Optimizing the Performance of Multi-threaded Linear Algebra Libraries, a Task Granularity based Approach

PhD Proposal

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Outline

Objective

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A compile-time and runtime solution to optimize the performance of a linear algebra library based on

- Machine architecture
- Number of cores to run the program on
- Expression to be evaluated
  - Type of operations
  - Number of matrices involved
  - Matrix sizes

Introduction

Introduction

- Current programming models would not be able to keep up with the advances toward exascale computing
  - More complex machine architectures, deeper memory hierarchies, heterogeneous nodes, complicated networks
- AMT(Asynchronous Many-Task) model and runtime systems
  - Examples: HPX, Charm++, Legion

Introduction

- Performance of HPC applications heavily rely on the linear algebra library they are using.
- Linear algebra libraries
  - BLAS(Basic Linear Algebra Subprograms) are the fundamental routines for basic vector and matrix operations.
  - Examples: ScaLAPACK, ATLAS, SPIRAL
- Our motivation:
  - Phylanx, a platform to run your python code in parallel and distributed with machine learning as the target application.

Background

Background: HPX

- HPX is a general purpose C++ runtime system for parallel and distributed applications of any scale.
- HPX is the first open source software runtime system implementing the concepts of the ParalleX execution model, on conventional systems including Linux clusters, Windows, Macintosh, Android, XeonPhi, and the Bluegene/Q.
- Fine-grained parallelism instead of heavyweight threads.

Background: HPX, Execution Model

Four major factors for performance degradation: SLOW

- Starvation
- Latency
- Overheads
- Waiting for contention resolution

Background: Blaze C++ Library

Blaze is a high performance C++ linear algebra library based on Smart Expression Templates.

- Expression Templates:
  - Creates a parse tree of the expression at compile time and postpone the actual evaluation to when the expression is assigned to a target
- Smart:
  - Integration with highly optimized compute kernels
  - Selecting optimal evaluation method automatically for compound expressions

Background: Blaze, Parallelization

Depending on the operation and the size of operands, the assignment could be parallelized through four different backends

- HPX
- OpenMP
- C++ threads
- Boost

Background: Blaze, Backend Implementation

In the current implementation, the work is equally divided between the cores at compile time.

- Parallel for loop

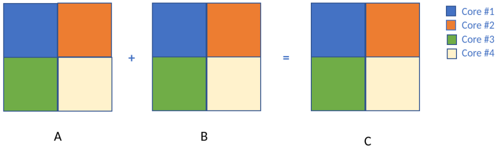


Figure 1: An example of how C=A+B is performed in parallel in Blaze with 4 cores

Background: Loop Scheduling

- Chunk size: Number of loop iterations executed by one thread

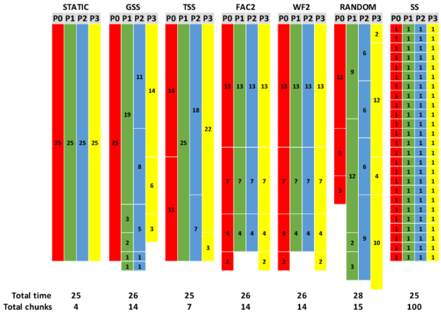


Figure 2: An example of effect of different loop scheduling methods[1]

[1] Ciorba, F. M., Iwainsky, C., & Buder, P. (2018, September). OpenMP loop scheduling revisited: making a case for more schedules. In International Workshop on OpenMP (pp. 21-36). Springer, Cham.

Background: Task Granularity

Grain size: The amount of work performed by one task

- What causes performance degradation?
  - Overheads
  - Starvation

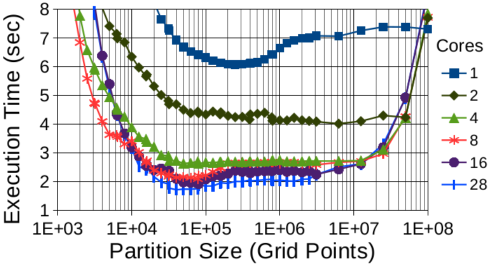


Figure 3: The effect of task size on execution time for Stencil application[2]

[2] Grubel, Patricia, et al. "The performance implication of task size for applications on the hpx runtime system." 2015 IEEE International Conference on Cluster Computing. IEEE, 2015.

Background: Modeling Performance based on number of cores

- Amdahl's Law

$$S(p) = \frac{p}{1 + \sigma(p-1)}$$

- Universal Scalability Law(USL)

$$S(p) = \frac{p}{1 + \sigma(p-1) + \kappa p(p-1)}$$

  - Models the effects of linear speedup, contention delay, and coherency delay due to crosstalk

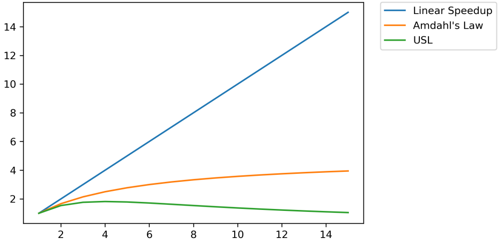


Figure 4: An example of speedup based on Amdahl's law and USL compared to the ideal linear speedup where $\sigma = 0.2$ and $\kappa = 0.05$.

Background: Modeling Performance based on number of cores: Other Models

- Quadratic model

$$S(p) = p - \gamma p(p-1)$$

- Exponential model

$$S(p) = p(1-\alpha)^{(p-1)}$$

- Geometric model

$$S(p) = \frac{1-\phi^p}{1-\phi}$$

Method

Method: Objective

Dynamically divide the work among the cores based on number of cores, matrix size, complexity of the operation, machine architecture. For this purpose two parameters have been introduced:

- block_size: at each loop iteration the assignment is performed on one block
- chunk_size: the number of loop iterations included in one task

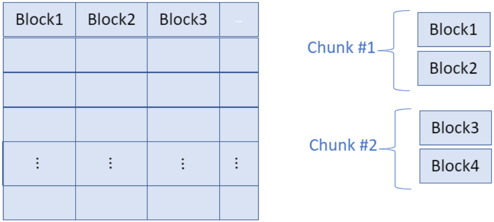


Figure 5: An example of blocking a matrix and creating chunks for chunk_size = 2

Method: Data Collection

- Starting from DMATDMATADD benchmark: $C = A + B$

Category	Configuration
Matrix sizes	200, 230, 264, 300, 396, 455, 523, 600, 690, 793, 912, 1048, 1200, 1380, 1587
Number of cores	1, 2, 3, 4, 5, 6, 7, 8
Number of rows in the block	4, 8, 12, 16, 20, 32
Number of columns in the block	64, 128, 256, 512, 1024
Chunk size	Between 1 and total number of blocks (logarithmic increase)

Table 1: List of different values used for each variable for running the *DMATDMATADD* benchmark

Method: Data Analysis

- For simplicity we look at each matrix size individually, one number of core at a time

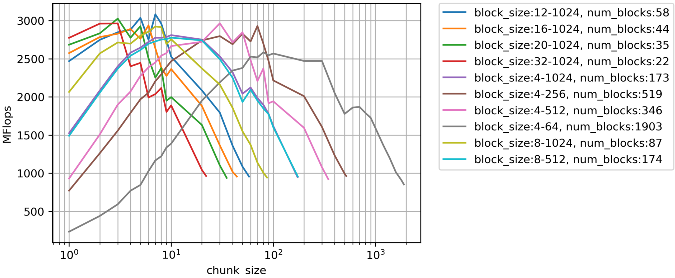
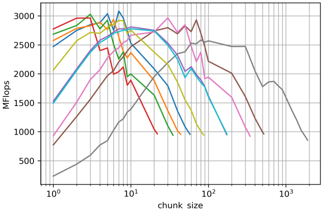


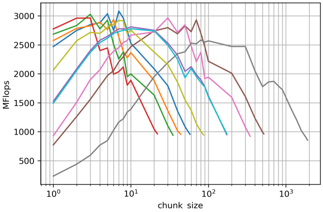
Figure 6: The results obtained from running *DMATDMATADD* benchmark for matrix sizes 690×690 with different combinations of block size and chunk size on 4 cores

Method: Observation



- For each selected block size, there is a range of chunk sizes that gives us the best performance.
- Except for some uncommon cases, no matter which block size we choose, we are able to achieve the maximum performance if we select the right chunk size.

Method: Observation

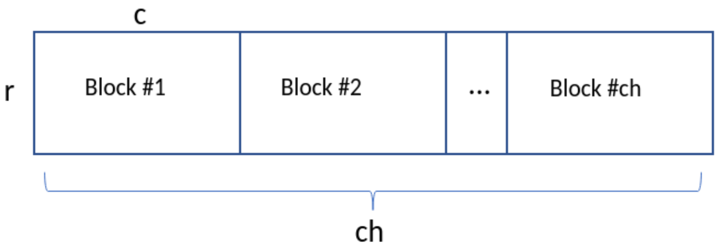


- For each selected block size, there is a range of chunk sizes that gives us the best performance.
- Except for some uncommon cases, no matter which block size we choose, we are able to achieve the maximum performance if we select the right chunk size.
- Instead of looking at block size and chunk size individually, look at grain size.

Method: Throughput vs. Grain Size

Grain size: The number of floating point operations performed by one thread

- Grain size represents the complexity of the expression
- For *DMATDMATADD*, with $block\_size = r \times c$ and $chunk\_size = ch$ Grain_size=$r \times c \times ch$



Method: Throughput vs. Grain Size

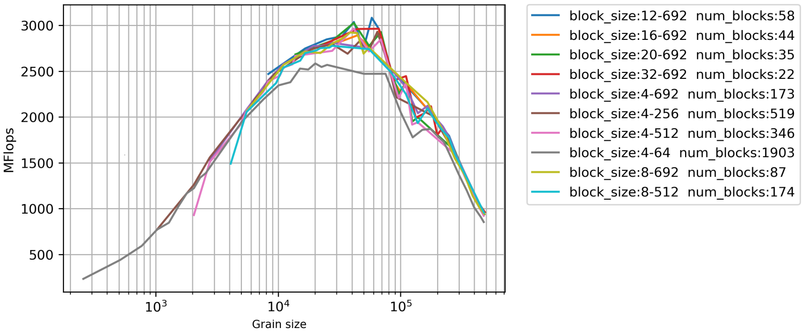


Figure 7: The results obtained from running *DMATDMATADD* benchmark through Blazemark for matrix size 690×690 on 4 cores.

Method: Throughput vs. Grain Size

The range of grain size for maximum performance

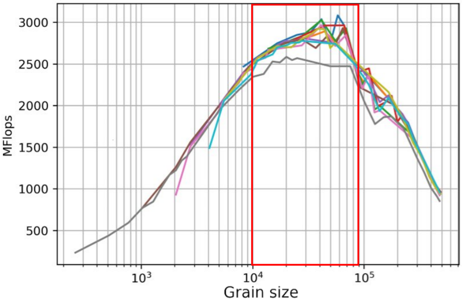


Figure 8: The results obtained from running *DMATDMATADD* benchmark through Blazemark for matrix size 690×690 on 4 cores.

Throughput vs. Grain Size and Number of Cores

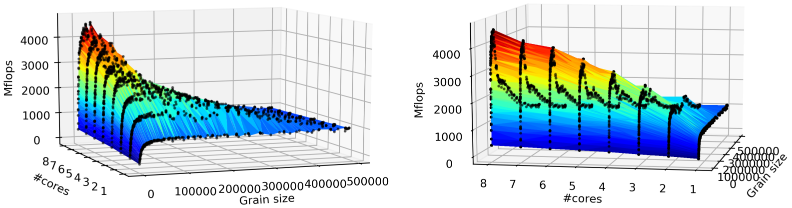


Figure 9: The results obtained from running *DMATDMATADD* benchmark through Blazemark for matrix size 690×690 based on grain size and number of cores.

Throughput vs. Grain Size and Number of Cores

Can we model the relationship between the **throughput** and the **grain size** and the **number of cores**?

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- First we try to model the relationship between throughput and grain size.

Throughput vs. Grain Size and Number of Cores

Can we model the relationship between the **throughput** and the **grain size** and the **number of cores**?

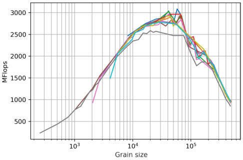
- First we try to model the relationship between throughput and grain size.
  - Polynomial Model
  - Bathtub Model

Method: Polynomial Model

In order to simplify the process and eliminate the effect of different possible factors, we started with limiting the problem to a fixed matrix size.

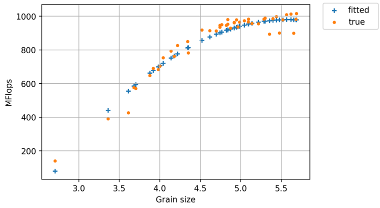
- Used a second order polynomial to model the relationship between the throughput and the grain size when number of cores is fixed.

$$P = ag^2 + bg + c$$

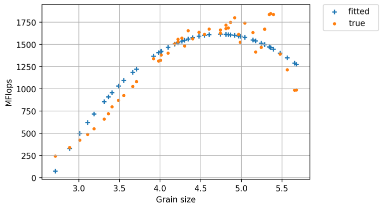


- Divide the data into training(60%) and test(40%)

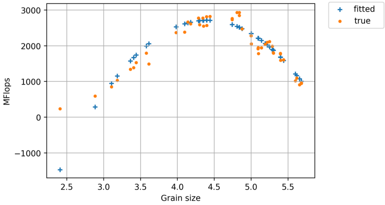
Method: Modeling Performance based on Grain Size



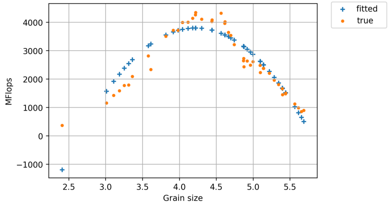
(a) 1 core



(b) 2 cores



(c) 4 cores



(d) 8 cores

Figure 10: The results of fitting the throughput vs grain size data into a 2d polynomial for *DMATDMATADD* benchmark for matrix size 690×690 with different number of cores on the test data.

Method: Modeling Performance based on Grain Size

$$Mean\_Absolute\_Error = \frac{1}{n}\sum_{i=1}^{n} abs(t_i - p_i)$$

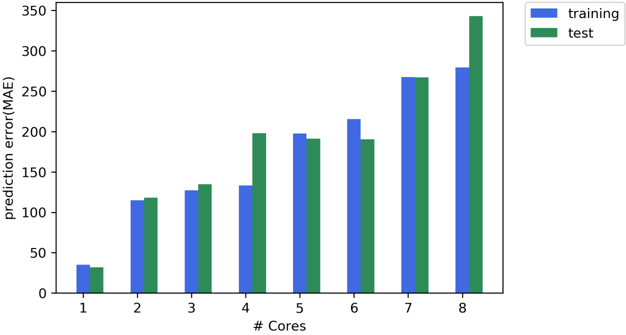


Figure 11: The training and test error for fitting data obtained from the *DMATDMATADD* benchmark for matrix size 690 $\times$ 690 against different number of cores cores.

Method: Modeling Performance based on Grain Size

$$R\_squared = 1 - \frac{\frac{1}{n}\sum_{i=1}^{n}\left(t_i - p_i\right)^2}{Var(t)}$$

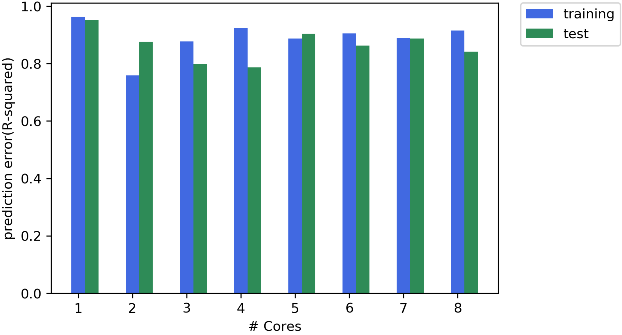
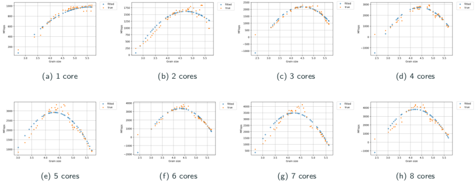


Figure 12: The training and test error for fitting data obtained from the *DMATDMATADD* benchmark for matrix size 690 × 690 against different number of cores cores.

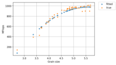
Method: Modeling Performance based on Grain Size



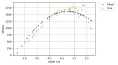
(a) 1 core (b) 2 cores (c) 3 cores (d) 4 cores

(e) 5 cores (f) 6 cores (g) 7 cores (h) 8 cores

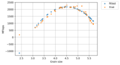
- We have developed a model for each number of cores:1, 2,.., 8
  $P = ag^2 + bg + c$



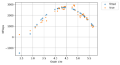
(a) 1 core



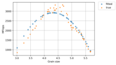
(b) 2 cores



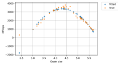
(c) 3 cores



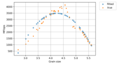
(d) 4 cores



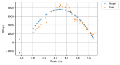
(e) 5 cores



(f) 6 cores



(g) 7 cores



(h) 8 cores

- We have developed a model for each number of cores:1, 2,.., 8
  $P = ag^2 + bg + c$
- Can we somehow integrate number of cores into the model?

Method: Modeling Performance based on Grain Size

- For $P = ag^2 + bg + c$, how do a, b, and c change with the number of cores?

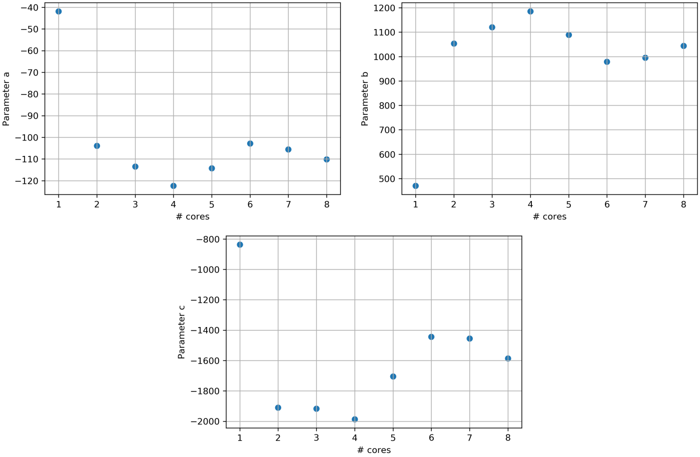


Figure 13: The parameters of the polynomial fit from the *DMATDMATADD* benchmark for matrix size 1587×1587 against different number of cores.

Method: Modeling Performance based on Grain Size

- Model the relationship with a 3rd degree polynomial

$$a = a_0 N^3 + a_1 N^2 + a_2 N + a_3, \quad b = b_0 N^3 + b_1 N^2 + b_2 N + b_3,$$

$$c = c_0 N^3 + c_1 N^2 + c_2 N + c_3$$

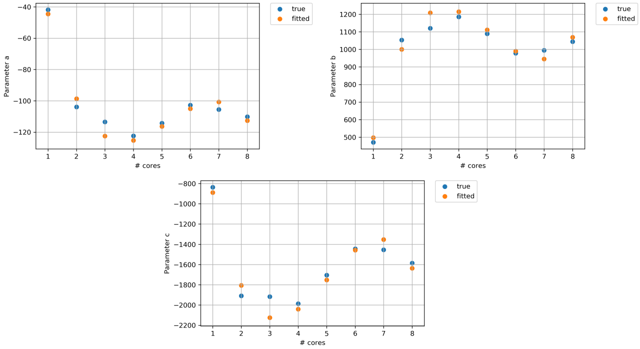
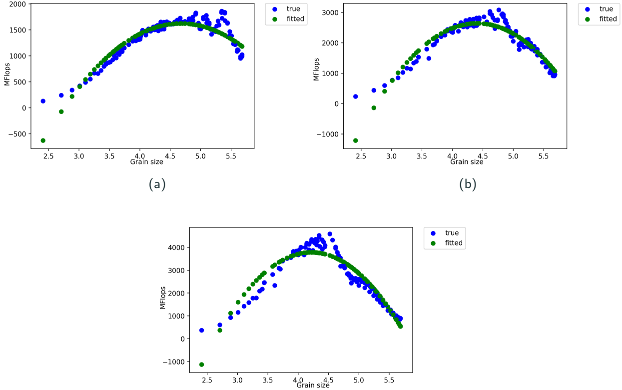


Figure 14: Fitting the parameters of the polynomial function with a 3rd degree polynomial from the *DMATDMATADD* benchmark for matrix size 1587 × 1587 against different number of cores.

Method: Modeling Performance based on Grain Size

The final model:

$$P = a_{11} g^2 N^3 + a_{10} g^2 N^2 + \dots + a_1 N + a_0$$



(a)

(b)

(c)

Figure 15: matrix size 690 × 690 for (a) 2 core, (b) 4 cores, (c) 8 cores.

Method: Modeling Performance based on Grain Size

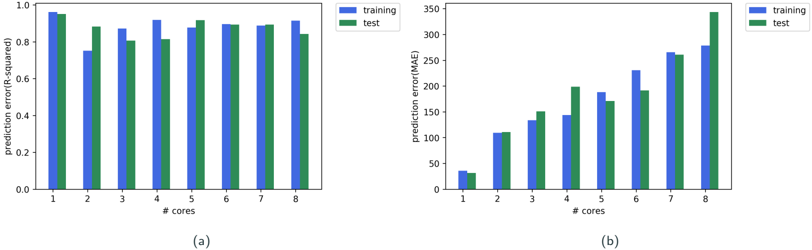
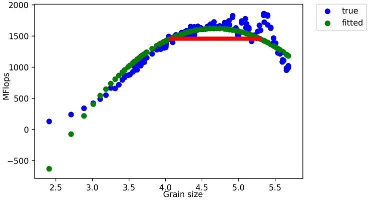
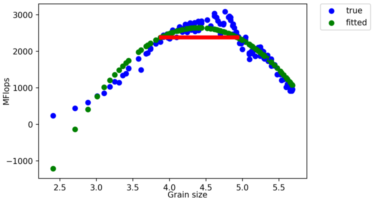


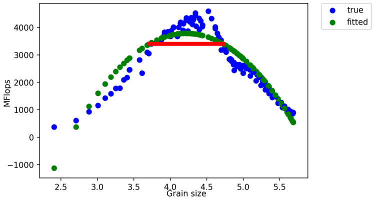
Figure 16: All the data points are included in calculation of error,(a) R_squared error (b) Mean Absolute Error(MAE) .

Method: Finding the Grain Size Range for Maximum Performance





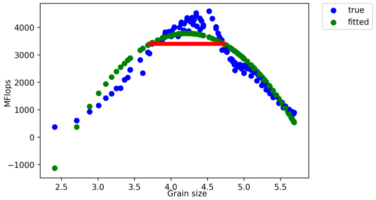
(a)



(b)

Figure 17: The range of grain size (shown as the red line) that leads to a performance within 10% of the maximum performance for (a) 2 cores, (b) 4 cores and (b) 8 cores.

Method: Finding the Grain Size Range for Maximum Performance



How do we use the calculated range?

- Select a reasonable block size, e.g. 4×256
- Find the range of chunk size that results in the calculated range of grain size, Grain_size=$r \times c \times ch$

Method: Finding the Grain Size Range for Maximum Performance

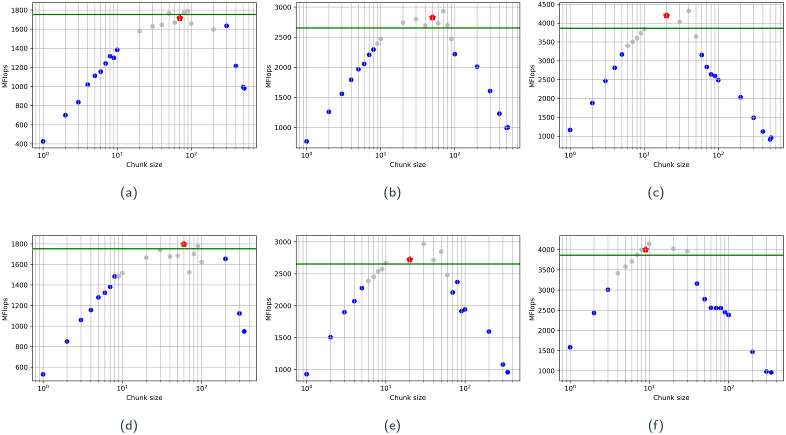


Figure 18: matrix size 690 × 690 with block size of 4 × 256 on (a) 2 cores, (b) 4 cores, and (c) 8 cores, and block size of 4 × 512 on (d) 2 cores, (e) 4 cores, and (f) 8 cores.

Method: Polynomial Model, Wrap up

- Simple model, can easily find the maximum.
- It is not physical.

Method: Bathtub Model

Can we create an analytic model for execution time based on grain size?

Method: Bathtub Model

- Overheads of creating tasks
- Starvation

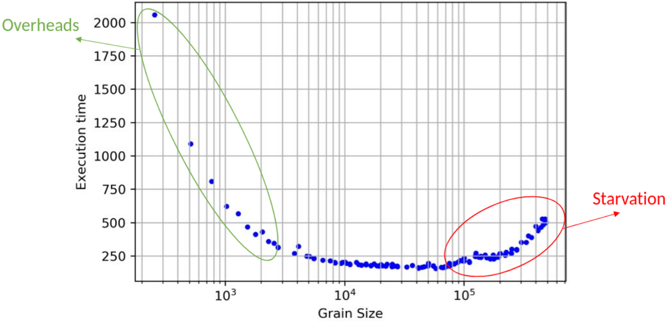


Figure 19: Results of running the *DMATDMATADD* benchmark on 8 cores matrix size 690 × 690(time unit is microseconds)

Method: Modeling Execution Time based on Grain Size

N: Number of cores

n_t: Number of created tasks

$$n_t = \frac{Total\ amount\ of\ work}{grain\_size}$$

t_s: sequential execution time

M: Number of cores actually doing the work

$$M = \begin{cases} n_t & \text{if } n_t < N \\ N & \text{otherwise} \end{cases}$$

Method: Modeling Execution Time based on Grain Size

N: Number of cores

n_t: Number of created tasks

$$n_t = \frac{Total\ amount\ of\ work}{grain\_size}$$

t_s: sequential execution time

M: Number of cores actually doing the work

$$M = \begin{cases} n_t & \text{if } n_t < N \\ N & \text{otherwise} \end{cases}$$

$Execution\_time = \frac{t_s}{M}$

Method: Modeling Execution Time based on Grain Size

N: Number of cores

n_t: Number of created tasks

$$n_t = \frac{Total\ amount\ of\ work}{grain\_size}$$

t_s: sequential execution time

M: Number of cores actually doing the work

$$M = \begin{cases} n_t & \text{if } n_t < N \\ N & \text{otherwise} \end{cases}$$

$Execution\_time = \frac{t_s}{M} + \alpha \frac{n_t}{M}$

Method: Modeling Execution Time based on Grain Size

N: Number of cores

n_t: Number of created tasks

$$n_t = \frac{Total\ amount\ of\ work}{grain\_size}$$

t_s: sequential execution time

M: Number of cores actually doing the work

$$M = \begin{cases} n_t & \text{if } n_t < N \\ N & \text{otherwise} \end{cases}$$

$Execution\_time = \frac{t_s}{M} + \alpha \frac{n_t}{M} + \gamma$

Method: Modeling Execution Time based on Grain Size

$$t = \begin{cases} \alpha + \dfrac{t_s}{n_t} + \gamma & \text{if } n_t < N \\ \dfrac{\alpha n_t + t_s}{N} + \gamma & \text{otherwise} \end{cases}$$

Method: Modeling Execution Time based on Grain Size

- Fixed matrix size, and number of cores
- Training set and test set (%60, %40)

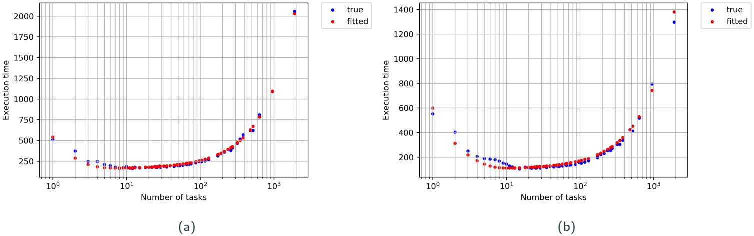
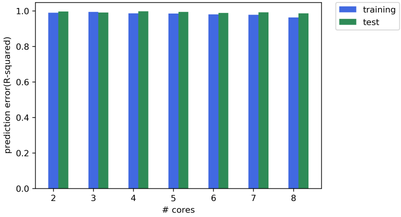
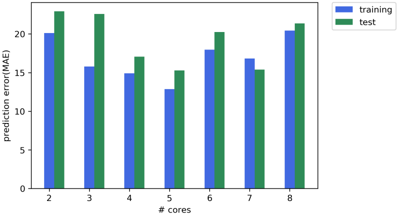


Figure 20: The prediction of execution time based on grain size using the bathtub model, for (a)4 cores and (b)8 cores for *DMATDMATADD* benchmark for matrix size 690 $\times$ 690.

Method: Modeling Performance based on Grain Size



(a)



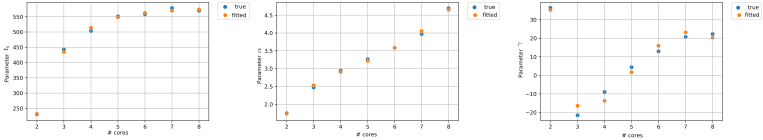
(b)

Figure 21: The error in fitting execution time with the bathtub formula for *DMATDMATADD* benchmark for matrix size 690×690 with different number of cores,(a) R_squared error (b) Mean Absolute Error(MAE).

Method: Modeling Execution Time based on Grain Size

- How do t_s, α, and γ change with number of cores?

$$f(N) = \frac{m_0 + m_1(N-1) + m_2(N-1)N + m_3(N^2)(N-1)}{N}$$



(a) (b) (c)

Figure 22: Fitting the three parameters (a)α, (b)t_s, and (c)γ for *DMATDMATADD* benchmark for matrix size 690×690.

Method: Modeling Execution Time based on Grain Size

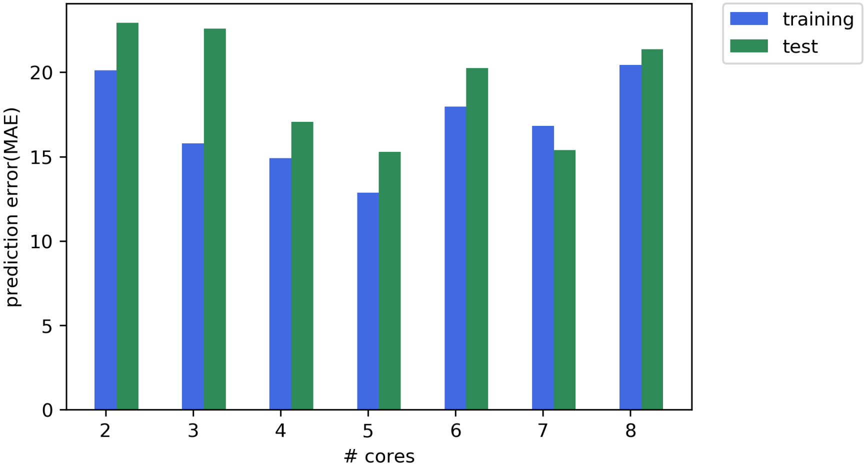


Figure 23: The error in fitting execution time with the bathtub formula integrated with the USL modelfor *DMATDMATADD* benchmark for matrix size 690 $\times$ 690 with different number of cores.

Method: Modeling Execution Time based on Grain Size

- The problem with the current model is that with this formula we know that the minimum occurs at $n_t = N$.
- Parameters t_s, α, γ do not behave the way we expect, they change with change of number of cores.
  What is the missing factor?

Setup: Blazemark

Blazemark is a benchmark suite provided by Blaze to compare the performance of Blaze with other linear algebra libraries.

```
Dense Vector/Dense Vector Addition:
  C-like implementation [MFlop/s]:
    100         1115.44
    10000000    206.317
  Classic operator overloading [MFlop/s]:
    100         415.703
    10000000    112.557
  Blaze [MFlop/s]:
    100         2602.56
    10000000    292.569
  Boost uBLAS [MFlop/s]:
    100         1056.75
    10000000    208.639
  Blitz++ [MFlop/s]:
    100         1011.1
    10000000    207.855
  GMM++ [MFlop/s]:
    100         1115.42
    10000000    207.699
  Armadillo [MFlop/s]:
    100         1095.86
    10000000    208.658
  MTL [MFlop/s]:
    100         1018.47
    10000000    209.065
  Eigen [MFlop/s]:
    100         2173.48
    10000000    209.899
```

```
N=100, steps=55116257
  C-like      = 2.33322  (4.94123)
  Classic     = 6.26062  (13.2586)
  Blaze       = 1        (2.11777)
  Boost uBLAS = 2.4628   (5.21565)
  Blitz++     = 2.57398  (5.4511)
  GMM++       = 2.33325  (4.94129)
  Armadillo   = 2.3749   (5.0295)
  MTL         = 2.55537  (5.41168)
  Eigen       = 1.19742  (2.53585)
N=10000000, steps=8
  C-like      = 1.41805  (0.387753)
  Classic     = 2.5993   (0.710753)
  Blaze       = 1        (0.27344)
  Boost uBLAS = 1.40227  (0.383437)
  Blitz++     = 1.40756  (0.384884)
  GMM++       = 1.40862  (0.385172)
  Armadillo   = 1.40215  (0.383403)
  MTL         = 1.39941  (0.382656)
  Eigen       = 1.39386  (0.381136)
```

Figure 24: An example of results obtained from Blazemark

Setup: Configuration

Category	Specification
CPU	2 x Intel(R) Xeon(R) CPU E5-2450 0 @ 2.10GHz
RAM	48 GB
Number of Cores	16
Hyperthreading	Off

Table 2: Specifications of the Marvin node from Rostam cluster at CCT.

Library	Version
HPX	1.3.0
Blaze	3.5

Table 3: Specifications of the libraries used to run our experiments.

Related Work

Related Work

- Liu et al. estimated the optimal number of cores to run the program on based on cache specific traces.
- Khatami et al. used logistic regression to find the best chunk size based on some static and dynamic features of the loop.
- Thoman et al. proposed a compile-time and runtime solution, using an effort estimation function set the chunk size.

Related Work

- Laberge et al. used machine learning to find the best chunk size to get the maximum performance, while block size was fixed statistically.
- Features included: matrix size, number of cores, number of floating point operations, number of iterations.

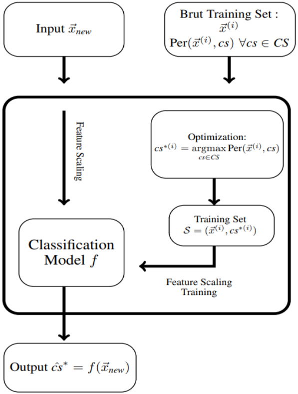


Figure 25: Laberge, G., Shirzad, S., Diehl, P., Kaiser, H., Prudhomme, S., Lemoine, A. (2019). Scheduling optimization of parallel linear algebra algorithms using Supervised Learning. arXiv preprint arXiv:1909.03947.

Our Contributions

- We propose a novel analytic model to represent how the execution time is expected to change based on grain size.
- To our knowledge, there has not been a work to create a 3D model of the throughput, grain size, and number of cores.
- We are proposing a method to apply the developed model to a linear algebra library, in a way specific to our application, and the machine architecture.

Proposed Study

Proposed Study

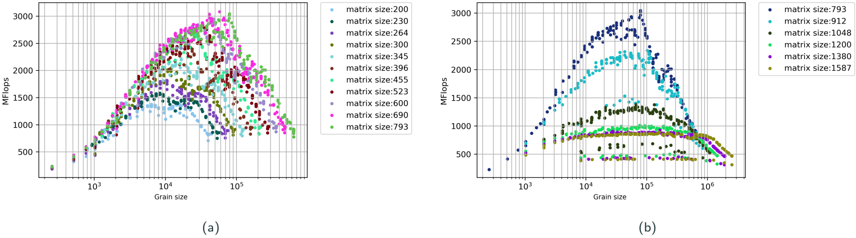


Figure 26: Throughput vs. grain size graph obtained from running *DMATDMATADD* benchmark on 4 cores for matrix sizes (a) smaller than 793×793 (b) larger than 793×793.

- Studying the bathtub model to find the missing factor, also location of the minimum
- Generalization for matrix size, adding runs for larger matrix sizes
- Generalization for complex expressions
- Generalization for different architectures

Thank you!

BLAS operations

Level 1	addition/scaling	$\alpha x, \quad \alpha x + y$
	dot products, norms	$x^T y, \quad \|x\|_2, \quad \|x\|_1$
Level 2	matrix/vector products	$\alpha Ax + \beta y, \quad \alpha A^T x + \beta y$
	rank 1 updates	$A + \alpha xy^T, \quad A + \alpha xx^T$
	rank 2 updates	$A + \alpha xy^T + \alpha yx^T$
	triangular solves	$\alpha T^{-1}x, \quad \alpha T^{-T}x$
Level 3	matrix/matrix products	$\alpha AB + \beta C, \quad \alpha AB^T + \beta C$
		$\alpha A^T B + \beta C, \quad \alpha A^T B^T + \beta C$
	rank-k updates	$\alpha AA^T + \beta C, \quad \alpha A^T A + \beta C$
	rank-$2k$ updates	$\alpha A^T B + \alpha B^T A + \beta C$
	triangular solves	$\alpha T^{-1}C, \quad \alpha T^{-T}C$

Figure 27: https://web.stanford.edu/class/ee392o/nlas-foils.pdf

Appendix

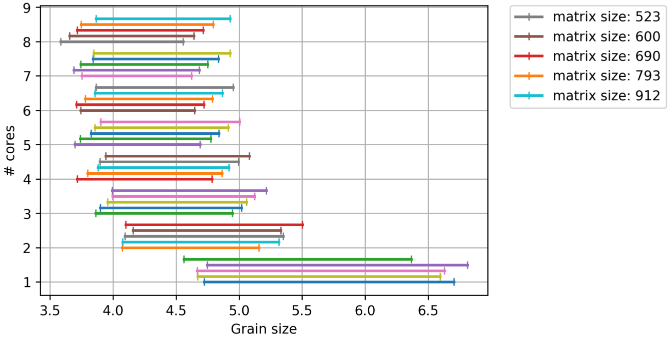


Figure 28: The range of grain size within 10% of the maximum performance of the fitted polynomial function for *DMATDMATADD* benchmark for different number of cores for matrix size 523×523 to 912×912.