2020 Fall EECS205002 Linear Algebra

2020/11/4 Quiz 2

1. (25%) What are the definitions of the following terms?

   (a) For a 3×3 matrix $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, the minor and the cofactor of a_{23}.

   (b) Skew symmetric matrix

   (c) Spanning set for a vector space V

   (d) Null space of a matrix A

   (e) Linear independence

2. (9%) What is $\det(EA)$ for an $n \times n$ matrix A with an elementary matrix E of type I, type II, and type III? Express your answer in terms of $\det(A)$.

3. (10%) Use mathematical induction to show that for an $n \times n$ matrix A and $\alpha \in \mathbb{R}$, $\det(\alpha A) = \alpha^n \det(A)$.

4. (10%) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 1 \end{bmatrix}$. What is the product of A and the adjoint matrix of A?

5. (16%) Let $A = \begin{bmatrix} -2 & 4 & 4 \\ 2 & -8 & 0 \\ 8 & -20 & -12 \end{bmatrix}$.

   (a) What is the null space of A?

   (b) Is the set of vectors $\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ a spanning set of the null space of A. Justify your answer.

6. (10%) Are vectors $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ linearly independent? Justify your answer.

7. (10%) Let A and B be $k \times k$ matrices and let

$$M = \begin{bmatrix} O & A \\ B & O \end{bmatrix}.$$

Show that $\det(M) = (-1)^k \det(A) \det(B)$.
(Hint: You can use the result $\det\left(\begin{bmatrix} A & O \\ O & B \end{bmatrix}\right) = \det(A)\det(B)$ directly.)

8. (10%) Let A be an $n \times n$ matrix and let $x_1, x_2, \ldots x_k$ be vectors in $\mathbb{R}^n$, where $k < n$. If the vectors $y_i = Ax_i$ for $i = 1, 2, \ldots k$ are linearly independent,

   (a) Show that the vectors $x_1, x_2, \ldots, x_k$ are linearly independent.

   (b) Given the conditions that $\{y_1, y_2, \ldots y_k\}$ are linearly independent, and $\{x_1, x_2, \ldots x_k\}$ are linearly independent. Under what kind of conditions that matrix A can be singular?