Enhancing project resilience: a risk-averse approach to payment delays

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Joint work with Öncü Hazır

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Late payments: the silent killer

intrum Services Industry Reports & Insights About Intrum

European Payment Report 2023

The European Payment Report (EPR) highlights the impact late payments has on the development and growth among European businesses. The insights are based on a survey of 10,556 companies across 29 European countries, conducted between November 2022 and March 2023. You can download the full European report from intrum.com/epr2023

Download the UK report

Inflation and interest rates are creating challenges

Across Europe, growth is slowing down while supply-chain disruption and soaring energy costs drive inflation at a rate not seen for decades. Almost 6 in 10 companies are worried that the risk of late

Strengthening cash flow and managing credit risk top the agenda

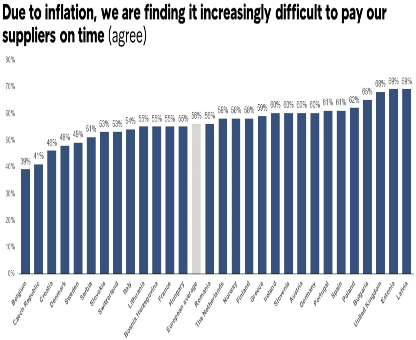
Liquidity, cash flow and credit risk management are the main strategic priorities for UK businesses as they seek to secure their financial positions.

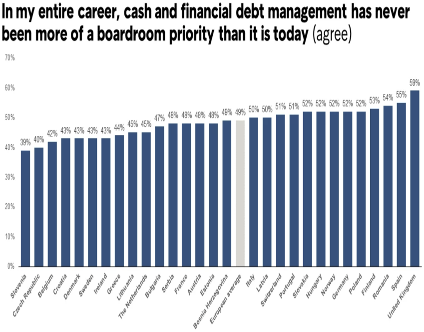
Late payments increasingly seen as a significant barrier to growth

Late payments are hindering the growth of UK companies, hampering the economic and social development of the economy. More than 8 in 10 UK businesses have been asked to accept longer payment terms than

Late payments: the silent killer

Nearly one in five (18%) of Europe's SMEs say that late payments threaten the survival of their business.

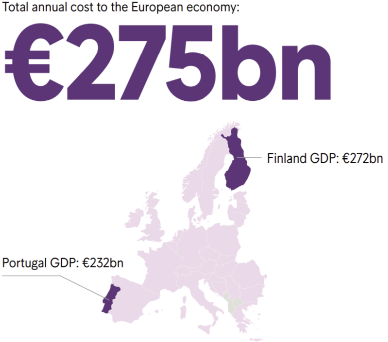


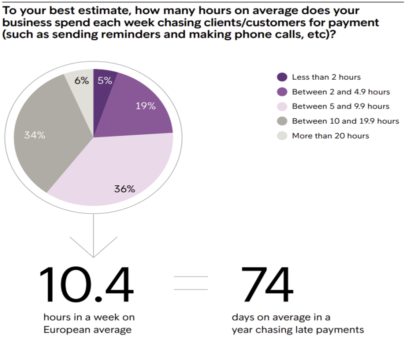


Source: Intrum's European Payment Report 2023

Late payments: the silent killer

Chasing payments is costing Europe a quarter of a trillion euros a year





Source: Intrum's European Payment Report 2023

Late payments: the silent killer

How do late payments impact EU businesses?

500 invoices are sent every second in the EU. However, only 200 of them will be paid on time. Here are some facts about late payments.

More than 60% of EU businesses
are still not paid on time and SMEs are most affected.

1 in 4 bankruptcies
are due to invoices not being paid on time.

€158 million in financing costs
for EU companies could be saved per day by reducing payment delays.

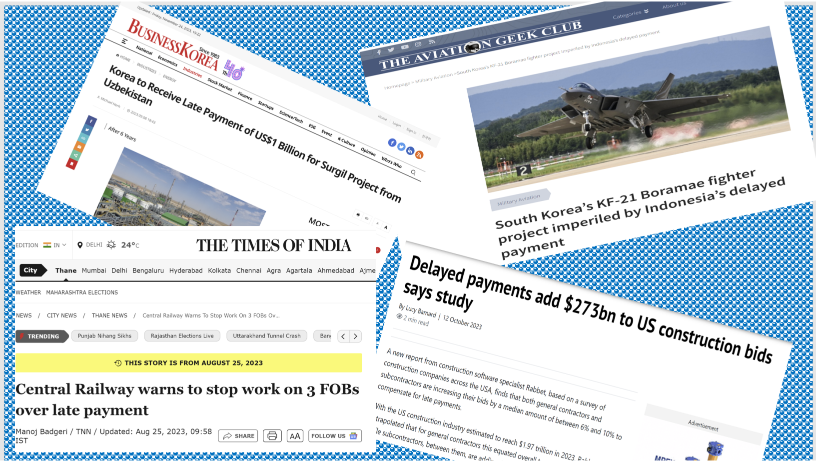
900,000 jobs
could be created in the public sector thanks to timely payments.

20% of businesses
consider payment delays as a barrier to their green transition.

Source: European Commission

Source: https://www.euronews.com/business/2023/09/05/

Projects



BUSINESSKOREA

Korea to Receive Late Payment of US$1 Billion for Surgil Project from Uzbekistan

After 6 Years

THE AVIATION GEEK CLUB

Military Aviation

South Korea's KF-21 Boramae fighter project imperiled by Indonesia's delayed payment

THE TIMES OF INDIA

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THIS STORY IS FROM AUGUST 25, 2023

Central Railway warns to stop work on 3 FOBs over late payment

Manoj Badgeri / TNN / Updated: Aug 25, 2023, 09:58 IST

Delayed payments add $273bn to US construction bids, says study

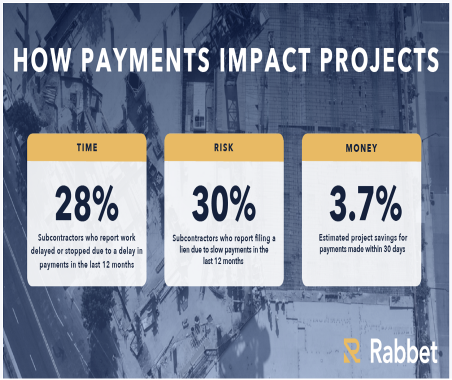
By Lucy Barnard | 12 October 2023

2 min read

A new report from construction software specialist Rabbet, based on a survey of construction companies across the USA, finds that both general contractors and subcontractors are increasing their bids by a median amount of between 6% and 10% to compensate for late payments.

Construction sector

Construction companies depend on positive cash flows for their daily functioning



Research motivation

How do firms compensate the lack of liquidity generated by (uncertain) payment delays?

Features of the problem

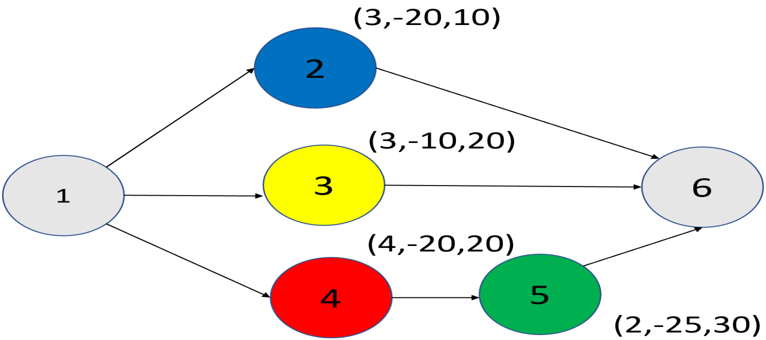
- NPV with delayed payments
- Financing costs
- Uncertainty
- Risk aversion

Literature review

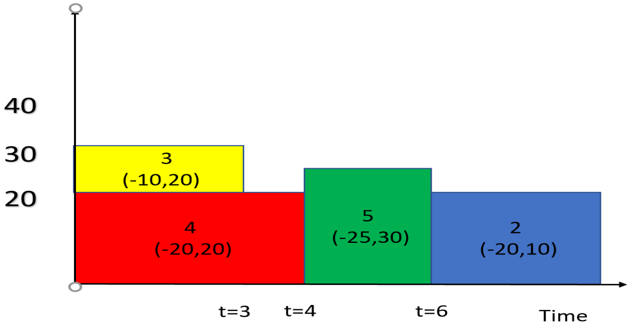
Year	Authors	Payments	Uncertainty	Risk	DR	Financing cost
2003	Vanhoucke et al.	Progress	X	X	X	X
2005	Mika et al.	Different modes	X	X	X	X
2005	Ke and Liu	X	Activity duration	Chance constraints	X	X
2010	Wiesemann e tal.	X	Activity duration & Cash flows	X	X	X
2016	Leyman and Vanhoucke	At compl. times	X	X	X	X
2017	Leyman and Vanhoucke.	At compl. times	X	X	X	X
2019	Liang et al.	X	Activity duration	X	X	X
2020	Rezaei et al.	X	Activity duration	CVaR	X	X
2021	Peymankar et al.	X	Cash Flows	X	X	X
2019	This paper	At compl. times	Payments delays	CVaR	Yes	Yes

Two-stage distributionally robust model with and without risk aversion for the NPV maximization with financing costs

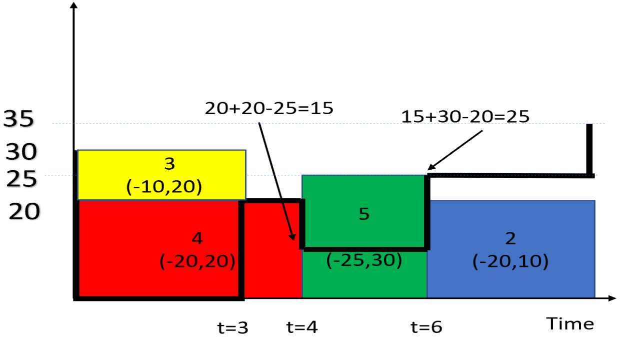
Example



Example

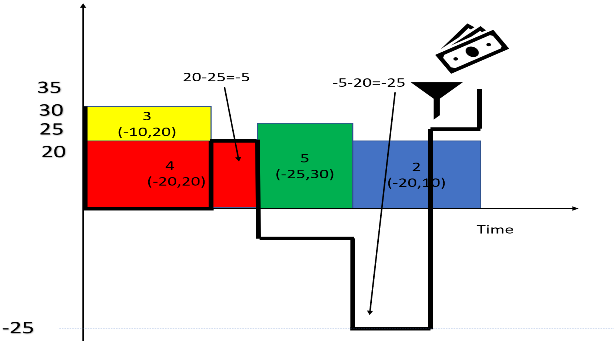


Example



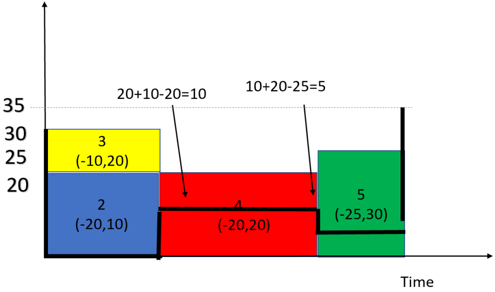
35
30
25
20
20+20-25=15
15+30-20=25
3
(-10,20)
4
(-20,20)
5
(-25,30)
2
(-20,10)
t=3
t=4
t=6
Time

Example



Delay in payment for activities 4 and 5

Example



Feature of the problem

Activities

- Precedence relations

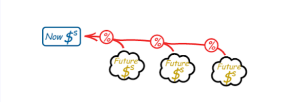
Time related concerns

- Deadlines

Cost related concerns

- Discounted cash flow analysis (NPV)

NPV is calculated by taking a sum of all the income and expenses over the time period chosen, and discounting it back to the present



Deterministic problem

Input

- V: set of all nodes
- E: set of all arcs (i.e. immediate precedence relationships)
- T: time horizon
- $\overline{d}$: deadline for the completion of the project
- β: discount rate
- p_i: duration of activity $i \in V$
- c_i^{in}/c_i^{out} : cash inflow/outflow of activity $i \in V$ $(c_i^{in} > 0)$,$(c_i^{out} < 0)$

$$\max \quad \sum_{i \in V} \sum_{t \in T} \frac{c_i^{out}}{(1+\beta)^t} x_{it} + \frac{c_i^{in}}{(1+\beta)^t} q_{it}$$

$$\sum_{t \in T} x_{it} = 1 \qquad \forall i \in V$$

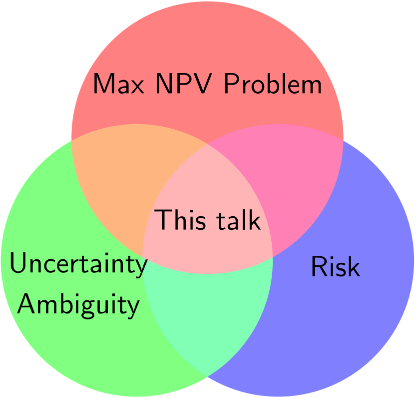
$$\sum_{t \in T} t x_{jt} \geq \sum_{t \in T} t x_{it} + p_i \qquad \forall (i,j) \in E$$

$$\sum_{t \in P^i} t q_{it} = \sum_{t \in T} t x_{it} + p_i \quad \forall i \in V$$

$$\sum_{t \in P^i} q_{it} = 1 \quad \forall i \in V$$

$$\sum_{t \in T} t x_{(n+1)t} + p_{n+1} \leq \overline{d}$$

$$q_{it}, x_{it} \in \{0,1\} \quad \forall i \in V, t \in T$$


Max NPV Problem
This talk
Uncertainty
Ambiguity
Risk

Setting the problem as a two-stage SIP

(Stage 1 decision) –Uncertainty—Stage 2 decision

Find a schedule

Goal: Maximize first stage objective+ Expected value of the second stage decisions

Hedge against uncertainty= Enhance resilience

Setting the problem as a two-stage SIP

(Stage 1 decision) –Uncertainty—Stage 2 decision

Goal: Maximize first stage objective+ Expected value of the second stage decisions

Hedge against uncertainty= Enhance resilience

Setting the problem as a two-stage SIP

(Stage 1 decision) –Uncertainty—Stage 2 decision

If, when, and how much to use short-term financing

Goal: Maximize first stage objective+ Expected value of the second stage decisions

Hedge against uncertainty= Enhance resilience

General SIP formulation

$$\begin{aligned} \max f(x) = \quad & c^T x + E[Q(x,\omega)] \\ s.t. \quad & Ax \geq b, x \in \mathcal{R}^{n_1-p_1} \times \mathcal{Z}^{p_1} \end{aligned}$$

$$\begin{aligned} Q(x,\omega) = \quad & max \;\; q(\omega)^T z \\ s.t. \quad & Wz \geq h(\omega) - T(\omega)x \\ & z \in \mathcal{R}^{n_2-p_2} \times \mathcal{Z}^{p_2} \end{aligned}$$

where

- ω random event
- $Q(x,\omega)$ is the second-stage objective function
- $T(\omega)$ is the recourse matrix
- x are first-stage decisions
- z are second-stage decisions

Decision Variables

First-stage

$$x_{it} = \begin{cases} 1 & \text{if activity } i \in V \text{ starts at time } t \in T; \\ 0 & \text{otherwise} \end{cases}$$

Second-Stage

$$q_{it}^{\omega} = \begin{cases} 1 & \text{if payment for activity } i \text{ is received at time } t \\ & \text{under scenario } \omega \in \Omega; \\ 0 & \text{otherwise} \end{cases}$$

$l_t^{\omega} \geq 0$ short-term loan contracted at period t in scenario ω

First-stage model

$$ENPV = \max \quad \sum_{i \in V} \sum_{t \in T} \frac{c_i^{out}}{(1+\beta)^t} x_{it} + E_\xi[\mathrm{Q}(\mathbf{x}, \xi(\omega))]$$

$$\sum_{t \in T} x_{it} = 1 \qquad \forall i \in V$$

$$\sum_{t \in T} t x_{jt} \geq \sum_{t \in T} t x_{it} + p_i \qquad \forall (i,j) \in E$$

$$\sum_{t \in T} t x_{(n+1)t} + p_{n+1} \leq \overline{d}$$

$$x_{it} \in \{0,1\} \qquad \forall i \in V, t \in T$$

Second stage constraints

$$\sum_{t \in T} t q_{it}^{\omega} = \sum_{t \in T} t x_{it} + p_i + \xi_i^{\omega} \qquad \forall i \in V$$

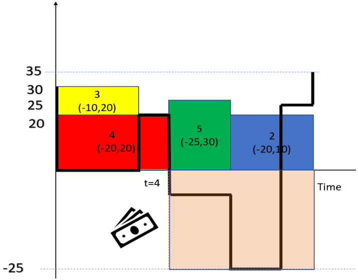
$$\sum_{t \in T} q_{it}^{\omega} = 1 \qquad \forall i \in V$$

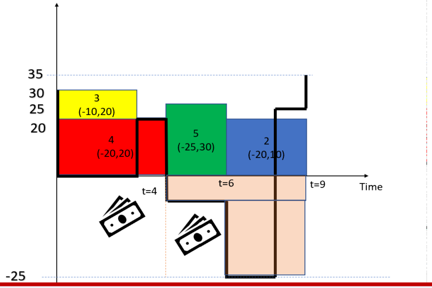
$$\sum_{i \in V} \sum_{t' \leq t} c_i^{out} x_{it'} + \sum_{i \in V} \sum_{t' \leq t} c_i^{in} q_{it'}^{\omega} + \sum_{t' \leq t} I_t^{\omega} \geq 0 \qquad \forall t \in T$$

$$q_{it}^{\omega} \in \{0,1\} \quad \forall i \in V, t \in T, \quad I_t^{\omega} \geq 0 \quad \forall t \in T$$

Borrowing policy

$$\frac{20}{(1+\beta)^4} - \frac{20(1+r)^{9-4}}{(1+\beta)^9} \qquad \frac{10}{(1+\beta)^4} + \frac{10}{(1+\beta)^6} - \frac{10(1+r)^{9-4}}{(1+\beta)^9} - \frac{10(1+r)^{9-6}}{(1+\beta)^9}$$





If $\beta > r$ then A is better, otherwise B

Recourse function

$$Q(\mathbf{x}, \xi(\omega)) =$$

$$= \max \sum_{i \in V} \sum_{t \in P^i} \frac{c_i^{in}}{(1+\beta)^t} q_{it}^{\omega} + \sum_{t \in T} \frac{I_t^{\omega}}{(1+\beta)^t} - \sum_{t \in T} \frac{I_t^{\omega}(1+r)^{T-t}}{(1+\beta)^T}$$

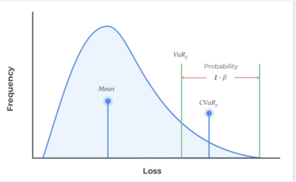
Well known risk and safety measures

$$VaR(\tilde{z}) = \min_{\eta}(\eta | \mathbf{P}(\tilde{z} \leq \eta) = F_{\tilde{z}}(\eta) \geq \beta)$$

Conditional Value at Risk

Quantifies the expected value of the random variable $\tilde{z}$ in the worst $1-\beta\%$ of cases described as follows:

$$CVaR = E[\tilde{z}|\tilde{z} \geq VaR].$$



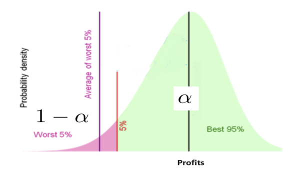
Well known risk and safety measures

$$SVaR(\tilde{z}) = \max_{\eta}(\eta | \mathbf{P}(\tilde{z} \geq \eta) \geq \alpha)$$

Conditional Value at Risk

Quantifies the expected value of the random variable $\tilde{z}$ in the worst $1 - \alpha\%$ of cases described as follows:

$$SCVaR = E[\tilde{z} | \tilde{z} \leq SVaR].$$



Reformulation for discrete set of scenarios

Conditional Value at Risk

$$SCVaR[Q(\mathbf{x}, \xi(\omega))] = \max_{\eta \in \mathcal{R}^+} \eta - \frac{E_\xi[(\eta - Q(\mathbf{x}, \xi(\omega)))^+]}{1 - \alpha}$$

Reformulation

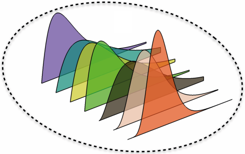
$$\max_{\eta \in \mathcal{R}^+} \eta - \frac{\sum_\omega \mathcal{P}^\omega \gamma^\omega}{1 - \alpha}$$

$$\gamma^\omega = \eta - Q(\mathbf{x}, \xi(\omega)) \qquad \forall \omega$$

$$\gamma^\omega \geq 0 \qquad \forall \omega$$

Ambiguity in the probability distribution

Let Ξ be a family of probability distributions with given characteristics.



Box Ambiguity Set

$$\Xi = \{\mathbf{p} = \mathbf{p_0} + \boldsymbol{\pi} | \mathbf{e}^{\mathbf{T}} \boldsymbol{\pi} = 0, \|\boldsymbol{\pi}\|_\infty \leq \Psi\}$$

where $\|\pi\|_\infty = \max_{\omega \in \Omega} |\pi_\omega|$; $\Psi \in [0, 1]$ is the upper bound of the fluctuation. vector; π a perturbation vector

$$\inf_{\xi \in \Xi} \max_{\eta \in \mathcal{R}^+} \eta - \frac{E_\xi[(\eta - \mathrm{Q}(\mathbf{x}, \xi(\omega)))^+]}{1-\alpha}$$

$$\inf_{\xi \in \Xi} \max_{\eta \in \mathcal{R}^+} \eta - \frac{E_\xi[(\eta - \mathsf{Q}(\mathbf{x}, \xi(\omega)))^+]}{1-\alpha}$$

$$\max_{\eta \in \mathcal{R}^+} \eta - \frac{1}{1-\alpha} \min_{\xi \in \Xi} E_\xi[(\eta - \mathsf{Q}(\mathbf{x}, \xi(\omega)))^+]$$

Toward a deterministic reformulation

$$\max_{\eta \in \mathcal{R}^+} \eta - \frac{1}{1-\alpha} \min_{\xi \in \Xi} E_\xi[(\eta - \mathrm{Q}(\mathbf{x}, \xi(\omega)))^+]$$

$$\min \gamma^T \mathbf{p} = \sum_\omega p^\omega \gamma^\omega$$

$$\gamma^\omega = \eta - \mathrm{Q}(\mathbf{x}, \xi(\omega)) \quad \forall \omega$$

$$\gamma^\omega \geq 0 \quad \forall \omega$$

$$\min_\pi \gamma^T \mathbf{p}$$

$$\gamma^\omega = \eta - \mathrm{Q}(\mathbf{x}, \xi(\omega)) \quad \forall \omega$$

$$\gamma^\omega \geq 0 \quad \forall \omega$$

$$\mathbf{p} = \mathbf{p_0} + \pi$$

$$\mathbf{e^T} \pi = 0$$

$$\|\pi\|_\infty \leq \Psi$$

Toward a deterministic reformulation

$$
\begin{aligned}
&\min_{\pi} \gamma^T \mathbf{p} \\
&\gamma^\omega = \eta - \mathrm{Q}(\mathbf{x}, \xi(\omega)) && \forall\omega \\
&\gamma^\omega \geq 0 && \forall\omega \\
&\mathbf{p} = \mathbf{p_0} + \pi \\
&\mathbf{e^T}\pi = 0 \\
&\|\pi\|_\infty \leq \Psi
\end{aligned}
$$

$$
\begin{aligned}
&\gamma^T \mathbf{p_0} + \min_{\pi} \gamma^T \pi \\
&\gamma^\omega = \eta - \mathrm{Q}(\mathbf{x}, \xi(\omega)) && \forall\omega \\
&\gamma^\omega \geq 0 && \forall\omega \\
&\mathbf{e^T}\pi = 0 \\
&-\pi \leq \boldsymbol{\Psi} \\
&\pi \leq \boldsymbol{\Psi} \\
&\boldsymbol{\Psi} = \Psi\mathbf{e}
\end{aligned}
$$

Toward a deterministic reformulation

$$\gamma^T \mathbf{p_0} + \min_{\pi} \gamma^T \pi$$

$$\mathbf{e^T} \pi = 0$$

$$-\pi \leq \mathbf{\Psi}$$

$$\pi \leq \mathbf{\Psi}$$

Toward a deterministic reformulation

$$\gamma^T \mathbf{p_0} + \min_{\pi} \gamma^T \pi$$

$$\mathbf{e}^{\mathbf{T}} \pi = 0 \qquad \mu$$

$$-\pi \leq \boldsymbol{\Psi} \qquad \beta$$

$$\pi \leq \boldsymbol{\Psi} \qquad \delta$$

Toward a deterministic reformulation

$$
\begin{aligned}
&\gamma^T \mathbf{p_0} + \min_{\pi} \gamma^T \pi && \\
&\mathbf{e^T} \pi = 0 && \mu \\
&-\pi \leq \mathbf{\Psi} && \beta \\
&\pi \leq \mathbf{\Psi} && \delta
\end{aligned}
$$

$$
\begin{aligned}
&\gamma^T \mathbf{p_0} + \max_{\beta,\delta,\mu} \mathbf{\Psi}^T \beta + \mathbf{\Psi}^T \delta \\
&\mathbf{e}\mu - \beta + \delta = \gamma \\
&\beta \geq 0 \\
&\delta \geq 0 \\
&\mu \quad \text{free}
\end{aligned}
$$

The risk-averse distributional robust model

$$\max \sum_{i \in V} \sum_{t \in T} \frac{c_i^{out}}{(1+\beta)^t} x_{it} + \eta - \frac{\gamma^T \mathbf{p_0} + \mathbf{\Psi}^T \beta + \mathbf{\Psi}^T \delta}{1-\alpha}$$

First and second stage constraints

$\mathbf{e}\mu - \beta + \delta = \gamma$

$\beta \geq 0$

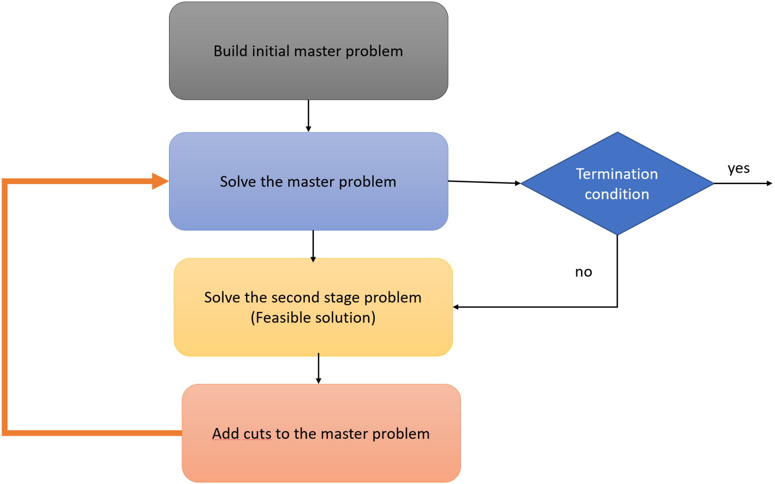
$\delta \geq 0$

$\gamma \geq 0$

μ free

	$\alpha = 0$	$\alpha = 1$
$\Psi = 0$	2S risk neutral	2S risk averse
$\Psi > 0$	2S-DR risk neutral	2S-DR risk averse

Heuristic


Build initial master problem
Solve the master problem
Termination condition
yes
no
Solve the second stage problem (Feasible solution)
Add cuts to the master problem

Master problem

$$MP^0 = \max \quad \sum_{i \in V} \sum_{t \in T} \frac{c_i^{out}}{(1+\beta)^t} x_{it} + \zeta$$

$$\zeta \leq \sum_{i \in V} \sum_{t \in T} \frac{c_i^{in}}{(1+\beta)^t} \hat{q}_{it} + \sum_{t \in T} \frac{\hat{l}_t}{(1+\beta)^t} - \sum_{t \in T} \frac{\hat{l}_t (1+r)^{T-t}}{(1+\beta)^T}$$

$$\sum_{t \in T} x_{it} = 1 \qquad \forall i \in V$$

$$\sum_{t \in T} t x_{jt} \geq \sum_{t \in T} t x_{it} + p_i \qquad \forall (i,j) \in E$$

$$\sum_{t \in T} t x_{(n+1)t} + p_{n+1} \leq \overline{d}$$

$$\sum_{t \in T} t \hat{q}_{it} = \sum_{t \in T} t x_{it} + p_i + \hat{\xi}_i \quad \forall i \in V$$

$$\sum_{t \in T} \hat{q}_{it} = 1 \quad \forall i \in V$$

$$C_0 + \sum_{i \in V} \sum_{t' \leq t} c_i^{out} x_{it'} + \sum_{i \in V} \sum_{t' \leq t} c_i^{in} \hat{q}_{it'} + \sum_{t' \leq t} \hat{l}_t \geq 0 \quad \forall t \in T$$

$$x_{it} \in \{0,1\} \qquad \forall i \in V, t \in T$$

$$\hat{q}_{it} \in \{0,1\} \quad \forall i \in V, t \in T$$

$$\hat{l}_t \geq 0 \quad \forall t \in T$$

Subproblem

$$q_{it}^{\kappa\omega} = \begin{cases} 1, & \text{if } \sum_{t' \in T}(t' x_{it'}^{\kappa} + p_i + \xi_i^{\kappa\omega}) = t \\ 0, & \text{otherwise.} \end{cases}$$

Subproblem

$$Q^{\kappa}_{LB} = \max \eta - \frac{\sum_{\omega\in\Omega}[\gamma^{\omega} p_0^{\omega} + \Psi^{\omega}\beta^{\omega} + \Psi^{\omega}\delta^{\omega}]}{1-\alpha}$$

$$\sum_{t'\leq t} I_t^{\omega} \geq \sum_{i\in V}\sum_{t'\leq t} c_i^{out} x_{it'}^{\kappa} + \sum_{i\in V}\sum_{t'\leq t} c_i^{in} q_{it'}^{\kappa\omega} \qquad \forall t\in T, \forall\omega\in\Omega$$

$$\gamma^{\omega} = \eta - \sum_{t\in T}\frac{c_i^{in}}{(1+\beta)^t} q_{it}^{\kappa\omega} + \sum_{t\in T}\frac{I_t^{\omega}}{(1+\beta)^t} - \sum_{t\in T}\frac{I_t^{\omega}(1+r)^{T-t}}{(1+\beta)^T} \qquad \forall\omega\in\Omega$$

$$\mu - \beta^{\omega} + \delta^{\omega} = \gamma^{\omega} \qquad \forall\omega\in\Omega$$

$$\beta^{\omega} \geq 0 \qquad \forall\omega\in\Omega$$

$$\delta^{\omega} \geq 0 \qquad \forall\omega\in\Omega$$

$$\gamma^{\omega} \geq 0 \qquad \forall\omega\in\Omega$$

$$I_t^{\omega} \geq 0 \qquad \forall t\in T, \forall\omega\in\Omega$$

$$\mu \qquad \text{free}$$

Cuts

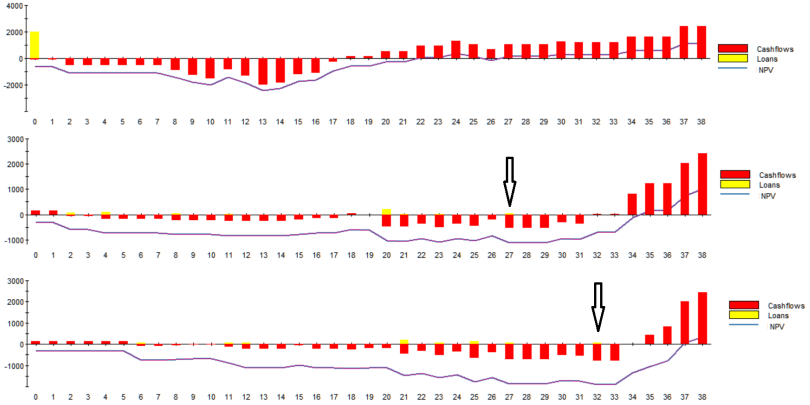
Integer cut

$$\zeta \leq (Q_{LB}^{\kappa})[1 + \sum_{i \in V, t \in T | x_{it}^{\kappa}=1} (1 - x_{it}) + \sum_{i \in V, t \in T | x_{it}^{\kappa}=0} x_{it}]$$

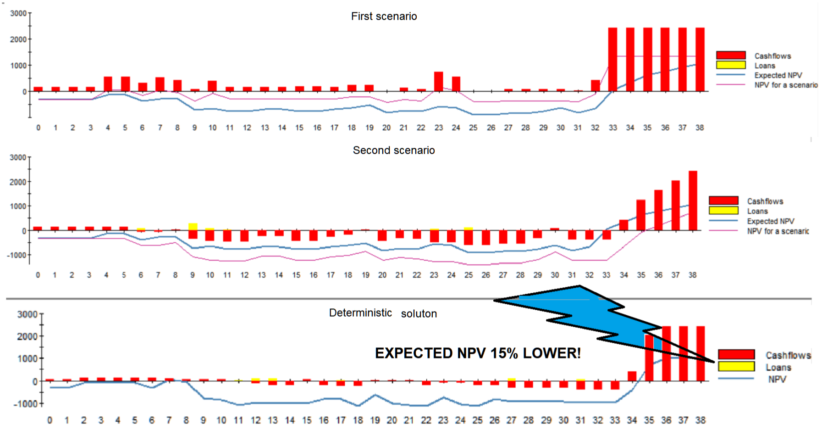
No-good cut

$$\sum_{(i,t) | x_{it}^{\kappa}=1} (1 - x_{it}) \geq 1$$

Sensitivity to the loan interest rate



Two-stage stochastic model



Sensitivity to ψ and α values

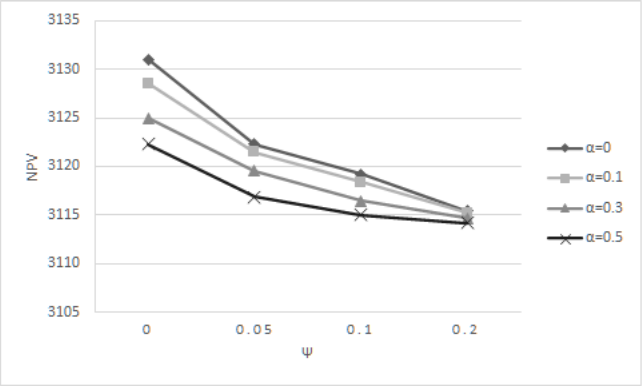


Figure: NPV function for different ψ and α values.

Price of distributional robustness

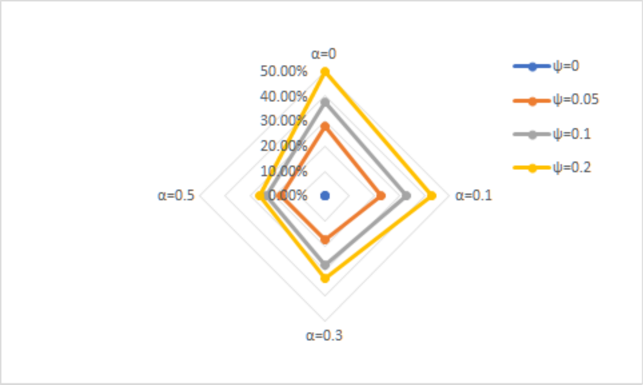


Figure: Price of distributional robustness for different values of ψ and α.

Heuristic performance

- 10 instances from the DC2 data set of Vanhoucke (2010)
- 9 combinations of the distribution of the cash flow
- Three values $(0.25, 0.50, 0.75)$ for the capital constrainedness (CC)
- Three scenarios cardinalities $|\Omega| = \{20, 40, 60\}$
- Four values for $\psi = 0, 0.05, 0.1, 0.2$
- Four values for $\alpha = 0, 0.1, 0.3, 0.5$

Heuristic performance

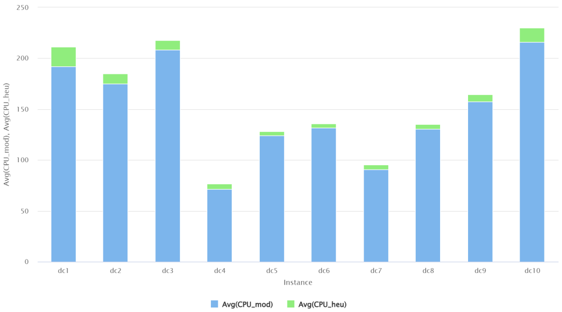


Figure: Heuristic versus exact solution: CPU time.

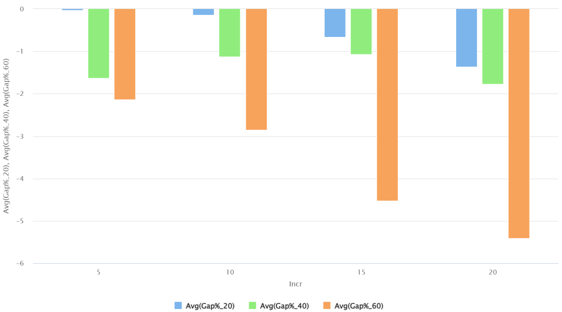


Figure: Heuristic Gap% for different scenario cardinalities.

Limitations

- The size of the model

Future research

- Design of a tailored solution approach
- Different payments modes
- Hybridization with forecasting techniques