

Constraint-Satisfaction Problems: Cryptarithm

This report is broken into two sections. The first section reiterates some of the logic outlined in the assignment. The second section shows the logic-steps taken to get to the solution. Note it appears that there are two separate solutions that meet the constraints as outlined in the problem statement, but only one that follows the CSP solver value assignment (smallest to largest).

Constraints Overview:

Problem:	
$\begin{array}{r} O D D \\ + O D D \\ \hline E V E N \end{array}$	
Constraints:	
1 $AllDiff(O, D, E, V, N)$	
2 $D + D = N + 10 * C10$	
3 $D + D + C10 = E + 10 * C100$	
4 $O + O + C100 = V + 10 * E$	
Variables:	Domains:
C10, C100	[0,1]
E	[1]
O	[1,2,3,4,5,6,7,8,9]
D, V, N	[0,1,2,3,4,5,6,7,8,9]

Logic Overview

Step	C10	C100	E	O	V	D	N	
Initial	0, 1	0, 1	1	1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	
Apply AllDiff Constraint	0, 1	0, 1	1	2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	
Choose variable C10, Choose value 0	0	0, 1	1	2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	
Inference (AllDiff and constraints 2 and 3)	0	0, 1	1	2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4	0,2,3,4,5,6,7,8,9	
Inference AllDiff D \neq 0 or D = N						2,3,4		
Inference on N from D						2,3,4	4,6,8	
Logical Impass >> If $D+D < 10$ then $E = V$ and $E \neq 1$ >> Therefore $D+D > 10$ and $C10 = 1$								
Reset to $C10 = 1$	1	0, 1	1	2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	
Infer from $C10 = 1$ that $D+D > 10$						5,6,7,8,9		
Infer N must be even							0,2,4,6,8	
Infer constraint 2 on D and N						5	0	Because $D+D+C10 = 1$, and we know $C10 = 1$ therefore $D = 5$
Choose Variable $C100 = 1^*$		1						*Cannot set $C100 = 0$; Because $D = 5$ there must be carryover onto $V >> 5 + 5 + 1 = 1 + 10^* C100$
Apply AllDiff Constraint	1	1	1	2,3,4,6,7,8,9	2,3,4,6,7,8,9	5	0	
Infer Constraint 4				6,7,8,9				$O + O + 1 = V + 10^*(1)$
Infer Constraint 4				6,8	3,7			$2^*O = V+9$ and Diff so cannot be 9 for O
Apply Solver Logic				6				CSP solver Value Assignment is smallest to largest --> $O = 6, V = 3$
Infer Constraint 4					3			
Finished	1	1	1	6	3	5	0	
	6	5	5					
+	6	5	5					
1	3	1	0					