

Bachelorarbeit
zur Erlangung des akademischen Grades
Bachelor of Science

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1 Introduction

2 The BGK Equation

3 Reduced Order Algorithms

3.1 Data Sampling

3.2 POD

The singular value decomposition of the input X [REF to Section 1] gives the optimal low-rank approximation \tilde{X} of X eq. (2)[Eckard-Young]. Figure 1 shows the singular values (left) and the cumulative energy (right) derived from eq. (1):

$$S_N = \sum_{k=1}^N a_k \quad \text{with a sequence} \quad \{a_k\}_{k=1}^n \quad (1)$$

$$\underset{\tilde{X}, s.t. rank(\tilde{X})=r}{\operatorname{argmin}} \quad ||X - \tilde{X}||_F = \tilde{U}\tilde{\Sigma}\tilde{V}^* \quad (2)$$

The first five singular values give an accurate approximation \tilde{X} of X . As a means to evaluate the low-rank approximation of X we will compare the density derived from eq. (5), computed from X and \tilde{X} .

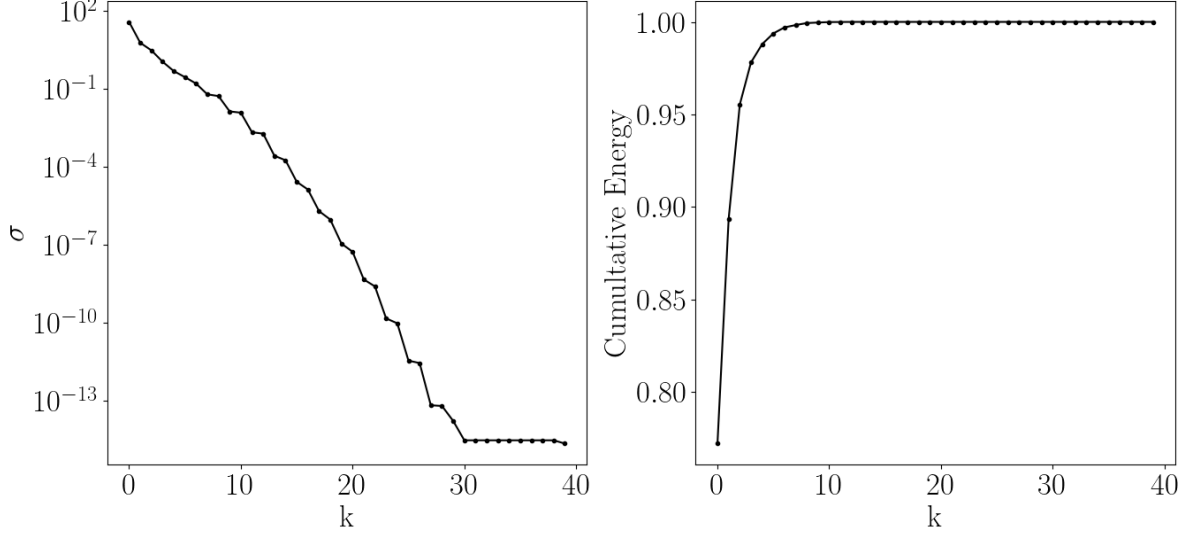


Figure 1: Singular Values (left) and cumulative enrgy (right) over the number of singular values

3.3 Autoencoders

The same matrix as in POD is used as input data for the autoencoder:

$$S = \begin{bmatrix} f(\xi_1, t_1, x_1) & \cdots & f(\xi_n, t_1, x_1) \\ f(\xi_1, t_1, x_2) & \cdots & f(\xi_n, t_1, x_2) \\ f(\xi_1, t_1, x_n) & \cdots & f(\xi_n, t_1, x_n) \\ f(\xi_1, t_2, x_1) & \cdots & f(\xi_n, t_2, x_1) \\ \vdots & \ddots & \vdots \\ f(\xi_1, t_n, x_n) & \cdots & f(\xi_n, t_n, x_n) \end{bmatrix}$$

During training every 1000 epochs a sample against its prediction was printed in order to link the value of the L1-Loss to a prediction. Using this method a first verification of the model was achieved. Continuing the search for any possible shortage of the models performance, that this method could not cover, eg. samples lying between every 1000 sample, that the model was not able to reconstruct correctly, a second verification process is conducted.

4 Results

4.1 Evaluation Methods

In order to evaluate the proposed dimensionality reduction algorithms, two methods are being introduced. The first one constitutes a qualitative analysis using eq. (3) which measures the euclidian distance of every reconstructed sample \tilde{X}_i against its corresponding original sample X .

$$|X_i - \tilde{X}_i| = \delta_i \quad \text{with } i \text{ being the } i^{\text{th}} \text{ sample} \quad (3)$$

$$\sum |\rho_i - \tilde{\rho}_i| = \delta_\rho \quad \text{with } i \text{ being the } i^{\text{th}} \text{ sample} \quad (4)$$

As a second quantitative approach a comparison of the density over space in time of the BGK model in eq. (5) is utilized. The sum over all euclidian distances from the original samples ρ_i to their reconstruction $\tilde{\rho}_i$ is evaluated in eq. (4).

$$\int_{\mathbb{R}^3} f(\mathbf{x}, \xi, t) \begin{pmatrix} 1 \\ \xi \\ \frac{\|\xi\|^2}{2} \end{pmatrix} d\xi = \begin{pmatrix} \rho(\mathbf{x}, t) \\ \rho(\mathbf{x}, t)U(\mathbf{x}, t) \\ E(\mathbf{x}, t) \end{pmatrix} \quad (5)$$

4.2 Results

A quantitative measure shows, that the linear Autoencoder performs slightly better than the POD in the input data.

Algorithm	Density	Samples
POD	3.79	50.61
AE	2.52	51.21
CAE	cell	cell

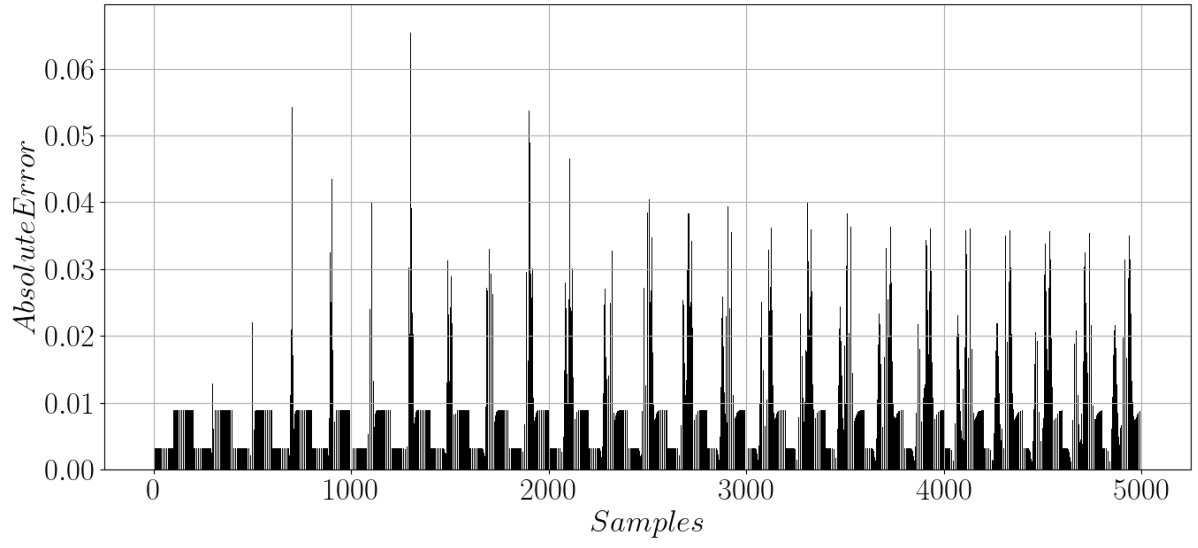


Figure 2: Absolute error for every sample in euclidean distance for the POD

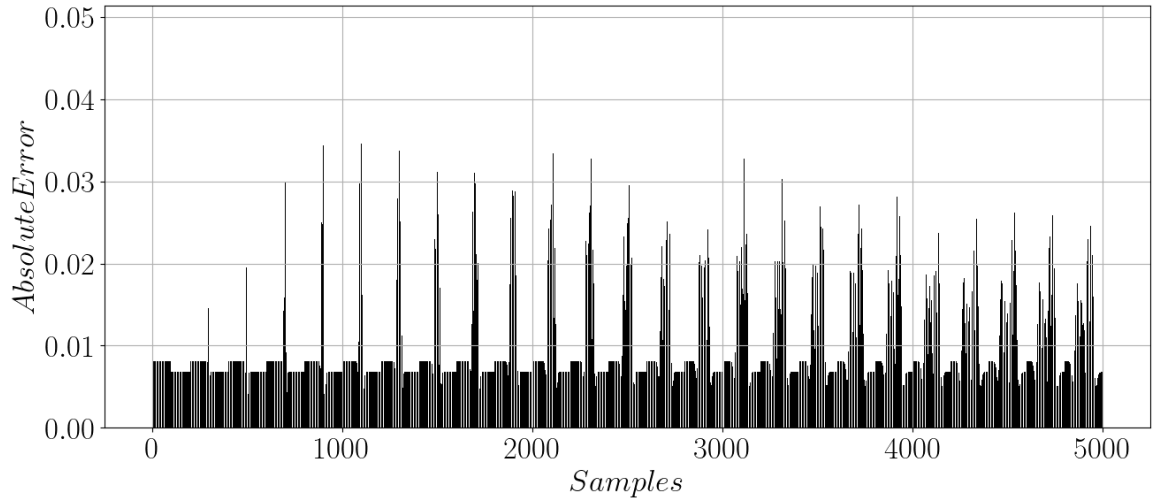


Figure 3: Absolute error for every sample in euclidean distance for the linear autoencoder

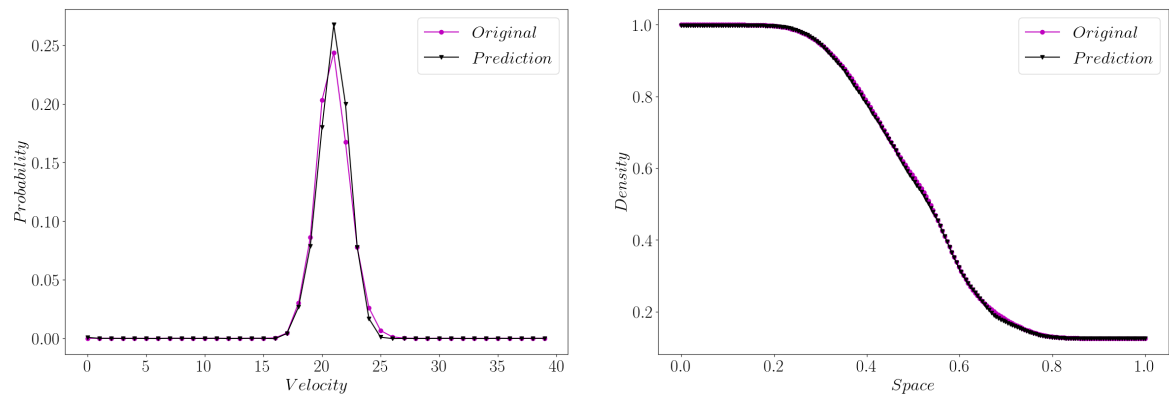


Figure 4: Error of each sample