

## EVAN'S CONJECTURE

Let  $v$  be a  $b$ -bit word.

- A *Type I* transformation replaces a 0 in  $v$  with a 1.

Example:  $0010101 \rightarrow 0011101$ .

- A *Type H* transformation replaces a substring in  $v$  of the form<sup>1</sup>  $011 \dots 10$  with its complement, i.e.,  $100 \dots 01$ .

Example:  $0011001 \rightarrow 0100101$ .

Recall that, in a poset,  $v$  is *covered* by  $w$  provided  $v \prec w$  and there is no element  $x$  with  $v \prec x \prec w$ .

**Conjecture 1.** *For  $v, w \in W_b$ ,  $v$  is covered by  $w$  if and only if  $w$  is obtained from  $v$  by a Type I or Type H transformation.*

In particular, if  $w_1 \prec w_2 \prec \dots \prec w_t$  is a maximum chain in  $W_b$ , then each  $w_{i+1}$  is obtained from  $w_i$  by a Type I or a Type H transformation.

Conjecture is well-supported by computer evidence.

Some chain examples:

```
julia> show_transformations(palindromes(7))
00000000 → 00010000      Type I
00010000 → 00101000      Type H
00101000 → 00111000      Type I
00111000 → 01000010      Type H
01000010 → 01010100      Type I
01010100 → 01101100      Type H
01101100 → 01111100      Type I
01111100 → 10000001      Type H
10000001 → 10010001      Type I
10010001 → 10101001      Type H
10101001 → 10111001      Type I
10111001 → 11000011      Type H
11000011 → 11010011      Type I
11010011 → 11101011      Type H
11101011 → 11111011      Type I
```

```
julia> show_transformations(max_chain(7))
00000000 → 00100000      Type I
```

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<sup>1</sup>The substring begins and ends with a 0 and has one or more consecutive 1s in between.

0010000	→	0011000	Type I
0011000	→	0100100	Type H
0100100	→	0110100	Type I
0110100	→	0111100	Type I
0111100	→	1000010	Type H
1000010	→	1010010	Type I
1010010	→	1011010	Type I
1011010	→	1011110	Type I
1011110	→	1100001	Type H
1100001	→	1101001	Type I
1101001	→	1101101	Type I
1101101	→	1110011	Type H
1110011	→	1111011	Type I
1111011	→	1111111	Type I

julia> show\_transformations(recursive\_chain(7))

0000000	→	0001000	Type I
0001000	→	0010100	Type H
0010100	→	0011100	Type I
0011100	→	0100010	Type H
0100010	→	0101010	Type I
0101010	→	0110110	Type H
0110110	→	0111110	Type I
0111110	→	1000001	Type H
1000001	→	1001001	Type I
1001001	→	1010101	Type H
1010101	→	1011101	Type I
1011101	→	1100011	Type H
1100011	→	1101011	Type I
1101011	→	1110111	Type H
1110111	→	1111111	Type I