{x} &= Dr\end{aligned} \tag{3}$$

# Autoencoder III

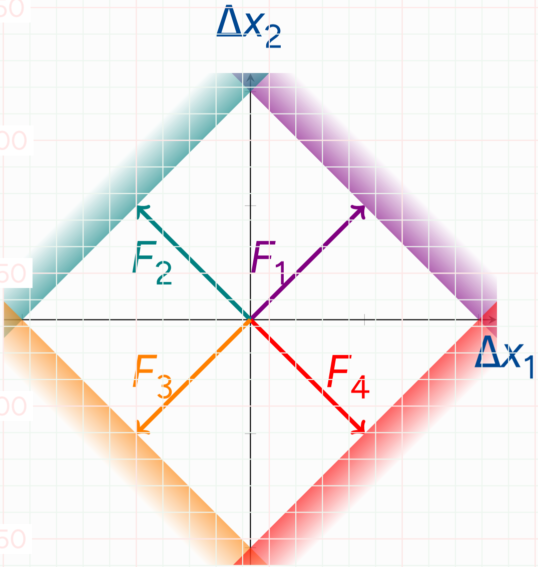


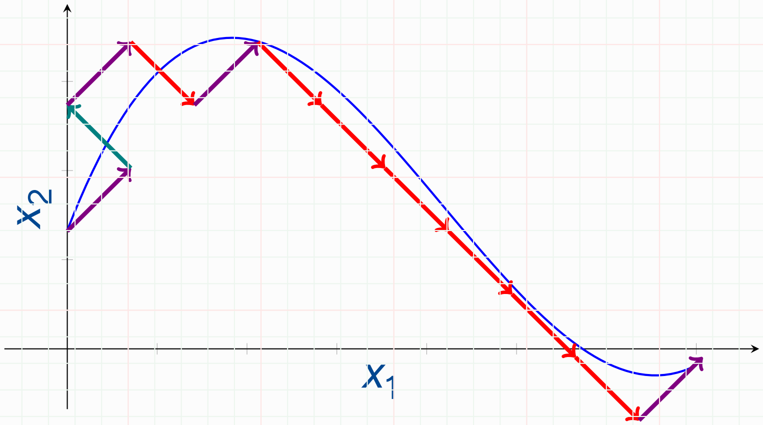

$$\dot{x} = c \quad (4)$$



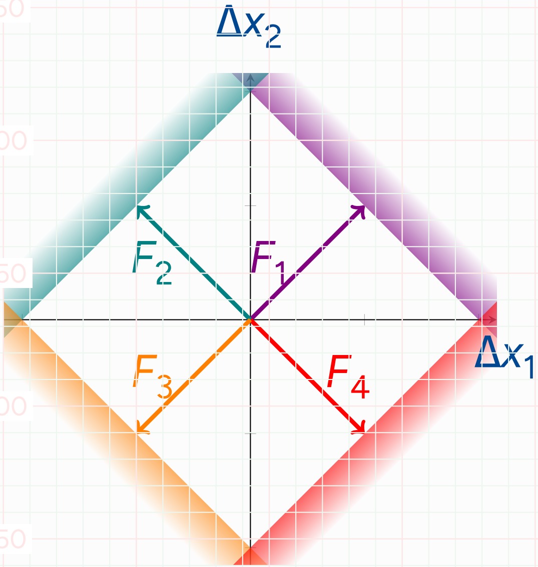

$$\dot{x} = Ax + c \quad (5)$$

# Geometric



# Geometric




Minimize the cost $J$ (Greedy)

$$J = \int_0^T \|x - \hat{x}\|_2^2 + C(r)\, dt \tag{6}$$

$$\begin{aligned} V_i &= F_i(x - \hat{x}) - \mu r_i \\ \dot{V}_i &= -\lambda_V V_i + F_i c(t) \\ &\quad + W^f o(t) + W^s r(t) + \sigma_V \eta(t) \\ W^f &= FF^T + \mu I \\ W^s &= F\,(A + \lambda_d I)\,F^T \end{aligned} \tag{7}$$

# Example Simple

content...

# Example Big

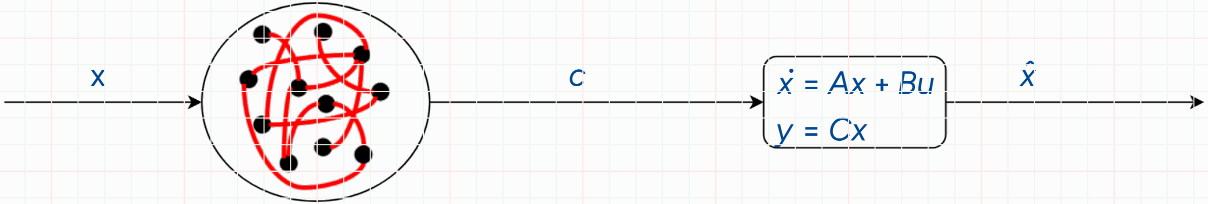
content...

# Conclusion

content...

# Control

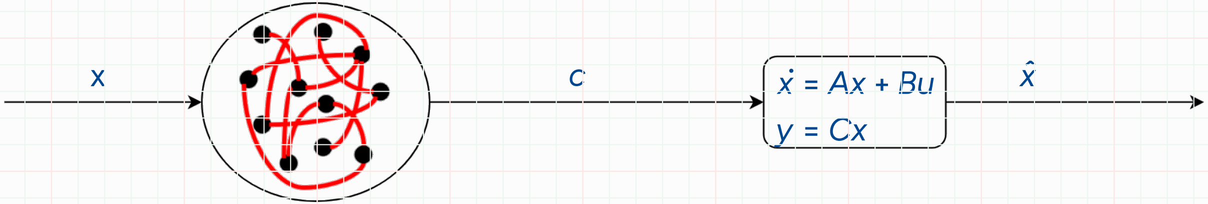
# Control Concept




[HC19]

Add a separator here

# Control Concept

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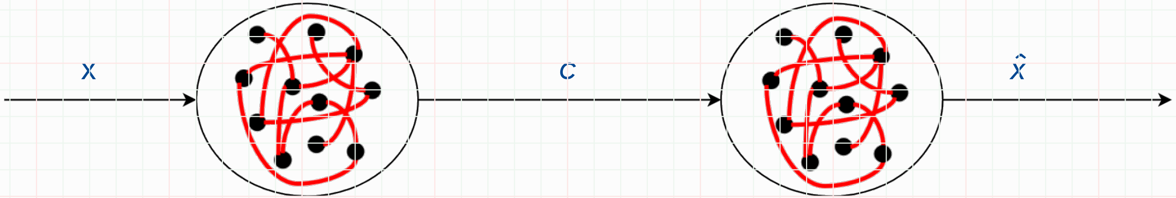

# Control with SNN

$$u = F^T r + \Omega o(t) \tag{8}$$

Slow and Instantaneous decoding

$$\dot{V}(t) = -\lambda_V V(t) + \Omega^T B^T A e(t) + \Omega^T B^T c(t) + W^s r(t) + W^f o(t) + \sigma_V \eta(t) \tag{9}$$

Requires full state information on $x$ and $\hat{x}$

$$c = \dot{x} - Ax \tag{10}$$

It is necessary on $B \in \mathbb{R}^{nxp}$

$$\operatorname{rank}(B^T C^T) = p \tag{11}$$

# Example in Ideal Conditions

works fine+ add plot

# Example with 2 networks

works bad+ add plot

# Conclusion

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- Acceptable results in ideal conditions

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- Rank condition is limiting factor

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# Conclusion

- Acceptable results in ideal conditions
- Rank condition is limiting factor
- Network noise is invisible to the control
- Simple open loop controller in the definition of $c$

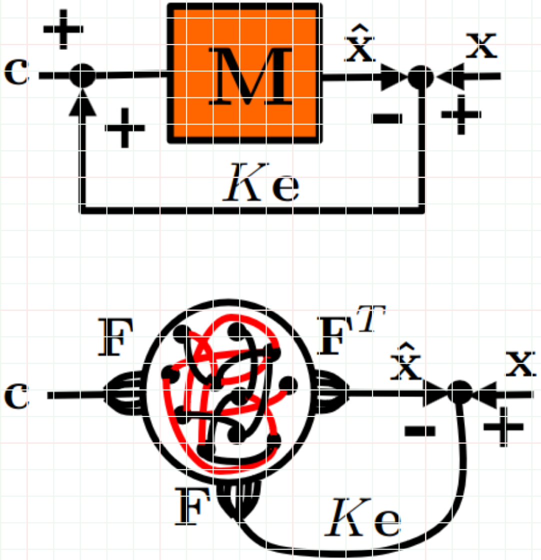
# Learning

# Fast Learning rule

**Slow Learning rule**
Online Teacher-Student Scheme

**Fast Learning rule**
content...

# Slow Learning rule




[BD15]

Online Teacher-Student Scheme for
$M$ under $\dot{x} = Mx + c$
Matrix update under squared loss

$$\delta M \propto e\hat{x}^T \longrightarrow \delta W^s \propto F^T\left(e\hat{x}^T\right)F \approx F^T e r \tag{12}$$

# Learned Control

# Conclusion

# Conclusion

- Very limited applicability
- Open loop + rank condition limiting factor
- Too inaccurate learning of slow weights $W^s$
- Too dependent on initial conditions in learning

- In ideal conditions useable results achievable
- Only of theoretical interest
- Impressive accuracy
- Results are somewhat translatable to NEF and LSMs

# Future Work

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# Frame title

## Block

Lorem ipsum!

# Bibliography


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