

## EVAN'S CONJECTURE

Let  $v$  be a  $b$ -bit word.

- A *Type I* transformation replaces a 0 in  $v$  with a 1.

Example:  $001\underline{0}101 \rightarrow 001\underline{1}01$ .

- A *Type H* transformation replaces a substring in  $v$  of the form<sup>1</sup> 011...10 with its complement, i.e., 100...01.

Example:  $001\underline{1}001 \rightarrow 01\underline{0}0101$ .

Recall that, in a poset,  $v$  is *covered by*  $w$  provided  $v \prec w$  and there is no element  $x$  with  $v \prec x \prec w$ .

**Conjecture 1.** *For  $v, w \in W_b$ ,  $v$  is covered by  $w$  if and only if  $w$  is obtained from  $v$  by a Type I or Type H transformation.*

In particular, if  $w_1 \prec w_2 \prec \dots \prec w_t$  is a maximum chain in  $W_b$ , then each  $w_{i+1}$  is obtained from  $w_i$  by a Type I or a Type H transformation.

Conjecture is well-supported by computer evidence.

Some chain examples:

```
julia> show_transformations(palindromes(7))
0000000 → 0001000      Type I
0001000 → 0010100      Type H
0010100 → 0011100      Type I
0011100 → 0100010      Type H
0100010 → 0101010      Type I
0101010 → 0110110      Type H
0110110 → 0111110      Type I
0111110 → 1000001      Type H
1000001 → 1001001      Type I
1001001 → 1010101      Type H
1010101 → 1011101      Type I
1011101 → 1100011      Type H
1100011 → 1101011      Type I
1101011 → 1110111      Type H
1110111 → 1111111      Type I
```

```
julia> show_transformations(max_chain(7))
0000000 → 0010000      Type I
```

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<sup>1</sup>The substring begins and ends with a 0 and has one or more consecutive 1s in between.

0010000 → 0011000	Type I
0011000 → 0100100	Type H
0100100 → 0110100	Type I
0110100 → 0111100	Type I
0111100 → 1000010	Type H
1000010 → 1010010	Type I
1010010 → 1011010	Type I
1011010 → 1011110	Type I
1011110 → 1100001	Type H
1100001 → 1101001	Type I
1101001 → 1101101	Type I
1101101 → 1110011	Type H
1110011 → 1111011	Type I
1111011 → 1111111	Type I

```
julia> show_transformations(recursive_chain(7))
0000000 → 0001000      Type I
0001000 → 0010100      Type H
0010100 → 0011100      Type I
0011100 → 0100010      Type H
0100010 → 0101010      Type I
0101010 → 0110110      Type H
0110110 → 0111110      Type I
0111110 → 1000001      Type H
1000001 → 1001001      Type I
1001001 → 1010101      Type H
1010101 → 1011101      Type I
1011101 → 1100011      Type H
1100011 → 1101011      Type I
1101011 → 1110111      Type H
1110111 → 1111111      Type I
```