ECEN 611 Homework 1

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```
clearvars
clc
```

Energy Stored in Magnetic Field

```
% syms lambda i
% Wfld = int(i,lambda,[0 lambda],'Hold',true)
```

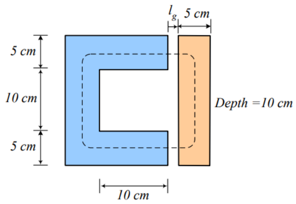
Exercise 1

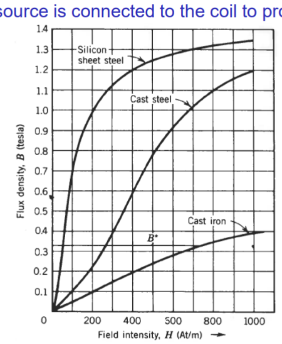
Exercise 1

The dimensions of a relay system are shown in figure below. The magnetic core is made of cast steel whose B-H characteristic is also shown. The coil has 300 turns, and the coil resistance is 6 ohms. For a fixed air-gap length l_g = 4 mm, a dc source is connected to the coil to produce a flux density of 1.1 Tesla in the air-gap. Calculate

(a) The voltage of the dc source.

(b) The stored field energy.






```
N = 300;   % number of turns of coil
Rcoil = 6; % coil total resistance [ohms]
lg = 4e-3; % airgap length [m]
Bg = 1.1;  % airgap flux density [T]
Depth = 10e-2; % [m]

uo = 4*pi*1e-7; % H/m
ug = uo;
Ag = 5e-2 * Depth % airgap cross-sectional area [m^2]
```

```
Ag = 0.0050
```

```
Ai = Ag;   % steel cross-sectional area facing airgap
```

(a) DC Voltage Source

B --> Φ --> H --> i --> V

Flux is the "currency" that holds constant across the steel and the airgap. Use this nature to obtain the flux density in the steel.

```
fluxg = Bg * Ag;  % flux in the airgap
fluxi = fluxg;    % flux in the steel

Bi = fluxi / Ai  % steel flux density
```

```
Bi = 1.1000
```

Read Hi corresponding to Bi from the characteristic

```
% Read from B-H characteristic
Hi = 800; % [A·turns/m]
```

Permeability of steel can hence be calculated

```
ui = Bi / Hi
```

```
ui = 0.0014
```

Construct flux paths and calculate reluctances for steel and airgap segments, respectively

```
%%%%%%%%%%%%%% cm %%%%%%%%%%%%%%
lccore_max = (5+10+5) + (5+10) + (5+10);
lccore_min = 10 + 10 + 10;
lccore_avg = (lccore_max + lccore_min) / 2;

licore_max = 5 + 5 + 10 + 5 + 5;
licore_min = 10;
licore_avg = (licore_max + licore_min) / 2;

%%%%%%%%%%%%%% m %%%%%%%%%%%%%%
li = (lccore_avg + licore_avg) * 1e-2;
Ri = li / (ui*Ai)
```

```
Ri = 8.7273e+04
```

```
Rg = 2*lg / (ug*Ag)
```

```
Rg = 1.2732e+06
```

Use Ohm's law in both magnetic and electric fields to calculate current and hence voltage

```
% fluxi = fluxg = flux
icoil = (Ri+Rg)*fluxi / N    % [A]
```

```
icoil = 24.9427
```

```
VDC = icoil * Rcoil           % [V]
```

```
VDC = 149.6563
```

(b) Stored Field Energy

```
L = N^2 / (Ri+Rg);
Wfld = 1/2 * L * icoil^2 % [J]
```

```
Wfld = 20.5777
```

```
% use more general method instead (safer!)
```

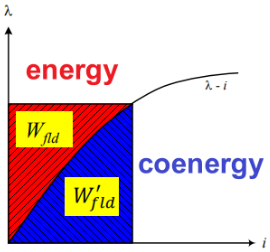
Exercise 2

Exercise 2

The λ-i relationship for an electromagnetic system is given by

$$i = \left(\frac{\lambda g}{0.09}\right)^2$$

which is valid for the limits $0 < i < 4$ A and $3 < g < 10$ cm. For current i = 3A and airgap length g = 5 cm, find the mechanical force on the moving part using coenergy and energy of the field.



The problem statement asks for a snapshot of the force under a specific circumstance dictated by the current and airgap length. This is calculated from the perspectives of energy and coenergy and the results match.

```
thisCurrent = 3;
thisAirGap = 5e-2;

syms i lambda g positive
% assume( (i>0) && (i<4) )
assume( 3e-2 < g < 10e-2 )
currentFun = (lambda*g/0.09)^2
```

currentFun =

$$\frac{10000\, g^2\, \lambda^2}{81}$$

```
expr = i == (lambda*g/0.09)^2;
lambdaFun(i,g) = solve(expr,lambda)
```

lambdaFun(i, g) =

$$\frac{9\ \sqrt{i}}{100\, g}$$

```
thisLambda = lambdaFun(thisCurrent,thisAirGap)
```

thisLambda =

$$\frac{9\ \sqrt{3}}{5}$$

Determination of Force from Energy

```
energyFun(g,lambda) = int(currentFun,lambda)
```

```
energyFun(g, lambda) =
```

$$\frac{10000\, g^2\, \lambda^3}{243}$$

```
forceFun(g,lambda) = -diff(energyFun,g)
```

```
forceFun(g, lambda) =
```

$$-\frac{20000\, g\, \lambda^3}{243}$$

```
force = double(forceFun(thisAirGap,thisLambda))
```

```
force = -124.7077
```

Determination of Force from Coenergy

```
coenergyFun = int(lambdaFun,i)
```

```
coenergyFun(i, g) =
```

$$\frac{3\, i^{3/2}}{50\, g}$$

```
forceFun = diff(coenergyFun,g)
```

```
forceFun(i, g) =
```

$$-\frac{3\, i^{3/2}}{50\, g^2}$$

```
force = double(forceFun(thisCurrent,thisAirGap))
```

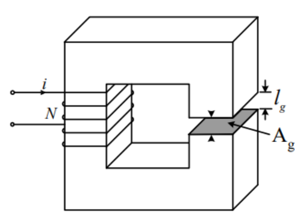
```
force = -124.7077
```

Q: In this case, forces obtained from energy and coenergy are the same. However, in Exercise 4 and 5, they differ in polarity. Why?

Exercise 3

Exercise 3

- The magnetic system shown in the figure has the following parameters:
  N = 400, i = 3 A
  Width of air-gap = 2.5 cm
  Depth of air-gap = 2.5 cm
  Length of air-gap = 1.5 mm



- Neglect the reluctance of the core, leakage flux and the fringing flux. Determine:
  (a) The force of attraction between both sides of the air-gap
  (b) The energy stored in the air-gap.
  (c) Coil Inductance

```
N = 400;
```

```
i = 3;
Width = 2.5e-2;  % cm --> m
Depth = 2.5e-2;  % cm --> m
lg = 1.5e-3; % mm --> m

MMF = N*i;
H = MMF / lg
```

```
H = 800000
```

```
% Neglect the reluctance of the core, leakage flux and the fringing flux
% H drops only across the air gap
Ag = Width * Depth
```

```
Ag = 6.2500e-04
```

```
Rg = lg / (ug*Ag)
```

```
Rg = 1.9099e+06
```

```
flux = MMF / Rg
```

```
flux = 6.2832e-04
```

```
Bg = flux / Ag
```

```
Bg = 1.0053
```

```
lambda = N*flux
```

```
lambda = 0.2513
```

```
% lambda-i relationship: linear
% k = lambda / i
```

(a) Coil Inductance

```
L = lambda / i % [H]
```

```
L = 0.0838
```

(b) Energy stored in the air-gap

```
Wfld = 1/2 * Bg^2/uo * Ag * lg
```

```
Wfld = 0.3770
```

(c) Force of attraction between both sides of the air-gap

```
% Wfld = 1/2 * L * i^2 % [J]
force = -Wfld / lg
```

```
force = -251.3274
```

```
syms g
RgFun = g/(ug*Ag);
Lfun = vpa(N^2/RgFun,4);
```

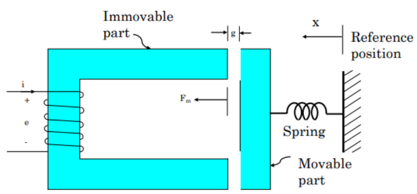
```
forceFun(g) = 1/2 * diff(Lfun,g) * i^2;
force = forceFun(lg)
```

force = -251.3

Exercise 4

Exercise 4

- The magnetic circuit shown below is made of high permeability steel so that its reluctance can be negligible. The movable part is free to move about an x-axis. The coil has 1000 turns, the area normal to the flux is (5 cm × 10 cm), and the length of a single air gap is 5 mm.
  - Derive an expression for the inductance, L, as a function of air gap, g.
  - Determine the force, F_m, for the current i =10 A.
  - The maximum flux density in the air gaps is to be limited to approximately 1.0 Tesla to avoid excessive saturation of the steel. Compute the maximum force.



(a) Inductance L = f(g)

```
% ui/uo > 1e3 ==> ui >> uo ==> H drops only in airgap
N = 1000;
A = (5*10) * 1e-4;
thisAirGap = 5e-3;

syms i g K positive
syms Rg(g)
Rg(g) = 2*g/(uo*A);

digits(4)
Lfun = vpa(N^2/Rg)
```

Lfun(g) =

$$\frac{0.003142}{g}$$

(b) Mechanical Force Fm

```
% i = 10;
% MMF = N*i;
% H = MMF/g;

% Determination of Force from Coenergy
% K*x from the spring side
coenergyFun(i,g) = 1/2 * Lfun * i^2
```

coenergyFun(i, g) =

$$\frac{0.001571\, i^2}{g}$$

```
forceFun = diff(coenergyFun,g)
```

forceFun(i, g) =

$$-\frac{0.001571\, i^2}{g^2}$$

```
thisCurrent = 10;
force = forceFun(thisCurrent,thisAirGap)
```

force = -6283.0

(c) Maximum Force before Saturation

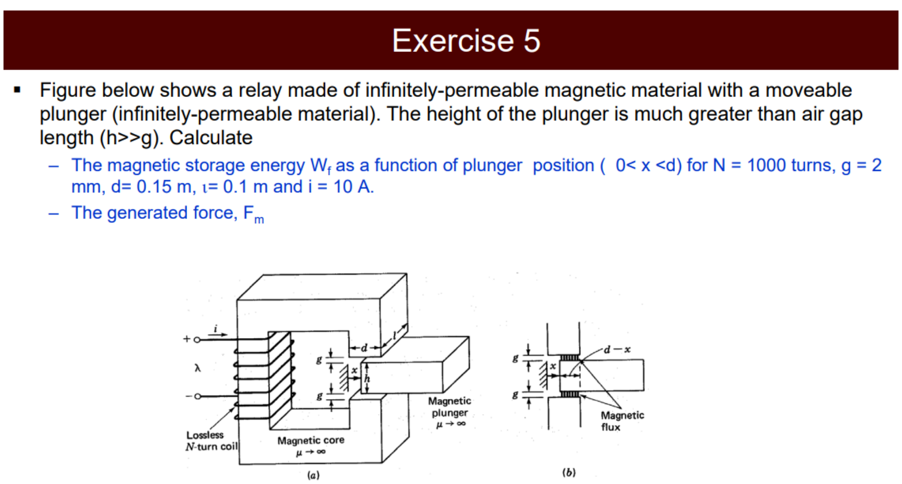
```
Bsat = 1;
fluxsat = Bsat * A;
% R*flux = Ni
isat = vpa(Rg(thisAirGap)*fluxsat/N)
```

isat = 7.958

```
maxForce = forceFun(isat,thisAirGap)
```

maxForce = -3979.0

Exercise 5



(a) Stored Magnetic Energy Wf = f(plunger position x)

```
% clear Rg g
N = 1000;
g = 2e-3;
d = 0.15;
l = 0.1;
```

```
i = 10;

syms x positive
Ag = (d-x)*l;
% no flux directly travels through the airgap
% without passing the plunger

Rg = vpa(2*g/(ug*Ag));
L = N^2 / Rg;
MMF = N*i;
flux = MMF/Rg;
lambda = N*flux;
coenergyFun(x) = 1/2 * L * i^2
```

coenergyFun(x) = $235.6 - 1571.0\,x$

```
Wf(x) = i*lambda - coenergyFun
```

Wf(x) = $235.6 - 1571.0\,x$

(b) Generated Force

```
Fm = -diff(Wf,x)
```

Fm(x) = 1571.0

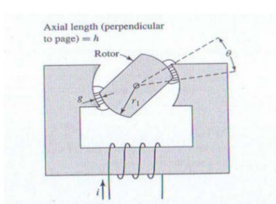
```
Fm = diff(coenergyFun,x)
```

Fm(x) = -1571.0

Exercise 6

Exercise 6

- The magnetic circuit shown is made of high-permeability electrical steel. Assume the reluctance of steel μ -- infinity. Derive the expression for the torque acting on the rotor .




```
syms N i g h theta r_1
r1 = r_1;

MMF = N*i;
Ag = r1 * theta * h;
Rg = 2*g/(uo*Ag);
flux = vpa( MMF/Rg );
lambdaFun = N*flux;
```

```
coenergyFun = int(lambdaFun,i);
T = diff(coenergyFun,theta)
```

T =

$$\frac{3.142\text{e-}7\, N^2\, h\, i^2\, r_1}{g}$$

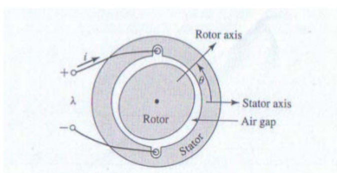
Q: Different coefficient

Exercise 7

Exercise 7

- The magnetic circuit below consists of a single coil stator and an oval rotor. Because of the air-gap is non uniform, the coil inductance varies with the rotor angular position. Given the coil inductance $L(\theta) = L_o + L_2\cos 2\theta$, where L_o= 10.6 mH and L_2= 2.7 mH,

  Find torque as a function of θ for a coil current of 2 A.




```
Lo = 10.6e-3;
L2 = 2.7e-3;

i = 2;

syms theta
L = Lo + L2*cos(2*theta)
```

L =

$$\frac{27\cos(2\,\theta)}{10000} + \frac{53}{5000}$$

```
T = i^2/2 * diff(L,theta)
```

T =

$$-\frac{27\sin(2\,\theta)}{2500}$$