2019 Fall EECS205002 Linear Algebra

Name: ID:

2019/10/09 Quiz 2

1. There are many properties of determinants. List four of them without any proof. But once you list here, they cannot be used in question 2 and question 3 directly or indirectly. (4 points)

2. A matrix A is skew-symmetric if $A^T = -A$. Show that if A is an $n \times n$ skew-symmetric matrix and n is odd, then A must be singular. (3 points)

3. A matrix A is called upper triangular if $A(i,j) = 0$ for $i > j$. Prove the inverse of a nonsingular and upper triangular matrix A is also an upper triangular matrix using the formula

$$A^{-1} = \frac{1}{\det(A)}\text{adj}(A)$$

where $\text{adj}(A)$ is the adjoint matrix of A defined as

$$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \vdots & & & \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{bmatrix},$$

in which A_{ij} is the cofactor associated with $A(i,j)$. (3 points)