Evan's Conjecture

Let v be a b-bit word.

- A *Type I* transformation replaces a 0 in v with a 1.

  Example: $001\underline{0}101 \to 001\underline{1}101$.

- A *Type H* transformation replaces a substring in v of the form[1] $011\ldots10$ with its complement, i.e., $100\ldots01$.

  Example: $0\underline{0110}01 \to 0\underline{1001}01$.

Recall that, in a poset, v is *covered by* w provided $v \prec w$ and there is no element x with $v \prec x \prec w$.

Proposition 1. *Suppose $v, w \in W_b$ and w is derived from v by an I transformation, than $v \prec w$.*

Proof. Let $v = (x, 0, y)$ and $w = (x, 1, y)$. Then $\operatorname{val} w = \operatorname{val} v + 2^k > \operatorname{val} v$ for some k. Likewise, $\operatorname{val} \rho w = \operatorname{val} \rho v + 2^j > \operatorname{val} \rho v$ for some j. Therefore $v \prec w$. □

Proposition 2. *Suppose $v, w \in W_b$ and w is derived from v by an H transformation, than $v \prec w$.*

Proof. Let $v = (x, 10\ldots01, y)$ and $w = (x, 01\ldots1, y)$. Note that $10\ldots01_{\text{TWO}} > 01\ldots10_{\text{TWO}}$ and therefore $\operatorname{val} w > \operatorname{val} v$. Likewise, $\operatorname{val} \rho w > \operatorname{val} \rho v$. Therefore $v \prec w$. □

Corrolary 3. *Let $v, w \in W_b$. If w is obtained from v by a series of I and H transformations, then $v \preceq w$.* □

The interesting part is the converse.

Theorem 4. *Let $v, w \in W_b$ with $v \prec w$. Then w is obtained from v by a sequence of I and H transformations.*

Proof. The proof is by strong induction on b. It is easy to verify for W_1, W_2, and W_3. Suppose the result has been shown for all W_i with $i < b$.

Let $v, w \in W_b$.

[1] The substring begins and ends with a 0 and has one or more consecutive 1s in between.

- Case I: Suppose that the first bits of v and w are the same, and the last bits of v and w are the same. That is $v = (sxt)$ and $w = (syt)$ where $s, t \in \{0, 1\}$. It follows that $x \prec y$ in W_{b-2} and therefore there is a sequence of H/I transformations that convert x to y (by induction). The same sequence converts v to w.

  Note that we cannot have $v = (1x0)$ and $w = (0y1)$ because $v \prec w$.

- Case II: Suppose that $v = (0x1)$ and $w = (1y0)$. Suppose, for the sake of contradiction, that there is no sequence of H/I transformations that converts v to w. Among all possible such pairs (v, w) choose one such that $\operatorname{val} y$ is smallest.

  THIS IS WHERE THE NEW MATERIAL WILL GO.

$\square$

Corrolary 5. *For $v, w \in W_b$, v is covered by w if and only if w is obtained from v by a Type I or Type H transformation.* $\square$

In particular, if $w_1 \prec w_2 \prec \cdots \prec w_t$ is a maximum chain in W_b, then each w_{i+1} is obtained from w_i by a Type I or a Type H transformation.