2019 Fall EECS205002 Linear Algebra

Name: ID:

2019/11/06 Quiz 4

1. Given a matrix $A = [a_1, a_2, a_3, a_4] = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 3 & 0 \end{bmatrix}$,

   (a) Are the row vectors of A linearly independent? Justify your answer. (2 points)

   (b) Show that (a_1, a_2, a_4) and (a_4, a_3, a_2) are two bases of A's column space. (2 points)

   (c) What is the transition matrix from the ordered basis (a_1, a_2, a_4) to the ordered basis (a_4, a_3, a_2) of A's column space? If you need to compute the inverse of a matrix, say B, you can just express your answer as B^{-1}. You do not need to compute the inverse explicitly. (2 points)

2. Prove the following statements. (4 points)

   (a) Show if U and V are subspaces of $\mathcal{R}^n$, then

   $$\dim(U + V) \leq \dim(U) + \dim(V).$$

   (b) Use (a) to show that for two $m \times n$ matrices A and B,

   $$\operatorname{rank}(A + B) \leq \operatorname{rank}(A) + \operatorname{rank}(B).$$