Optimizing the Performance of Multi-threaded Linear Algebra Libraries, a Task Granularity based Approach

PhD Proposal

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Outline

Objective

Objective

- A compile-time and runtime solution to optimize the performance of a linear algebra library based on a specific application
- These parameters could be: Machine architecture, number of cores to run the program on, the expression to be evaluated: the type of operations, the number of matrices involved, the matrix sizes.

Introduction

Introduction

- AMT(Asynchronous Many-task) model and runtime systems
    - HPX, Charm++, Uintah, Legion
- Linear algebra libraries
    - Scalapack, ATLAS, SPIRAL
- Our motivation:
    - Phylanx

Background

HPX

- HPX is a general purpose C++ runtime system for parallel and distributed applications of any scale.
- HPX is the first open source software runtime system implementing the concepts of the ParalleX execution model, on conventional systems including Linux clusters, Windows, Macintosh, Android, XeonPhi, and the Bluegene/Q.
- Fine-grained Parallelism instead of Heavyweight Threads.

HPX: Execution Model

- SLOW
    - Starvation
    - Latency
    - Overheads
    - Waiting for contention resolution

Blaze C++ Library

Linear Algebra Library based on Smart Expression Templates

- Expression Templates:
    - Creates a parse tree of the expression at compile time and postpone the actual evaluation to when the expression is assigned to a target
- Smart:
    - Creation of intermediate temporaries when needed
    - Integration with highly optimized compute kernels
    - Selecting optimal evaluation method automatically for compound expressions

Blaze: Parallelization

Depending on the operation and the size of operands, the assignment could be parallelized through four different backends

- HPX
- OpenMP
- C++ threads
- Boost

Blaze: Backend Implementation

In the current implementation, the work is equally divided between the cores at compile time.

- Parallel for loop

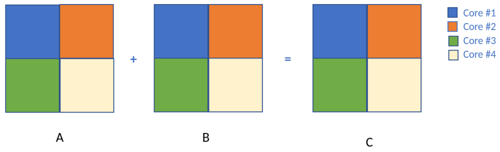


Figure 1: An example of how C=A+B is performed in parallel in Blaze with 4 cores

Loop Scheduling

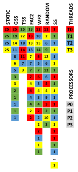


Figure 2: An example of loop chunking[1]

Chunk size: Number of loop iterations executed by one thread

- Static
- Dynamic
- Other methods including Guided, Factoring

[1]Ciorba, Florina M., Christian Iwainsky, and Patrick Buder. "OpenMP loop scheduling revisited: making a case for more schedules." International Workshop on OpenMP. Springer, Cham, 2018.

Task Granularity

Grain size: The amount of work performed by one HPX thread

- What causes performance degradation?
  - Overheads
  - Starvation

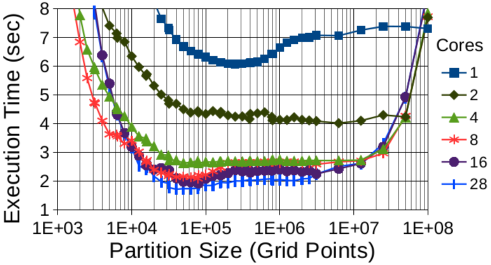


Figure 3: The effect of task size on execution time for Stencil application[2]

[2] Grubel, Patricia, et al. "The performance implication of task size for applications on the hpx runtime system." 2015 IEEE International Conference on Cluster Computing. IEEE, 2015.

Modeling Performance

- Amdahl's Law

$$S(p) = \frac{p}{1 + \sigma(p-1)}$$

- Universal Scalibility Law

$$X(p) = \frac{\gamma p}{1 + \sigma(p-1) + \kappa p(p-1)}$$

  - Models the effects of linear speedup, contention delay, and coherency delay due to crosstalk

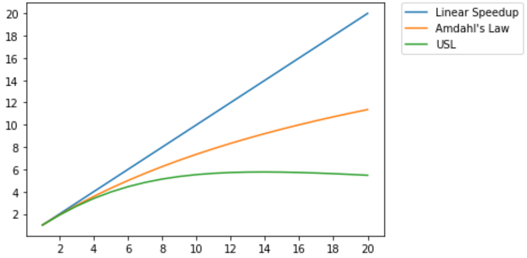


Figure 4: An example of the achievable speedup based on Amdahl's

Modeling Performance: Other Models

- Quadratic model

$$S(p) = p - \gamma p(p-1)$$

- Exponential model

$$S(p) = p(1-\alpha)^{(p-1)}$$

- Geometric model

$$S(p) = \frac{1-\phi^p}{1-\phi}$$

Objective

Dynamically divide the work among the cores based on number of cores, matrix size, complexity of the operation, machine architecture. For this purpose two parameters have been introduced:

- block_size: at each loop iteration the assignment is performed on one block
- chunk_size: the number of loop iterations included in one task

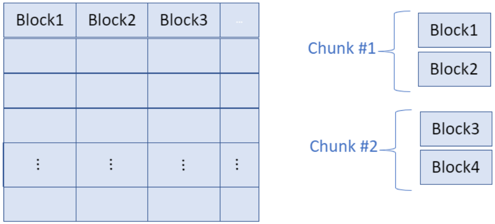


Figure 5: An example of blocking and creating chunks for chunk_size = 2

Method

Data Collection

- Starting from DMATDMATADD benchmark: $C = A + B$

Category	Configuration
Matrix sizes	200, 230, 264, 300, 396, 455, 523, 600, 690, 793, 912, 1048, 1200, 1380, 1587
Number of cores	1, 2, 3, 4, 5, 6, 7, 8
Number of rows in the block	4, 8, 12, 16, 20, 32
Number of columns in the block	64, 128, 256, 512, 1024
Chunk size	Between 1 and total number of blocks (logarithmic increase)

Table 1: List of different values used for each variable for running the *DMATDMATADD* benchmark

Data

- For simplicity we look at each matrix size individually, one number of core at a time,

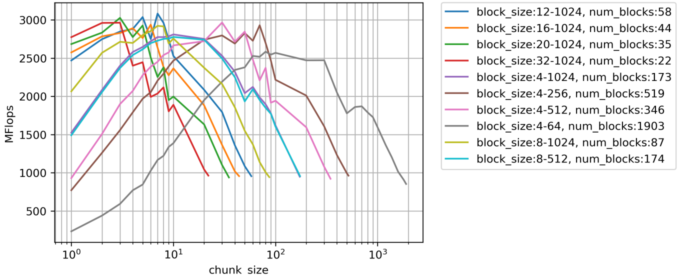
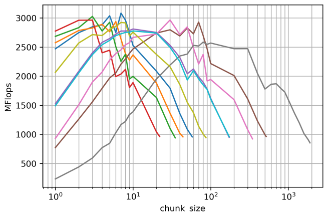


Figure 6: The results obtained from running *DMATDMATADD* benchmark through Blazemark for matrix sizes from 690×690 with different combinations of block size and chunk size on 4 cores

Observation



- For each selected block size, there is a range of chunk sizes that gives us the best performance.
- Except for some uncommon cases, no matter which block size we choose, we are able to achieve the maximum performance if we select the right chunk size.

Throughput vs. Grain Size

Grain size: The amount of work performed by one HPX thread

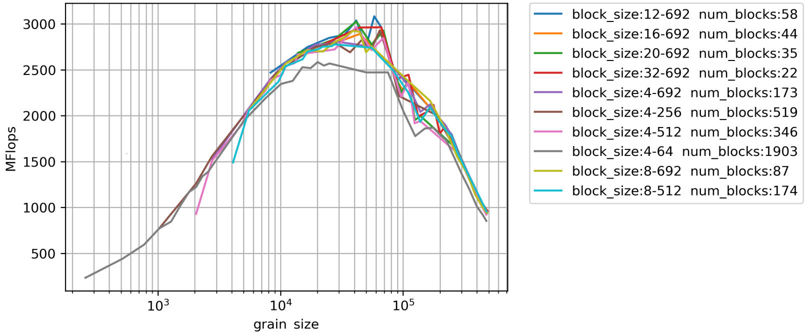


Figure 7: The results obtained from running *DMATDMATADD* benchmark through Blazemark for matrix size 690×690 on 4 cores.

Throughput vs. Grain Siz

Can we model the relationship between the throughput and the grain size?

Modeling

1- Polynomial Model

2- Bathtub Model

Method: Polynomial Model

In order to simplify the process and eliminate the effect of different possible factors, we started with limiting the problem to a fixed matrix size.

- Used a second order polynomial to model the relationship between the throughput and the grain size when number of cores is fixed.

$$P = ag^2 + bg + c$$

- Divide the data into training(60%) and test(40%)

Method: Modeling Performance based on Grain Size

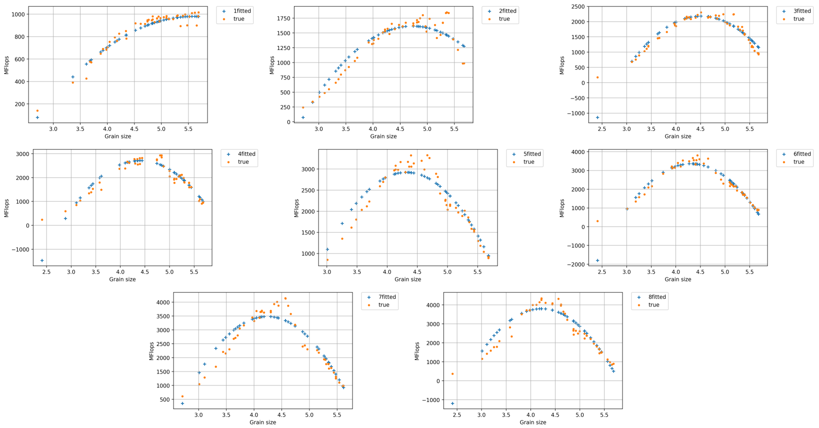


Figure 8: The results of fitting the throughput vs grain size data into a 2d polynomial for *DMATDMATADD* benchmark for matrix size 690×690 with different number of cores on the test data set (a) 1 core, (b) 2 cores, (c) 3 cores, (d) 4 cores, (e) 5 cores, (f) 6 cores, (g) 7 cores, (h) 8 cores.

Method: Modeling Performance based on Grain Size

$$Relative_error = \frac{1}{n}\sum_{i=1}^{n} 1 - p_i/t_i$$

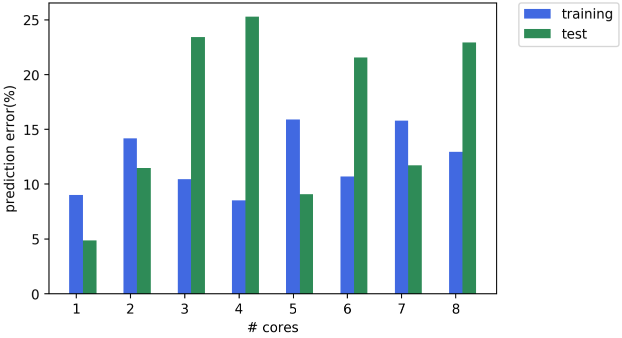


Figure 9: The training and test error for fitting data obtained from the *DMATDMATADD* benchmark for matrix size 690 × 690 against different number of cores cores.

Method: Modeling Performance based on Grain Size

Can we somehow integrate number of cores into the model?

Method: Modeling Performance based on Grain Size

- For $P = ag^2 + bg + c$, see how a, b, and c change with the number of cores
- Model the relationship with a 3rd degree polynomial

$$a = a_0 N^3 + a_1 N^2 + a_2 N + a_3,\ b = b_0 N^3 + b_1 N^2 + b_2 N + b_3,$$

$$c = c_0 N^3 + c_1 N^2 + c_2 N + c_3$$

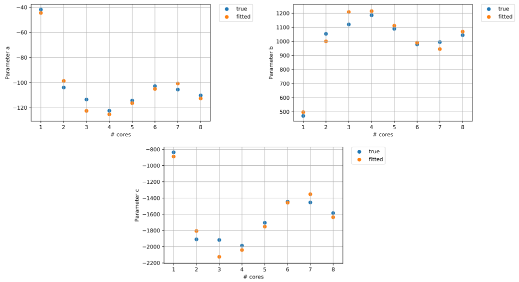


Figure 10: Fitting the parameters of the polynomial function with a 3rd degree polynomial from the *DMATDMATADD* benchmark for matrix size 690×690 against different number of cores.

Method: Modeling Performance based on Grain Size

The final model:

$$P = a_{11}g^2N^3 + a_{10}g^2N^2 + \ldots + a_1N + a_0$$

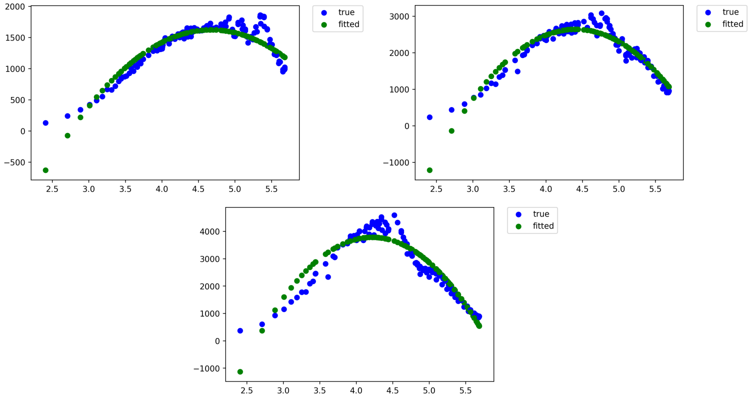


Figure 11: matrix size 690 × 690 for (a) 2 core, (b) 4 cores, (c) 8 cores.

Method: Modeling Performance based on Grain Size

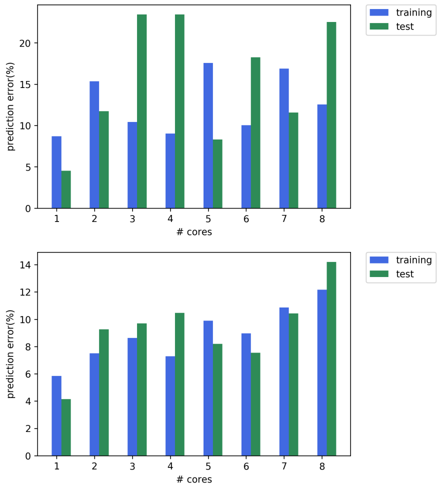
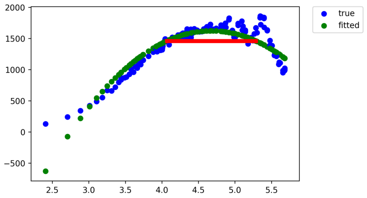
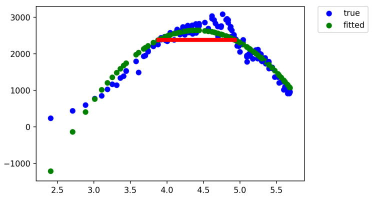


Figure 12: (a) All the data points are include in calculation of error, (b) the leftmost sample was removed from error calculation.

Method: Finding the Grain Size Range for Maximum Performance





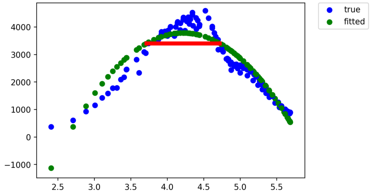


Figure 13: The range of grain size (shown as the red line) that leads to a performance within 10% of the maximum performance for (a) 2 cores, (b) 4 cores and (b) 8 cores.

Method: Finding the Grain Size Range for Maximum Performance

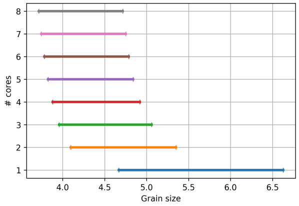


Figure 14: The range of grain size within 10% of the maximum performance of the fitted polynomial function for *DMATDMATADD* benchmark for different number of cores for (a) matrix size 690×690

Method: Finding the Grain Size Range for Maximum Performance

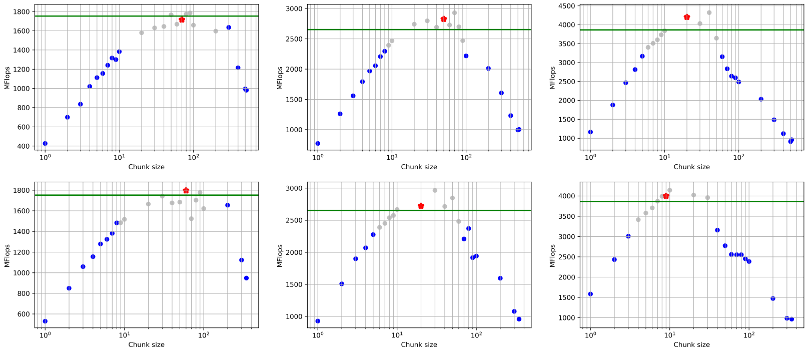


Figure 15: matrix size 690 × 690 with block size of 4 × 256 on (a) 2 cores, (b) 4 cores, and (c) 8 cores, and block size of 4 × 512 on (d) 2 cores, (e) 4 cores, and (f) 8 cores.

Method: Bathtub Model

Creating a analytic model for execution time based on grain size

Method: Bathtub Model

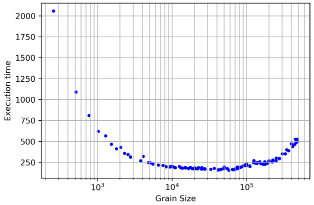


Figure 16: The execution time vs. grain size graph for *DMATDMATADD* benchmark for matrix size 690×690 ran on 4 cores.

Modeling Execution Time based on Grain Size

- Overheads of creating tasks
- Starvation

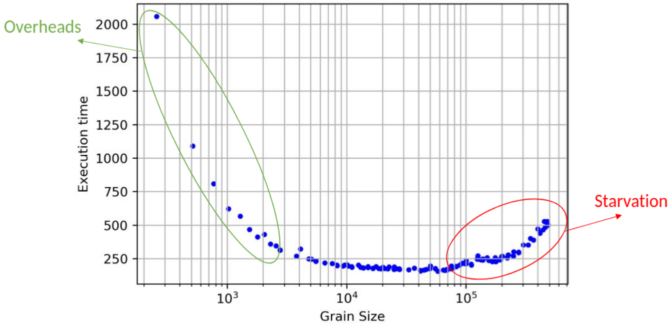


Figure 17: Results of running the *DMATDMATADD* benchmark on 8 cores matrix size 690 $\times$ 690(time unit is microseconds)

Modeling Execution Time based on Grain Size

$$t = \begin{cases} \alpha + \frac{t_s}{n_t} + \gamma & \text{if } n_t < N \\ \frac{\alpha n_t + t_s}{N} + \gamma & \text{otherwise} \end{cases}$$

n_t: number of tasks

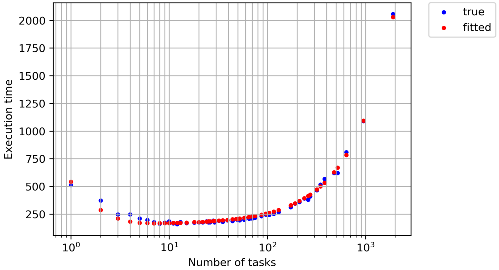
N: number of cores

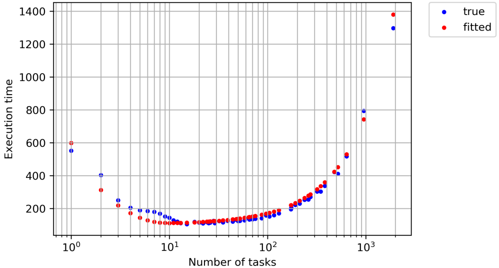
t_s: sequential execution time

γ: parallelization constant

Modeling Execution Time based on Grain Size

- Fixed matrix size, and number of cores
- Training set and test set (%60, %40)





Modeling Execution Time based on Grain Size

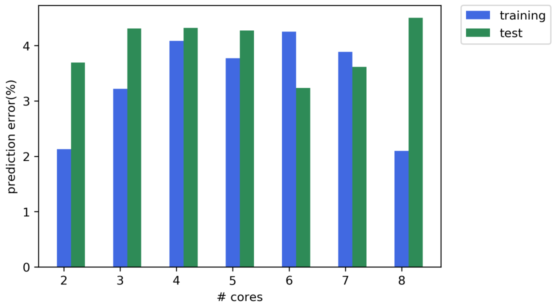


Figure 19: The error in fitting execution time with the bathtub formula for *DMATDMATADD* benchmark for matrix size 690×690 with different number of cores.

Modeling Execution Time based on Grain Size

How do α, t_s, and γ change with number of cores?

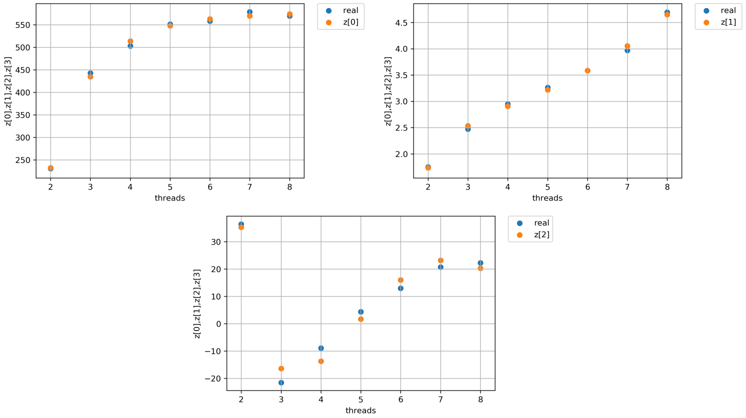


Figure 20: Fitting the three parameters (a)α, (b)t_s, and (c)γ for *DMATDMATADD* benchmark for matrix size 690×690.

Modeling Execution Time based on Grain Size

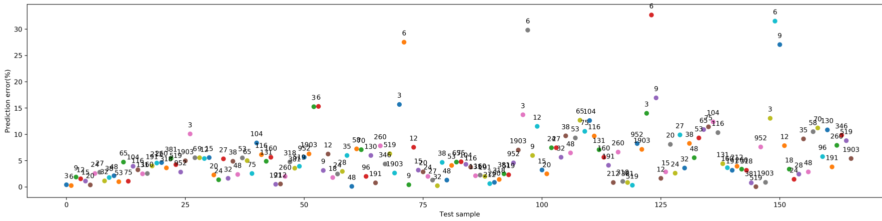


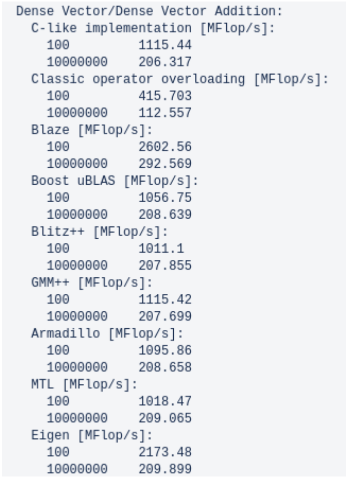
Figure 21: The error in fitting execution time with the bathtub formula for *DMATDMATADD* benchmark for matrix size 690 × 690 with different number of cores.

Modeling Execution Time based on Grain Size

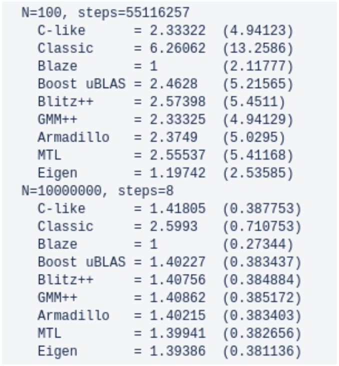
The problem with the current model is that with this formula we know that the minimum occurs at $n_t = N$.

Setup: Blazemark

Blazemark is a benchmark suite provided by Blaze to compare the performance of Blaze with other linear algebra libraries.



```
Dense Vector/Dense Vector Addition:
  C-like implementation [MFlop/s]:
    100         1115.44
    10000000    206.317
  Classic operator overloading [MFlop/s]:
    100         415.703
    10000000    112.557
  Blaze [MFlop/s]:
    100         2602.56
    10000000    292.569
  Boost uBLAS [MFlop/s]:
    100         1056.75
    10000000    208.639
  Blitz++ [MFlop/s]:
    100         1011.1
    10000000    207.855
  GMM++ [MFlop/s]:
    100         1115.42
    10000000    207.699
  Armadillo [MFlop/s]:
    100         1095.86
    10000000    208.658
  MTL [MFlop/s]:
    100         1018.47
    10000000    209.065
  Eigen [MFlop/s]:
    100         2173.48
    10000000    209.899
```



```
N=100, steps=55116257
  C-like      = 2.33322  (4.94123)
  Classic     = 6.26062  (13.2586)
  Blaze       = 1        (2.11777)
  Boost uBLAS = 2.4628   (5.21565)
  Blitz++     = 2.57398  (5.4511)
  GMM++       = 2.33325  (4.94129)
  Armadillo   = 2.3749   (5.0295)
  MTL         = 2.55537  (5.41168)
  Eigen       = 1.19742  (2.53585)
N=10000000, steps=8
  C-like      = 1.41805  (0.387753)
  Classic     = 2.5993   (0.710753)
  Blaze       = 1        (0.27344)
  Boost uBLAS = 1.40227  (0.383437)
  Blitz++     = 1.40756  (0.384884)
  GMM++       = 1.40862  (0.385172)
  Armadillo   = 1.40215  (0.383403)
  MTL         = 1.39941  (0.382656)
  Eigen       = 1.39386  (0.381136)
```

Figure 22: An example of results obtained from Blazemark

Setup: Configuration

Category	Specification
CPU	2 x Intel(R) Xeon(R) CPU E5-2450 0 @ 2.10GHz
RAM	48 GB
Number of Cores	16
Hyperthreading	Off

Table 2: Specifications of the Marvin node from Rostam cluster at CCT.

Library	Version
HPX	1.3.0
Blaze	3.5

Table 3: Specifications of the libraries used to run our experiments.

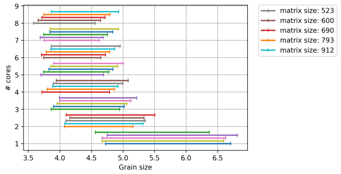
Related Work

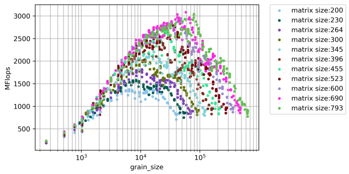
Related Work

- Estimating the optimal number of cores to run the program on
- Zahra Used logostic regsression to find the best chunk size
- Gabriel Used machine learning to find the best chunk size, while block size was fixed statically
- Peter thoman proposed a compile-time and runtime solution

Proposed Study

Proposed Study





- Studying the bathtub model
- Generalization for matrix size
- Adding runs for larger matrix sizes
- Generalization for complex expressions
- Generalization for different architectures

Thank you!