

2020 Fall EECS205002 Linear Algebra

Name:

ID:

2020/10/7 Quiz 1

1. (40%) Multiple choice questions. Each question may have 0, 1, or more correct choices. For each question, you need to choose all the correct items to get the credit.

- (a) Which matrices are in the row echelon form?

$$A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 0 \\ 3 & 0 & 4 \end{bmatrix}$$

- (b) Which statement(s) indicates that an $n \times n$ matrix A is nonsingular.

- A. There exists a matrix B such that BA equals to an $n \times n$ identity matrix.
- B. $Ax = b$ is consistent.
- C. A is row equivalent to I .
- D. $Ax = 0$ has a nontrivial solution.
- E. $A^T = A$.

- (c) Which matrices are elementary matrices?

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

- (d) For a linear system $Ax = b$, which condition guarantees it is consistent.

- A. A is nonsingular.
- B. $Ax = b$ is an underdetermined system.
- C. $Ax = b$ is an overdetermined system.
- D. $b = 0$.
- E. $Ax = 0$ has a solution.

- (e) Which statements are true?

- A. All types of elementary matrices are invertible, and the inverse matrix is also an elementary matrix of the same type.
- B. A symmetric upper triangular matrix must be a diagonal matrix.
- C. A matrix in the row echelon form must be an upper triangular matrix.
- D. An overdetermined system must be inconsistent.
- E. If $E_k \dots E_2 E_1 A = I$, then $A^{-1} = E_1^{-1} E_2^{-1} \dots E_k^{-1}$.

- (f) In the following coefficient matrix of a linear system, which variables are free variables?

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix}$$

- (g) Let $F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Which of the following matrix X makes $XF = FX$?

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, E = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

(h) For matrix A , B , C , and scalar α , β , which statements are true?

- A. $A(B + C) = AB + AC$.
- B. If A is nonsingular, $(\alpha A)^{-1} = \alpha^{-1} A^{-1}$.
- C. If A and B are nonsingular, $(BA)^{-1} = B^{-1} A^{-1}$.
- D. $(ABC)^T = C^T(AB)^T = (CB)^T A^T$.
- E. $(A + B + C)^T = A^T + C^T + B^T$.

2. (20%) For the following linear system

$$2x_1 + 2x_2 + x_3 = 2$$

$$x_1 + x_2 + x_3 = 3$$

$$3x_1 + 4x_2 + 2x_3 = 6$$

- (a) Write its augmented matrix.
 - (b) Convert the augmented matrix to the row echelon form.
 - (c) Indicate which elementary matrices are used for (b).
 - (d) Find all the solutions of the linear system.
3. (20%) Prove the following statements
- (a) If A is nonsingular, then A^T is nonsingular.
 - (b) If A is a symmetric nonsingular matrix, then A^{-1} is also symmetric. (You can use the result in (a))
4. (10%) Let U be an $n \times n$ matrix in the row echelon form. Show that if any diagonal element of U is 0, the last row of U must be entirely 0.
5. (10%) In the movie Tenet, there is a sator matrix, as shown in the figure below. Show it is a symmetric matrix.

