2019 Fall EECS205002 Linear Algebra

Name: ID:

2019/9/25 Quiz 1

1. True or False: (7 points)

   (a) The following matrix is in the reduced row echelon form,

   $$A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

   (b) If a system is consistent and the nonzero rows of the row echelon form of the matrix form a strictly triangular system, the system has a unique solution.

   (c) An underdetermined homogeneous system of linear equations has a unique solution.

   (d) Let E be a type III elementary matrix as defined below, where $m \neq 0$. Its inverse E^{-1} is

   $$E = \begin{bmatrix} 1 & & & & & & \\ \vdots & \ddots & & & & & \\ 0 & \cdots & 1 & & & & \\ \vdots & & & \ddots & & & \\ 0 & \cdots & m & \cdots & 1 & & \\ \vdots & & & & & \ddots & \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \quad E^{-1} = \begin{bmatrix} 1 & & & & & & \\ \vdots & \ddots & & & & & \\ 0 & \cdots & 1 & & & & \\ \vdots & & & \ddots & & & \\ 0 & \cdots & 1/m & \cdots & 1 & & \\ \vdots & & & & & \ddots & \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}$$

   (e) All types of elementary matrices are invertible, and the inverse matrix is also an elementary matrix of the same type.

   (f) If A is nonsingular, $A\vec{x} = \vec{0}$ has a unique solution.

   (g) A symmetric upper triangular matrix must be a diagonal matrix.

   (h) For nonzero matrices A, B, and C, if $AC = BC$, then $A = B$.

   (i) If $E_k \dots E_2 E_1 A = I$, then $A^{-1} = E_1^{-1} E_2^{-2} \dots E_k$.

   (j) A linear system $A\vec{x} = \vec{b}$ has a unique solution if and only if A is nonsingular.

(k) Let A be a 3×3 matrix with column vectors $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$. If $2\vec{a}_1 + \vec{a}_2 - 4\vec{a}_3 = 0$, then A is singular.

(l) Let A be an $n \times n$ matrix. If $A\vec{x} = A\vec{y}$ and $\vec{x} \neq \vec{y}$, then A is nonsingular.

(m) Let A and B be $n \times n$ matrices, and $C = AB$. If A is singular, C must be singular too.

(n) If A and B are nonsingular, then $(AB)^T$ is nonsingular, and

$$((AB)^T)^{-1} = (B^{-1})^T(A^{-1})^T.$$

2. Prove that if A is nonsingular then A^T is nonsingular. (3 points)