Spiking Neural Networks for Control

Max Schaufelberger

February 9, 2024 —– KTH Royal Institute of Technology

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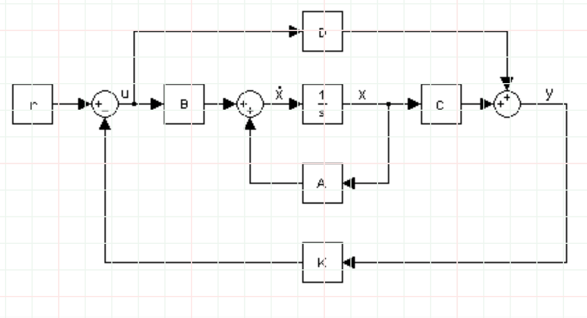
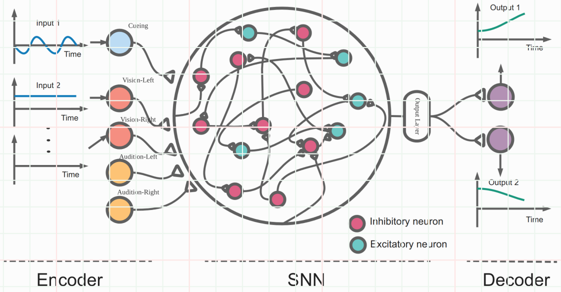
Introduction

What are we talking about

Control a Linear system



Use Spiking neural networks



[Xue+22]

What are we talking about

Control a Linear system

- Tracking of reference trajectory

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \qquad (1)$$

Only stable systems

Use Spiking neural networks

- Third Generation of NN
- Working with discrete spikes
- Inherently fit for temporal data

Goal / Motivation

Artificial SNN can already solve various cognitive task such as

- Memorization
- Basic Logic
- Simulation of Dynamic Systems
- Control

Although with varying levels of biologic plausibility. We set out to build a controlled dynamic system based on SNN using learning and biologic plausibility

- Allow for black-box deployment without manual parameter tuning
- "Limit ourselves to use the brains capabilities to design a controller"

Method

1. Simulate

Use a spiking network to simulate a dynamic system

2. Control

Devise a control scheme to control the network output

3. Learn

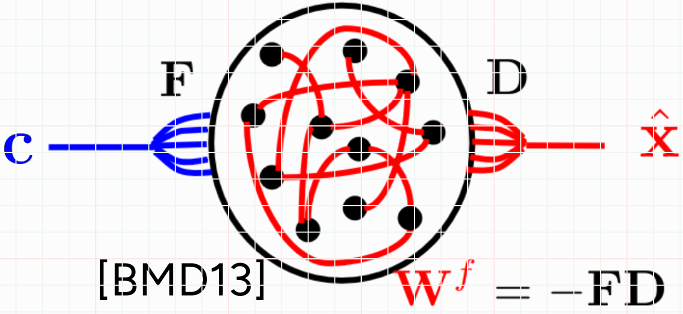
Apply biologically plausible learning rules to our network

4. Combine

Integrate all three steps into a single controller

Simulation

Simulation of Linear systems

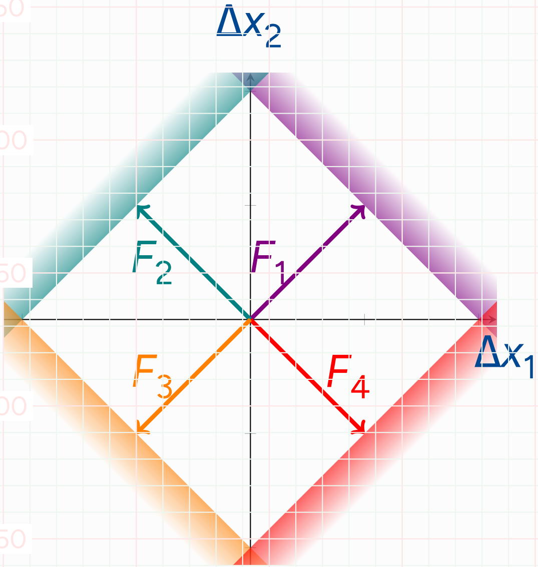


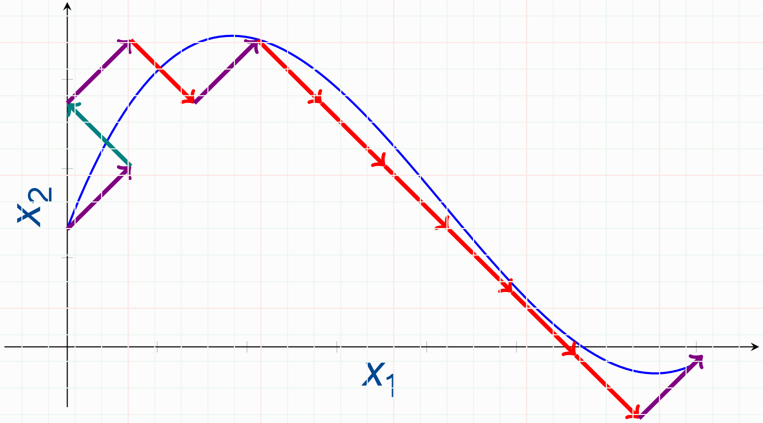
[BMD13]

- Build NN that outputs $\hat{x}$ from the system $\dot{x} = Ax + c$ given c
- Group of LIF neurons with with intrinsic Voltage, tracking the projected error $V_i = F(x - \hat{x}) + \mu r_i$
- Network decoding $\hat{x} = F^T r$

$$\dot{V} = -\lambda_V V + Fc + W^f o(t) + W^s r(t) + \sigma_V \eta(t)$$

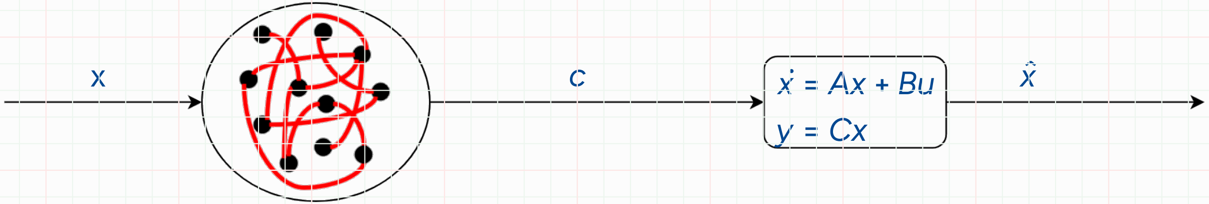
Geometric





Control

Control Concept



$\dot{x} = Ax + Bu$
$y = Cx$

[HC19]

(Almost) identical network architecture

- Network output is external input into (previous) simulating network ⟷ Network state contains control signal
- Governed by PD-control as $c = \dot{x} - Ax$
- In presence of output matrix $C \neq \mathbf{I} \leftrightarrow \mathrm{rank}(B^T C^T) = rank(B^T)$

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Examples

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Learning

$V_i = F_i(x-\hat{x})--\mu r_i$

Learning rules [BD15]

Slow Learning rule $W^s = F(A + \lambda_d \mathbf{I})F^T$

- Online Learning of Student teacher dynamics $\dot{\hat{x}} = M\hat{x} + c$
- Error Feedback Ke during Training
- $\delta M \propto e\hat{x}^T \longrightarrow \delta W^s \propto F(e\hat{x}^T)F^T \approx Fer^T$
- Error alignment?
- Supervised Learning rule

Fast Learning rule $W^f = FF^T + \mu \mathbb{I}$

- Voltage measures system error
- Minimize average Voltage outside of Neuron Threshold
- Biologically plausible pre×post locally
- Unsupervised Learning Rule

Examples

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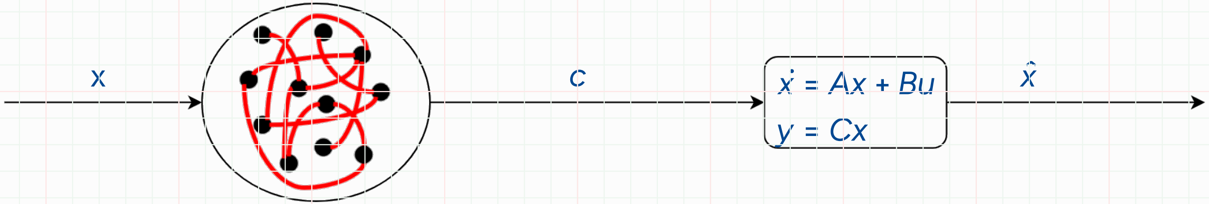
Combined Learning

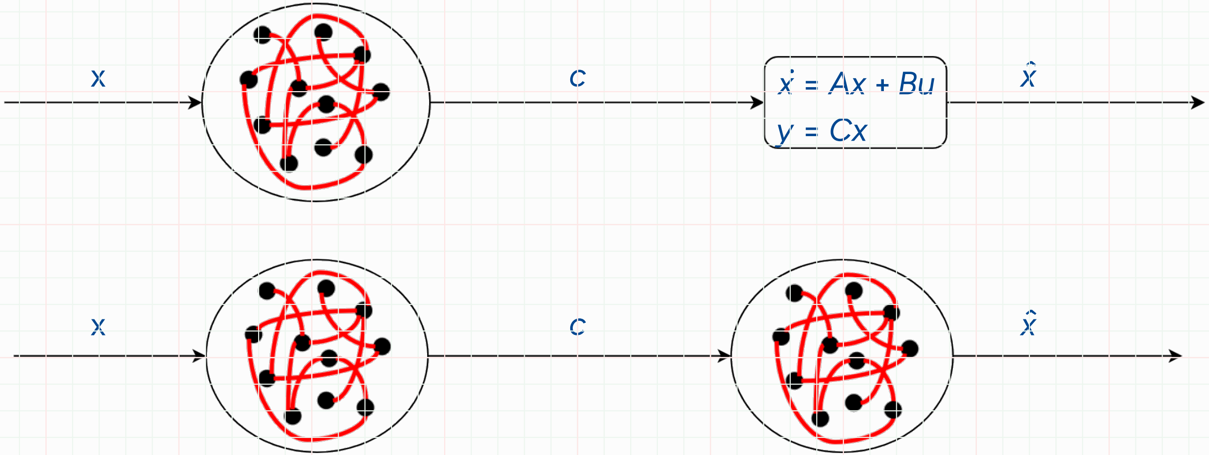
Control Concept



$$\dot{x} = Ax + Bu$$
$$y = Cx$$

[HC19]

Control Concept



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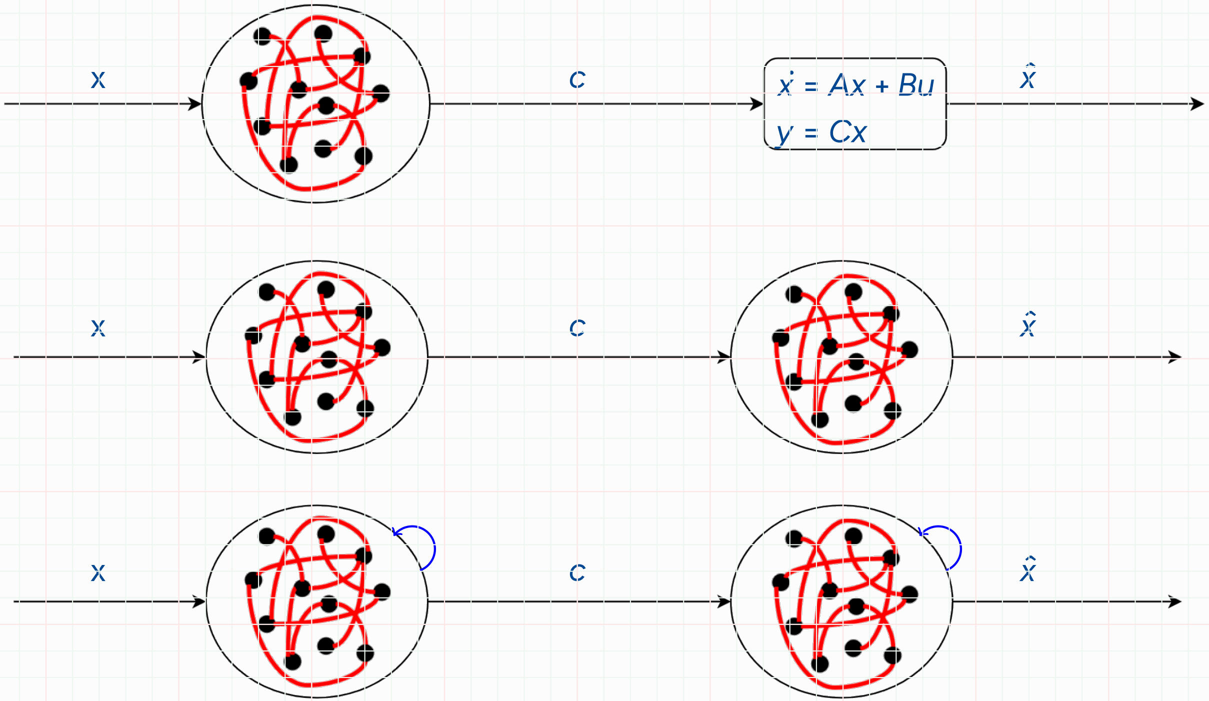
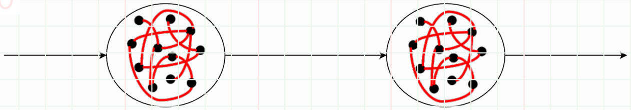
Control Concept

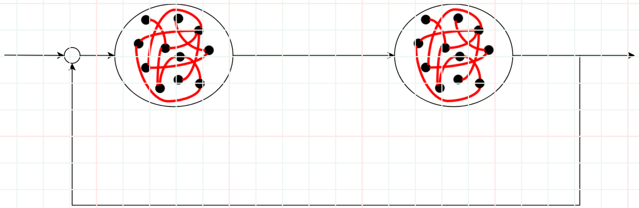


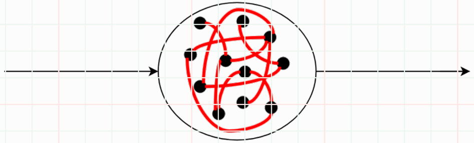
$$\dot{x} = Ax + Bu$$
$$y = Cx$$

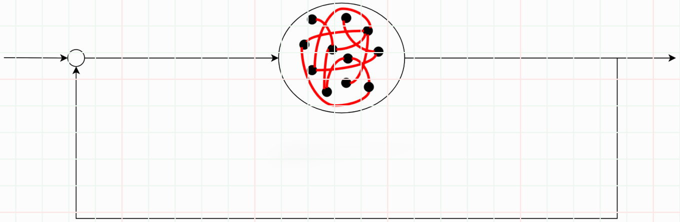
[HC19]

Control Concept II





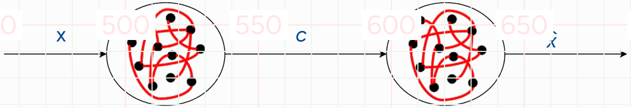




Problems

In conjunction, problems can arise:

- Divergence in Learning
- Control with Noise
- Reliance on analytic results
- Biologically implausible Learning



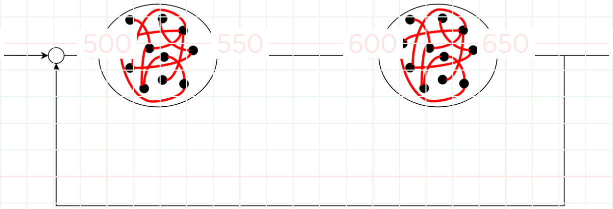
Dual Network

No Learning rule for control network available

Open Loop Control:

- Incapable of noise detection or correction
- No Compensation of Training error

Highly dependent on governing dynamics from $c_{contr} = \dot{x}_{ref} - Ax_{ref}$



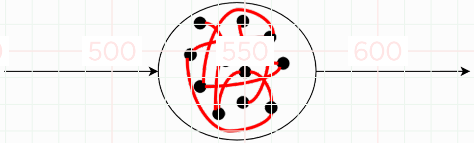
Dual Network with Feedback

No Learning rule for control network available

~~Open loop Control:~~

- ~~Incapable of noise detection or correction~~
- ~~No Compensation of Training error~~

~~Highly dependent on governing dynamics from $e_{contr} = \dot{x}_{ref} - Ax_{ref}$~~



Single Network

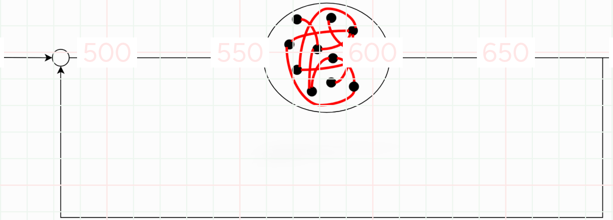
~~No Learning rule for control network available~~

Open Loop Control:

- Incapable of noise detection or correction
- No Compensation of Training error

Highly dependent on governing dynamics from $c_{\text{contr}} = \dot{x}_{\text{ref}} - Ax_{\text{ref}}$

Orthonormality restriction on Input Matrix $B \in \mathbb{B} := \{M \mid M^T M = \mathbb{I}\}$



Single Network with Feedback

~~No Learning rule for control network available~~

~~Open loop Control:~~

- ~~Incapable of noise detection or correction~~
- ~~No Compensation of Training error~~

~~Highly dependent on governing dynamics from $c_{contr} = \dot{x}_{ref} - Ax_{ref}$~~

Orthonormality restriction on Input Matrix $B \in \mathbb{B} := \{M \mid M^T M = \mathbb{I}\}$

Examples

1 Example working with single Network working

1 Example of 2 networks working

Conclusion

Conclusion

- Open loop and inaccurate learning of slow weights W^s need to be addressed.
- Highly dependent on initial conditions in learning
- Impressive accuracy

- In ideal conditions useable results achievable
- Limited Applicability → Only of theoretical Interest
- Results are somewhat translatable to NEF and LSMs

Choice between biologic plausibility or and Input Matrix Restriction for accurate results

Future Work

- Enable non-linear dynamics
- Obey Dale’s Law for neuron excitation and inhibition
- Optimize Control
- Learning of En- and Decoder Γ
- Allow for synaptic delays

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[BD15] Ralph Bourdoukan and Sophie Denève. “Enforcing balance allows local supervised learning in spiking recurrent networks”. In: **Advances in Neural Information Processing Systems**. Ed. by C. Cortes, N. Lawrence, D. Lee, M. Sugiyama, and R. Garnett. Vol. 28. Curran Associates, Inc., 2015. URL: `https://proceedings.neurips.cc/paper_files/paper/2015/file/3871bd64012152bfb53fdf04b401193f-Paper.pdf`.

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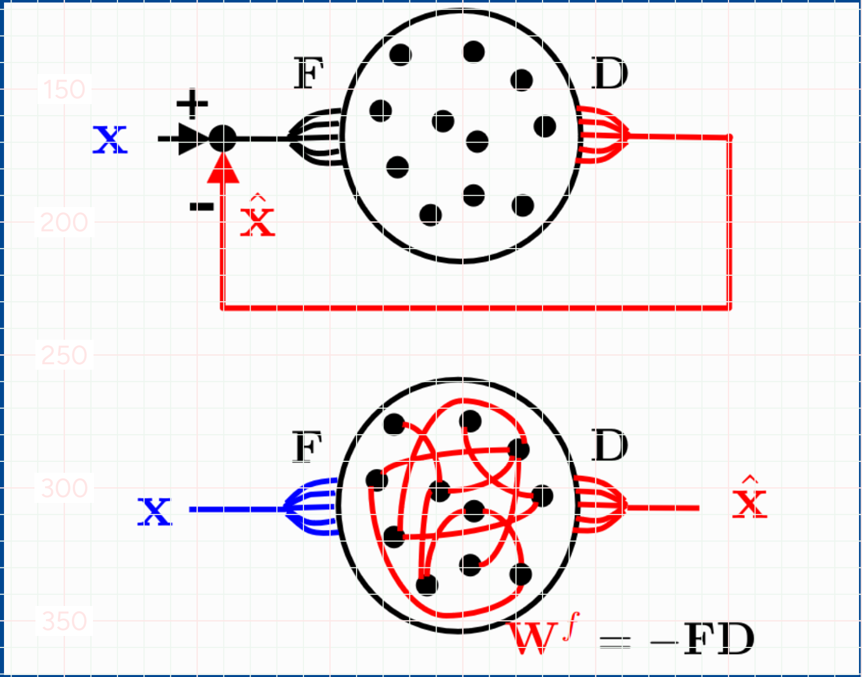
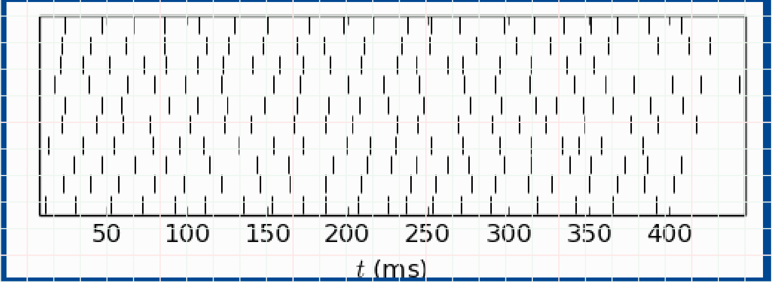
BackupSlides

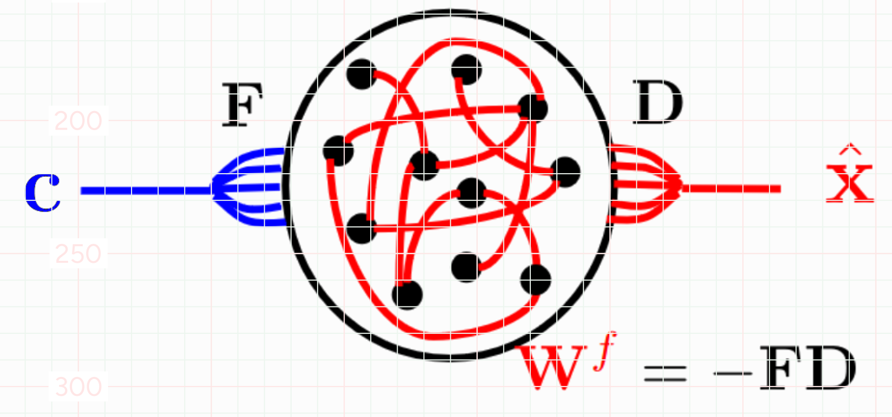
Autoencoder





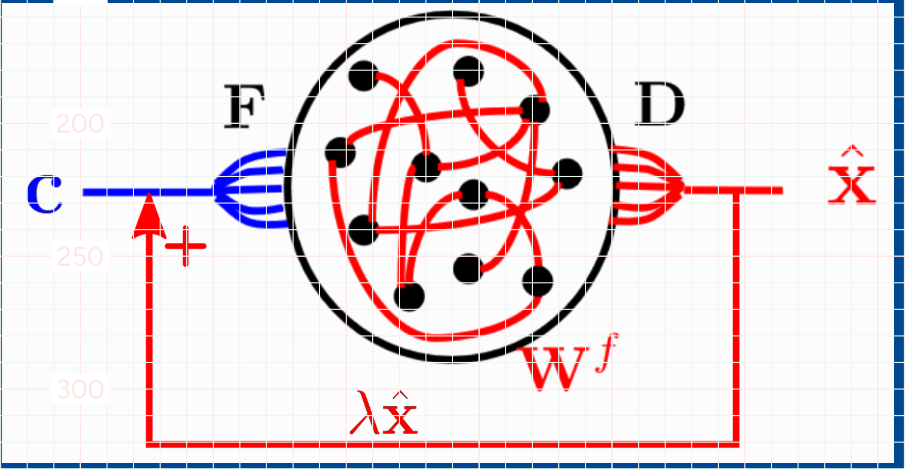
$\dot{r} = -\lambda r + \sigma(t)$

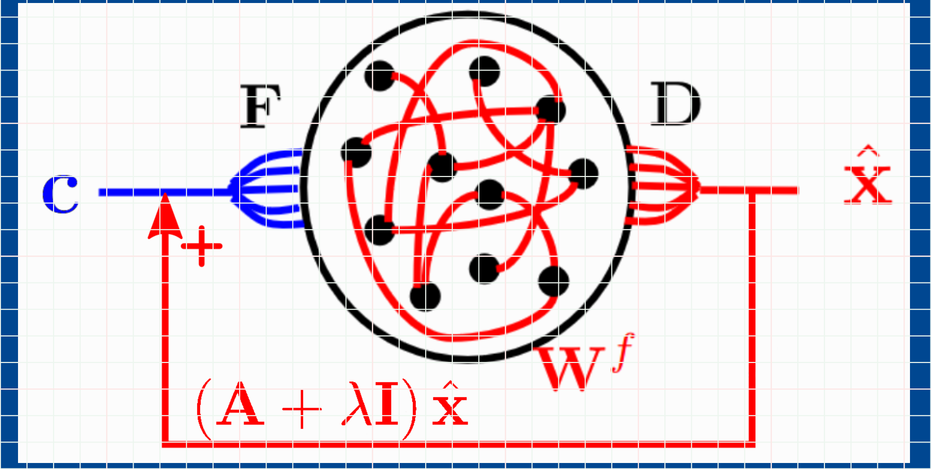
Autoencoder II



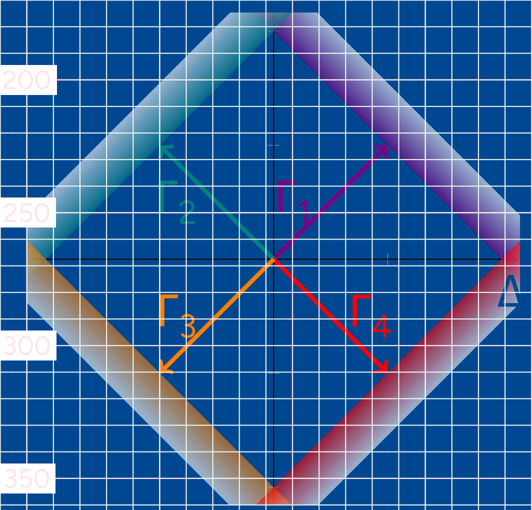
$$\dot{r} = -\lambda r + o(t)$$
$$x = Dr$$

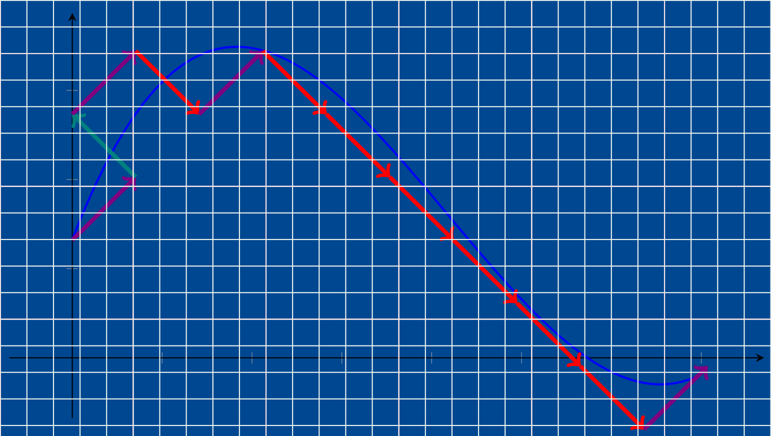
Autoencoder III





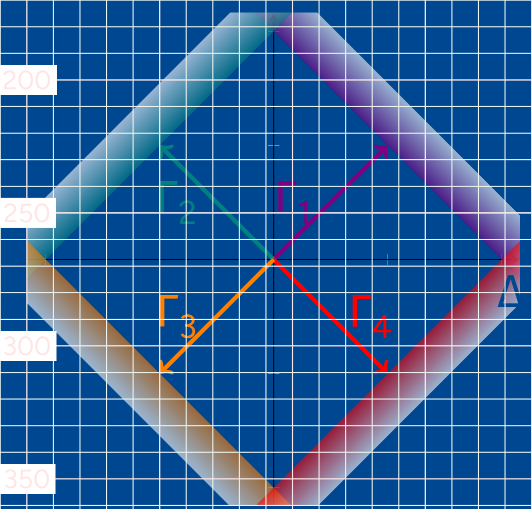
Geometric





Geometric

Minimize the cost (Greedy)



Example Simple

content...

Example Big

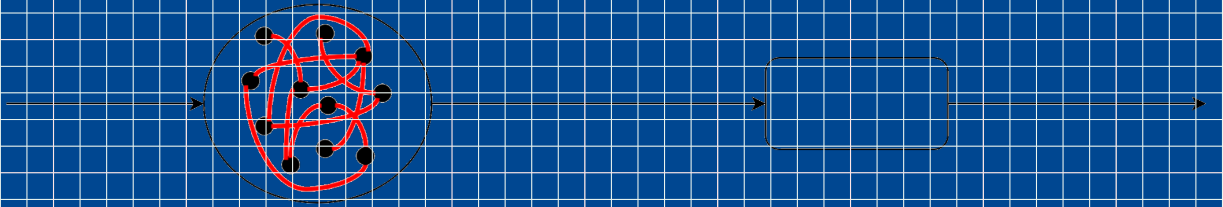
content...

Conclusion

content...

Control

Control Concept



[HC19]

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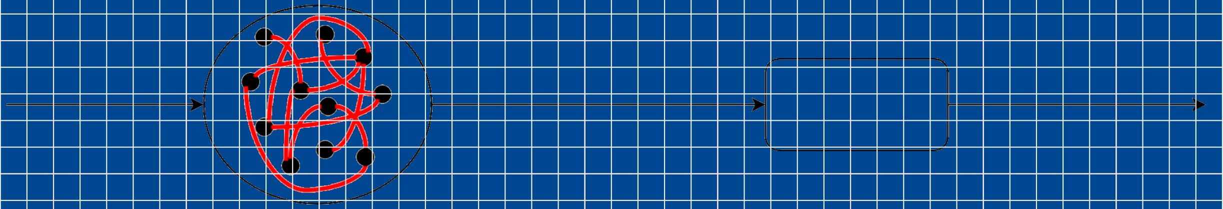
Control Concept



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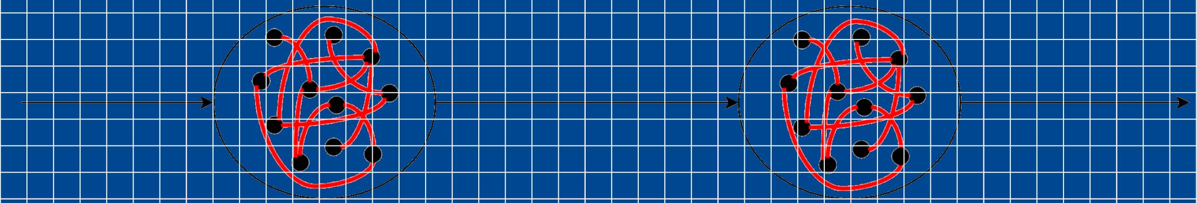


Control with SNN

Slow and Instantaneous decoding

Requires full state information on
and

It is necessary on

Example in Ideal Conditions

works fine+ add plot

Example with 2 networks

works bad+ add plot

Conclusion

Conclusion

- Acceptable results in ideal conditions

Conclusion

- Acceptable results in ideal conditions
- Rank condition is limiting factor

Conclusion

- Acceptable results in ideal conditions
- Rank condition is limiting factor
- Network noise is invisible to the control

Conclusion

- Acceptable results in ideal conditions
- Rank condition is limiting factor
- Network noise is invisible to the control
- Simple open loop controller in the definition of

Learning

Fast Learning rule

Slow Learning rule

Online Teacher-Student Scheme

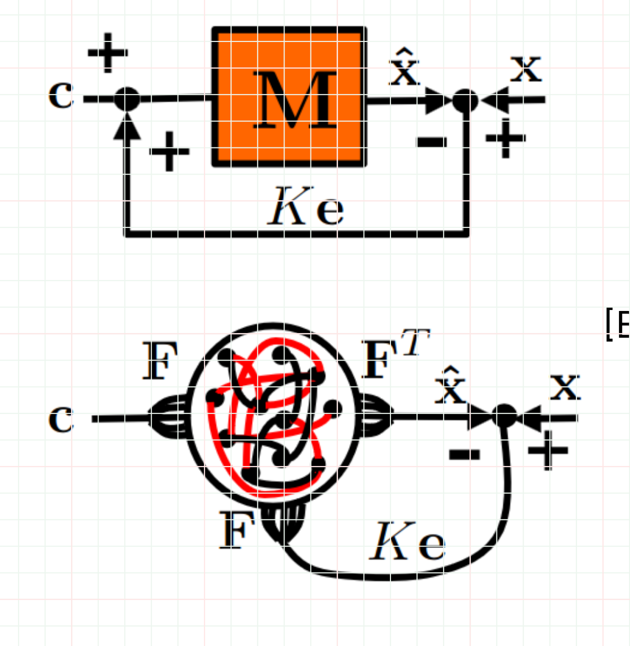
Fast Learning rule

content...

$$\dot{\hat{x}} = (M - K\mathbf{I})\hat{x} + c + Kx$$

$$W^s = \Gamma^T (A + \lambda_d \mathbf{I})\, \Gamma$$

Slow Learning rule



[BD15]

Online Teacher-Student Scheme for under

Matrix update under squared loss

replace the F with Γ in the picture!

Learned Control

Conclusion

Conclusion

- Very limited applicability
- Open loop + rank condition limiting factor
- Too inaccurate learning of slow weights
- Too dependent on initial conditions in learning

- In ideal conditions useable results achievable
- Only of theoretical interest
- Impressive accuracy
- Results are somewhat translatable to NEF and LSMs

Future Work

Future Work

- Enable non-linear dynamics

Future Work

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- Obey Dale's Law for neuron excitation and inhibition

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Bibliography

[BD15] Ralph Bourdoukan and Sophie Denève. “Enforcing balance allows local supervised learning in spiking recurrent networks”. In: Advances in Neural Information Processing Systems. Ed. by C. Cortes, N. Lawrence, D. Lee, M. Sugiyama, and R. Garnett. Vol. 28. Curran Associates, Inc., 2015. URL: .

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