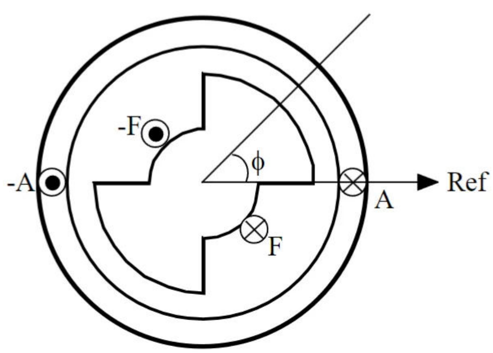
ECEN 611 Homework 4: Gap Function and Mutual Inductance for Salient Pole Rotor

Table of Contents

Problem 2



(pole arc = 90 degree)

```
clearvars
clc
```

Inverse Gap Function

if you could make Problem 1 a function...

```
syms g_min g_max     positive real
syms alpha           positive real
syms phi                      real
syms g_phi(phi)
syms theta_r theta

assume( (alpha > 0) & (alpha <= deg2rad(180)) )
```

```
T = 2*pi;
POLEARC = deg2rad(90);
THETA_R = deg2rad(45);

poleArc = alpha;
midArc = T/2 - poleArc;

rotor_profile_theta = [poleArc/2 midArc poleArc midArc poleArc/2];
rotor_profile_turning_phi = [theta_r, theta_r + cumsum(rotor_profile_theta)];

rotor_profile_turning_phi = subs( ...
    rotor_profile_turning_phi, ...
    poleArc, POLEARC ...
)
```

rotor_profile_turning_phi =

$$\left(\theta_r \quad \theta_r+\frac{\pi}{4} \quad \theta_r+\frac{3\,\pi}{4} \quad \theta_r+\frac{5\,\pi}{4} \quad \theta_r+\frac{7\,\pi}{4} \quad \theta_r+2\,\pi\right)$$

```
% phi_ref = 0;
% rotor_profile_turning_phi = [phi_ref, rotor_profile_turning_phi]
```

```
gap_profile = [g_min, g_max, g_min, g_max, g_min];

firstPhi = rotor_profile_turning_phi(1);
secondPhi = rotor_profile_turning_phi(2);

firstGap = gap_profile(1);

g_phi(phi) = piecewise(firstPhi <= phi < secondPhi, firstGap)
```

g_phi(phi) = $\left\{ g_{\min} \quad \text{if } \theta_r \le \phi \wedge 4\,\phi < 4\,\theta_r + \pi \right.$

```

for k = 2 : length(rotor_profile_turning_phi) - 1

    thisPhi = rotor_profile_turning_phi(k);
    nextPhi = rotor_profile_turning_phi(k+1);
    thisGap = gap_profile(k);

    g_phi(phi) = piecewise( ...
        thisPhi <= phi < nextPhi, thisGap, ...
        phi <= thisPhi, g_phi(phi) ...
    );
end

g_phi(phi) = simplify(g_phi)
```

g_phi(phi) =

$$\begin{cases} g_{\min} & \text{if } \phi < \theta_r + 2\,\pi \wedge \theta_r + \frac{7\,\pi}{4} \leq \phi \\ g_{\max} & \text{if } \phi < \theta_r + \frac{7\,\pi}{4} \wedge \theta_r + \frac{5\,\pi}{4} \leq \phi \\ g_{\min} & \text{if } \phi < \theta_r + \frac{5\,\pi}{4} \wedge \theta_r + \frac{3\,\pi}{4} \leq \phi \\ g_{\max} & \text{if } \phi < \theta_r + \frac{3\,\pi}{4} \wedge \theta_r + \frac{\pi}{4} \leq \phi \\ g_{\min} & \text{if } \theta_r \leq \phi \wedge 4\,\phi < 4\,\theta_r + \pi \end{cases}$$

```
g_phi_inv = 1 / g_phi
```

g_phi_inv(phi) =

$$\begin{cases} \frac{1}{g_{\min}} & \text{if } \phi < \theta_r + 2\,\pi \wedge \theta_r + \frac{7\,\pi}{4} \leq \phi \\ \frac{1}{g_{\max}} & \text{if } \phi < \theta_r + \frac{7\,\pi}{4} \wedge \theta_r + \frac{5\,\pi}{4} \leq \phi \\ \frac{1}{g_{\min}} & \text{if } \phi < \theta_r + \frac{5\,\pi}{4} \wedge \theta_r + \frac{3\,\pi}{4} \leq \phi \\ \frac{1}{g_{\max}} & \text{if } \phi < \theta_r + \frac{3\,\pi}{4} \wedge \theta_r + \frac{\pi}{4} \leq \phi \\ \frac{1}{g_{\min}} & \text{if } \theta_r \leq \phi \wedge 4\,\phi < 4\,\theta_r + \pi \end{cases}$$

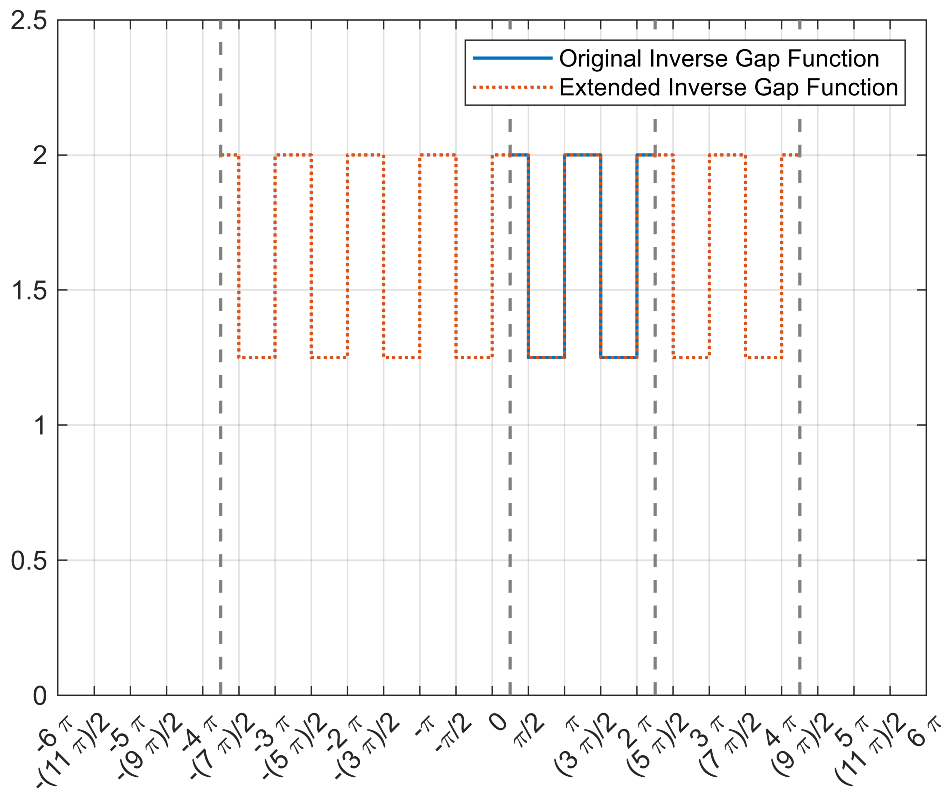
```
g_phi_inv_ext(phi) = piecewise( ...
    theta_r - 2*T <= phi < theta_r - T,  g_phi_inv(phi+2*T), ...
    theta_r - T <= phi < theta_r,        g_phi_inv(phi+T), ...
    theta_r <= phi < theta_r + T,        g_phi_inv(phi), ...
    theta_r + T <= phi <= theta_r + 2*T, g_phi_inv(phi-T) ...
)
```

g_phi_inv_ext(phi) =

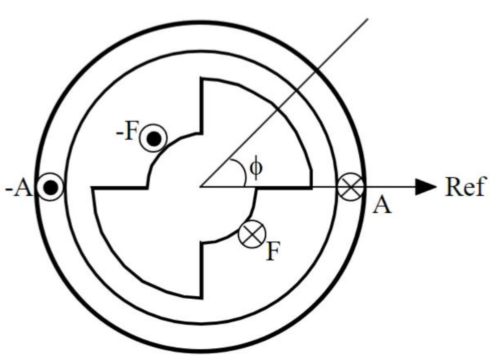
$$
\begin{cases}
\frac{1}{g_{\min}} & \text{if } \phi + 2\pi < \theta_r \wedge \theta_r \le \phi + \frac{9\pi}{4} \wedge \theta_r \in \mathbb{R} \\
\frac{1}{g_{\max}} & \text{if } \phi + \frac{9\pi}{4} < \theta_r \wedge \theta_r \le \phi + \frac{11\pi}{4} \wedge \phi \in \mathbb{R} \wedge \theta_r \in \mathbb{R} \\
\frac{1}{g_{\min}} & \text{if } \phi + \frac{11\pi}{4} < \theta_r \wedge \theta_r \le \phi + \frac{13\pi}{4} \wedge \phi \in \mathbb{R} \wedge \theta_r \in \mathbb{R} \\
\frac{1}{g_{\max}} & \text{if } \phi + \frac{13\pi}{4} < \theta_r \wedge \theta_r \le \phi + \frac{15\pi}{4} \wedge \phi \in \mathbb{R} \wedge \theta_r \in \mathbb{R} \\
\frac{1}{g_{\min}} & \text{if } (\phi < \theta_r \wedge 4\theta_r \le 4\phi + \pi) \vee (4\phi + 15\pi < 4\theta_r \wedge \theta_r \le \phi + 4\pi \wedge \phi \in \mathbb{R}) \\
\frac{1}{g_{\max}} & \text{if } 4\theta_r \le 4\phi + 3\pi \wedge \phi \in \mathbb{R} \wedge \theta_r \in \mathbb{R} \wedge 4\phi + \pi < 4\theta_r \\
\frac{1}{g_{\min}} & \text{if } 4\phi + 3\pi < 4\theta_r \wedge 4\theta_r \le 4\phi + 5\pi \wedge \phi \in \mathbb{R} \wedge \theta_r \in \mathbb{R} \\
\frac{1}{g_{\max}} & \text{if } 4\phi + 5\pi < 4\theta_r \wedge 4\theta_r \le 4\phi + 7\pi \wedge \phi \in \mathbb{R} \wedge \theta_r \in \mathbb{R} \\
\frac{1}{g_{\min}} & \text{if } (4\phi + 7\pi < 4\theta_r \wedge \theta_r \le \phi + 2\pi \wedge \phi \in \mathbb{R}) \vee (4\theta_r + 7\pi \le 4\phi \wedge \phi < \theta_r + 2\pi \wedge \theta_r \in \mathbb{R}) \\
\frac{1}{g_{\max}} & \text{if } 4\phi < 4\theta_r + 7\pi \wedge 4\theta_r + 5\pi \le 4\phi \wedge \phi \in \mathbb{R} \wedge \theta_r \in \mathbb{R} \\
\frac{1}{g_{\min}} & \text{if } 4\phi < 4\theta_r + 5\pi \wedge 4\theta_r + 3\pi \le 4\phi \wedge \phi \in \mathbb{R} \wedge \theta_r \in \mathbb{R} \\
\frac{1}{g_{\max}} & \text{if } 4\phi < 4\theta_r + 3\pi \wedge \phi \in \mathbb{R} \wedge \theta_r \in \mathbb{R} \wedge 4\theta_r + \pi \le 4\phi \\
\frac{1}{g_{\min}} & \text{if } (\theta_r \le \phi \wedge 4\phi < 4\theta_r + \pi) \vee (4\theta_r + 15\pi \le 4\phi \wedge \phi < \theta_r + 4\pi \wedge \theta_r \in \mathbb{R}) \\
\frac{1}{g_{\max}} & \text{if } \phi < \theta_r + \frac{15\pi}{4} \wedge \theta_r + \frac{13\pi}{4} \le \phi \wedge \phi \in \mathbb{R} \wedge \theta_r \in \mathbb{R} \\
\frac{1}{g_{\min}} & \text{if } \phi < \theta_r + \frac{13\pi}{4} \wedge \theta_r + \frac{11\pi}{4} \le \phi \wedge \phi \in \mathbb{R} \wedge \theta_r \in \mathbb{R} \\
\frac{1}{g_{\max}} & \text{if } \phi < \theta_r + \frac{11\pi}{4} \wedge \theta_r + \frac{9\pi}{4} \le \phi \wedge \phi \in \mathbb{R} \wedge \theta_r \in \mathbb{R} \\
\frac{1}{g_{\min}} & \text{if } 4\phi < 4\theta_r + 9\pi \wedge \theta_r + 2\pi \le \phi \wedge \phi \in \mathbb{R}
\end{cases}
$$

```
% it was [0 T], now it becomes [theta_r theta_r+T]

vars = [poleArc g_min g_max theta_r];
VALS = [POLEARC 0.5   0.8   THETA_R];

figure
fplot(subs(g_phi_inv, vars, VALS), [-3*T 3*T], ...
      "DisplayName", "Original Inverse Gap Function", ...
      "LineWidth", 1.2)
hold on
fplot(subs(g_phi_inv_ext, vars, VALS), [-3*T 3*T], ...
```

```
        "DisplayName", "Extended Inverse Gap Function", ...
        "LineStyle",":", ...
        "LineWidth", 1.2 ...
        )
hold off
ylim([0 2.5])
grid on
legend
ax = gca;
S = sym(ax.XLim(1):pi/2:ax.XLim(2));
ax.XTick = double(S);
ax.XTickLabel = arrayfun(@texlabel,S,'UniformOutput',false);
```



Stator Counting & Winding Function




```
numberOfConductors = 2;
```

```
syms I positive real
syms N_s

NS = 1;

% close the "loop": one turn is at phi=0 and the other T/2
% however, closing the "loop" by winding up to T (same as phi=0) makes
% programming easier because that corresponds to a full period
turnsStatorNum = [1 -1 1] * N_s;   % number of turns at each location: replace Nt
with 1 for simplicity
turnsStatorPhi = [0 T/2 T];  % angle of each location

% Initialize the piecewise function with the first interval
phiReference = sym(0); % unique and universal "yardstick" for all angles associated
with rotor and stator

turnsStatorNumLevel = cumsum(turnsStatorNum)
```

turnsStatorNumLevel = $\begin{pmatrix} N_s & 0 & N_s \end{pmatrix}$

```
% Initialize the piecewise function with the first interval
statorCountingFunction(phi) = turnsStatorNumLevel(1);

for k = 1:length(turnsStatorPhi)-1
    thisAngle = turnsStatorPhi(k);
    nextPhi = turnsStatorPhi(k+1);
    statorCountingFunction(phi) = piecewise(thisAngle <= phi < nextPhi,
turnsStatorNumLevel(k), ...
                                            phiReference <= phi < thisAngle,
statorCountingFunction);
end
```

Original Stator Counting Function φ ∈ [0 T]

```
disp("Original Stator Counting Function ϕ ∈ [0 T] :")
```

Original Stator Counting Function ϕ ∈ [0 T] :

```
disp(statorCountingFunction(phi))
```

$$\begin{cases} 0 & \text{if } \phi \in [\pi, 2\,\pi) \\ N_s & \text{if } \phi \in [0, \pi) \end{cases}$$

Extended Stator Counting Function φ ∈ [-2T 2T]

```
statorCountingFunction_ext(phi) = piecewise( ...
    -2*T <= phi < -T, statorCountingFunction(phi+2*T), ...
    -T <= phi < 0, statorCountingFunction(phi+T), ...
    0 <= phi < T, statorCountingFunction(phi), ...
    T <= phi < 2*T, statorCountingFunction(phi-T) ...
);
```

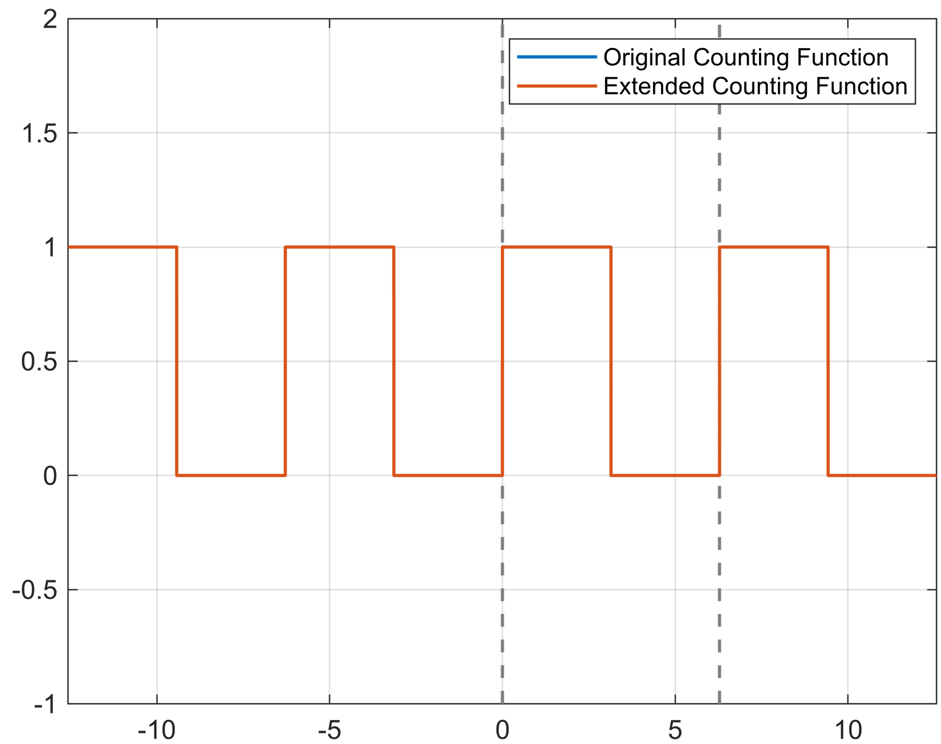
```
disp("Extended Stator Counting Function ϕ ∈ [-2T 2T] :")
```

```
Extended Stator Counting Function ϕ ∈ [-2T 2T] :
```

```
disp(statorCountingFunction_ext(phi))
```

$$\begin{cases} 0 & \text{if } \phi \in [-3\,\pi, -2\,\pi) \\ N_s & \text{if } \phi \in [-4\,\pi, -3\,\pi) \\ 0 & \text{if } \phi \in [-\pi, 0) \\ N_s & \text{if } \phi \in [-2\,\pi, -\pi) \\ 0 & \text{if } \phi \in [\pi, 2\,\pi) \\ N_s & \text{if } \phi \in [0, \pi) \\ 0 & \text{if } \phi \in [3\,\pi, 4\,\pi) \\ N_s & \text{if } \phi \in [2\,\pi, 3\,\pi) \end{cases}$$

```
figure
fplot(subs(statorCountingFunction(phi), N_s, NS), [-2*T 2*T], ...
    "DisplayName", "Original Counting Function", ...
    "LineWidth", 1.2)
hold on
fplot(subs(statorCountingFunction_ext(phi), N_s, NS), [-2*T 2*T], ...
    "DisplayName", "Extended Counting Function", ...
    "LineWidth", 1.2)
hold off
ylim([-1 2])
grid on
legend
```



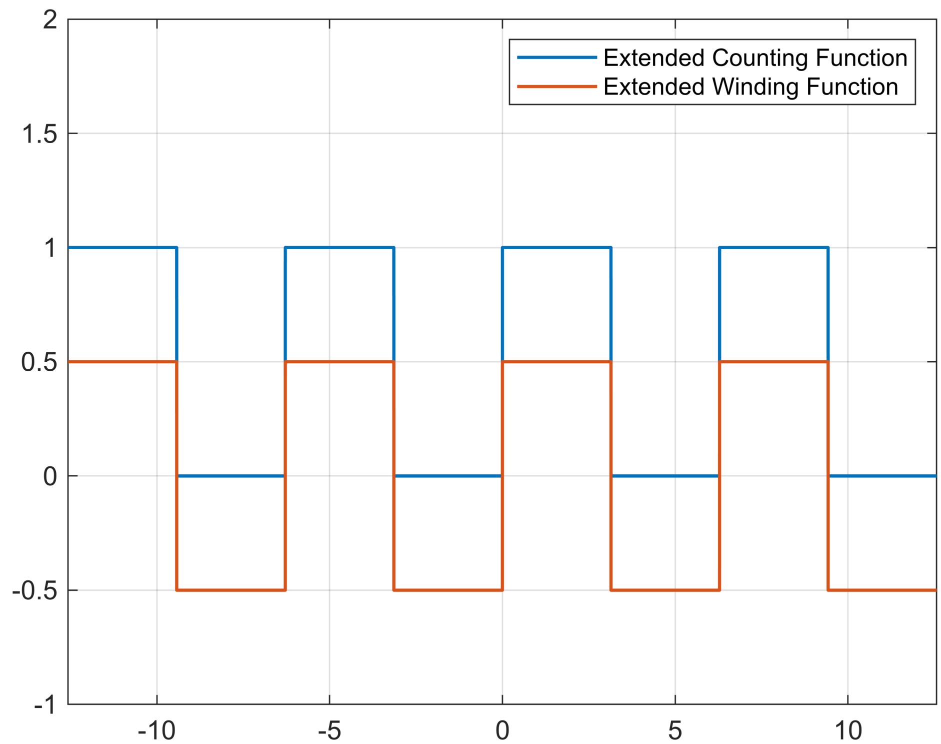
Extended Stator Winding Function φ ∈ [-2T 2T]

```
statorCountingFunction_ext_avg = 1/T * int(statorCountingFunction_ext, phi, [0 T]);
statorWindingFunction_ext = statorCountingFunction_ext -
statorCountingFunction_ext_avg
```

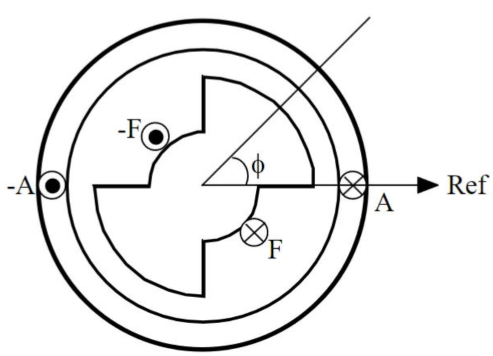
statorWindingFunction_ext(phi) =

$$\begin{cases} -\frac{5734161139222659\,\pi\,N_s}{36028797018963968} & \text{if } \phi \in [-3\,\pi, -2\,\pi) \\ N_s - \frac{5734161139222659\,\pi\,N_s}{36028797018963968} & \text{if } \phi \in [-4\,\pi, -3\,\pi) \\ -\frac{5734161139222659\,\pi\,N_s}{36028797018963968} & \text{if } \phi \in [-\pi, 0) \\ N_s - \frac{5734161139222659\,\pi\,N_s}{36028797018963968} & \text{if } \phi \in [-2\,\pi, -\pi) \\ -\frac{5734161139222659\,\pi\,N_s}{36028797018963968} & \text{if } \phi \in [\pi, 2\,\pi) \\ N_s - \frac{5734161139222659\,\pi\,N_s}{36028797018963968} & \text{if } \phi \in [0, \pi) \\ -\frac{5734161139222659\,\pi\,N_s}{36028797018963968} & \text{if } \phi \in [3\,\pi, 4\,\pi) \\ N_s - \frac{5734161139222659\,\pi\,N_s}{36028797018963968} & \text{if } \phi \in [2\,\pi, 3\,\pi) \end{cases}$$

```
figure
fplot(subs(statorCountingFunction_ext(phi), N_s, NS), [-2*T 2*T], ...
    "DisplayName", "Extended Counting Function", ...
    "LineWidth", 1.2)
hold on
fplot(subs(statorWindingFunction_ext(phi), N_s, NS), [-2*T 2*T], ...
    "DisplayName", "Extended Winding Function", ...
    "LineWidth", 1.2)
hold off
ylim([-1 2])
grid on
legend
```



Rotor Counting & Winding Function



```
% rotor_profile_theta = [poleArc/2 midArc poleArc midArc poleArc/2];
% rotor_profile_turning_phi = [theta_r, theta_r + cumsum(rotor_profile_theta)];
%
% rotor_profile_turning_phi = subs( ...
%     rotor_profile_turning_phi, ...
%     poleArc, POLEARC ...
% )
syms N_r
NR = 1;

turnsRotorNum = [0    -1     1    0 ] * N_r;   % number of turns at each location:
replace Nt with 1 for simplicity
turnsRotorPhi = [0   T/4   T/2  T/4 ];  % angle of each location

turnsRotorNumLevel = cumsum(turnsRotorNum)
```

turnsRotorNumLevel = $\begin{pmatrix} 0 & -N_r & 0 & 0 \end{pmatrix}$

```
rotor_winding_turning_phi = theta_r + cumsum(turnsRotorPhi)
```

rotor_winding_turning_phi =

$$\begin{pmatrix} \theta_r & \theta_r + \frac{\pi}{2} & \theta_r + \frac{3\,\pi}{2} & \theta_r + 2\,\pi \end{pmatrix}$$

```
% Initialize the piecewise function with the first interval
rotorCountingFunction(phi) = turnsRotorNumLevel(1);

for k = 1:length(rotor_winding_turning_phi)-1

    thisPhi = rotor_winding_turning_phi(k);
    nextPhi = rotor_winding_turning_phi(k+1);

    rotorCountingFunction(phi) = piecewise(thisPhi <= phi < nextPhi,
turnsRotorNumLevel(k), ...
                                           phiReference <= phi < thisPhi,
rotorCountingFunction);
end
```

Original Rotor Counting Function φ ∈ [θr θr+T]

```
disp("Original Rotor Counting Function: ");
```

Original Rotor Counting Function:

```
disp(rotorCountingFunction(phi));
```

$$\begin{cases} 0 & \text{if } \phi < \theta_r + 2\pi \wedge \theta_r + \frac{3\pi}{2} \le \phi \\ -N_r & \text{if } \phi < \theta_r + \frac{3\pi}{2} \wedge \theta_r + \frac{\pi}{2} \le \phi \wedge 0 \le \phi \\ 0 & \text{if } \phi < \theta_r + \frac{\pi}{2} \wedge 0 \le \phi \wedge (\phi < \theta_r \vee (\theta_r \le \phi \wedge 2\phi < 2\theta_r + \pi)) \end{cases}$$

Extended Rotor Counting Function φ ∈ [θr-2T θr+2T]

```
rotorCountingFunction_ext(phi) = piecewise( ...
    theta_r - 2*T <= phi < theta_r - T, rotorCountingFunction(phi+2*T), ...
    theta_r - T <= phi < theta_r,       rotorCountingFunction(phi+T), ...
    theta_r <= phi < theta_r + T,       rotorCountingFunction(phi), ...
    theta_r + T <= phi < theta_r + 2*T, rotorCountingFunction(phi-T) ...
)
```

`rotorCountingFunction_ext(phi) =`

$$\begin{cases} 0 & \text{if } \phi + 2\pi < \theta_r \wedge \theta_r \le \phi + \frac{5\pi}{2} \wedge \theta_r \in \mathbb{R} \\ -N_r & \text{if } 2\phi + 5\pi < 2\theta_r \wedge 2\theta_r \le 2\phi + 7\pi \wedge \theta_r \in \mathbb{R} \wedge -4\pi \le \phi \\ 0 & \text{if } (\phi < \theta_r \wedge 2\theta_r \le 2\phi + \pi) \vee (2\phi + 7\pi < 2\theta_r \wedge \theta_r \le \phi + 4\pi \wedge (\phi + 4\pi < \theta_r \vee (2\phi + 7\pi < 2 \\ -N_r & \text{if } 2\theta_r \le 2\phi + 3\pi \wedge \theta_r \in \mathbb{R} \wedge -2\pi \le \phi \wedge 2\phi + \pi < 2\theta_r \\ 0 & \text{if } (2\theta_r + 3\pi \le 2\phi \wedge \phi < \theta_r + 2\pi \wedge \theta_r \in \mathbb{R}) \vee (2\phi + 3\pi < 2\theta_r \wedge \theta_r \le \phi + 2\pi \wedge (\phi + 2\pi < \theta_r \\ -N_r & \text{if } 2\phi < 2\theta_r + 3\pi \wedge \theta_r \in \mathbb{R} \wedge 2\theta_r + \pi \le 2\phi \wedge 0 \le \phi \\ 0 & \text{if } (2\theta_r + 7\pi \le 2\phi \wedge \phi < \theta_r + 4\pi \wedge \theta_r \in \mathbb{R}) \vee (\theta_r \le \phi \wedge 2\phi < 2\theta_r + \pi \wedge 0 \le \phi \wedge (\phi < \theta_r \vee (\theta \\ -N_r & \text{if } \phi < \theta_r + \frac{7\pi}{2} \wedge \theta_r + \frac{5\pi}{2} \le \phi \wedge \theta_r \in \mathbb{R} \wedge 2\pi \le \phi \\ 0 & \text{if } \phi < \theta_r + \frac{5\pi}{2} \wedge \theta_r + 2\pi \le \phi \wedge (\phi < \theta_r + 2\pi \vee (2\phi < 2\theta_r + 5\pi \wedge \theta_r + 2\pi \le \phi)) \wedge 2\pi \le \phi \end{cases}$$

```
% figure
% fplot(subs(rotorCountingFunction(phi), [theta_r N_r], [0 NR]), [-2*T 2*T], ...
%     "DisplayName", "Original Counting Function", ...
%     "LineWidth", 1.2)
% hold on
% fplot(subs(rotorCountingFunction_ext(phi), [theta_r N_r], [0 NR]), [-2*T 2*T], ...
%     "DisplayName", "Extended Counting Function", ...
%     "LineWidth", 1.2)
% hold off
% ylim([-2 2])
% grid on
%
% ax = gca;
% S = sym(ax.XLim(1):pi/2:ax.XLim(2));
% ax.XTick = double(S);
% ax.XTickLabel = arrayfun(@texlabel,S,'UniformOutput',false);
```

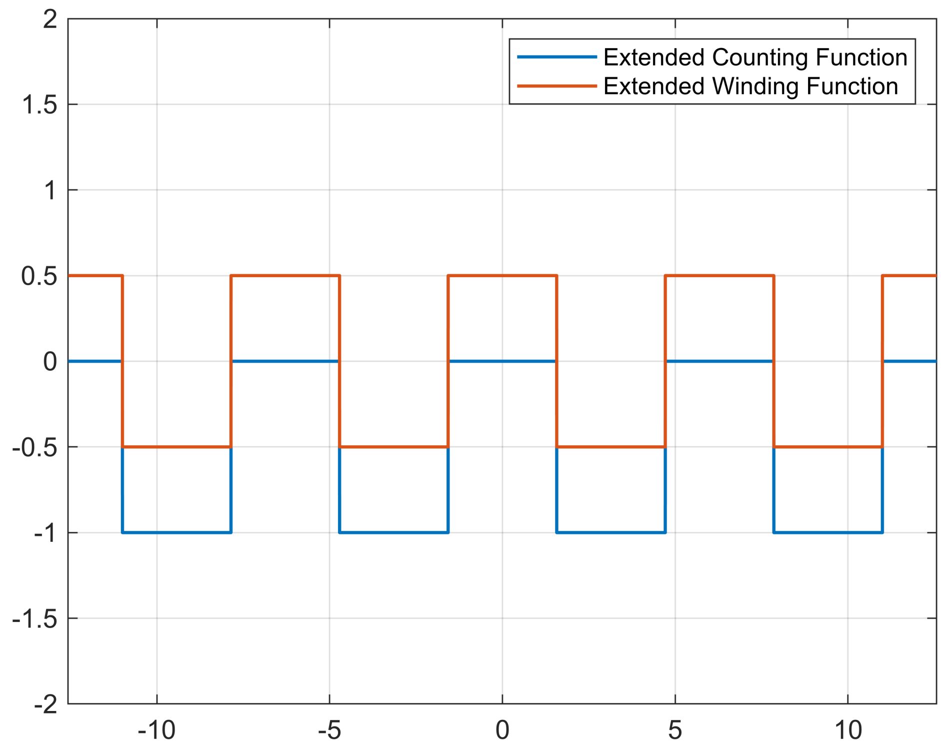
Extended Rotor Winding Function φ ∈ [θr-2T θr+2T]

```
% assume( theta < T )

rotorCountingFunction_ext_avg = 1/T * int( subs(rotorCountingFunction_ext, theta_r, THETA_R), phi, [0 T]);
rotorWindingFunction_ext = rotorCountingFunction_ext - rotorCountingFunction_ext_avg
```

rotorWindingFunction_ext(phi) =

$$\begin{cases} \frac{5734161139222659\,\pi\,N_r}{36028797018963968} & \text{if } \phi + 2\,\pi < \theta_r \wedge \theta_r \le \phi + \frac{5\,\pi}{2} \wedge \theta_r \in \mathbb{R} \\ \frac{5734161139222659\,\pi\,N_r}{36028797018963968} - N_r & \text{if } 2\,\phi + 5\,\pi < 2\,\theta_r \wedge 2\,\theta_r \le 2\,\phi + 7\,\pi \wedge \theta_r \in \mathbb{R} \wedge -4\,\pi \le \phi \\ \frac{5734161139222659\,\pi\,N_r}{36028797018963968} & \text{if } (\phi < \theta_r \wedge 2\,\theta_r \le 2\,\phi + \pi) \vee (2\,\phi + 7\,\pi < 2\,\theta_r \wedge \theta_r \le \phi + 4\,\pi \wedge (\phi + \\ \frac{5734161139222659\,\pi\,N_r}{36028797018963968} - N_r & \text{if } 2\,\theta_r \le 2\,\phi + 3\,\pi \wedge \theta_r \in \mathbb{R} \wedge -2\,\pi \le \phi \wedge 2\,\phi + \pi < 2\,\theta_r \\ \frac{5734161139222659\,\pi\,N_r}{36028797018963968} & \text{if } (2\,\theta_r + 3\,\pi \le 2\,\phi \wedge \phi < \theta_r + 2\,\pi \wedge \theta_r \in \mathbb{R}) \vee (2\,\phi + 3\,\pi < 2\,\theta_r \wedge \theta_r \\ \frac{5734161139222659\,\pi\,N_r}{36028797018963968} - N_r & \text{if } 2\,\phi < 2\,\theta_r + 3\,\pi \wedge \theta_r \in \mathbb{R} \wedge 2\,\theta_r + \pi \le 2\,\phi \wedge 0 \le \phi \\ \frac{5734161139222659\,\pi\,N_r}{36028797018963968} & \text{if } (2\,\theta_r + 7\,\pi \le 2\,\phi \wedge \phi < \theta_r + 4\,\pi \wedge \theta_r \in \mathbb{R}) \vee (\theta_r \le \phi \wedge 2\,\phi < 2\,\theta_r + \\ \frac{5734161139222659\,\pi\,N_r}{36028797018963968} - N_r & \text{if } \phi < \theta_r + \frac{7\,\pi}{2} \wedge \theta_r + \frac{5\,\pi}{2} \le \phi \wedge \theta_r \in \mathbb{R} \wedge 2\,\pi \le \phi \\ \frac{5734161139222659\,\pi\,N_r}{36028797018963968} & \text{if } \phi < \theta_r + \frac{5\,\pi}{2} \wedge \theta_r + 2\,\pi \le \phi \wedge (\phi < \theta_r + 2\,\pi \vee (2\,\phi < 2\,\theta_r + 5\,\pi \wedge \end{cases}$$

```
figure
fplot(subs(rotorCountingFunction_ext(phi),[theta_r N_r], [0 NR]), [-2*T 2*T], ...
    "DisplayName", "Extended Counting Function", ...
    "LineWidth", 1.2)
hold on
fplot(subs(rotorWindingFunction_ext(phi),[theta_r N_r], [0 NR]), [-2*T 2*T], ...
    "DisplayName", "Extended Winding Function", ...
    "LineWidth", 1.2)
hold off
ylim([-2 2])
grid on
legend
```



(a) Magnetizing inductance of winding A on the stator

$$L_{AA} = \mu_0 r l \int_0^{2\pi} n_A(\phi) N_A(\phi) g^{-1}(\phi) d\phi \qquad (1.167)$$

```
digits(4)

syms mu_o r l
Lss = mu_o*r*l * int( statorCountingFunction * statorWindingFunction_ext *
subs(g_phi_inv_ext, theta_r, [0 THETA_R T/2]), ...
                            phi, [0 T]);

Lss = simplify( vpa(Lss') )
```

Lss =

$$\begin{pmatrix} \frac{0.7854\ \left(\overline{N_s}\right)^2\ \overline{l}\ \overline{\mu_o}\ \overline{r}\ (g_{\min} + g_{\max})}{g_{\min}\ g_{\max}} \\ \frac{0.7854\ \left(\overline{N_s}\right)^2\ \overline{l}\ \overline{\mu_o}\ \overline{r}\ (g_{\min} + g_{\max})}{g_{\min}\ g_{\max}} \\ \frac{0.7854\ \left(\overline{N_s}\right)^2\ \overline{l}\ \overline{\mu_o}\ \overline{r}\ (g_{\min} + g_{\max})}{g_{\min}\ g_{\max}} \end{pmatrix}$$

(b) Magnetizing inductance of winding F on the rotor

```
THETA_R_ARRAY = 0: T/8 : T;

rotorIntegrand = subs( ...
    rotorCountingFunction * rotorWindingFunction_ext * g_phi_inv_ext, ...
    theta_r, THETA_R_ARRAY);
Lrr = mu_o*r*l * int( rotorIntegrand, phi, [0 T]);

Lrr = simplify( vpa(Lrr') )
```

Lrr =

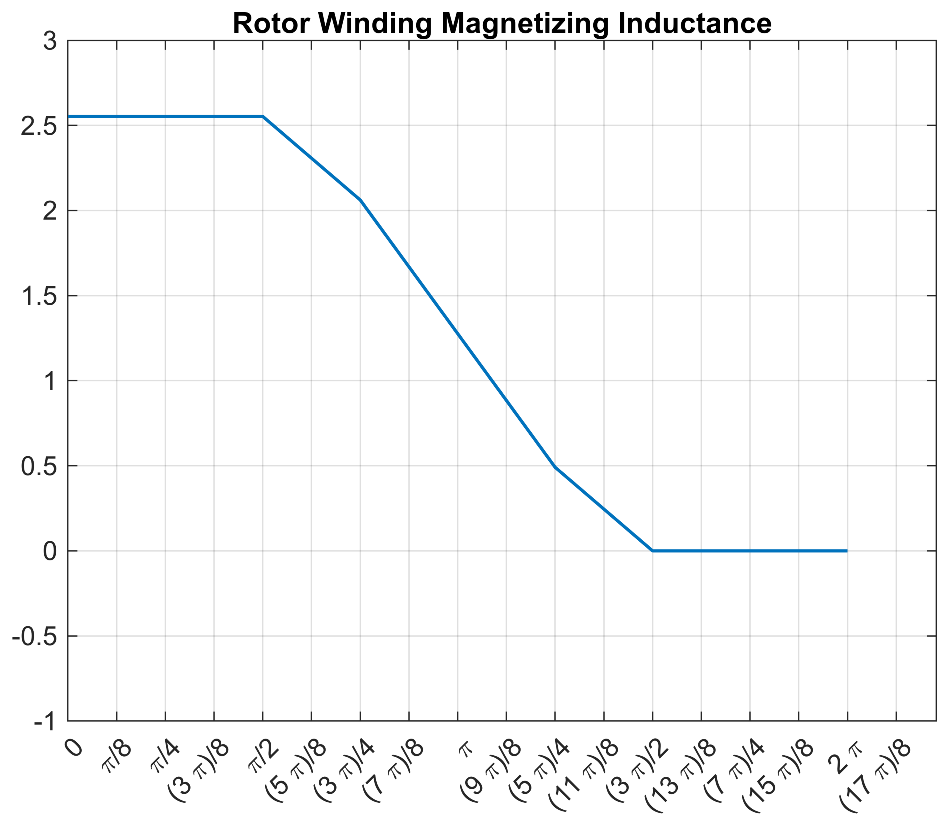
$$\begin{pmatrix} \frac{0.7854\ (\overline{N_r})^2\ \overline{l}\ \overline{\mu_o}\ \overline{r}\ (g_{\min} + g_{\max})}{g_{\min}\, g_{\max}} \\ \frac{0.7854\ (\overline{N_r})^2\ \overline{l}\ \overline{\mu_o}\ \overline{r}\ (g_{\min} + g_{\max})}{g_{\min}\, g_{\max}} \\ \frac{0.7854\ (\overline{N_r})^2\ \overline{l}\ \overline{\mu_o}\ \overline{r}\ (g_{\min} + g_{\max})}{g_{\min}\, g_{\max}} \\ \frac{0.3927\ (\overline{N_r})^2\ \overline{l}\ \overline{\mu_o}\ \overline{r}\ (g_{\min} + 2.0\, g_{\max})}{g_{\min}\, g_{\max}} \\ \frac{0.3927\ (\overline{N_r})^2\ \overline{l}\ \overline{\mu_o}\ \overline{r}\ (g_{\min} + g_{\max})}{g_{\min}\, g_{\max}} \\ \frac{0.3927\ (\overline{N_r})^2\ \overline{l}\ \overline{\mu_o}\ \overline{r}}{g_{\max}} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

```
syms K
Lrr_simplified = double( subs(Lrr, [(N_r^2*l*mu_o*r)' vars], [1 VALS]) )
```

```
Lrr_simplified = 9×1
    2.5526
    2.5526
    2.5526
    2.0617
    1.2763
    0.4909
         0
         0
         0
```

```
figure
plot(THETA_R_ARRAY, Lrr_simplified, ...
    "LineWidth", 1.2)
ylim([-1 3])
grid on
```

```
title("Rotor Winding Magnetizing Inductance")
ax = gca;
S = sym(ax.XLim(1):pi/8:ax.XLim(2));
ax.XTick = double(S);
ax.XTickLabel = arrayfun(@texlabel,S,'UniformOutput',false);
```



(c) Mutual inductance between stator and rotor for the following salient pole synchronous machine

$$L_{AB} = \mu_0 r l \int_0^{2\pi} n_A(\phi) N_B(\phi) g^{-1}(\phi) d\phi \qquad (1.165)$$

```
THETA_R_ARRAY = 0: T/8 : T;

Lsr_integrand = subs( ...
    statorCountingFunction * rotorWindingFunction_ext * g_phi_inv_ext, ...
    theta_r, THETA_R_ARRAY);
Lsr = mu_o*r*l * int( Lsr_integrand, phi, [0 T]);

Lsr = simplify( vpa(Lsr') )
```

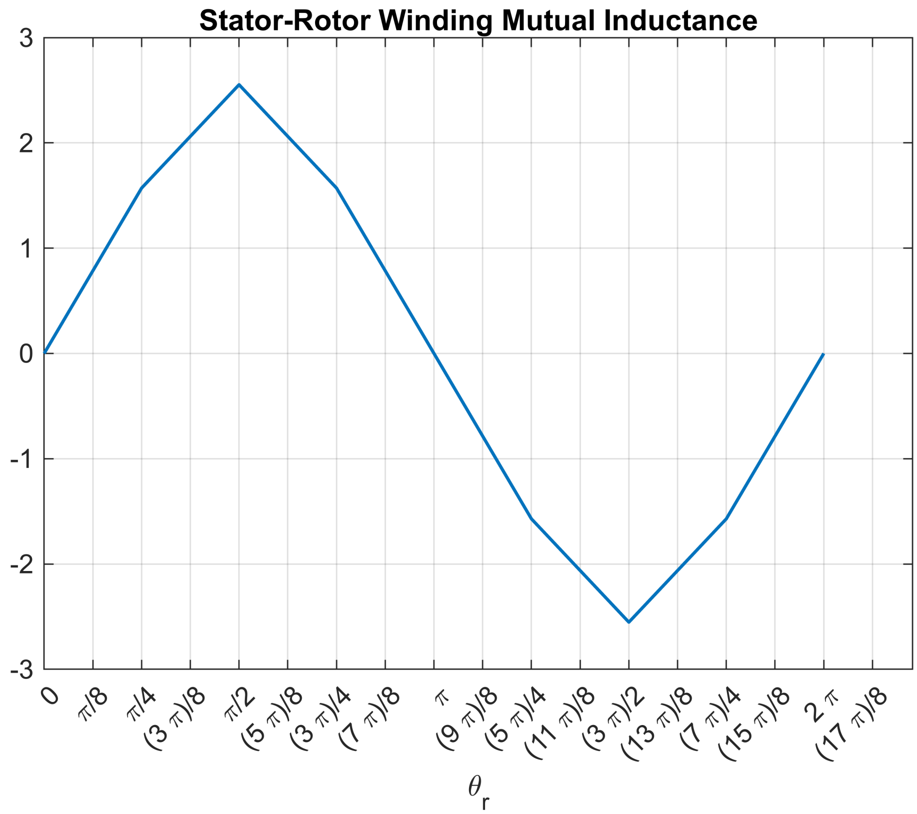
```
Lsr =
```

$$\begin{pmatrix} \dfrac{90949\,\overline{N_r}\,\overline{N_s}\,\overline{l}\,\overline{\mu_o}\,\overline{r}\,(g_{\min}+g_{\max})}{100000000000000000\,g_{\min}\,g_{\max}} \\ \dfrac{1.0\text{e-}17\,\overline{N_r}\,\overline{N_s}\,\overline{l}\,\overline{\mu_o}\,\overline{r}\,(90950.0\,g_{\min}+7.854\text{e+}16\,g_{\max})}{g_{\min}\,g_{\max}} \\ \dfrac{3927\,\overline{N_r}\,\overline{N_s}\,\overline{l}\,\overline{\mu_o}\,\overline{r}\,(g_{\min}+g_{\max})}{5000\,g_{\min}\,g_{\max}} \\ \dfrac{1.0\text{e-}17\,\overline{N_r}\,\overline{N_s}\,\overline{l}\,\overline{\mu_o}\,\overline{r}\,(90950.0\,g_{\min}+7.854\text{e+}16\,g_{\max})}{g_{\min}\,g_{\max}} \\ \dfrac{90949\,\overline{N_r}\,\overline{N_s}\,\overline{l}\,\overline{\mu_o}\,\overline{r}\,(g_{\min}+g_{\max})}{100000000000000000\,g_{\min}\,g_{\max}} \\ \dfrac{1.0\text{e-}17\,\overline{N_r}\,\overline{N_s}\,\overline{l}\,\overline{\mu_o}\,\overline{r}\,(90950.0\,g_{\min}-7.854\text{e+}16\,g_{\max})}{g_{\min}\,g_{\max}} \\ -\dfrac{3927\,\overline{N_r}\,\overline{N_s}\,\overline{l}\,\overline{\mu_o}\,\overline{r}\,(g_{\min}+g_{\max})}{5000\,g_{\min}\,g_{\max}} \\ \dfrac{1.0\text{e-}17\,\overline{N_r}\,\overline{N_s}\,\overline{l}\,\overline{\mu_o}\,\overline{r}\,(90950.0\,g_{\min}-7.854\text{e+}16\,g_{\max})}{g_{\min}\,g_{\max}} \\ \dfrac{90949\,\overline{N_r}\,\overline{N_s}\,\overline{l}\,\overline{\mu_o}\,\overline{r}\,(g_{\min}+g_{\max})}{100000000000000000\,g_{\min}\,g_{\max}} \end{pmatrix}$$

```
Lsr_simplified = double( subs(Lsr, [(N_r*N_s*l*mu_o*r)' vars], [1 VALS]) )
```

```
Lsr_simplified = 9×1
    0.0000
    1.5708
    2.5526
    1.5708
    0.0000
   -1.5708
   -2.5526
   -1.5708
    0.0000
```

```
figure
plot(THETA_R_ARRAY, Lsr_simplified, ...
    "LineWidth", 1.2)
ylim([-3 3])
title("Stator-Rotor Winding Mutual Inductance")
xlabel("\theta_r")
grid on

ax = gca;
S = sym(ax.XLim(1):pi/8:ax.XLim(2));
ax.XTick = double(S);
ax.XTickLabel = arrayfun(@texlabel,S,'UniformOutput',false);
```


Stator-Rotor Winding Mutual Inductance
3
2
1
0
-1
-2
-3
0
π/8
π/4
(3 π)/8
π/2
(5 π)/8
(3 π)/4
(7 π)/8
π
(9 π)/8
(5 π)/4
(11 π)/8
(3 π)/2
(13 π)/8
(7 π)/4
(15 π)/8
2 π
(17 π)/8
θ_r