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# Template

## KTH Thesis Report

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# Abstract

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The report needs to be compiled using XeLaTeX as different fonts are needed for the project to look like the original report. You might have to change this manually in overleaf.

This template  
was created by Hannes Rabo <hannes.rabo@gmail.com or hrabo@kth.se> from the template provided by KTH. You can send me an email if you need help in making it work for you.

Write an abstract. Introduce the subject area for the project and describe the problems that are solved and described in the thesis. Present how the problems have been solved, methods used and present results for the project. Use probably one sentence for each chapter in the final report.

The presentation of the results should be the main part of the abstract. Use about 1/2 A4-page. English abstract

## Keywords

Template, Thesis, Keywords ...

# Abstract

Svenskt abstract Svensk version av abstract – samma titel på svenska som på engelska.

Skriv samma abstract på svenska. Introducera ämnet för projektet och beskriv problemen som löses i materialet. Presentera

## Nyckelord

Kandidat examensarbete, ...

# Acknowledgements

Write a short acknowledgements. Don't forget to give some credit to the examiner and supervisor.

# Acronyms

<b>AD</b>	Automatic Differentiation
<b>DS</b>	Dynamic System
<b>RNN</b>	Recurrent Neural Network
<b>NN</b>	Neural Network
<b>GD</b>	Gradient Descent
<b>ANN</b>	Artificial Neural Network
<b>SNN</b>	Spiking neural network
<b>GPU</b>	Graphics Processing Unit
<b>SRDP</b>	Spike Rate Dependent Plasticity
<b>LSM</b>	Liquid State Machine
<b>SOP</b>	Synaptic Operation
<b>IF</b>	Integrate and Fire
<b>LIF</b>	Leaky-integrate-and-fire
<b>ODE</b>	ordinary differential equation
<b>LHS</b>	Left Hand Side
<b>RHS</b>	Right Hand Side
<b>HH</b>	Hodgkin–Huxley
<b>NLP</b>	Natural Language Processing
<b>LQG</b>	Linear Quadratic Gaussian
<b>LQR</b>	Linear Quadratic Regulator

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Background . . . . .	3
1.2	Problem . . . . .	3
1.3	Purpose . . . . .	6
1.4	Goal . . . . .	7
1.5	Benefits, Ethics and Sustainability . . . . .	7
1.6	Methodology . . . . .	8
1.7	Stakeholders . . . . .	9
1.8	Delimitations . . . . .	10
1.9	Outline . . . . .	10
<b>2</b>	<b>&lt;Theoretical Background&gt;</b>	<b>11</b>
2.1	Use headings to break the text . . . . .	11
2.2	Related Work . . . . .	11
2.3	Dynamic systems . . . . .	12
2.4	Autoencoder . . . . .	12
2.5	Neuron model . . . . .	13
2.5.1	Biological Neuron model . . . . .	13
2.5.2	"IF and LIF" . . . . .	14
2.5.3	Izhikevich Neuron . . . . .	15
2.6	Neural Networks . . . . .	15
2.6.1	Artificial Neural Networks . . . . .	15
2.6.2	Plasticity . . . . .	15
2.7	Spiking Neural Networks . . . . .	16
2.7.1	Rate Networks . . . . .	16
2.7.2	Liquid state machines . . . . .	18

2.7.3	Balanced Networks . . . . .	19
2.7.4	Plasticity . . . . .	20
<b>3</b>	<b>&lt;Engineering-related content, Methodologies and Methods&gt;</b>	<b>23</b>
3.1	Choice of Network architecture . . . . .	23
3.2	Simulation of Dynamic systems using Graphics Processing Units (SNNs)	24
3.2.1	Balanced network simulation . . . . .	24
3.2.2	Greedy optimization of the cost . . . . .	25
3.2.3	Neuron Voltage . . . . .	27
3.2.4	Regularization . . . . .	28
3.3	Control of Dynamic systems using SNNs . . . . .	31
3.3.1	Balanced networks as a controller . . . . .	31
3.3.2	Dynamics . . . . .	31
3.3.3	The instantaneous decoding weights . . . . .	32
3.3.4	Extension with direct Error feedback . . . . .	33
3.3.5	Direct error . . . . .	34
3.4	Learning of network parameters . . . . .	34
3.4.1	Optimal weights . . . . .	34
3.4.2	Learning of fast connection weights $\mathbf{W}^f$ . . . . .	34
3.4.3	Learning of slow connection weights $\mathbf{W}^s$ . . . . .	34
3.5	Engineering-related and scientific content: . . . . .	35
<b>4</b>	<b>&lt;The work&gt;</b>	<b>36</b>
4.1	Writing the balanced network simulation . . . . .	36
4.2	Writing the balanced network controller . . . . .	36
4.3	Learning the dynamics . . . . .	36
<b>5</b>	<b>&lt;Result&gt;</b>	<b>37</b>
<b>6</b>	<b>&lt;Conclusions&gt;</b>	<b>38</b>
6.1	Discussion . . . . .	38
6.1.1	Future Work . . . . .	38
6.1.2	Final Words . . . . .	38
	<b>References</b>	<b>41</b>



# Chapter 1

## Introduction

Provide a general introduction to the area for the degree project. Use references!

Link things together with references. This is a reference to a section:

The human brain is a brilliant computing unit comprised of around 86 billion[6] neurons. Each of these neurons can have thousands of connections to other neurons. Between these connections, information travels through the network as electrical impulses that interact with the neurons own electrical potential. With this network, the human brain is capable of performing vastly different and complex tasks. Machines and robots beat raw human computing power by several orders of magnitude, yet some tasks are next to impossible to solve by machines and classical algorithms alone. Moreover many machine implementations lack the speed, precision or flexibility of the human counterpart.

Researchers tried to remedy this by mimicking the brain's internal network structure to solve problems deemed unsuitable for classic algorithms.

Artificial Neural Networks (ANNs) have shown a great success in previously hard to solve problems.

Furthermore with the newer more biologically inspired networks we are able to solve a broader range of problems. One aspect of these are SNNs which simulate the discrete spiking behaviour of natural neural networks. We set out to design such a network in order to control a linear system.

Maybe separate this in intro and background

A variety of different architectures for these ANNs have been proposed, with the most prevalent design being a feed-forward network. In these networks, information travels only in one direction and is not propagated by spikes but gradients usually set in  $[0, 1]$  or  $[-1, 1]$ . These ANNs have made impressive progress in the fields of image recognition, autonomic driving, medical diagnosis[46] or Natural Language Processing (NLP) (using Transformers[54]).

This abstract representation bears advantages e.g in modelling and implementation but also gives away some key features of the human brain. Due to the information travelling only towards the output, feed-forward networks cannot build a memory or easily process temporal data. Recurrent models exist which allow for memory [19] and sequential data input but lose some of the advantages compared to the Feed-Forward due to its increased complexity.

A third generation[39] of network architectures has risen, which aims to be even more biologically plausible. Inspired from nature, they implement spiking behaviour and recurrence found in the human brain. This newer form of SNN is as powerful as the classic feed-forward but suited for temporal data.

While state of the art feed-forward networks are still outperforming SNNs, in some cases modern SNNs are on par or more performant with older feed-forward implementations. This comes with the added benefit of consuming much less power. Usually deep ANNs are run on Graphics Processing Units (GPUs), especially for training, in which the energy consumption can exceed 300W for modern chips<sup>1</sup>. The brain however is estimated to only consume about 20W [11] for immense computing capacity. Accompanying the SNN with neuromorphic hardware can yield a similar boost in efficiency with processors energy consumption in the pJ per Synaptic Operation (SOP)[27] offering a huge potential.

This is a separator

Present the background for the area. Give the context by explaining the parts that are needed to understand the degree project and thesis. (Still, keep in mind that this is an introductory part, which does not require too detailed description).

Use references<sup>2</sup>

Detailed description of the area should be moved to Chapter 2, where detailed

---

<sup>1</sup>e.g. a NVidia RTX 3090

<sup>2</sup>You can also add footnotes if you want to clarify the content on the same page.

information about background is given together with related work.

This background presents background to writing a report in latex.

Look at sample table 1.0.1 for a table sample.

Table 1.0.1: Sample table. Make sure the column with adds up to 0.94 for a nice look.

SAMPLE	TABLE
One	Stuff 1
Two	Stuff 2
Three	Stuff 3

Boxes can be used to organize content

Development environment for prototype
<div>Operating systems</div> <div>computer: Linux - kernel 4.18.5-arch1-1-ARCH</div> <div>android phone: 8.1.0</div> <div>Build tools</div> <div>exp (build tool): version 55.0.4</div> <div>...</div>

## 1.1 Background

Background.

## 1.2 Problem

Now list the goal: We want to do it for DS and check how good they are. Then method and then work. Take from below

Conventional Feed-Forward neural networks do are not designed to work with temporal data. They are static input output machines. This makes sense in the context of many tasks but at the same time limits the power of these networks. There are

workarounds to fit temporal data, for example by sampling the previous values back into the network used for example in time series forecasting [52][56][53] or to quantize the whole input if the complete time horizon is available. For example with recorded audio data.

Instead, recurrent neural networks are often proposed for these kinds of tasks. However recurrent neural networks experience problems when training with back-propagation[7]. For Recurrent Neural Networks (RNNs) and deep Feed-forward Neural Networks the gradients used in the back-propagation algorithm can explode or vanish. Different methods have been proposed to combat this problem, e.g. batch normalization[28], using alternative activation functions(ReLU)[43] or gradient clipping[45] to name a few. For recurrent models in particular different architectures have been suggested, most prominently among them the LSTM cell [21] with enormous success [37, 42, 49].

Yet, recurrent methods do not represent a very plausible formulation for biological networks. or biological movement. For the control of biological movements (often Linear Quadratic Regulator (LQR) is a proposed model for biological control [36] Secondly when it comes to simulation of biologic dynamic systems usually Linear Quadratic Gaussian (LQG) control is used. LQG has been used widely for modelling biological movements and control.

this is another smaller separator

SNNs are inherently designed in time since neural impulses are integrated over time. This makes the use for temporal data natural. The use of snn is online, can deal with much noise and offers an alternative way to reason for motor movement.

add other snn models. e.g reach, sorn maybe or others,FORCE

We hope that with SNN we have even better performance

From very biological to very abstract there have been many proposals

Cost performance trade off.

Spiking networks have gained similar or exceeding performance compared to the artificial one in some areas-> refs

Key advantage is in the temporal dimensional gain.

One field they are suited well is the control of dynamic systems

In this thesis we use a spiking neural network to control a linear dynamic system

The usual way to simulate biological dynamic systems is using LQG control -> ref

I believe because of the energy minimization

So we can compare them with usual NN and control in terms of performance... i guess

We start by giving an intro into spiking neural networks

Then spiking neural networks for dynamic systems

After control theory with SNN and maybe regular LQG control

Lately the learnign of SNNs for the control of dynamic system

Further work:

Maybe learning methods to control nonlinear dynamic systems

Maybe we can even do the adversarial attack to try to screw with the network.

Implement this on neuromorphic hardware

Problem:

Problem is that it is unnatural for classic NN to use temporal data.

They usually quantize it and make a big input layer -> ref

There are recurrent networks but ... they need to have smth bad as well

The LQG control is also not great for some reason I need to find

There are many spiking network archetypes like poisson and GLM and balanced

Also problem is that for some spiking networks learning rules could be hard to come by.

There are many prospects though as for example .....->refs

Also usually learning rules smth of an inverse and that the brain does not have or do I believe arvind said

Method:

We use a SNN to to control any arbitrary DS

Balanced networks show a some key motives seen in the brain like poisson distribution and smth else ->ref

The SNN is to be trained with a STDP rule

Then compared to optimal weights

Then investigated about robustness and other things as many before

One part of robustness is trying to get the most essential nodes of the snn to function

well.

, Then we have the potential to find a classic nn and train it with that ???

With that out of the way we can compare the performance of all the methods.

Then we could study the usability for biological interpretation.

Maybe even train time over performance or smth whatever

Work:

Explain the controller method aka what the math of the controller

In method explain the balanced and the derivation

In work summarize the implementation

Same for the conventional NN

Summarize the training method

Explain and derive the training method in method

Results:

To everthign mentioned in method for performance and so on

Answer the questions of the problem!!!!

NN have excelled at many fields

Fields where they are not fit

aka temporal data

They have ways to compromise on that

-> reference

Spiking nn inherently temporal

more natural choice

However they also have problems

like the following:::: reference!!

### 1.3 Purpose

The purpose of the degree project/thesis is the purpose of the written material, i.e., the thesis. The thesis presents the work / discusses / illustrates and so on.

It is not “The project is about” even though this can be included in the purpose. If so, state the purpose of the project after purpose of the thesis).

Probably delete as a own paragraph but mention smth like that.

## 1.4 Goal

The goal means the goal of the degree project. Present following: the goal(s), deliverables and results of the project.

The goal of this project is to create a SNN that can control any given linear Dynamic System (DS). Furthermore should the Neural Network (NN) be robust against failing neurons or connections.

We expect better results to conventional NNs because of the SNN’s natural way to use temporal data. For the SNN itself we desire similarities to the brain, such as high precision or Poisson distributed spiking. To mimic the brain’s learning, we want to use local training rules that are biologically plausible. The network should be converge to the optimal parameters.

Ideally, optimality should be reached, making sure that the network found the best parameters for the given system. This is usually a strong condition, as with conventional NNs using Gradient Descent (GD) only guarantee a local optimum. This requirement would allow us to give a justified measure on how close we are replicating the brain’s control power compared to our artificial implementation in specific circumstances.

Lastly, adjusting neural networks to a specific task is usually done by hand and requires time consuming hand tuning of parameters to achieve optimal results. Goal here is to automate as much of the process as possible i.e. the user does not need to adjust hyper-parameters himself. The network should be able to set itself up to find the best set of hyper-parameters given the task at hand, independent of the given control command or size of the system.

What do we  
else want  
like the  
brain. Maybe  
low spike  
count? And  
what can  
we do? Also  
References!

sounds vague

## 1.5 Benefits, Ethics and Sustainability

Benefits?

Paper [47] lesen fuer benefits und

Neuromorphic hardware & Energy efficiency?

Event driven though not much computatuin needed

, SNN are fast, energy efficient, high data transmission rate

SNNs possess more power[39]

Smart wearables?

"SNN used in event sensing" maybe useful in benefits

AI on low power products

Possibility to explain biologic inner workings of motor tasks

Describe who will benefit from the degree project, the ethical issues (what ethical problems can arise) and the sustainability aspects of the project.

Use references!

## 1.6 Methodology

Introduce, theoretically, the methodologies and methods that can be used in a project and, then, select and introduce the methodologies and methods that are used in the degree project. Must be described on the level that is enough to understand the contents of the thesis.

Use references!

Preferably, the philosophical assumptions, research methods, and research approaches are presented here. Write quantitative / qualitative, deductive / inductive / abductive. Start with theory about methods, choose the methods that are used in the thesis and apply.

Detailed description of these methodologies and methods should be presented in Chapter 3. In chapter 3, the focus could be research strategies, data collection, data analysis, and quality assurance.

To achieve our set out goal we first investigate what kind of Spiking network architecture to use. There are many different ways to design a SNN each with its



pros and cons. An architecture that allows for some approximations to enhance performance while retaining biologic similarities.

After that we implemented a LQG controller a baseline reference. Part of the goal is that the user does not have to touch the neural network working underneath the control problem. The user should submit the state, input, and output matrices and a target trajectory. The network underneath works independently and shall not be designed manually. After training, the network finds a control signal to the given system.

Firstly, we set out to achieve this goal by implementing a SNN that allows to simulate, not control, any given linear system with given external inputs. With this network in place it was set out to a second SNN that acts as the controller and combining those two. The controller would generate a control signal that would be used as external input to return the system behaviour to the controller. This approach however depended on finding magic numbers to set the controller to give usable results which was in opposition to our goals.

To find these magic numbers we tried to implement a learning regime based on a similar approach. However this was unfruitful.

Instead the original learning SNN approach is used to control the system directly.

How do we compare the end performance?

We build a SNN for a control problem and check it for performance as mentioned above. In addition we design a conventional controller and compare the result. IF we have the time for it we put a conventional NN to it too. We see the performance compared to the others and look at the specs we mentioned above. The SNN is trained by learning using STDP rule. We can compare the learned weights with the optimal weights when we have our own optimal controller/ we simulate our trajectory. For our approach we use a balanced spiking network.

## 1.7 Stakeholders

Present the stakeholders for the degree project.

## 1.8 Delimitations

Explain the delimitations. These are all the things that could affect the study if they were examined and included in the degree project. Use references!

## 1.9 Outline

In text, describe what is presented in Chapters 2 and forward. Exclude the first chapter and references as well as appendix.

make the outline in the end!

# Chapter 2

## <Theoretical Background>

In this chapter, a detailed description about background of the degree project is presented together with related work. Discuss what is found useful and what is less useful. Use valid arguments.

Explain what and how prior work / prior research will be applied on or used in the degree project /work (described in this thesis). Explain why and what is not used in the degree project and give valid reasons for rejecting the work/research.

Use references!

### 2.1 Use headings to break the text

Do not use subtitles after each other without text in between the sections.

### 2.2 Related Work

You should probably keep a heading about the related work here even though the entire chapter basically only contains related work.

Here just what has been done for each of the headlines

Previous efforts were already made to control dynamic systems with SNNs.

List here also efforts with other concepts apart from Balanced Networks

Neural networks in general spiking neural networks and their differences and what

they are better for. neuron models, iwazishi neuron and maybe one more mein neuron model und warum ich es ausgewaelt habe: einfach zu implementieren. Bereits fuer dynamische systeme verwendet, Nachteile dieses modells. Vlt vergleich mit einem anderen modell. Ganz kurzer ausflug in die regelung von dynamischen systemen.

What is a neural network? -> not here ref a paper. kurze erkl'ung in der einfuehrung in der einfuehrung vlt auch hodgekin huxley erwaehen :)

## 2.3 Dynamic systems

## 2.4 Autoencoder

An autoencoder is a type of neural network for learning a representation of its input. It consists of an encoder and decoder function  $z = f(x)$  &  $\hat{x} = g(z)$ . The encoder function from the input space, e.g  $\mathbb{R}^n$ , to a encoded space  $\mathcal{Z}$ . The decoder is then decoding the data from  $\mathcal{Z}$  back to  $\mathbb{R}^n$ . The goal is that the representation is as accurate as possible. This is easy in case the if  $\mathcal{Z}$  is equal or larger than the input space. Then every possible input can be encoded by its own value as

$$\begin{aligned} z &= f(x) = x \\ \hat{x} &= g(z) = z = x \end{aligned} \tag{2.1}$$

which is not useful.  $\mathcal{Z}$  has to many DOF and can "memorize" each input. In most cases however  $\mathcal{Z}$  is constrained that the autoencoder has to find the relevant properties of the input. This is often done by reducing the dimension if  $\mathcal{Z}$  or by regularization. Regularization is added to the Loss

$$L(x, \hat{x}) + C(z) \tag{2.2}$$

where L can be e.g. MSE and the regularization term  $C$  can enforce sparsity or other properties[18].

## 2.5 Neuron model

### 2.5.1 Biological Neuron model

The first biologically accurate model of neuron spiking behaviour is the Hodgkin–Huxley (HH) model from 1952[23]. Since then the HH model has been extended in multiple ways to cover more e.g. different ion channels. The HH-model considers the neuron with its ion channels. The membrane acts as a capacitance and the travelling ions in each ion channel contribute a current to the overall membrane potential. These ion gates are voltage dependent and are defined positive in direction out of the cell.

A particular ion channel for ion  $X$  can be modelled as

$$I_X = g_X \cdot (V - V_X) \quad (2.3)$$

These currents are summed for the different ion channels in question, most commonly for Sodium, Potassium and a leak current. In reality there are a plethora of different channels and channel properties<sup>1</sup>. The  $V_X$  are the equilibrium potentials for each of the channels and can be computed using the Nernst equation [33].

$$C \frac{dV}{dt} = g_{Na} \cdot (V - V_{Na}) + g_K \cdot (V - V_K) + g_l \cdot (V - V_l) \quad (2.4)$$

To model the voltage dependency of the ion channels, the conductances are described with gating variables, usually called  $n$ ,  $h$  and  $g$  for Na-Activation, Na-Inactivation and K-activation respectively. One gating variable is set between  $[0, 1]$  and models the permeability of said gate. Multiple gates are used to fit to each ion channel in order to match experimental data and the model behaviour.

Gates have first order dynamics of the form

$$\frac{dn}{dt} = \alpha_n(1 - n) - \beta_n n \quad (2.5)$$

for e.g. the  $n$  gate. The other gates' dynamics are analogous. The functions  $\alpha$  and  $\beta$  are voltage but not time dependent. The discussion of initial values as well as functions for  $\alpha_p$ ,  $\beta_p$   $p = (n, h, m)$  can be found in [22] or [33]. The gates for each ion channel's

---

<sup>1</sup>See `channelpedia.epfl.ch` for an extensive list

conductance are found to be

$$\begin{aligned} g_{Na} &= \bar{g}_{Na} n^4 \\ g_K &= \bar{g}_K m^3 h \end{aligned} \tag{2.6}$$

and give form to the final model

$$\begin{aligned} C \frac{dV}{dt} &= I(t) - \bar{g}_{Na} n^4 (V - V_{Na}) - \bar{g}_K m^3 h (V - V_K) - g_L (V - V_L) \\ \frac{dn}{dt} &= (1 - n) \alpha_n(V) - \beta_n n(V) \\ \frac{dm}{dt} &= (1 - m) \alpha_m(V) - \beta_m m(V) \\ \frac{dh}{dt} &= (1 - h) \alpha_h(V) - \beta_h h(V) \end{aligned} \tag{2.7}$$

We did not define a gate for the leak term as it is assumed constant.

### 2.5.2 "IF and LIF"

In contrast of the HH model in eq. (2.7), the simplest models of neurons are the Integrate and Fire (IF) and Leaky-integrate-and-fire (LIF) models.

**IF Neurons** IF Neurons, as the name implies, integrate the incoming current over time.

$$\frac{dV(t)}{dt} = \frac{1}{C} I(t) \tag{2.8}$$

The membrane voltage is governed by the incoming current spikes of connected neurons and the membrane capacitance. The neuron potential does not change without a change of input current and thus presents as a perfect integrator of the input.

**LIF Neurons** In contrast to that the LIF neuron contains a leak term on the RHS which brings the voltage back to its resting potential over time. The model can be expressed as

$$\tau \frac{dV(t)}{dt} = -(V(t) - E_r) + RI(t), \tag{2.9}$$

where  $\tau = RC$  is the time constant the composed of the membrane resistance  $R$  and the membrane capacitance  $C$  and the resting potential  $E_r$ . In the absence of input  $I(t)$  the voltage settles on the membrane potential  $E_r$ .

The input  $I(t)$  encapsulates external inputs as well as a sum of Dirac functions indicating a spiking neuron

$$I(t) = \sum_k \delta(t - t^k) \quad (2.10)$$

and  $t_k$  being the time of the  $k$ -th spike. When the membrane voltage exceeds the threshold potential  $\bar{v}$ , a spike is sent out by the neuron and the voltage sets back to its reset voltage  $v_{res}$ .

This is not correct. For weights, but the same time only when there are more than one neuron

### 2.5.3 Izhikevich Neuron

While the above models deliver a useful and cheap simplification, they lack in accuracy. The Izhikevich model [29] of the neuron tries to be the of both worlds in terms of efficiency and accuracy. It is comprised of 2D ODEs with the membrane potential  $v$  as

$$\begin{aligned} \frac{dv}{dt} &= 0.04v^2 + 5v + 140 - u + I(t) \\ \frac{du}{dt} &= a(bv - u). \end{aligned} \quad (2.11)$$

With the chosen factors, the neuron experiences a spike when  $u \geq 30\text{mV}$ , in which case the neuron resets to

$$\begin{aligned} u &\leftarrow u + d \\ v &\leftarrow c \end{aligned} \quad (2.12)$$

The parameters describe  $a$  scale of recovery,  $b$  sensitivity,  $c$  the reset potential of  $v$  and  $d$  the reset of variable  $u$ . Depending on these parameters one can achieve different behaviours of the neuron e.g. regular spiking, fast spiking and low threshold spiking to name a few [29].

Maybe shift explanation which could be extended on

## 2.6 Neural Networks

### 2.6.1 Artificial Neural Networks

#### 2.6.2 Plasticity

STDP

Gradient descent

Hebbian

Make clear distinction between forward nn and ann. Both are not the same!

maybe?

Key to give any NN the ability to solve a task is to learn/train the it. The adaption of weights and biases is necessary to accomplish any functionality based on the underlying data[59]. There are various ways to train a network. For ANNs, gradient based algorithms are often the method of choice. They use the error from the loss function do compute the derivative  $\frac{\partial L}{\partial \theta_{ij}}$  to adjust each parameter  $\theta$ , i.e. weights and biases with

$$\theta_{ij} = \theta_{ij} - \eta \frac{\partial L}{\partial \theta_{ij}} \quad (2.13)$$

until a local minima is reached. The computation of gradients is done efficiently using the backpropagation algorithm (reverse accumulation from Automatic Differentiation (AD)), in which the gradients are propagated from the output backwards towards the input by making use of the chain rule and the fact that the NN has a layered structure. An in-debt explanation is given in e.g [18] or [44].

There are a plethora of different learning techniques available, see [1][50] for a review. The most fundamental distinction can be made between supervised, unsupervised and reinforcement learning rules. One needs to remember that ANNs and SNNs require completely different learning algorithms because of their different transport of information. A short summary of learning rules in SNN is given in a later chapter.

## 2.7 Spiking Neural Networks

A spiking Neural network is one step closer to a biologic representation of a brain. Instead of conveying information using a gradient in conventional NNs, information is propagated using discrete spikes of excitation, similar to biological neurons. Hereby one can distinguish between several ideas of implementation.

### 2.7.1 Rate Networks

Poisson networks are built around the idea that information is encoded in the firing rate of a neuron. The precise timing of a spike is essentially meaningless[10]. This makes a strong contrast to the approach chosen in this paper, where every spike will be timed exactly to minimize a cost function. The encoding of a value, e.g. four, is set



by endowing the input neurons with a Poisson point process with a suitable encoding rate  $r_i$ [13].

One typically uses probabilistic stimuli because observations in the spiking of the human brain do are different in a trial by trial basis.

Input spikes are travelling through the recurrent network with weighted connections. The decoding is done by counting the spikes of output neurons over a certain time window. The time window plays a crucial rule in the decoding. If it is set smaller, spatio temporal patterns can be captured which can convey information about the input. Equally the sensitivity to noise becomes higher. If the time window is set to large, the firing patterns are lost due to exceedingly large averaging though the impact of random spikes is reduced.

Using this method is comparatively simple way of encoding as firing rates are used in favour of the individual spikes which are modelled by random processes. Additionally this approach is biologically not completely unsound. In nature it has been shown that the firing rate does convey information about the stimuli's magnitude. [2].

The connection weights are subject to change over the learning/training period[3] and can be trained with different training algorithms e.g. (anti-)Hebbian, STDP, gradient based or more involved training methods[12]. See [57] for an overview.

The issue with a Poisson process to put out spikes for its respective rate is that the Poisson process needs many spikes to transmit a signal accurately. For  $N$  neurons representing a given value the error or variance scales with  $\frac{1}{\sqrt{N}}$ [8]. This means to get accurate results a huge amount of spikes need to be fired. This is intuitive as the time between spikes is exponentially distributed and the more samples are available the better the rate parameter can be estimated. Theoretically using a large number of spikes is unproblematic and even with large spike counts neuromorphic hardware is still much more energy efficient than deep neural networks when deployed[27].

I can deliver the derivative of that number if necessary

Though there are more issues to this method. Firstly, this approach has the problem that responses are limited by the time window in which spikes are counted[4]. This means that the rate decoding is too slow to capture the fast travelling information[20] and there need to be faster ways to transmit information. The second problem is that due to the Poisson process one needs more spikes to represent an average firing rate. An action potential consumes a lot of the cells energy[5], thus making it unfeasible to use a large number of spikes if one tries to model the brain. There are more efficient

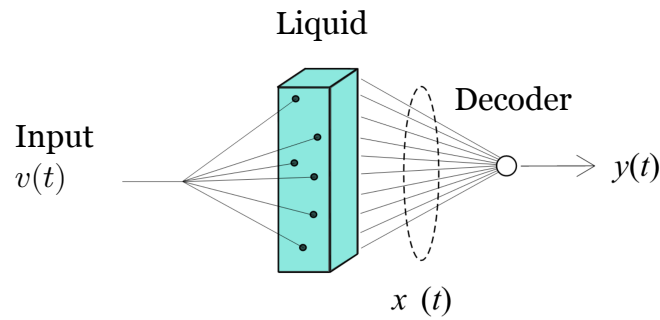


Figure 2.7.1: Abstract idea of Reservoir Computing. Adapted from [38]

ways to transmit information

Although the above mentioned problems, rate encoded SNNs have seen interest by research. A big hurdle of deploying SNNs is the lack of performant learning algorithms. For this there efforts have been made to train recurrent or convolutional ANNs using backpropagation and afterwards convert the trained network to a SNN[47] using rate encoding[16][15].

## 2.7.2 Liquid state machines

write how the offline computing is pretty bad for brain things, but good for chess for example. The online computing is what the brain does and it is not yet as developed.

One alternative method has been the use of Liquid State Machine (LSM) or more general Reservoir computing.

The term Reservoir computing was introduced by Benjamin Schrauwen and describes a general group of recurrent network approach[55].

The "reservoir" is a non-linear map from input to outputs that combines the input in various, even random ways. These contain but are not limited to sums, differences, multiplications, division and exponentiation. In general the output  $\mathbf{x}(t)$  is higher dimensional than the input  $\mathbf{v}(t)$ , in order to allow for sufficient variety in the mapping. The output of the reservoir, which is usually treated as a black box, is fed in a linear decoder in order to retrieve the desired output signal.

The liquid can be made of any system that fulfils two properties.

- Non-linear nodes of computation
- Fading memory

To these points it is usually set for the system to be time invariant[38]. A reservoir can be a mathematical abstract formulation or physical object, e.g. a literal bucket of water [51].

After the choice of "liquid" in the reservoir is fixed, its dynamics are not altered. Only the linear decoder is trained to return the desired decoded output[30]. This is a considerable time saver since the training of recurrent networks is expensive. On the contrary the linear decoder can be learned relatively cheaply.

A reservoir computer is called a LSM if one chooses a spiking neural network as the reservoir. The requirements mentioned above are fulfilled by the recurrent structure to retain information of the neurons and its non-linear spiking behaviour.

LSMs are capable of computing any dynamical system of any order of the form of

$$z^{(n)} = G(z, z^{(1)}, z^{(2)}, \dots, z^{(n-1)}) + u \quad (2.14)$$

given a sufficiently large liquid and a suitable feedback and decoder[41] and have been used for speech recognition[31][58]. The systematic structure can be in fig. 2.7.2. The feedback  $K(x, u)$  is a function of the dynamical system input  $u(t)$  and the output  $x(t)$ . The result of  $K(x, u)$  is fed back replaces the previous input  $v(t)$  into the Liquid. The decoder  $h(x)$  is not linear but can be simplified to be in a cost-performance trade-off when using a sufficiently large Liquid.

### 2.7.3 Balanced Networks

The idea of tightly balanced spiking networks was first proposed by Boerlin et al.[8]. It uses predictive coding in combination with spikes to simulate arbitrary linear systems. The technical derivation will be described in section 3.2.1. The approach defines a cost function measuring the networks' accuracy in addition with regularization terms that moderates the spiking behaviour. Using a greedy algorithm this cost function determines the voltage threshold and therefore the neurons' spiking behaviour.

For each neuron voltage can be understood as a projection of the global system error to a local error. One neuron is only tracking the system error under this projection.

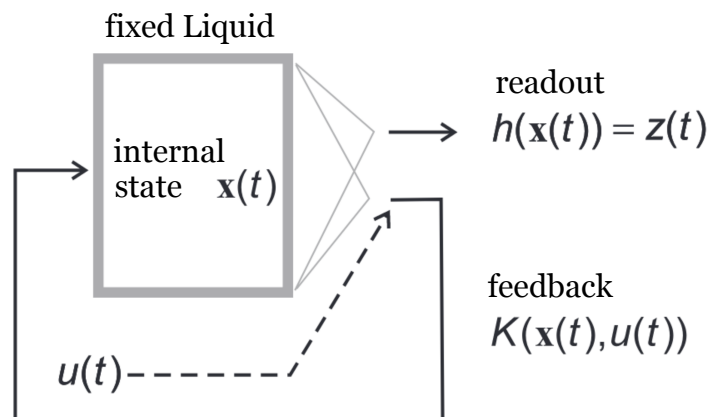


Figure 2.7.2: Adding suitable feedback allows LSMs to be universal approximator. Adapted from [40]

When the error under this projection reaches a threshold, a spike is fired.

The firing of a neuron resets its voltage as well as correcting the system to reset the perceived projected error.

Balanced networks differ from the previous rate encoding in that excitation and inhibition is closely tracked. In rate encoded networks both inhibitory and excitatory spikes are received by a single neuron. A change of the variable is then governed by which type dominates. Here a rate coding is used, however in the matter of control, a combination with instantaneous decoding is [32] utilized.

Additionally, every neuron is given an specific area of the system to surveil. The projection of the system error therefore has therefore immense influence on an individual neuron's spiking behaviour. On the other hand on rate networks a population of neurons is given a value to represent and the population spiking rate encodes this value. An individual spike in this population does not have a direct relevance to the system and a discrete impact on its behaviour.

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## 2.7.4 Plasticity

It's important to remember that ANNs and SNNs require completely different learning algorithms because of their different transport of information.

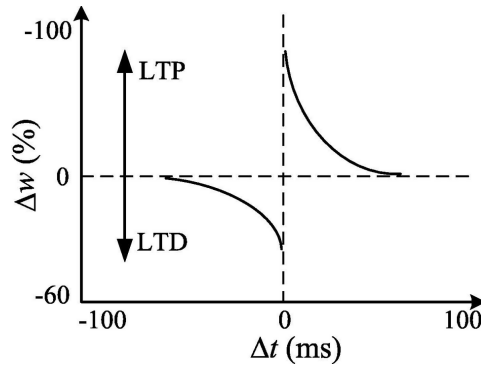


Figure 2.7.3: Graphical representation of STDP learning rule. Weight change depending on the time between pre- and postsynaptic activation. Negative time is the time between the postsynaptic neuron firing before the presynaptic. Graphic taken from [57].

Gradient based methods require differentiability and therefore continuity, thus are only applicable for ANNs. This means that they cannot be used for SNNs since spikes introduce discontinuities. Methods have been proposed to use backpropagation in spiking networks[35] yet they do not provide a biologically plausible way to learn. The problem is that in biology neurons do not have access to the global error from the loss function but only to their pre-synaptic neurons.

### (Anti-) Hebbian Learning and STDP

The Hebbian learning rule is one of the oldest learning rules for neural networks with large experimental evidence in biology(see for example [17] for a summary). Its key idea can be summarized as "Neurons that fire together, wire together". If the postsynaptic neuron fires shortly after the presynaptic neuron, the connection strength is increased. Oppositely, if the postsynaptic neuron fires before the presynaptic, their connection strength is decreased.

The longer the delay between firing activation, the smaller the increase. This behaviour of potentiation and depression is pictured in fig. 2.7.3 and build the bases for the spike time dependent plasticity rule (STDP). In addition to the Hebbian rule, there is also the anti-Hebbian rule that is reverses the aforementioned behaviour. This means that regular firing of pre- and postsynaptic neurons is discouraged and more irregular and distributed spiking is favoured. This can be understood as mirroring fig. 2.7.3 along the x-axis.

These rules are based in biology and satisfy the restrictions found in nature. Many

learning rules adapted from ANNs lack these properties e.g. locality. Locality demands that the basis of learning is restricted to having only the information of the direct pre- and postsynaptic neuron available. This already rules out most backpropagation algorithms as they come derive the derivative with respect to a global cost function.

Of course there many variations and extensions have been proposed, e.g. Spike Rate Dependent Plasticity (SRDP)[34], which have been summarized in [57].

# Chapter 3

## <Engineering-related content, Methodologies and Methods>

Research question: Develop a biologically sensible SNN to control any linear dynamical system.

Research  
question:  
Develop a  
biologically  
sensible SNN  
to control  
any linear  
dynamical  
system.

### 3.1 Choice of Network architecture

The field of SNNs is under ongoing research. Therefore many different network models and learning approaches have been proposed e.g. LSMs[14]. In order to stay biologically more realistic we ignore purely rate based spiking networks as there is evidence that the precise spike timing is relevant in nature[10][48]. On the other side of the spectrum it makes little sense to use HH's model. Even though it is very biologically plausible its increased complexity and little abstraction makes it more suited for solely accurate biological neural simulation and less for the engineering task at hand. Additionally there are no training/learning rules available to solve such a high level problem.

Explain choice of SNN architecture

HH doesn't make sense because too complex.

Recurrent models are possible but also not biologic enough because we want to use the spiking property

LSM was considered but in a LSM you only learn the decoder. For the problem at hand

is is more natural to have the dynamics in a neural network. Plus with this approach we learn the decoder as well so there it makes it for a more independent approach.

Design a LQG controller to start with as reference First found how to simulate a dynamical system with given input  $c$  from Boerlin

Then finding the controller structure to find external input  $c$  to control the system in a desired way

Then notice you need to have magic numbers to get it to work properly.

Then trying to bring the learning mechanics to that approach.

Deemed difficult

finding way to control a dynamical system with the learning SNN framework

Also not so far to do nonlinear systems

Describe the engineering-related contents (preferably with models) and the research methodology and methods that are used in the degree project.

Most likely it generally describes the method used in each step to make sure that you can answer the research question.

## 3.2 Simulation of Dynamic systems using SNNs

In the following sections the simulation of dynamic systems using SNNs is derived and explained. This serves as the a basic building block for the attempted method on how to solve our target set out in section

add reference to the goal section

. We begin with the formal derivation of the network dynamics.

### 3.2.1 Balanced network simulation

This section follows the derivation found in [8] and [24]. The goal is to describe a dynamical system of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{c}(t) \tag{3.1}$$



with  $J$  state variables. The estimation is done by leaky integration of spike trains  $\mathbf{o}(t)$  in

$$\dot{\hat{\mathbf{x}}} = -\lambda_d \hat{\mathbf{x}} + \mathbf{\Gamma} \mathbf{o}(t). \quad (3.2)$$

$\mathbf{\Gamma}$  is a given Matrix of size  $\mathbb{R}^{J \times N}$ ,  $N$  being the number of neurons. This matrix is given as initial and can be optimized by training later on[9].

In addition to the estimate  $\hat{\mathbf{x}}$  we define a spiking rate variable  $\mathbf{r}$  following the dynamics of

$$\dot{\mathbf{r}} = -\lambda_d \mathbf{r} + \mathbf{o}(t). \quad (3.3)$$

The rate variable is connected to the state vector in the decoding with

$$\hat{\mathbf{x}} = \mathbf{\Gamma} \mathbf{r}. \quad (3.4)$$

explain how this is better than just rate encoding

The spiking dynamics arise from the minimization of a cost function. A spike is fired if it minimizes the cost function that tracks the error between the true and estimated value over time

$$E(t) = \int_0^t \|\mathbf{x}(u) - \hat{\mathbf{x}}(u)\|_2^2 du. \quad (3.5)$$

### 3.2.2 Greedy optimization of the cost

The cost function eq. (3.5) is minimized using a greedy optimization i.e. a spike is fired if it reduces the cost. For the derivation we use the cost function eq. (3.22) which is identical to setting  $\mu = 0, \nu = 0$ .

We express this as

$$E(t|i \text{ spike}) < E(t, i \overline{\text{spike}}) \quad (3.6)$$

If there is no spike fired, the rate and estimated state variable in eq. (3.2) and eq. (3.3) respectively behave as

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= -\lambda_d \hat{\mathbf{x}} \\ \dot{\mathbf{r}} &= -\lambda_d \mathbf{r} \end{aligned} \quad (3.7)$$

and therefore decay exponentially with  $e^{-\lambda_d t}$ .

If a spike is fired at time  $t^k$ , the inhomogeneous solution is found by variation of

constants in eq. (3.3) to

$$\begin{aligned}
 r_i^h &= c_i(t)e^{-\lambda_d t} \\
 c_i'(t)e^{-\lambda_d t} - c_i(t)\lambda_d e^{-\lambda_d t} &= -\lambda_d c_i(t)e^{-\lambda_d t} + \delta(t - t^k) \\
 c_i'(t) &= \delta(t - t^k)e^{\lambda_d t} \\
 c_i(t) &= e^{\lambda_d t^k} \mathbf{H}(t - t^k) \\
 r_i &= e^{-\lambda_d t} + e^{-\lambda_d(t-t^k)} \mathbf{H}(t - t^k).
 \end{aligned} \tag{3.8}$$

The last equation is the identical the solution of eq. (3.7) with the addition of a decaying exponential added at time  $t_i^k$ .  $\mathbf{H}(t)$  denotes the Heaviside step function . Analogously the estimate is updated at time  $t^k$  to

$$\mathbf{x} = \mathbf{x} + \mathbf{\Gamma}_i e^{-\lambda_d(t-t^k)} \mathbf{H}(t - t^k). \tag{3.9}$$

We look at the error a  $\epsilon$  time in the future of  $t^k$  and check eq. (3.6)

$$\begin{aligned}
 &\int_0^{t^k+\epsilon} \left( \underbrace{\|\mathbf{x}(u) - \hat{\mathbf{x}}(u) - \mathbf{\Gamma}_i h(u - t^k)\|_2^2}_I + \underbrace{\nu \|\mathbf{r}(u) + \lambda_d \mathbf{e}_i h(u - t^k)\|_1}_{II} \right. \\
 &\quad \left. + \underbrace{\mu \|\mathbf{r}(u) + \lambda_d \mathbf{e}_i h(u - t)\|_2^2}_{III} \right) du \\
 &< \int_0^{t^k+\epsilon} (\|\mathbf{x}(u) - \hat{\mathbf{x}}(u)\|_2^2 + \nu \|\mathbf{r}(u)\|_1 + \mu \|\mathbf{r}(u)\|_2^2) du
 \end{aligned} \tag{3.10}$$

where we abbreviated  $h(u) = e^{-\lambda_d(u)} \mathbf{H}(u)$ . To treat each term individually we start with I. Simplifying the norm we obtain

$$I = \|\mathbf{x}(u) - \hat{\mathbf{x}}(u)\|_2^2 - 2h(u - t^k) \mathbf{\Gamma}_i^T (\mathbf{x}(u) - \hat{\mathbf{x}}(u)) + h^2(u - t^k) \mathbf{\Gamma}_i^T \mathbf{\Gamma}_i. \tag{3.11}$$

For II the 1-norm and the rate holds that the  $r_i(u) > 0 \quad \forall i$ . Thus we can simplify  $\|\mathbf{r}\|_1 = \sum_k r_k$  resulting in

$$II = \nu (\|\mathbf{r}\|_1 + h(u - t^k)). \tag{3.12}$$

Similarly to I, III can be simplified by  $\|\mathbf{r}\|_2^2 = \mathbf{r}^T \mathbf{r}$ , giving

$$III = \mu \|\mathbf{r}\|_2^2 + \mu h^2(u - t^k) + 2\mathbf{r} \cdot \mathbf{e}_i h(u - t^k). \tag{3.13}$$

After cancellation the remaining terms are grouped grouped by time dependency to yield

$$\begin{aligned} & \int_0^{t^k+\epsilon} h(u-t^k) \mathbf{\Gamma}_i^T (\mathbf{x}(u) - \hat{\mathbf{x}}(u)) - \mu r_i(u) du \\ & > \frac{1}{2} \int_0^{t^k+\epsilon} h^2(u-t^k) \mathbf{\Gamma}_i^T \mathbf{\Gamma}_i + \nu h(u-t^k) + \mu h^2(u-t^k) du \end{aligned} \quad (3.14)$$

Using the fact that the Heaviside function in eq. (3.8) and subsequently in  $h(u)$  allow us to change the borders of integration to  $\int_{t^k}^{t^k+\epsilon}$ . Lastly we simplify  $h(t) = 1$  if  $t \approx \epsilon$  and have

$$\mathbf{\Gamma}_i^T (\mathbf{x} - \hat{\mathbf{x}}) - \mu r_i > \frac{\|\mathbf{\Gamma}\|^2 + \nu + \mu}{2} \quad (3.15)$$

We notate the Left Hand Side (LHS) as the voltage and the constant Right Hand Side (RHS) as the voltage threshold  $T_i$

$$V_i > T_i = \frac{\|\mathbf{\Gamma}_i\|^2 + \nu + \mu}{2}. \quad (3.16)$$

### 3.2.3 Neuron Voltage

As mentioned above, a neuron spikes if it meets the condition eq. (3.16). But so far it is unclear how neuron voltage evolves over time. Denote  $\mathbf{L}$  the the left pseudo-inverse of  $\mathbf{\Gamma}$

$$\mathbf{L} = (\mathbf{\Gamma} \mathbf{\Gamma}^T)^{-1} \mathbf{\Gamma} \quad (3.17)$$

such that  $\mathbf{L} \mathbf{\Gamma}^T = \mathbf{I}$ .

Next, taking the derivative of eq. (3.29) yielding

$$\dot{\mathbf{V}}(t) = \mathbf{\Gamma}^T (\dot{\mathbf{x}}(t) - \dot{\hat{\mathbf{x}}}(t)) - \mu \dot{\mathbf{r}}(t). \quad (3.18)$$

Now using the pseudo-inverse to rewrite the voltage equation eq. (3.29) as

$$\begin{aligned} \mathbf{V}(t) &= \mathbf{\Gamma}^T (\mathbf{x}(t) - \hat{\mathbf{x}}(t)) - \mu \mathbf{r}(t) \\ \mathbf{L} \mathbf{V}(t) &= (\mathbf{x}(t) - \hat{\mathbf{x}}(t)) - \mu \mathbf{L} \mathbf{r}(t) \\ \mathbf{x}(t) &= \mathbf{L} \mathbf{V}(t) + \hat{\mathbf{x}}(t) + \mu \mathbf{L} \mathbf{r}(t) \end{aligned} \quad (3.19)$$

Remember that i read somewhere that the no is necessary Maybe men that here to And find th reference

Now the derivative terms in eq. (3.18) are replaced with their respective equations eq. (3.1), eq. (3.2) and eq. (3.3). Lastly we substitute eq. (3.19) in eq. (3.18) and obtain

$$\begin{aligned}\dot{\mathbf{V}} &= \mathbf{\Gamma}^T \mathbf{A} \mathbf{L} \mathbf{V} \\ &+ (\mathbf{\Gamma}^T \mathbf{A} \mathbf{\Gamma} + \mu \mathbf{\Gamma}^T \mathbf{A} \mathbf{L} + \lambda_d \mathbf{\Gamma}^T \mathbf{\Gamma} + \mu \lambda_d) \mathbf{r} \\ &+ (\mathbf{\Gamma}^T \mathbf{\Gamma} + \mu) \mathbf{o} + \mathbf{\Gamma}^T \mathbf{c}.\end{aligned}\tag{3.20}$$

The last argument is to consider the network behaviour for larger networks. We increase the number of neurons  $N \rightarrow \infty$  and require that the network output as well as the firing rates remains constant.

When looking at the decoding at eq. (3.4) we therefore need to scale  $\mathbf{\Gamma}$  by  $\frac{1}{N}$ . To make sure that the threshold in eq. (3.16) will not get dominated by cost terms  $\mu$  &  $\nu$ , they should also scale with  $\frac{1}{N^2}$ . As the threshold decreases with  $\frac{1}{N^2}$  so does the Voltage itself. With this in mind, all terms that scale with  $\frac{1}{N^2}$  are neglected. As a substitute for the neglected voltage term, a generic leak term is added making these LIFs neurons. The dynamics are therefore

$$\begin{aligned}\dot{\mathbf{V}} &= -\lambda_V \mathbf{V} + \mathbf{W}^s \mathbf{r} + \mathbf{W}^f \mathbf{o} + \mathbf{\Gamma}^T \mathbf{c} \\ \mathbf{W}^s &= \mathbf{\Gamma}^T (\mathbf{A} + \lambda_d \mathbf{I}) \mathbf{\Gamma} \\ \mathbf{W}^f &= -(\mathbf{\Gamma}^T \mathbf{\Gamma} + \mu \mathbf{I})\end{aligned}\tag{3.21}$$

find a coherent name for the matrix

### 3.2.4 Regularization

Two regularization terms are added to influence spiking behaviour.

$$E(t) = \int_0^t (\|\mathbf{x}(u) - \hat{\mathbf{x}}(u)\|_2^2 + \nu \|\mathbf{r}(u)\|_1 + \mu \|\mathbf{r}(u)\|_2^2) du\tag{3.22}$$

The parameter  $\nu$  controls the amount of spiking by penalizing the total number of spikes as

$$\|\mathbf{r}(t)\|_1 = \sum_i |r_i(t)| = \sum_i r_i(t).\tag{3.23}$$

The firing rate is directly related to the number of spiking and therefore the cost is reduced by fewer spikes.

The second term solves different issues at the same time. One problem concerns networks that have decoding kernels with the same direction but opposite sign. To show this we imagine a network of only two neurons. A network of two neurons is sufficient to simulate a scalar ordinary differential equation (ODE) i.e  $\mathbf{A} \in \mathbb{R}$ . We further assume that the kernel has the form

$$\Gamma = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad (3.24)$$

Ignoring the cost terms in eq. (3.16) the threshold is set at

$$V_i > \frac{\|\Gamma_i\|^2}{2} \quad (3.25)$$

after which that a spike is fired and the voltage of neuron  $i$  resets to

$$V_i = V_i + \mathbf{W}_{ii}^s = V_i + \|\Gamma_i\|_2^2 \quad \text{with } \mathbf{W}^f = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \quad (3.26)$$

ideally setting the Voltage to  $-T_i$ . This can be seen when looking at the threshold as

$$T_i = \frac{\|\Gamma_i\|^2}{2} = \frac{-\text{diag}(\mathbf{W}^f)}{2}. \quad (3.27)$$

The repolarization of the spiking neuron acts as a depolarization or pushing the voltage towards its threshold for neurons with opposing sign. The problem now is that for neurons with the same kernel magnitude the depolarization is larger enough to push this neuron over the threshold. The subsequent spike re-polarizes the neuron but in turn excites the first neuron over its threshold. This pattern repeats and destroys network performance.

For the given example above, the threshold is given by 0.5 for both. The neurons' voltages of are identical up to the sign, since they are tracking the error for the same variable. At the time one neurons reaches the threshold of 0.5 the second neuron's voltage is close to -0.5 considering noise (in a perfect system minus the value of the spiking neuron). After the spike is fired, the first neuron is reset to -0.5 stemming

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$$\begin{aligned} \mathbf{V} &= \mathbf{V} + \mathbf{W}^f \mathbf{o} = \mathbf{V} + \mathbf{W}^f \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \mathbf{V} + \mathbf{W}_{:0}^f \\ \mathbf{V} &= \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} \end{aligned} \quad (3.28)$$

$\mathbf{W}_{00}^f$  whereas the second neuron gets pushed up to 0.5 , causing a spike. This in turn reverts the changes of eq. (3.28) resulting in a loop. This problem is caused by the greedy optimization, looking only at the immediate future to decrease the cost.

To fix this we set the threshold slightly higher. As seen above in eq. (3.16), this can be done by either raising the linear or quadratic cost.

The second issue fixed by adding quadratic cost is when there are neurons with similar kernel direction but non normalized. In a perfect noise free scenario the neuron with the smaller threshold will always fire first. The neurons reset after the spike will reset the neurons with similar direction, inhibiting the second neuron from ever firing. The linear cost do not make a difference since it is penalizing the global number of spikes but does not discern where the spikes are fired. With the quadratic cost the norm of spike rates distributed among many neurons is reduced compared to few.

---

The first was termed "ping-pong" effect and is described in the supplementary material of [8]. To understand the issue, we imagine a minimal network consisting of 2 neurons with equal kernel but opposite sign.

The second regularization comes into play when there are kernels with different magnitude. Kernels with small kernel magnitude reach their threshold sooner and therefore fire more frequently. In the extreme case, only small number of neurons fire rapidly while the majority remains idle. By penalizing the rate in the 2-norm it forces the network to spread the firing among the whole network.

The dynamic variable  $\mathbf{x}$  is tracked by firing spikes in when the defined "pseudo voltage" of a neuron surpasses its threshold. The voltage for each neuron is defined by

$$V_i(t) = \mathbf{\Gamma}^T(\mathbf{x}(t) - \hat{\mathbf{x}}(t)) - \mu \lambda_d r_i(t) \quad i = 1 \dots N. \quad (3.29)$$

Add figure

Figure 3.3.1: Schematic to illustrate the use of balanced networks as controllers.

For negligible quadratic cost  $\mu$  the voltage can be understood as measure of the error projected on  $\Gamma_i$ . The explicit derivation of the above equation is found in [8] and will be adapted .

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## 3.3 Control of Dynamic systems using SNNs

### 3.3.1 Balanced networks as a controller

We now make the step to use the balanced network approach from above as a controller mechanism.

The idea was taken from [26] and is illustrated in fig. 3.3.1. With the given reference signal, the network receives the feedback error of the system. The networks spikes are decoded into a control signal which is further fed into the dynamical system.

The system itself is simulated using a common numerical method i.e. explicit Euler. Yet the goal is to capture the entire problem using SNNs. The control signal  $u$  is generated using the an independent SNN which is in turn the command  $c$  for a separate SNN simulating the states with feedback to the controlling SNN.

### 3.3.2 Dynamics

The derivation of this method is similar to the one in section 3.2. Names and variables are reused if not stated here.

The system in question has the form

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}. \quad (3.30)$$

The basic definitions of the SNN remain the same with rate  $\mathbf{r}$  as well as decoding weights  $\mathbf{\Gamma}$ . Additionally, [26] defines instantaneous decoding weights  $\mathbf{\Omega}$  with the same shape as  $\mathbf{\Gamma} \in \mathbb{R}^{J \times N}$ . It is important to note that  $J$  does not represent the number of state variables but the number of inputs. The decoding is the same as in eq. (3.4) with the added  $\mathbf{\Omega}$  giving.

$$\mathbf{u}(t) = \mathbf{\Gamma r} + \mathbf{\Omega o}. \quad (3.31)$$

The derivation of the network dynamics in [24] is similar to [8] and the derivation presented above. Differences arise in the computation of the cost function as the spike changes the system to

$$\begin{aligned} \mathbf{u} &= \mathbf{u} + h(t - t^k)\mathbf{\Gamma}_k + \mathbf{\Omega}_k \\ \mathbf{r} &= \mathbf{r} + h(t - t^k)\mathbf{e}_k \\ \hat{\mathbf{x}} &= \hat{\mathbf{x}} + h(t - t^k) \int_0^{t-t^k} e^{(\mathbf{A} + \lambda_d \mathbf{I})\zeta} d\zeta \mathbf{B}\mathbf{\Gamma}_k + e^{A(t-t^k)} \mathbf{B}\mathbf{\Omega}_k \end{aligned} \quad (3.32)$$

where  $\mathbf{\Gamma}_k$  and  $\mathbf{\Omega}_k$  correspond to the  $k$ -th column of  $\mathbf{\Gamma}$ ,  $\mathbf{\Omega}$  and  $h$  the same as defined above. Results are similar for the rate and control signal whereas the state update is obtained by formally integrating the system. The rest of the derivations are analogous and completely derived in [26]. The results summarize to eqs. (3.33) to (3.38).

$$\mathbf{V} = \mathbf{\Omega}^T \mathbf{B}^T (\mathbf{x} - \hat{\mathbf{x}}) - \mu \mathbf{r} \quad (3.33)$$

$$\dot{\mathbf{V}} = -\lambda_V \mathbf{V} + \mathbf{\Omega}^T \mathbf{B}^T \mathbf{c}(t) + \mathbf{W}^f \mathbf{o} + \mathbf{W}^s \mathbf{r} \quad (3.34)$$

$$\mathbf{c} = \dot{\mathbf{x}} - \mathbf{A}\mathbf{x} \quad (3.35)$$

$$\mathbf{W}^f = -(\mathbf{\Omega}^T \mathbf{B}^T \mathbf{B}\mathbf{\Omega} + \mu \mathbf{I}) \quad (3.36)$$

$$\mathbf{W}^s = -\mathbf{\Omega}^T \mathbf{B}^T \mathbf{B}\mathbf{\Gamma} \quad (3.37)$$

$$T_i = \frac{\mathbf{\Omega}_i^T \mathbf{B}^T \mathbf{B}\mathbf{\Omega}_i + \nu + \mu}{2} \quad (3.38)$$

Note that the notation differs in the original paper and the reference signal is denoted by  $\hat{\mathbf{x}}$  instead of  $\mathbf{x}$  here and  $\mathbf{\Omega}_i$  again refers to the  $i$ -th column of  $\mathbf{\Omega}$ .

### 3.3.3 The instantaneous decoding weights

The necessity of instantaneous decoding is necessary otherwise no spiking can occur. In eq. (3.32) the control signal is integrated with the matrix exponential. The problem is that the integral

$$\lim_{t \rightarrow 0} \int_0^t e^{(\mathbf{A} + \lambda_d \mathbf{I})\zeta} d\zeta = 0 \quad (3.39)$$



for our small  $\epsilon$  time horizon.

This is true for any matrix exponential  $e^{\Lambda\zeta}$  seen by Taylor expansion

$$\begin{aligned}\lim_{t \rightarrow 0} \int_0^t e^{\Lambda\zeta} d\zeta &= \lim_{t \rightarrow 0} \int_0^t \sum_{k=0}^{\infty} \frac{(\Lambda\zeta)^k}{k!} d\zeta \\ &= \lim_{t \rightarrow 0} \sum_{k=1}^{\infty} t \frac{(\Lambda t)^{k-1}}{k!} = 0.\end{aligned}\tag{3.40}$$

This means that the rate decoding vanishes in the derivation of eqs. (3.33) to (3.38). Therefore the firing threshold condition becomes

$$-\mu \mathbf{r}_i > \frac{\nu + \mu}{2}\tag{3.41}$$

if  $\Omega$  is ignored which is an insatiable condition since  $\mathbf{r}$  is always non-negative.

### 3.3.4 Extension with direct Error feedback

The same group of [26] later published a new but very similar approach in [25] which is based on the same idea, however the approach in [25] makes the error a direct part of the voltage dynamics. The difference arises from the an new derivation avoiding the pseudo-inverse  $\mathbf{L}$ .

Instead, during the analogous step of eq. (3.18) they set  $\hat{\mathbf{x}}$  and  $\mathbf{x}$  to follow the same dynamics, namely

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{c}(t)\tag{3.42}$$

for the reference signal and

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u}\tag{3.43}$$

for the system.

In total this adjustment changes the dynamics of eq. (3.34) to

$$\dot{\mathbf{V}} = -\lambda_V \mathbf{V} + \Omega^T \mathbf{B}^T \mathbf{A} \mathbf{e}(t) + \Omega^T \mathbf{B}^T \mathbf{c}(t) + \mathbf{W}^f \mathbf{o} + \mathbf{W}^s \mathbf{r}.\tag{3.44}$$

Important to note is that due to the investigation of the network's limit behaviour in [26], similarly done in eq. (3.20) and the subsequent neglect of certain terms, changes the definition of  $\mathbf{W}^s$ . In the derivation in [25] this is not done and therefore terms

remain changing  $\mathbf{W}^s$ 's definition to

$$\mathbf{W}^s = -\boldsymbol{\Omega}^T \mathbf{B}^T \mathbf{B} \boldsymbol{\Gamma} + \mu \mathbf{I}. \quad (3.45)$$

add that there is always noise somewhere

### 3.3.5 Direct error

The important feature of the approach in eqs. (3.36) and (3.42) is that the signal  $\mathbf{c}(t)$  is explicitly calculated from the system  $\mathbf{A}$  as well as the information of the reference signal and its derivative.

## 3.4 Learning of network parameters

All the methods described above work on the optimally ideal weights for the dynamics. However in nature often new skills or dynamics are learned and not optimally tuned for the problem at hand. In the following chapter the optimality of derived is explained and local learning rules for the weights are introduced.

### 3.4.1 Optimal weights

**Fast connection weights  $\mathbf{W}^f$**

**Slow connection weights  $\mathbf{W}^s$**

### 3.4.2 Learning of fast connection weights $\mathbf{W}^f$

### 3.4.3 Learning of slow connection weights $\mathbf{W}^s$

The recurrent weights  $\mathbf{W}^s$  are called slow because they are in conjunction with the filtered spike train  $\mathbf{r}(t)$  in eq. (3.20) in contrast to  $\mathbf{W}^f$  which are reset the neuron voltage after a spike.

### 3.5 Engineering-related and scientific content:

Applying engineering related and scientific skills; modelling, analysing, developing, and evaluating engineering-related and scientific content; correct choice of methods based on problem formulation; consciousness of aspects relating to society and ethics (if applicable).

As mentioned earlier, give a theoretical description of methodologies and methods and how these are applied in the degree project.

was ist meine research question?

zusammensetzung von den beiden systeme: dynamisches system und neuronales netz. mehr oder weniger die herleitung kopieren aus dem paper. Dann mit learning von den gewichten.

Here I describe what how it needs to be done. So this is the place for the derivation The concept and the process whatever that means Later there comes the how I implemented it. Here is what we needs to be implemented.

Here very detailed explanation of the Balanced network for this problem

Very detailed way for the regular NN for this problem Basics of the controller design used in this comparison aka LQG controller

Method of learning the weights for the SNN Method of comparison

# Chapter 4

## <The work>

Describe the degree project. What did you actually do? This is the practical description of how the method was applied. How do we make the SNN MATLAB Balanced spiking network (say why to use that )

maybe pseudo code Ideally some theorem (convergence???) Simulation? nein kommt in den naechsten part

### 4.1 Writing the balanced network simulation

### 4.2 Writing the balanced network controller

### 4.3 Learning the dynamics

Write that the most of all the control is pretty picky for parameters. This means that it is not very versatile. So instead we want to learn our own parameters so we have adaptability. Enter in the learning paper of Deneve and Brendel and why it would be nice. It would be nice bcs we can learn the parameters independently and can preserve some structure or the fact that we want to include B matrix somewhere This doesnt work for some reasons I need to point out. Then the bourdoukan paper Then we set this for control. And results

# Chapter 5

## <Result>

Describe the results of the degree project. Analyses of results

How does the optimal network simulate? Influence of parameters

What are the magic parameters?  $\mu$  etc

The control with optimal works fine too. Influence of the parameters Magic number tuning required for x,y,z etc

Learning works fine with magic numbers Examples. What works better? What works worse? What if I only learn 1 part? What is the bigger error. Convergence over time usually very bad Compared to the eigenvalues maybe convergence improves How do the parameters look? Heatmap How to judge the accuracy of the closeness of the parameters to the optimal What is the influence of the input You have to juggle the values of learning rate and input amplitude to reach ideal results

# Chapter 6

## <Conclusions>

Describe the conclusions (reflect on the whole introduction given in Chapter 1).

Discuss the positive effects and the drawbacks.

Describe the evaluation of the results of the degree project.

Describe valid future work.

The sections below are optional but could be added here.

### 6.1 Discussion

#### 6.1.1 Future Work

find nonlinearity

end point control

#### 6.1.2 Final Words

# Todo list

■ Provide a general introduction to the area for the degree project. Use references!	
Link things together with references. This is a reference to a section: . . .	1
■ Maybe separate this in intro and background . . . . .	1
■ This is a separator . . . . .	2
■ Background. . . . .	3
■ Now list the goal: We want to do it for DS and check how good they are. Then method and then work. Take from below . . . . .	3
■ true? . . . . .	4
■ find refs . . . . .	4
■ Add that LQR made many improvements but is also trash because of the matrix inversion, and nonlinear things in biology. Maybe also that it is an offline thing? Not sure whether that's true or if it is already online/can do noise but I believe yes. . . . .	4
■ this is another smaller separator . . . . .	4
■ backprop is not bio feasible bcs neurons are local and cannot traverse whole network . . . . .	4
■ Feedforward have no memory. Makes it hard for many tasks to be useful. With recurrence you get memory. . . . .	4
■ add other snn models. e.g reach, sorn maybe or others, FORCE . . . . .	4
■ What do we else want like the brain. Maybe low spike count? And what can we do? Also References! . . . . .	7
■ sounds vague . . . . .	7
■ Benefits? . . . . .	7
■ Paper [47] lesen fuer benefits und . . . . .	7

■ Instead the original learning SNN approach is used to control the system directly. . . . .	9
■ How do we compare the end performance? . . . . .	9
■ make the outline in the end! . . . . .	10
■ List here also efforts with other concepts apart from Balanced Networks . . .	11
■ This is not truly correct. Forgot weights, but at the same time only when there are more than 1 neuron . . . . .	15
■ Maybe shitty explanation, which could be extended on. . . . .	15
■ Make clear distinction between forward nns and ann. Bcs apparently they are not the same! . . . . .	15
■ The main difference is the motion of time . . . . .	16
■ Maybe add that our approach does not rule rate out completely. . . . .	16
■ I can deliver the derivation of that number if necessary . . . . .	17
■ write how the offline computing is pretty bad for brain things, but good for chess for example. The online computing is what the brain does and it is not yet as developed. . . . .	18
■ Add more differences prob and explain more why we use this one. . . . .	20
■ Research question: Develop a biologically sensible SNN to control any linear dynamical system. . . . .	23
■ Research question: Develop a biologically sensible SNN to control any linear dynamical system. . . . .	23
■ add reference to the goal section . . . . .	24
■ explain how this is better than just rate encoding . . . . .	25
■ Remember that i read somewhere that the noise is necessary. Maybe mention that here too. And find the reference . . . . .	27
■ find a coherent name for the matrix . . . . .	28
■ maybe a picture . . . . .	29
■ Write better the ping pong effect! Maybe later . . . . .	30
■ find the right place to explain that! . . . . .	30
■ Add figure . . . . .	31
■ Where? Here, in the appendix of at all? . . . . .	31
■ add that there is always noise somewhere . . . . .	34



**If you are using mendeley to manage references, you might have to export them manually in the end as the automatic ways removes the "date accessed" field**

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# Appendix - Contents

<b>A First Appendix</b>	<b>53</b>
<b>B Second Appendix</b>	<b>54</b>

# **Appendix A**

## **First Appendix**

This is only slightly related to the rest of the report

# **Appendix B**

## **Second Appendix**

this is the information