2018 Fall EECS205002 Linear Algebra

Name: ID:

2018/10/15 Quiz 2

1. If A is 3×3 and $I = E_1(2,3)E_2(2,9)E_3(3,1,-2)A$, what is A?

2. An upper triangular matrix A has element $a_{ij} = 0$ for $i > j$. Show that for a nonsingular upper triangular matrix A, A^{-1} is upper triangular. The outline of proof using induction is given. You need to help completing the proof of step (b) and (c).

   (a) For 1x1 matrix, it is trivial.

   (b) Assume for $n \leq k$, it holds. For any $n \geq 2$, A can be expressed as

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix},$$

where A_{11} and A_{22} are square submatrices. Show that A_{11} and A_{12} are upper triangular and invertible. (hint: using determinant.)

(c) With explicit form of C, show that

$$A^{-1} = \begin{bmatrix} A_{11}^{-1} & C \\ 0 & A_{22}^{-1} \end{bmatrix}.$$

(d) Since the size of A_{11} and A_{22} are less than or equal to k, by induction, A_{11}^{-1} and A_{22}^{-1} are upper triangular, so is A^{-1}.