ROB 530 Project Notes

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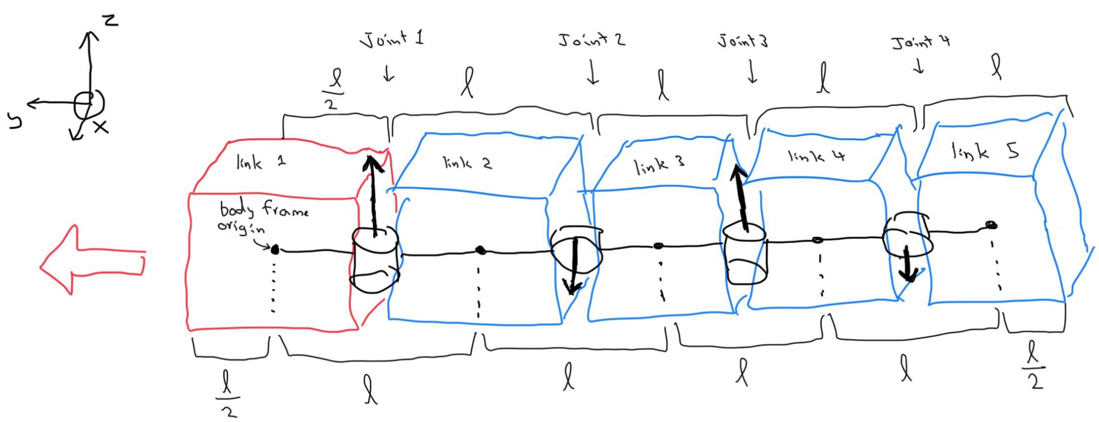

Figure 1: Example diagram of 4 joint snake robot.

1 Forward Kinematics

$$
\begin{aligned}
g_{st}(\theta) &= e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} \cdots e^{\hat{\xi}_N\theta_N} g_{st}(0) \\
g_{si}(\theta) &= e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} \cdots e^{\hat{\xi}_{i-1}\theta_{i-1}} g_{si}(0) \\
\xi_i &= \begin{bmatrix} -\omega_i \times q_i \\ \omega_i \end{bmatrix} \\
\omega_i &= \begin{cases} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^\top & i \text{ is even} \\ \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^\top & i \text{ is odd} \end{cases} \\
q_i &= \begin{bmatrix} 0 \\ \frac{l}{2} + \sum_{j=1}^{i-1} l \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ l(i - \frac{1}{2}) \\ 0 \end{bmatrix} \\
\xi_i &= \begin{cases} \begin{bmatrix} 0 & 0 & -l(i-\frac{1}{2}) & 1 & 0 & 0 \end{bmatrix}^\top & i \text{ is even} \\ \begin{bmatrix} l(i-\frac{1}{2}) & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^\top & i \text{ is odd} \end{cases} \\
g_{si}(0) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l(i-1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

2 Prediction Step

$$
\begin{aligned}
\mathbf{x}_k &= \begin{bmatrix} a_k \\ q_k \\ \omega_k \end{bmatrix} \in \mathbb{R}^{10} \\
\mathbf{x}_k &= f(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{w}_k \\
&= \begin{bmatrix} e^{-\tau \Delta t} a_{k-1} \\ \exp\left(-\frac{1}{2}\Psi(\omega_{k-1})\Delta t\right) q_{k-1} \\ \omega_{k-1} \end{bmatrix} \\
q_k &= \exp\left(-\frac{1}{2}\Psi(\omega_{k-1})\Delta t\right) q_{k-1} \\
&= \left(I \cos\left(\frac{\|\omega_{k-1}\Delta t\|}{2}\right) - \frac{1}{2} \begin{bmatrix} 0 & -\omega^x \Delta t & \omega^y \Delta t & \omega^z \Delta t \\ -\omega^x \Delta t & 0 & -\omega^z \Delta t & -\omega^y \Delta t \\ -\omega^y \Delta t & \omega^z \Delta t & 0 & -\omega^x \Delta t \\ -\omega^z \Delta t & -\omega^y \Delta t & \omega^x \Delta t & 0 \end{bmatrix} \frac{\sin\left(\|\omega_{k-1}\Delta t\|\right)}{\|\omega_{k-1}\Delta t\|} \right) q_{k-1}
\end{aligned}
$$

$$F_k = \frac{\partial f}{\partial \mathbf{x}_{k-1}} = \begin{bmatrix} Ie^{-\tau\Delta t} & 0 & 0 \\ 0 & \frac{\partial q_k}{\partial q_{k-1}} & \frac{\partial q_k}{\partial \omega_{k-1}} \\ 0 & 0 & I \end{bmatrix}$$

$$\frac{\partial q_k}{\partial q_{k-1}} = TODO$$

$$\frac{\partial q_k}{\partial \omega_{k-1}} = TODO$$

3 Update Step

$$\mathbf{z}_k = \begin{bmatrix} \alpha_k \\ \gamma_k \end{bmatrix} \in \mathbb{R}^{6N}$$

$$\hat{\mathbf{z}}_k = h(\mathbf{x}_k)$$

$$= \begin{bmatrix} W_k^1 R_k(g + a_k) + \hat{a}^1_{\text{internal}} \\ \vdots \\ W_k^N R_k(g + a_k) + \hat{a}^N_{\text{internal}} \\ (W_k^1)^\top \omega_k + \hat{\omega}^1_{\text{internal}} \\ \vdots \\ (W_k^N)^\top \omega_k + \hat{\omega}^N_{\text{internal}} \end{bmatrix}$$

$$g_{si}(\theta_k) = \begin{bmatrix} R_k^i & p_k^i \\ 0 & 1 \end{bmatrix}$$

$$W_k^i = C_k R_k^i(\theta_k)$$

$$\hat{\omega}^i_{\text{internal}} = \left(\dot{W}_k^i (W_k^i)^\top\right)^\vee \approx \frac{\operatorname{Log}\left(W_k^i (W_{k-1}^i)^\top\right)}{\Delta t} \approx \left(\frac{W_k^i (W_{k-1}^i)^\top}{\Delta t} - I\right)^\vee$$

$$\hat{a}^i_{\text{internal}} = \ddot{p}_k^i$$

$$H_k = \frac{\partial h}{\partial \mathbf{x}_k} = \begin{bmatrix} \frac{\partial \hat{\alpha}_k^1}{\partial a_k} & \frac{\partial \hat{\alpha}_k^1}{\partial q_k} & \frac{\partial \hat{\alpha}_k^1}{\partial \omega_k} \\ \vdots & \vdots & \vdots \\ \frac{\partial \hat{\alpha}_k^N}{\partial a_k} & \frac{\partial \hat{\alpha}_k^N}{\partial q_k} & \frac{\partial \hat{\alpha}_k^N}{\partial \omega_k} \\ \frac{\partial \hat{\gamma}_k^1}{\partial a_k} & \frac{\partial \hat{\gamma}_k^1}{\partial q_k} & \frac{\partial \hat{\gamma}_k^1}{\partial \omega_k} \\ \vdots & \vdots & \vdots \\ \frac{\partial \hat{\gamma}_k^N}{\partial a_k} & \frac{\partial \hat{\gamma}_k^N}{\partial q_k} & \frac{\partial \hat{\gamma}_k^N}{\partial \omega_k} \end{bmatrix} = \begin{bmatrix} \frac{\partial \hat{\alpha}_k^1}{\partial a_k} & \frac{\partial \hat{\alpha}_k^1}{\partial q_k} & 0 \\ \vdots & \vdots & \vdots \\ \frac{\partial \hat{\alpha}_k^N}{\partial a_k} & \frac{\partial \hat{\alpha}_k^N}{\partial q_k} & 0 \\ 0 & 0 & \frac{\partial \hat{\gamma}_k^1}{\partial \omega_k} \\ \vdots & \vdots & \vdots \\ 0 & 0 & \frac{\partial \hat{\gamma}_k^N}{\partial \omega_k} \end{bmatrix}$$