ROB 530 Project Notes

rlybrdgs

April 14, 2024

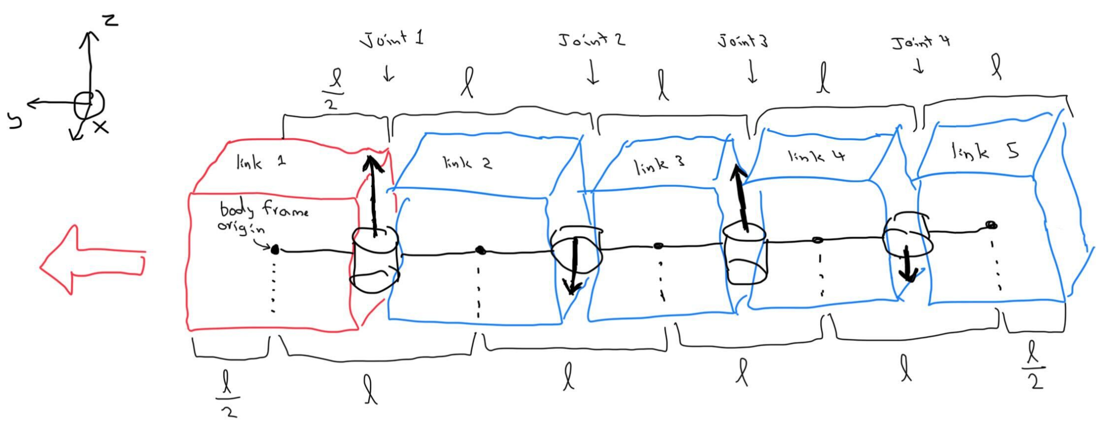

Figure 1: Example diagram of 4 joint snake robot.

1 Forward Kinematics

$$\begin{aligned}
g_{st}(\theta) &= e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}\cdots e^{\hat{\xi}_N\theta_N}g_{st}(0) \\
g_{si}(\theta) &= e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}\cdots e^{\hat{\xi}_{i-1}\theta_{i-1}}g_{si}(0) \\
\xi_i &= \begin{bmatrix} -\omega_i \times q_i \\ \omega_i \end{bmatrix} \\
\omega_i &= \begin{cases} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^\top & i \text{ is even} \\ \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^\top & i \text{ is odd} \end{cases} \\
q_i &= \begin{bmatrix} 0 \\ \frac{l}{2} + \sum_{j=1}^{i-1} l \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ l(i - \frac{1}{2}) \\ 0 \end{bmatrix} \\
\xi_i &= \begin{cases} \begin{bmatrix} 0 & 0 & -l(i-\frac{1}{2}) & 1 & 0 & 0 \end{bmatrix}^\top & i \text{ is even} \\ \begin{bmatrix} l(i-\frac{1}{2}) & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^\top & i \text{ is odd} \end{cases} \\
g_{si}(0) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l(i-1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

2 Prediction Step

$$\begin{aligned}
\mathbf{x}_k &= \begin{bmatrix} a_k \\ q_k \\ \omega_k \end{bmatrix} \in \mathbb{R}^{10} \\
\mathbf{x}_k &= f(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{w}_k \\
&= \begin{bmatrix} e^{-\tau\Delta t}a_{k-1} \\ \exp\left(-\frac{1}{2}\Psi(\omega_{k-1})\Delta t\right) q_{k-1} \\ \omega_{k-1} \end{bmatrix} \\
q_k &= \exp\left(-\frac{1}{2}\Psi(\omega_{k-1})\Delta t\right) q_{k-1} \\
&= \left(I\cos\left(\frac{\|\omega_{k-1}\Delta t\|}{2}\right) - \frac{1}{2}\begin{bmatrix} 0 & -\omega^x\Delta t & \omega^y\Delta t & \omega^z\Delta t \\ -\omega^x\Delta t & 0 & -\omega^z\Delta t & -\omega^y\Delta t \\ -\omega^y\Delta t & \omega^z\Delta t & 0 & -\omega^x\Delta t \\ -\omega^z\Delta t & -\omega^y\Delta t & \omega^x\Delta t & 0 \end{bmatrix} \frac{\sin\left(\|\omega_{k-1}\Delta t\|\right)}{\|\omega_{k-1}\Delta t\|} \right) q_{k-1}
\end{aligned}$$

$$F_k = \frac{\partial f}{\partial \mathbf{x}_{k-1}} = \begin{bmatrix} Ie^{-\tau\Delta t} & 0 & 0 \\ 0 & \frac{\partial q_k}{\partial q_{k-1}} & \frac{\partial q_k}{\partial \omega_{k-1}} \\ 0 & 0 & I \end{bmatrix}$$

$$\frac{\partial q_k}{\partial q_{k-1}} = TODO$$

$$\frac{\partial q_k}{\partial \omega_{k-1}} = TODO$$

3 Update Step

Variable Conventions:

- q_k or R_k is rotation from VC frame to world frame at time k
- a_k is acceleration of VC frame in world frame, expressed in the world frame at time k
- ω_k is angular velocity of VC frame in world frame, expressed in the VC frame at time k
- $g_{si}(\theta)$ is transform from link i to snake head frame, given joint angles θ
- C_k is rotation from VC frame to snake head frame at time k
- W_k^i is rotation from link i to VC frame at time k
- α_k is the measured acceleration of link i in the world frame, expressed in the link frame at time k
- γ_k is the measured angular velocity of link i in the world frame, expressed in the link frame at time k

$$g_{si}(\theta_k) = \begin{bmatrix} R_k^i & p_k^i \\ 0 & 1 \end{bmatrix}$$

$$W_k^i = C_k^\top R_k^i$$

$$\mathbf{z}_k = \begin{bmatrix} \alpha_k \\ \gamma_k \end{bmatrix} \in \mathbb{R}^{6(N+1)}$$

$$\hat{\alpha}_k^i = (W_k^i)^\top R_k^\top (g + a_k) + \hat{a}_{\text{internal}}^i$$

$$
\begin{aligned}
\hat{a}^i_{\text{internal}} &= C_k^\top \ddot{p}^i_k \approx C_k^\top \frac{v^i_k - v^i_{k-1}}{\Delta t} \approx C_k^\top \frac{p^i_k - 2p^i_{k-1} + p^i_{k-2}}{\Delta t^2} \\
\hat{\gamma}^i_k &= (W^i_k)^\top \omega_k + \hat{\omega}^i_{\text{internal}} \\
\hat{\omega}^i_{\text{internal}} &= C_k^\top \left(\dot{R}^i_k (R^i_k)^\top \right)^\vee \\
&= \left((W^i_k)^\top \dot{W}^i_k \right)^\vee \approx \frac{\text{Log}\left((W^i_k)^\top W^i_{k-1} \right)}{\Delta t} \approx \left(\frac{(W^i_k)^\top W^i_{k-1}}{\Delta t} - I \right)^\vee
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mathbf{z}}_k &= h(\mathbf{x}_k) \\
&= \begin{bmatrix} (W^1_k)^\top R_k^\top (g + a_k) + \hat{a}^1_{\text{internal}} \\ \vdots \\ (W^{N+1}_k)^\top R_k^\top (g + a_k) + \hat{a}^{N+1}_{\text{internal}} \\ (W^1_k)^\top \omega_k + \hat{\omega}^1_{\text{internal}} \\ \vdots \\ (W^{N+1}_k)^\top \omega_k + \hat{\omega}^{N+1}_{\text{internal}} \end{bmatrix} \\
H_k = \frac{\partial h}{\partial \mathbf{x}_k} &= \begin{bmatrix} \frac{\partial \hat{\alpha}^1_k}{\partial a_k} & \frac{\partial \hat{\alpha}^1_k}{\partial q_k} & \frac{\partial \hat{\alpha}^1_k}{\partial \omega_k} \\ \vdots & \vdots & \vdots \\ \frac{\partial \hat{\alpha}^{N+1}_k}{\partial a_k} & \frac{\partial \hat{\alpha}^{N+1}_k}{\partial q_k} & \frac{\partial \hat{\alpha}^{N+1}_k}{\partial \omega_k} \\ \frac{\partial \hat{\gamma}^1_k}{\partial a_k} & \frac{\partial \hat{\gamma}^1_k}{\partial q_k} & \frac{\partial \hat{\gamma}^1_k}{\partial \omega_k} \\ \vdots & \vdots & \vdots \\ \frac{\partial \hat{\gamma}^{N+1}_k}{\partial a_k} & \frac{\partial \hat{\gamma}^{N+1}_k}{\partial q_k} & \frac{\partial \hat{\gamma}^{N+1}_k}{\partial \omega_k} \end{bmatrix} = \begin{bmatrix} \frac{\partial \hat{\alpha}^1_k}{\partial a_k} & \frac{\partial \hat{\alpha}^1_k}{\partial q_k} & 0 \\ \vdots & \vdots & \vdots \\ \frac{\partial \hat{\alpha}^{N+1}_k}{\partial a_k} & \frac{\partial \hat{\alpha}^{N+1}_k}{\partial q_k} & 0 \\ 0 & 0 & \frac{\partial \hat{\gamma}^1_k}{\partial \omega_k} \\ \vdots & \vdots & \vdots \\ 0 & 0 & \frac{\partial \hat{\gamma}^{N+1}_k}{\partial \omega_k} \end{bmatrix} \\
\frac{\partial \hat{\alpha}^i_k}{\partial a_k} &= \frac{\partial}{\partial a_k} \left((W^i_k)^\top R_k^\top (g + a_k) \right) + \frac{\partial}{\partial a_k} \hat{a}^i_{\text{internal}} \\
&= \frac{\partial}{\partial a_k} \left((W^i_k)^\top R_k^\top g \right) + \frac{\partial}{\partial a_k} \left((W^i_k)^\top R_k^\top a_k \right) + 0 \\
&= 0 + \frac{\partial}{\partial a_k} \left((W^i_k)^\top R_k^\top a_k \right) \\
&= (W^i_k)^\top R_k^\top \\
\frac{\partial \hat{\alpha}^i_k}{\partial q} &= \begin{bmatrix} \frac{\partial \hat{\alpha}^i_k}{\partial q_x} & \frac{\partial \hat{\alpha}^i_k}{\partial q_y} & \frac{\partial \hat{\alpha}^i_k}{\partial q_z} & \frac{\partial \hat{\alpha}^i_k}{\partial q_w} \end{bmatrix} \\
\frac{\partial \hat{\alpha}^i_k}{\partial q_j} &= \frac{\partial}{\partial q_j} \left((W^i_k)^\top R_k^\top (g + a_k) \right) + \frac{\partial}{\partial q_j} \hat{a}^i_{\text{internal}} \\
&= (W^i_k)^\top \frac{\partial}{\partial q_j} \left(R_k^\top (g + a_k) \right) + 0
\end{aligned}
$$

$$
\begin{aligned}
&= (W_k^i)^\top \frac{\partial R_k^\top}{\partial q_j}(g + a_k) = (W_k^i)^\top \left(\frac{\partial R_k}{\partial q_j}\right)^\top (g + a_k) \\
\frac{\partial \hat{\gamma}_k^i}{\partial \omega_k} &= \frac{\partial}{\partial \omega_k} \left((W_k^i)^\top \omega_k\right) + \frac{\partial}{\partial \omega_k} \hat{\omega}_{\text{internal}}^i \\
&= (W_k^i)^\top
\end{aligned}
$$