A Subexponential Time Algorithm for Makespan Scheduling of Unit Jobs with Precedence Constraints

Jesper Nederlof

UU

Céline Swennenhuis

TU/e

Karol Węgrzycki

MPI

Department of Mathematics and Computer Science, Eindhoven University of Technology

$P3|prec, p_j = 1|C_{\max}$

$$P3 \left| prec, p_j = 1 \right| C_{\max}$$

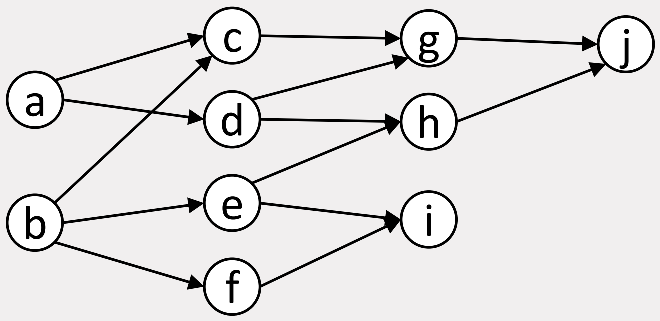
3 identical
parallel machines

$P3|prec, p_j = 1|C_{\max}$

Given:

- n jobs of length 1

$P3|prec, p_j = 1|C_{\max}$



Given:

- n jobs of length 1
- A precedence graph G

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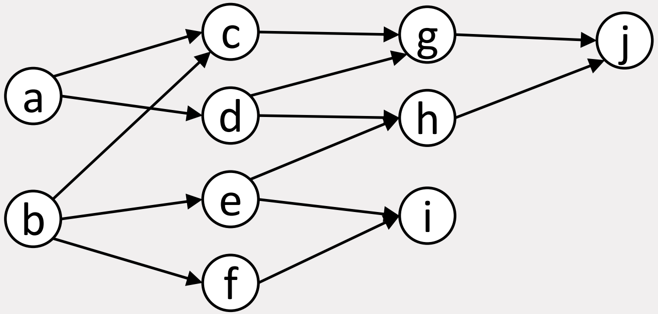

Given:

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- $T \in \mathbb{N}$

Q: Is there a schedule of makespan T?

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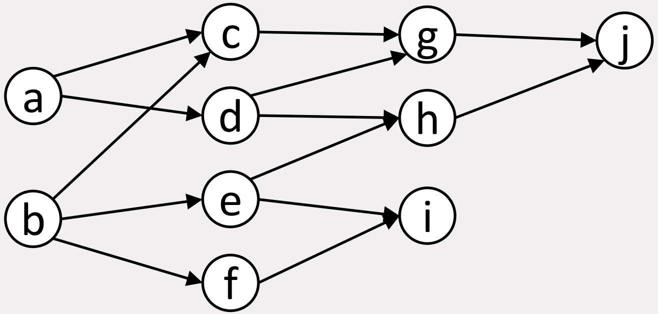

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time →

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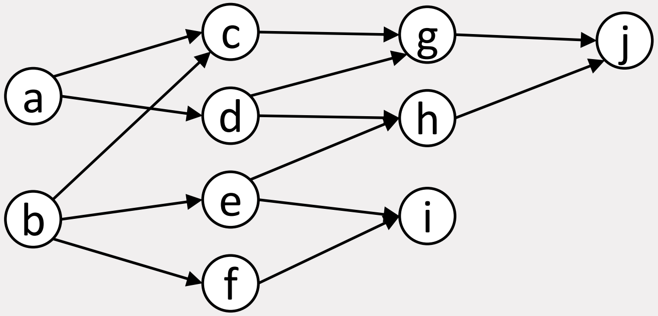

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G defines the problem

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Observation:
Jobs of length one ⇒ `timeslots’

List of Open Problems by Garey and Johnson 1979

1. Graph Isomorphism
2. Subgraph Homeomorphism
3. Graph genus
4. Chordal graph completion
5. Chromatic index
6. Spanning tree parity problem
7. Partial order dimension
8. Precedence constrained 3-processor scheduling
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$2^{O\left((\log n)^3\right)}$ time
[Babai 2017]

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$2^{O\left(\sqrt{n}\cdot\log n\right)}$ time
This talk

Literature overview $Pm|prec, p_j = 1|C_{\max}$

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Before:

$Pm|prec, p_j = 1|C_{\max}$ can be solved in $O\left(2^n \cdot \binom{n}{m}\right)$ time.

Our Result

Our result:

$Pm|prec, p_j = 1|C_{\max}$ can be solved in $\left(1 + \frac{n}{m}\right)^{O(\sqrt{nm})}$ time.

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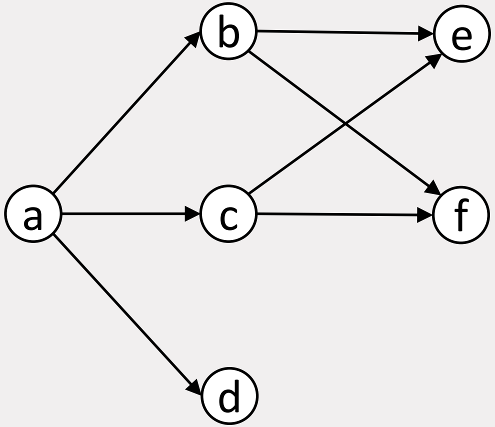
Two ways to explain, but main insights:

1. Use of look-up table
2. Keeping track of number of isolated vertices
3. Finding win-win strategy

Definitions

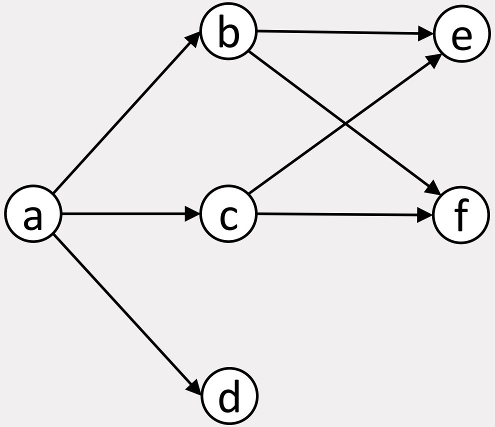


Precedence Constraints Graph G

Definitions

$G \Rightarrow$ partial order:

- $i \prec j$ if $(i,j) \in G$



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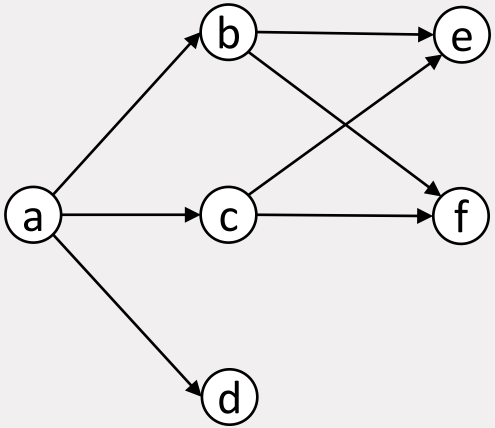

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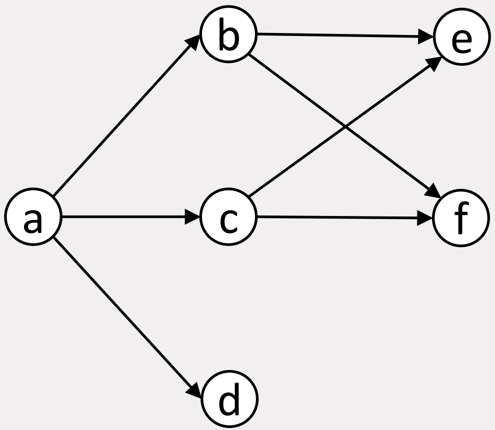

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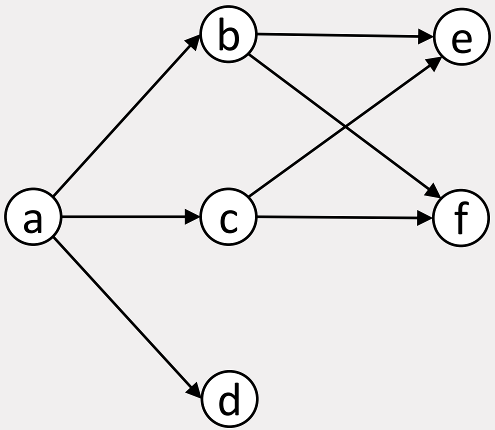

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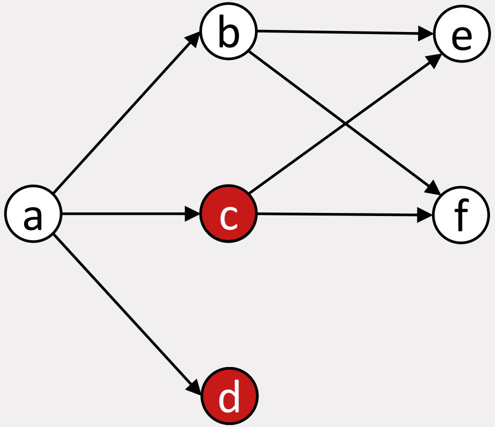

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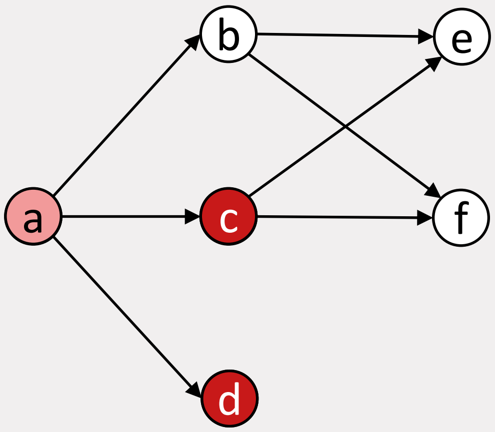

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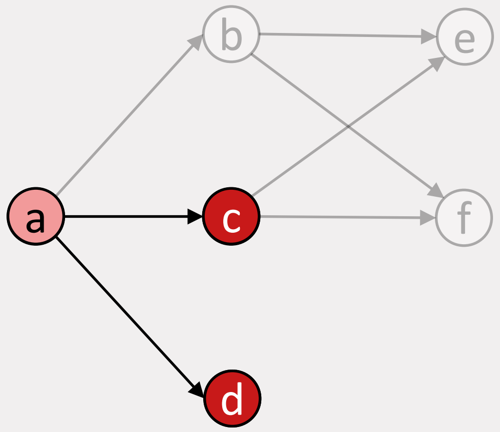

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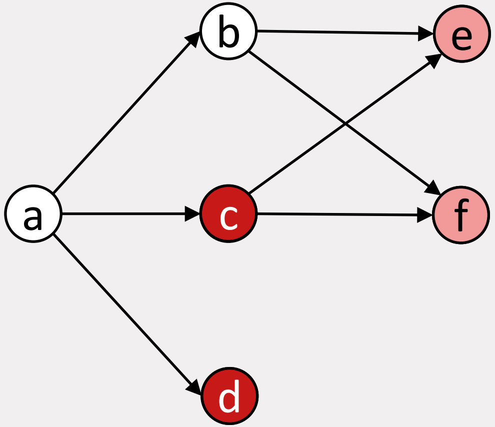

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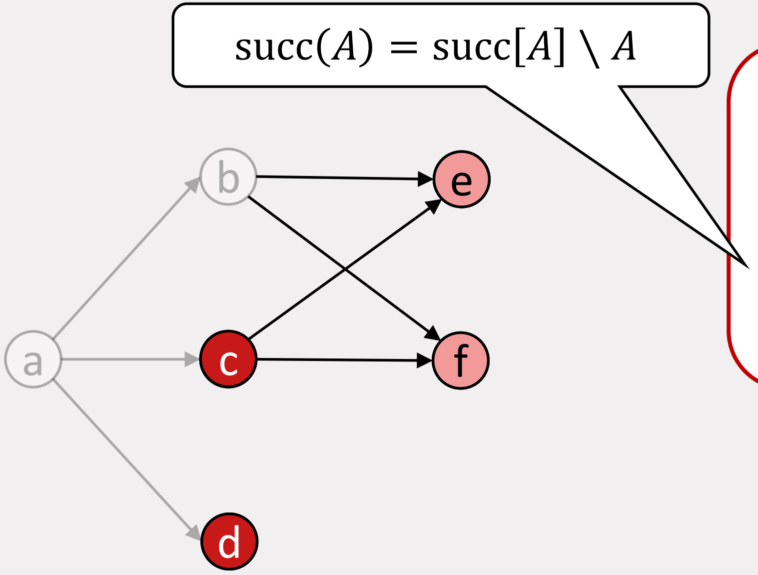

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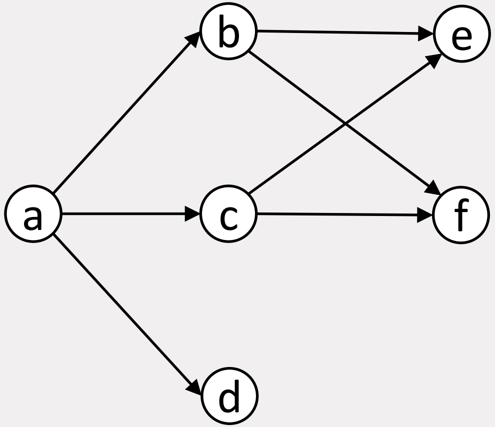

Precedence Constraints Graph G

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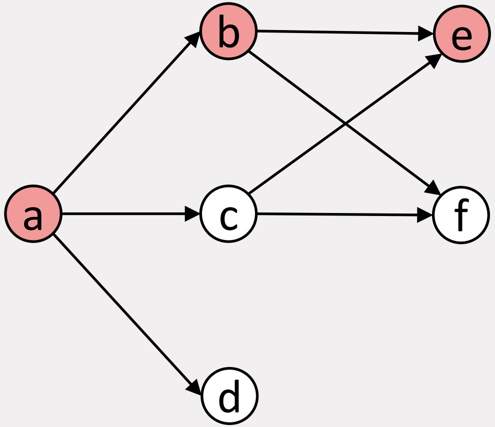
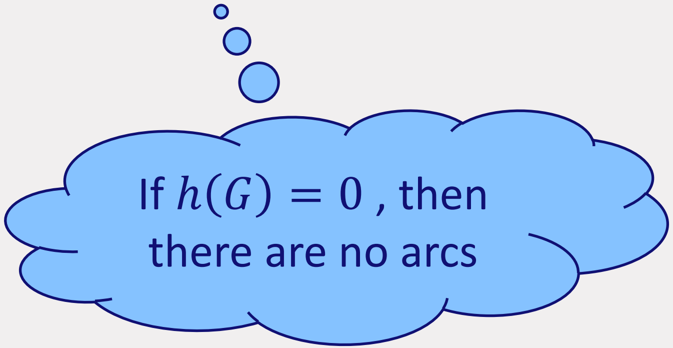
Def: The *height* $h(G)$ is the size of the longest chain (in #arcs).



Precedence Constraints Graph G

In the example $h(G) = 2$



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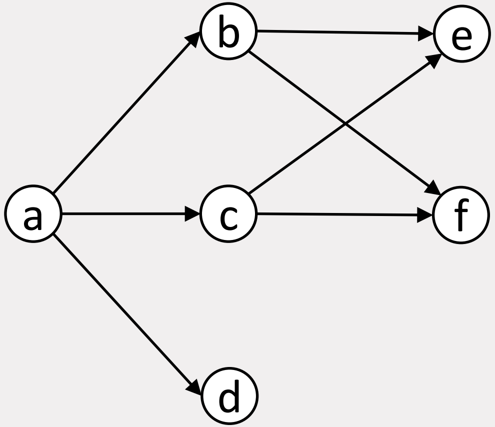


Precedence Constraints Graph G

Def: A *chain* is a set A whose elements are pairwise **comparable**.
Def: The *height* $h(G)$ is the size of the longest chain (in #arcs).

In the example $h(G) = 2$

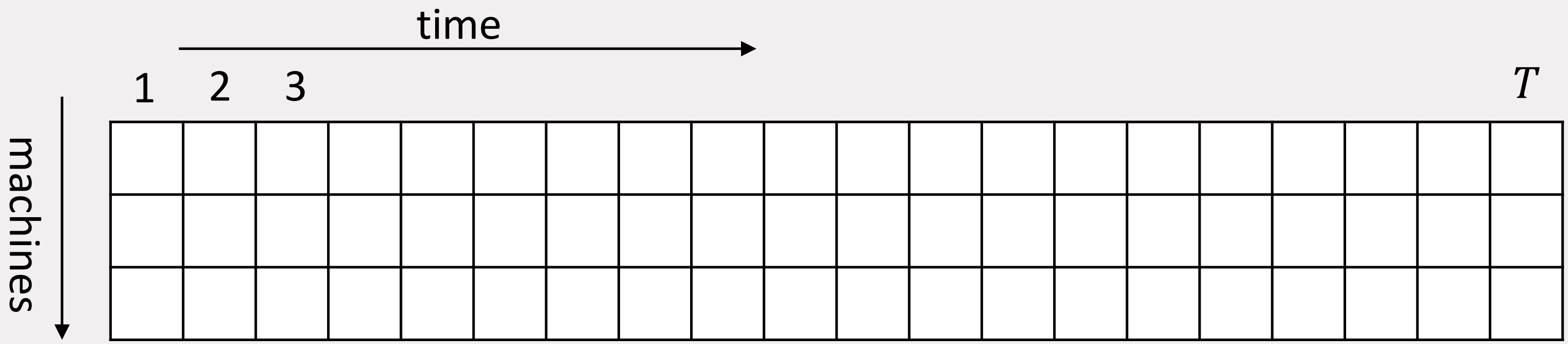
Def: An *antichain* is a set A whose elements are pairwise **incomparable**.

Examples of antichains in G

- ✓ $\{b, c, d\}$
- ✓ $\{b, c\}$
- ✓ $\{d, f\}$

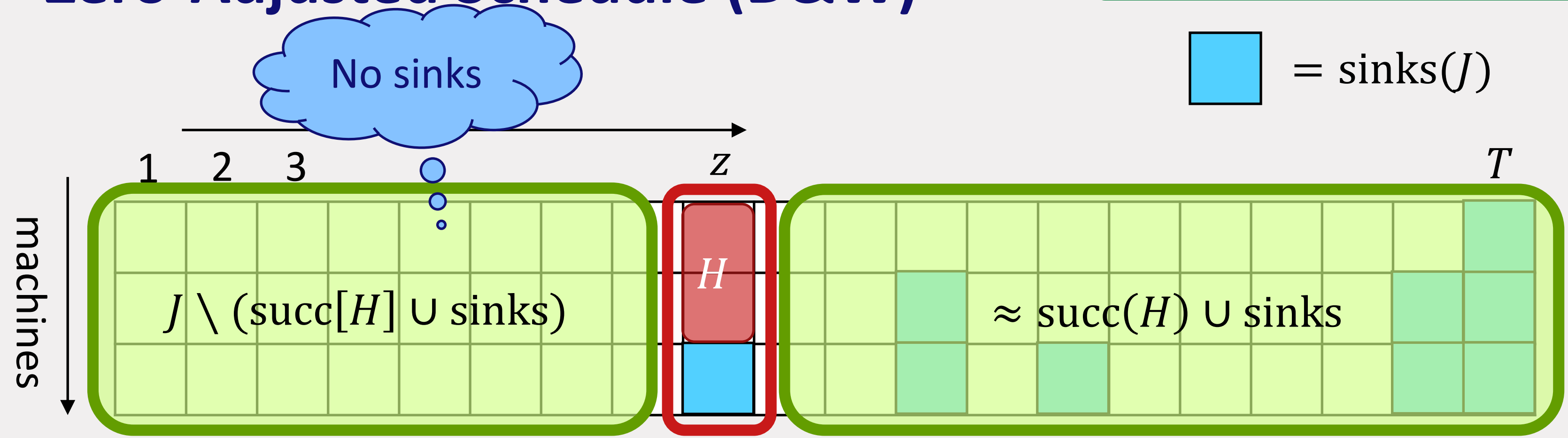
Jobs in one timeslot always form an antichain

Zero-Adjusted Schedule (D&W)



Zero-Adjusted Schedule (D&W)

Assumption: $n = 3 \cdot T$



Let $z \in [1, T]$ be the first moment with a sink.
D&W: W.m.a. Each job x **after *z*** is a sink or a successor of a job at time z.

Dolev and Warmuth

Recursive

Schedule(J):

1. **if** $h(G[J]) \leq 0$ (i.e. $\text{sinks}(J) = J$) **return** $\left\lceil \frac{|J|}{3} \right\rceil$
2. **else return** $\min_{H \in \text{Sep}(J)} \{\text{Schedule}(\text{left}(J,H)) + \text{Schedule}(\text{right}(J,H)) + 1\}$

$\text{Sep}(J) \coloneqq \{\, H \subseteq J$ s.t.
(1) $|H| \leq 3$,
(2) H is antichain,
(3) $|H \setminus \text{sinks}(J)| < 3\}$

$\text{left}(J,H) \coloneqq J \setminus (\text{succ}[\text{H}] \cup \text{sinks}(J))$
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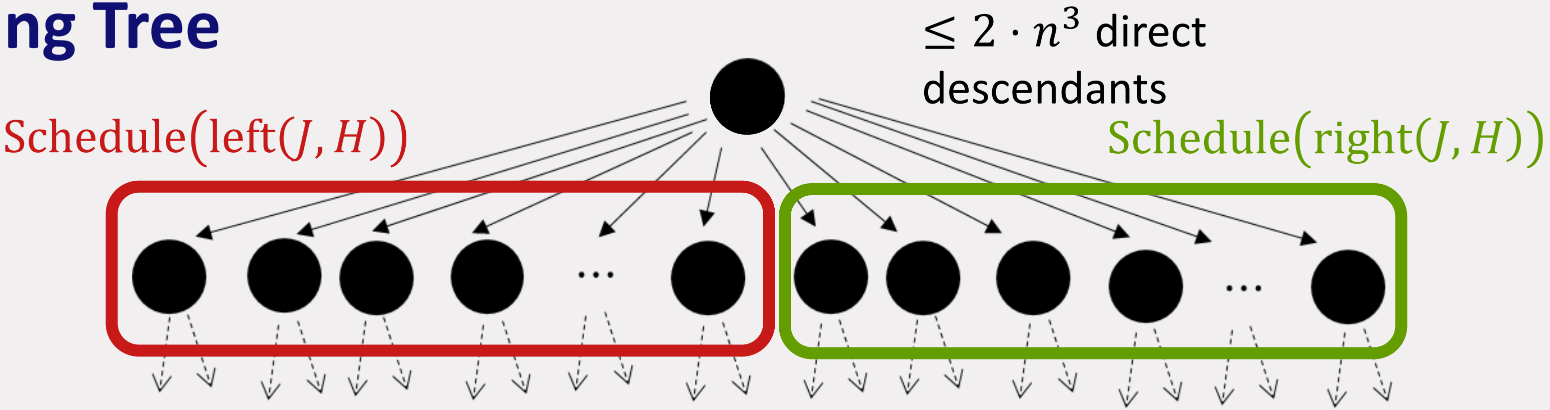
Each subproblem: height decreases by ≥ 1!

Branching Tree



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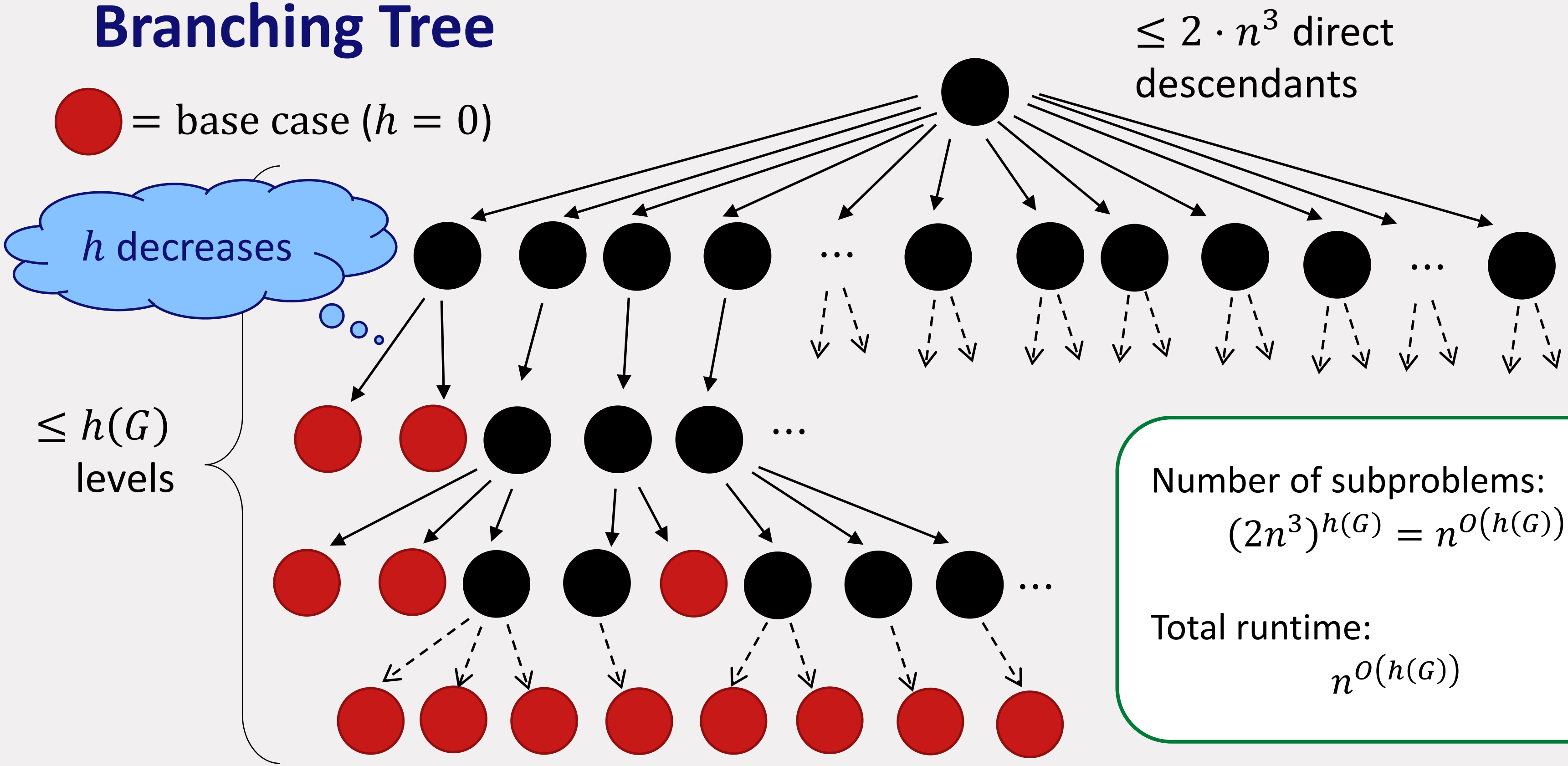

Number of subproblems:

$$(2n^3)^{h(G)} = n^{O(h(G))}$$

Total runtime:

$$n^{O(h(G))}$$

D&W

Schedule(J):

1. **if** $h(G[J]) \leq 0$ **return** $\left\lceil \frac{|J|}{3} \right\rceil$
2. **for each** $H \in \text{Sep}(J)$ **do:**
3. $\quad\quad \text{OPT}[\text{left}(J,H)] \coloneqq \text{Schedule}\big(\text{left}(J,H)\big)$
4. $\quad\quad \text{OPT}[\text{right}(J,H)] \coloneqq \text{Schedule}(\text{right}(J,H))$
5. $\text{OPT}[J] := \min_{H \in \text{Sep}(J)} \{\text{OPT}[\text{left}(J,H)] + \text{OPT}[\text{right}(J,H)] + 1\}$
6. Return $\text{OPT}[J]$

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D&W + LookUp Table

Schedule(J):

1. **return** LUT[J] if it was already set
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7. $\text{LUT}[J] = \text{OPT}[J]$
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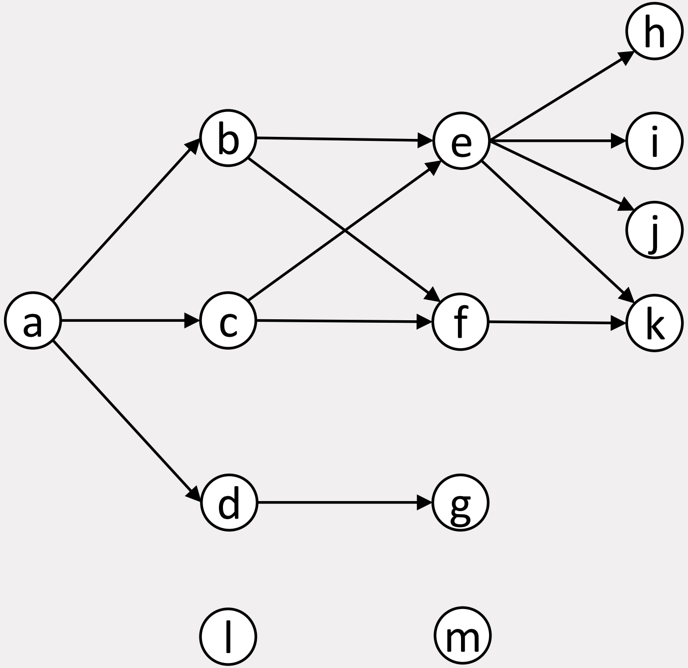
Too many different problems!

What to store?



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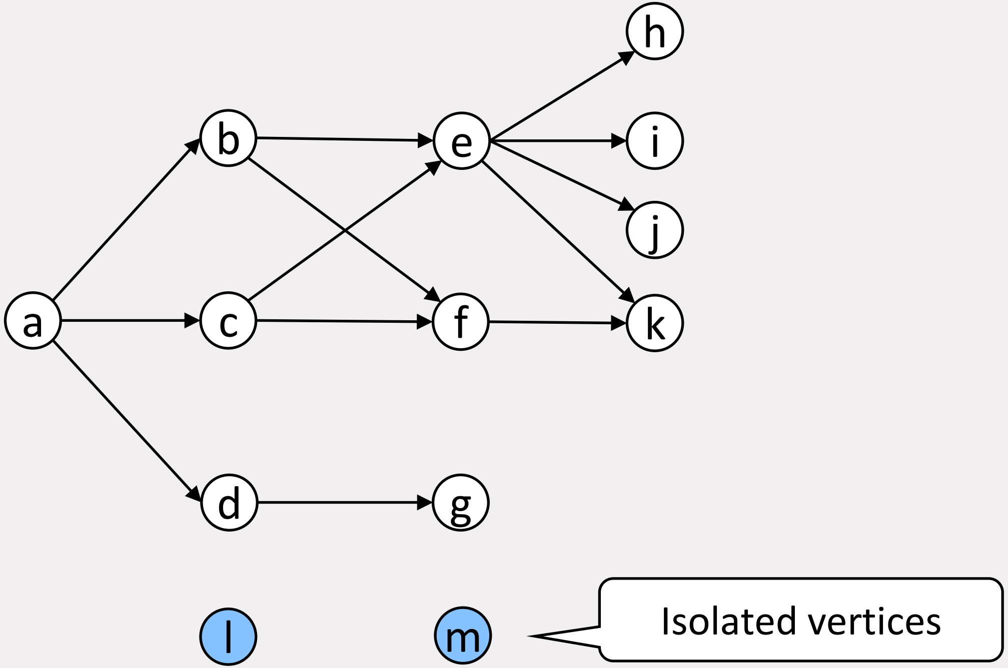

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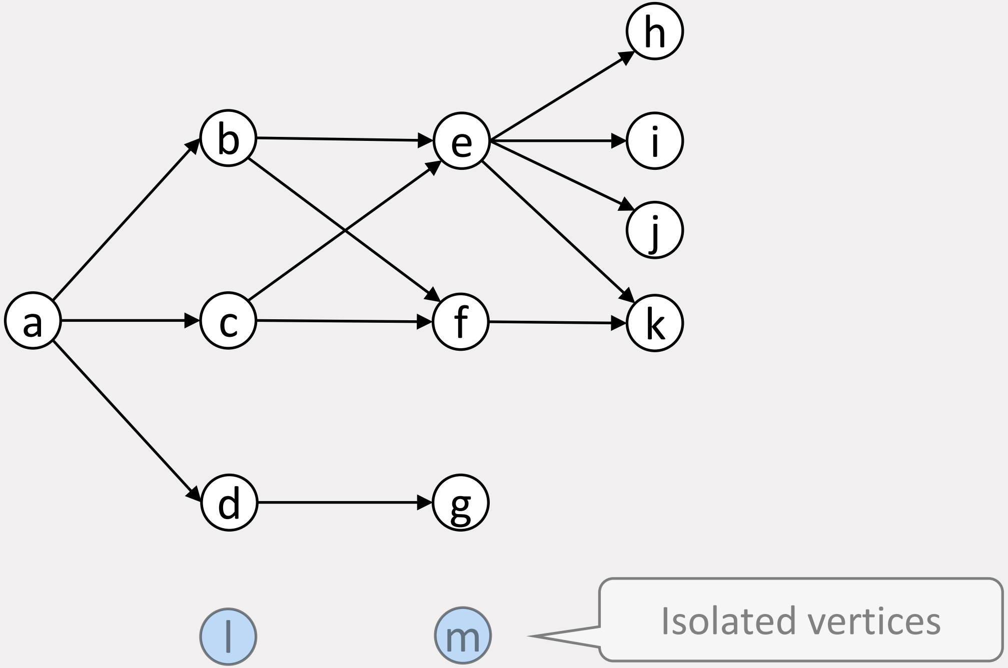

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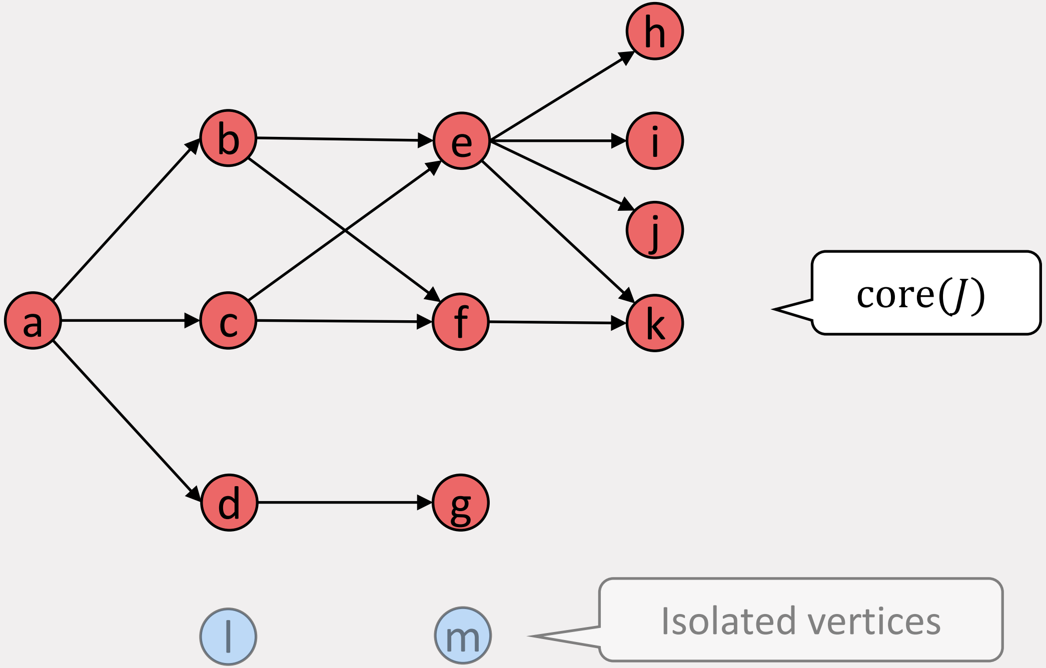

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$\mathrm{Schedule}(J)$:

return $\mathrm{LUT}[\mathrm{core}(J), \#\mathrm{iso}(J)]$ if it was already set
if $J = \emptyset$ **return** 0
for each $H \in \mathrm{Sep}(J)$ **do:**
  $\mathrm{OPT}[\mathrm{left}(J,H)] \coloneqq \mathrm{Schedule}\big(\mathrm{left}(J,H)\big)$
  $\mathrm{OPT}[\mathrm{right}(J,H)] \coloneqq \mathrm{Schedule}(\mathrm{right}(J,H))$
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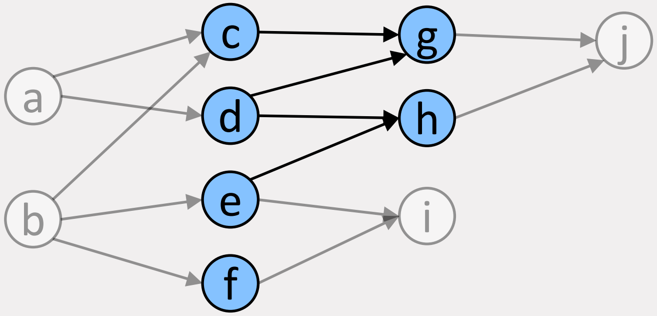
$\mathrm{LUT}[\mathrm{core}(J), \#\mathrm{iso}(J)] = \mathrm{OPT}[J]$
Return $\mathrm{OPT}[J]$

How does this help?

Feasible Job sets

Let J be a *feasible set of jobs*.



time

	1	2	3	4
1	a	c	f	i
2	b	d	g	j
3		e	h	

Feasible Job sets

Let J be a *feasible set of jobs*.

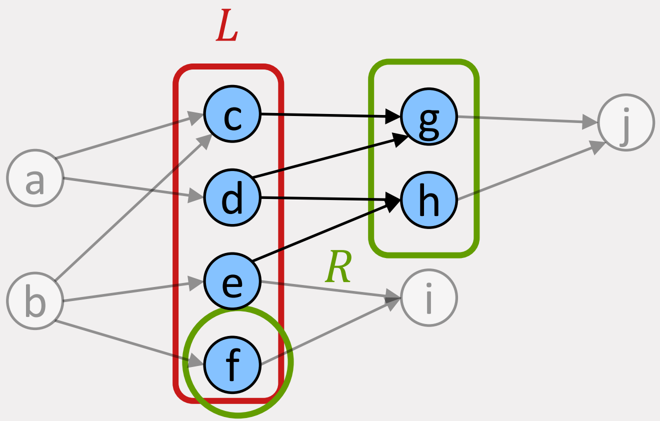
Jobs J can be described as

$$J = \mathrm{succ}[L] \cap \mathrm{pred}[R]$$

where

L = minimal elements = sources of J

R = maximal elements = sinks of J



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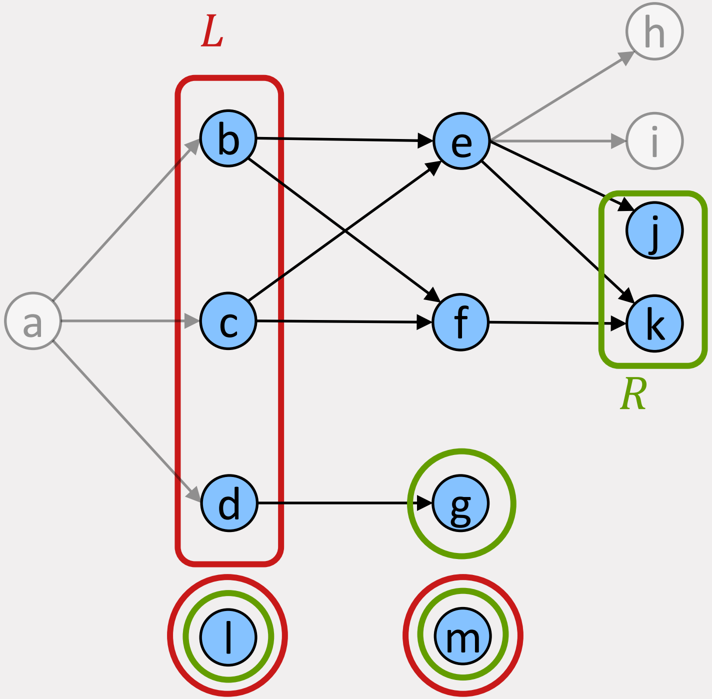

Feasible Job sets

Let J be a *feasible set of jobs*.

~~Jobs J~~ $\mathbf{core}(J)$ can be described as

$$\cancel{J}\ \mathbf{core}(J) = \mathrm{succ}[L] \cap \mathrm{pred}[R]$$

where

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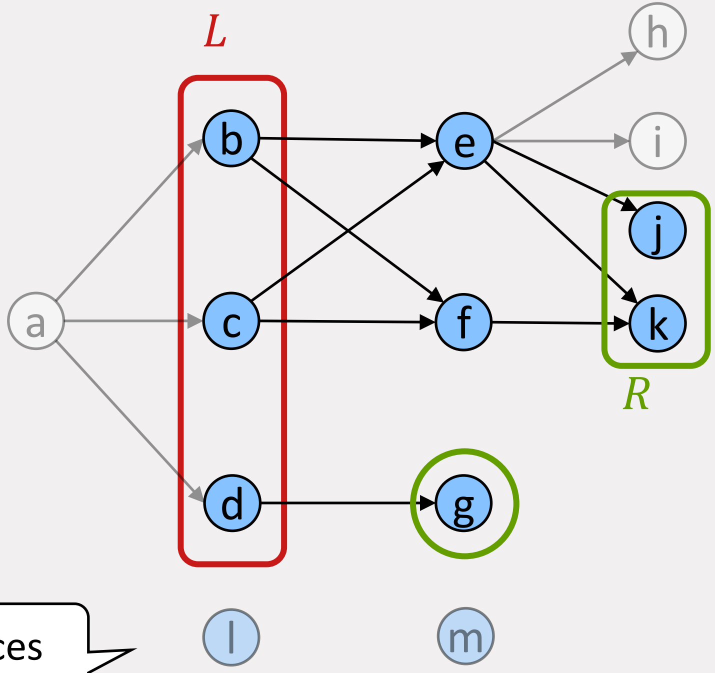
2 isolated vertices



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~~Jobs J~~ can be described as
$\mathbf{core}(J)$

$$\mathbf{succ}(L)$$

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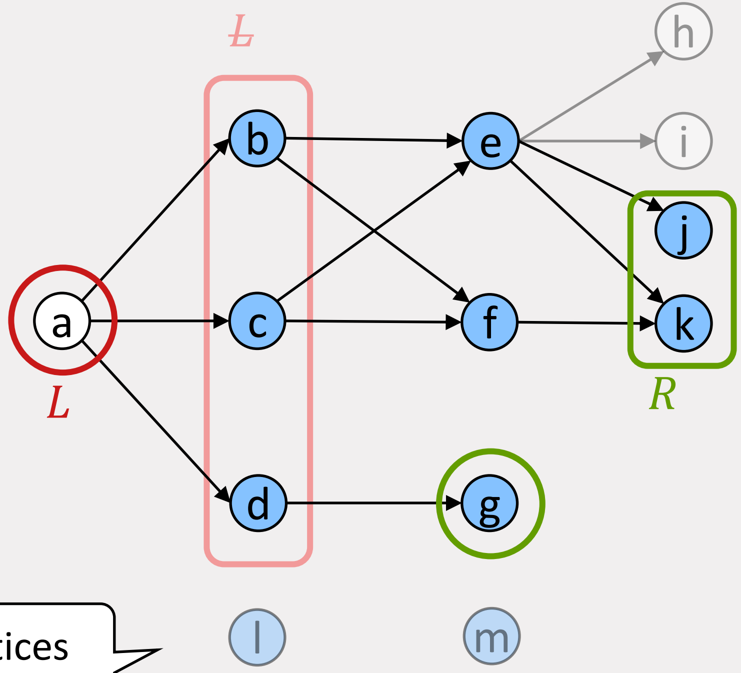
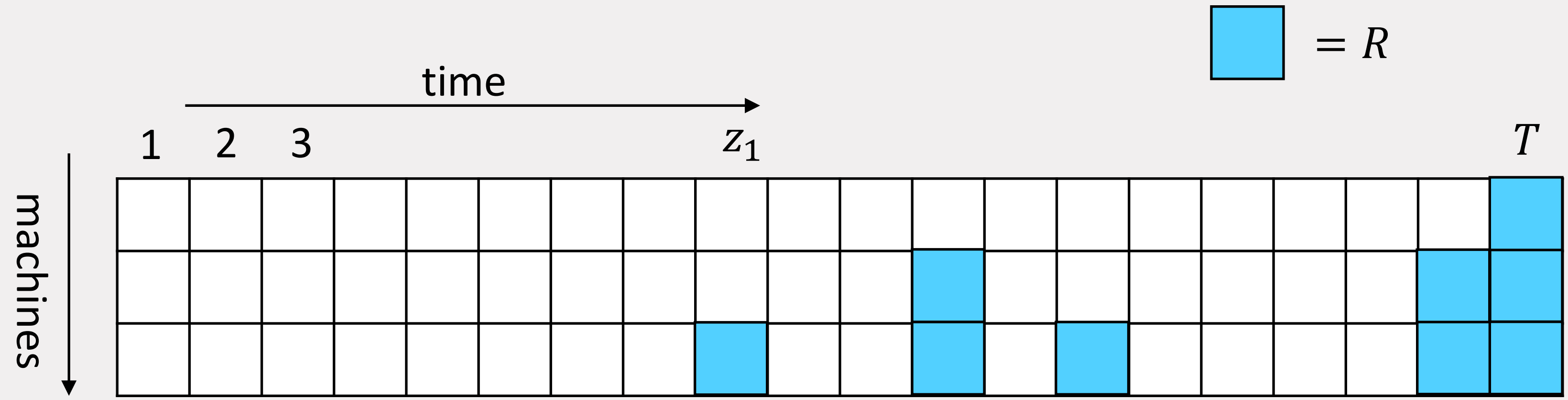


2 isolated vertices

Going to the right

Assumption: $n = 3 \cdot T$



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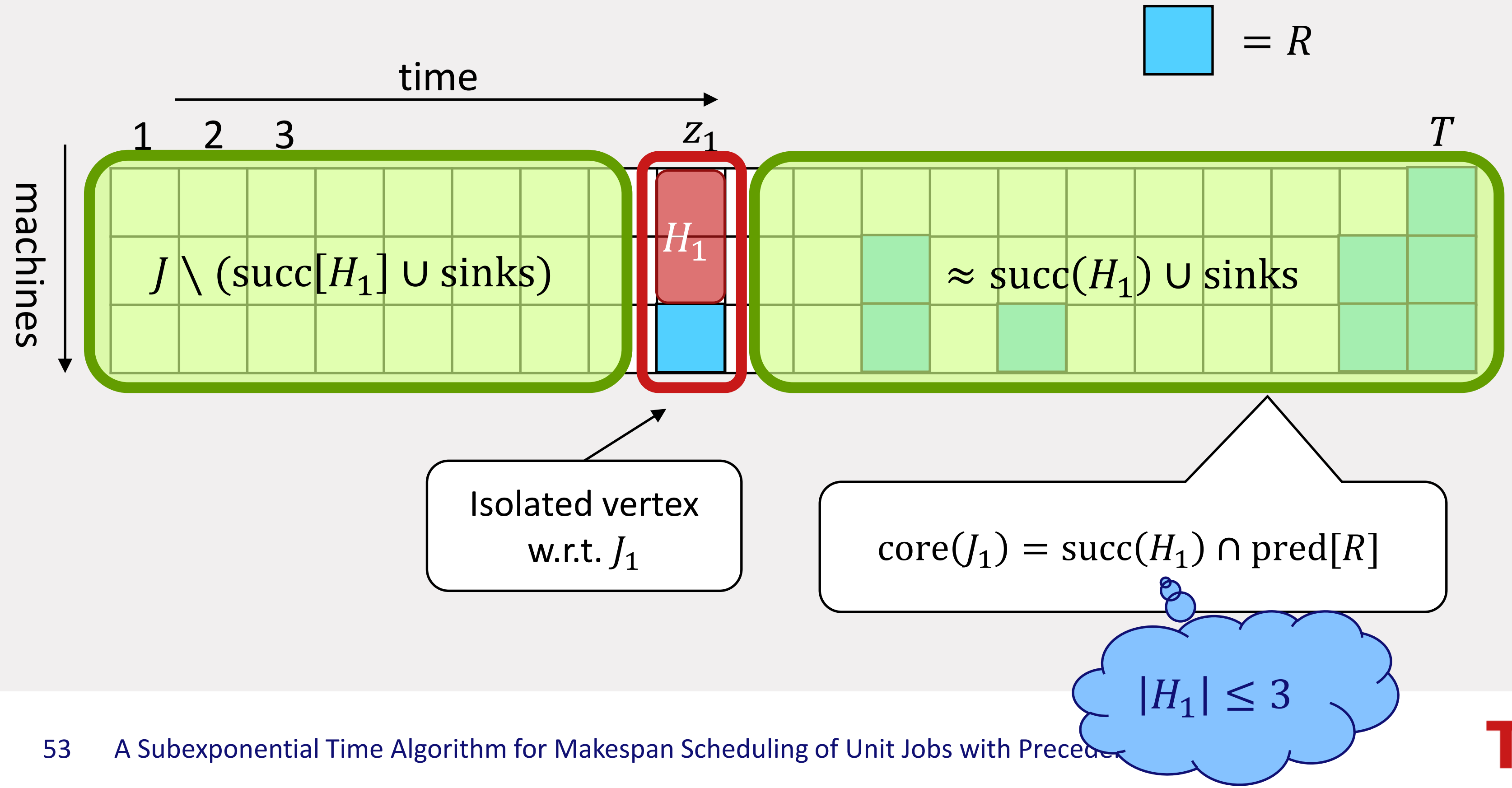

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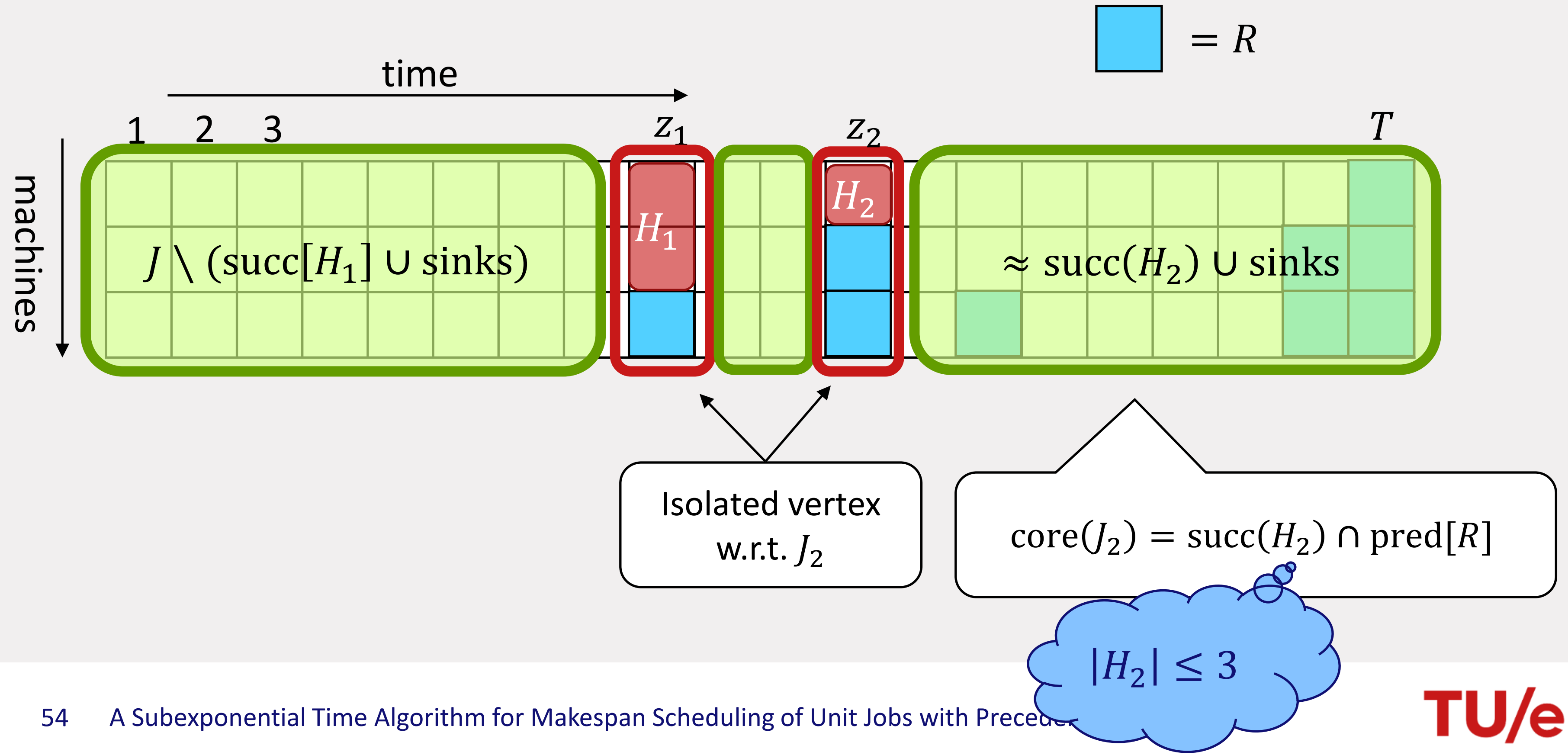

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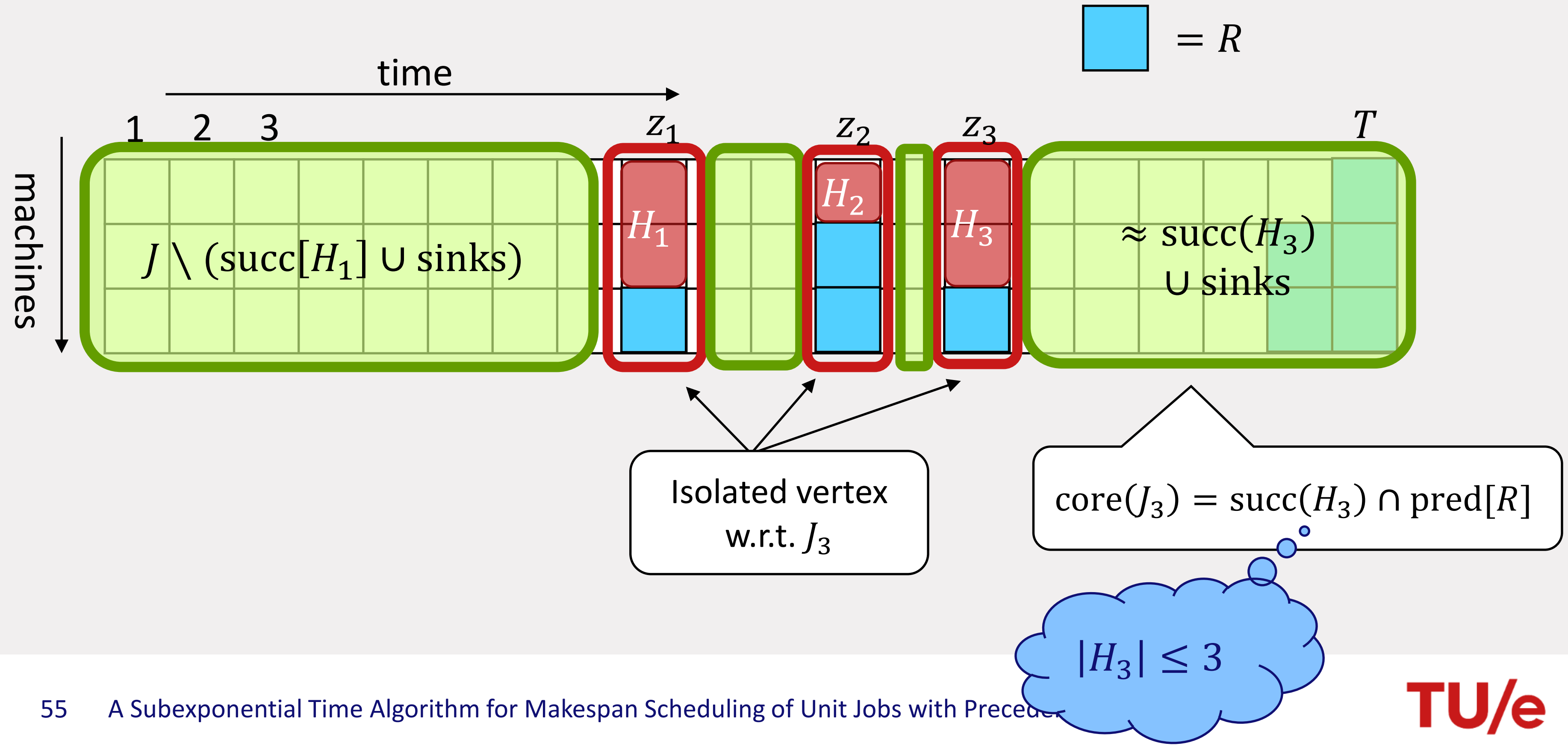

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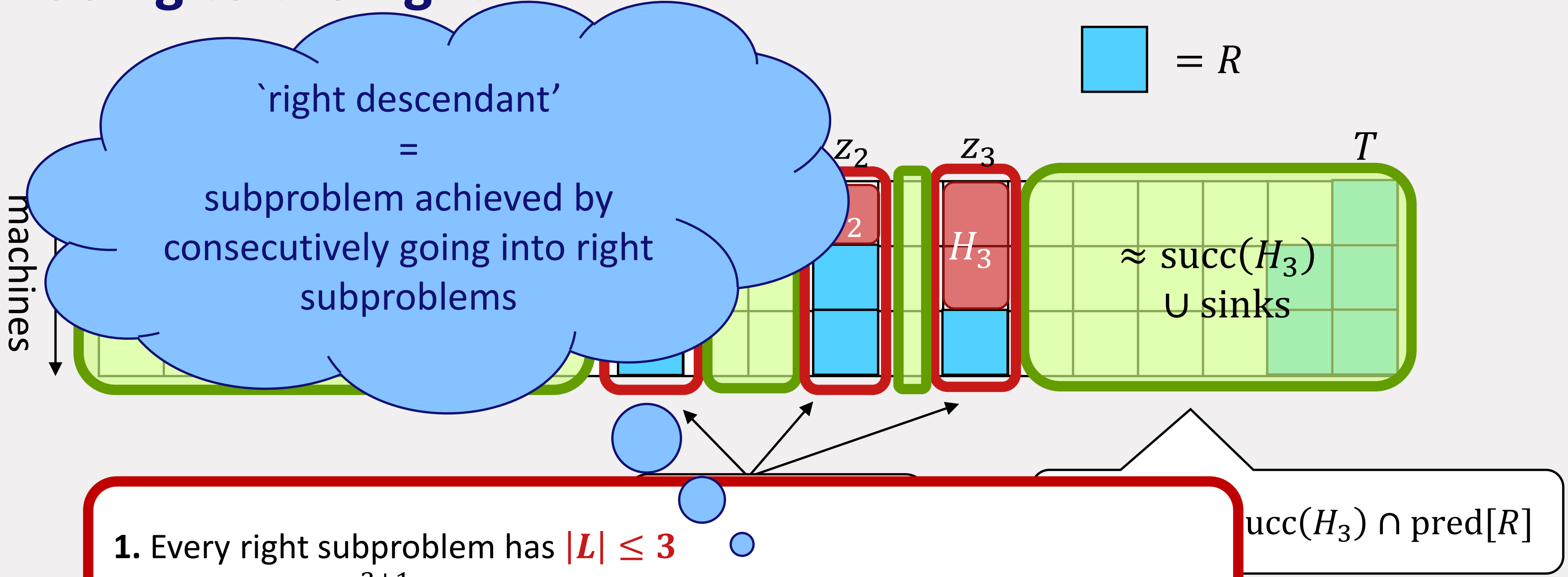
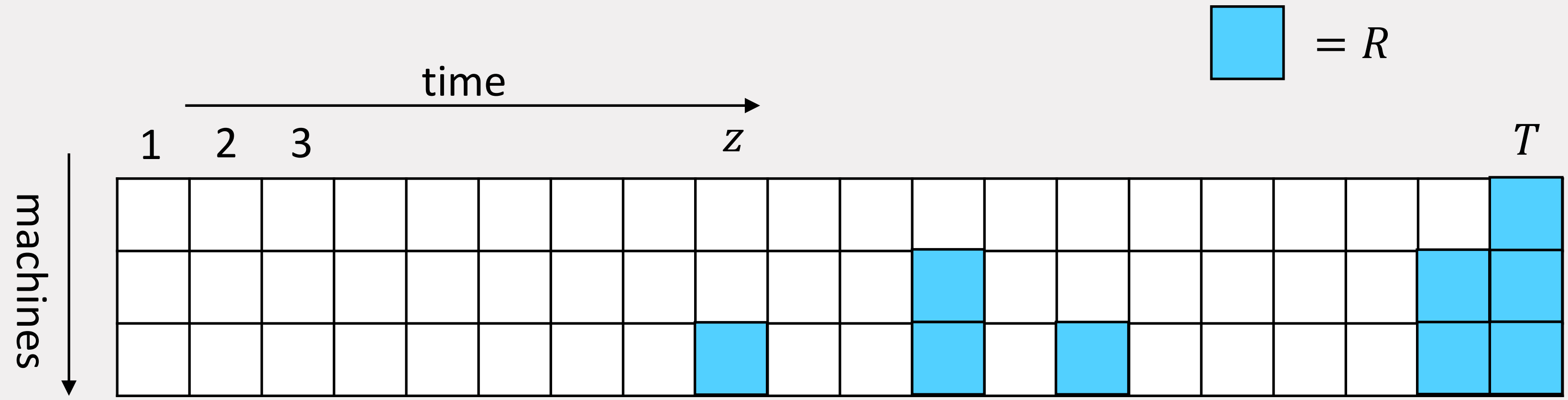


1. Every right subproblem has $|L| \leq 3$
2. There are $\leq n^{3+1}$ different **`right descendants'**

Going to the left

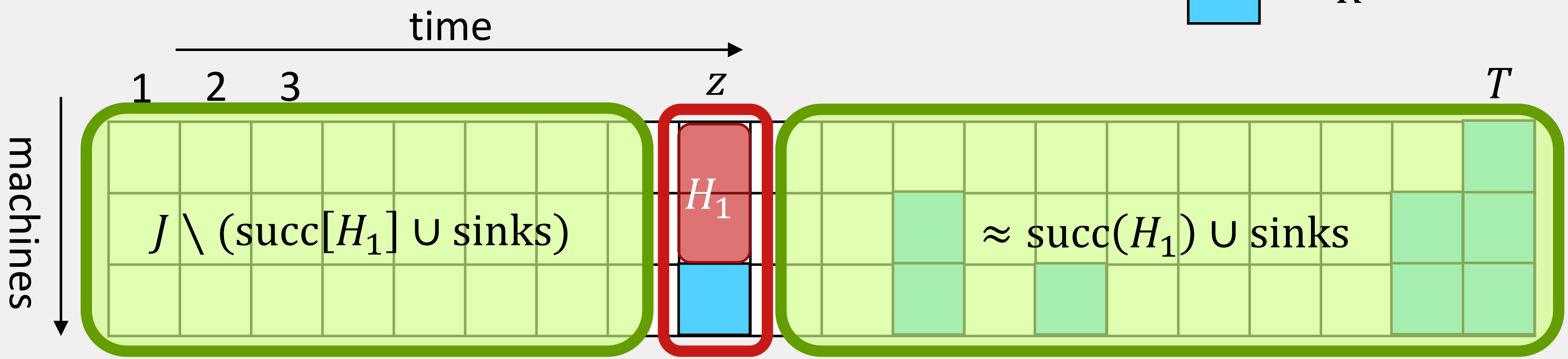
Assumption: $n = 3 \cdot T$



Going to the left

Assumption: $n = 3 \cdot T$

$= R$



core $= \mathbf{succ}(\boldsymbol{L}) \cap \mathrm{pred}[R_{\mathrm{new}}]$

1. Every left subproblem has $|\boldsymbol{L}| \leq \mathbf{3}$
2. Problem size decreases by $|\boldsymbol{R}|$

Win-Win strategy

Invariant: $J = (\mathrm{succ}(L) \cap \mathrm{pred}[R]) \cup k$ isolated vertices, $|L| \leq 3$

$\leq n^3$

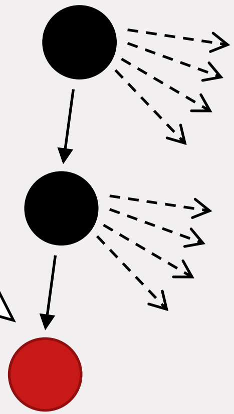
Win-win strategy

$\leq n$

Case $\lvert R\rvert \leq \sqrt{n}$	Case $\lvert R\rvert > \sqrt{n}$
$\Rightarrow$ only $\binom{n}{\sqrt{n}} = 2^{O(\sqrt{n}\cdot\log n)}$ different R's	In next left step: make $\sqrt{n}$ jobs progress!

TU/e

Branching Tree

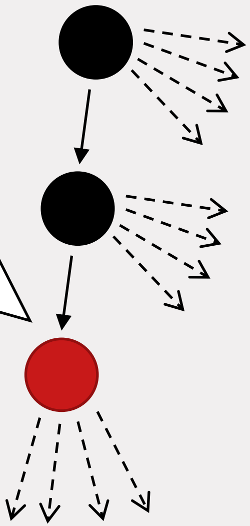


Either already in Lookup Table:
- Base case

$= |R| \leq \sqrt{n}$

$= |R| > \sqrt{n}$

Branching Tree



Either already in Lookup Table:
- Base case

Or not yet in Lookup Table:

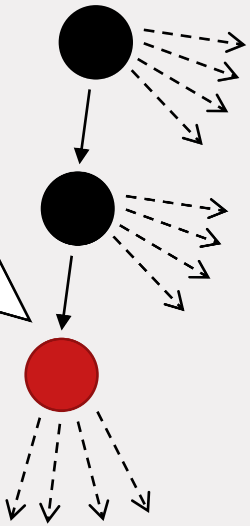
$= |R| \leq \sqrt{n}$

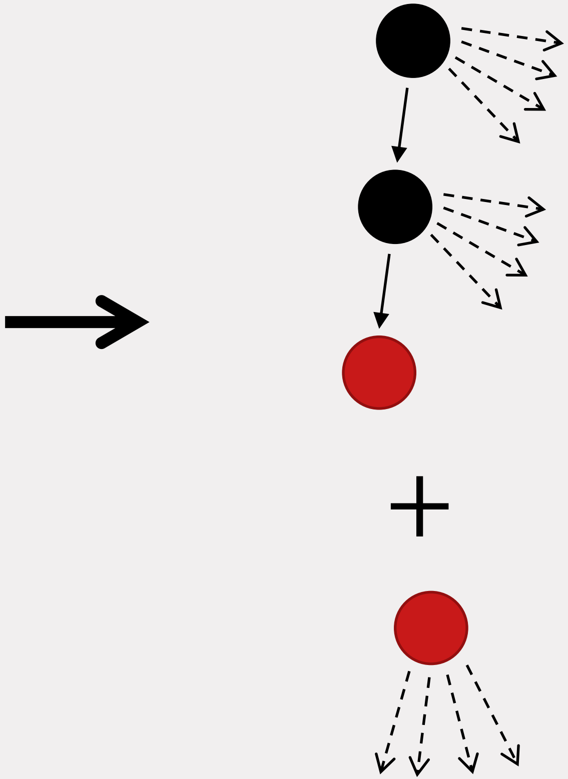
$= |R| > \sqrt{n}$

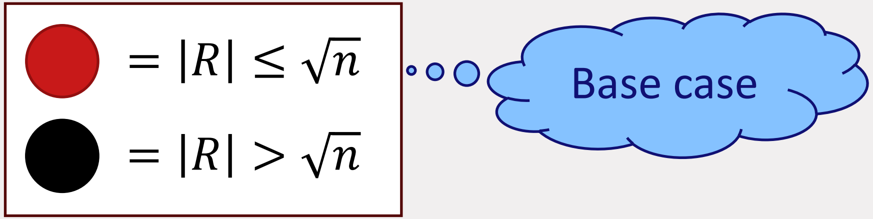
Branching Tree

Either already in Lookup Table:
- Base case

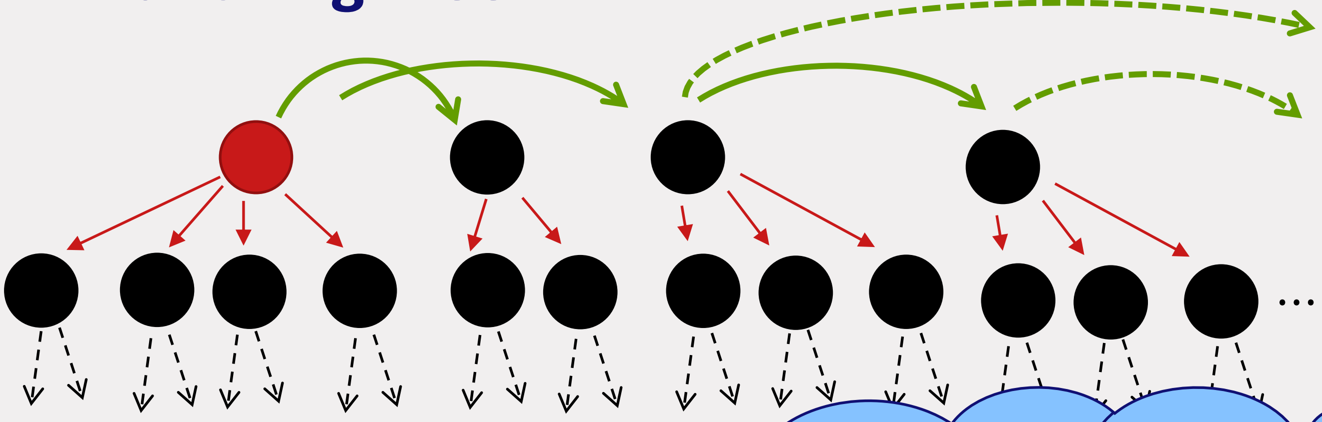
Or not yet in Lookup Table:
- View as its `own tree'
- $\Rightarrow n^{\sqrt{n}}$ such trees







Branching Tree

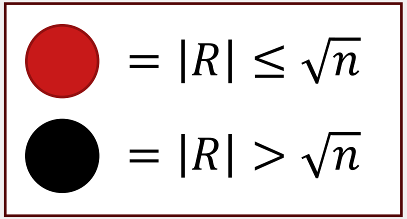


$\leq n^4$ right descendants

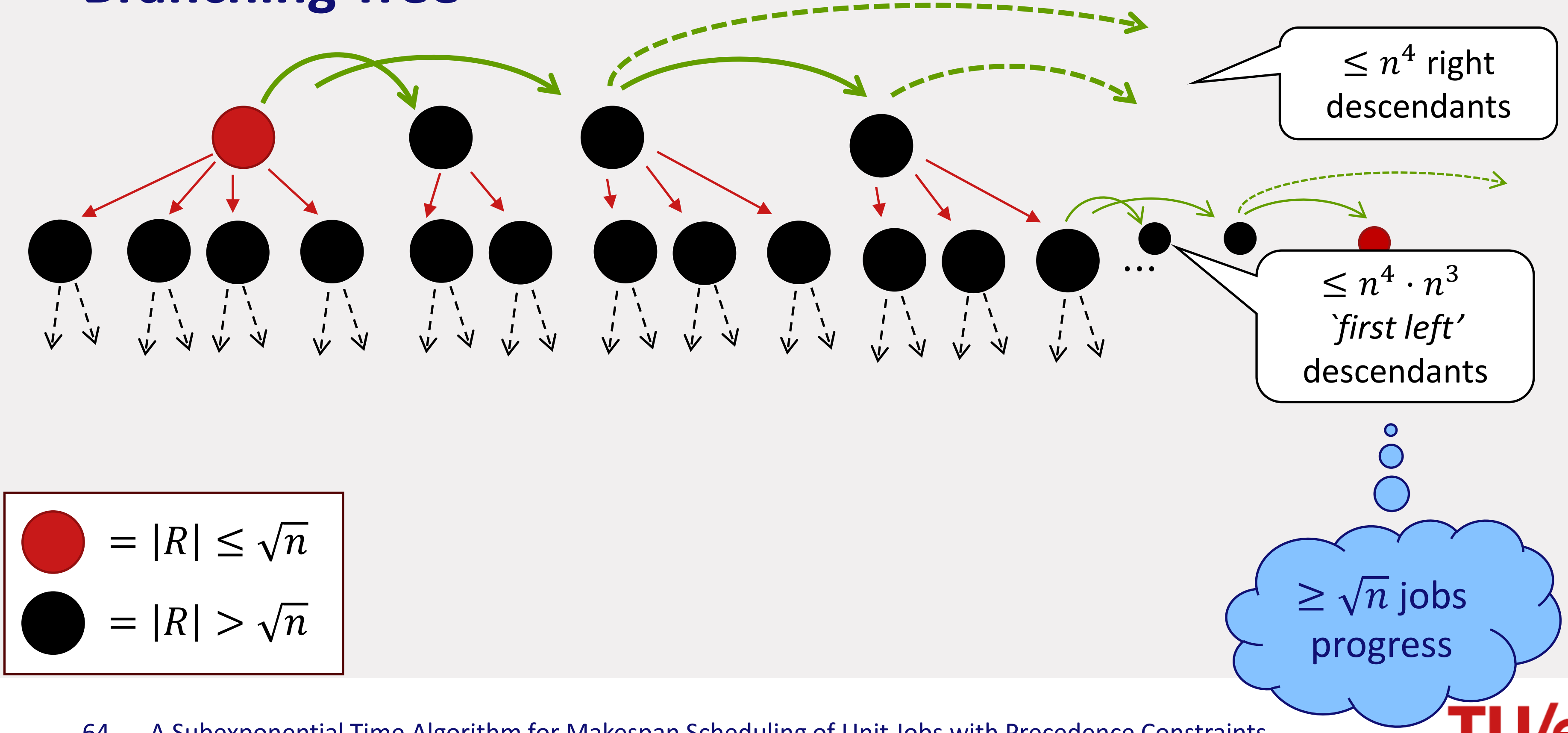
$\leq n^4 \cdot n^3$ `first left' descendants

`first left descendant'
=
subproblem achieved by consecutively going into right subproblems, then **once left**

$\geq \sqrt{n}$ jobs progress



Branching Tree



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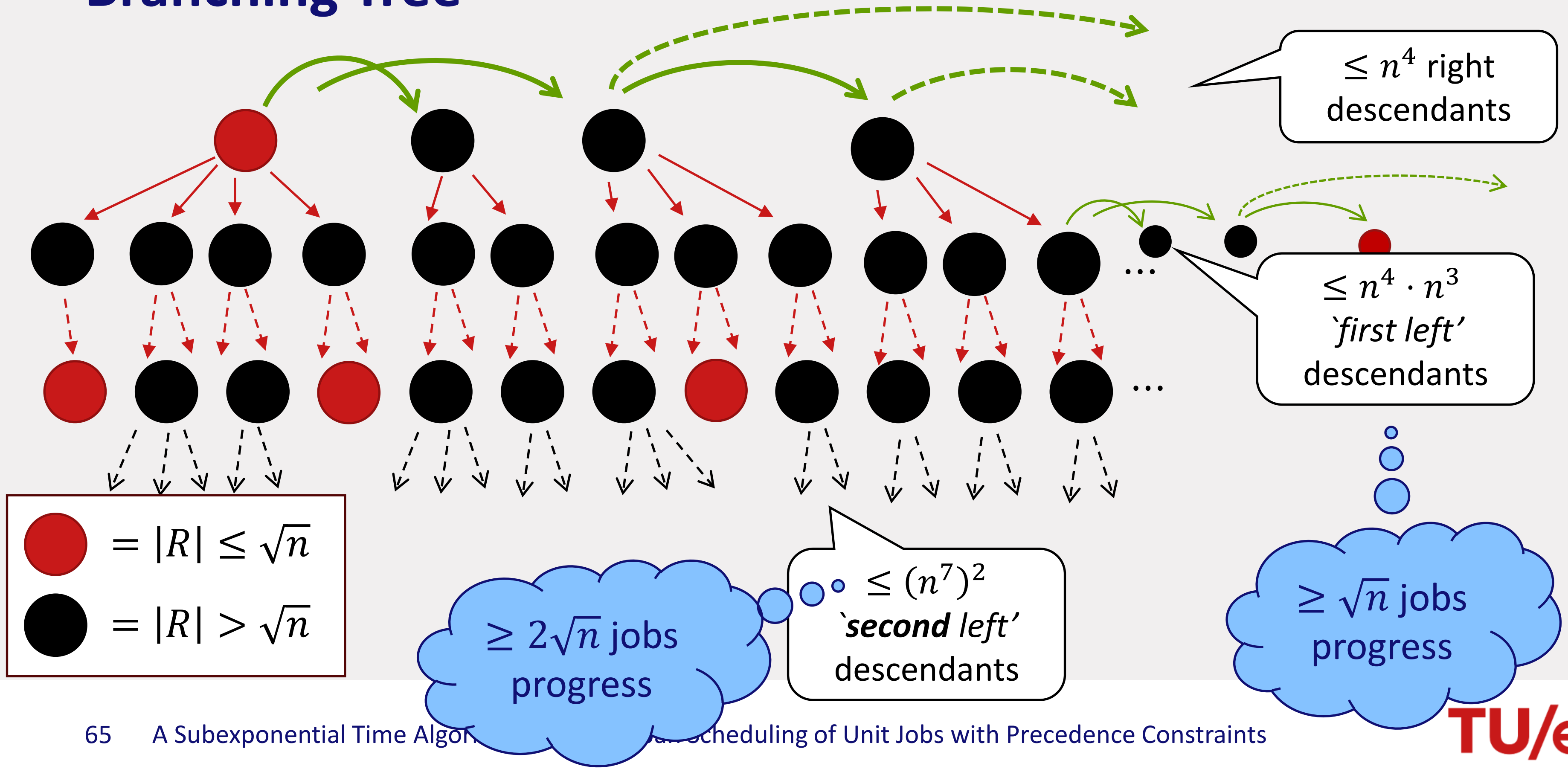
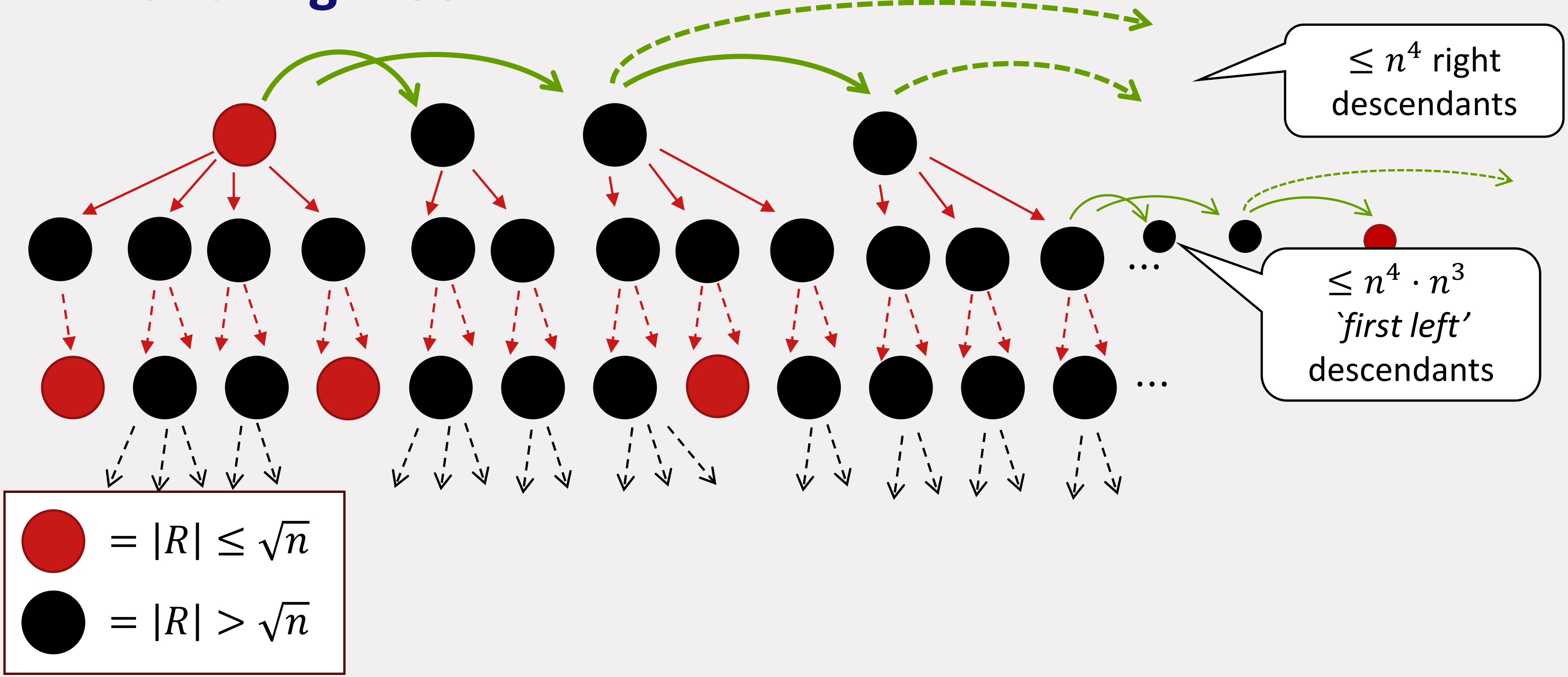


Branching Tree



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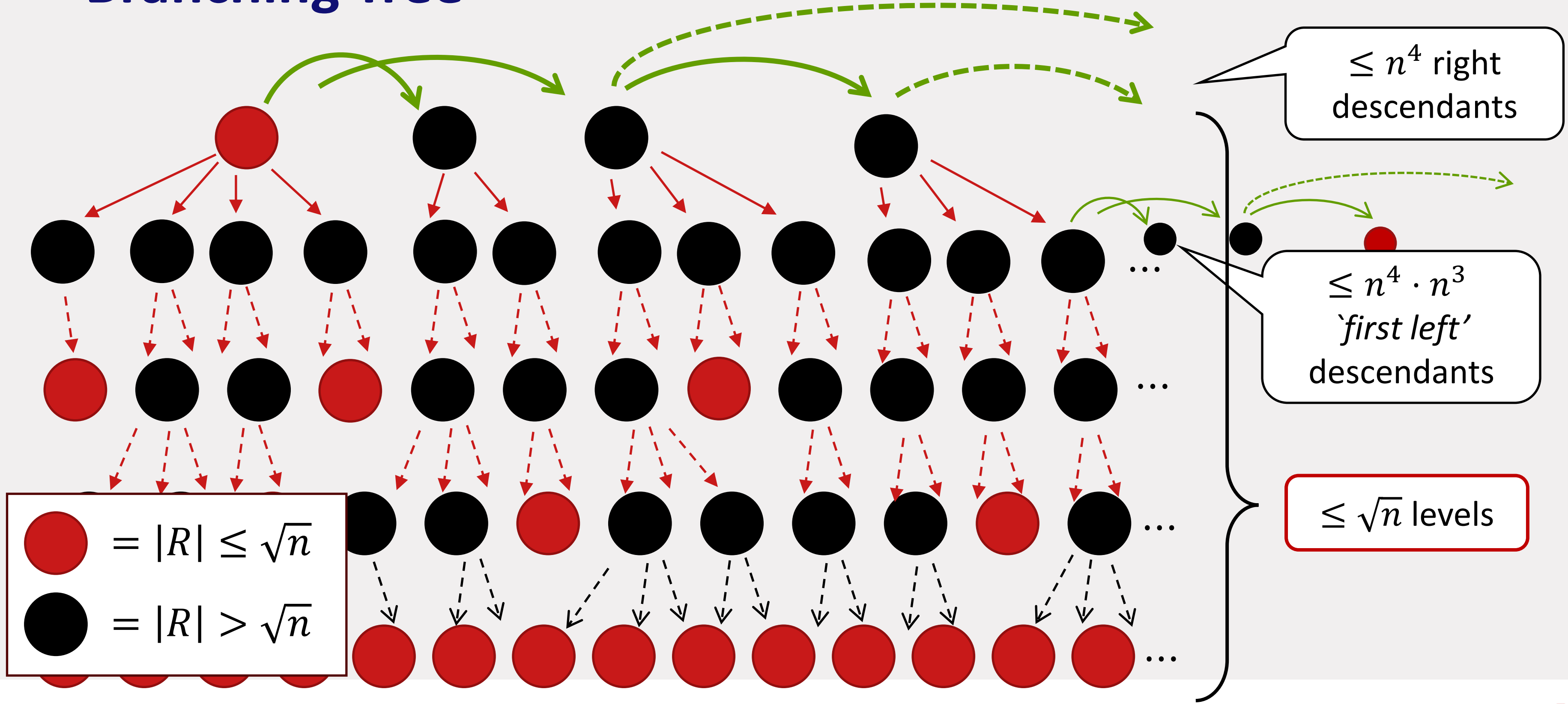
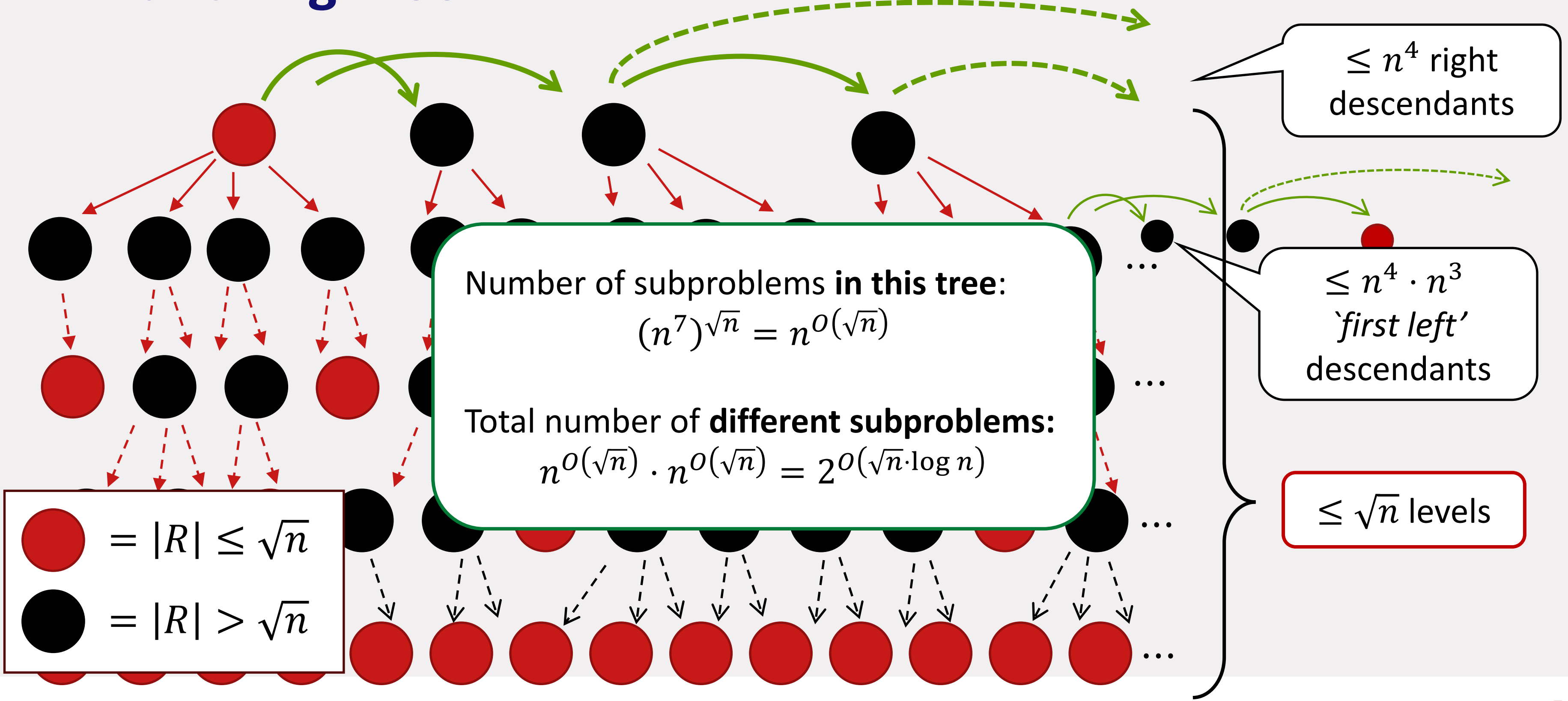
Branching Tree



Branching Tree



Algorithm

Schedule(J):

1. **return** $\mathrm{LUT}[\mathrm{core}(J), \#\mathrm{iso}(J)]$ if it was already set
2. **if** $J = \emptyset$ **return** 0
3. **for each** $H \in \mathrm{Sep}(J)$ **do:**
4. $\qquad \mathrm{OPT}[\mathrm{left}(J,H)] \coloneqq \mathrm{Schedule}\big(\mathrm{left}(J,H)\big)$
5. $\qquad \mathrm{OPT}[\mathrm{right}(J,H)] \coloneqq \mathrm{Schedule}(\mathrm{right}(J,H))$
6. $\mathrm{OPT}[J] := \min_{H \in \mathrm{Sep}(J)} \{\mathrm{OPT}[\mathrm{left}(J,H)] + \mathrm{OPT}[\mathrm{right}(J,H)] + 1\}$
7. $\mathrm{LUT}[\mathrm{core}(J), \#\mathrm{iso}(J)] = \mathrm{OPT}[J]$
8. Return $\mathrm{OPT}[J]$

$\mathrm{Sep}(J) \coloneqq \{\, H \subseteq J$ s.t.
(1) $|H| \leq 3$,
(2) H is antichain,
(3) $|H \setminus \mathrm{sinks}(J)| < 3\}$

$\mathrm{left}(J,H) \coloneqq J \setminus \big(\mathrm{succ}[\mathrm{H}] \cup \mathrm{sinks}(J)\big)$
$\mathrm{right}(J,H) \coloneqq J \cap \big((\mathrm{succ}(H) \cup \mathrm{sinks}(J)\big) \setminus H$

Only $2^{O(\sqrt{n}\cdot\log n)}$ different problems encountered

TU/e

Corollaries

Our result:

$Pm|prec, p_j = 1|C_{\max}$ can be solved in $\left(1 + \frac{n}{m}\right)^{O(\sqrt{nm})}$ time.

Corollary 1

$Pm|prec, p_j = 1|C_{\max}$ can be solved in subexponential time whenever $m = o(n)$.

Corollary 2

$\boldsymbol{P}|prec, p_j = 1|C_{\max}$ can be solved in $1.997^n \cdot poly(n)$ time.

Conclusion

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$P3|prec, p_j = 1|C_{\max}$ in $2^{O(\sqrt{n}\cdot\log n)}$ time.

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Future Research:

$P3|prec, p_j = 1|C_{\max}$ in quasi-polynomial time?

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Thanks for your attention!