

ROB 530 Project Notes

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1 Prediction Step

$$\mathbf{x}_k = \begin{bmatrix} a_k \\ q_k \\ \omega_k \\ \theta_k \\ \dot{\theta}_k \end{bmatrix} \in \mathbb{R}^{10+2N}$$

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{w}_k$$

$$= \begin{bmatrix} e^{-\tau \Delta t} a_{k-1} \\ \exp\left(-\frac{1}{2} \Psi(\omega_{k-1}) \Delta t\right) q_{k-1} \\ \omega_{k-1} \\ \theta_{k-1} + \dot{\theta}_{k-1} \Delta t \\ (1 - \lambda) \dot{\theta}_{k-1} + \lambda \mathbf{u}_k \end{bmatrix}$$

$$q_k = \exp\left(-\frac{1}{2} \Psi(\omega_{k-1}) \Delta t\right) q_{k-1}$$

$$= \left(I \cos\left(\frac{\|\omega_{k-1} \Delta t\|}{2}\right) - \frac{1}{2} \begin{bmatrix} 0 & -\omega^x \Delta t & \omega^y \Delta t & \omega^z \Delta t \\ -\omega^x \Delta t & 0 & -\omega^z \Delta t & -\omega^y \Delta t \\ -\omega^y \Delta t & \omega^z \Delta t & 0 & -\omega^x \Delta t \\ -\omega^z \Delta t & -\omega^y \Delta t & \omega^x \Delta t & 0 \end{bmatrix} \frac{\sin(\|\omega_{k-1} \Delta t\|)}{\|\omega_{k-1} \Delta t\|} \right) q_{k-1}$$

$$F_k = \frac{\partial f}{\partial \mathbf{x}_{k-1}} = \begin{bmatrix} e^{-\tau \Delta t} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial q_k}{\partial q_{k-1}} & \frac{\partial q_k}{\partial \omega_{k-1}} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial q_k}{\partial q_{k-1}} =$$

$$\frac{\partial q_k}{\partial \omega_{k-1}} =$$

2 Update Step

$$\begin{aligned}\mathbf{z}_k &= \begin{bmatrix} \phi_k \\ \alpha_k \\ \gamma_k \end{bmatrix} \in \mathbb{R}^{7N} \\ \hat{\mathbf{z}}_k &= h(\mathbf{x}_k) \\ &= \begin{bmatrix} \theta_k \\ W_k^1 R_k g + \hat{a}_{\text{motion}}^1 \\ \vdots \\ W_k^N R_k g + \hat{a}_{\text{motion}}^N \\ \bar{\omega}^1 + (W_k^1)^\top \omega_k \\ \vdots \\ \bar{\omega}^N + (W_k^N)^\top \omega_k \end{bmatrix}\end{aligned}$$