Coordenadas.

Seja V um espaço vetorial, $\{b_1, \dots, b_n\} = B$ base

$v \in V$ pode ser escrito unicamente como

$$v = \alpha_1 b_1 + \dots + \alpha_n b_n$$

$$\alpha = (\alpha_1, \dots, \alpha_n) = [v]_B$$ vetor das coordenadas

<u>Exemplo</u> $V = \mathbb{R}^2$ $B = \{(1,-1), (1,1)\}$

$$v = (2, 3) \qquad [v]_B = \left(-1/2, \, 5/2\right)$$

Matriz da transformação linear

Seja $T: V \to W$ T L

Sejam $B = \{b_1, \dots, b_n\}$, $C = \{c_1, \dots, c_m\}$ bases de V e W

Ponha
$$T(b_i) = \sum_{k=1}^{m} \alpha_{ik} c_k$$

Escreva $[T]_C^B = \begin{pmatrix} \alpha_{11} & \alpha_{21} & \cdots & \alpha_{n1} \\ \vdots & & & \vdots \\ \alpha_{1m} & \alpha_{2m} & & \alpha_{nm} \end{pmatrix} = \left([T(b_1)]_C \cdots [T(b_n)]_C \right)$

A matriz de T relativa às bases B e C.

Temos que $[T(v)]_C = [T]_C^B [v]_B$.

Exemplo $V = \mathbb{R}^3$, $W = \{(x,y,z) \in \mathbb{R}^3 \mid x+y+z=0\}$

$$T(x,y,z) = (x-y,\ y-z,\ z-x)$$

$\mathbb{R}^3 = \langle e_1, e_2, e_3 \rangle$ base canônica

$W = \langle (1,0,-1), (0,1,-1) \rangle$

$e_1 \mapsto (1,0,-1)$

$e_2 \mapsto (-1,1,0)$

$e_3 \mapsto (0,-1,1)$

$$[T]_C^B = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

Mudança de bases

Seja V um espaço com bases $B = \{b_1, \ldots, b_n\}$ e $C = \{c_1, \ldots, c_n\}$

Ponha $c_i = \sum_{k=1}^{n} \alpha_{ik} b_k$

Considere $id : V \to V$, $v \mapsto v$

$$[id]_B^C = \begin{pmatrix} \alpha_{11} & \cdots & \alpha_{n1} \\ \vdots & & \vdots \\ \alpha_{1n} & & \alpha_{nn} \end{pmatrix} = \left([c_1]_B, \ldots, [c_n]_B \right)$$

A matriz $[id]_B^C$ é dita matriz mudança de base

$$[id]_B^C = \left([c_1]_B \ \cdots \ [c_n]_B \right)$$

Temos que $[v]_B = [id]_B^C \, [v]_C$

$[id]_C^B = \left([id]_B^C\right)^{-1}$, pois $[id]_B^C [id]_C^B [v]_B = [id]_B^C [v]_C = [v]_B$

Exemplo: $V = \mathbb{R}^2$ $B = \{e_1, e_2\}$ $C = \{(1,1), (1,-1)\}$

$[id]_B^C = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $[id]_C^B = \left([id]_B^C\right)^{-1} = \frac{1}{2} (id)_C^B$

$$[v]_B = [id]_C^B \, [v]_C \qquad v = (-1, 2) \,, \; [v]_B = (-1, 2)$$

$$[v]_C = [id]_C^B \quad [v]_B \quad = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = (1/2, -3/2)$$

$$(1/2, 1/2) + (-3/2, 3/2) = (-1, 2) \quad \text{OK.}$$

Transformações lineares

$T: V \to W$

$B = \{b_1 \cdots b_n\}$ bases de V

$B' = \{b_1' \cdots b_n'\}$

$C = \{c_1, \cdots, c_m\}$ bases de W

$C' = \{c_1', \cdots, c_m'\}$

$$[T]_C^{B'} = [T]_C^B [id]_B^{B'}, \quad [T]_{C'}^B = [id]_{C'}^C [T]_C^B$$

$$[T]_{C'}^{B'} = [id]_{C'}^C \; [T]_C^B \; [id]_B^{B'}$$

$$[T]_{C'}^{B'}[w]_{B'} = [T(w)]_{C'} = [id]_{C'}^{C}[T(w)]_{C} = [id]_{C'}^{C}[T]_{C}^{B}[w]_{B} =$$

$$= [id]_{C'}^{C}[T]_{C}^{B}[id]_{B}^{B'}[w]_{B'}$$

Seja $B = \{b_1, \dots, b_n\}$ $C = \{c_1, \dots, c_n\}$ bases de V. $T : V \to V$

$$[T]_{C}^{C} = [id]_{C}^{B}[T]_{B}^{B}[id]_{B}^{C} = [id]_{C}^{B}[T]_{B}^{B}\left([id]_{C}^{B}\right)^{-1}$$

Exemplo: Seja $v = (a, b)$ $\|v\| = 1$

$$R_v(w) = w - 2\left(\frac{v \cdot w}{}\right) v$$

$v' = (b, -a)$

R_v é linear. $R_v(v) = -v$

$R_v(v') = v'$

v, v' formam uma base $B = \{v, v'\}$

$$[R_v]_B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$[R_v]_c = [id]_c^B \, [R_v]^B \left([id]_B^c\right)^{-1} =$$

$$\begin{pmatrix} a & b \\ b & -a \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ b & -a \end{pmatrix} = \begin{pmatrix} -a & b \\ -b & -a \end{pmatrix} \begin{pmatrix} a & b \\ b & -a \end{pmatrix} =$$

$$= \begin{pmatrix} -a^2 + b^2 & -2ab \\ -2ab & -b^2 + a^2 \end{pmatrix}$$

$R_v(1,0) = (1,0) - 2a(a,b) = (1 - 2a^2, \; 2ab)$

$R_v(0,1) = (0,1) \; 2b(a,b) = (2ba, \; 1 - 2b^2)$