

Chapter 6

RAREFIED GAS DYNAMICS

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INTRODUCTION

Rarefied gas dynamics is concerned with flows at such low density that the molecular mean free path is not negligible. Under these conditions, the gas no longer behaves as a continuum. Important modifications in aerodynamic and heat transfer characteristics occur which are ascribable to the basic molecular structure of the gas.

This branch of gas dynamics has been the subject of many investigations since the time of Maxwell. Most of the early studies were related to very low speed flows, and usually to “internal” geometries—pipes, ducts, orifices, and the like—in connection with vacuum problems. The results of this classical research are to be found in the standard texts on kinetic theory. Since World War II, however, a revival of interest in the field has occurred due to applications to very high-altitude, high-speed flight.

It is convenient to subdivide rarefied gas dynamics into four different flow regimes. These are called “free-molecular flow,” “near-free-molecular flow,” “transition flow,” and “slip flow,” corresponding, respectively, to extremely rarefied, highly rarefied, moderately rarefied, and only slightly rarefied gas flows. This subdivision is desirable because the four flow regimes exhibit quite different phenomena and the basic theoretical approaches are entirely different. Since “rarefied” is a relative term, the demarcation of these four subdivisions is not characterized by absolute pressure or gas density levels, but rather in terms of the ratio of the mean free path λ to some dimension L characteristic of the flow field. The ratio λ/L is called the Knudsen number, Kn . Free-molecular flow corresponds to very large Knudsen number, $Kn > 10$; slip flow corresponds to Knudsen numbers in the range $0.01 < Kn < 0.1$; while the transitional regimes lies in between, with $0.10 < Kn < 10$. These values are, of course, arbitrary.

Phenomena do not change abruptly. However, they seem to correspond pretty well with present experimental evidence.

The mean free path, the sound speed a , and the kinematic viscosity ν are related by $\nu \sim \lambda a$. Hence the Knudsen number is expressible in terms of the two basic parameters of ordinary continuum gas dynamics, namely the Mach number $M = V/a$ and the Reynolds number $Re = VL/\nu$, where V is the reference gas velocity. This relation is $Kn \sim M/Re$, which is basic for a discussion of rarefied gas dynamics.

The literature on rarefied gas dynamics has become much too extensive to cover in a chapter in this book. Detailed summaries are available in ^(1,2) and in the proceedings of the first five symposia on rarefied gas dynamics ⁽³⁾.

1. ELEMENTS OF KINETIC THEORY

The basic assumption of kinetic theory is that a gas can be idealized as consisting of simple molecules moving freely with respect to each other except during collisions. Most of the more advanced results are also confined to the assumption that the molecules possess no internal degrees of freedom. The basic concept of kinetic theory is the molecular distribution function $f(\mathbf{x}, \mathbf{c}, t)$, which is the number density in phase space of molecules with position \mathbf{x} , velocity \mathbf{c} , at the time t . The distribution function obeys a conservation equation, known as the Maxwell-Boltzmann equation,

$$\frac{\partial f}{\partial t} + \sum_{i=1}^3 c_i \frac{\partial f}{\partial x_i} = \iint_{-\infty}^{\infty} d\mathbf{c}' \iint d\omega \{ |\mathbf{c}' - \mathbf{c}| I(\mathbf{c}' - \mathbf{c}, \theta) (f_1 f_1' - f f') \} \quad (1)$$

where $I(\mathbf{c}' - \mathbf{c}, \theta)$ is the differential cross section for the scattering of a molecule with velocity \mathbf{c}' through an angle θ within the solid angle $d\omega$, and \mathbf{c}_1 and \mathbf{c}_1' are the velocities after collision of two molecules whose velocities before collision were \mathbf{c} and \mathbf{c}' . It has also been assumed that no external forces are acting.

The Maxwell-Boltzmann equation is a nonlinear integro-partial differential equation. The left side represents the rate of change of f due to convection. The right-hand side, known as the collision integral, represents the rate of change of f due to molecular collisions. The validity of the equation requires that the gas density be not so great that multiple collisions or the volume occupied by the molecules become important, nor so small that ensemble averages fluctuate too much. Both of these requirements are generally satisfied for gas flows of interest to the mechanics of rarefied gases.