# 5 CONTINUUM MODELS

## 5.1 INTRODUCTION

When the gas is slightly rarefied or, to be more exact, when the appropriately defined $Kn$ number is in the scope less than 0.1 and larger than 0.01, the discrete molecular effects do not manifest themselves sharply (see the discussion in 0.1 of the Chapter Introduction), the gas flow can be investigated from the point of view of the continuum model. An established practice has been and remains to start from the Navier-Stokes equations with the employment of the slip boundary conditions to obtain the solutions of certain problems. The asymptotic theory starts from the Boltzmann equation by using the asymptotic analysis to obtain the equations of fluid mechanics more exact than the Navier-Stokes equations and corresponding slip boundary conditions to solve the problems in this regime. Qian Xuesen (H. S. Tsien) [1] pointed out early in the forties of $20^{th}$ century, that the Burnett equations obtained as the second order terms of the Chapman-Eskog expansion should give results butter than those of the Navier-Stokes equations (first order terms of the Chapman-Enskog expansion) when $Kn$ is not so small and $Ma$ is large, and for the Burnett equations, as equations of order higher than the Navier-Stoke equations, more boundary conditions should be proposed. Own to the complexity of the Burnett equations and the instability problem related to the high frequency disturbances there have been suspicions concerning reliability of the Burnett equations. But recent investigations and the comparison with the DSMC method and the experimental results show that the Burnett equations in the slip flow regime are indeed superior than the Navier-Stokes equations. Grad [2] employed the Hermite polynomials expansion up to the third order terms to solve the Boltzmann equation yielding the thirteen moment equations, or the Grad equations. To the Grad equations had been also attached great importance, unfortunately the experimental and theoretical investigation did not verify their validity. In section 5.2 the Navier-Stokes equations, the Burnett equations and the Grad thirteen moment equations are presented, and the reliability of them as the basic equations of the continuum model is discussed. In section 5.3 the derivations of the slip velocity and temperature jump boundary conditions are given and the problem of the formulation of the boundary conditions in slip flow regime is discussed. In section 5.4 solutions of some simple problems are discussed starting from the Navier-Stokes equations and usual slip conditions. In section 5.5 the problem of thermal creep is discussed.

## 5.2 BASIC EQUATIONS

### 5.2.1 EQUATIONS OF MASS, MOMENTUM AND ENERGY CONSERVATION

In section 2.7 of Chapter 2 the moment equation or the Maxwellian transport equation (2.183) of the Boltzmann equation (2.152) is obtained

$$\frac{\partial}{\partial t}\left(n\overline{Q}\right)+\nabla\cdot n\overline{cQ}-nF\cdot\overline{\frac{\partial Q}{\partial c}}=\Delta[Q], \tag{5.1}$$

where the collision integral $\Delta[Q]$ can be expressed in the form of Eq. (2.160). From there it is seen that when $Q$ is taken as the mass $m$, the momentum $mc$ and kinetic energy $(1/2)mc^2$, the right hand side of Eq. (5.1) is zero. And the moments of $Q$ and $cQ$ are various macroscopic quantities in gas dynamics discussed in detail in section 2.2 of Chapter 2 (see formulae collected at the end of section 2.2). Thus, the following equations of mass, momentum and energy conservation are obtained (cf. reference [3], section 2 and section 3 of Chapter 3)

$$\begin{cases} \dfrac{d\rho}{dt}+\rho\dfrac{\partial u_i}{\partial x_i}=0 \\ \rho\dfrac{du_i}{dt}+\dfrac{\partial p}{\partial x_i}+\dfrac{\partial \tau_{ij}}{\partial x_j}=0 \\ \rho\dfrac{de}{dt}+p\dfrac{\partial u_i}{\partial x_i}+\tau_{ij}\dfrac{\partial u_i}{\partial x_j}+\dfrac{\partial q_i}{\partial x_i}=0. \end{cases} \tag{5.2}$$