

2019 Fall EECS205002 Linear Algebra

Name:

ID:

2019/12/25 Quiz 7

1. (3 points) Give a brief geometry explanation about the least square problem $\min_{\vec{x}} \|A\vec{x} - \vec{b}\|$. And show how the fundamental subspace theorem can help to solve this problem.

2. (2 points) For 2-norm, $\|\vec{u} + \vec{v}\| = \|\vec{u}\| + \|\vec{v}\|$ if $\vec{u} = \alpha\vec{v}$. For $\vec{u}, \vec{v} \in \mathbb{R}^2$, under what kind of conditions that $\|u + v\| = \|u\| + \|v\|$ for 1-norm and ∞ -norm.

3. (5 points) Let $A = \begin{bmatrix} 6/7 & 3/7 \\ 3/7 & -2/7 \\ -2/7 & 6/7 \end{bmatrix}$

- (a) (1 point) Show A is an orthogonal matrix.
- (b) (1 point) Solve the linear least square problem $A\vec{x} = \vec{b}$ for $\vec{b} = [2, 0, -1]^T$.
- (c) (2 points) Find an orthonormal basis for $N(A^T)$.
- (d) (1 point) What is the projection matrix Q that projects vectors in \mathbb{R}^3 onto $N(A^T)$. Write your answer in terms of A .