2019 Fall EECS205002 Linear Algebra

Name: ID:

2019/10/23 Quiz 3

1. Which of the following structures are vector spaces? If it is not a vector space, list at least one axiom or closure property that it cannot satisfy. Note that all the scalars in the questions are real numbers. (5 points)

Closure properties
C1 For all $x \in V$, $\alpha x \in V$.
C2 For all $x, y \in V$, $x + y \in V$.
Axioms of vector spaces
A1 $x + y = y + x$ for any x and y in V.
A2 $(x + y) + z = x + (y + z)$ for any x, y, z in V.
A3 There exists an element 0 in V such that $x + 0 = x$ for each $x \in V$.
A4 For each $x \in V$, there exists an element $-x$ in V such that $x + (-x) = 0$.
A5 $\alpha(x + y) = \alpha x + \alpha y$ for each scalar α and any x and y in V.
A6 $(\alpha + \beta)x = \alpha x + \beta x$ for any scalars α and β and any $x \in V$.
A7 $(\alpha\beta)x = \alpha(\beta x)$ for any scalars α and β and any $x \in V$.
A8 $1x = x$ for all $x \in V$.

(a) Let V be the set of all integers with addition defined in the usual way and the scalar multiplication $\circ$ defined as

$$\alpha \circ k = \lfloor \alpha \rfloor k,$$

where $\lfloor \alpha \rfloor$ returns the greatest integer that is less than or equal to α.

(b) Let V be the set of real numbers with scalar multiplication defined in the usual way and the addition $\oplus$ defined as

$$x \oplus y = \max(x, y).$$

(c) Let $V = A \cup B$, where A, B are two subspaces of a vector space W.

(d) For a nonzero matrix A, $V = \{x | Ax \neq 0\}$ with addition and scalar multiplication defined in the usual ways.

(e) Let V be the set of all ordered pairs of real numbers. Define scalar multiplication $\circ$ and addition $\oplus$ on S as

$$\alpha \circ (x_1, x_2) = (\alpha x_1, \alpha x_2)$$

$$(x_1, x_2) \oplus (y_1, y_2) = (x_1 + y_1, 0)$$

2. What is the null space of $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & -1 \\ 4 & 3 & 1 \end{bmatrix}$? Write your derivation and the answer. (3 points)

3. Let A be a 3×3 matrix, and x_1, x_2, x_3 be three linearly independent vectors in $\mathcal{R}^3$. Show that A is nonsingular if and only if Ax_1, Ax_2, and Ax_3 are linearly independent. (2 points)