2019 Fall EECS205002 Linear Algebra

Name: ID:

2020/01/08 Quiz 8

1. (4 points) True or False

   (a) A matrix is singular if and only if it has zero eigenvalues.

   (b) A defective matrix must have at least two identical eigenvalues.

   (c) If $B = S^{-1}AS$ and $(\lambda, \vec{x})$ is an eigenpair of A, $(\lambda, S\vec{x})$ is an eigenpair of B.

   (d) The eigenvectors of a real symmetric matrix are orthogonal to each other.

   (e) The sum of all eigenvalues of a matrix equals to the trace of the matrix.

   (f) If an eigenvector of a real matrix A is complex, its conjugate must be also A's eigenvector.

   (g) An $n \times n$ matrix is diagonalizable if and only if it has n distinct eigenvalues.

   (h) All the eigenvalues of a real symmetric matrix are real.

2. (2 points) If a 2×2 matrix A has eigenvalues/eigenvectors as follows, what is A?

$$\left(\lambda_1 = 1, x_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}\right), \left(\lambda_2 = -4, x_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$$

3. (2 points) Can the matrix $A = \begin{bmatrix} \alpha+1 & 1 \\ 0 & 2\alpha \end{bmatrix}$ be defective? If yes, what α should be? If not, what are the eigenvectors of A (expressing them in terms of functions of α)?

4. (2 points) Let u be a unit vector in $\mathbb{R}^n$ and $H = I - 2uu^T$. Show the eigenvalues of H are either 1 or -1.

5. (5 points bonus) Gaussian elimination is a versatile numerical method in linear algebra. List all the applications or proofs you have learned in this semester that use Gaussian elimination or (reduced) row echelon form.