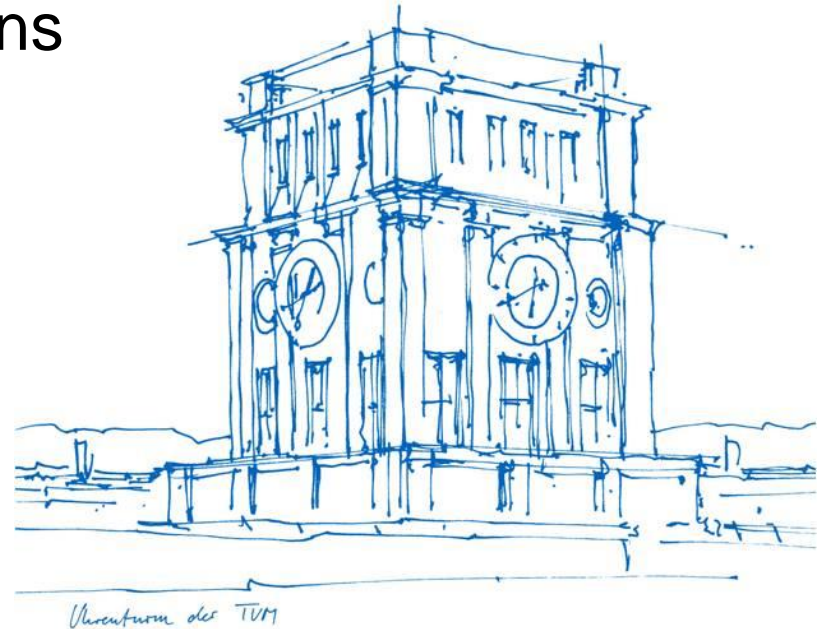


The Resource-Constrained Project Scheduling Problem with Flexible Resource Profiles: Models, Methods, and Applications

Rainer Kolisch
Technical University of Munich

Scheduling Seminar - April 26, 2023



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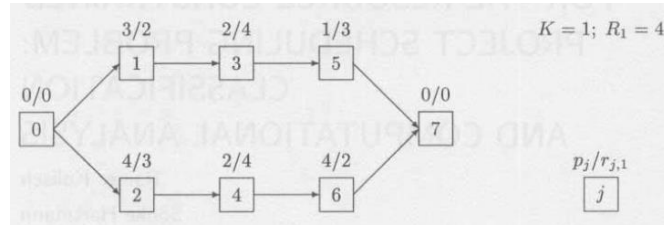
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1. Introduction of the FRCPSP
2. Solution Approaches
 - Comparison of MIPs
 - Hybrid Metaheuristic
3. Instances
4. Applications
 - Lead Optimization in Pharmaceutical Research
 - Engineering-to-Order Production Planning
5. Research Opportunities

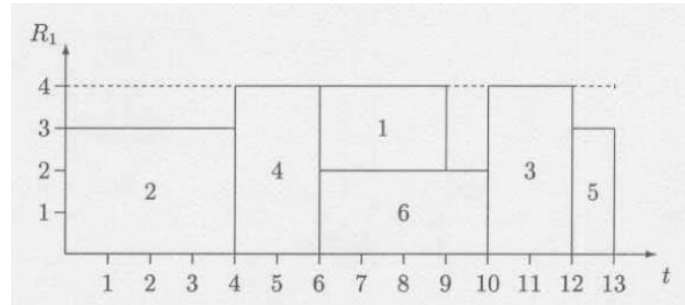
Resource-Constrained Project Scheduling Problem (RCPSP)

Decisions:

Scheduling (start time of activities)



Source: Kolisch and Hartmann (1999)

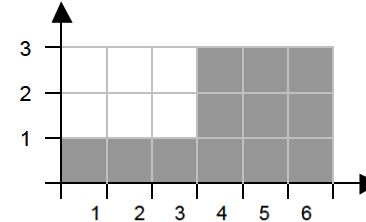
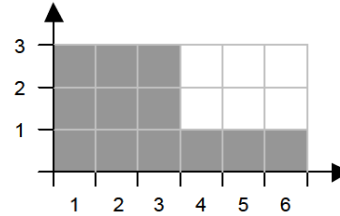
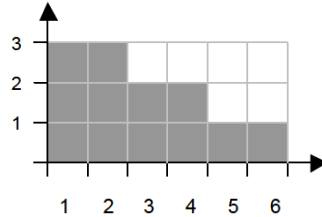
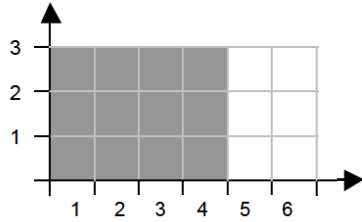


Flexible RCPSP (FRCPSP)

Decisions:

- **Scheduling**: Start time of activities (RCPSP)
- **Resource Allocation and activity demand profile for each activity j** :
Resources have to be allocated to the periods such that $[\underline{u}_j, \overline{u}_j]$
 - 1) the work content w_j is covered within the time window $[ES_j, LF_j]$
 - 2) there are no idling periods between the start and the finish period of the activity (no preemption)
 - 3) if resources are allocated in a period the amount has to be within interval
 - 4) resource allocation has to be constant for at least l periods except the final periods (minimum block length)

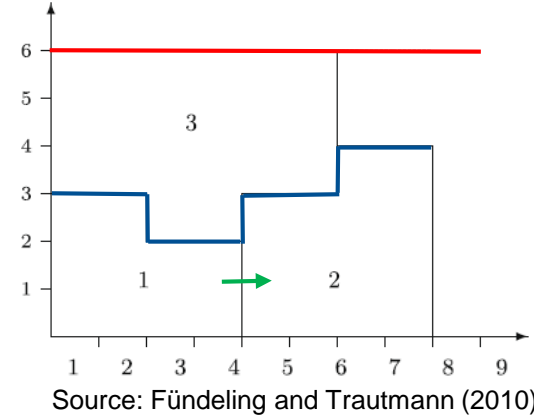
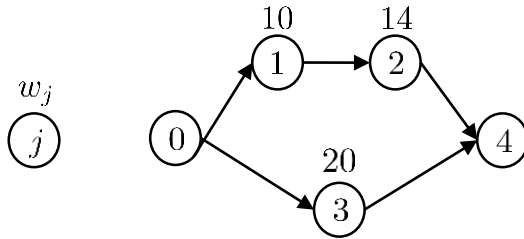
Resource Allocation: Example



$w = 12$
 $l = 2$
 $\underline{u} = 1, \bar{u} = 3$
 $ES = 1, LF = 6$

Decisions of the FRCPSP

Resource allocation and scheduling subject to precedence and resource constraints.



$$l = 2$$
$$\underline{u} = 2, \bar{u} = 4$$

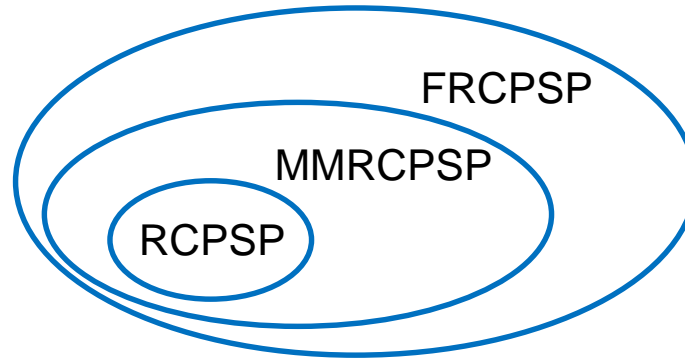
Trautmann et al. term the FRCPSP as “RCPSP with work content constraints”.

Variants of the FRCPSP

		Time	
		Continuous	Discrete
Resource Allocation	Discrete		<p>Operational problem (periods days)</p> <ul style="list-style-type: none">• Kolisch et al. (2003)• Fündeling and Trautmann (2010)• Baumann et al. (2015)
	Continuous	<ul style="list-style-type: none">• Naber (2017)	<p>Tactical problem (periods weeks)</p> <ul style="list-style-type: none">• Tritschler et al. (2017)• Naber and Kolisch (2014)

FRCPSP in the Project Scheduling Landscape

- The FRCPSP is a general case of the (Multi-Mode) RCPSP and thus many scheduling problems such as the JSP, the FSP, the simple ALB, SP, as well as single and parallel machine scheduling problems.
- A feasible solution can be obtained in polynomial time if the time windows are sufficiently large.
- Solving the FRCPSP to optimality is NP-hard.



Solution Approaches for the FRCPSP



MIPs

- Kolisch et al. (2003)
- Baumann and Trautmann (2013)
- [Naber and Kolisch \(2014\)](#)

Heuristics

- Kolisch et al. (2003): Priority Rule
- Fündeling and Trautmann (2010): Priority Rule
- Ranjbar and Kianfar (2010): Genetic Algorithm
- [Tritschler et al. \(2017\): Hybrid Metaheuristic](#)
- Zimmermann (2017): Matheuristic

Binary variables:

$z_{j,t} = 1$, if j is processed in t (On/Off)

$x_{j,t} = 1$, if j starts at the beginning of t (Pulse)

$x_{j,t}^a = 1$, if j starts at the beginning of t or before (Step)

$y_{j,t} = 1$, if j is completed at the end of t (Pulse)

$y_{j,t}^a = 1$, if j is completed at the end of $t - 1$ or before (Step)

$\delta_{j,r,t} = 1$, if resources allocated to j change from $t - 1$ to t

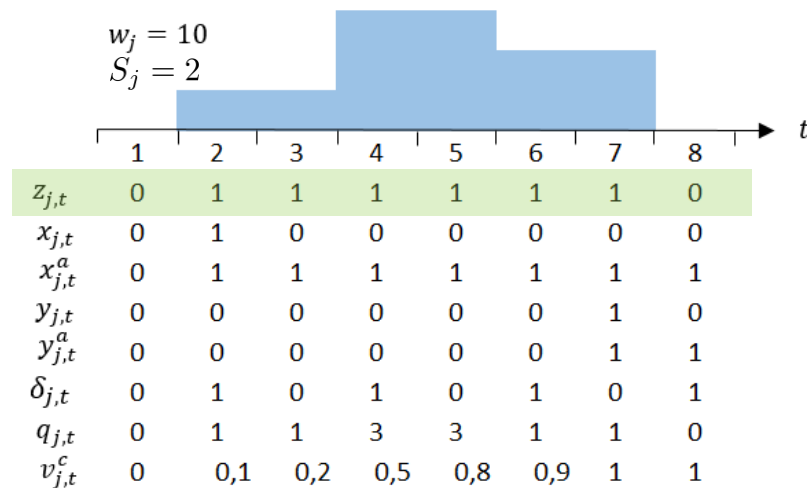
Continuous variables:

$q_{j,r,t} \geq 0$ Resources allocated to j in t

$v_{j,t}^c \in [0, 1]$ Cumulated intensity of j in period t

$$v_{j,t}^c = v_{j,t-1}^c + \frac{q_{j,r,t}}{w_{j,r}}$$

$S_j \geq 0$ Starting period of j



Binary variables:

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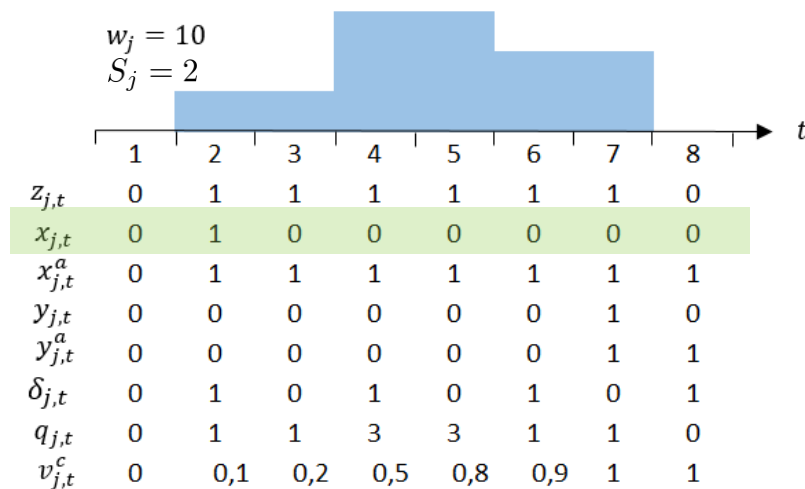
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MIP-Formulations: Variables

Binary variables:

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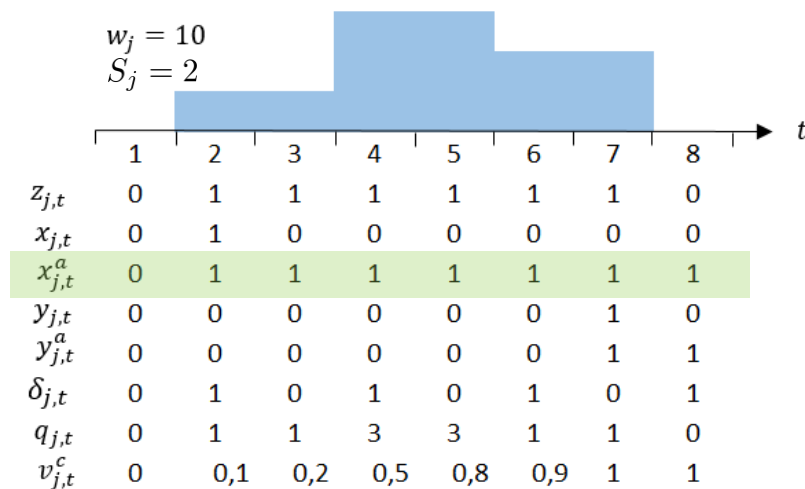
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$$v_{j,t}^c = v_{j,t-1}^c + \frac{q_{j,r,t}}{w_{j,r}}$$

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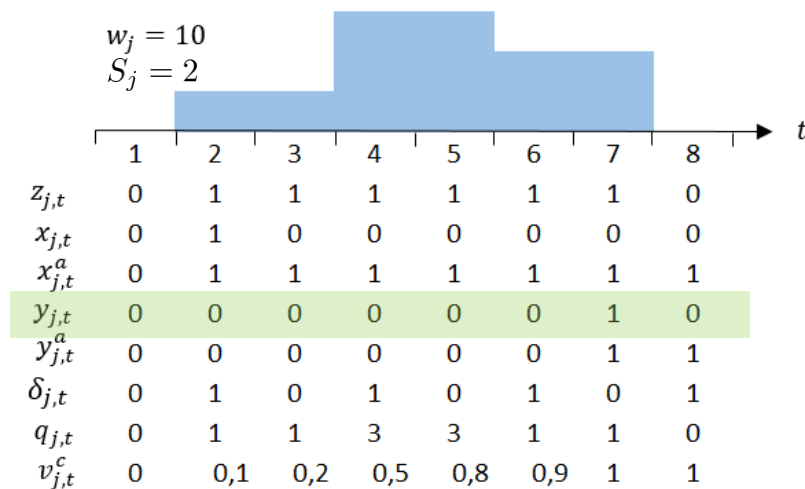
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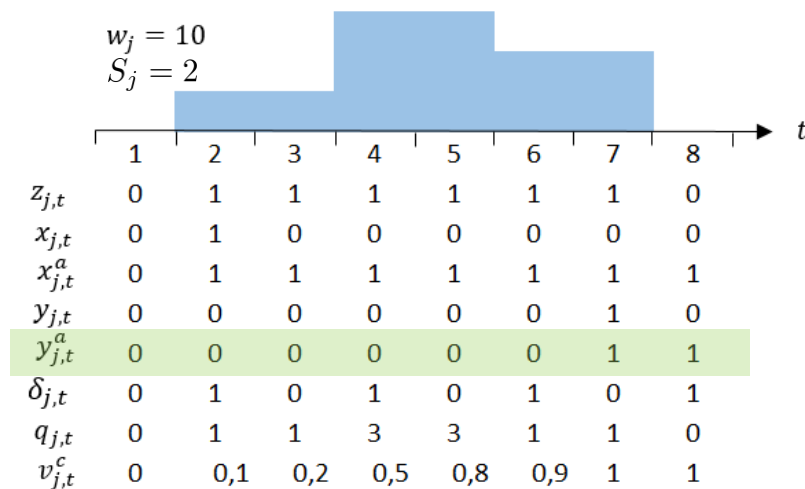
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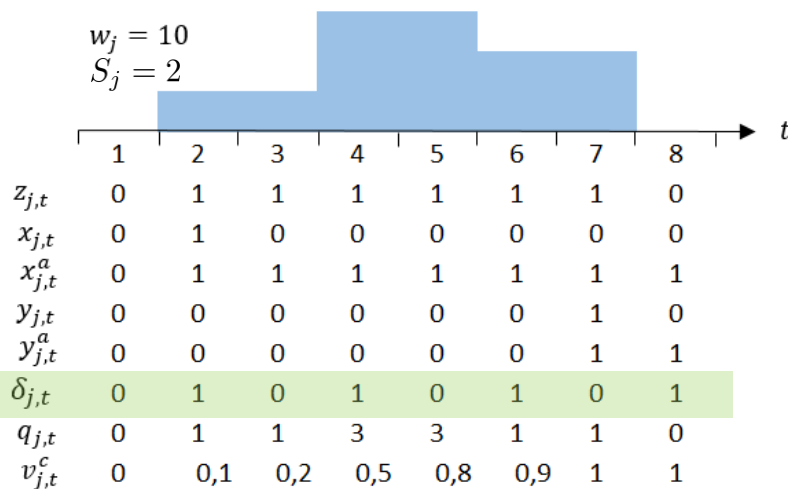
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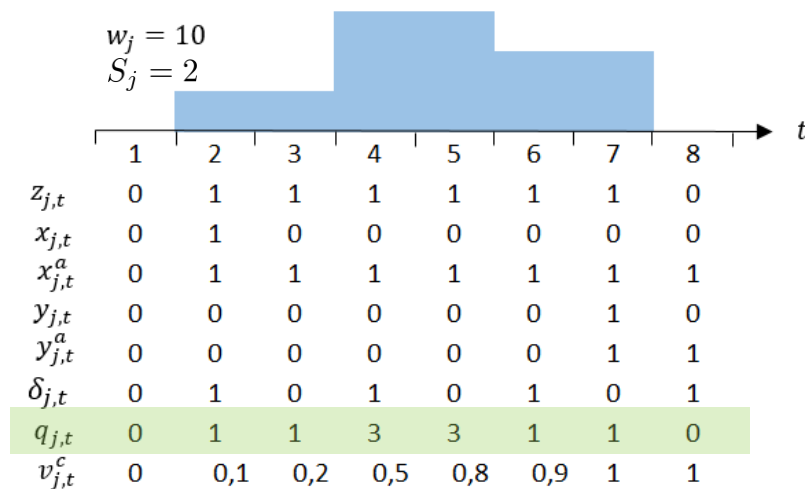
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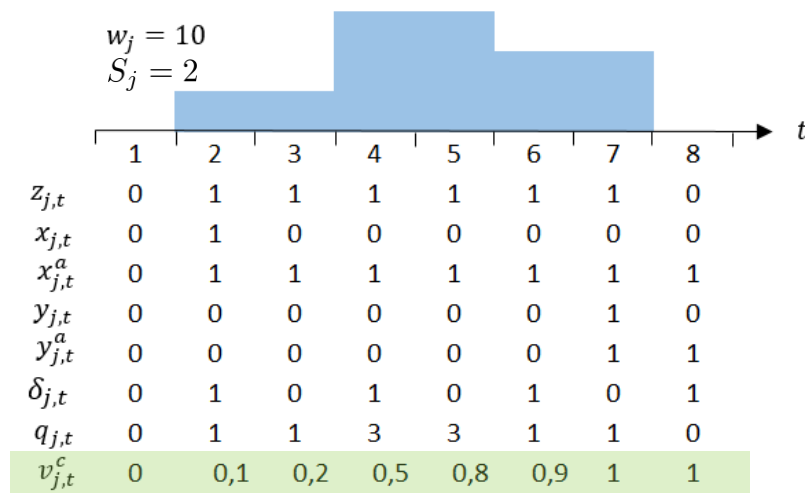
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Model FP-DT	Based on	Binary variables						Continuous variables			
		x_{jt}	x_{jt}^a	y_{jt}	y_{jt}^a	z_{jt}	δ_{rjt}	v_{jt}^c	s_j	q_{rjt}	C_{max}
1	Formulation 2 of Klein (2000)					•	•			•	•
2	Ranjbar and Kianfar (2010)	•		•			•			•	•
3	Bianco and Caramia (2013) and Kis (2005)		•		•		•	•		•	•
4	Sabzehparvar and Seyed-Hosseini (2008)					•	•		•	•	•

Source: Naber and Kolisch (2014)

FP-DT1:

Based on Klein (2000), same binary variables in Baumann and Trautmann (2013).

On/Off variable $z_{j,t}$ (1, if j is processed in t).

FP-DT2

Based on Ranjbar and Kianfar (2010).

Pulse variables $x_{j,t}$ (1, if j starts at the beginning of t) and $y_{j,t}$ (1, if j is completed by the end of period t).

FP-DT3

Based on Bianco and Caramia (2013) and Kis (2005).

Step variables $x_{j,t}^a$ (1, if j starts at the beginning of t or before) and

$y_{j,t}^a$ (1, if j is completed by the end of period $t - 1$ or before) and continuous variable $v_{j,t}^c$ (cumulated intensity)

FP-DT4

Based on Sabzehparvar and Seyed-Hosseini (2008)

On/Off variable $z_{j,t}$ (1, if j is processed in t) and continuous start variable s_j

Comparison of MIP-Formulations: Model Sizes

Bechnmark instances of Fündeling and Trautmann (2010)

Model FP-DT	1	2	3	4
<i>2–10 Activities with 1 resource (46.3%), 3 resources (0.5%), and 4 resources (53.2%) in total 551 instances</i>				
Binary variables	403	622	710	575
Continuous variables	216	424	588	435
Rows	1417	2837	3411	2913
Nonzero A's (%)	0.475%	0.786%	0.204%	0.394%
<i>11–20 Activities with 1 resource (35%) and 4 resources (65%) in total 569 instances</i>				
Binary variables	1770	2039	2217	1771
Continuous variables	1294	1294	1796	1314
Rows	8161	8813	10,392	8912
Nonzero A's (%)	0.141%	0.452%	0.069%	0.162%
<i>21–53 Activities with 1 resource (27.3%) and 4 resources (72.7%) in total 553 instances</i>				
Binary variables	6165	7358	7683	6166
Continuous variables	4543	4543	6180	4582
Rows	28,878	31,410	35,790	31,070
Nonzero A's (%)	0.058%	0.251%	0.021%	0.065%

Source: Naber and Kolisch (2014)

MIP-Formulations: Performance

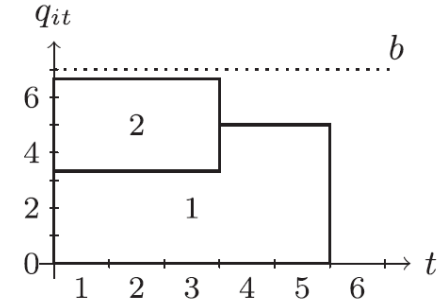
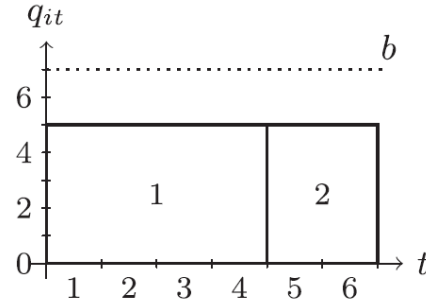
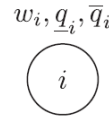
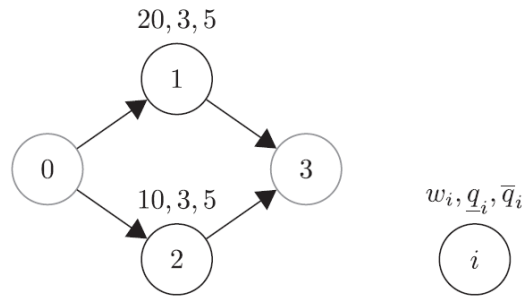
Model FP-DT	1 (%)	2 (%)	3 (%)	4 (%)
<i>2–10 Activities with 1 resource (46.3%), 3 resources (0.5%), and 4 resources (53.7%) in total of 551 instances</i>				
Instances attaining optimality	98.73	97.82	100.00	97.64
Instances with good solutions ($\leq 5\%$)	99.46	99.64	100.00	99.64
Instances with feasible solutions	100.00	100.00	100.00	100.00
Avg. gap of best solution – best known	0.06	0.04	0.00	0.09
Avg. gap of LP – best known (9.15%)	2.92	2.91	1.30	3.67
Instances with LP = best known	65.15	65.34	79.31	59.89
<i>11–20 Activities with 1 resource (35%) and 4 resources (65%) in total of 569 instances</i>				
Instances attaining optimality	86.12	80.84	92.79	81.72
Instances with good solutions ($\leq 5\%$)	89.46	84.71	99.82	85.59
Instances with feasible solutions	99.47	93.32	100.00	94.02
Avg. gap of best solution – best known	1.35	2.26	0.02	2.00
Avg. gap of LP – best known (8.83%)	3.39	3.33	2.38	3.93
Instances with LP = best known	57.29	58.00	64.85	51.14
<i>21–53 Activities with 1 resource (27.3%) and 4 resources (72.7%) in total of 553 instances</i>				
Instances attaining optimality	65.34	55.96	71.25	59.03
Instances with good solutions ($\leq 5\%$)	70.52	62.39	86.62	64.56
Instances with feasible solutions	74.73	66.06	86.98	68.77
Avg. gap of best solution – best known ¹	1.40	2.47	0.08	2.71
Avg. gap of LP – best known ¹ (5.68%)	4.12	4.11	3.56	4.82
Instances with LP = best known ¹	60.87	60.25	67.49	44.72
<i>All instances in total of 1673 instances</i>				
Instances attaining optimality	83.40	78.21	88.05	79.46
Instances with good solutions ($\leq 5\%$)	86.49	82.25	95.52	83.26
Instances with feasible solutions	91.47	86.51	95.70	87.65
Avg. gap of best solution – best known ²	0.92	1.56	0.03	1.56
Avg. gap of LP – best known ² (7.99%)	3.45	3.42	2.36	4.11
Instances with LP = best known ²	61.07	61.20	70.62	52.21

Source: Naber and Kolisch (2014)

Hybrid Metaheuristic of Tritschler et al. (2017)

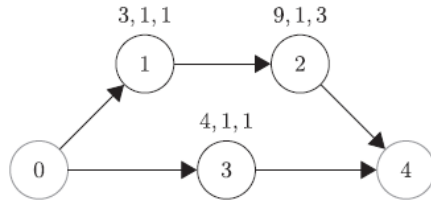
- FRCPSP-adapted parallel schedule generation scheme of Kolisch (1996)
- FRCPSP-adapted GA of Hartmann (1998)
- VNS-based transfer of resources

Example: Greedy Resource Assignment May Not Be Optimal

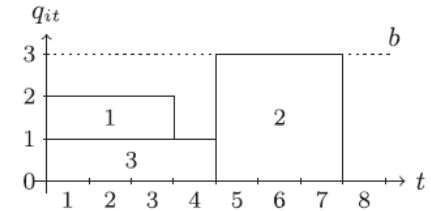
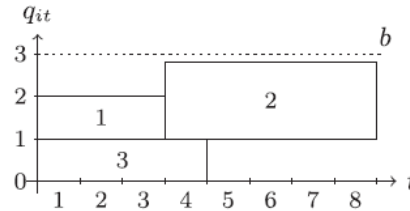
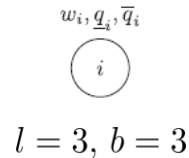


Source: Tritschler et al. (2017)

Example: Early Start May Not Be Optimal



Source: Tritschler et al. (2017)



Solution Encoding of the GA

- Activity list $\lambda = (j_1, j_2, \dots, j_n)$
- Resource allocation limit list $\rho = (\rho_1, \rho_2, \dots, \rho_n)$ with $\rho_i \in \{0, \dots, \bar{\rho}_i\}$

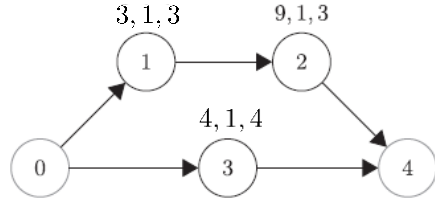
$$\bar{\rho}_i = \min(\bar{d}_i - \underline{d}_i, LF_i - ES_i)$$

$$q_{i,r,t} = \min\left(\bar{b}_{r,t}, \frac{w_{i,r}}{\underline{d}_i + \rho_i}\right)$$

$$\bar{b}_{r,t} = \text{Left-over capacity in period } t$$

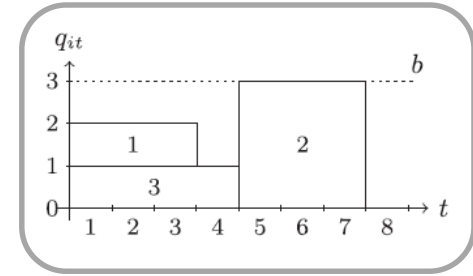
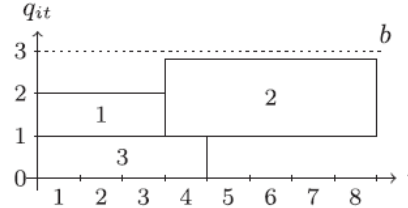
$$\underline{d}_i = \text{Min duration of activity } i$$
- Start delay list $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_n)$ with $\sigma_i \in \{0, 1, \dots, LS_i - ES_i\}$

Mapping of (λ, ρ, σ) Into a Solution: Adapted SGS



Source: Tritschler et al. (2017)

$l = 3, b = 3$



$\lambda = (1, 3, 2)$

$\rho = (2, 0, 3)$

$\sigma = (0, 1, 0)$

Period $t = 1$:

Select activity 1, $S_1 = ES'_1 + \sigma_1 = 1 + 0 = 1$, $q_{1,1} = \min(\bar{b}_1, \frac{w_1}{\underline{d}_1 + \rho_1}) = \min(3, \frac{3}{1+2}) = 1$

Select activity 3, $S_3 = ES'_3 + \sigma_3 = 1 + 0 = 1$, $q_{3,1} = \min(\bar{b}_1, \frac{w_3}{\underline{d}_3 + \rho_3}) = \min(2, \frac{4}{1+3}) = 1$

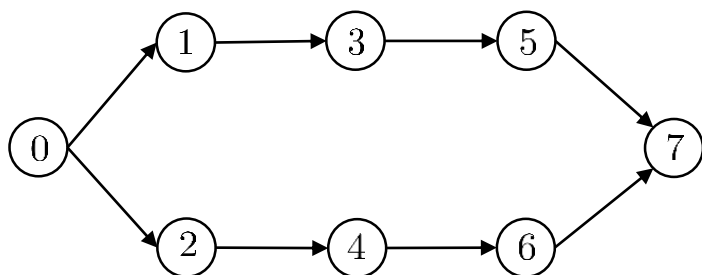
....

Period $t = 4$:

...

Select activity 2, $S_2 = ES'_2 + \sigma_2 = 4 + 1 = 5$, $q_{2,5} = \min(\bar{b}_5, \frac{w_2}{\underline{d}_2 + \rho_2}) = \min(3, \frac{9}{3+0}) = 3$

Adapted Two-Point Crossover



Parents

$$\lambda_1^P = (1, 3, 5, 2, 4, 6)$$

$$\rho_1^P = (1, 2, 4, 3, 1, 2)$$

$$\sigma_1^P = (4, 5, 6, 1, 2, 3)$$

$$\lambda_2^P = (2, 4, 6, 1, 3, 5)$$

$$\rho_2^P = (0, 0, 1, 2, 3, 3)$$

$$\sigma_2^P = (0, 0, 0, 2, 2, 2)$$

Children

$$\lambda_1^C = (1, 3, 2, 4, 5, 6)$$

$$\rho_1^C = (1, 2, 1, 2, 1, 2)$$

$$\sigma_1^C = (4, 5, 0, 2, 2, 3)$$

$$\lambda_2^C = (2, 4, 1, 3, 5, 6)$$

$$\rho_2^C = (0, 0, 4, 3, 1, 2)$$

$$\sigma_2^C = (0, 0, 6, 1, 2, 2)$$

Resource Transfers:

- Determine the set of critical activities (on the critical path) \mathcal{V}^c
- Determine pair of activities (i, j) , $i \in \mathcal{V}^c$, $j \in \mathcal{V} \setminus \mathcal{V}^c$
- For each pair (i, j) determine the max resource flow from j to i
- Update list ρ to ρ' to consider the resource flow
- Obtain the new schedule with the adapted SGS and lists λ , ρ' and σ

Variable Neighborhood Search: Consider the neighborhood of $k = 1, \dots, k^{max}$ resource transfers.

Computational Results

Based on instance sets of
Fündeling and Trautmann (2010)

HM = Hybrid Method (GA + VNS)

SGA = Self adapting SGS based on Hartmann (2002)

PRS = Parallel SGS with random sampling

SRS = Serial SGS with random sampling

Set / Ω	HM		SGA		PRS		SRS	
	Δ_{mip}	Δ_{lb}	Δ_{mip}	Δ_{lb}	Δ_{mip}	Δ_{lb}	Δ_{mip}	Δ_{lb}
A₅₅	0.03	5.60	2.12	7.82	2.64	8.41	3.89	9.83
1000	0.64	6.27	2.52	8.30	3.26	9.09	4.66	10.68
5000	0.01	5.58	2.13	7.83	2.67	8.45	3.95	9.89
15,000	-0.24	5.30	1.95	7.63	2.37	8.10	3.57	9.47
25,000	-0.27	5.25	1.88	7.53	2.26	7.98	3.40	9.28
B₁₀	0.24	5.40	1.31	6.57	1.55	6.81	1.59	6.88
1000	0.44	5.62	1.34	6.61	1.56	6.83	1.62	6.91
5000	0.22	5.38	1.30	6.56	1.55	6.82	1.58	6.87
15,000	0.15	5.31	1.30	6.56	1.55	6.81	1.58	6.87
25,000	0.14	5.28	1.30	6.56	1.55	6.81	1.58	6.87
B₂₀	0.28	4.24	0.75	4.77	1.28	5.34	1.31	5.40
1000	0.60	4.60	0.86	4.90	1.63	5.73	1.70	5.84
5000	0.27	4.23	0.75	4.78	1.31	5.37	1.30	5.39
15,000	0.15	4.10	0.70	4.71	1.12	5.16	1.15	5.23
25,000	0.10	4.04	0.69	4.70	1.07	5.10	1.09	5.16
B₄₀	-1.88	4.08	-1.73	4.29	-0.05	6.21	0.32	6.74
1000	-1.66	4.35	-1.53	4.55	0.49	6.84	0.93	7.44
5000	-1.88	4.09	-1.72	4.31	-0.01	6.26	0.42	6.85
15,000	-1.98	3.96	-1.82	4.18	-0.29	5.94	0.04	6.41
25,000	-2.01	3.93	-1.85	4.13	-0.40	5.81	-0.09	6.26
B₁₀₀		3.94		4.05		7.15		8.09
1000		4.07		4.21		7.68		8.68
5000		3.94		4.08		7.21		8.14
15,000		3.89		3.97		6.92		7.84
25,000		3.87		3.93		6.80		7.71
B₂₀₀		3.41		3.55		7.12		8.20
1000		3.46		3.65		7.53		8.67
5000		3.40		3.57		7.16		8.27
15,000		3.39		3.51		6.95		7.98
25,000		3.39		3.48		6.83		7.88
Overall	-0.33	4.46	0.63	5.20	1.37	6.86	1.81	7.55

Impact of the VNS

Inst% = percentage of instances where the VNS improves the solution

ΔC_{max} = Absolute improvement of the makespan

$\Delta C_{max}\%$ = Improvement of the makespan in percent

Set	Inst %	ΔC_{max}	$\Delta C_{max}\%$
A _{≤55}	5.59	1.06	2.27
B ₁₀	1.56	1.06	2.44
B ₂₀	7.57	1.23	1.39
B ₄₀	20.63	1.34	0.85
B ₁₀₀	54.24	2.12	0.57
B ₂₀₀	61.84	3.84	0.55
Overall	22.91	2.58	1.34

Fündeling and Trautmann (2010)

- Test set A derived from PSPLIB-instances (553 instances)
- Test set B generated with ProGen/max (2.400 instances)

Brachmann et al. (2023)

- Test set with 360 instances derived from real-world ETO production planning problems

Instances of Brachmann et al. (2023)

- Time horizon 1.5 years
- 144 customer orders

Parameter	Values	Levels
No. of projects (No. of activities)	38 (622), 76 (1,219), 114 (1,839)	3
Resource availability	1.7, 1.6, 1.5, 1.4, 1.3, 1.2	5
Due date tightness	1.6, 1.5, 1.4, 1.3, 1.2	4
Resource flexibility	[0.10, 0.70], [0.15, 0.65], [0.20, 0.60], [0.25, 0.55]	4

=> $3 \cdot 5 \cdot 4 \cdot 4 = 360$ instances

- Resource availability: *Scaling Factor* · *Average Resource Capacity in ERP Data*
- Due date tightness: *Project Arrival* + *Scaling Factor* · *Critical Path Duration*
- Resource flexibility: *Scaling Factor* · *Activity Work Content*

We calculate the following instance measures for the generated instances:

Network topology:

- Coefficient of Network Complexity (CNC)
- Order Strength (OS)
- Serial-Parallel-Indicator (SPI)
- Activity Distribution (AD)

Resource scarcity:

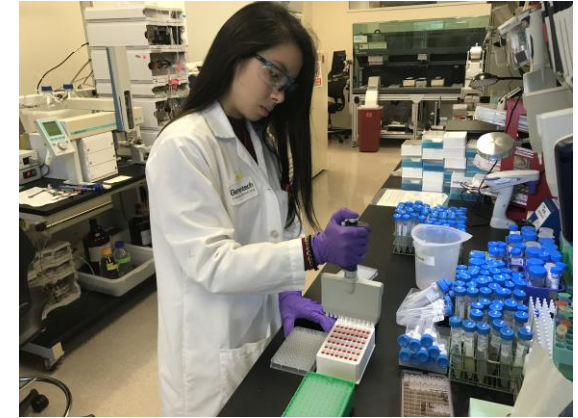
- Resource Strength (RS)
- Normalized Average Resource Loading Factor (NARLF)
- Utilization Factor (UF)
- Variance of Utilization Factor (σ_{UF}^2)
- Resource Sharing Ratio (RSR)

Projects	Activities	CNC	OS	SPI	AD	NARLF	RSR
38	622	1.52	0.0079	0.0064	0.48	-16.93	0.066
76	1,219	1.52	0.0040	0.0033	0.48	9.79	0.079
114	1,839	1.52	0.0027	0.0022	0.48	1.07	0.083

- Kolisch et al. (2003): Scheduling lead optimization in pharmaceutical research
- Braune and Dörner (2017): Scheduling testing of input material for chemical production.
- Kis and Drótos (2017): Scheduling in digital factories.
- Baur and Rieck (2020): Scheduling of disaster response projects.
- Brachmann and Kolisch (2021): Scheduling engineer-to-order (ETO) production planning with working time accounts.
- Nouredine et al. (2023): Scheduling ship overhauls.
- Brachmann et al. (2023): Real-world instance set for ETO production planning.

Lead Optimization in Pharmaceutical Research

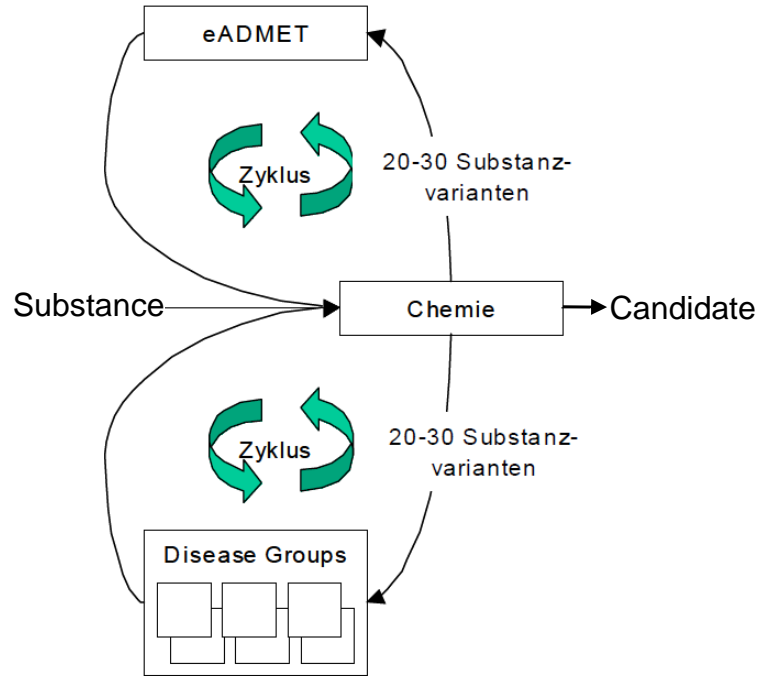
- Kolisch, Meyer, Mohr, Schwindt, Urmann (2003): Ablaufplanung für die Leistrukturoptimierung in der Pharmaforschung, Zeitschrift für Betriebswirtschaft 78(8) 825-848.



Source:
<https://www.genengnews.com/insights/improved-assays-and-predictive-tools-for-drug-lead-optimization/>



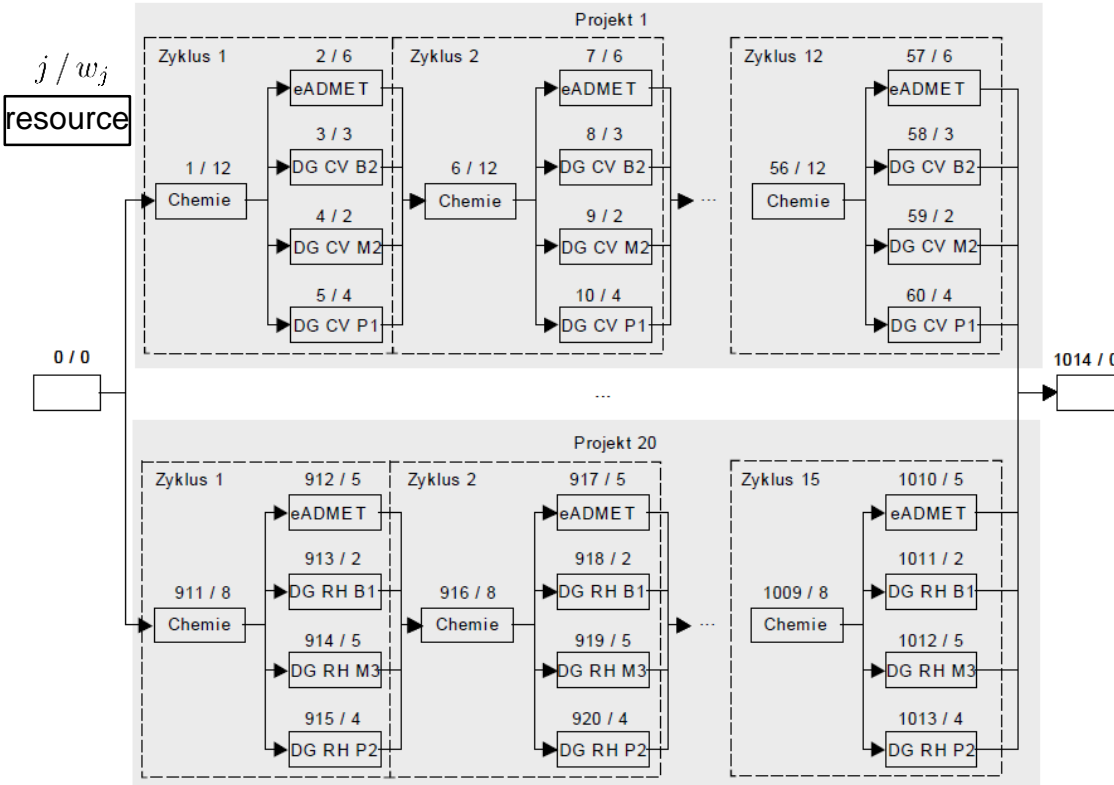
Research Cycle for Lead Optimization



Source: Kolisch et al. (2003)

- Iterative improvement from substance to candidate
- 15 – 20 cycles
- „Chemistry“ → alternation of substance
- „eADMET“ (early Administration, Distribution, Metabolism, Kinetics, Toxicology) → test of tolerability by the human body
- „DG“ (disease groups) → test of effectiveness of the substance

Modeling as FRCMPSP



Source: Kolisch et al. (2003)

- ~ 20 parallel research projects
- ~ 60 activities for each project
- Series-parallel network structure
- Work content w_j in laboratory unit weeks
- One activity requires one specific resource (specific type of laboratory unit)
- One laboratory unit consists of 1 chemical engineer, 2 chemical lab assistants and one laboratory including research equipment
- 27 DG resources with capacity [4,7]
- 1 eADMET resource with capacity 14
- 1 chemistry resource with capacity 50
- Periods are weeks
- Objective: Min makespan

Engineering-to-Order Production Planning

Brachmann, R. and Kolisch, R. (2021): The impact of flexibility on engineer-to-order production planning, International, Journal of Production Economics, Vol. 239, 108183.

Weber, H., Brachmann, R. and Kolisch R. (2023): Generation and characterization of real-world instances for the flexible resource-constrained multi-project scheduling problem, Working Paper.

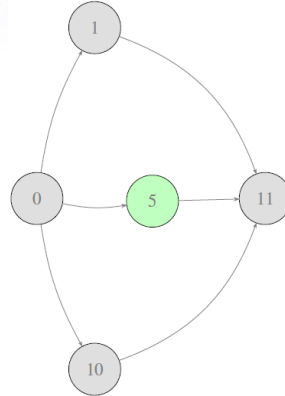
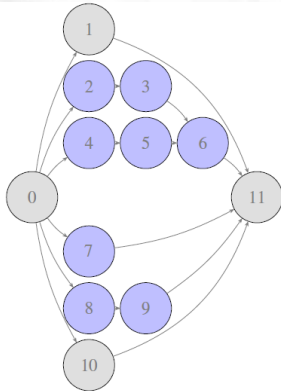


Source: <https://www.uhlmann.de/news-media/newsroom/news-detail/sustainability-from-nice-to-haves-to-must-haves-with-innovative-solutions-from-uhlmann-pac-systeme-339/>

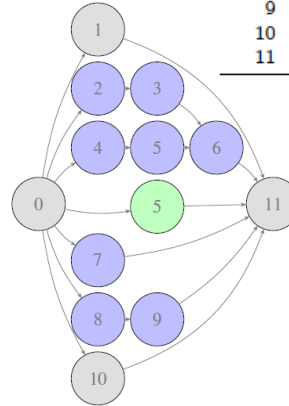
Single Project

Module 1 (M1)

Module 2 (M2)



M1 U M2

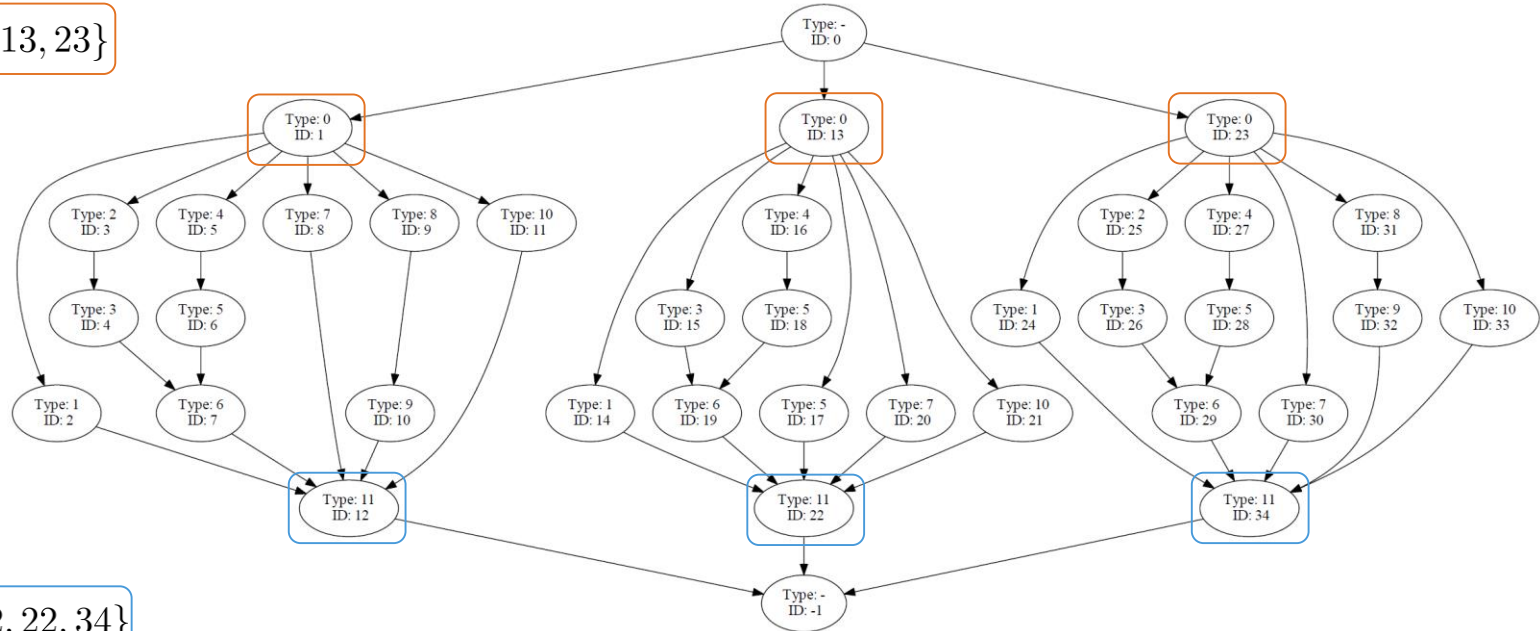


Activity		Resource	
Type	Description	Number	Description
0	Planning	1	Project office
1	Management	1	Project office
2	Mechanical design	2	Mechanical engineering
3	Mechanical pre-assembly	3	Mechanical assembly
4	Electrical engineering hardware	4	Electrical engineering
5	Electrical pre-assembly	5	Electrical assembly
6	Machine assembly	6	Machine assembly
7	Electrical software engineering	7	Software engineering
8	Tools engineering	8	Tools engineering
9	Tools assembly	9	Tools assembly
10	Layout design	10	Layout design
11	Synchronization	11	Synchronization

- Period length week
- Work content in [??]
- Planning horizon 1.5 years

Multi-Project Network

$$\mathcal{V}^S = \{1, 13, 23\}$$



$$\mathcal{V}^F = \{12, 22, 34\}$$

$$\text{Min} \sum_{j \in \mathcal{V}^F} w_j (\alpha \cdot E_j + \beta \cdot L_j)$$

Period length week

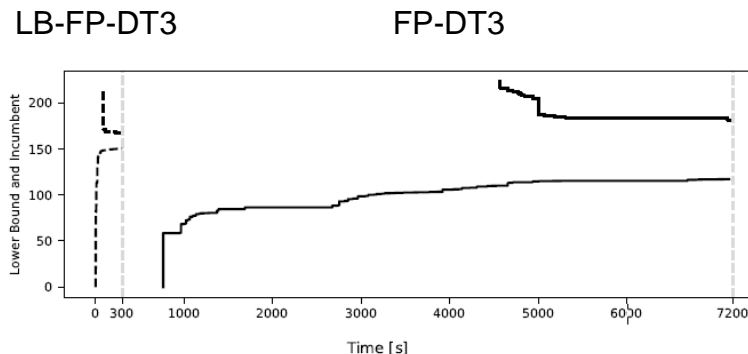
Solution Approach

- MIP-Formulation based on FP-DT3.
- Use of the Lower Bound as detailed below.

Lower Bound

- MIP-Formulation based on FP-DT3.
- Observation: For each project, resource-constraints of activities have a block-angular structure.
- Determine processing time window for finish activities \mathcal{V}^F
- Delete all activities from the MIP $\mathcal{V} \setminus \mathcal{V}^F$ and solve it to optimality.

Number of	FP-DT3	LB-FP-DT3
Continuous variables	21,938	1,256
Integer variables	53,143	2,368
Binary variables	52,991	2,216



Computational Results

- Python-Gurobi implementation
- i7-8700K with 3.7GHz, 32 GigaByte RAM
- 2 hours computation time

MIP

MIP+LB

Size	Measure	Resource flexibility				
		All	Level 1 [0.1, 0.7]	Level 2 [0.15, 0.65]	Level 3 [0.2, 0.6]	Level 4 [0.25, 0.55]
624	Optimality (%)	92.50	100.00	100.00	83.33	86.67
	Solved (%)	100.00	100.00	100.00	100.00	100.00
	Objective value	4.35	1.57	2.57	4.73	8.53
	Best bound	4.22	1.57	2.57	4.53	8.23
	Gap (%)	1.47	0.00	0.00	2.29	3.57
	Improved optimality (%)	98.33	100.00	100.00	93.33	100.00
	Improved best bound	4.33	1.57	2.57	4.67	8.53
	Improved gap (%)	0.21	0.00	0.00	0.83	0.00
	CPU time FRCMPSP	716.39	21.36	230.04	1361.37	1252.78
	CPU time FRCMPSP-Rel	6.74	0.00	0.00	23.97	3.00
1221	Optimality (%)	68.33	86.67	76.67	60.00	50.00
	Solved (%)	100.00	100.00	100.00	100.00	100.00
	Objective value	27.93	20.17	24.00	29.90	37.63
	Best bound	22.52	16.67	19.77	23.90	29.77
	Gap (%)	6.61	3.25	4.34	8.24	10.58
	Improved optimality (%)	70.83	86.67	80.00	63.33	53.33
	Improved best bound	26.08	19.33	22.97	27.83	34.20
	Improved gap (%)	2.10	0.76	1.04	2.60	4.00
	CPU time FRCMPSP	2635.11	1469.43	2079.91	3213.30	3777.82
	CPU time FRCMPSP-Rel	86.42	40.02	48.63	111.45	145.56
1841	Optimality (%)	31.67	40.00	40.00	33.33	13.33
	Solved (%)	87.50	93.33	90.00	90.00	76.67
	Objective value	69.60	64.25	65.37	81.00	67.70
	Best bound	62.20	48.70	57.67	64.63	77.80
	Gap (%)	14.77	15.05	12.04	16.47	15.64
	Improved optimality (%)	35.83	46.67	40.00	36.67	20.00
	Improved best bound	73.90	63.57	68.77	80.13	83.13
	Improved gap (%)	7.17	7.46	5.54	6.70	9.26
	CPU time FRCMPSP	5492.80	4850.44	5281.24	5393.97	6445.54
	CPU time FRCMPSP-Rel	122.02	83.25	105.12	143.20	156.51

Research Opportunities

- Improving MIP-formulations
- Improving metaheuristics
- New applications

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- [1] P. Baumann, C.-U. Fündeling, and N. Trautmann. The resource-constrained project scheduling problem with work-content constraints. In C. Schwindt and J. Zimmermann, editors, *Handbook on Project Management and Scheduling Vol. 1*, chapter 24, pages 533–544. Springer, 2015.
- [2] P. Baumann and N. Trautmann. Optimal scheduling of work-content constrained projects. In T. Laosirihongthong, R. Jiao, M. Xie, and Sirovetnukul R., editors, *Proceedings of the IEEE international Conference on Industrial Engineering and Engineering Management*, pages 395–399. Bangkok, 2013.
- [3] N.-F. Baur and J. Rieck. Project management with scarce resources in disaster response. In J.S. Neufeld, U. Buscher, R. Lasch, D. Möst, and J. Schönberger, editors, *Operations Research Proceedings 2019*, pages 607–614. Springer, 2020.
- [4] L. Bianco and M. Caramia. A new formulation for the project scheduling problem under limited resources. *Flexible Services and Manufacturing Journal*, 25(1–2):6–24, 2013.
- [5] R. Brachmann and R. Kolisch. The impact of flexibility on engineer-to-order production planning. *International Journal of Production Economics*, 239:108183, 2021.
- [6] R. Brahmann, H. Weber, and R. Kolisch. Generation and characterization of real-world instances for the flexible resource-constrained multi-project scheduling problem. Technical report, Technical University of Munich, 2023.
- [7] R. Braune and K.F. Doerner. Real-world flexible resource profile scheduling with multiple criteria: learning scalarization functions for MIP and heuristic approaches. *Journal of the Operational Society*, 68:952?972.
- [8] C.-U. Fündeling and N. Trautmann. A priority-rule method for project scheduling with work-content constraints. *European Journal of Operational Research*, 203:568–574, 2010.

- [9] S. Hartmann. A competitive genetic algorithm for resource-constrained project scheduling. *Naval Research Logistics*, 45:733–750, 1998.
- [10] S. Hartmann. A self-adapting genetic algorithm for project scheduling under resource constraints. *Naval Research Logistics*, 49:433–448, 2002.
- [11] T. Kis. A branch-and-cut algorithm for scheduling of projects with variable-intensity activities. *Mathematical Programming*, 103:515–539.
- [12] T. Kis and M. Drotos. Hard planning and scheduling problems in the digital factory. In L. Ghezzi, P. Hömberg, and C. Landry, editors, *Math for the Digital Factory*, pages 3–19. Springer, 2017.
- [13] R. Kolisch. Serial and parallel resource-constrained project scheduling methods revisited: Theory and computation. *European Journal of Operational Research*, 90:320–333, 1996.
- [14] R. Kolisch and S. Hartmann. Heuristic algorithms for solving the resource-constrained project scheduling problem - classification and computational analysis. In J. Weglarz, editor, *Project Scheduling Recent Models, Algorithms and Applications*, pages 147–178, Boston. Kluwer.
- [15] R. Kolisch, K. Meyer, R. Mohr, C. Schwindt, and M. Urmann. Ablaufplanung für die Leitstrukturoptimierung in der Pharmaforschung. *Zeitschrift für Betriebswirtschaft*, 73(8):825–848, 2003.
- [16] A. Naber. Resource-constrained project scheduling with flexible resource profiles in continuous time. *Computers & Operations Research*, 84:33–45, 2017.
- [17] A. Naber and R. Kolisch. MIP models for resource-constrained project scheduling with flexible resource profiles. *European Journal of Operational Research*, 239(2):335 – 348, 2014.
- [18] A. Nouredine, F. Soumis, and R. Pellerin. Strategies for improving ship overhaul project planning. Technical report, 2023. SSRN-paper.

- [19] M. Ranjbar and F. Kianfar. Resource-constrained project scheduling problem with flexible work profiles: A genetic algorithm approach. *Transaction E: Industrial Engineering*, 17:25–35, 2010.
- [20] M. Sabzehparvar and S.M. Seyed-Hosseini. A mathematical model for the multi-mode resource-constrained project scheduling problem with mode dependent time lags. *The Journal of Supercomputing*, 44:257–273.
- [21] M. Tritschler, A. Naber, and R. Kolisch. A hybrid metaheuristic for resource-constrained project scheduling with flexible resource profiles. *European Journal of Operational Research*, 262(1):262 – 273, 2017.
- [22] A. Zimmermann. A mixed-integer programming-based heuristic for project scheduling with work-content constraints. *European Journal of Industrial Engineering*, 11:752–773, 2017.