ECEN 611 Homework 4: Gap Function and Mutual Inductance for Salient Pole Rotor

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Problem 1

Problem 1) Find the Fourier series of the inverse gap function, $g^{-1}(\phi)$, for a salient pole synchronous machine. Find the rotor pole arc such that there are no multiple of 3^{rd} harmonic in the Fourier series of the inverse gap function.

Express gap length as a function of the angle Φ.

```
clearvars
clc
```

```
syms g_min g_max     positive real
syms alpha           positive real
syms phi                      real
syms g_phi(phi)
syms theta_r theta

assume( (alpha > 0) & (alpha <= deg2rad(180)) )

T = 2*pi;
POLEARC = deg2rad(90);
THETA_R = deg2rad(45);

poleArc = alpha;
midArc = T/2 - poleArc;

rotor_profile_theta = [poleArc/2 midArc poleArc midArc poleArc/2];
rotor_profile_turning_theta_r = theta_r + cumsum(rotor_profile_theta)
```

rotor_profile_turning_theta_r =

$$\left(\frac{\alpha}{2}+\theta_r \quad \theta_r-\frac{\alpha}{2}+\pi \quad \frac{\alpha}{2}+\theta_r+\pi \quad \theta_r-\frac{\alpha}{2}+2\,\pi \quad \theta_r+2\,\pi\right)$$

```
% disp(rotor_profile_turning_theta_r)
% The location of a point of symmetry on one of the two-pole faces
% is now aligned with the reference position for the ϕ
```

```
% which is selected as the (magnetic) axis of phase A
phi_ref = 0;
rotor_profile_turning_phi = subs(rotor_profile_turning_theta_r, theta_r,
-deg2rad(phi_ref))
```

rotor_profile_turning_phi =

$$\left(\frac{\alpha}{2} \quad \pi - \frac{\alpha}{2} \quad \frac{\alpha}{2} + \pi \quad 2\,\pi - \frac{\alpha}{2} \quad 2\,\pi\right)$$

```
% add phi_ref to rotor_profile_turning_phi for better consistency
rotor_profile_turning_phi = [phi_ref, rotor_profile_turning_phi]
```

rotor_profile_turning_phi =

$$\left(0 \quad \frac{\alpha}{2} \quad \pi - \frac{\alpha}{2} \quad \frac{\alpha}{2} + \pi \quad 2\,\pi - \frac{\alpha}{2} \quad 2\,\pi\right)$$

```
gap_profile = [g_min, g_max, g_min, g_max, g_min];

firstPhi = rotor_profile_turning_phi(1);
secondPhi = rotor_profile_turning_phi(2);

firstGap = gap_profile(1);

g_phi(phi) = piecewise(firstPhi <= phi < secondPhi, firstGap)
```

g_phi(phi) = $\begin{cases} g_{\min} & \text{if } 2\,\phi < \alpha \wedge 0 \le \phi \end{cases}$

```

for k = 2 : length(rotor_profile_turning_phi) - 1

    thisPhi = rotor_profile_turning_phi(k);
    nextPhi = rotor_profile_turning_phi(k+1);
    thisGap = gap_profile(k);

    % Both lower and upper boundaries for phi are 'closed'
    % i.e. [lower boundary, upper boundary]
    % if k == length(rotor_profile_turning_phi) - 1
    %     g_phi(phi) = piecewise( ...
    %         thisPhi <= phi <= nextPhi, thisGap, ...
    %         phi <= thisPhi, g_phi(phi) ...
    %     );
    % end

    % Only lower boundary is closed
    g_phi(phi) = piecewise( ...
        thisPhi <= phi < nextPhi, thisGap, ...
        phi <= thisPhi, g_phi(phi) ...
    );
end

g_phi(phi) = simplify(g_phi)
```

```
g_phi(phi) =
```

$$
\begin{cases}
g_{\min} & \text{if } \phi < 2\,\pi \wedge 4\,\pi \leq \alpha + 2\,\phi \\
g_{\max} & \text{if } \dfrac{\alpha}{2} + \pi \leq \phi \wedge \alpha + 2\,\phi < 4\,\pi \\
g_{\min} & \text{if } \alpha + 2\,\phi \in [2\,\pi, 4\,\pi] \wedge 2\,\phi < \alpha + 2\,\pi \\
g_{\max} & \text{if } \alpha \leq 2\,\phi \wedge \alpha + 2\,\phi < 2\,\pi \\
g_{\min} & \text{if } 2\,\phi < \alpha \wedge 0 \leq \phi
\end{cases}
$$

```
vars = [poleArc g_min g_max theta_r];
VALS = [POLEARC 0.5   0.8   THETA_R];

% figure
% fplot(subs(g_phi, vars, VALS), [0 2*T])
% hold on
% ylim([0 1])
% grid on
% ax = gca;
% S = sym(ax.XLim(1):pi/4:ax.XLim(2));
% ax.XTick = double(S);
% ax.XTickLabel = arrayfun(@texlabel,S,'UniformOutput',false);
```

Original Inverse Gap Function

```
g_phi_inv = 1 / g_phi
```

```
g_phi_inv(phi) =
```

$$
\begin{cases}
\dfrac{1}{g_{\min}} & \text{if } \phi < 2\,\pi \wedge 4\,\pi \leq \alpha + 2\,\phi \\
\dfrac{1}{g_{\max}} & \text{if } \dfrac{\alpha}{2} + \pi \leq \phi \wedge \alpha + 2\,\phi < 4\,\pi \\
\dfrac{1}{g_{\min}} & \text{if } \alpha + 2\,\phi \in [2\,\pi, 4\,\pi] \wedge 2\,\phi < \alpha + 2\,\pi \\
\dfrac{1}{g_{\max}} & \text{if } \alpha \leq 2\,\phi \wedge \alpha + 2\,\phi < 2\,\pi \\
\dfrac{1}{g_{\min}} & \text{if } 2\,\phi < \alpha \wedge 0 \leq \phi
\end{cases}
$$

```

% figure
% fplot(subs(g_phi, vars, VALS), [0 2*T], ...
%     "DisplayName", "Gap Function", ...
%     "LineWidth", 1.2)
% hold on
% fplot(subs(g_phi_inv, vars, VALS), [0 2*T], ...
%     "DisplayName", "Inverse Gap Function", ...
%     "LineWidth", 1.2)
% hold off
% ylim([0 2.5])
```

```
% grid on
% legend
%
% ax = gca;
% S = sym(ax.XLim(1):pi/4:ax.XLim(2));
% ax.XTick = double(S);
% ax.XTickLabel = arrayfun(@texlabel,S,'UniformOutput',false);
```

Extended Even Inverse Gap Function

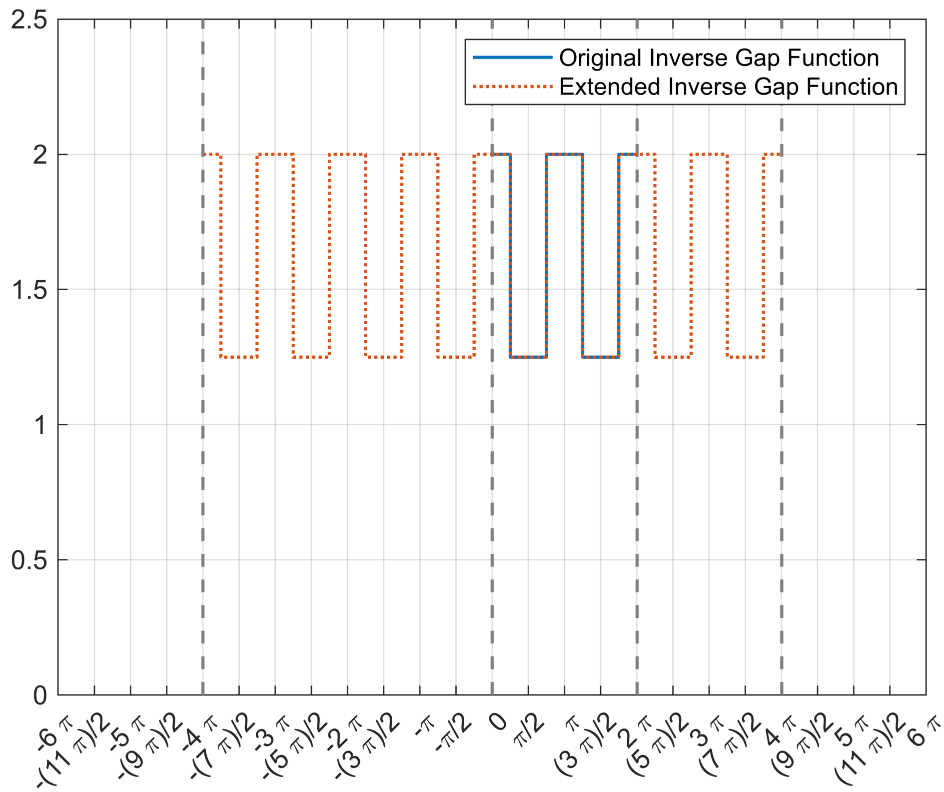
```
g_phi_inv_ext(phi) = piecewise( ...
    -2*T <= phi < -T, g_phi_inv(phi+2*T), ...
    -T <= phi < 0, g_phi_inv(phi+T), ...
    0 <= phi < T, g_phi_inv(phi), ...
    T <= phi <= 2*T, g_phi_inv(phi-T) ...
)
```

```
g_phi_inv_ext(phi) =
```

$$
\begin{cases}
\frac{1}{g_{\min}} & \text{if } \phi < -2\pi \wedge -4\pi \leq \alpha + 2\phi \\
\frac{1}{g_{\max}} & \text{if } \phi < -2\pi \wedge \alpha + 2\phi < -4\pi \wedge \alpha \leq 2\phi + 6\pi \\
\frac{1}{g_{\min}} & \text{if } \alpha + 2\phi + 8\pi \in [2\pi, 4\pi] \wedge \phi < -2\pi \wedge 2\phi + 6\pi < \alpha \\
\frac{1}{g_{\max}} & \text{if } \phi < -2\pi \wedge \alpha + 2\phi < -6\pi \wedge \alpha \leq 2\phi + 8\pi \\
\frac{1}{g_{\min}} & \text{if } (\phi < 0 \wedge 0 \leq \alpha + 2\phi) \vee (\phi \in [-4\pi, -2\pi) \wedge 2\phi + 8\pi < \alpha) \\
\frac{1}{g_{\max}} & \text{if } \alpha \leq 2\phi + 2\pi \wedge \phi < 0 \wedge \alpha + 2\phi < 0 \\
\frac{1}{g_{\min}} & \text{if } \alpha + 2\phi + 4\pi \in [2\pi, 4\pi] \wedge 2\phi + 2\pi < \alpha \wedge \phi < 0 \\
\frac{1}{g_{\max}} & \text{if } \alpha + 2\phi < -2\pi \wedge \alpha \leq 2\phi + 4\pi \wedge \phi < 0 \\
\frac{1}{g_{\min}} & \text{if } (\phi < 2\pi \wedge 4\pi \leq \alpha + 2\phi) \vee (\phi \in [-2\pi, 0) \wedge 2\phi + 4\pi < \alpha) \\
\frac{1}{g_{\max}} & \text{if } \phi < 2\pi \wedge \alpha + 2\phi < 4\pi \wedge \alpha + 2\pi \leq 2\phi \\
\frac{1}{g_{\min}} & \text{if } \alpha + 2\phi \in [2\pi, 4\pi] \wedge \phi < 2\pi \wedge 2\phi < \alpha + 2\pi \\
\frac{1}{g_{\max}} & \text{if } \phi < 2\pi \wedge \alpha \leq 2\phi \wedge \alpha + 2\phi < 2\pi \\
\frac{1}{g_{\min}} & \text{if } (\phi \in [0, 2\pi) \wedge 2\phi < \alpha) \vee (\phi < 4\pi \wedge 8\pi \leq \alpha + 2\phi) \\
\frac{1}{g_{\max}} & \text{if } \phi \leq 4\pi \wedge \alpha + 2\phi < 8\pi \wedge \alpha + 6\pi \leq 2\phi \\
\frac{1}{g_{\min}} & \text{if } \alpha + 2\phi - 4\pi \in [2\pi, 4\pi] \wedge \phi \leq 4\pi \wedge 2\phi < \alpha + 6\pi \\
\frac{1}{g_{\max}} & \text{if } \phi \leq 4\pi \wedge \alpha + 2\phi < 6\pi \wedge \alpha + 4\pi \leq 2\phi \\
\frac{1}{g_{\min}} & \text{if } \phi \in [2\pi, 4\pi] \wedge 2\phi < \alpha + 4\pi
\end{cases}
$$

```
figure
fplot(subs(g_phi_inv, vars, VALS), [-3*T 3*T], ...
      "DisplayName", "Original Inverse Gap Function", ...
      "LineWidth", 1.2)
hold on
fplot(subs(g_phi_inv_ext, vars, VALS), [-3*T 3*T], ...
      "DisplayName", "Extended Inverse Gap Function", ...
      "LineStyle",":", ...
      "LineWidth", 1.2 ...
      )
```

```
hold off
ylim([0 2.5])
grid on
legend
ax = gca;
S = sym(ax.XLim(1):pi/2:ax.XLim(2));
ax.XTick = double(S);
ax.XTickLabel = arrayfun(@texlabel,S,'UniformOutput',false);
```



Fourier Series of Even Inverse Gap Function

```
digits(4)

a0 = vpa( (1/T) * int( subs(g_phi_inv, theta_r, THETA_R), phi, [0 T], ...
    'IgnoreSpecialCases', true) )
```

a0 =

$$\frac{0.3183\,\alpha}{g_{\min}} - \frac{0.3183\,(\alpha - 3.142)}{g_{\max}}$$

```
syms n

an = (2/T) * int( subs(g_phi_inv, theta_r, THETA_R) * cos(n * phi), phi, [0 T]);
bn = (2/T) * int( subs(g_phi_inv, theta_r, THETA_R) * sin(n * phi), phi, [0 T]);

orderOfHarmonics = 1:13;
```

```
an_13 = vpa( simplify(subs(an, n, orderOfHarmonics)) )
```

an_13 =

$$\left(0 \quad -\frac{0.6366\sin(\alpha)\ (g_{\min} - 1.0\,g_{\max})}{g_{\min}\,g_{\max}} \quad 0 \quad -\frac{0.3183\sin(2.0\,\alpha)\ (g_{\min} - 1.0\,g_{\max})}{g_{\min}\,g_{\max}} \quad 0 \quad -\frac{0.2122\sin(3.0\,\alpha)\ (g}{g_{\min}\,g_{m}}\right.$$

```
bn_13 = vpa( simplify(subs(bn, n, orderOfHarmonics)) )
```

bn_13 = $(0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$

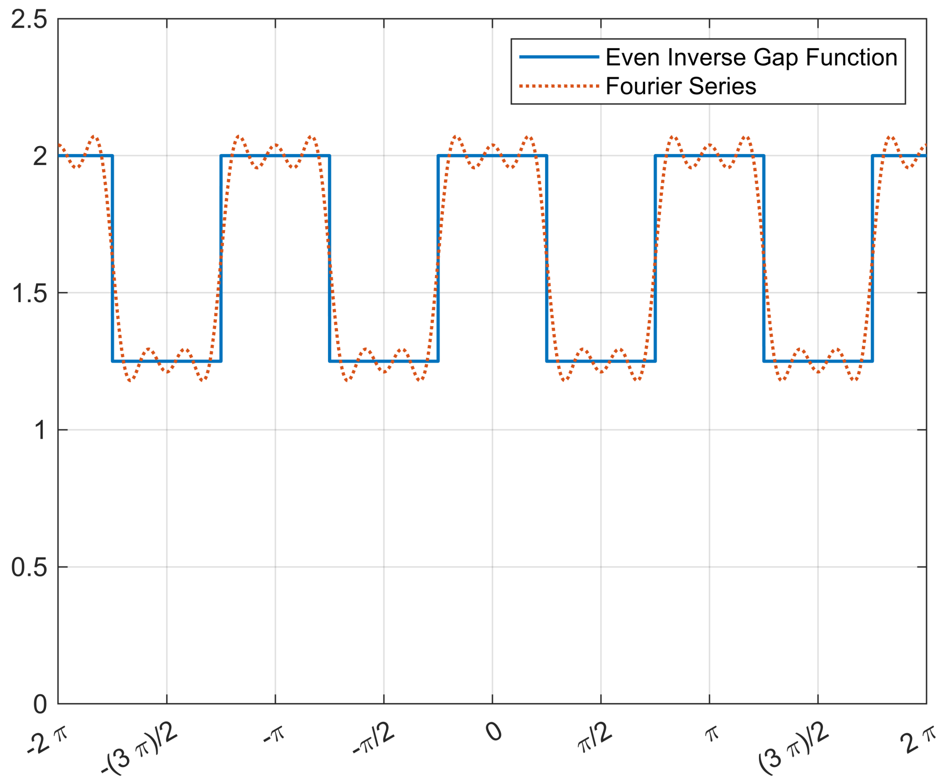
```
g_phi_inv_even_fourier = a0 + ...
                    sum( an_13 .* cos(orderOfHarmonics .* phi) + ...
                         bn_13 .* sin(orderOfHarmonics .* phi) )
```

g_phi_inv_even_fourier =

$$\frac{0.3183\,\alpha}{g_{\min}} - \frac{0.3183\ (\alpha - 3.142)}{g_{\max}} - \frac{0.3183\cos(4\,\phi)\sin(2.0\,\alpha)\ (g_{\min} - 1.0\,g_{\max})}{g_{\min}\,g_{\max}} - \frac{0.1592\cos(8\,\phi)\sin(4.0\,\alpha)}{g_{\min}\,g_{\max}}$$

```
figure
fplot(subs(g_phi_inv_ext, vars, VALS), [-T T], ...
      "DisplayName", "Even Inverse Gap Function", ...
      "LineWidth", 1.2)
hold on
fplot(subs(g_phi_inv_even_fourier, vars, VALS), [-T T], ...
      "DisplayName", "Fourier Series", ...
      "LineStyle",":", ...
      "LineWidth", 1.2 ...
      )
hold off
ylim([0 2.5])
grid on
legend

ax = gca;
S = sym(ax.XLim(1):pi/2:ax.XLim(2));
ax.XTick = double(S);
ax.XTickLabel = arrayfun(@texlabel,S,'UniformOutput',false);
```



Removal of Multiples of 3rd Harmonics in Fourier Series of Even Inverse Gap Function g(φ)^(-1)

```
terms = children(g_phi_inv_even_fourier);
multiplesOf3rd = find(mod(orderOfHarmonics,3)==0)
```

```
multiplesOf3rd = 1×4
     3     6     9    12
```

```
% Initialize a symbolic zero to accumulate matching terms
matching_terms = [];

for i = 1:length(terms)
    if has(terms{i}, cos(multiplesOf3rd*phi))
        matching_terms = [matching_terms terms{i}]; % Accumulate matching terms
    end
end

disp(matching_terms);
```

$$\left(-\frac{0.2122\cos(6\,\phi)\,\sin(3.0\,\alpha)\,\left(g_{\min}-1.0\,g_{\max}\right)}{g_{\min}\,g_{\max}} \quad -\frac{0.1061\cos(12\,\phi)\,\sin(6.0\,\alpha)\,\left(g_{\min}-1.0\,g_{\max}\right)}{g_{\min}\,g_{\max}} \right)$$

It is clear that when $\alpha = \frac{m\pi}{3}$ where m is any positive integer, both $\cos(6\,\phi)$ and $\cos(12\,\phi)$ are eliminated as both $\sin(3.0\,\alpha)$ and $\sin(6.0\,\alpha)$ are zero.