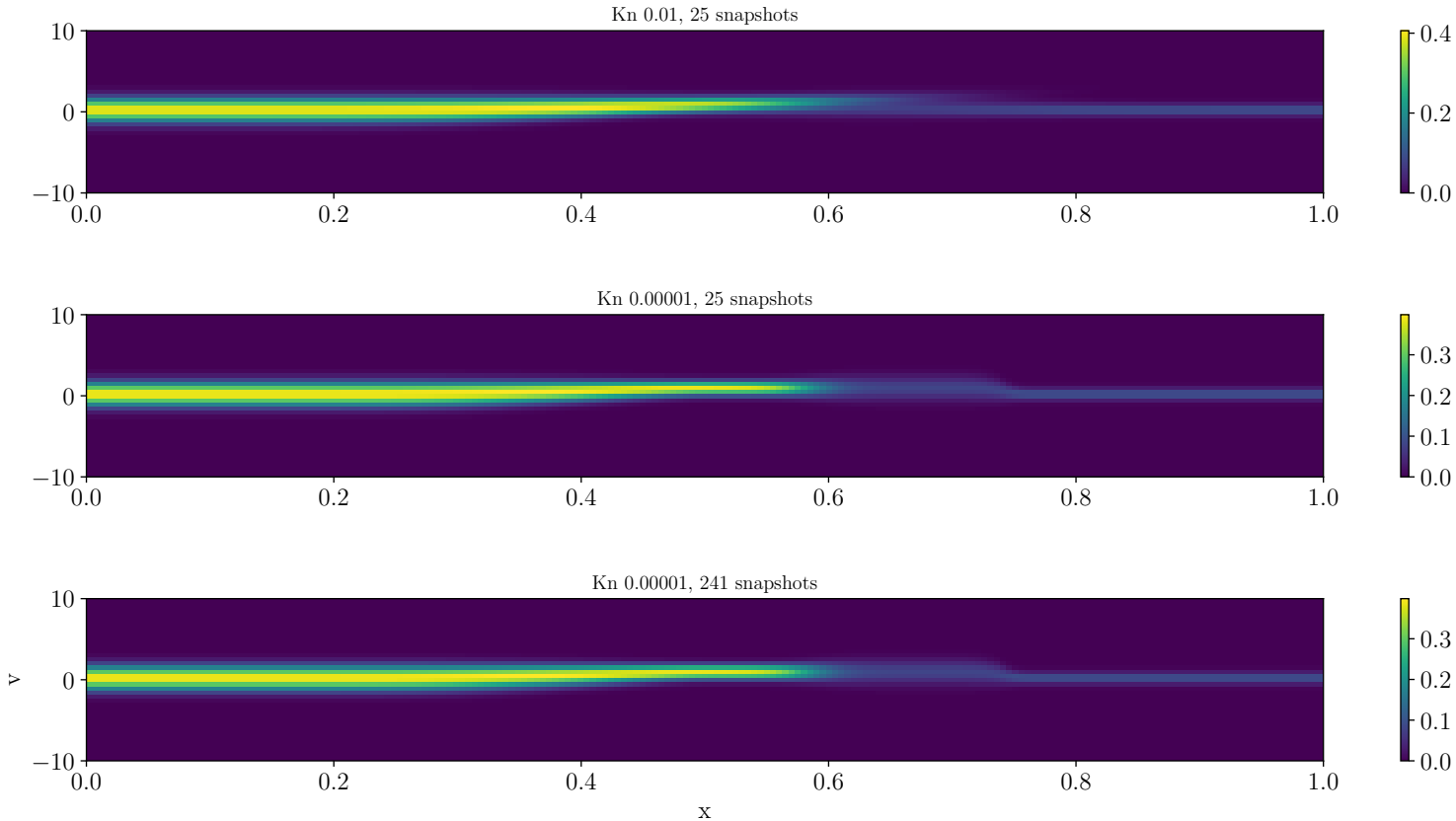
ROM for BGK equation

BGK velocityspace

Original snapshot data in velocity space at time t_{end}.



ROM for BGK equation

Eckard-Young Theorem and snapshot matrix S

Alignment of tensorvalued data $f^{t \times v \times x}$ into an 2D-array S for decomposition:

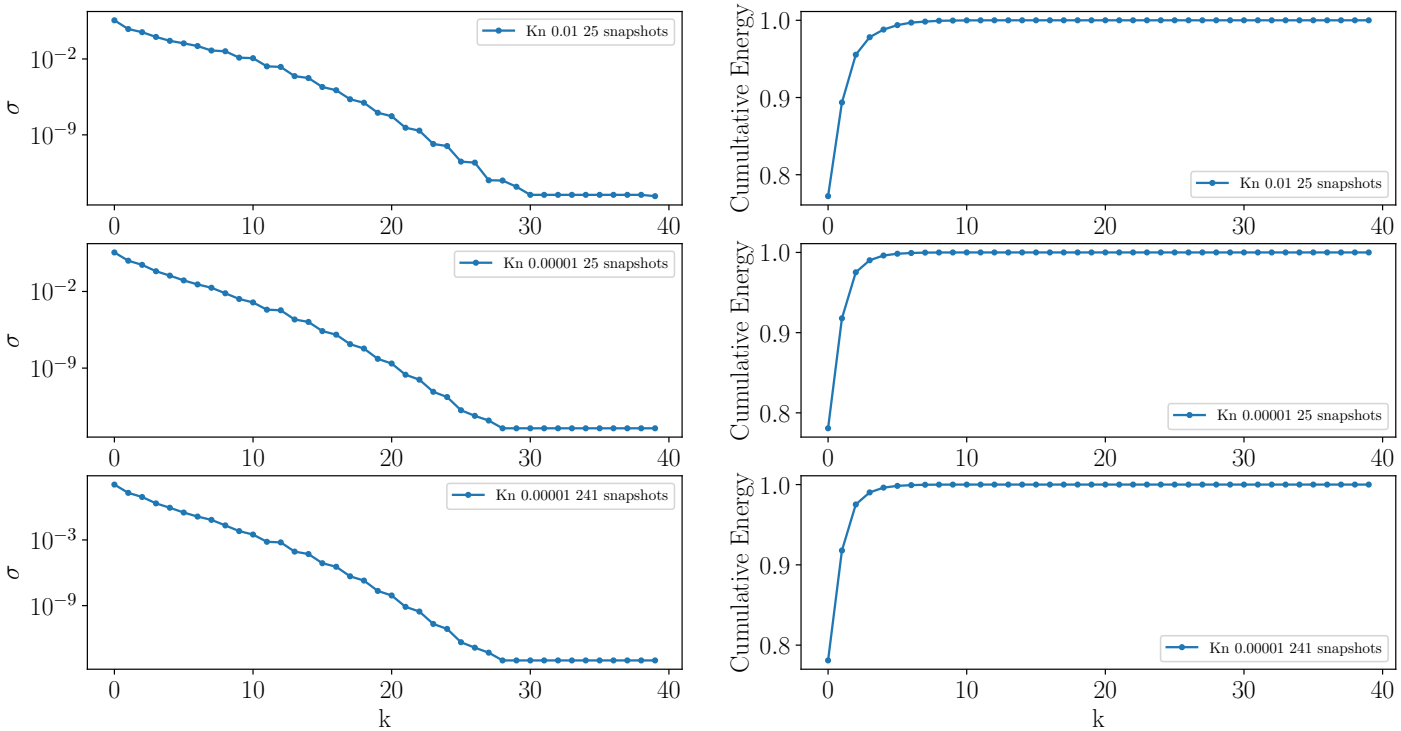
$$S = \begin{bmatrix} f_1(\xi_1) & f_2(\xi_1) & \cdots & f_{N_{snaps}}(\xi_1) \\ f_1(\xi_2) & f_2(\xi_2) & \cdots & f_{N_{snaps}}(\xi_2) \\ \vdots & \vdots & \ddots & \vdots \\ f_1(\xi_{N_\xi}) & f_2(\xi_{N_{xi}}) & \cdots & f_{N_{snaps}}(\xi_{N_\xi}) \end{bmatrix}$$

- The SVD decomposes the data into three unitary matrices $S = U\Sigma V^*$
- Eckart-Young Theorem : The optimal rank-r approximation to S, in a least squares sense, is given by the rank-r SVD truncation $\hat{S}$: argmin $||S - \hat{S}||_F = \hat{U}\hat{\Sigma}\hat{V}^*$

ROM for BGK equation

Cumulative energy and singular values

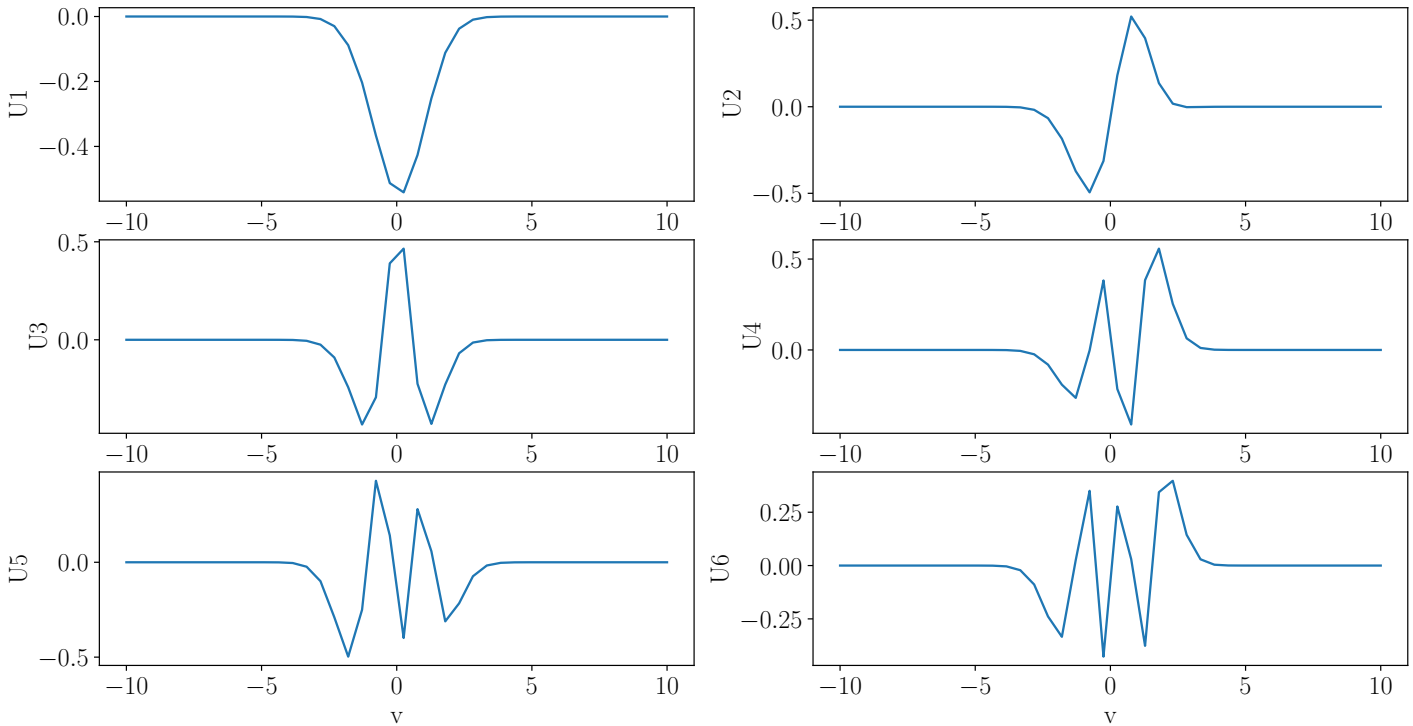
Comparison of of the singular values σ_k for the microscopic and macroscopic data with $\Sigma = diag(\sigma_k)$.



Cumulative sum of given data $\{a, b, c\} \rightarrow a, a + b, a + b + c$

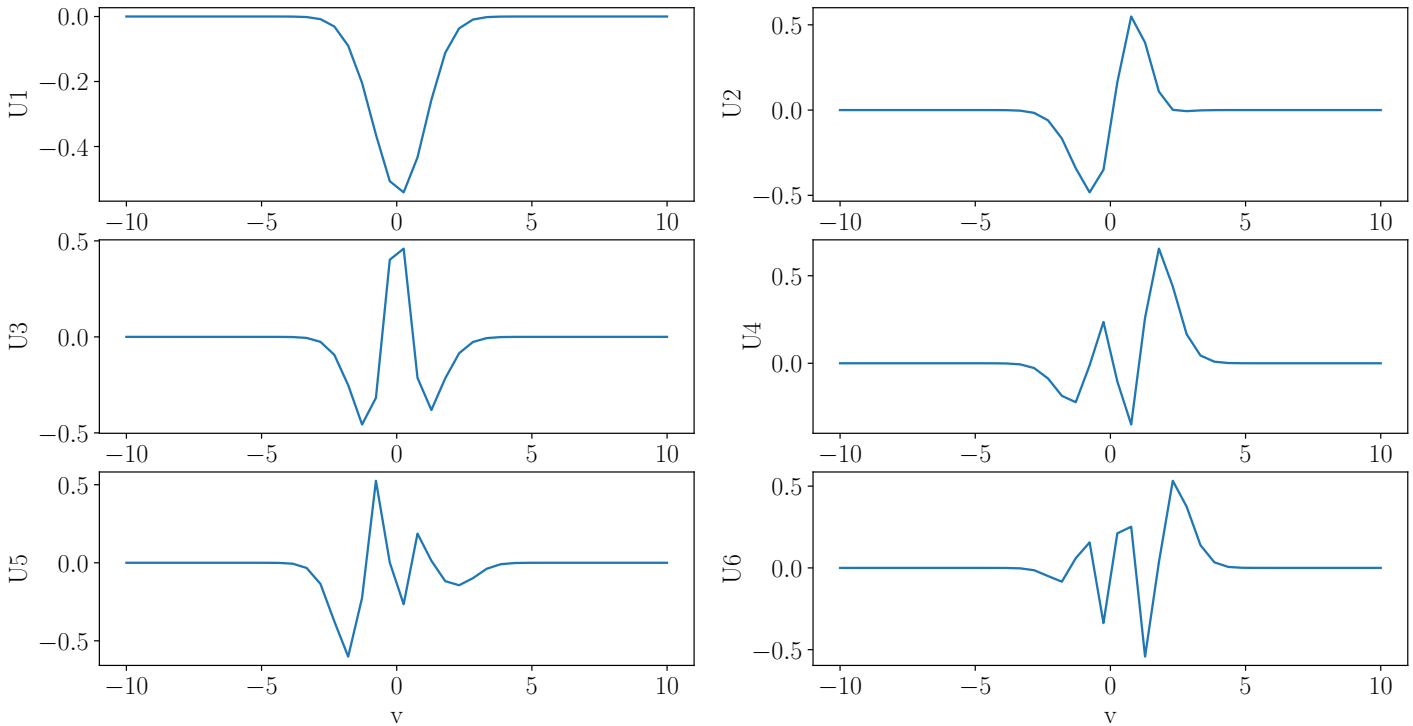
ROM for BGK equation

Eigenmodes of the microscopic data



ROM for BGK equation

Eigenmodes of the macroscopic data.



ROM for BGK equation

Compressed Data

Density over x for the compressed (dotted red) and original (blue) data at time t_{end}. The compressed data consists of the first three eigenmodes.