2019 Fall EECS205002 Linear Algebra

Name: ID:

2019/12/11 Quiz 6

1. (2 points) If A and B are similar to each other, and $(A - \alpha I)$ is nonsingular, show that $(B - \alpha I)$ is also nonsingular.

2. (3 points) Find the matrix representation of the linear transformation $L([x_1, x_2, x_3]^T) = [x_1 + x_2, x_3]^T$ with respect to the ordered bases $E = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ and $F = \{\vec{b}_1, \vec{b}_2\}$, where

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \text{ and } \vec{b}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

3. (3 points) For a point $p = (5, 2)$ and a line $L : y = 2x + 2$.

   (a) Find the closet point on L to p.

   (b) Find the distance from p to L.

   (c) Find the line equation that passes p and orthogonal to L.

4. (2 points) Let U and V be subspaces in $\mathbb{R}^2$. Give an example for each pair of operations to illustrate the difference between

   (a) $U \cup V$ and $U + V$,

   (b) $U + V$ and $U \oplus V$.