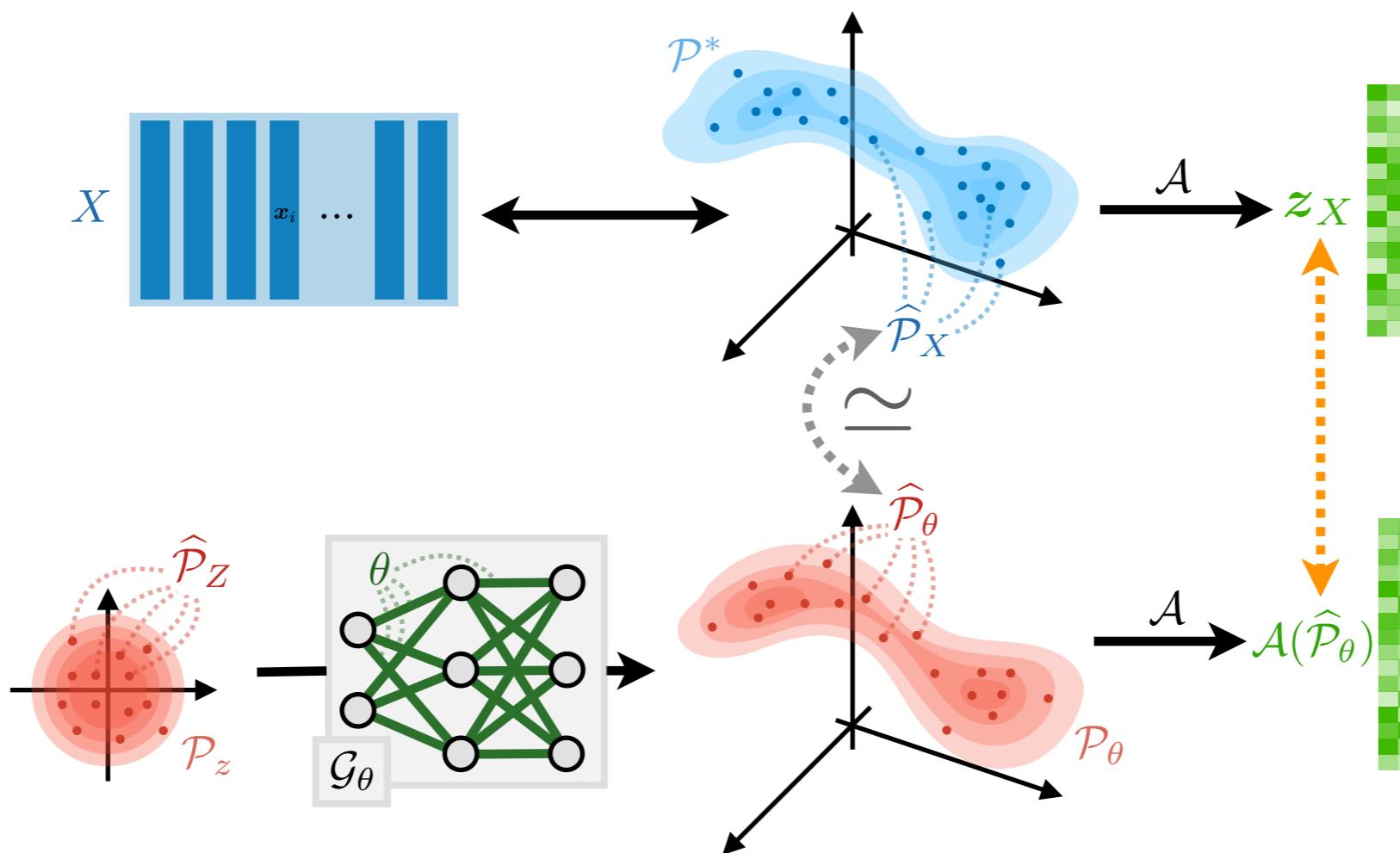


# Compressive Learning of Generative Networks

Vincent Schellekens & Laurent Jacques

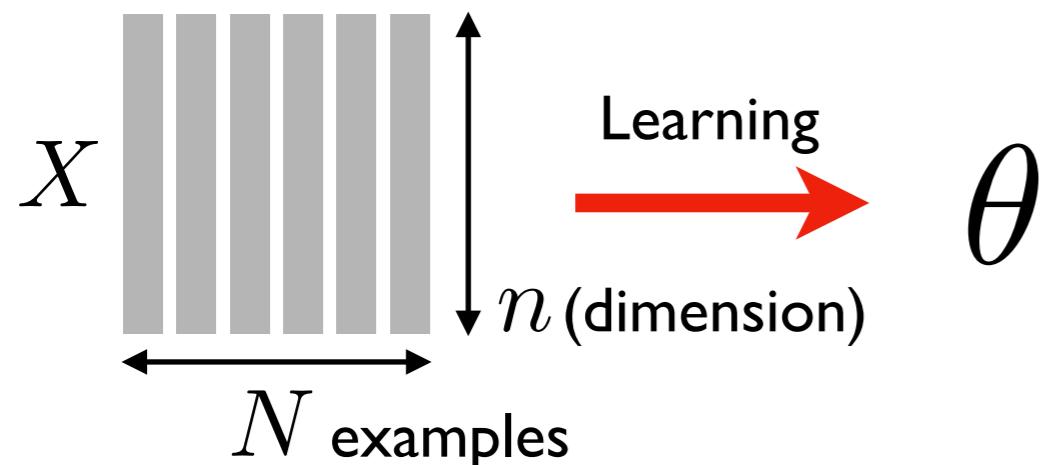
UCLouvain



# Compressive Learning of Generative Networks

# Compressive Learning: Why?

Usual machine learning

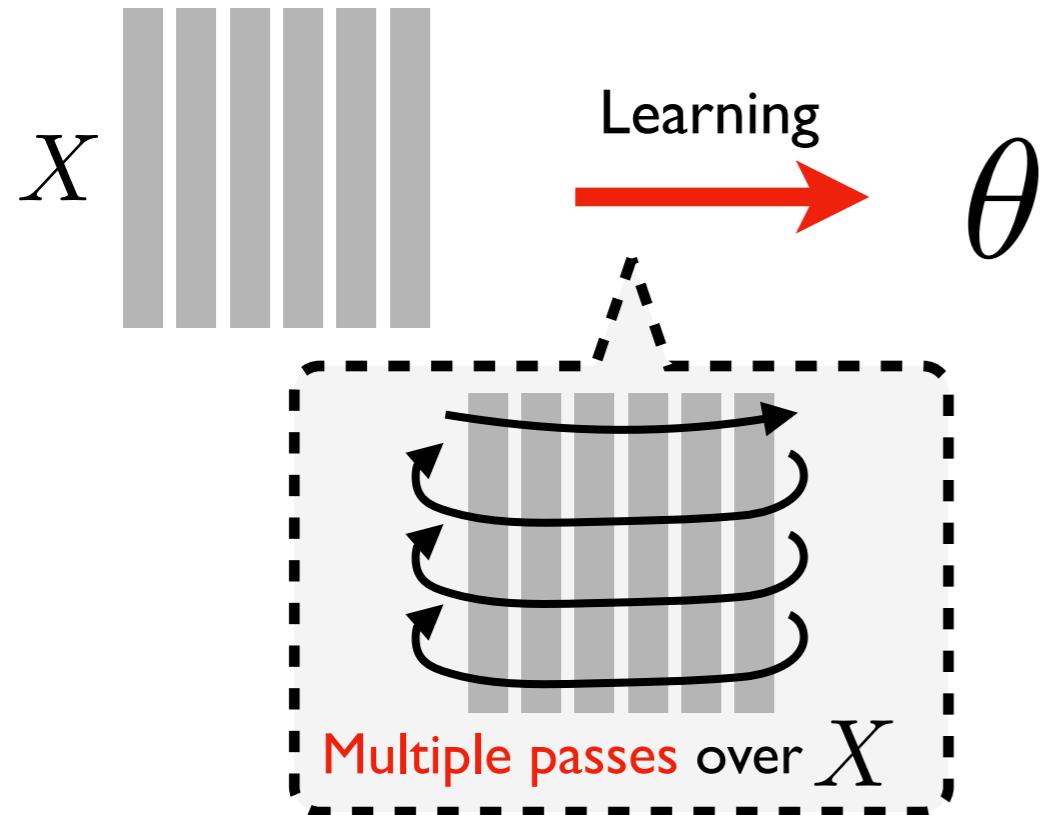


Dataset

Parameters

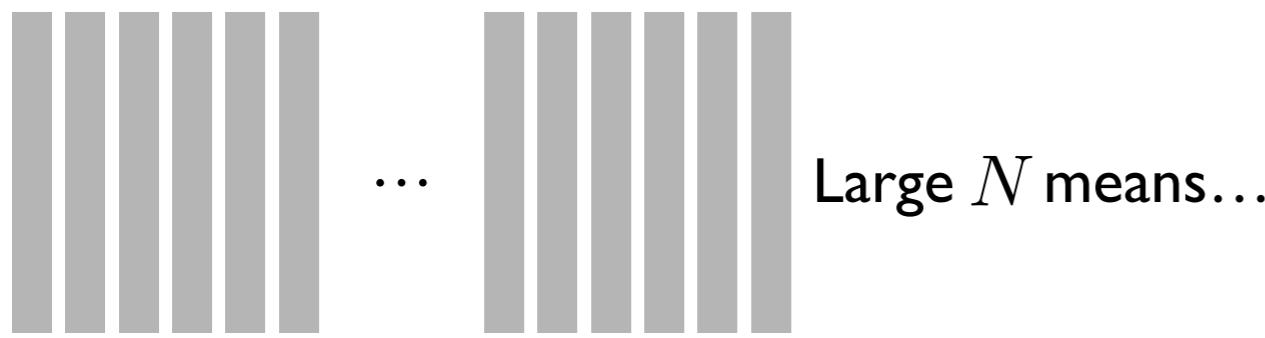
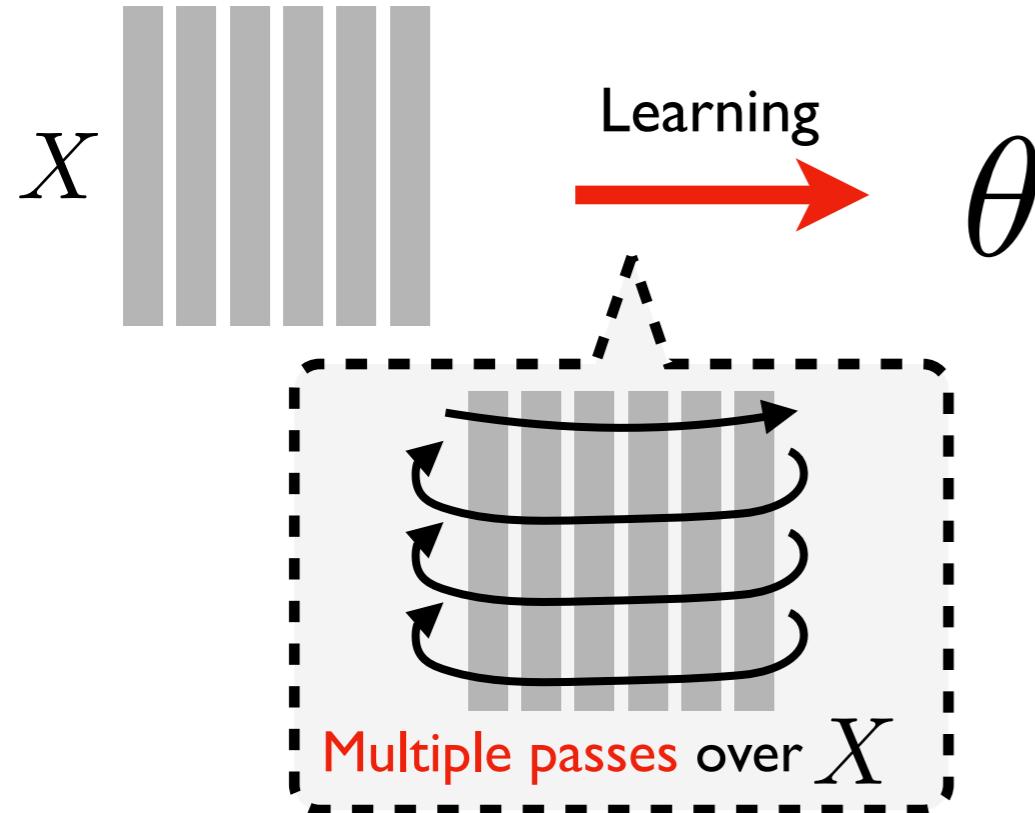
# Compressive Learning: Why?

Usual machine learning



# Compressive Learning: Why?

Usual machine learning

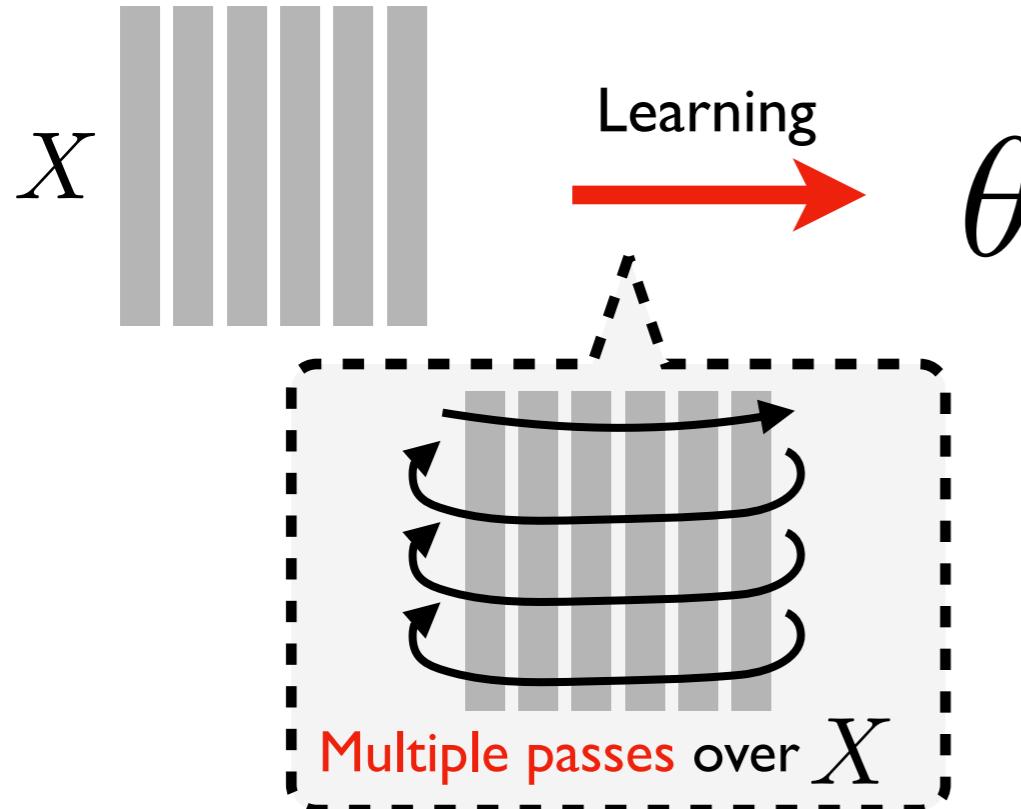


... large  
memory &  
training time!

Context: large-scale learning

# Compressive Learning: What?

## Usual machine learning



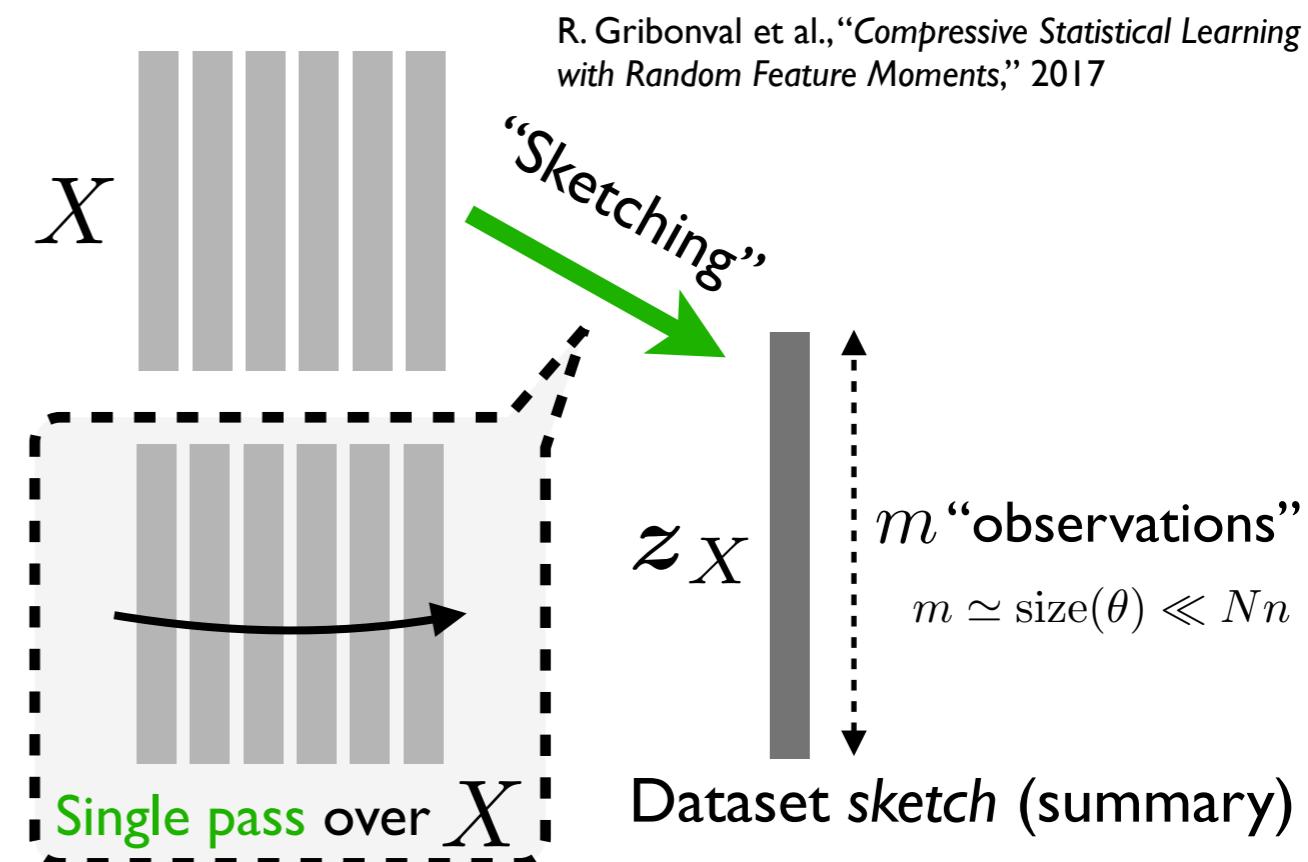
Large  $N$  means...

This section shows two sets of data structures. The first set consists of two vertical stacks of gray bars, each with a red cylinder icon below it. The second set consists of two vertical stacks of gray bars, each with a red hourglass icon below it. Ellipses between the two sets indicate that there are more such pairs.

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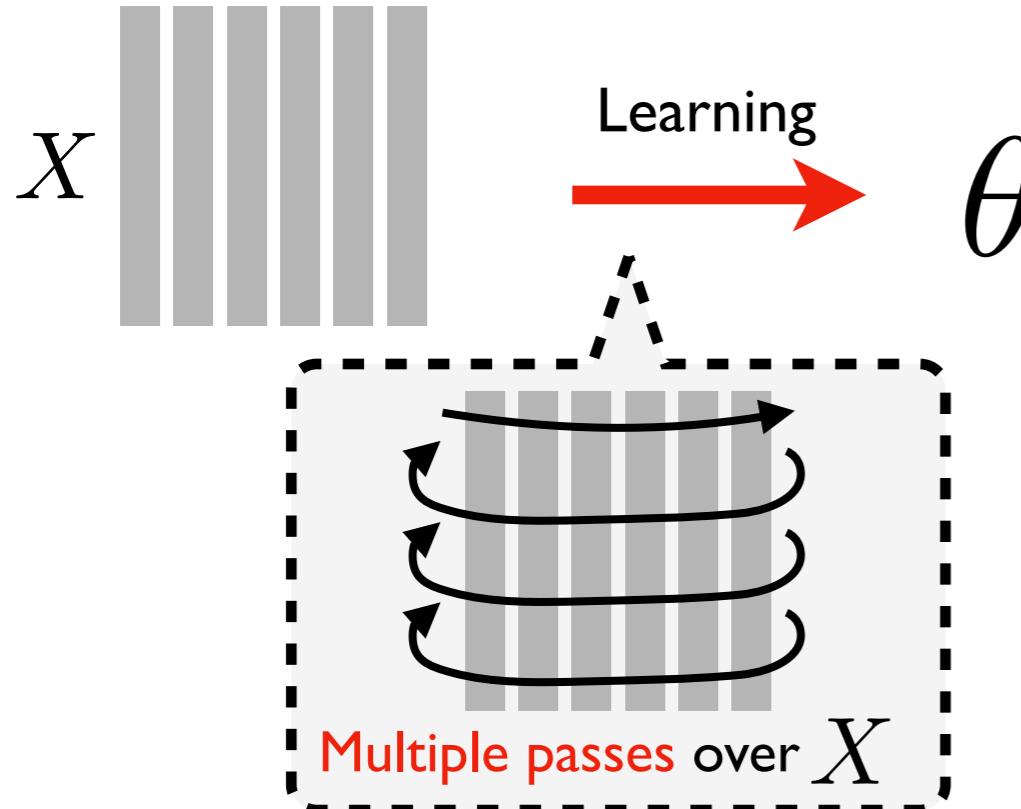
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## Compressive Learning



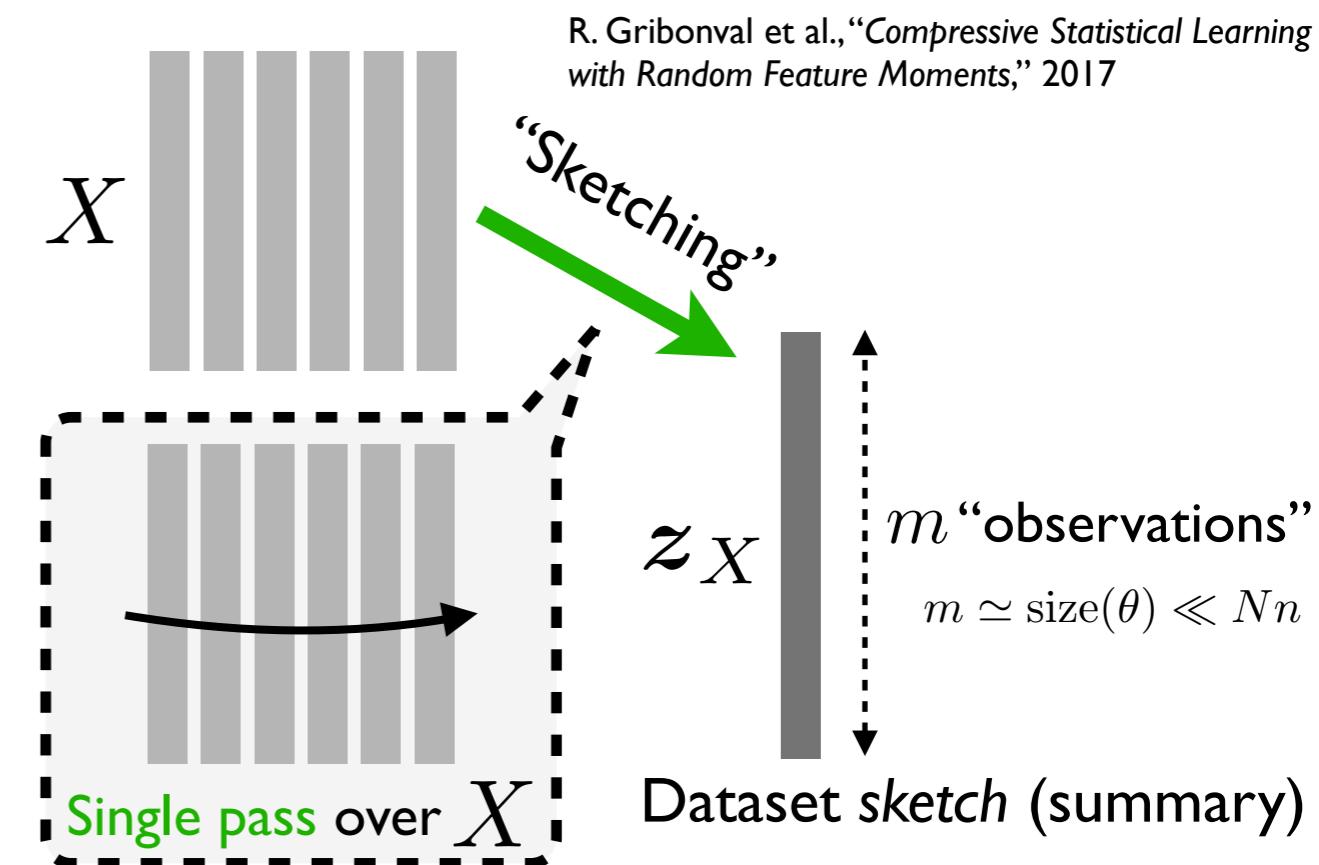
# Compressive Learning: What?

## Usual machine learning



... Large  $N$  means...  
 ... ... ... ... ... ...   
... large memory & training time!

## Compressive Learning

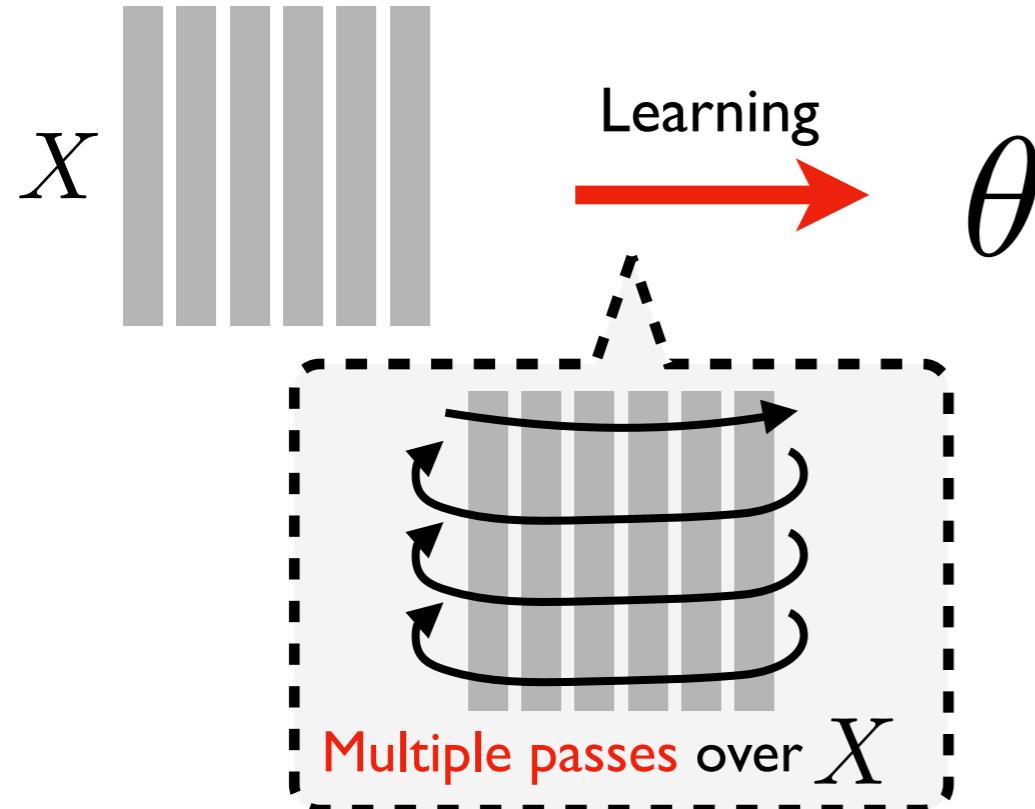


$$z_X = \frac{1}{N} \sum_{i=1}^N \Phi(x_i)$$

"Generalized moments"  
(average of features of the examples)

# Compressive Learning: What?

## Usual machine learning



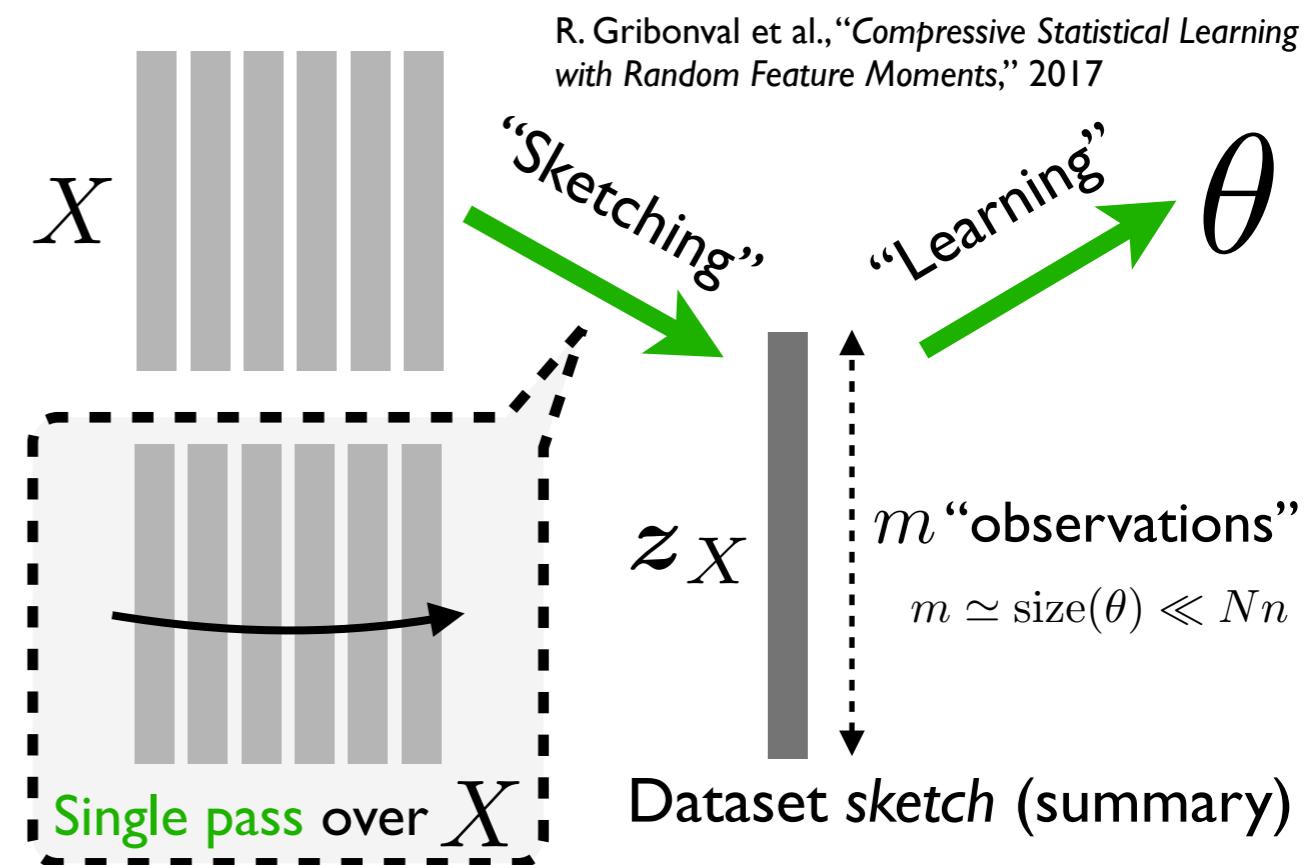
Large  $N$  means...

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## Compressive Learning



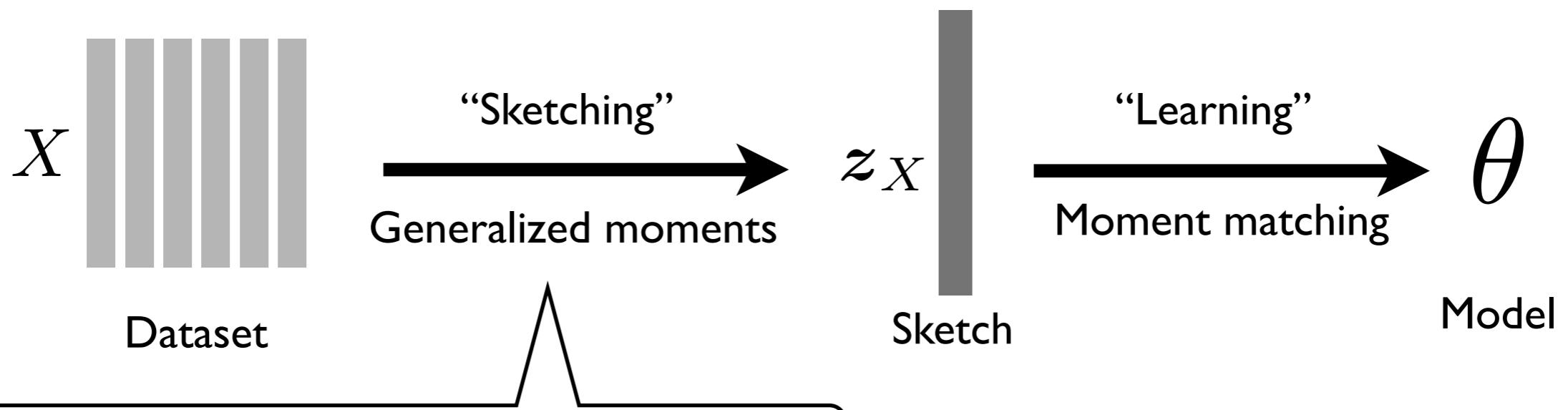
Large  $N$  means...

...  
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... constant  
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training time!

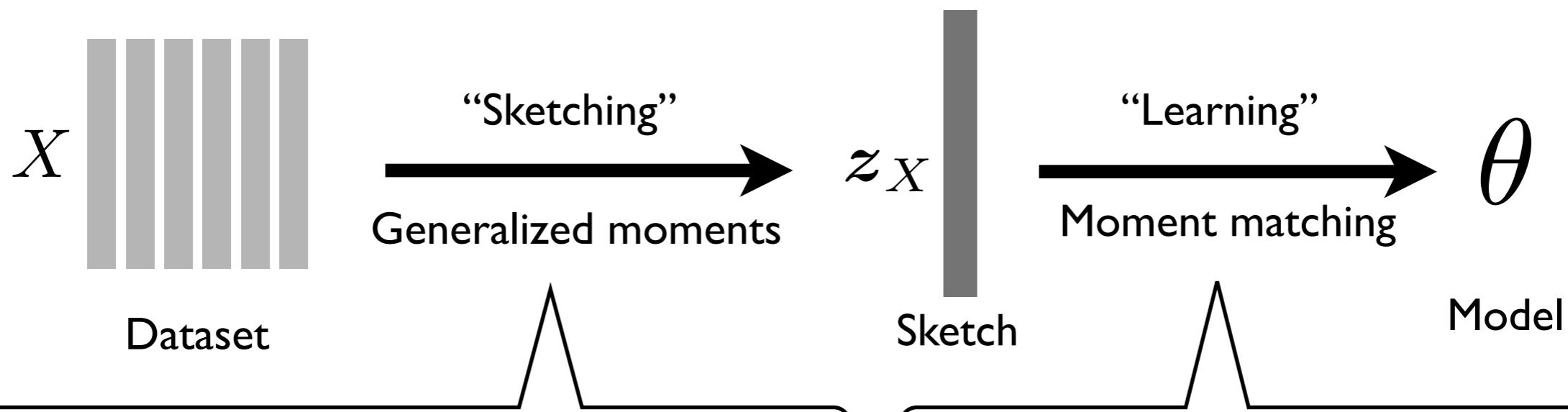
# Compressive Learning in a nutshell



## Advantages:

- **Harsh compression** (ideal for large-scale datasets, data streams)
- Only a **single pass on dataset** (parallelizable, distributable)
- Handy for **privacy preservation**

# Compressive Learning in a nutshell



Advantages:

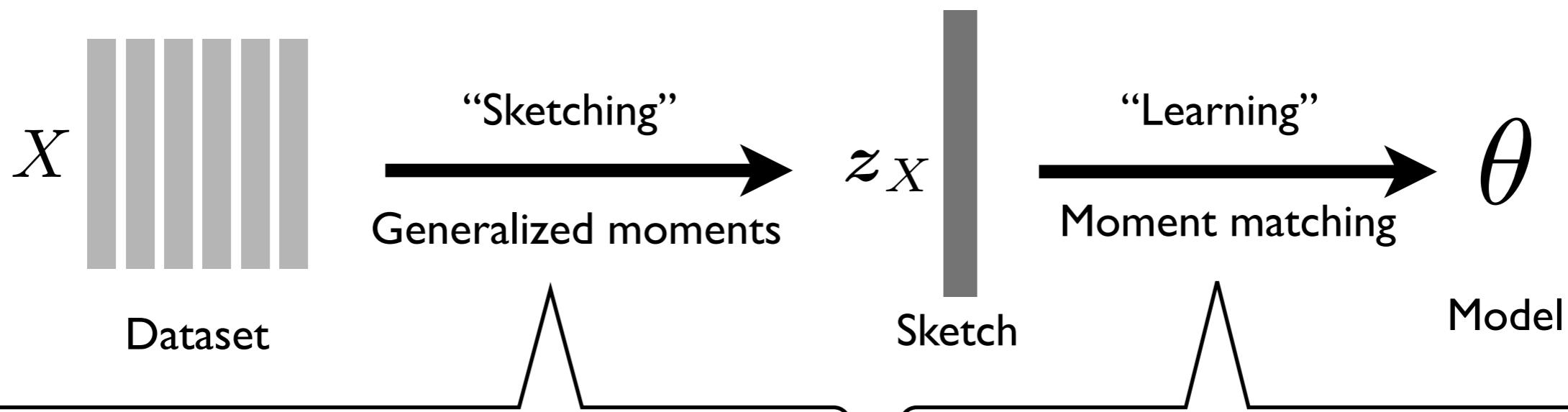
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Existing algorithms for:

- k-means clustering
- mixture model estimation: GMM, alpha-stable distributions
- Principal Component Analysis
- Independent Component Analysis

**Difficult to extend the framework to new tasks!**

# Compressive Learning in a nutshell



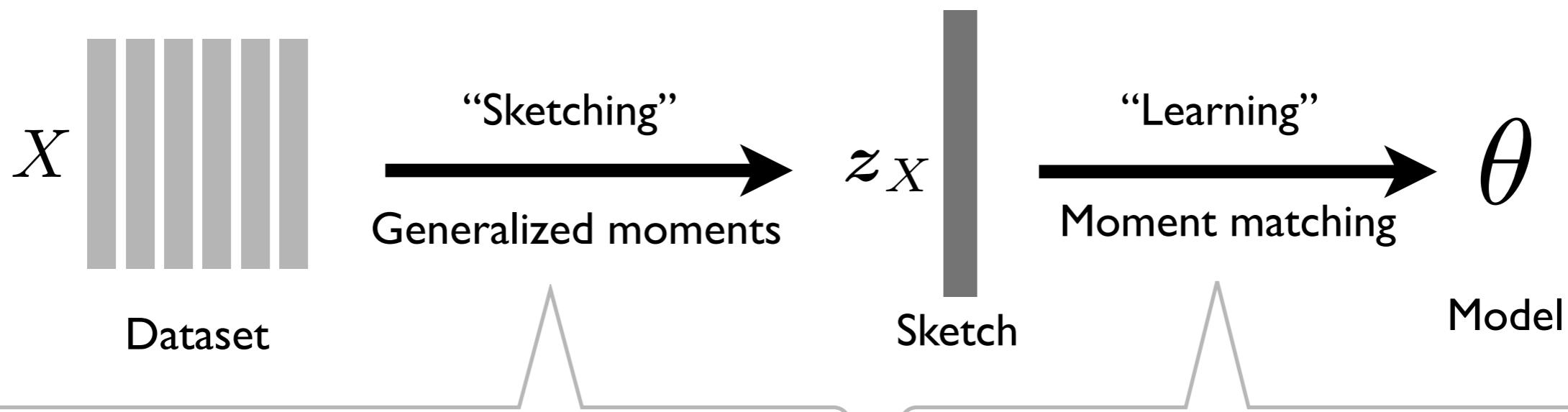
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# Compressive Learning in a nutshell



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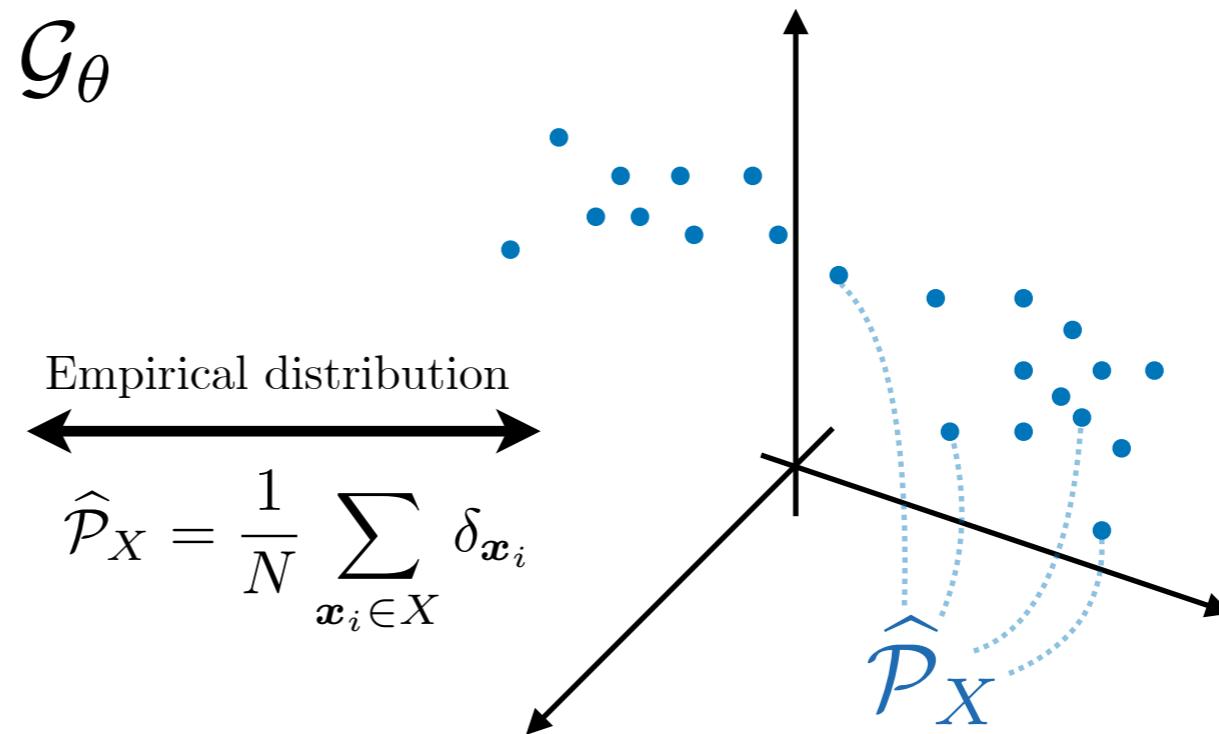
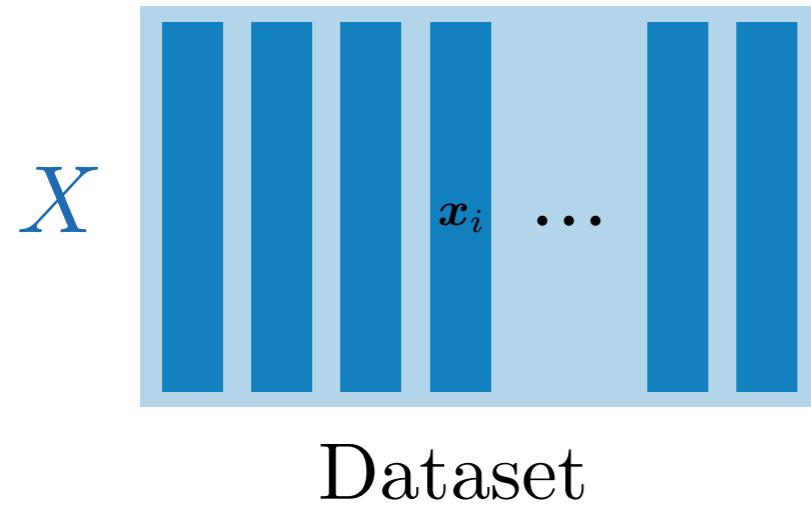
Contributions:

- Connecting literature (Compressive Learning & Generative Networks)
- Practical learning algorithm
- Qualitative proof-of-concept (toy example)
- Quantitative experimental validation (real data)

# Compressive Learning of Generative Networks

# Generative Networks: What?

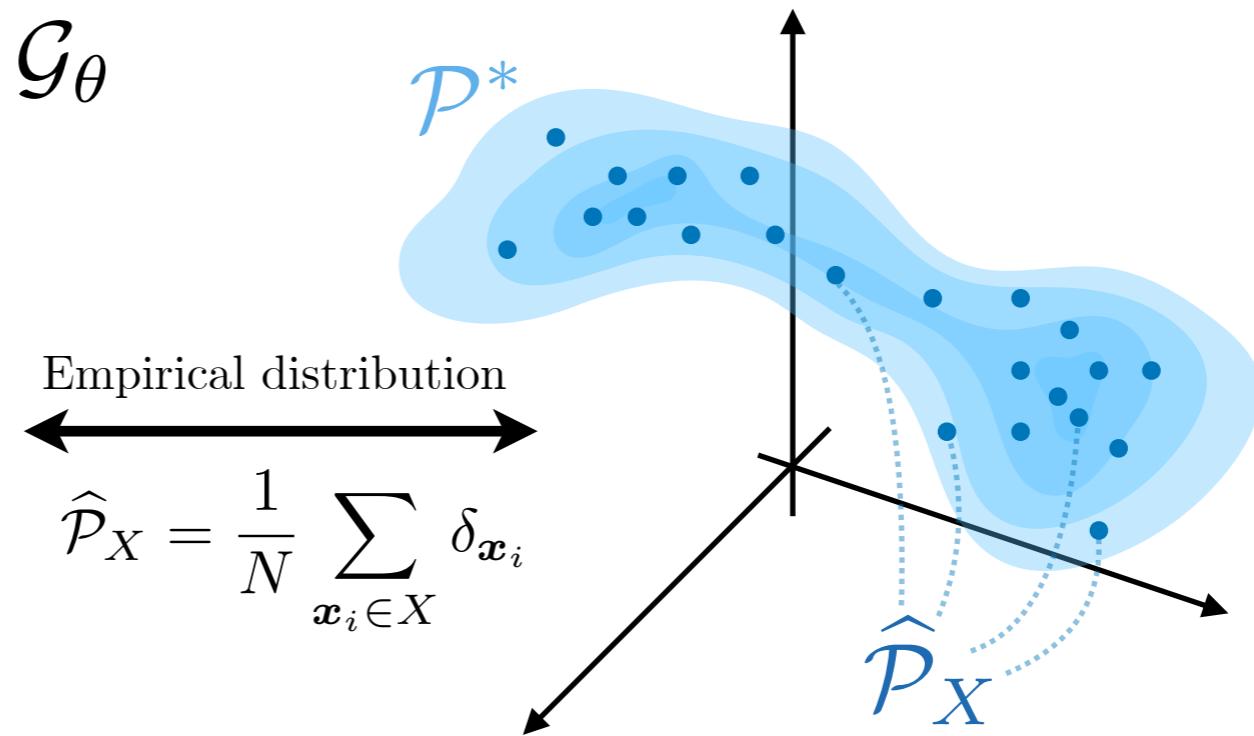
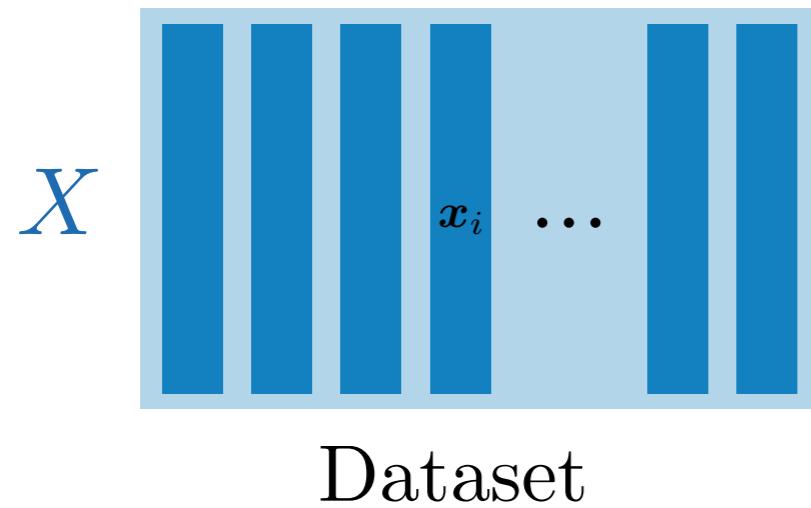
Learn a generative network  $\mathcal{G}_\theta$



We have **samples** (signals)

# Generative Networks: What?

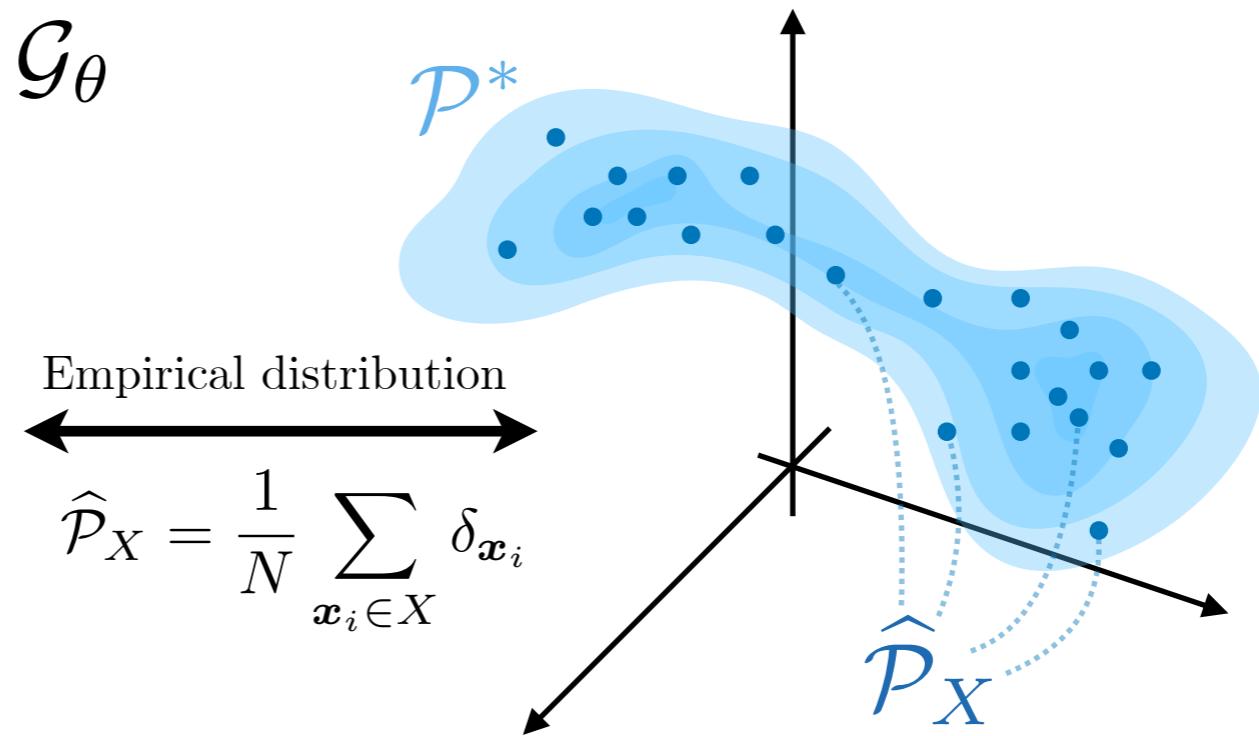
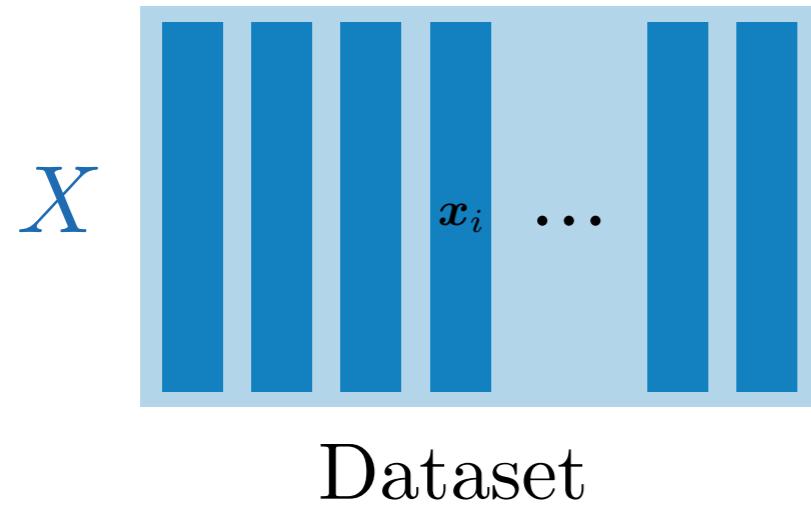
Learn a generative network  $\mathcal{G}_\theta$



We have **samples** (signals) from a high-dimensional **distribution**...

# Generative Networks: What?

Learn a generative network  $\mathcal{G}_\theta$

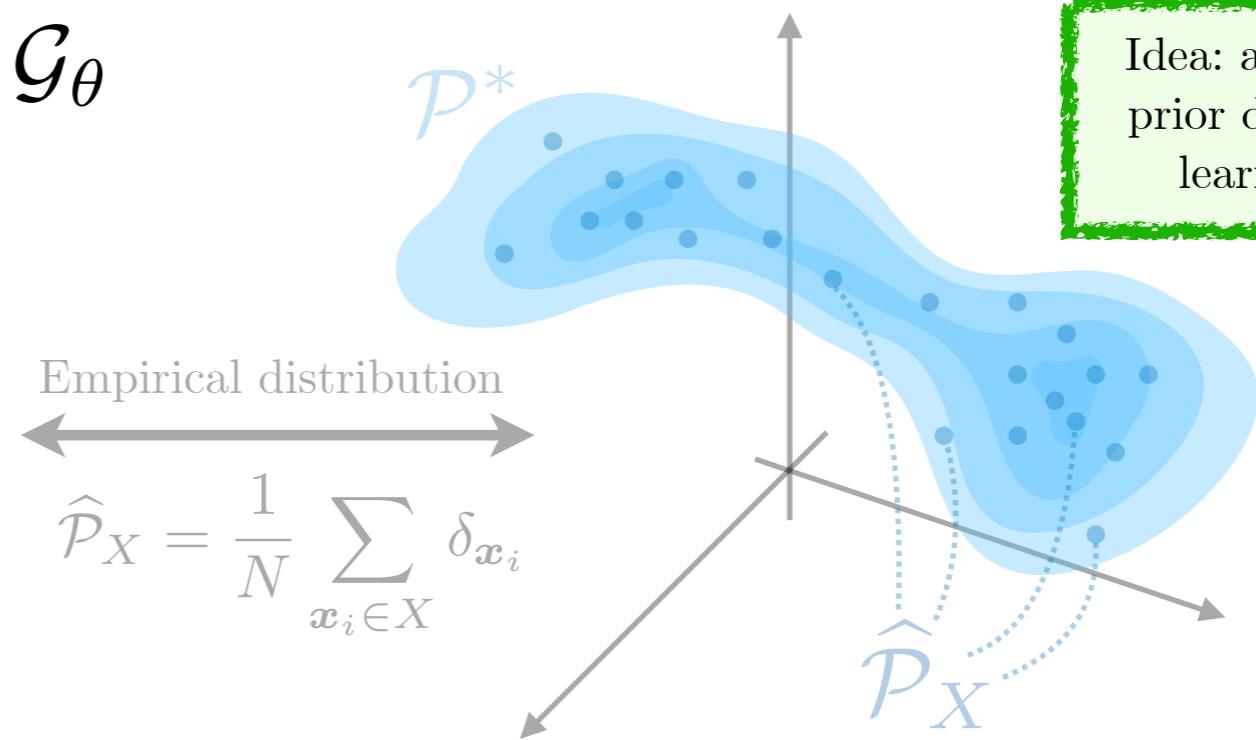
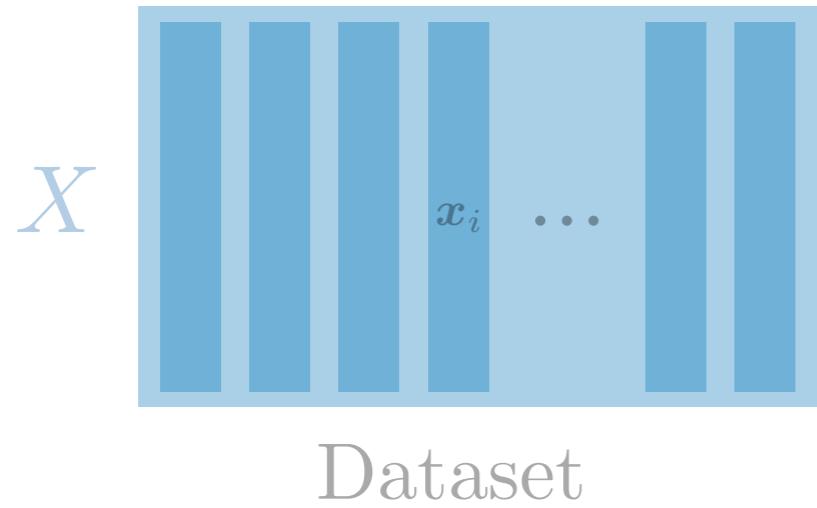


We have **samples** (signals) from a high-dimensional **distribution**...

Idea: approximate true prior distribution from learning examples

# Generative Networks: What?

Learn a generative network  $\mathcal{G}_\theta$

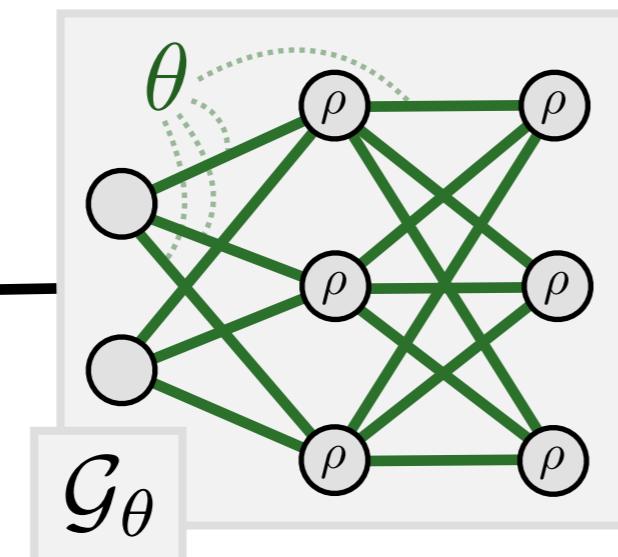
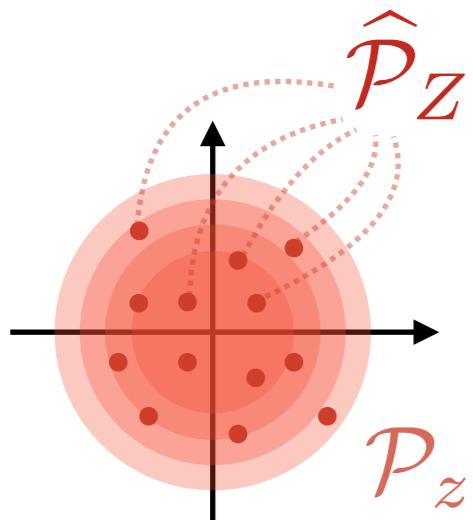


Idea: approximate true prior distribution from learning examples

We have **samples** (signals) from a high-dimensional **distribution**...

... and a way to generate “artificial” samples...

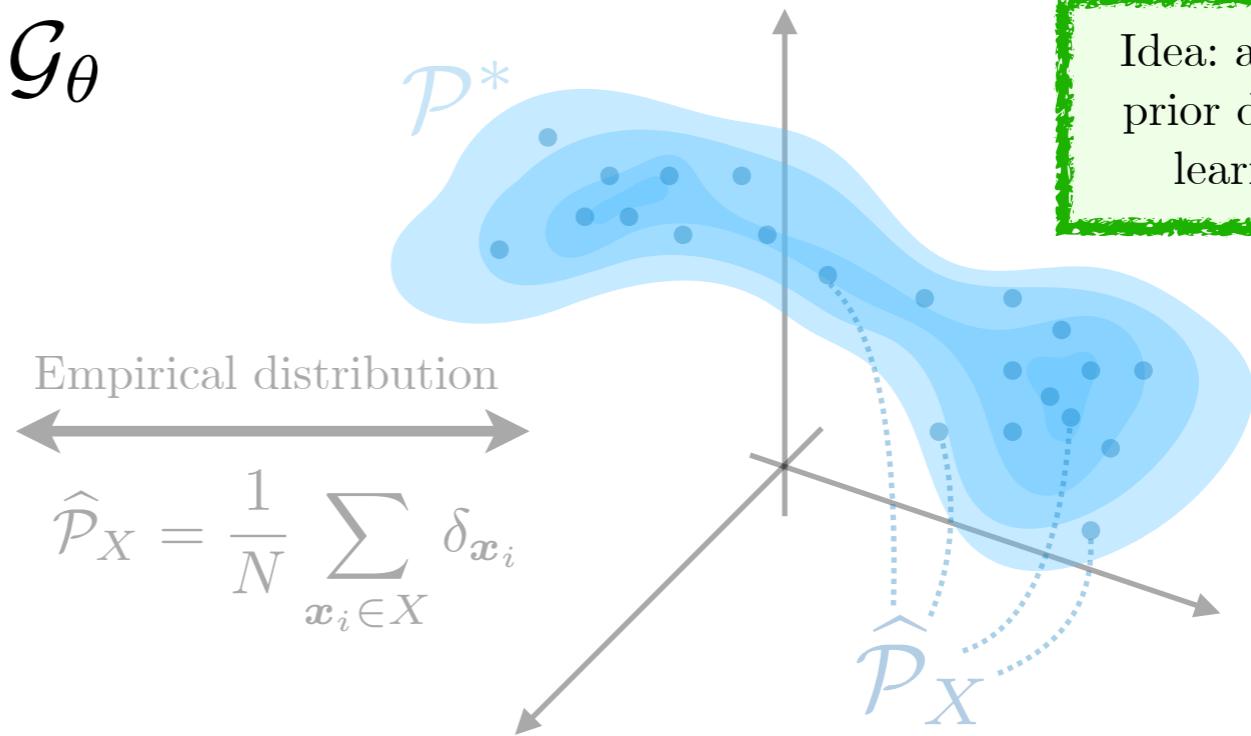
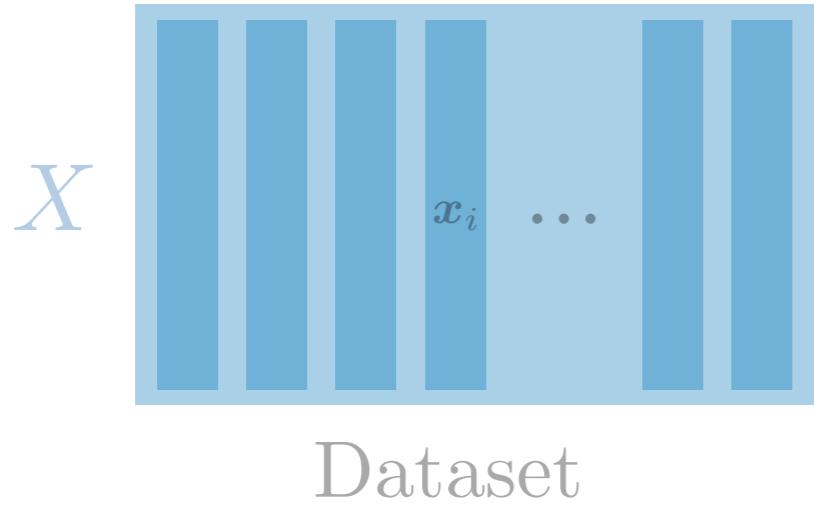
$$\mathcal{G}_\theta(\mathbf{z}) = \rho(\Theta_L \cdot \rho(\Theta_{L-1} \cdots \rho(\Theta_2 \cdot \rho(\Theta_1 \cdot \mathbf{z}))))$$



Generative network

# Generative Networks: What?

Learn a generative network  $\mathcal{G}_\theta$

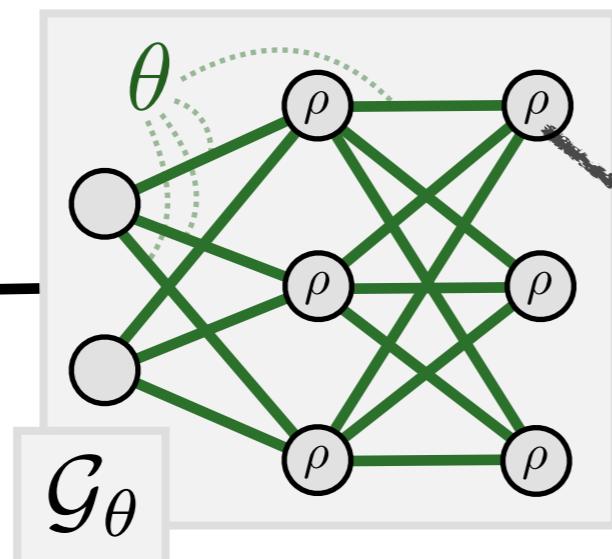
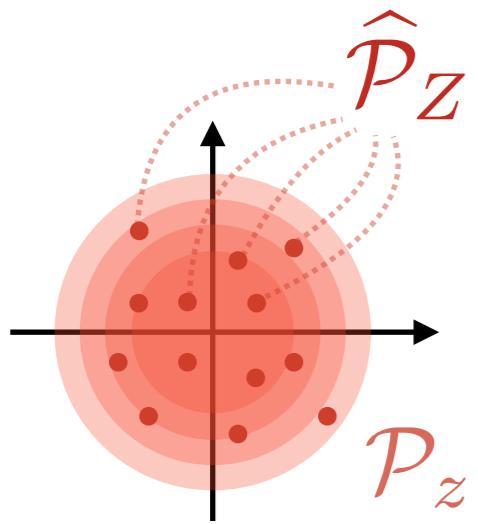


Idea: approximate true prior distribution from learning examples

We have **samples** (signals) from a high-dimensional **distribution**...

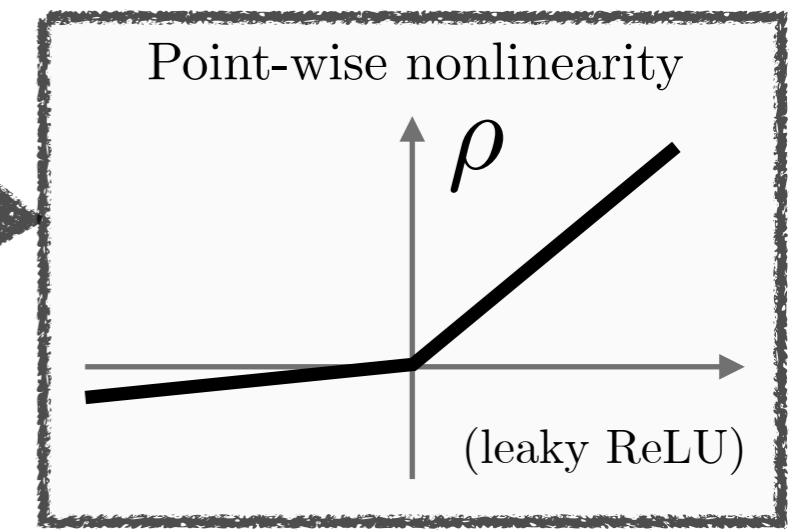
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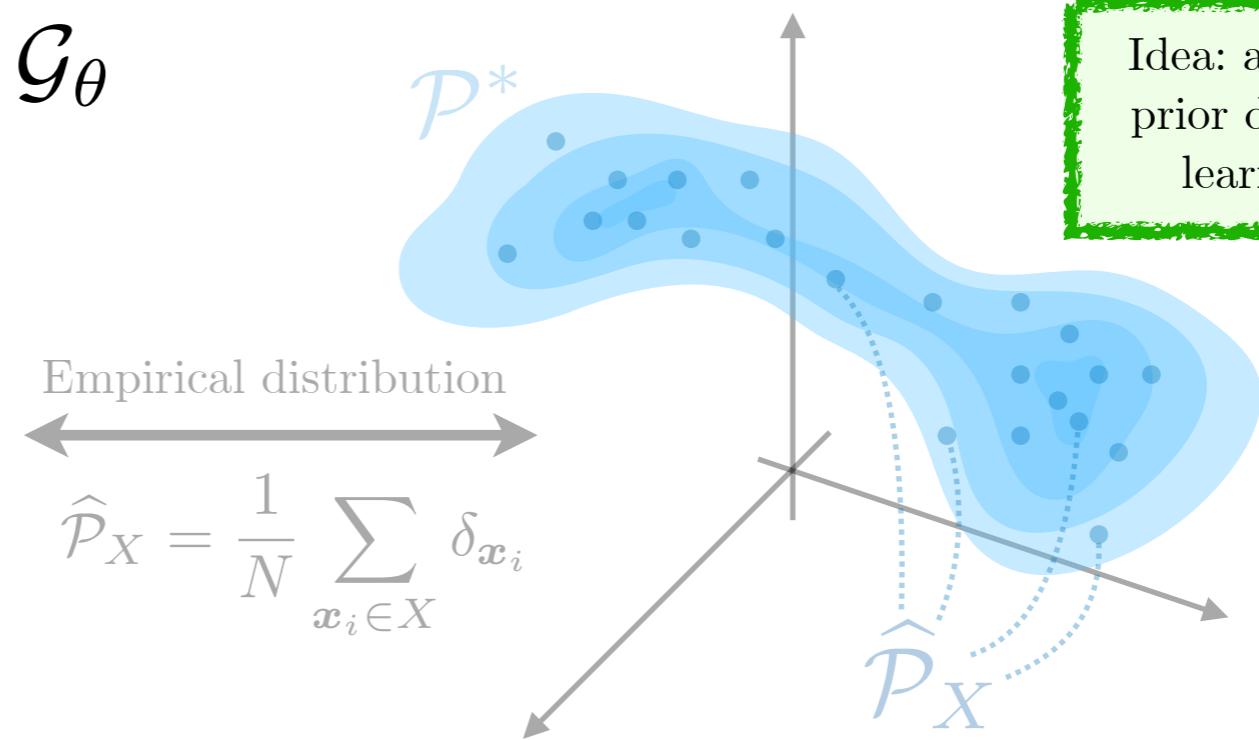
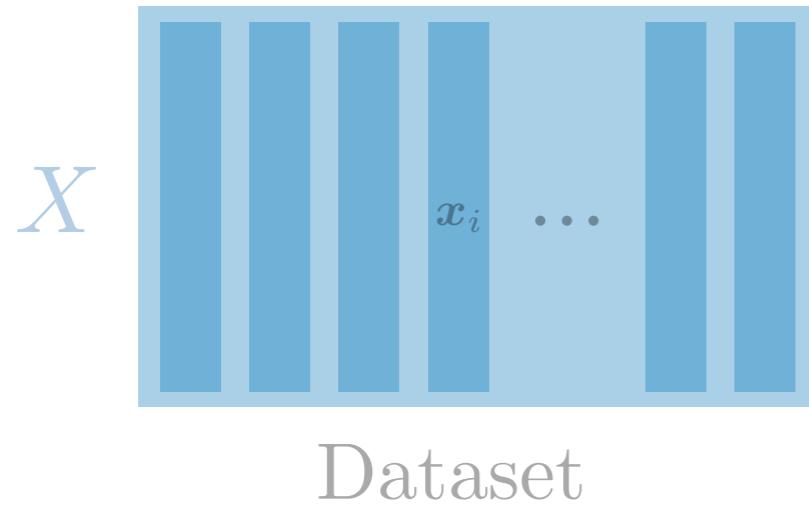
Latent space  
Random “noise”

Generative network



# Generative Networks: What?

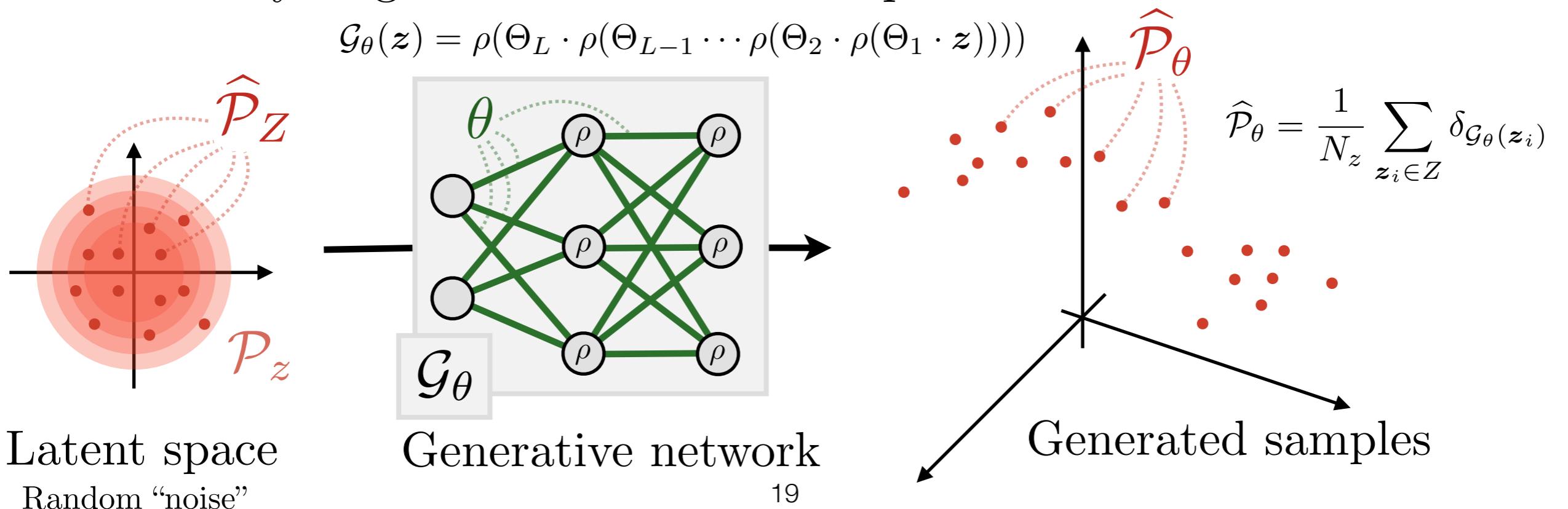
Learn a generative network  $\mathcal{G}_\theta$



Idea: approximate true prior distribution from learning examples

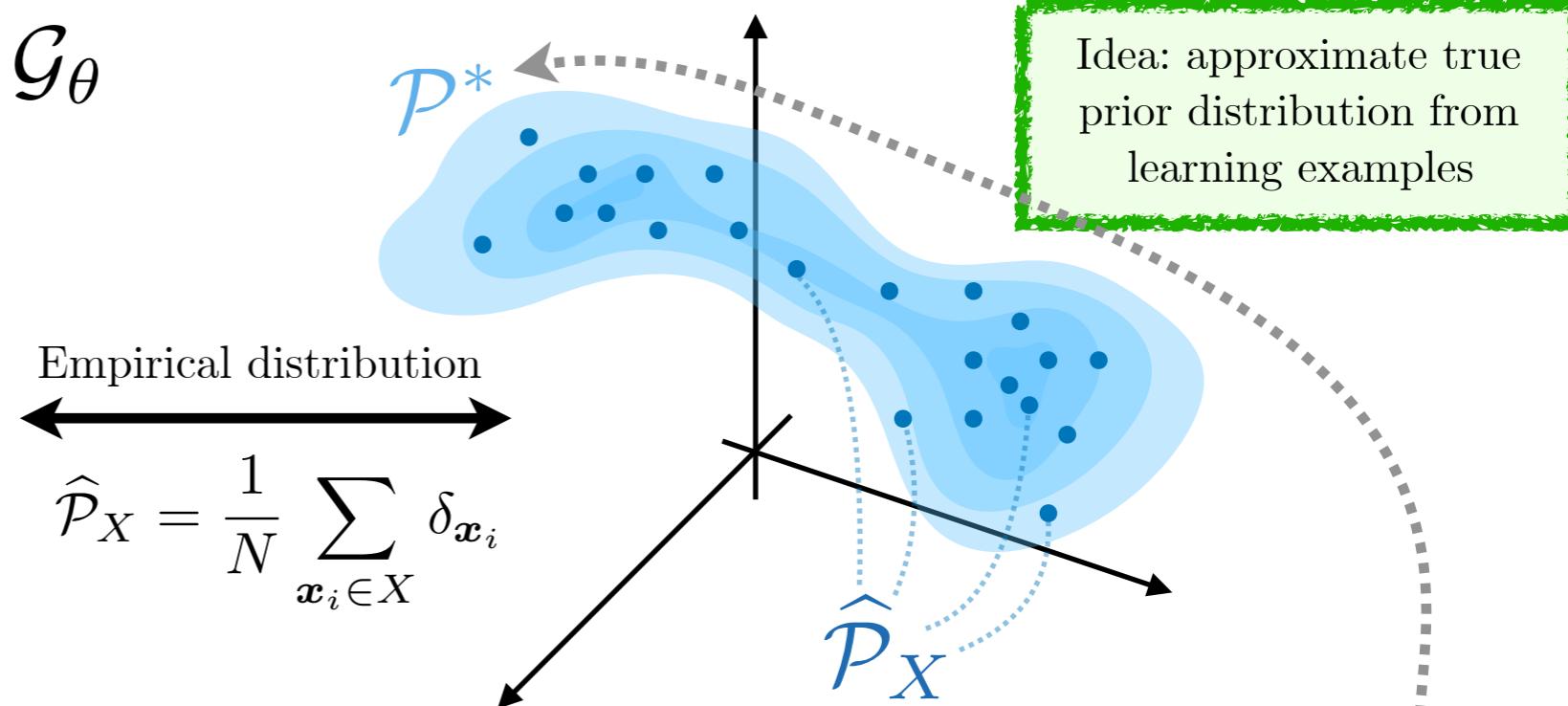
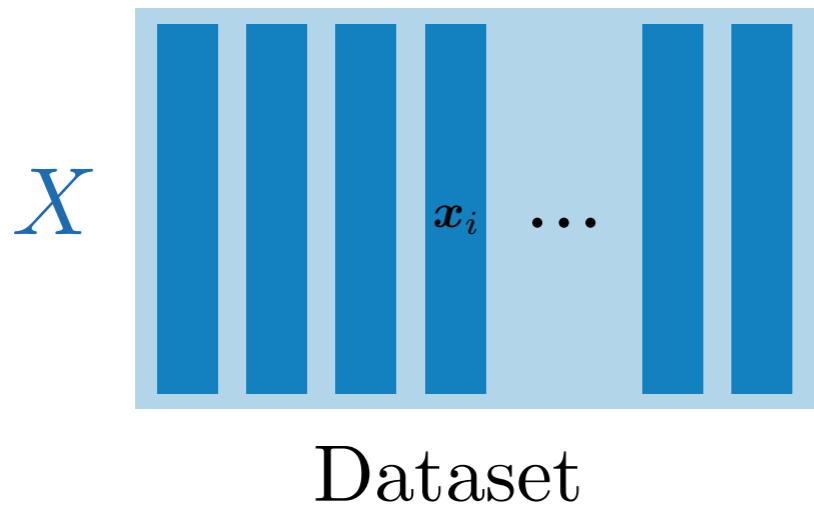
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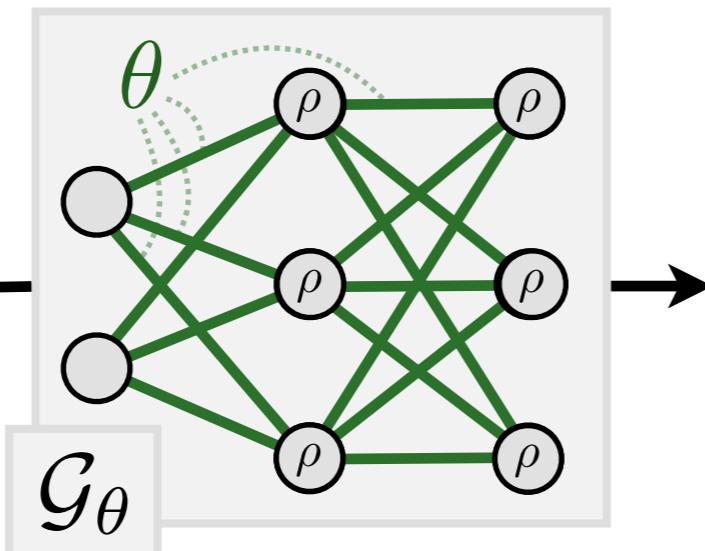
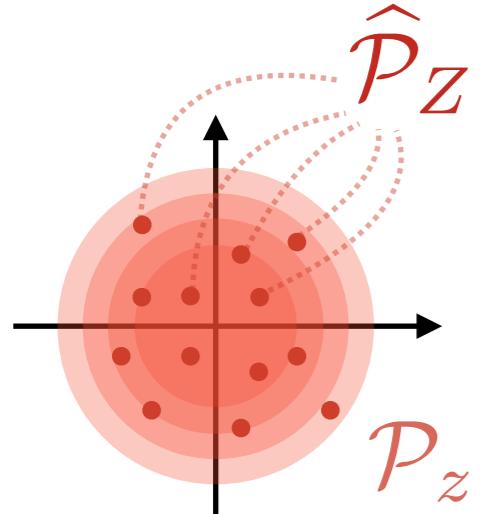


# Generative Networks: What?

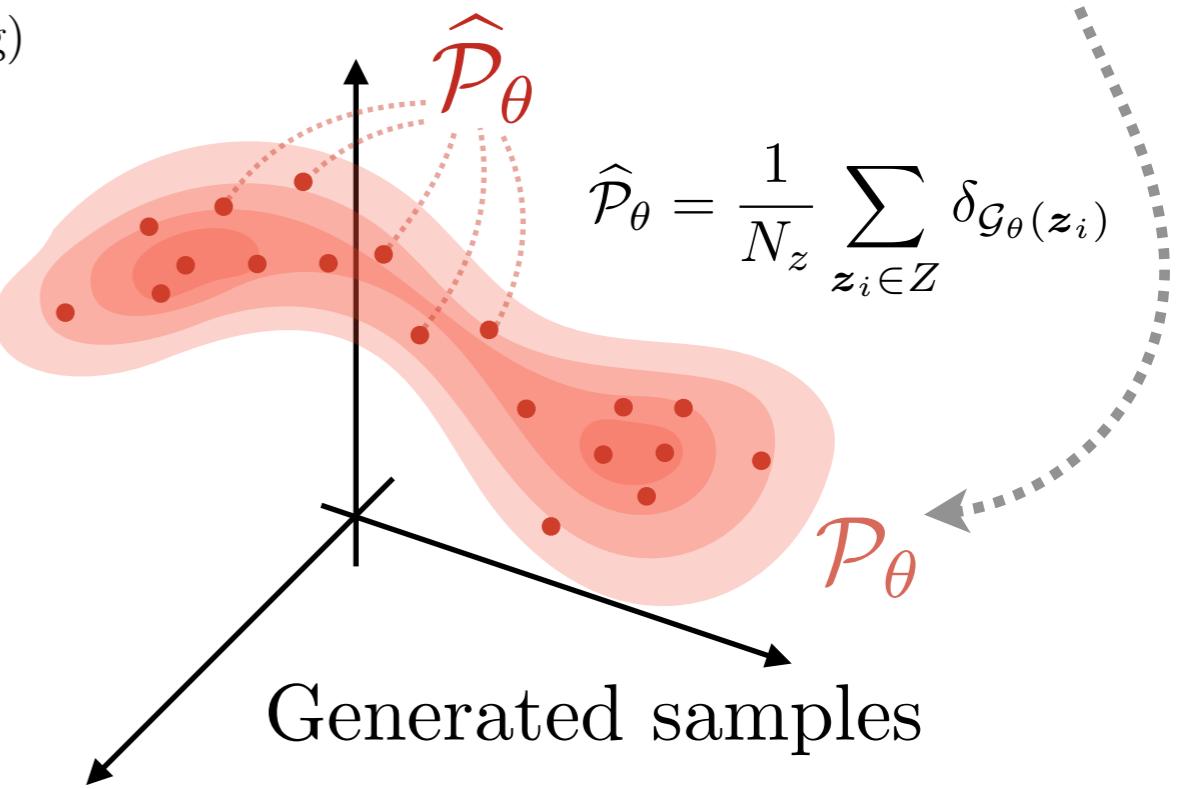
Learn a generative network  $\mathcal{G}_\theta$



Goal: mimic sampling from the data-generating distribution  
(implicit manifold learning)

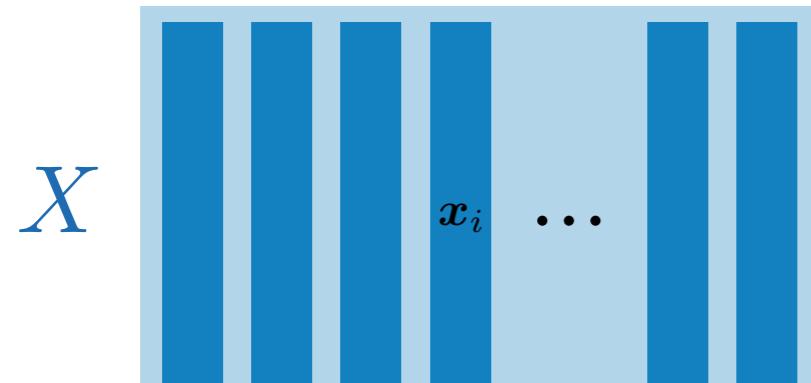


Latent space  
Random “noise”

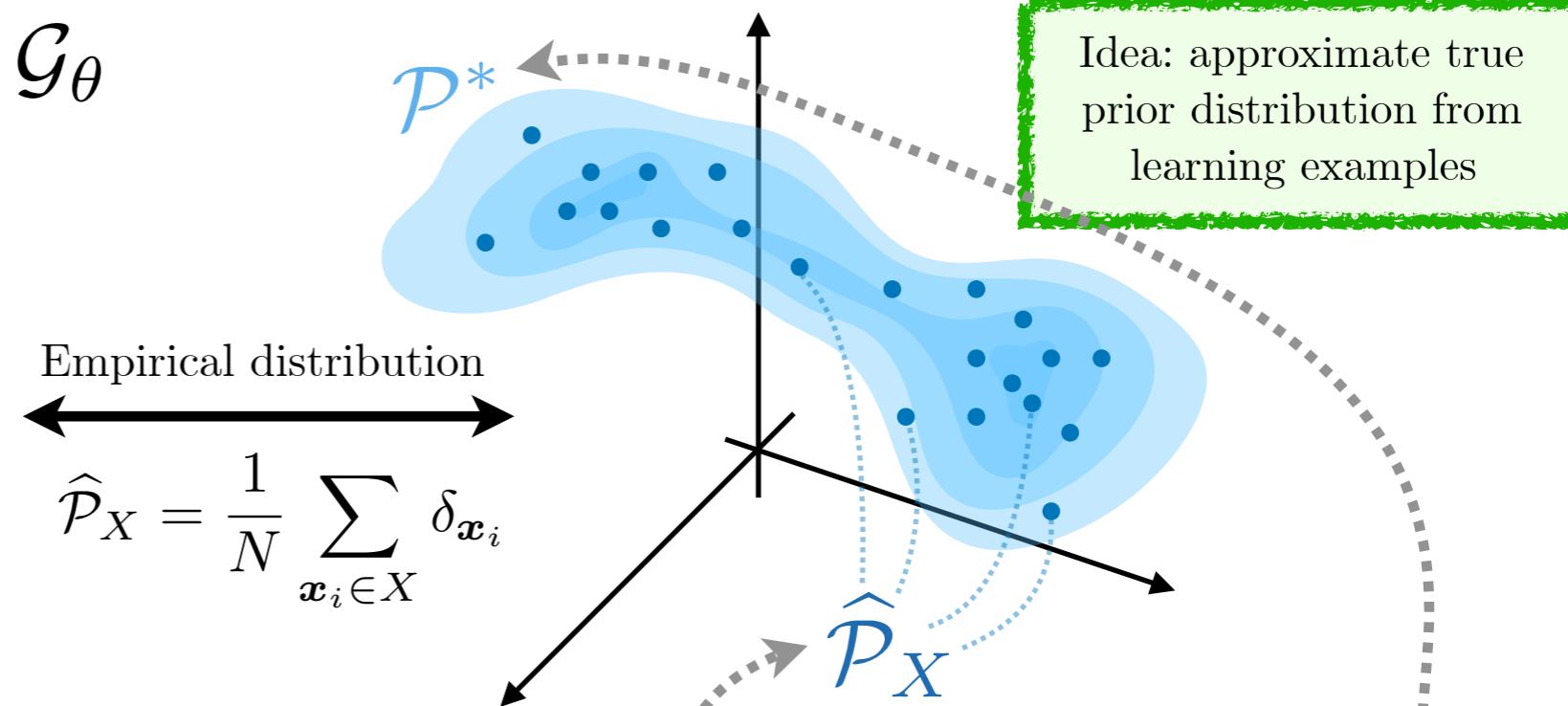


# Generative Networks: What?

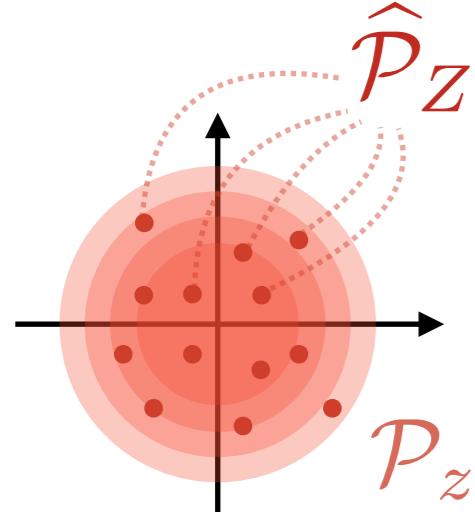
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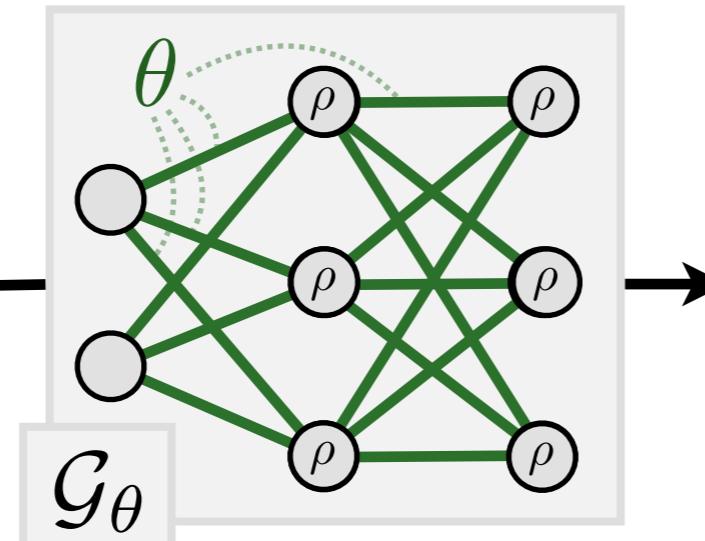
Dataset



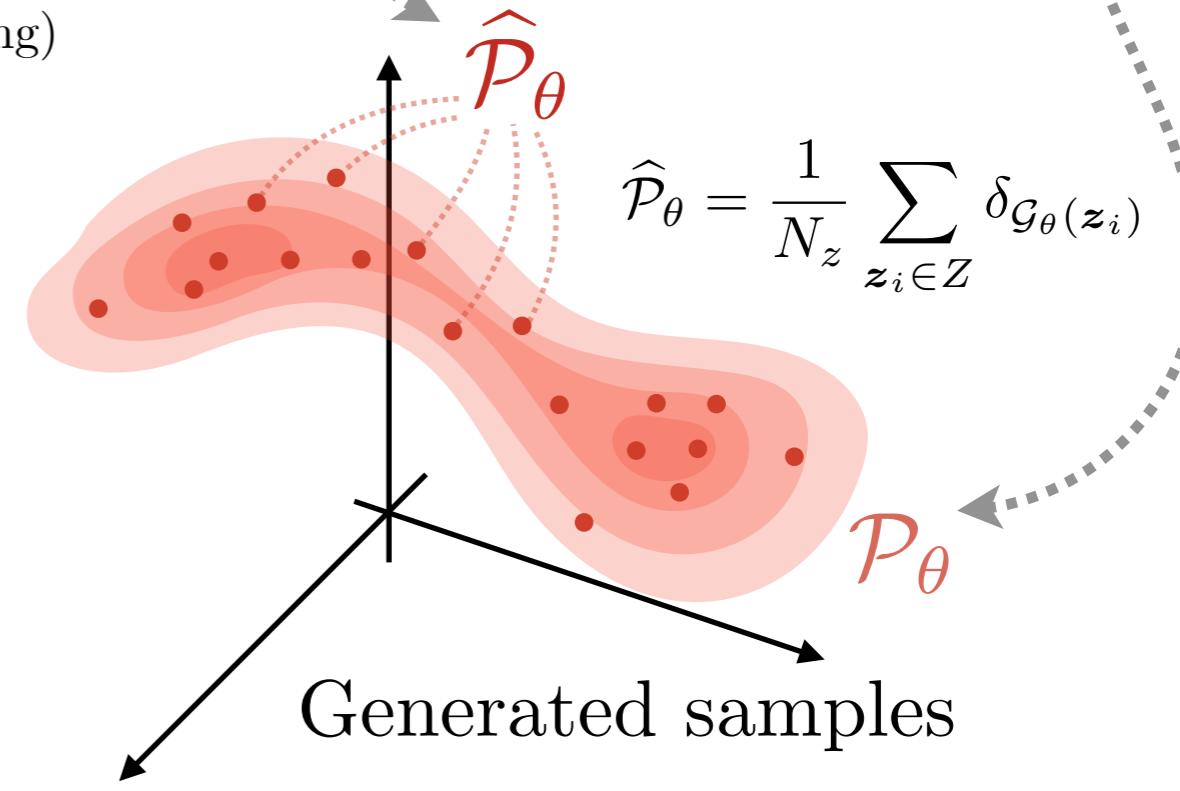
Goal: mimic sampling from the  
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Latent space  
Random “noise”



Generative network



Generated samples

# Generative Networks: Why?

Motivation (amongst others): priors in inverse problems

## SUNLayer: Stable denoising with generative networks

Dustin G. Mixon\* Soledad Villar†

### Abstract

It has been experimentally established that deep neural networks can be used to produce generative models for real world data that can efficiently solve classical inverse problems in signal processing, like compressed sensing and super resolution. In this paper, we propose a theoretical setting that uses the properties of the activation functions will allow signal denoising.

## 1 Introduction

Deep neural networks, in particular generative adversarial networks, have been used to produce generative models for real world data that can efficiently solve classical inverse problems in signal processing, like compressed sensing ([Bora et al., 2017]). The latter numerically solves the compressed sensing problem with ten times fewer measurements than required. Follow-up work by [Hand and Voroninski, 2017] recasts empirical risk minimization in the compressed sensing task by training a network with random weights and ReLU activation functions.

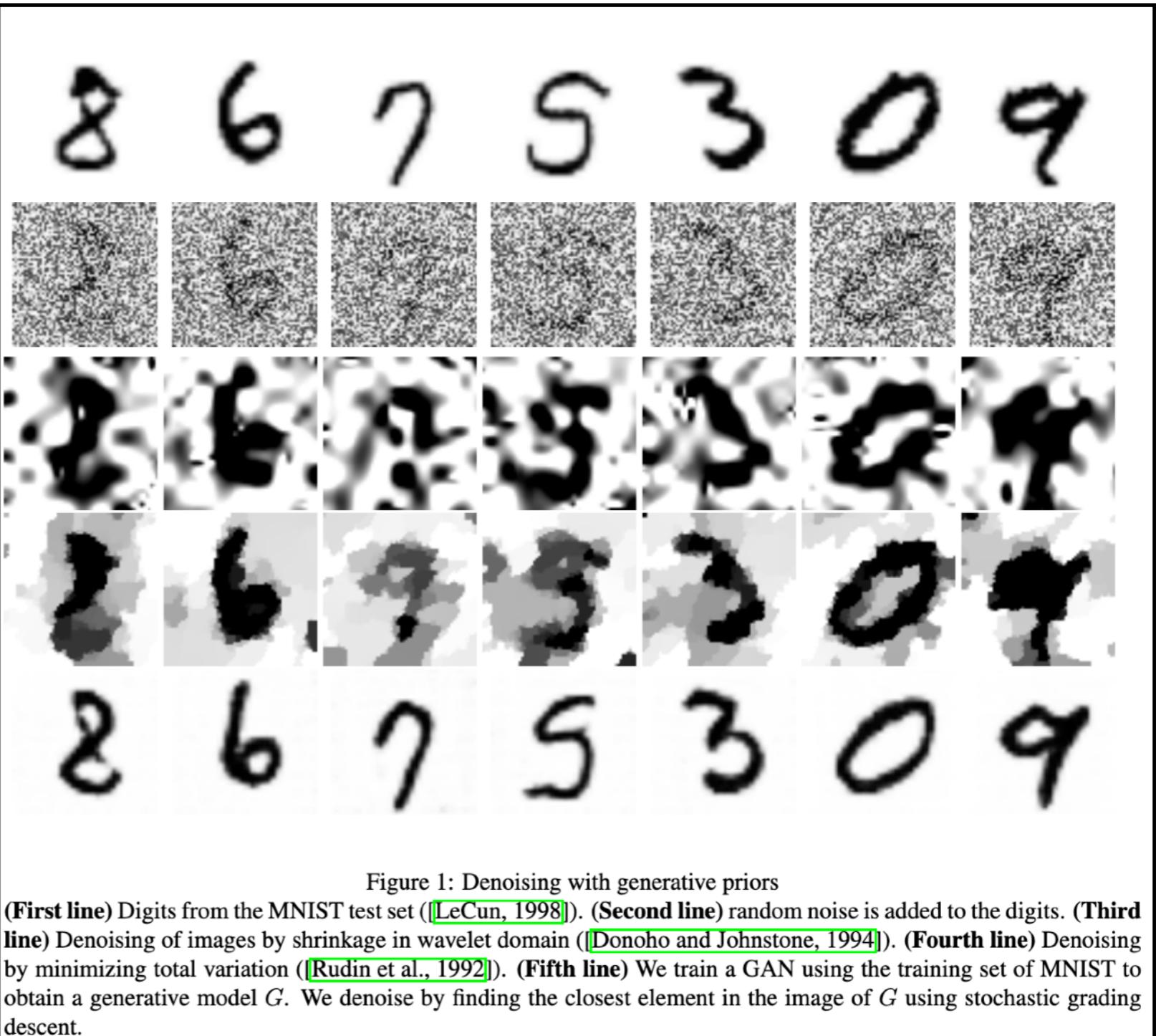


Figure 1: Denoising with generative priors

**(First line)** Digits from the MNIST test set ([LeCun, 1998]). **(Second line)** random noise is added to the digits. **(Third line)** Denoising of images by shrinkage in wavelet domain ([Donoho and Johnstone, 1994]). **(Fourth line)** Denoising by minimizing total variation ([Rudin et al., 1992]). **(Fifth line)** We train a GAN using the training set of MNIST to obtain a generative model  $G$ . We denoise by finding the closest element in the image of  $G$  using stochastic gradient descent.

# Generative Networks: Why?

Motivation (amongst others): priors in inverse problems

## Blind Image Deconvolution using Deep Generative Priors

Muhammad Asim\*, Fahad Shamshad\*, and Ali Ahmed

**Abstract**—This paper proposes a novel approach to regularize the *ill-posed* and *non-linear* blind image deconvolution (blind deblurring) using deep generative networks as priors. We employ two separate generative models — one trained to produce sharp images while the other trained to generate blur kernels from lower-dimensional parameters. To deblur, we propose an alternating gradient descent scheme operating in the latent lower-dimensional space of each of the pretrained generative models. Our experiments show promising deblurring results on images even under large blurs, and heavy noise. To address the shortcomings of generative models such as mode collapse, we augment our generative priors with classical image priors and report improved performance on complex image datasets. The deblurring performance depends on how well the range of the generator spans the image class. Interestingly, our experiments show that even an untrained structured (convolutional) generative networks acts as an image prior in the image deblurring context allowing us to extend our results to more diverse natural image datasets.

**Index Terms**—Blind image deblurring, generative adversarial networks, variational autoencoders, deep image prior.

### I. INTRODUCTION

**B**LIND image deblurring aims to recover a true image  $\mathbf{B}_i$  and a blur kernel  $k$  from blurry and possibly noisy observation  $y$ . For a uniform and spatially invariant blur, it can be mathematically formulated as

$$y = i \otimes k + n, \quad (1)$$

where  $\otimes$  is a convolution operator and  $n$  is an additive Gaussian noise. In its full generality, the inverse problem (1) is severely ill-posed as many different instances of  $i$ , and  $k$  fit the observation  $y$ . See [1] for a thorough discussion on

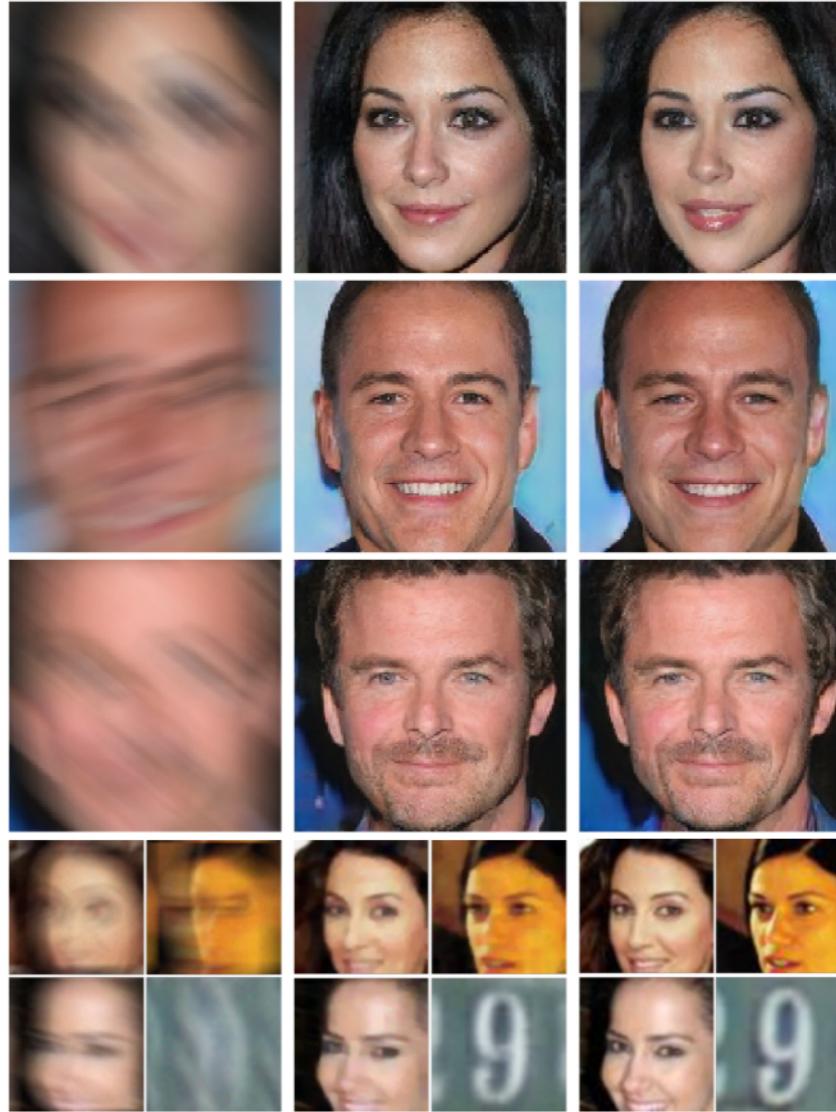
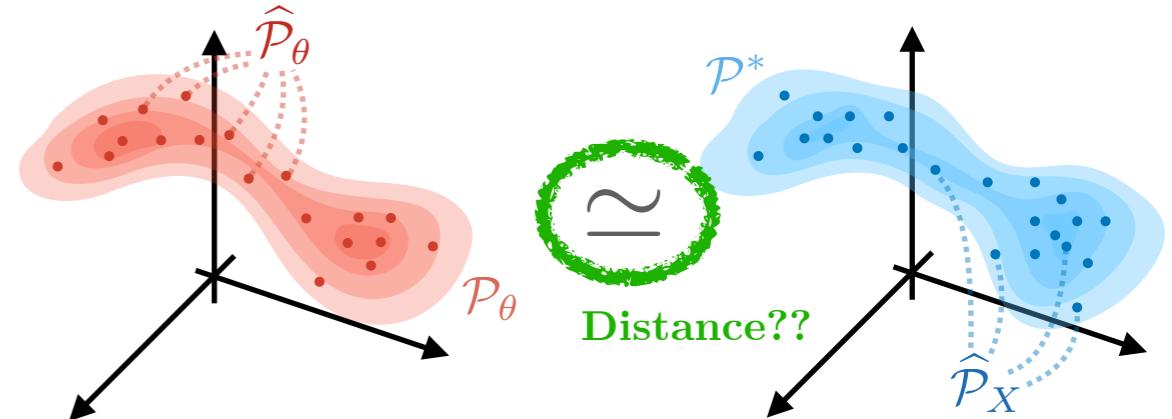


Fig. 1: Blind image deblurring using deep generative priors.

# Generative Networks: How?

How to learn the generative network  $\mathcal{G}_\theta$

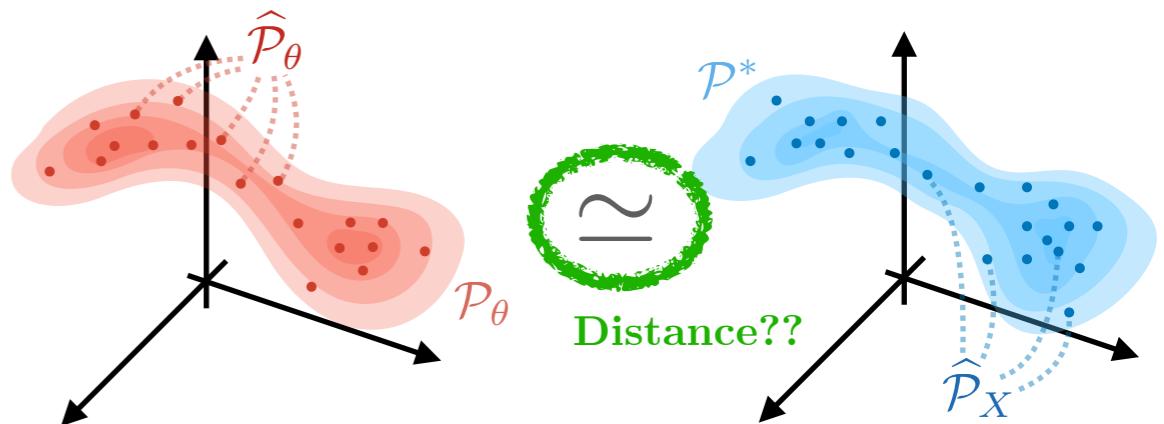
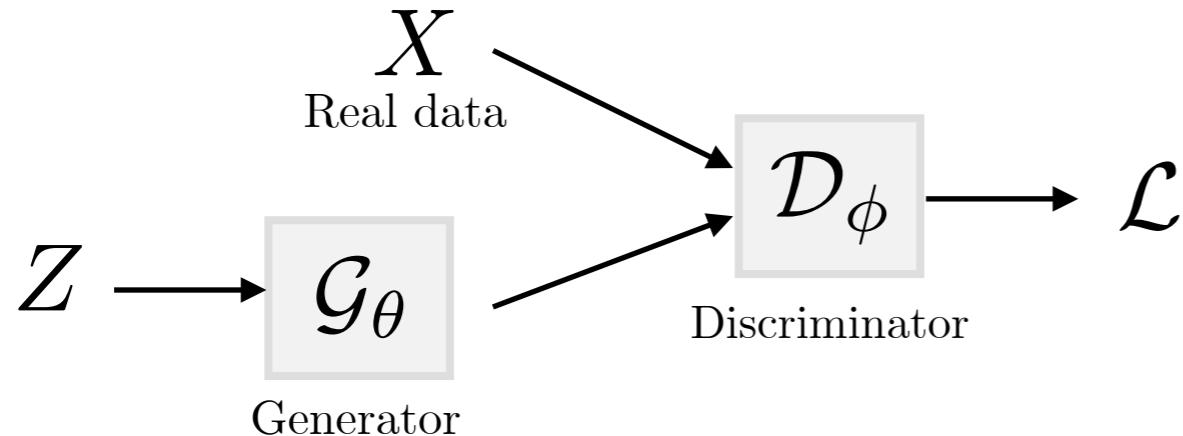


# Generative Networks: How?

How to learn the generative network  $\mathcal{G}_\theta$

1) Golden standard: Generative Adversarial Networks

Learn a second “discriminator” network that classifies real/fake at the same time as the generator

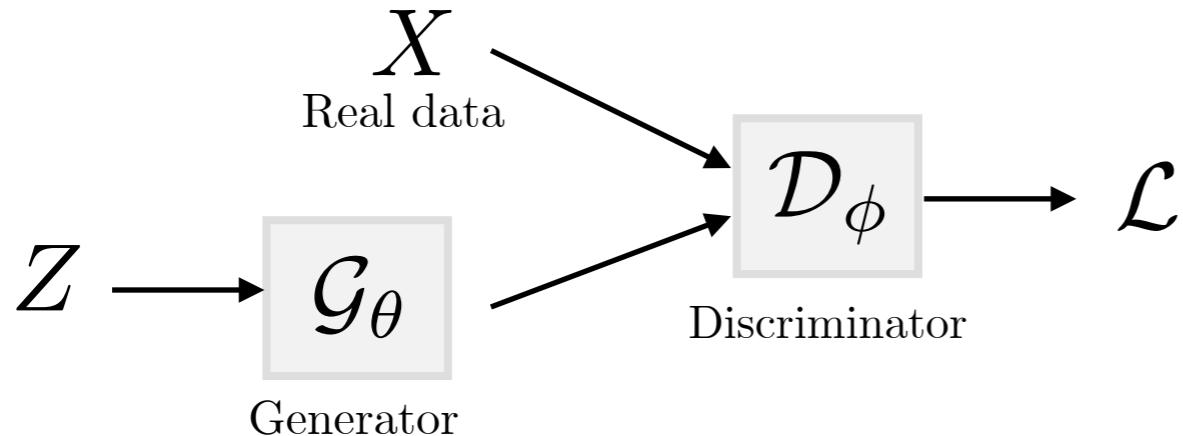
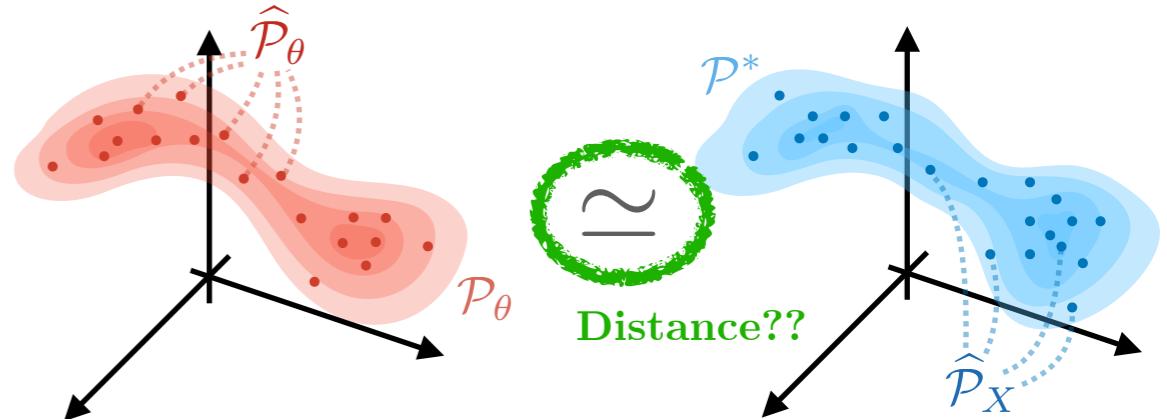


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Very difficult to train (due to balancing of training discriminator/generator)

$$\min_{\theta} \max_{\phi} \mathcal{L}(\theta, \phi)$$

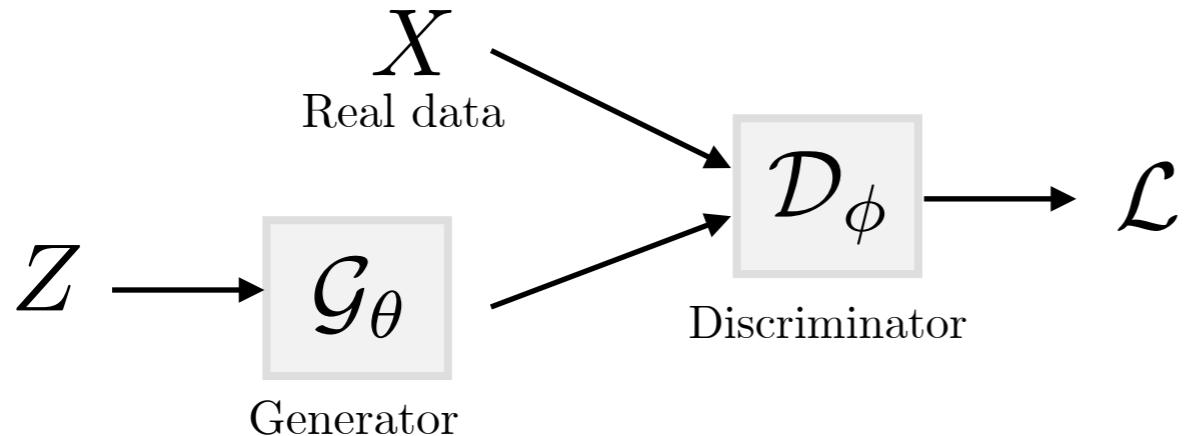
Lots of research interest in other (easier) methods

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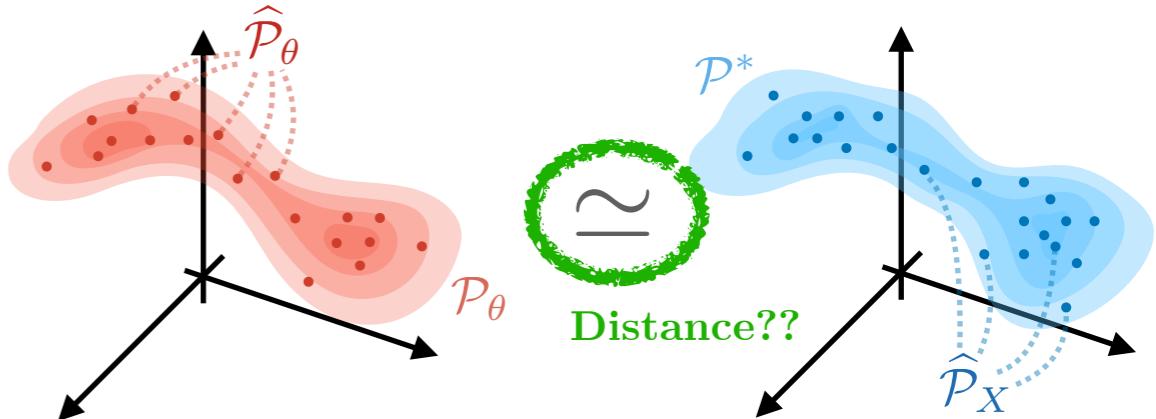
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$$\min_{\theta} \max_{\phi} \mathcal{L}(\theta, \phi)$$

2) Maximum Mean Discrepancy

$$\min_{\theta} \text{MMD}_\kappa(\hat{\mathcal{P}}_X, \hat{\mathcal{P}}_\theta)$$

Easier to train (no balancing)...

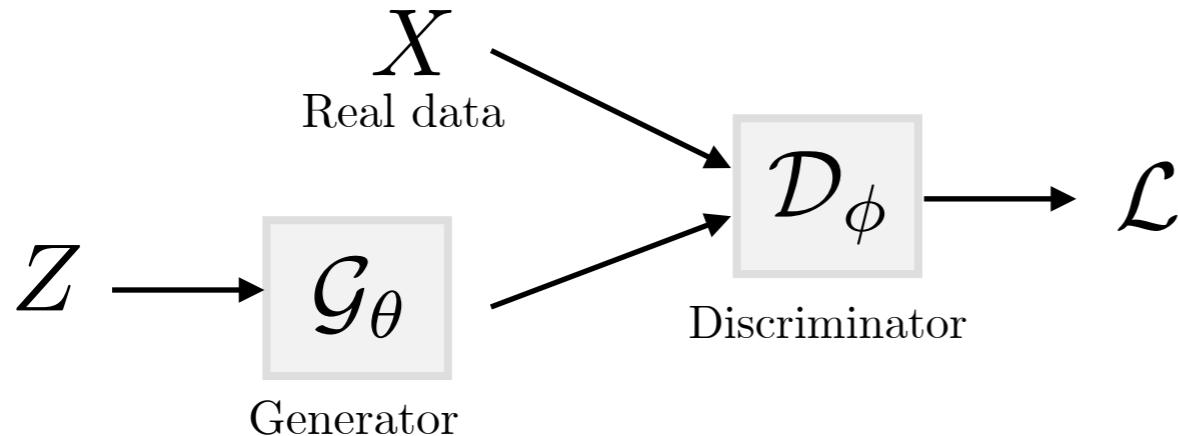
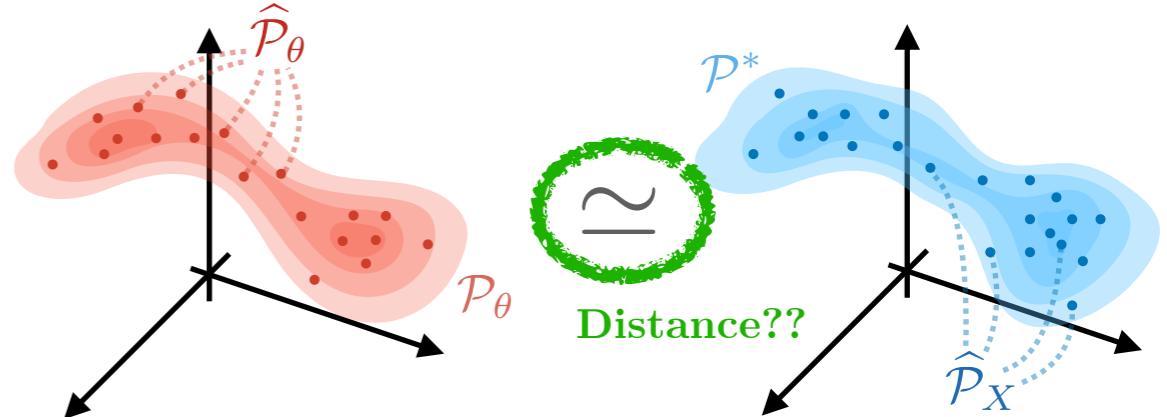


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$$\sum_{\substack{\mathbf{x}_i \in X \\ \mathbf{x}_j \in X}} \kappa(\mathbf{x}_i, \mathbf{x}_j) - 2 \sum_{\substack{\mathbf{x}_i \in X \\ \mathbf{z}_j \in Z}} \kappa(\mathbf{x}_i, \mathcal{G}_\theta(\mathbf{z}_j)) + \sum_{\substack{\mathbf{z}_i \in Z \\ \mathbf{z}_j \in Z}} \kappa(\mathcal{G}_\theta(\mathbf{z}_i), \mathcal{G}_\theta(\mathbf{z}_j))$$

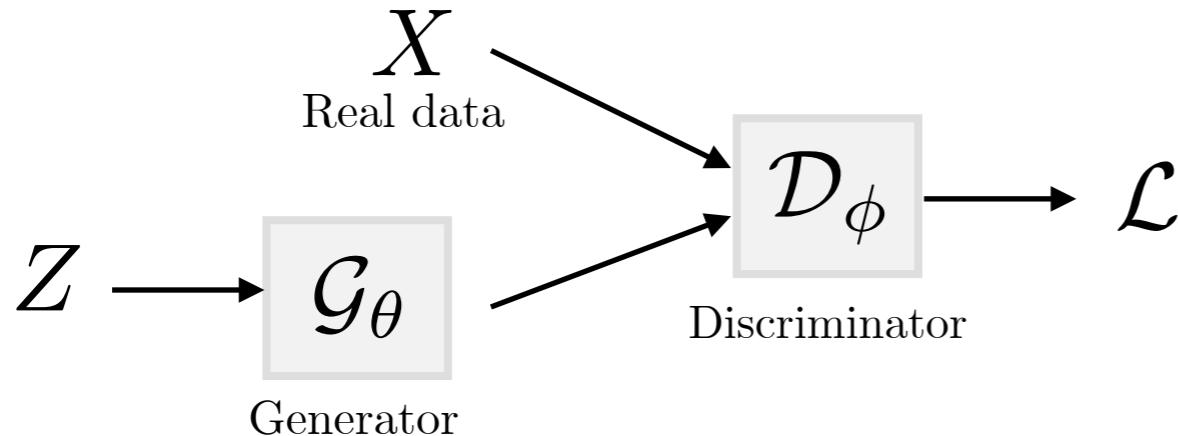
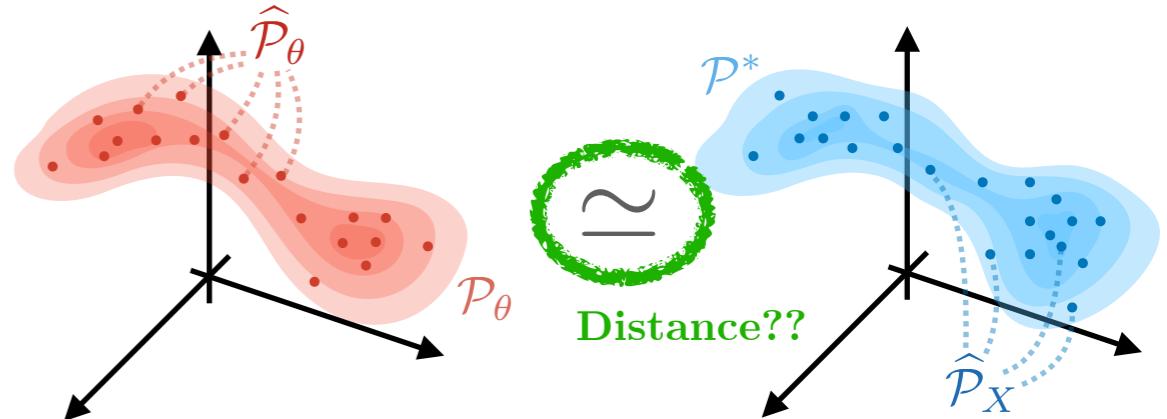
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2) Maximum Mean Discrepancy

$$\min_{\theta} \text{MMD}_{\kappa}(\hat{\mathcal{P}}_X, \hat{\mathcal{P}}_\theta)$$

Similarity between samples

$$\sum_{\mathbf{x}_i \in X} \sum_{\mathbf{x}_j \in X} \kappa(\mathbf{x}_i, \mathbf{x}_j) - 2 \sum_{\mathbf{x}_i \in X} \sum_{z_j \in Z} \kappa(\mathbf{x}_i, \mathcal{G}_\theta(z_j)) + \sum_{z_i \in Z} \sum_{z_j \in Z} \kappa(\mathcal{G}_\theta(z_i), \mathcal{G}_\theta(z_j))$$

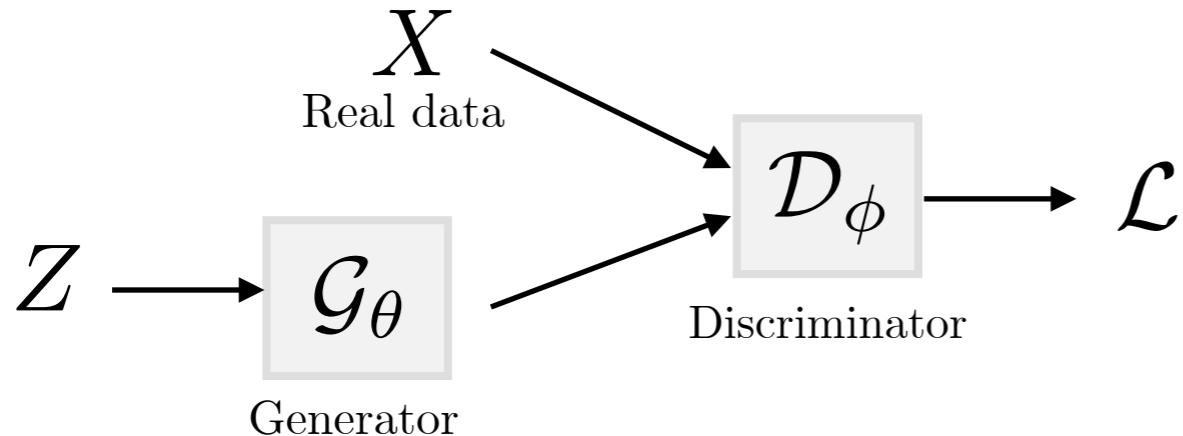
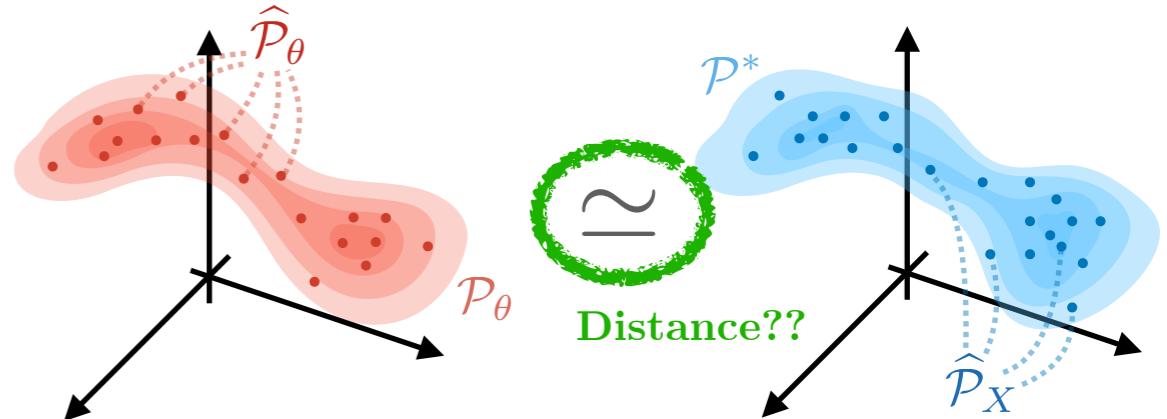
Easier to train (no balancing)...

# Generative Networks: How?

How to learn the generative network  $\mathcal{G}_\theta$

1) Golden standard: Generative Adversarial Networks

Learn a second “discriminator” network that classifies real/fake at the same time as the generator



Very difficult to train (due to balancing of training discriminator/generator)

$$\min_{\theta} \max_{\phi} \mathcal{L}(\theta, \phi)$$

2) Maximum Mean Discrepancy

$$\min_{\theta} \text{MMD}_{\kappa}(\hat{\mathcal{P}}_X, \hat{\mathcal{P}}_\theta)$$

Similarity between samples

$$\sum_{\mathbf{x}_i \in X} \sum_{\mathbf{x}_j \in X} \kappa(\mathbf{x}_i, \mathbf{x}_j) - 2 \sum_{\mathbf{x}_i \in X} \sum_{\mathbf{z}_j \in Z} \kappa(\mathbf{x}_i, \mathcal{G}_\theta(\mathbf{z}_j)) + \sum_{\mathbf{z}_i \in Z} \sum_{\mathbf{z}_j \in Z} \kappa(\mathcal{G}_\theta(\mathbf{z}_i), \mathcal{G}_\theta(\mathbf{z}_j))$$

equivalent to (for later)

$$\mathbb{E}_{\omega \sim \Lambda} \left| \sum_{\mathbf{x}_i \in X} e^{i\omega^T \mathbf{x}_i} - \sum_{\mathbf{z}_i \in Z} e^{i\omega^T \mathcal{G}_\theta(\mathbf{z}_i)} \right|^2$$

Easier to train (no balancing)...

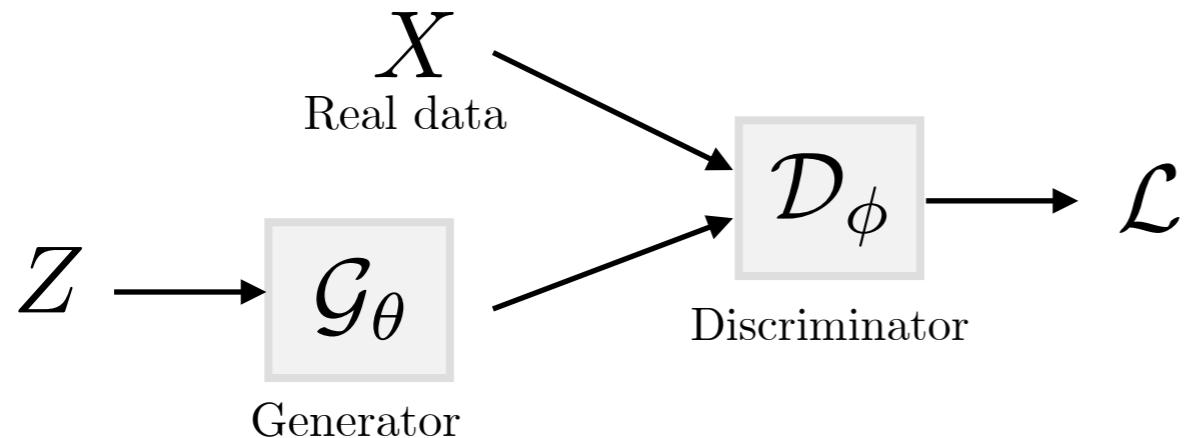
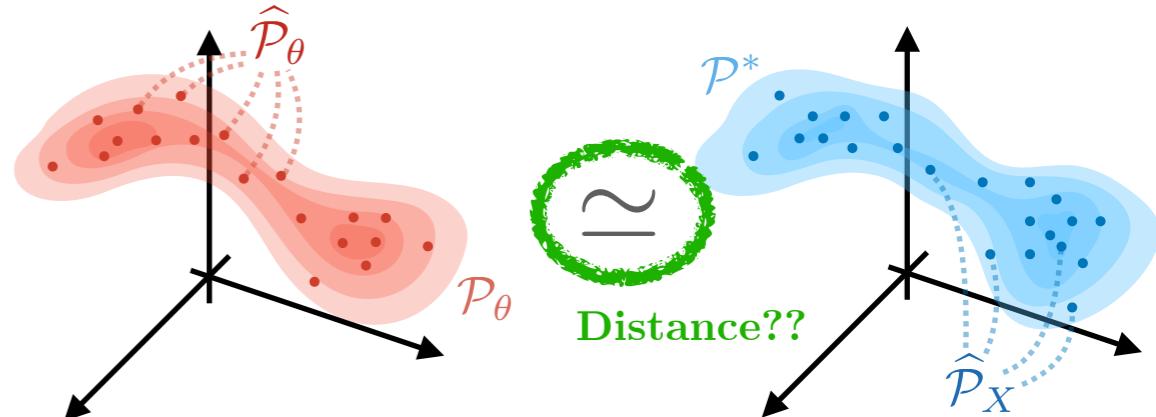
With  $\Lambda = \mathcal{F}\kappa$

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Easier to train (no balancing) but quadratic complexity...

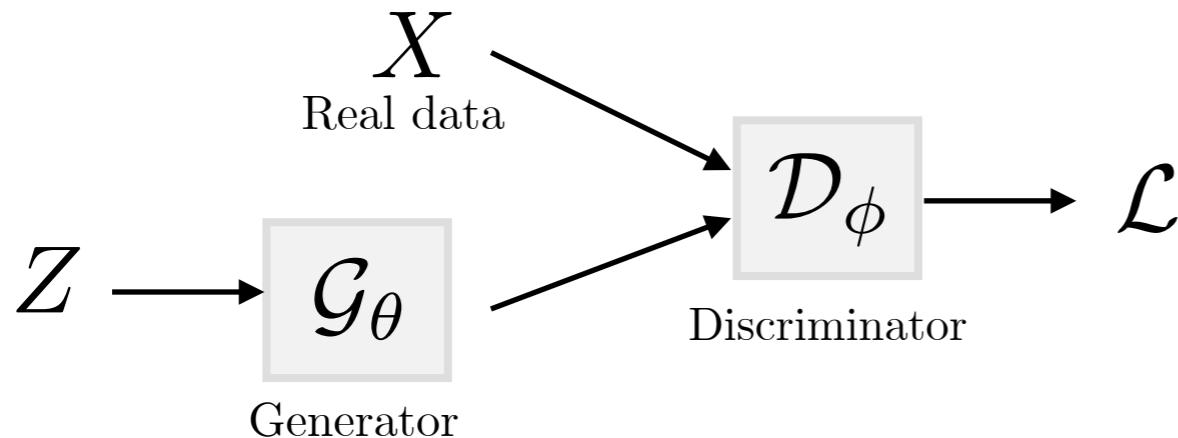
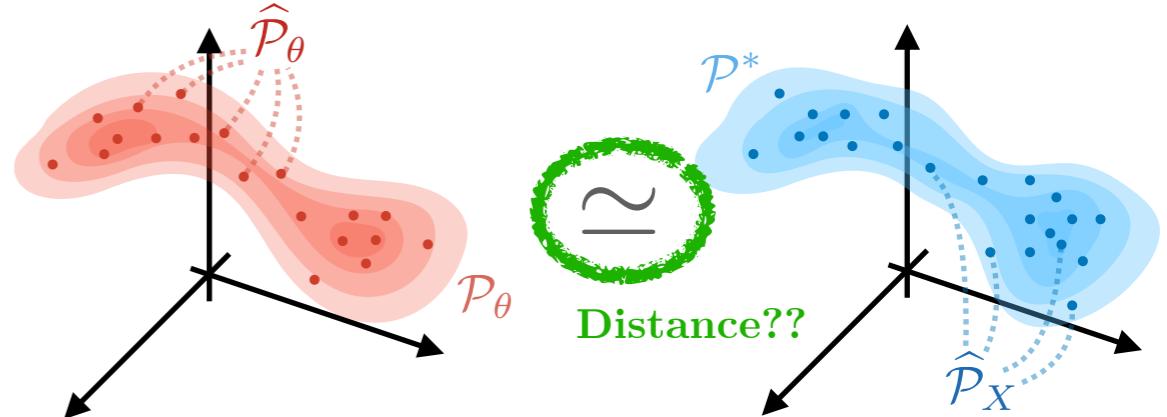
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$$\mathcal{O}(N'(N + N'))$$

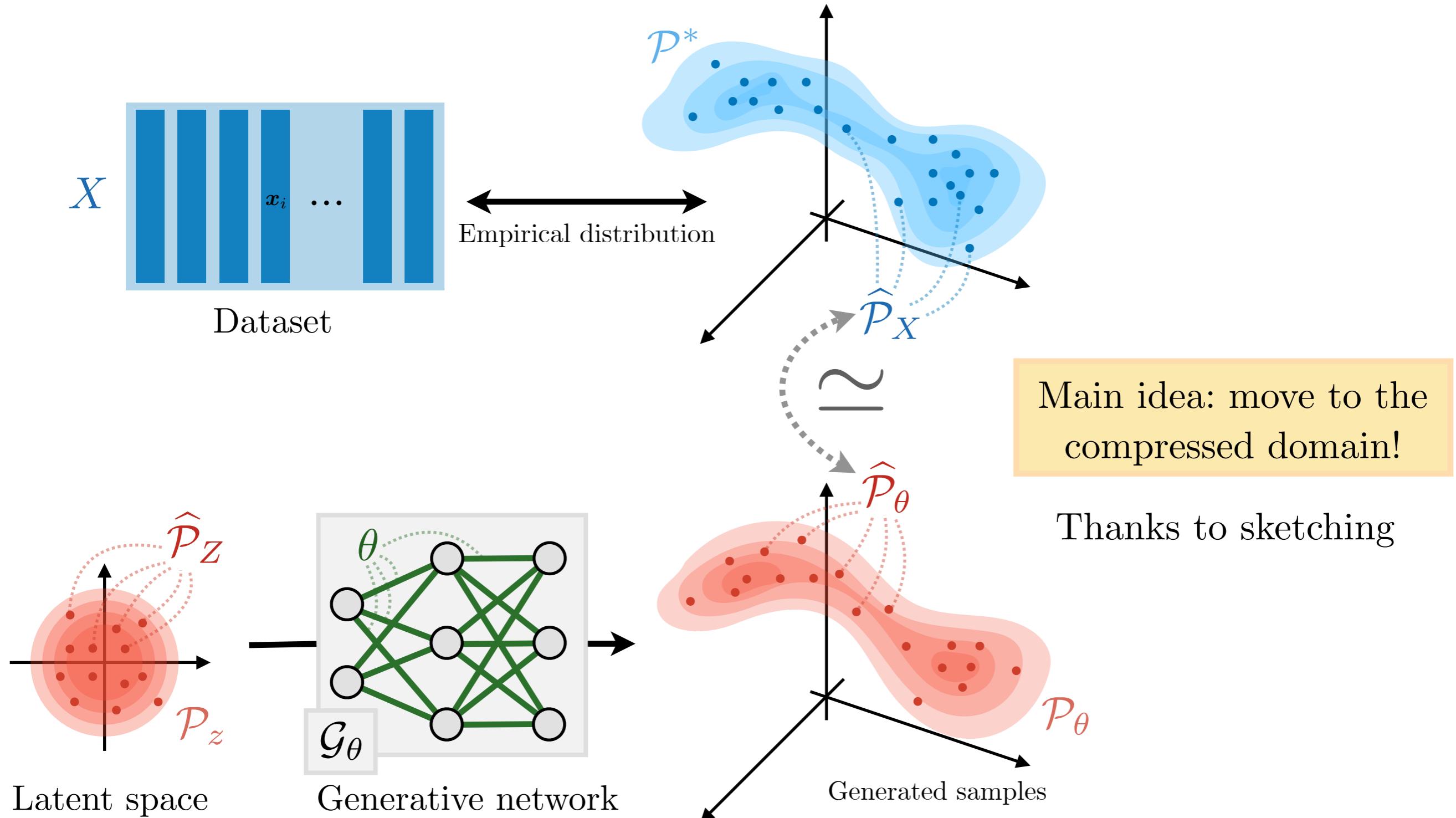
$$\sum_{x_i \in X} \sum_{x_j \in X} \kappa(x_i, x_j) - 2 \sum_{x_i \in X} \sum_{z_j \in Z} \kappa(x_i, \mathcal{G}_\theta(z_j)) + \sum_{z_i \in Z} \sum_{z_j \in Z} \kappa(\mathcal{G}_\theta(z_i), \mathcal{G}_\theta(z_j))$$

Similarity between samples

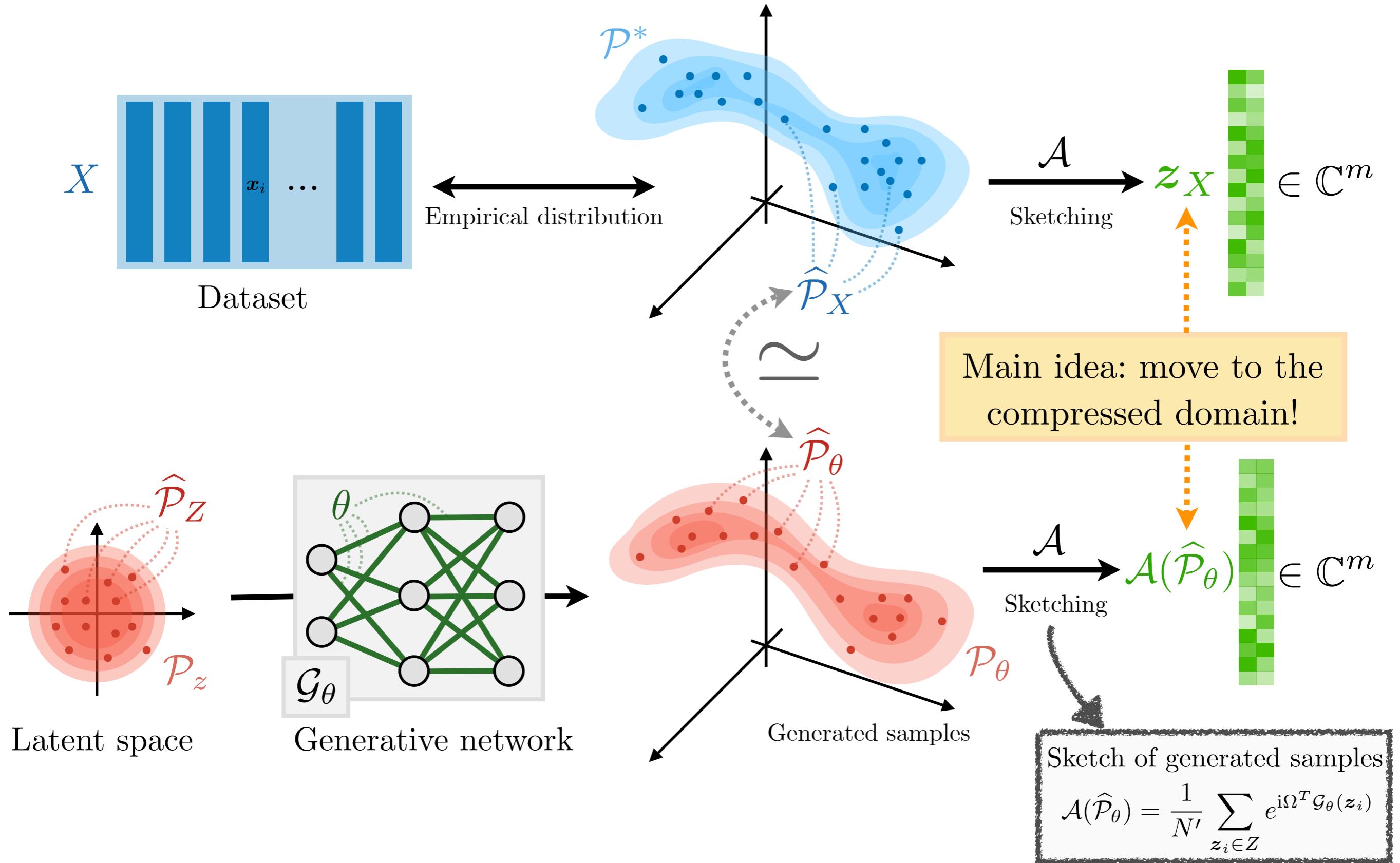
Training generative networks typically requires massive amounts of data!

# Compressive Learning of Generative Networks

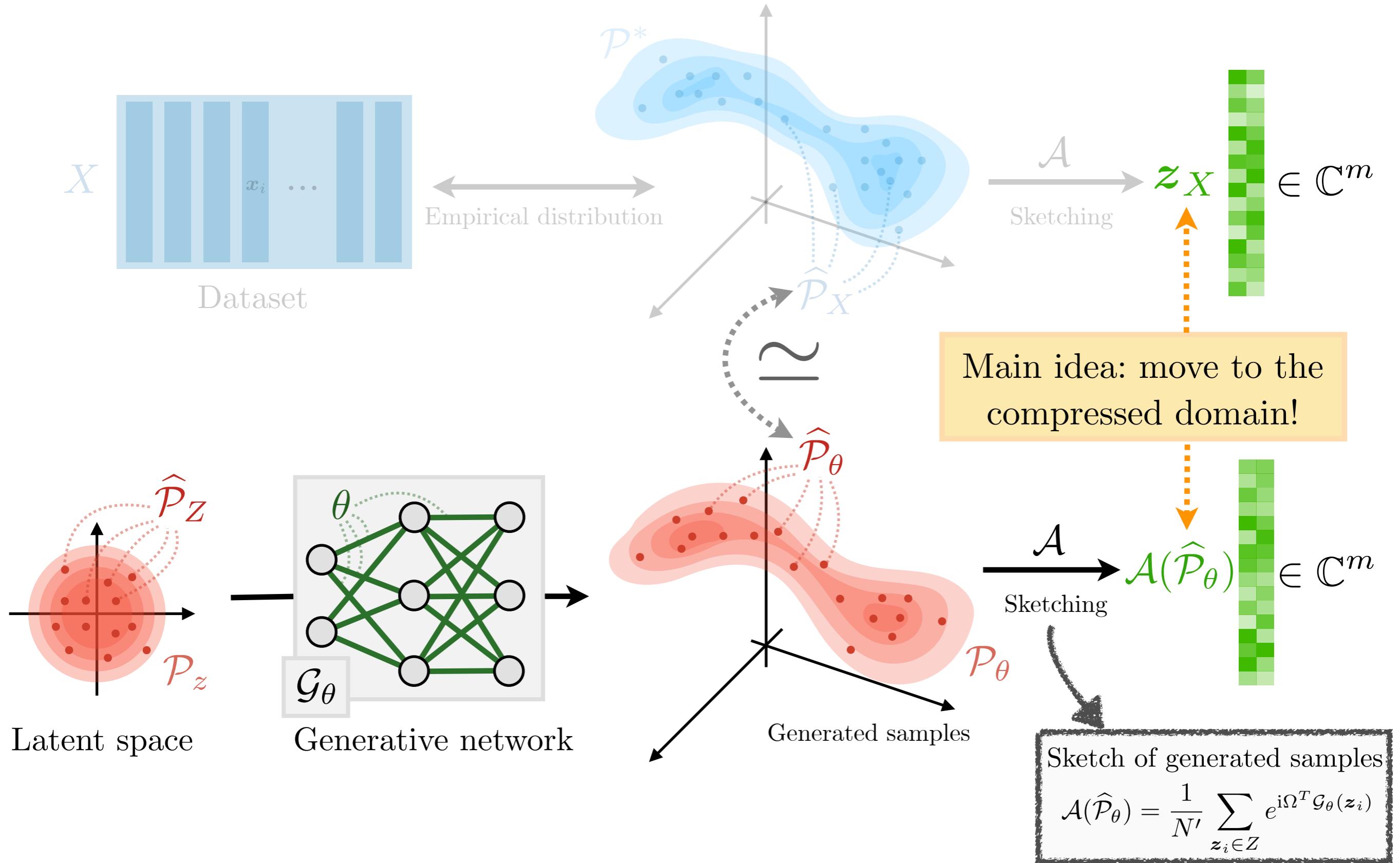
# Compressively learning generative networks



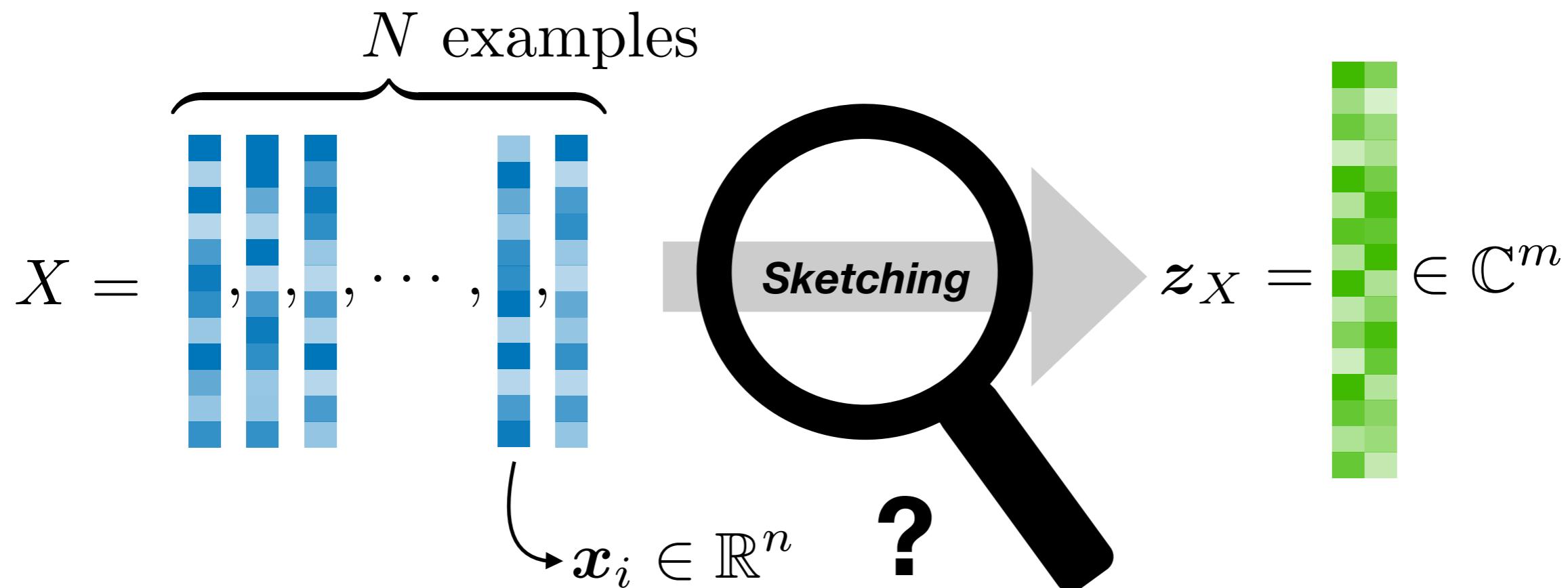
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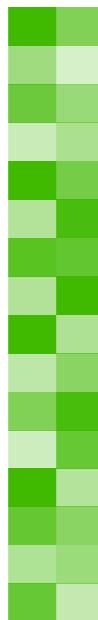


# Sketching a dataset

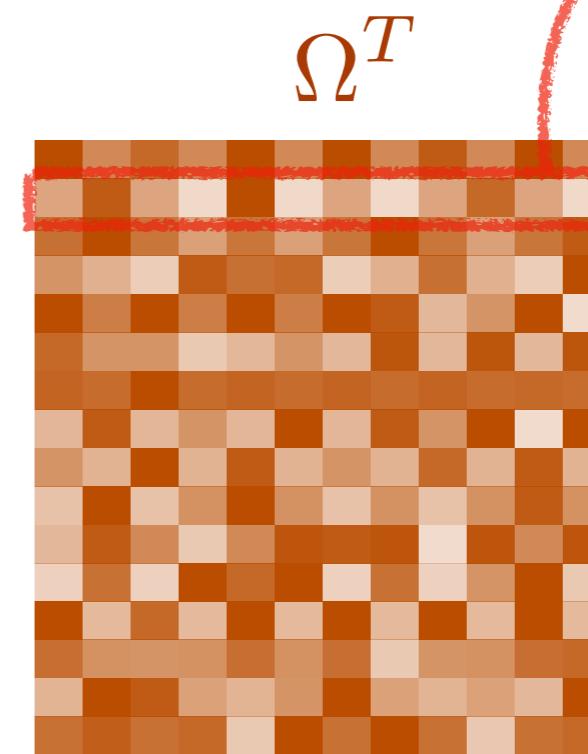


# Sketching a dataset

$z_X$



=



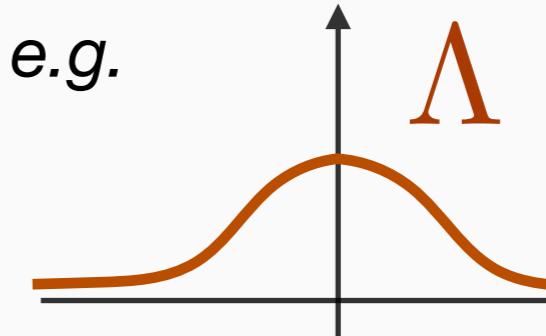
$x_i$



$\omega_j \sim \Lambda$

1. Project on  $m$  (random) vectors

e.g.



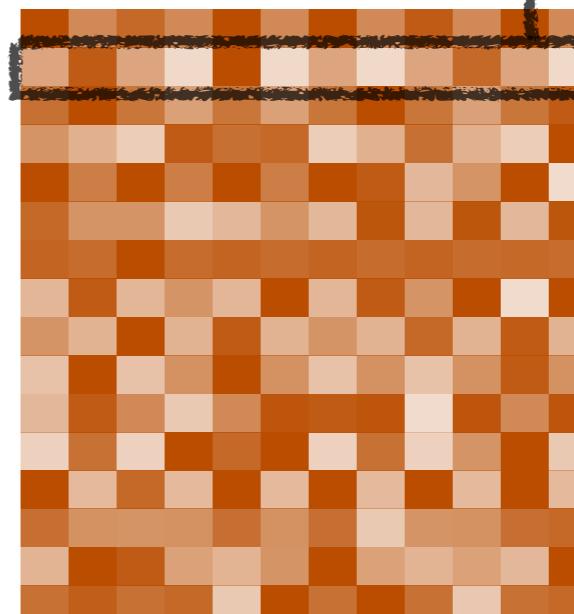
Controls the *cluster scale*

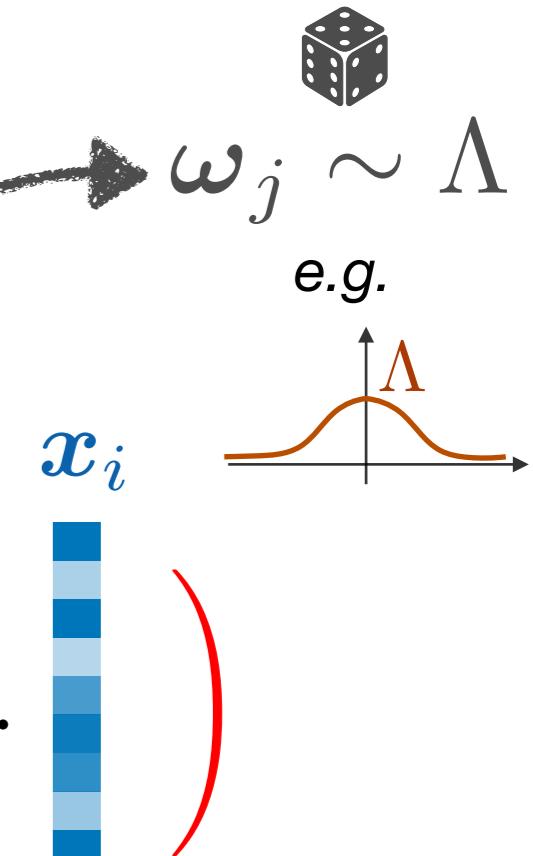
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$$z_X$$


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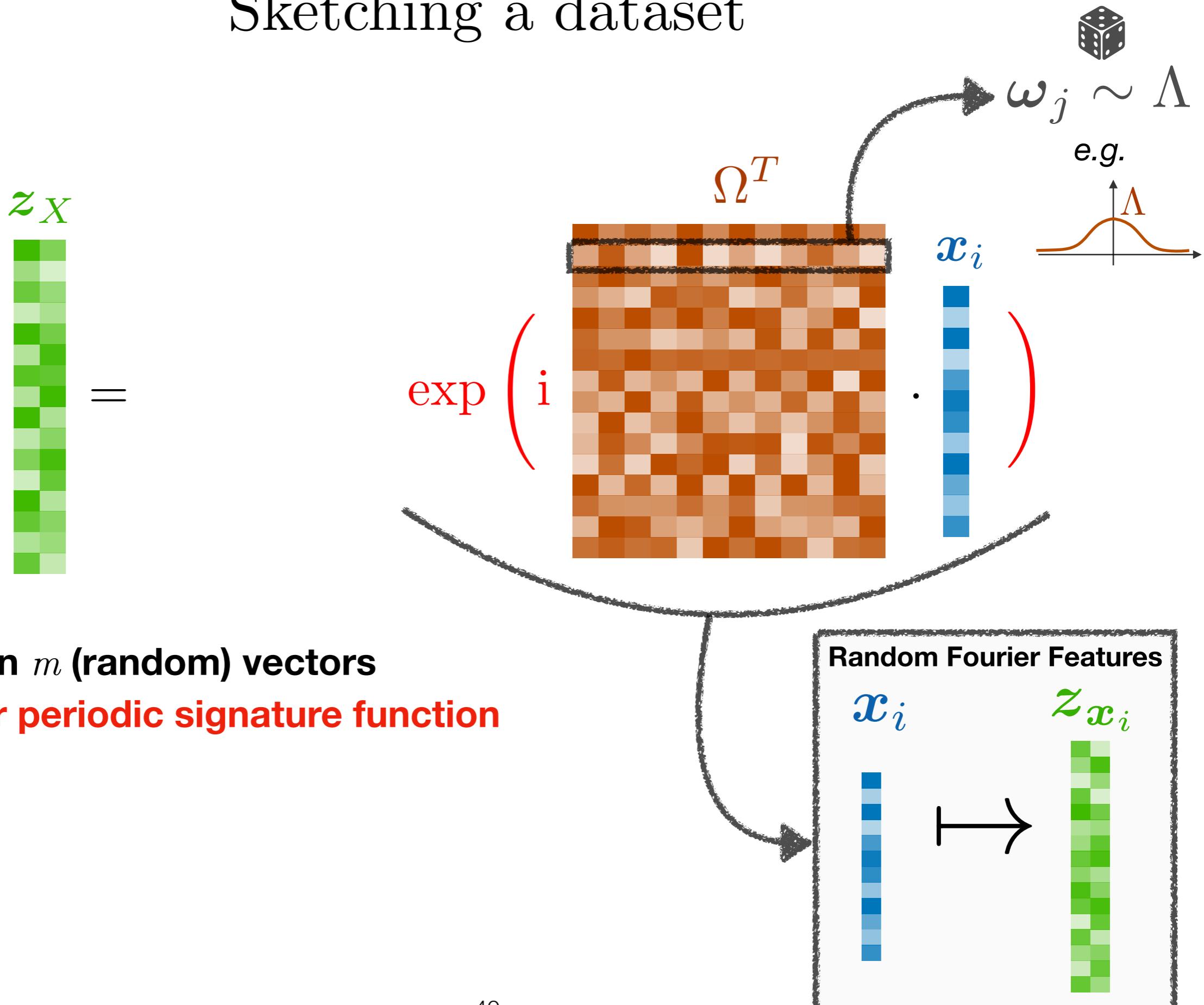
$$\exp \left( i \right)$$

$$\Omega^T \cdot$$


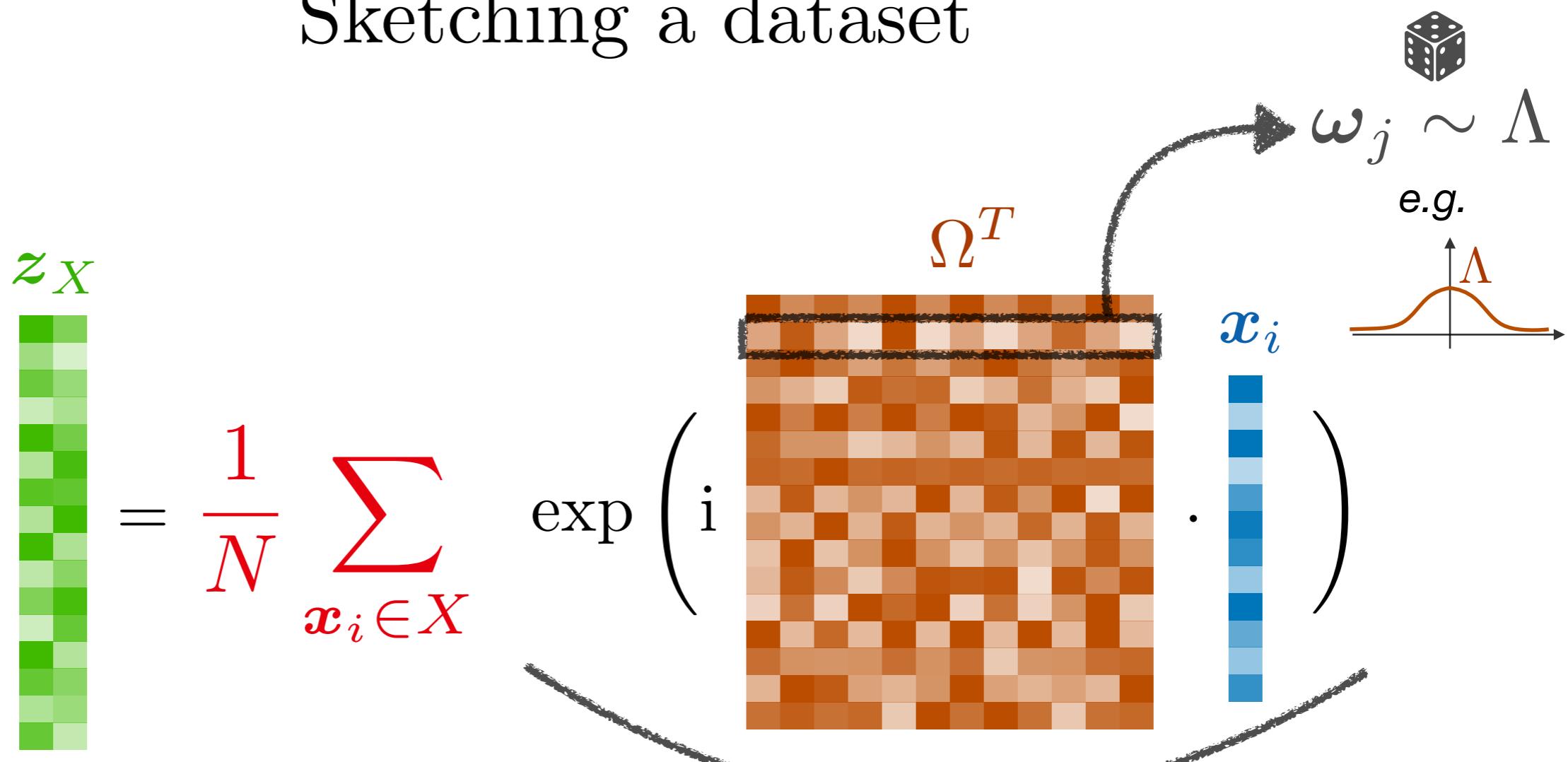


1. Project on  $m$  (random) vectors
2. Nonlinear periodic signature function

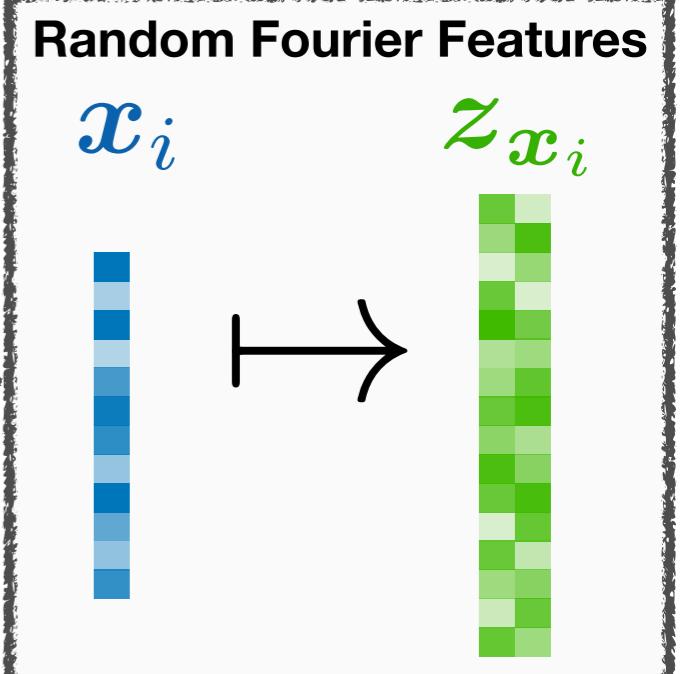
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# Sketching a dataset



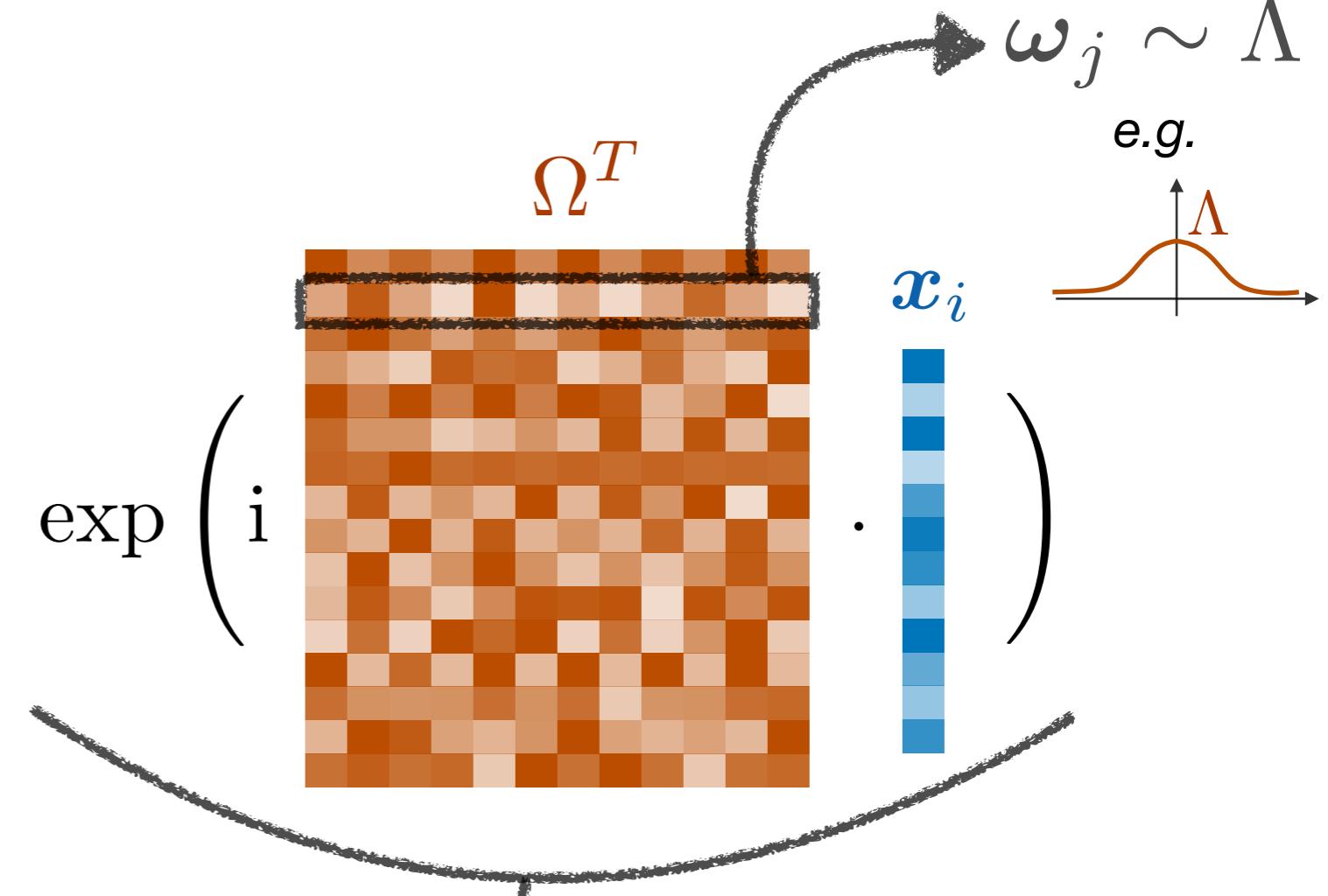
1. Project on  $m$  (random) vectors
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3. Pooling (average)



# Sketching a dataset

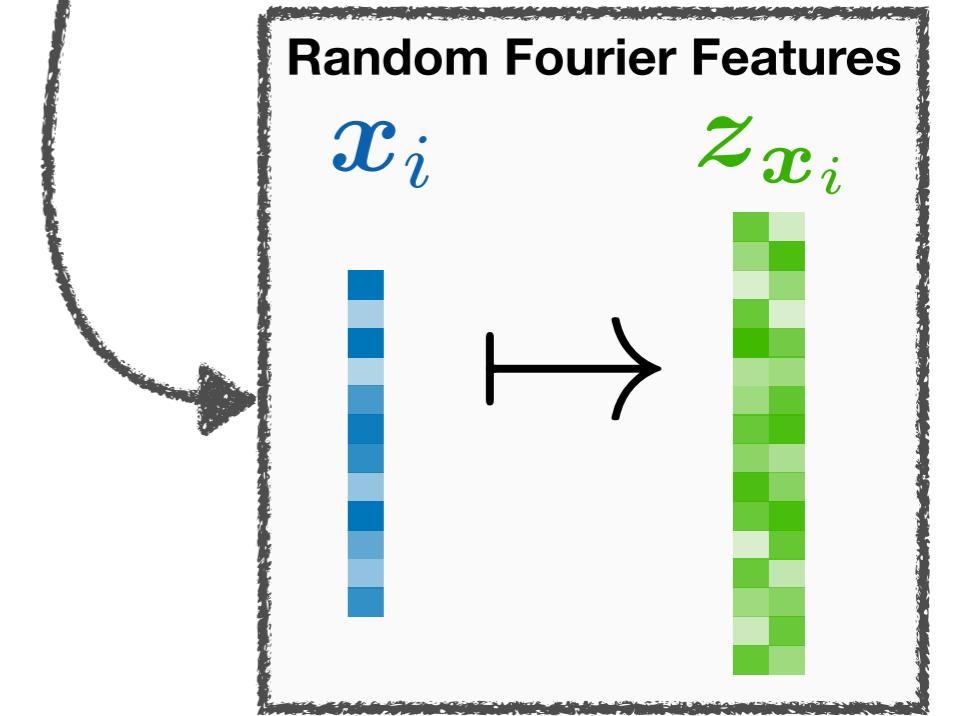
$$z_X$$

$$= \frac{1}{N} \sum_{x_i \in X} \exp \left( i$$



1. Project on  $m$  (random) vectors
2. Nonlinear periodic signature function
3. Pooling (average)

$$z_X = \left[ \frac{1}{N} \sum_{x_i \in X} e^{i\omega_j^T x_i} \right]_{j=1}^m \in \mathbb{C}^m$$



# Compressively learning generative networks

Proposed approach: match the sketches of real and generated data

$$\min_{\theta} \left\| \mathbf{z}_X - \frac{1}{N'} \sum_{\mathbf{z}_i \in Z} e^{i\Omega^T \mathcal{G}_{\theta}(\mathbf{z}_i)} \right\|_2^2$$

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Sampled (Monte Carlo)  
estimation to the MMD!

$$\omega_j \sim \Lambda$$

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...but where dataset is accessed only once 

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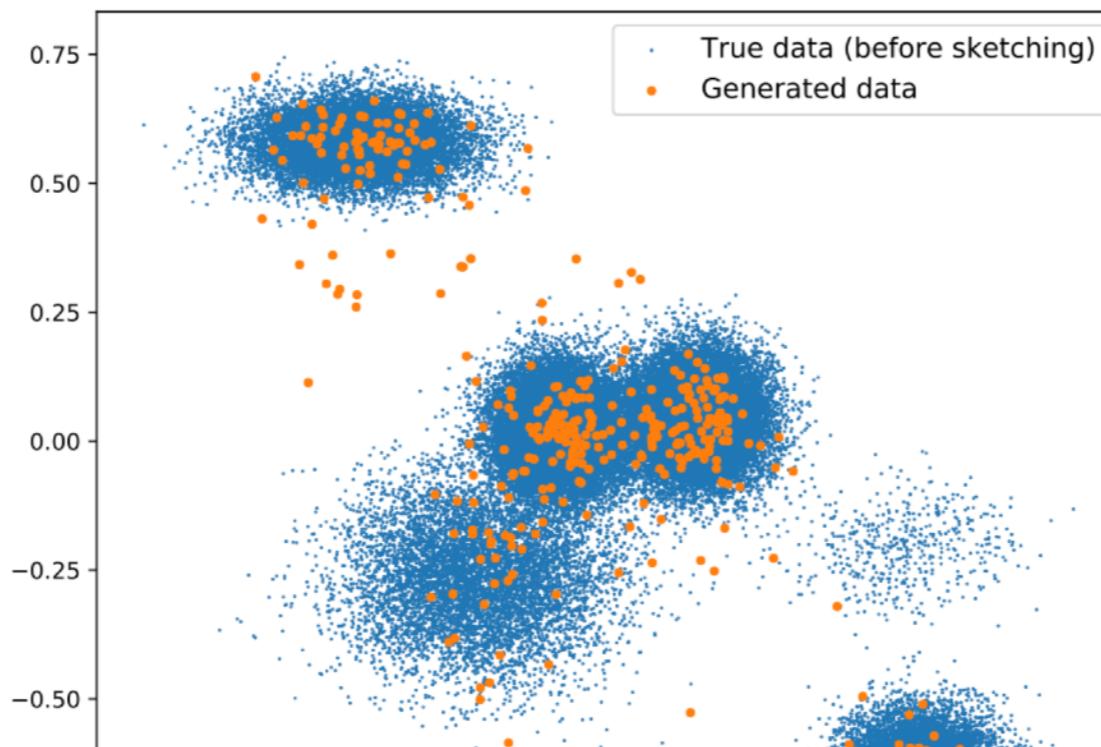
Practical learning algorithm?

Differentiable by chain rule (feat. backprop) 

$$\nabla_{\theta} \widehat{\mathcal{L}}(\theta; \mathbf{z}_X) = -2 \cdot \frac{1}{n'} \sum_{i=1}^{n'} \Re \left[ \mathbf{r}^H \left( \frac{\partial \Phi(\mathbf{u})}{\partial \mathbf{u}} \Big|_{\mathbf{u}=\mathcal{G}_{\theta}(\mathbf{z}_i)} \cdot \frac{\partial \mathcal{G}_{\theta}(\mathbf{z}_i)}{\partial \theta} \right) \right]$$

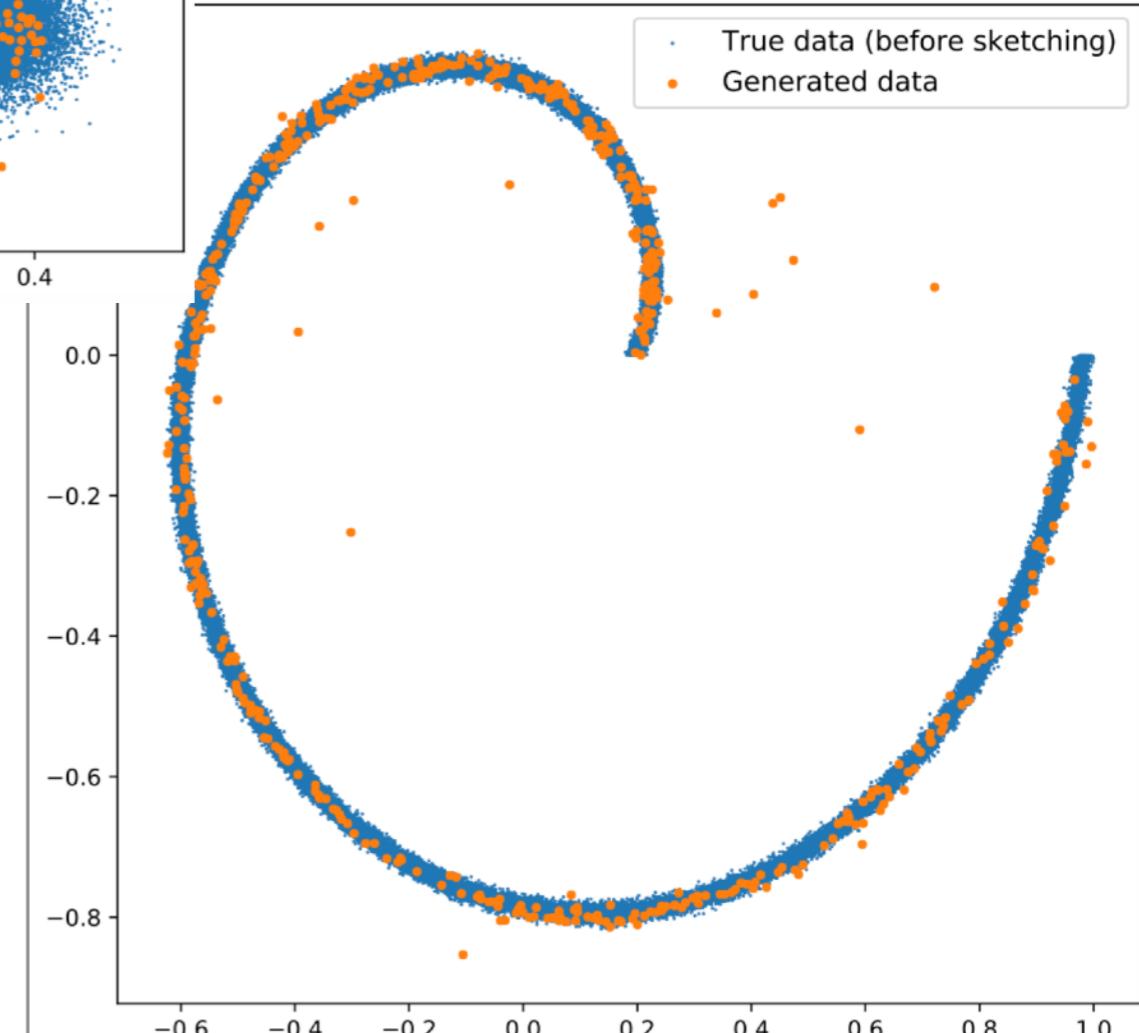
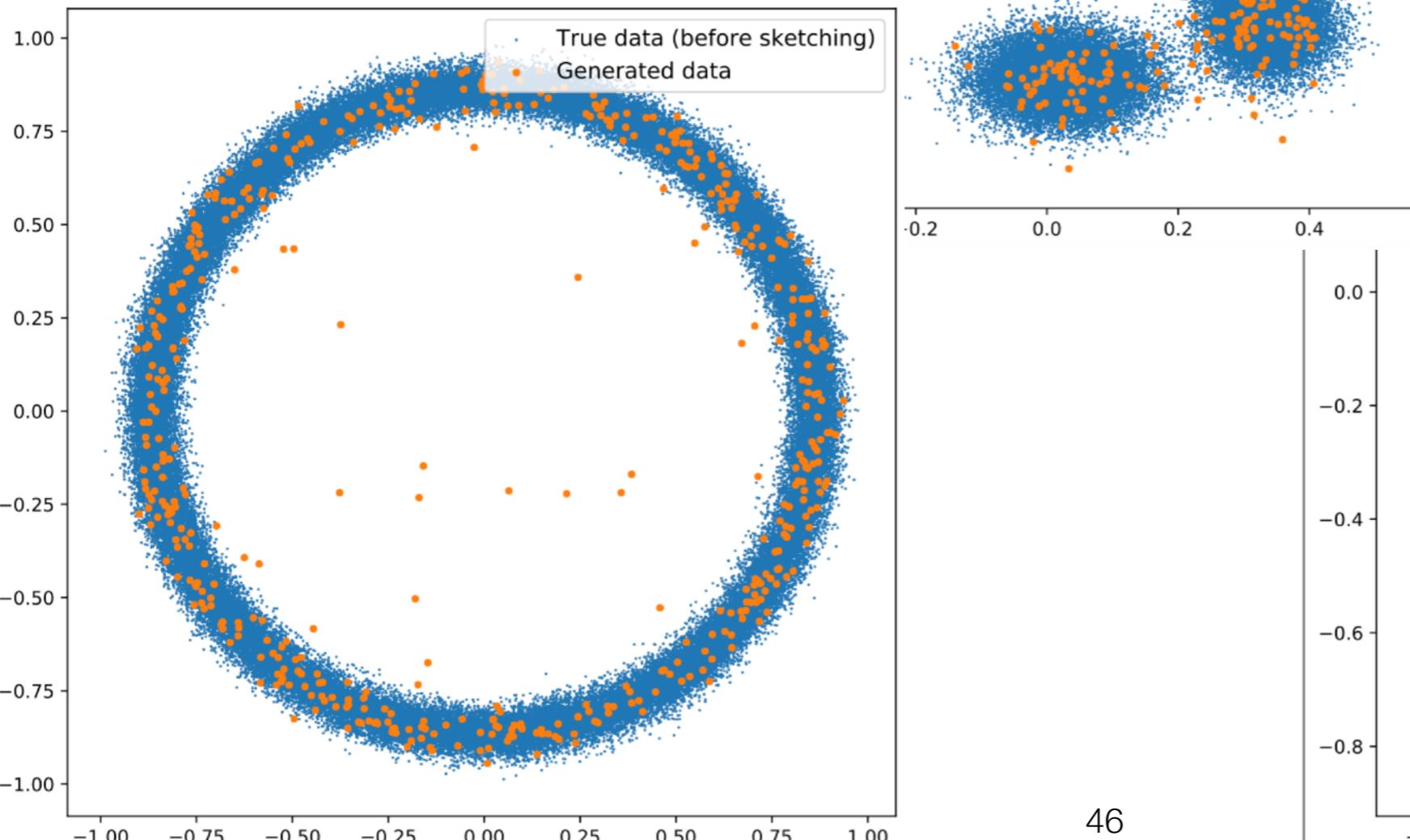
$\mathcal{O}(N')$

# Results: preliminary toy example



Simple 2d signals...

...it works (in principle)!



# Results (from other authors)

## Differentially Private Mean Embeddings with Random Features for Synthetic Data Generation

Frederik Harder<sup>\*,1,2</sup>, Kamil Adamczewski<sup>\*,1,3</sup>, Mijung Park<sup>1,2</sup>

\*Equal Contribution

<sup>1</sup>Max Planck Institute for Intelligent Systems

<sup>2</sup>University of Tübingen

<sup>3</sup>ETH Zürich

{fharder|kadamczewski|mpark}@tue.mpg.de

### Abstract

We present a differentially private data generation paradigm using random feature representations of kernel mean embeddings when comparing the distribution of true data with that of synthetic data. We exploit the random feature representations for two important benefits. First, we require a very low privacy cost for training deep generative models. This is because unlike kernel-based distance metrics that require computing the kernel matrix on all pairs of true and synthetic data points, we can detach the data-dependent term from the term only dependent on synthetic data. Hence, we need only to compute the data-dependent term once and then use it until the end. The analytic sensitivity of the kernel mean embedding is bounded by construction. Thus, we can use a clipping norm to handle the sensitivity. We provide several variants of our method for generating synthetic data with random features (DP-MERF) for datasets such as heterogeneous datasets. DP-MERF achieves better privacy-utilitiy trade-offs than several datasets.

### Submission history

From: Frederik Harder [view email]

[v1] Wed, 26 Feb 2020 16:41:41 UTC (1,629 KB)

- Use the same cost function
- Rely on privacy-friendly benefits of sketching
- Rely on pre-trained autoencoders to handle images

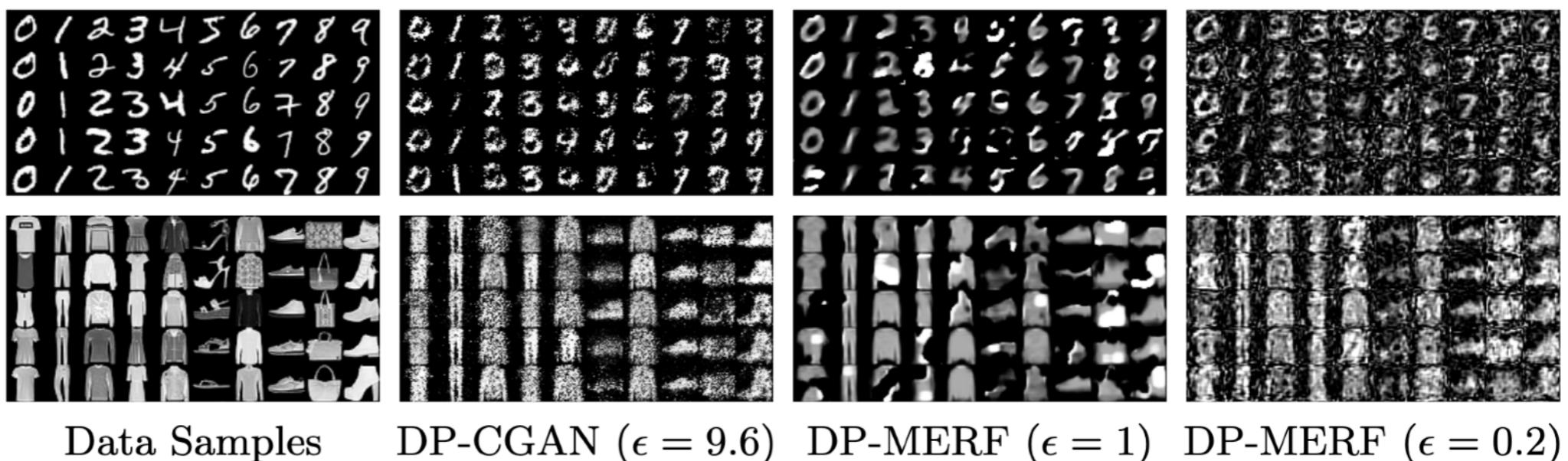
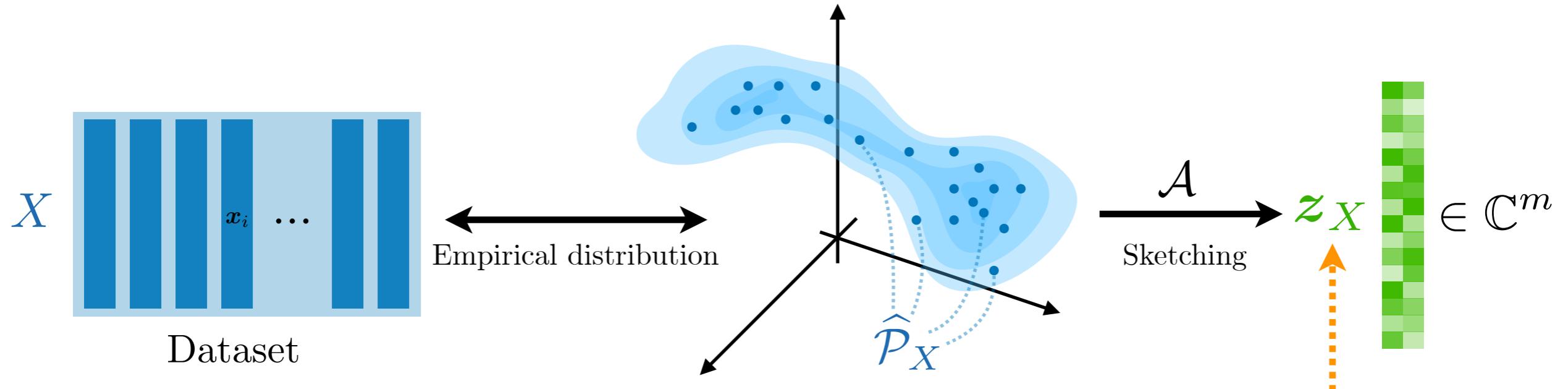


Figure 1: Generated samples with different levels of privacy

# To conclude: open challenges



$$\min_{\theta} \left\| \mathbf{z}_X - \frac{1}{N'} \sum_{\mathbf{z}_i \in Z} e^{i \Omega^T \mathcal{G}_{\theta}(\mathbf{z}_i)} \right\|_2^2$$

Sketch size?      Batch size?      Kernel?

