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Section: G2
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Lab Report #4

MAE 107L – Dynamic Systems Laboratory
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(Please note that all answers to questions asked in the laboratory manual are marked within this lab report as (**QUESTION**)).

OBJECTIVE

The overall objective of this experiment was to observe the impulse and step responses for the low-pass filter, high-pass filter, resonator, and anti-alias filter. Using four different impulse vectors, the impulse responses were measured and graphed. The output of the impulse responses for the low-pass filter and high-pass filter were compared with theoretical, calculated analytical models based on the exponential decay of the response. The behavior of the high-pass filter with regard to the input was also observed. Finally, the step response output was graphed and compared with the theoretical model, and specific parameters for the step response were calculated based on the output graphs. The experiment verified if the experimental impulse and step responses for the four different inputs were equivalent, or within error, of the theoretical models that we expected to occur.

FIGURES 1-4: EXPERIMENTAL IMPULSE RESPONSES

We first list the four vectors and the area sum under the vectors that we used in order to measure the experimental impulses responses for the low-pass filter, high-pass filter, resonator, and anti-alias filter. The four vectors that were utilized within this part of the experiment are as follows: (**QUESTION)**

Vector 1: [0 4 6 8 8 8 8 8 6 4 0]
Vector 2: [0 7 9 7 9 7 9 7 9 7 0]
Vector 3: [0 -2 9 9 9 9 9 2 9 8 0]
Vector 4: [0 9 9 9 9 9 9 9 9 9 0]

This corresponds to an area sum as follows:

Vector 1: 0.0060
Vector 2: 0.0071
Vector 3: 0.0062
Vector 4: 0.0081

As such, we will be graphing out the vectors in the following order: Vector 1, Vector 3, Vector 2, and then Vector 4, in order to graph out the vectors from the smallest area to the largest area underneath the input vector.

Now that we have listed the vectors that we use, we want to graph out the experimental impulse response for the low-pass filter for the four vectors. We also calculate an analytical solution by taking the natural log of the low-pass filter output and then estimating the parameters that we wish to determine from the slope and the y-intercept of the graph that follows. We note that the analytical impulse response to a first order low-pass filter is given by the following equation:

$$h(t) = be^{at}$$

We then take the natural log of the output. In order to determine the parameters, the slope of the natural log of the output is the parameter a, and the y-intercept of this graph is the natural log of the parameter b.

We measured an a value of -8.935320869 from the slope of the natural log output graph, and found the y-intercept to be 2.068. As such, the analytical response for the low-pass filter is: (**QUESTION)**

$$h(t) = e^{2.068}e^{-8.935320869t}$$
$$h(t) = 7.908989311e^{-8.935320869t}$$

We can then graph the impulse responses for the low-pass filter and the analytical response that we have just determined. We note that the figure should show an impulse spike at t = 0 seconds, and then a decay down from there, following the analytical response.

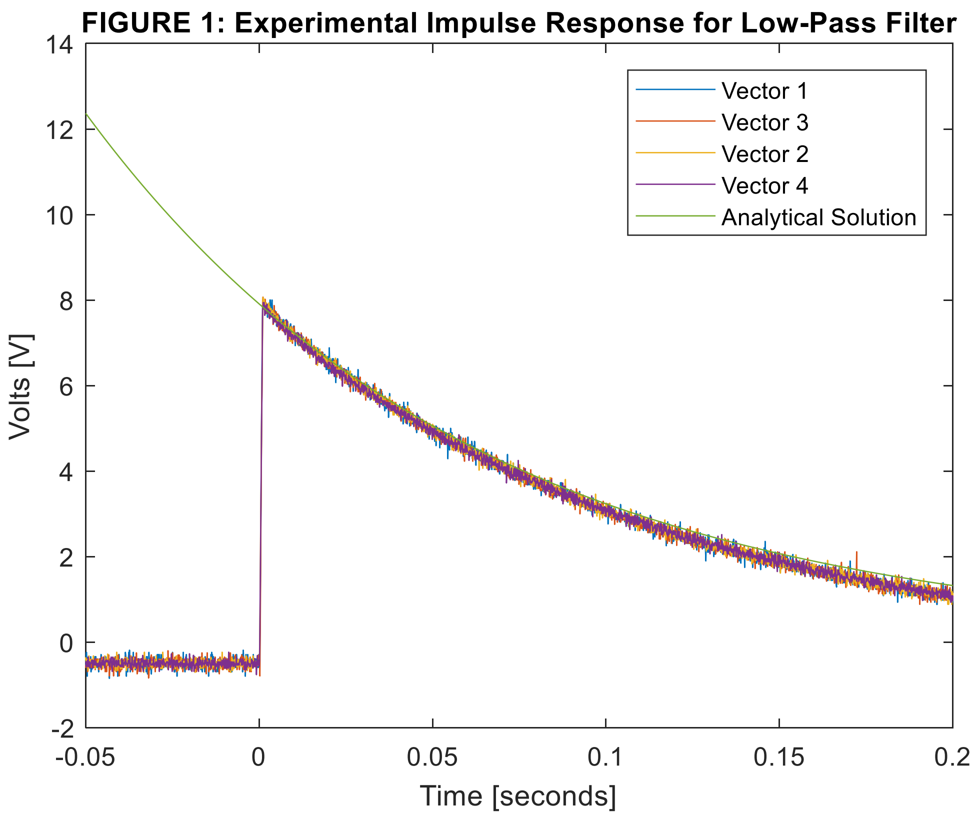

FIGURE 1: This figure shows the experimental impulse response for the low-pass filter, including the analytical solution that was determined via the natural log output.

We then graph the high-pass filter output response for the four different vectors. We graph it for the x range of [-0.0001, 0.0011]. We also note that this figure is supposed to mimic the input vectors that we put in, although with different orders of magnitude. The figure is as follows

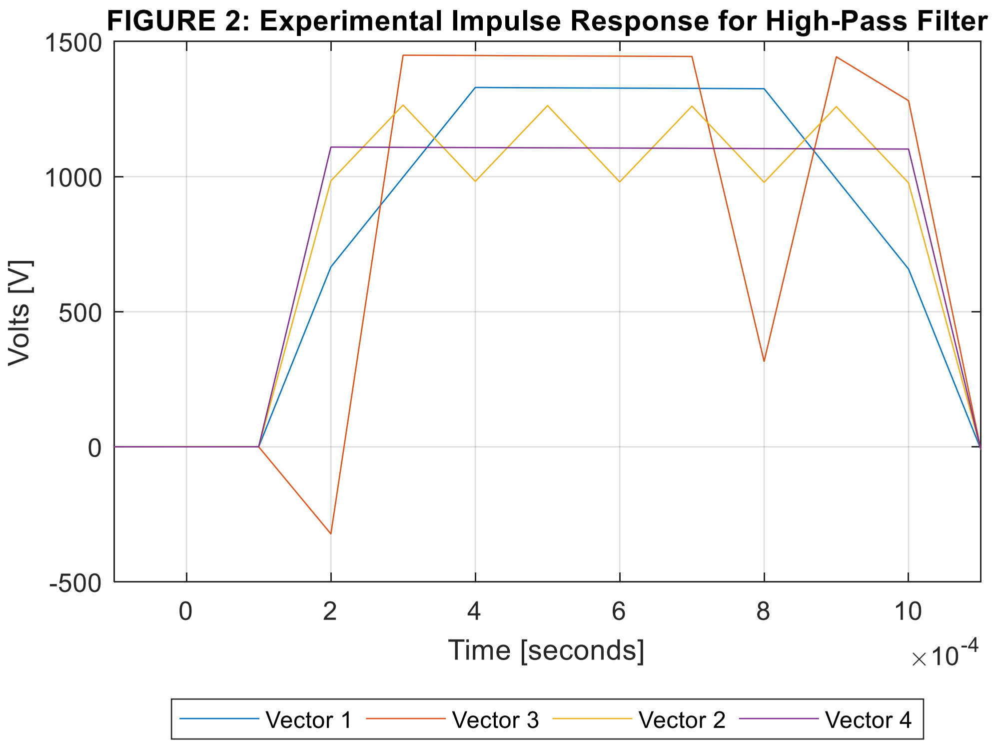

FIGURE 2: This figure shows the experimental impulse response for a high-pass filter. Note how the graph mimics the input impulse vectors.

Now that we have graphed this out, we then restrict the x-axis to the range [-0.1, 4] and the y-axis to the range [-12, 4] and graph it out. We note that even though the input vectors are different, the response at this range is almost exactly the same. We can also determine the analytical solution to this high-pass filter response. We know that the analytical solution is of the form:

$$h(t) = b\delta(t) + abe^{at}$$

We can then determine the values of a and b again by taking the natural log of the high-pass filter response and then finding the slope and the y-intercept. We note that the y-intercept, in this case, will equal the natural log of parameter a times parameter b, whereas for the low-pass filter it was simply parameter a. Graphing this out, we find that the slope is -7.76975052 and the y-intercept is 2.055. This means that parameter a is equal to -7.76975052, and parameter b is equal to the natural log of 2.055 divided by -7.76975052, which is the natural log of -0.264487257. This means the analytical response has the form: (**QUESTION)**

$$h(t) = 0.767599942\delta(t) - 7.806837873e^{-7.769975052t}$$

The graph of the figure, including all four vector responses and the analytical solution, are as follows. Note that the impulse spike can be seen at t = 0 seconds, and then the gradual slope based on the exponential function back to a 0 V response after approximately 0.4 seconds.

FIGURE 2B: Experimental Impulse Response for High-Pass Filter

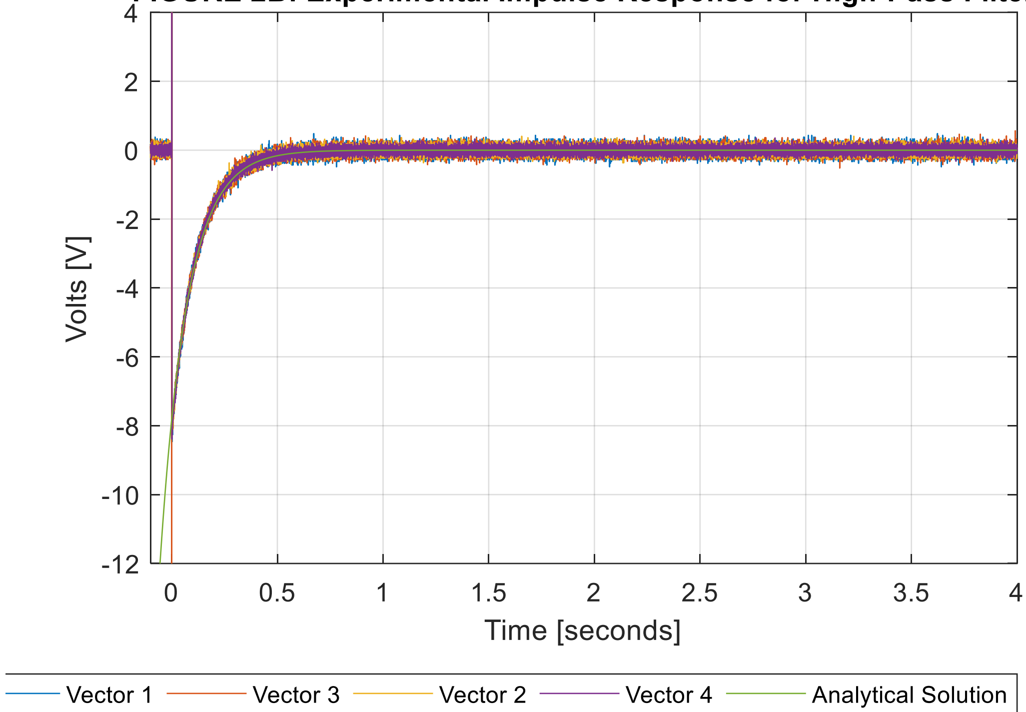

FIGURE 2B: This figure shows the experimental impulse response for a high-pass filter for a limited range, including all four vectors and the analytical solution that was determined above. Note the legend at the bottom, and the analytical solution (which can be seen as the line that threads through the rest of the noise).

For the next two figures within the laboratory experiment, we are graphing the normalized resonator response and the normalized anti-alias filter response within the x-limits of [-0.01, 0.05]. We graph out the four vector responses for both the resonator and the anti-alias filter and expect them to line up almost perfectly with one another.

We note that for the resonator, the samples tend towards the negative direction initially since the input doesn't influence the first equation, but rather the derivative of it. Also, the anti-alias filter response is a fourth order system that is supposed to show oscillation characteristic of a higher order system. We expect to see these features in the graphs as follows:

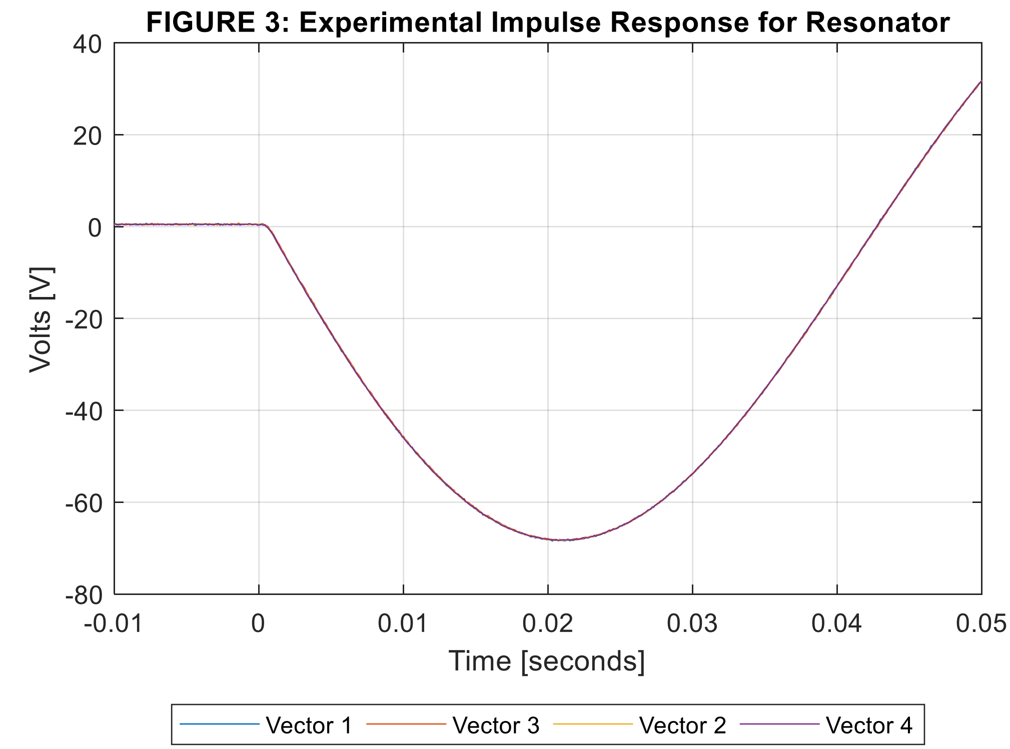

FIGURE 3: This figure shows the experimental impulse response for resonator.

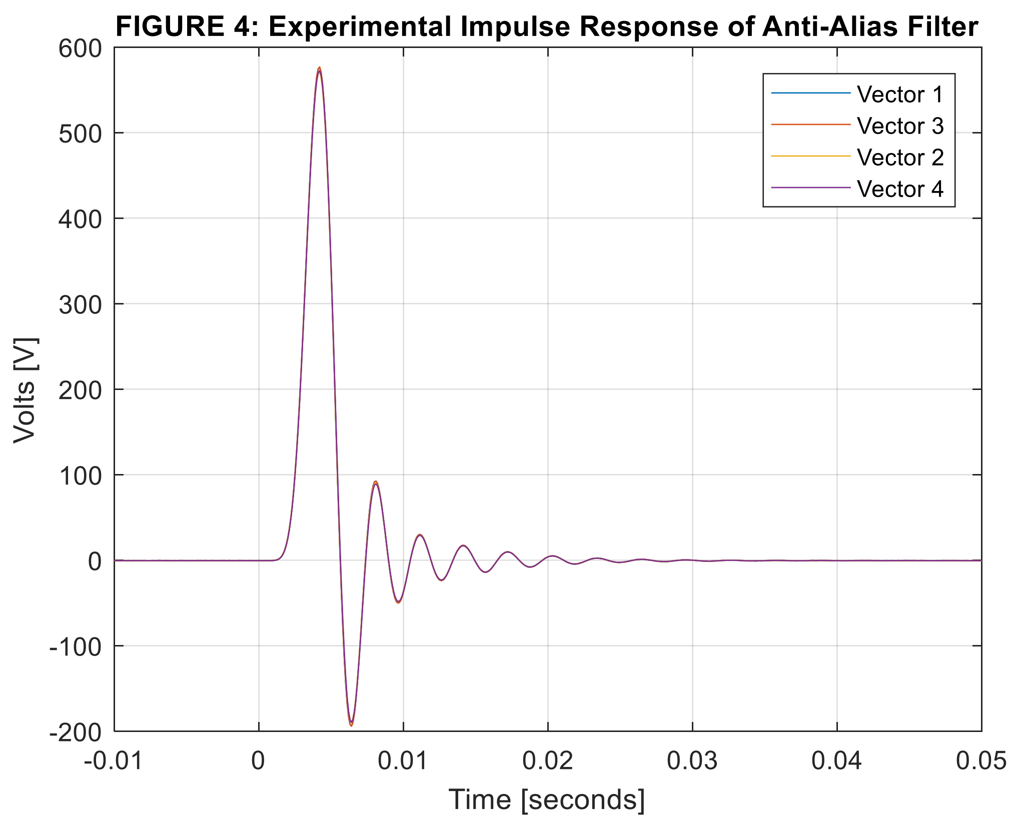

FIGURE 4: This figure shows the experimental impulse response for the anti-alias filter. Note the oscillations characteristic of a higher order system.

FIGURES 5A-5B: INPUT VS HIGH-PASS FILTER RESPONSE

Now we can graph one of the input vectors and compare it with the high-pass filter response. We choose the vector [0 4 6 8 8 8 8 8 6 4 0] in order to graph. Note that this vector correlates with vector 1, as stated above. We also plot the points of the vector, and do not allow MATLAB to interpolate between the points.

The figure is as follows:

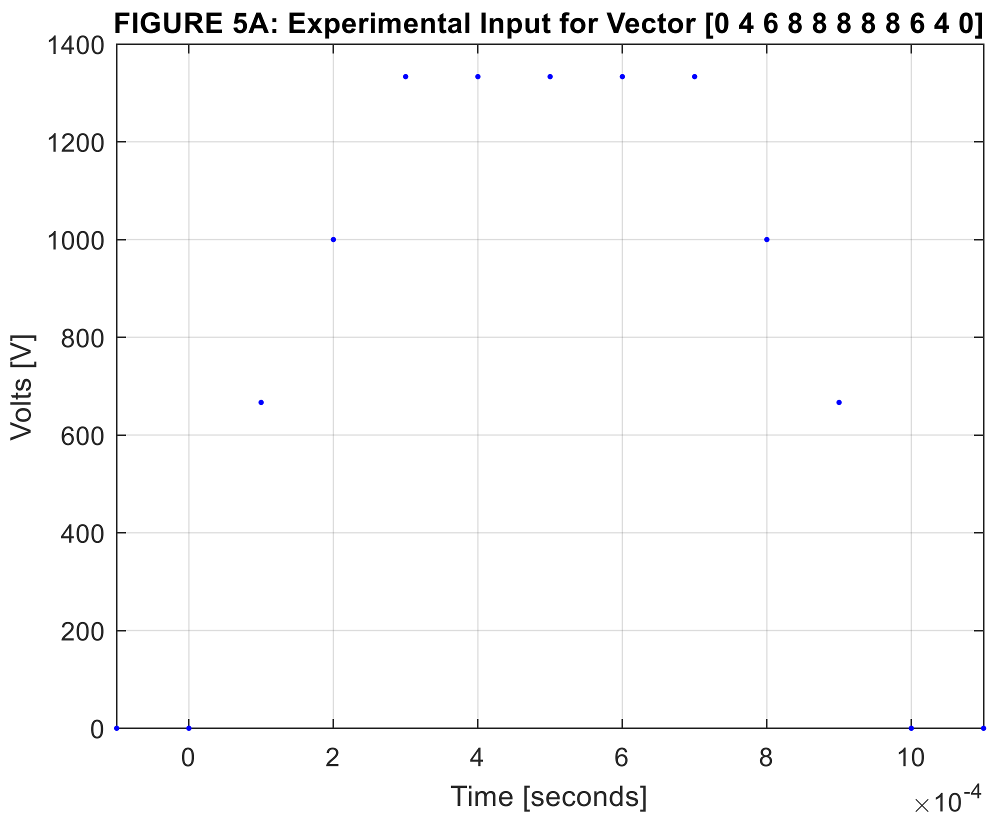

FIGURE 5A: This figure shows the input vector 1, corresponding to the vector stated in the title. Note that the points follow the general shape of the vector – in this case, a pyramidal shape.

Now that we have graphed out the input, we can also graph out the high-pass filter response to this same vector 1 for the same length of time. We expect the high-pass filter to follow the same "pattern" as Figure 5A. This will be explained after the graph, which is as follows:

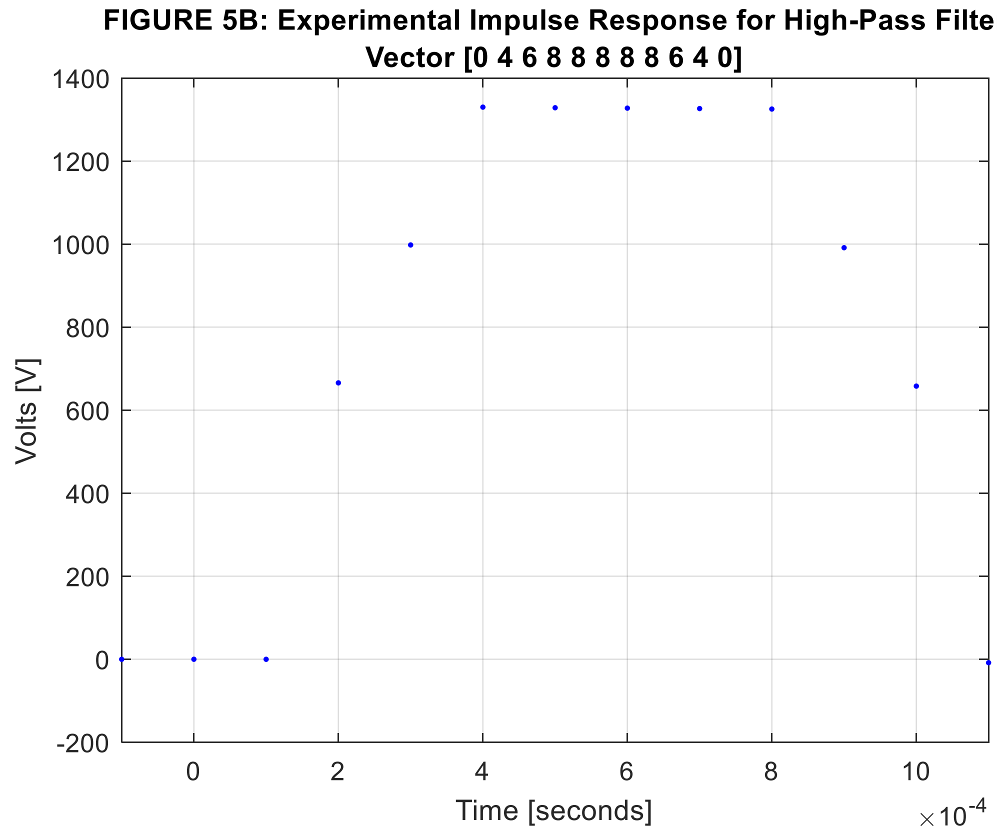

FIGURE 5B: This figure shows the experimental output of the high-pass filter for vector 1, which was the vector that was graphed out in Figure 5A. Note that the pattern follows the same pattern as Figure 5A.

We note that the plot in Figure 5B follows the same pattern as that in Figure 5A. This is because we know that the high-pass filter is supposed to follow the same impulse pattern as the input impulse. If we examine the analytical solution of the high-pass filter, we note that this is actually an impulse term: (**QUESTION**)

$$h(t) = b\delta(t) + abe^{at}$$

This means that the output of the high-pass filter will be the impulse multiplied by the parameter b, and that the decay of this (assuming no other impulse input) will be characterized by the second term in the analytical output response. However, we can say that the high-pass filter will follow the same pattern as the input vector because the response of the high-pass filter actually has the delta-dirac impulse function within it, and thus will follow the input vector because the input vectors are all impulses. We also note that the high-pass filter graph in Figure 5B does not show the second portion of the output analytical solution, because we plotted points and thus there was no decay possible. For this reason, the high-pass filter will have the same pattern as the input vector.

FIGURES 6-9: EXPERIMENTAL STEP RESPONSE

Now that we have graphed out the experimental impulse responses, we can actually also graph out the experimental step response for the low-pass, high-pass, resonator, and anti-alias filters.

From the graph, we calculated a slope of approximately **-7.78** for the decay after the two unit steps. This is compared to the time decay constant of **-8.935320869** for Figure 1. We note that the time decay constant for this case is actually smaller than the time decay constant for Figure 1. We calculated the slope of this exponential function by taking the natural log of the step response output for the low-pass filter and finding the slope after the step inputs. (**QUESTION**)

We note that the two decay constants are supposed to be similar – however, there may have been other sources of error that were introduced that caused the low-pass filter to decay more slowly than the impulse response.

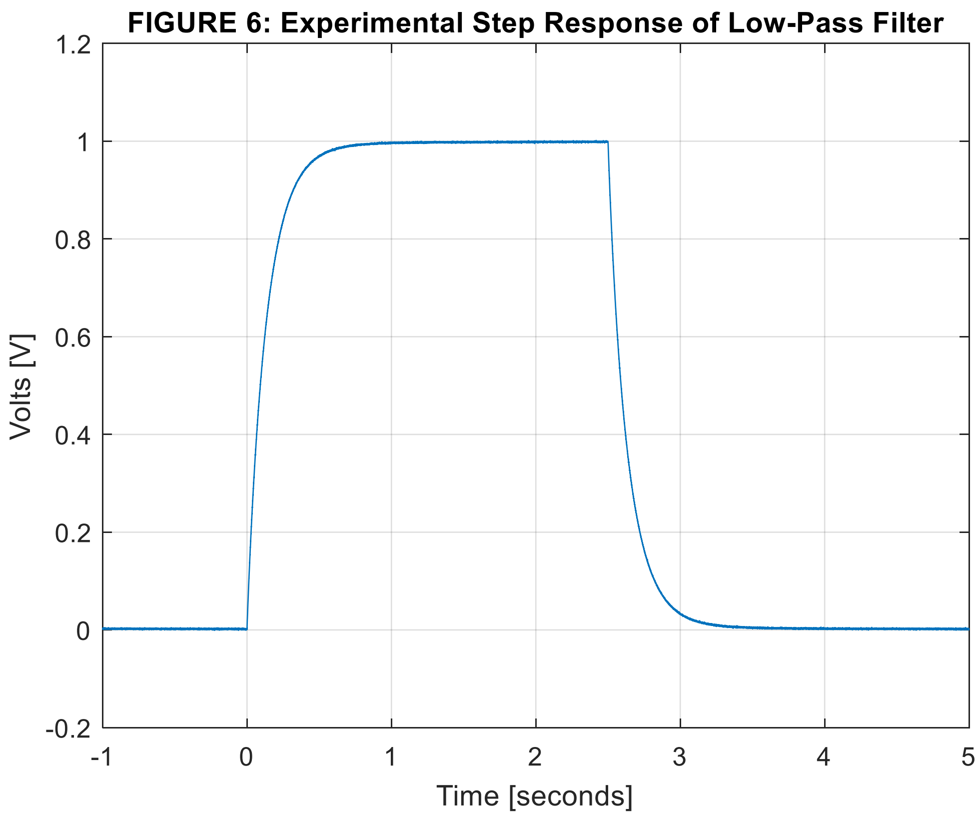

FIGURE 6: This figure shows the experimental step response for the low-pass filter.

We also note that after the first unit step, the output converges to the steady state value of 1, and after the second unit step, the output converges to the steady state value of 0. (**QUESTION**).

Then, we can graph out the high-pass filter step response. We notice that there is indeed no DC offset in the signal, and that it always settles to 0 regardless of the magnitude of the input. The graph is as follows:

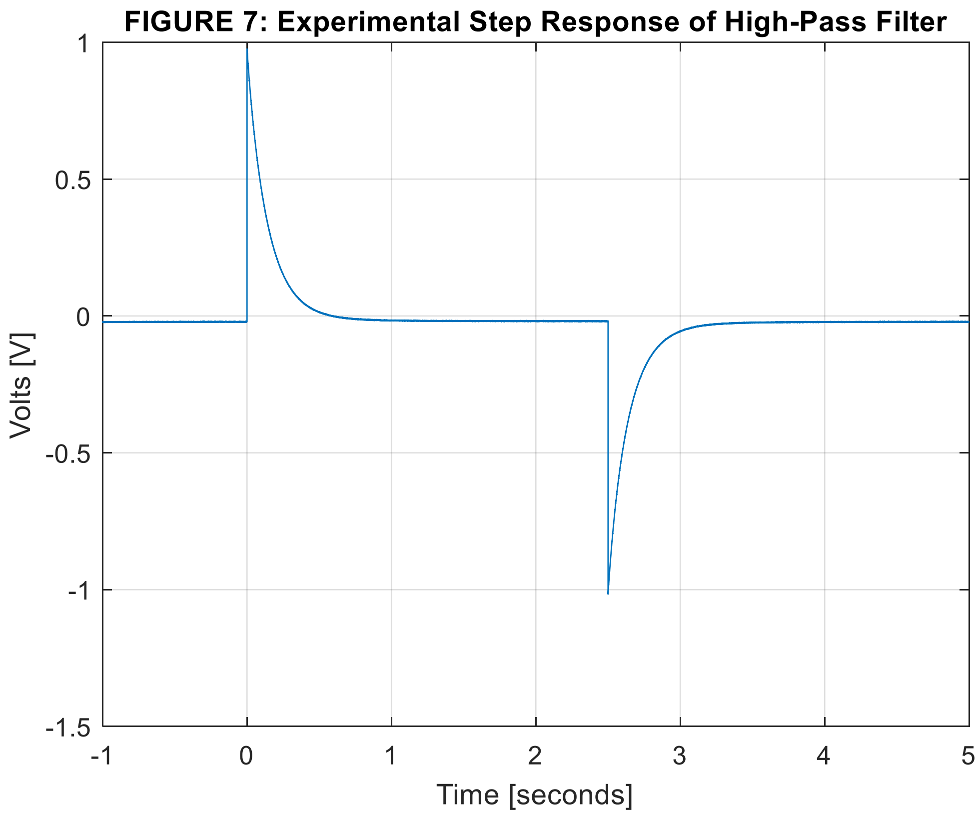

FIGURE 7: This figure shows the experimental step response for the high-pass filter.

If we examine the wave form immediately after each unit step, we notice that they are exponential functions (e). By taking the natural log of the output for the high-pass filter, we can actually calculate the decay constant for this graph.

We calculate the slope to be -7.485658409, as compared against the slope decay constant for Figure 2, which was -7.76975052. We note that these values are very close together to one another, and that as such, we can say that the decay values for these two cases are the same. When input with a step response and an impulse response, the output will spike and then decrease exponentially with the same decay constant. (**QUESTION**).

Now that we have graphed out the high-pass filter and the low-pass filter, we can graph out the resonator step response. We notice that the positive unit step has a steady state value of -1 and after the negative unit step, it settles to a steady state value of 0.

The graph is as follows:

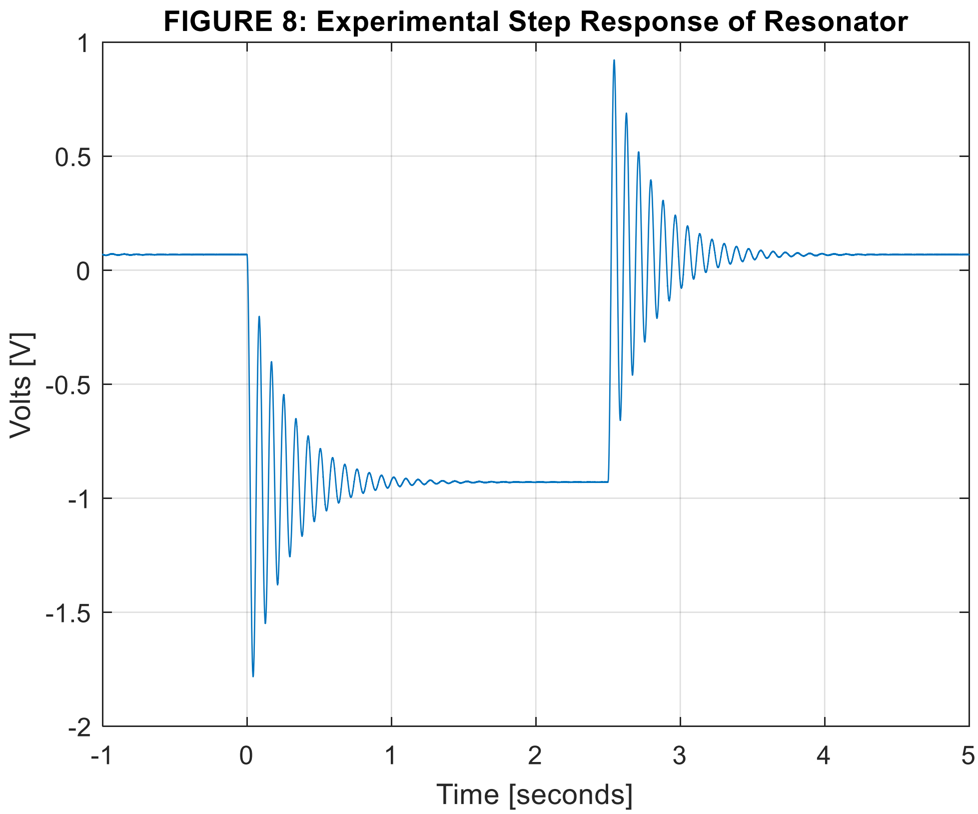

FIGURE 8: This figure shows the experimental step response for the resonator.

Now that we have this, we can actually calculate the resonant frequency in the following manner – we must first calculate the Q factor of the resonator. When a step response is input into a resonator, it rings it at its natural frequency. As such, we can calculate the Q factor by noticing when the resonator step response is 50% of its initial amplitude, and finding out how many cycles it takes to get there. From the graph above, we notice that it takes 2 cycles in order for the resonator to get to 50%. Then, we can find the Q factor by multiplying 2 cycles by 4.53, which is a constant. This gives us a Q factor of approximately 9.06.

In order to convert the Q factor into the resonant frequency, we take the following equation:

$$Envelope = \frac{1}{C}e^{-at}u(t)$$

$$a = \frac{1}{2RC}$$

$$Q = \omega_o RC$$

$$\omega_o = \frac{Q}{2a}$$

We can then find the envelope fit for our resonator response. We then note that our a fits with a slope of -0.3788, and as such we discover that the equation simplifies to:

$$\omega_o = \frac{9.06}{2(0.3788)}$$

This then reduces to a resonant frequency of:

$$\boldsymbol{\omega_o = 11.96\ Hz}$$

As such, we have determined the resonant frequency based off of the experimental resonator step response plot. (**QUESTION**)

Now, we graph the experimental step response of the anti-alias filter. This graph is as follows:

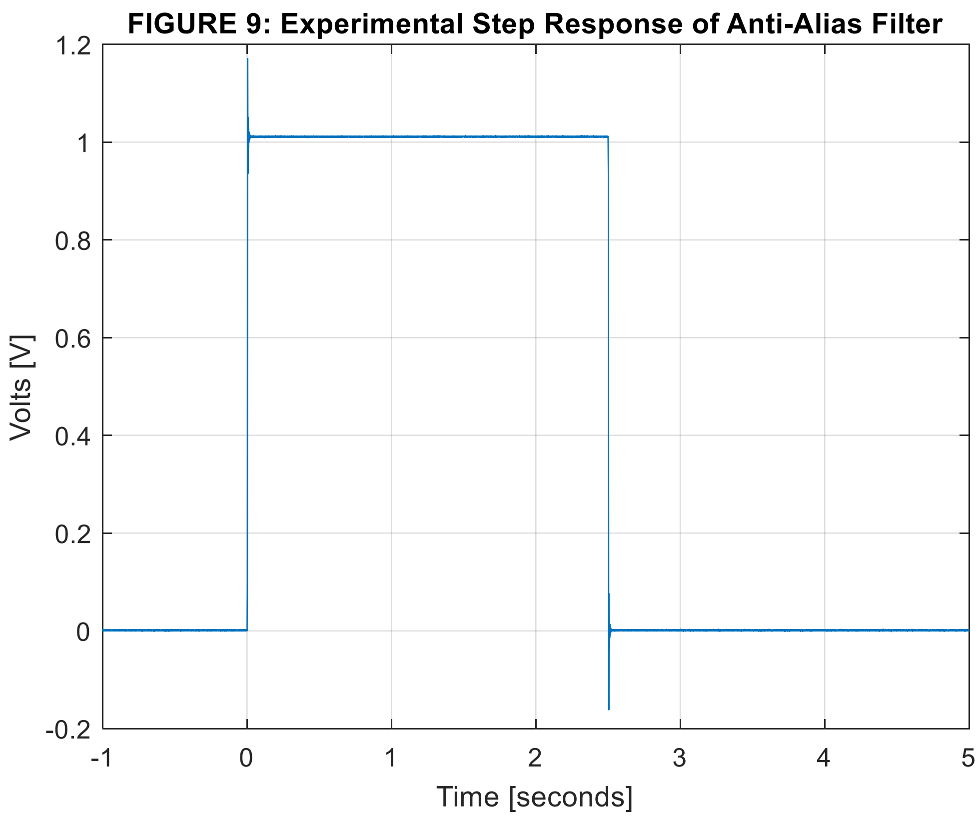

FIGURE 9: This figure shows the anti-alias filter experimental step response.

When compared to the step input, we can say that the anti-alias filter step response is almost exactly the same as the step input – there is a step input at 0 of magnitude 1 (although the anti-alias filter overshoots slightly), and then the step comes down around 2.5 seconds, just like the input step function. The output for the anti-alias filter and the input step are essentially the same. (**QUESTION**).

Now, we zoom into the region just after the step, and we can observe sinusoidal behavior, or oscillations, as the amplitude dies back down to a steady state value. We can thus say that the transient of the fourth order system is sinusoidal in nature, as in there is a exponential function or a sine or cosine term within the transient of this system. The graph is as follows:

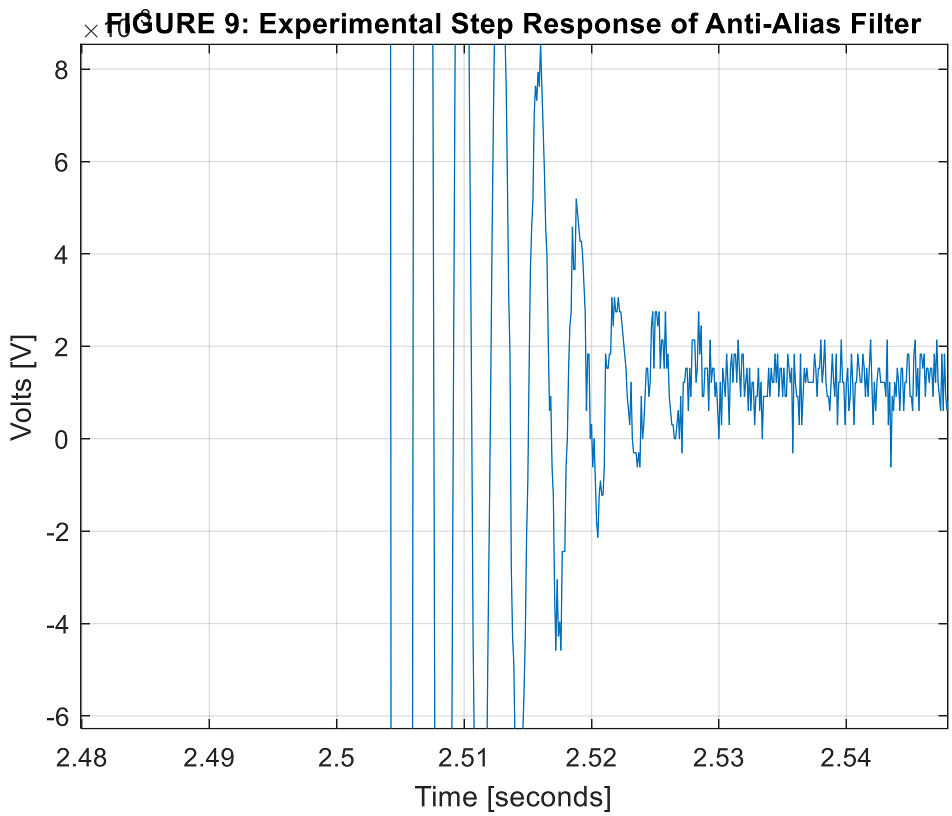

FIGURE 9A: This figure shows the zoomed-in graph of the experimental step response for the anti-alias filter after the step input, showing the oscillations.

As such, we can say that the oscillations are because of a transient sinusoidal term.

Based on the step response for each system, we can actually quantify the rise time, the settle time, and the overshoot of each of the systems. The rise time is the time it takes to reach the maximum value; the settle time is the time it takes to reach the steady state value, and the overshoot is how much the system goes over the theoretical maximum. We list these values for each of these graphs as follows:

TABLE 1: Rise Time, Settle Time, and Overshoot

FILTER	RISE TIME	SETTLE TIME	OVERSHOOT
Low-Pass	0.6285 sec	0.231 sec	0 V
High-Pass	0 sec	0.7488 sec	0.014 V
Resonator	0 sec	1.413 sec	0 V
Anti-Alias	0 sec	0.0314 sec	0.171 V

TABLE 1: This table shows the rise time, settle time, and overshoot for each of the four filters for the experimental step response.

CONCLUSION

Thus, within this experiment, the experimental impulse and step responses were measured for the low-pass filter, the high-pass filter, the resonator, and the anti-alias filter. The graphs of the experimental impulse and step responses were created and compared against the theoretical, expected models. First, the low-pass and high-pass impulse responses were graphed against the analytical models derived from the equations. Then, we compared the high-pass filter output with the input vector, and found them to follow the same exact pattern. We then graphed out the step response, and found that the decay constant for the low-pass was slightly smaller for the step response than the impulse response. When we graphed out the high-pass step response, we found that the decay constant was essentially the same for both the impulse and the step responses. We found a resonant frequency of approximately 11.96 Hz by using the Q-factor of the resonator, and found that the anti-alias filter step response output was essentially the same as the input step. We note that there was some noise within our circuit that could have caused error, and that human error was introduced when measuring the slope of the natural log graphs. However, we can say that this experiment was overall a success, because the graphs and patterns for the experimental impulse and step responses are essentially the same as the theoretical ones.

APPENDIX

```
%%%%%%%%%%%%%%%%%%%%%%%%%
%
% MAE 107 LAB 5
% Samuel Chen
% UID: 704-453-763
%
%%%%%%%%%%%%%%%%%%%%%%%%%

close all; clc;

%%%%%%%% LOAD DATA
load imp1; load imp2; load imp3; load imp4; load step_resp;


%%%%%% FIRST IMPULSE %%%%%%
imp1_time = imp1.X(1).Data;
qq = imp1_time<1.0;
imp1_input = imp1.Y(1).Data;

imp1_aaf = imp1.Y(2).Data;
offset = mean(imp1_aaf(qq));
imp1_aaf = imp1_aaf - offset;

imp1_hpf = imp1.Y(3).Data;
offset = mean(imp1_hpf(qq));
imp1_hpf = imp1_hpf - offset;

imp1_lpf = imp1.Y(4).Data;
offset = mean(imp1_lpf(qq));
```

```
imp1_lpf = imp1_lpf - offset;

imp1_res = imp1.Y(5).Data;
offset = mean(imp1_res(qq));
imp1_res = imp1_res - offset;



%%%%%% SECOND IMPULSE %%%%%%
imp2_time = imp2.X(1).Data;
imp2_input = imp2.Y(1).Data;

imp2_aaf = imp2.Y(2).Data;
offset = mean(imp2_aaf(qq));
imp2_aaf = imp2_aaf - offset;

imp2_hpf = imp2.Y(3).Data;
offset = mean(imp2_hpf(qq));
imp2_hpf = imp2_hpf - offset;

imp2_lpf = imp2.Y(4).Data;
offset = mean(imp2_lpf(qq));
imp2_lpf = imp2_lpf - offset;

imp2_res = imp2.Y(5).Data;
offset = mean(imp2_res(qq));
imp2_res = imp2_res - offset;



%%%%%% THIRD IMPULSE %%%%%%
imp3_time = imp3.X(1).Data;
imp3_input = imp3.Y(1).Data;

imp3_aaf = imp3.Y(2).Data;
offset = mean(imp3_aaf(qq));
imp3_aaf = imp3_aaf - offset;

imp3_hpf = imp3.Y(3).Data;
offset = mean(imp3_hpf(qq));
imp3_hpf = imp3_hpf - offset;

imp3_lpf = imp3.Y(4).Data;
offset = mean(imp3_lpf(qq));
imp3_lpf = imp3_lpf - offset;

imp3_res = imp3.Y(5).Data;
offset = mean(imp3_res(qq));
imp3_res = imp3_res - offset;


%%%%%% FOURTH IMPULSE %%%%%%
imp4_time = imp4.X(1).Data;
imp4_input = imp4.Y(1).Data;

imp4_aaf = imp4.Y(2).Data;
offset = mean(imp4_aaf(qq));
imp4_aaf = imp4_aaf - offset;

imp4_hpf = imp4.Y(3).Data;
offset = mean(imp4_hpf(qq));
imp4_hpf = imp4_hpf - offset;
```

```
imp4_lpf = imp4.Y(4).Data;
offset = mean(imp4_lpf(qq));
imp4_lpf = imp4_lpf - offset;

imp4_res = imp4.Y(5).Data;
offset = mean(imp4_res(qq));
imp4_res = imp4_res - offset;


%%%%%% STER_RESP %%%%%%
step_time = step_resp.X(1).Data;
step_input = step_resp.Y(1).Data;
step_aaf = step_resp.Y(2).Data;
step_hpf = step_resp.Y(3).Data;
step_lpf = step_resp.Y(4).Data;
step_res = step_resp.Y(5).Data;

%%%%% FIRST IMPULSE %%%%%
dt = 0.0001;    % sample period
a1 = sum(imp1_input)*dt;
imp1_input = imp1_input/a1;
imp1_lpf = imp1_lpf/a1;
imp1_hpf = imp1_hpf/a1;
imp1_res = imp1_res/a1;
imp1_aaf = imp1_aaf/a1;

%%%%% SECOND IMPULSE %%%%%
a2 = sum(imp2_input)*dt;
imp2_input = imp2_input/a2;
imp2_lpf = imp2_lpf/a2;
imp2_hpf = imp2_hpf/a2;
imp2_res = imp2_res/a2;
imp2_aaf = imp2_aaf/a2;


%%%%% THIRD IMPULSE %%%%%
a3 = sum(imp3_input)*dt;
imp3_input = imp3_input/a3;
imp3_lpf = imp3_lpf/a3;
imp3_hpf = imp3_hpf/a3;
imp3_res = imp3_res/a3;
imp3_aaf = imp3_aaf/a3;

%%%%% FOURTH IMPULSE %%%%%
a4 = sum(imp4_input)*dt;
imp4_input = imp4_input/a4;
imp4_lpf = imp4_lpf/a4;
imp4_hpf = imp4_hpf/a4;
imp4_res = imp4_res/a4;
imp4_aaf = imp4_aaf/a4;


disp(a1);
disp(a2);
disp(a3);
disp(a4);

%%%%% FIGURE 1 %%%%%
% b = e^2.068
% a = -8.935320869
func1 = 7.908989311*exp(-8.935320869.*(imp1_time));

figure(1)
```

```
plot(imp1_time, imp1_lpf); hold on;     % 0.0060
plot(imp1_time, imp3_lpf);              % 0.0062
plot(imp1_time, imp2_lpf);              % 0.0071
plot(imp1_time, imp4_lpf);
plot(imp1_time, func1); hold off;    % 0.0081
xlim([-0.05 0.2]);
title('FIGURE 1: Experimental Impulse Response for Low-Pass Filter');
xlabel('Time [seconds]');
ylabel('Volts [V]');
legend('Vector 1', 'Vector 3', 'Vector 2', 'Vector 4', 'Analytical Solution');
grid on;

%imp4_lpf_log = log(imp4_lpf);
%figure(2)
%plot(imp4_time, imp4_lpf_log);


figure(2)
plot(imp1_time, imp1_hpf); hold on;
plot(imp1_time, imp3_hpf);
plot(imp1_time, imp2_hpf);
plot(imp1_time, imp4_hpf);
xlim([-0.0001, 0.0011]);
title('FIGURE 2: Experimental Impulse Response for High-Pass Filter');
xlabel('Time [seconds]');
ylabel('Volts [V]');
legend('Vector 1', 'Vector 3', 'Vector 2', 'Vector 4', 'orientation', 'horizontal',
'location', 'southoutside');
grid on;

a2 = -7.76975052;
b2 = exp(-0.264487257);
ab = exp(2.055);

impulse = imp1_input==0;
%impulse = dirac(imp1_time);
func2 = -ab*exp(a2*imp1_time);

figure(3)   %2b
plot(imp1_time, imp1_hpf); hold on;
plot(imp1_time, imp3_hpf);
plot(imp1_time, imp2_hpf);
plot(imp1_time, imp4_hpf);
plot(imp1_time, func2); hold off;
xlim([-0.1 4]);
ylim([-12 4]);
title('FIGURE 2B: Experimental Impulse Response for High-Pass Filter');
xlabel('Time [seconds]');
ylabel('Volts [V]');
legend('Vector 1', 'Vector 3', 'Vector 2', 'Vector 4', 'Analytical Solution',
'location', 'eastoutside');
grid on;

%imp3_hpf_log = log(imp3_hpf);
%figure(4)
%plot(imp3_time, imp3_hpf_log);
%xlim([-0.1 4]);
%ylim([-12 4]);

figure(4) % figure 3
plot(imp1_time, imp1_res); hold on;
plot(imp1_time, imp3_res);
plot(imp1_time, imp2_res);
```

```
plot(imp1_time, imp4_res); hold off;
xlim([-0.01 0.05]);
title('FIGURE 3: Experimental Impulse Response for Resonator');
xlabel('Time [seconds]');
ylabel('Volts [V]');
legend('Vector 1', 'Vector 3', 'Vector 2', 'Vector 4', 'orientation', 'horizontal',
'location', 'southoutside');
grid on;

figure(5) % figure 4
plot(imp1_time, imp1_aaf); hold on;
plot(imp1_time, imp3_aaf);
plot(imp1_time, imp2_aaf);
plot(imp1_time, imp4_aaf); hold off;
xlim([-0.01 0.05]);
grid on;


%%%%% FIGURES 5A AND 5B %%%%%
% let's plot input 1

figure(6) % figure 5a
plot(imp1_time, imp1_input, '.b');
xlim([-0.0001 0.0011]);
title('FIGURE 5A: Experimental Input for Vector [0 4 6 8 8 8 8 8 6 4 0]');
xlabel('Time [seconds]');
ylabel('Volts [V]');
grid on;

figure(7)  % figure 5b
plot(imp1_time, imp1_hpf, '.b');
xlim([-0.0001 0.0011]);
title({'FIGURE 5B: Experimental Impulse Response for High-Pass Filter', 'Vector [0 4 6
8 8 8 8 8 6 4 0]'});
xlabel('Time [seconds]');
ylabel('Volts [V]');
grid on;

func1_step = 7.908989311*exp(-8.935320869.*(step_time));

%%%%% FIGURE 6-9 %%%%%
figure(8) % figure 6
plot(step_time, step_lpf); %hold on;
%plot(step_time, func1_step); hold off;
title('FIGURE 6: Experimental Step Response of Low-Pass Filter');
xlabel('Time [seconds]');
ylabel('Volts [V]');
grid on;

figure(9)
plot(step_time, step_hpf);
title('FIGURE 7: Experimental Step Response of High-Pass Filter');
xlabel('Time [seconds]');
ylabel('Volts [V]');
grid on;

% resonance at 11.9 Hz
figure(10)
plot(step_time, step_res); hold on;
plot(step_time, exp(-1.38*step_time));
title('FIGURE 8: Experimental Step Response of Resonator');
xlabel('Time [seconds]');
ylabel('Volts [V]');
```

```
grid on;

figure(11)
plot(step_time, step_aaf);


figure(12)
plot(step_time, log(step_lpf));
grid on;

figure(13)
plot(step_time, log(step_hpf));
grid on;

figure(14)
plot(step_time, step_input);
grid on;
```