ECEN 611 Homework 3: Winding Factor and Inductance

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```
clearvars
```

```
clc
```

Problem 1: Winding Factor

Verify the harmonic winding factor for the fractional pitch, uniformly distributed winding.

Symbol	Description
Nc(φ*) (c might stand for ‘concentrated’)	winding function for concentrated winding expressed in Fourier series
Nch	coefficients of each term in Nc(φ*) (h = 1, 2, 3, …, n) of **concentrated** winding function
Nh (without c compared to above)	coefficients of each term of **fractional/short** pitch winding
kh	Nh/Nch

```
T = 2*pi;
NT = 1;    % number of turns
```

Full Pitch, Concentrated Two-Pole Winding

Counting Function

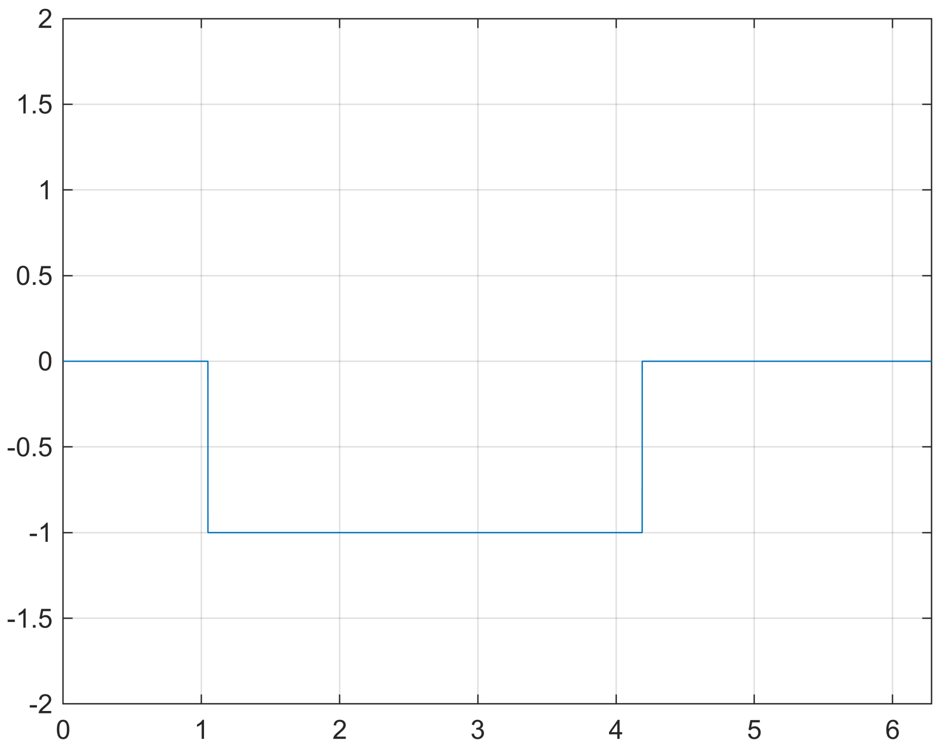
```
syms phi
countingFun = piecewise( phi >= 0      & phi <  pi/3,    0, ...
                         phi >= pi/3   & phi <  4*pi/3, -NT, ...
                         phi >= 4*pi/3 & phi <= T,       0)
```

countingFun =

$$\begin{cases} 0 & \text{if } \phi \in \left[0, \frac{\pi}{3}\right) \\ -1 & \text{if } \phi \in \left[\frac{\pi}{3}, \frac{4\,\pi}{3}\right) \\ 0 & \text{if } \phi \in \left[\frac{4\,\pi}{3}, 2\,\pi\right] \end{cases}$$

(Figure) Counting Function

```
fplot(countingFun, [0 2*pi])
ylim([-2 2])
grid
```



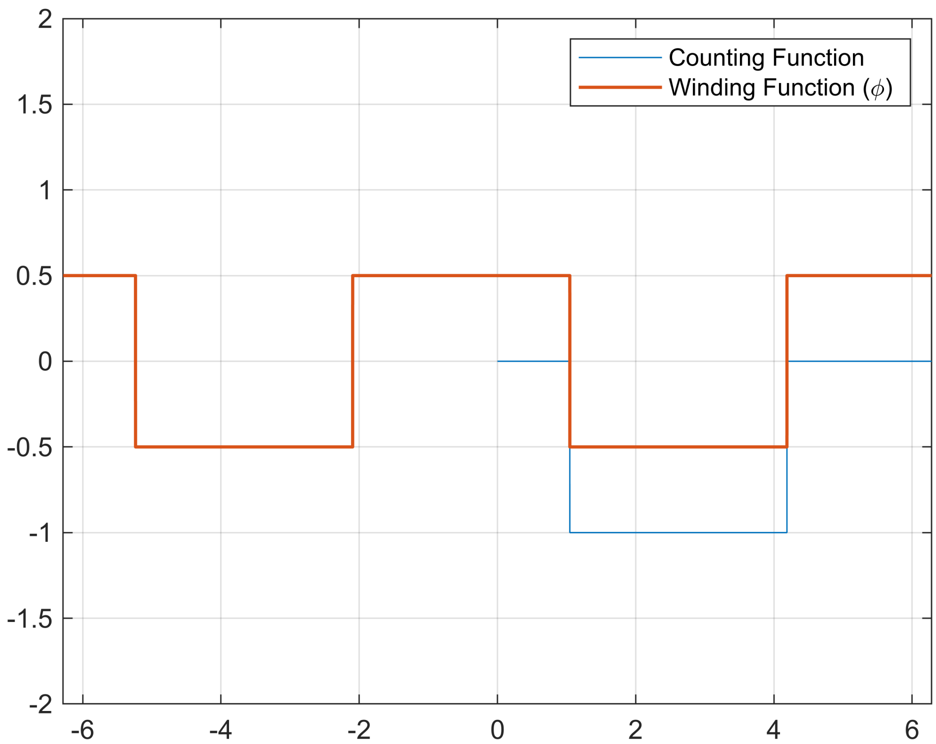

```
countingFun_avg = 1/T * int(countingFun, phi, [0 T]);
```

Winding Function

```
windingFun = countingFun - countingFun_avg;
windingFun = piecewise( ...
    phi <= 0,            subs(windingFun, phi, phi+T), ...
    phi >= 0 & phi <= T, windingFun, ...
    phi > T,             subs(windingFun, phi, phi-T) ... % I'm a genius!!
);
```

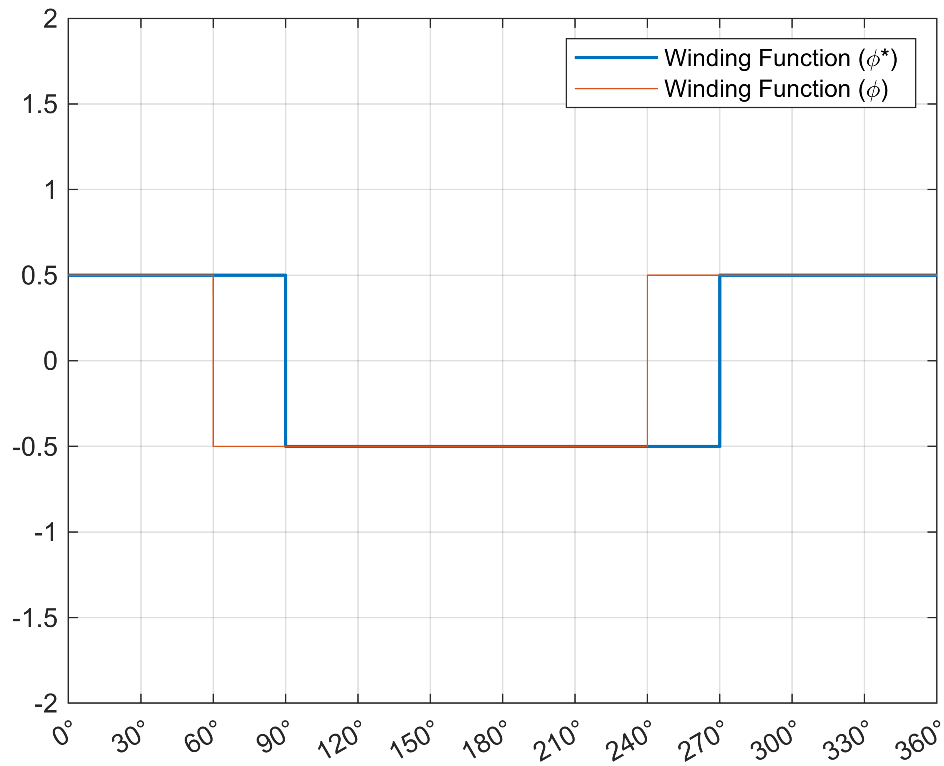
(Figure) Counting Function and Winding Function

```
figure
fplot(countingFun, [0 2*pi], "DisplayName", "Counting Function")
hold on
fplot(windingFun, [-2*pi 2*pi], ...
    "LineWidth", 1.2, ...
    "DisplayName", "Winding Function (\phi)")
hold off
ylim([-2 2])
legend
grid
```



Centralized Winding Function

```
shiftAngle = T/4 - pi/3; % How to relate 4 to the number of poles?
windingFun_star = subs(windingFun, phi, phi - shiftAngle);

figure
fplot(windingFun_star, [0 2*pi], "LineWidth", 1.2, "DisplayName", "Winding Function
(\phi*)")
hold on
fplot(windingFun, [0 2*pi], "DisplayName", "Winding Function (\phi)")
ylim([-2 2])
grid
legend
% Customize x-tick labels to show degrees with increments of 30 degrees
xticks([0 pi/6 pi/3 pi/2 2*pi/3 5*pi/6 pi 7*pi/6 4*pi/3 3*pi/2 5*pi/3 11*pi/6
2*pi]);
xticklabels({'0°', '30°', '60°', '90°', '120°', '150°', '180°', '210°', '240°',
'270°', '300°', '330°', '360°'});
```



Winding Function in Fourier Series

```
num_harmonics = 7;
h = 1:num_harmonics;
digits(4)

% a0 = vpa( (1/T)*int(windingFun, phi, 0, T), 4 )
% b = (2/T)*int(expression*sin(k*theta), theta, 0, T);
% Nch = a

% windingFun_star = piecewise( phi >= 0      & phi <  pi/2,    NT/2, ...
%                              phi >= pi/2   & phi <  3*pi/2, -NT/2, ...
%                              phi >= 3*pi/2 & phi <= T,       NT/2);

a0 = vpa( (1/T)*int(windingFun, phi, 0, T), 4 )
```

a0 = 1.819e-12

```
a  = vpa( (2/T)*int(windingFun_star*cos(h*phi), phi, 0, T) );
b  = vpa( (2/T)*int(windingFun_star*sin(h*phi), phi, 0, T) )
```

b = $(0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$

Nch: My Result

```
Nch = a
```

Nch = $(0.6366 \quad 0 \quad -0.2122 \quad 0 \quad 0.1273 \quad 0 \quad -0.09095)$

$$N_{ch} = (-1)^{\frac{h-1}{2}}\left(\frac{2N_t}{h\pi}\right) \qquad \text{for} \quad h = 1,3,5,7.... \qquad (1.28)$$

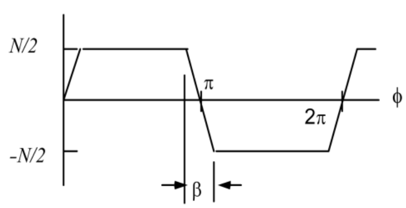
$$N_{ch} = 0 \qquad \text{for} \quad h{=}2,4,6,... \qquad (1.29)$$

Nch: Textbook Result

```
Nch_ref = real( (-1).^((h-1)/2).*(2*NT./(h*pi)) )
```

```
Nch_ref = 1×7
    0.6366         0   -0.2122         0    0.1273         0   -0.0909
```

Full Pitch, Uniformly Distributed 2-Pole Winding

Winding Type	**Winding Function**	**Harmonic Winding Factor** k_h **(**h **odd)**
e) Full Pitch Uniformly Distributed		$\dfrac{\sin\left(\frac{h\beta}{2}\right)}{\frac{h\beta}{2}}$

Counting Function

```
% windingFun_star
syms beta N_t
windingDensity = N_t/beta
```

windingDensity =

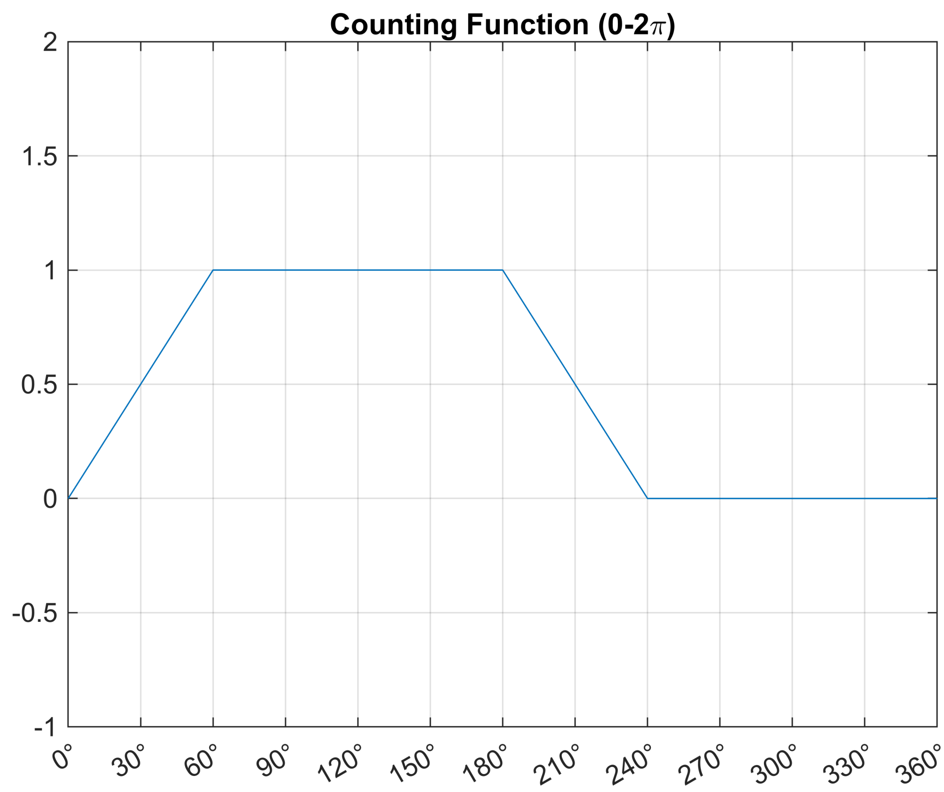
$$\frac{N_t}{\beta}$$

```
countingFun = piecewise( ...
    phi >= 0          & phi <      beta, windingDensity * phi, ... % for 0 ≤ ϕ ≤ β
    phi >= beta       & phi <        pi, N_t, ...
    phi >= pi         & phi < pi + beta, -windingDensity*(phi-pi) + N_t, ...
    phi >= pi + beta  & phi <=     2*pi, 0 ...
)
```

countingFun =

$$
\begin{cases}
\frac{N_t\,\phi}{\beta} & \text{if } \phi < \beta \wedge 0 \le \phi \\
N_t & \text{if } \phi < \pi \wedge \beta \le \phi \\
N_t - \frac{N_t\,(\phi - \pi)}{\beta} & \text{if } \pi \le \phi \wedge \phi < \beta + \pi \\
0 & \text{if } \phi \le 2\,\pi \wedge \beta + \pi \le \phi
\end{cases}
$$

```
countingFun = subs(countingFun, [beta N_t], [pi/3 1]);

figure
fplot(countingFun, [0 2*pi])
title("Counting Function (0-2\pi)")
ylim([-1 2])
grid
% Customize x-tick labels to show degrees with increments of 30 degrees
xticks([0 pi/6 pi/3 pi/2 2*pi/3 5*pi/6 pi 7*pi/6 4*pi/3 3*pi/2 5*pi/3 11*pi/6
2*pi]);
xticklabels({'0°', '30°', '60°', '90°', '120°', '150°', '180°', '210°', '240°',
'270°', '300°', '330°', '360°'});
```




```
% countingFun = piecewise( ...
%     phi >= 0 & phi <= T, countingFun, ...
%     phi > T, subs(countingFun, phi, phi-T) ... % I'm a genius!!
% )
% figure
```

```
% fplot(countingFun, [0 4*pi])
% title("Counting Function Extended (0-4\pi)")
% ylim([-1 2])
% grid

% Customize x-tick labels to show degrees with increments of 30 degrees
% xticks([0 pi/6 pi/3 pi/2 2*pi/3 5*pi/6 pi 7*pi/6 4*pi/3 3*pi/2 5*pi/3 11*pi/6
2*pi]);
% xticklabels({'0°', '30°', '60°', '90°', '120°', '150°', '180°', '210°', '240°',
'270°', '300°', '330°', '360°'});
```

```
countingFun_avg = 1/T * int(countingFun, phi, [0 T]);
```

Winding Function

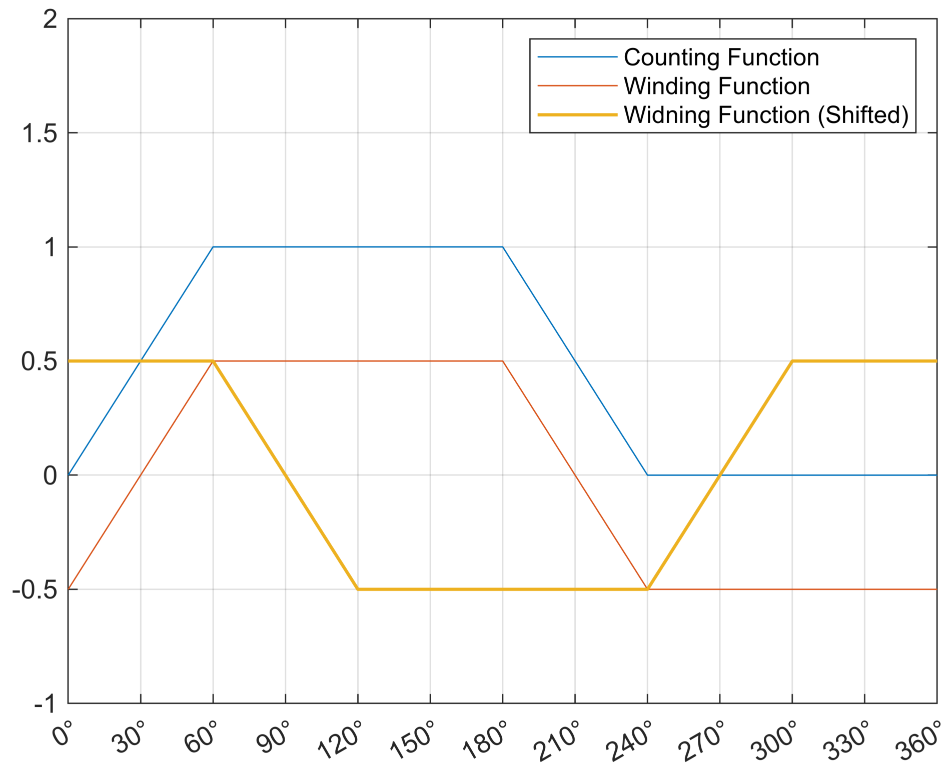
```
windingFun = countingFun - countingFun_avg;
windingFun = piecewise( ...
    phi <= 0,              subs(windingFun, phi, phi+T), ...
    phi >= 0 & phi <= T, windingFun, ...
    phi > T,               subs(windingFun, phi, phi-T) ... % I'm a genius!!
);
```

Centralized Winding Function

```
windingFun_star = subs(windingFun, phi, phi+pi/2+beta/2); % need to generalize this
shift angle
windingFun_star = subs(windingFun_star, beta, pi/3);
```

(Figure) Counting Function and Winding Function

```
figure
fplot(countingFun, [0 2*pi], "DisplayName", "Counting Function")
hold on
fplot(windingFun, [0 2*pi], "DisplayName", "Winding Function")
fplot(windingFun_star, [0 2*pi], "LineWidth", 1.2, "DisplayName", "Widning Function
(Shifted)")
hold off
ylim([-1 2])
legend
grid

% Customize x-tick labels to show degrees with increments of 30 degrees
xticks([0 pi/6 pi/3 pi/2 2*pi/3 5*pi/6 pi 7*pi/6 4*pi/3 3*pi/2 5*pi/3 11*pi/6
2*pi]);
xticklabels({'0°', '30°', '60°', '90°', '120°', '150°', '180°', '210°', '240°',
'270°', '300°', '330°', '360°'});
```



Nh: My Result

```
Nh = vpa( (2/T)*int(windingFun_star*cos(h*phi), phi, 0, T) )
```

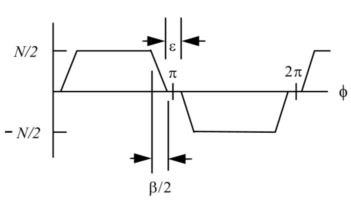
`Nh =` $\begin{pmatrix} 0.6079 & 0 & -0.1351 & 0 & 0.02432 & 0 & 0.01241 \end{pmatrix}$

Nh: Textbook Result

```
Nh_ref = (-1).^((h-1)/2).*(2*NT./(h*pi)).*sin(h*beta/2)./(h*beta/2);
Nh_ref = vpa( real( subs(Nh_ref, beta, pi/3) ) )
```

`Nh_ref =` $\begin{pmatrix} 0.6079 & 0 & -0.1351 & 0 & 0.02432 & 0 & 0.01241 \end{pmatrix}$

Fractional Pitch, Uniformly Distributed Two-Pole Winding

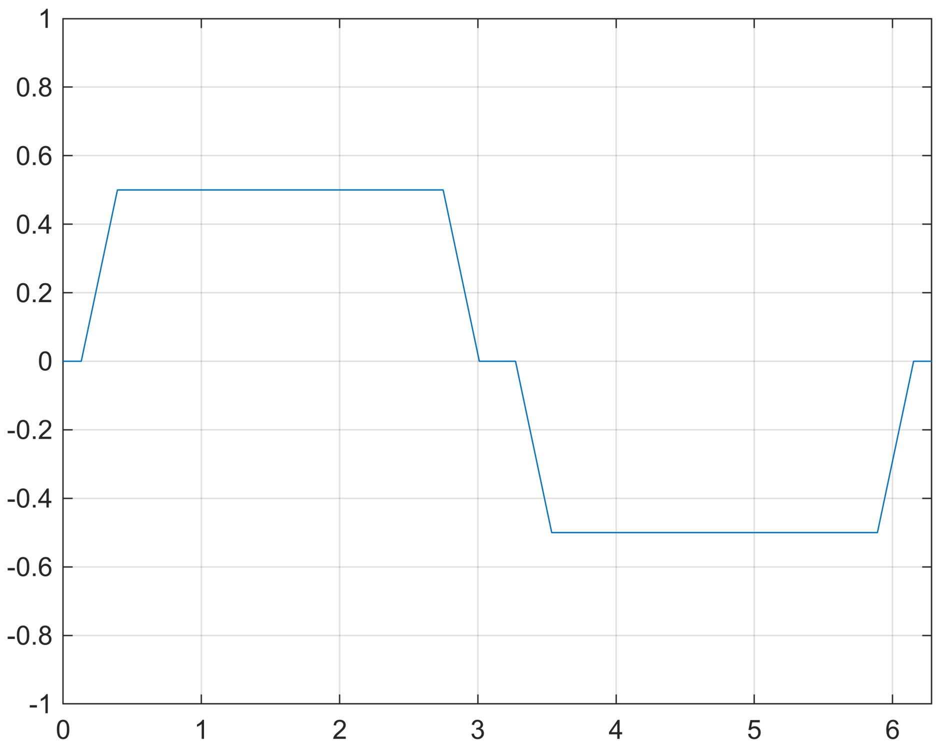
Winding Type	**Winding Function**	**Harmonic Winding Factor** k_h **(h odd)**
g) Fractional Pitch, Uniformly Distributed		$\dfrac{\sin\left(\frac{h\beta}{4}\right)}{\frac{h\beta}{4}}\cos\left[h\left(\frac{\beta}{4}+\frac{\varepsilon}{2}\right)\right]$

Winding Function

```
syms epsilon
windingFun_star = piecewise( ...
    phi >= 0                    & phi < epsilon/2, 0, ...
    phi >= epsilon/2            & phi < epsilon/2 + beta/2, N_t/beta*(phi-
epsilon/2), ...
    phi >= epsilon/2 + beta/2      & phi < pi - epsilon/2 - beta/2, N_t/2, ...
    phi >= pi - epsilon/2 - beta/2 & phi < pi - epsilon/2, -N_t/beta*(phi-(pi-
epsilon/2)), ...
    phi >= pi - epsilon/2          & phi < pi, 0 ...
);

% windingFun_star = subs(windingFun_star, [N_t beta epsilon], [NT pi/6 pi/12]);
% figure
% fplot(windingFun_star, [0 2*pi])
% ylim([-1 1])
% grid
windingFun_star = piecewise( ...
    phi >= 0  & phi <= T/2, windingFun_star, ...
    phi > T/2 & phi <= T,  -subs(windingFun_star, phi, phi-pi) ... % I'm a genius!!
);
```

(Figure) Winding Function

```
figure
fplot(subs(windingFun_star, [N_t beta epsilon], [NT pi/6 pi/12]), [0 2*pi]);
ylim([-1 1])
grid
```

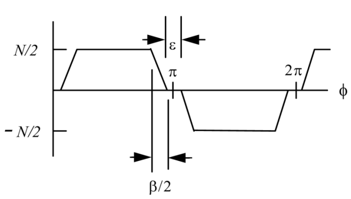


Nh: My Result

```
windingFun_star = subs(windingFun_star, [N_t beta epsilon], [NT pi/6 pi/12]);

Nh = vpa( (2/T)*int(windingFun_star*sin(h*phi), phi, 0, T) )
```

Nh = $(0.6132 \quad 0 \quad 0.1462 \quad 0 \quad 0.03065 \quad 0 \quad -0.02038)$

```
windingFun_star_rec = sum(Nh.*sin(h.*phi));
```

Winding Type	**Winding Function**	**Harmonic Winding Factor** k_h **(h odd)**
g) Fractional Pitch, Uniformly Distributed		$\dfrac{\sin\left(\dfrac{h\beta}{4}\right)}{\dfrac{h\beta}{4}}\cos\left[h\left(\dfrac{\beta}{4}+\dfrac{\varepsilon}{2}\right)\right]$

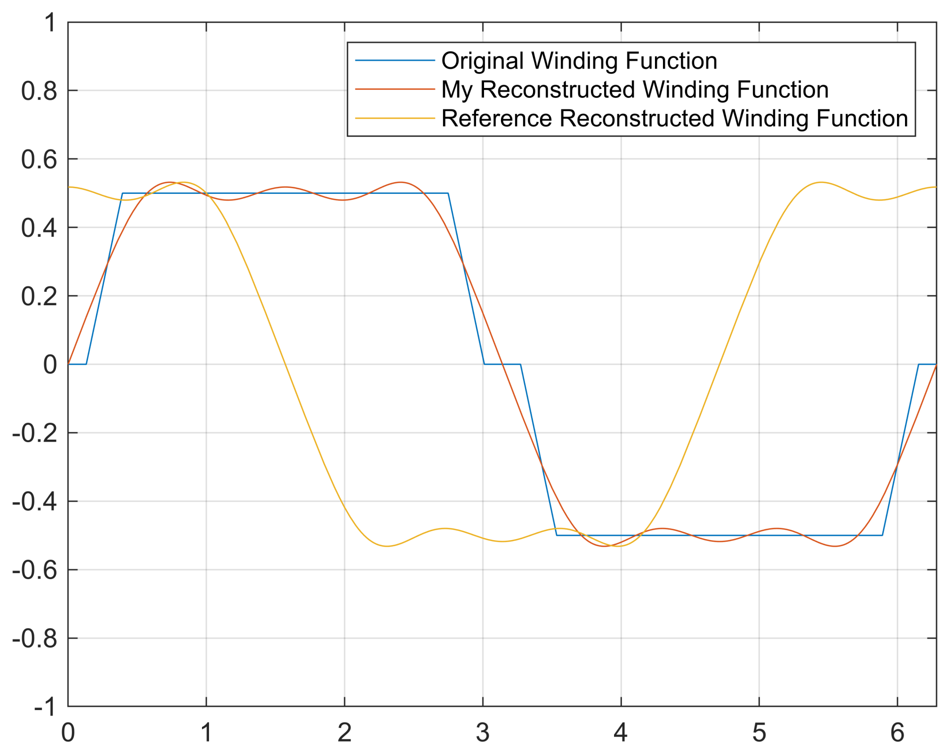
Nh: Textbook Result

```
Nh_ref = sin(h*beta/4)./(h*beta/4) .* cos(h*(beta/4+epsilon/2)); % <-- What's its reference?
Nh_ref = vpa( subs(Nh_ref.*Nch_ref, [beta epsilon], [pi/6 pi/12]) )
```

Nh_ref = $(0.6132 \quad 0 \quad -0.1462 \quad 0 \quad 0.03065 \quad 0 \quad 0.02038)$

(Figure) Validation of Fourier Series through Reconstruction

```
figure
fplot(windingFun_star, [0 2*pi], "DisplayName", "Original Winding Function")
hold on
fplot(windingFun_star_rec, [0 2*pi], "DisplayName", "My Reconstructed Winding
Function")
fplot(sum(Nh_ref.*cos(h.*phi)), [0 2*pi], "DisplayName", "Reference Reconstructed
Winding Function")
hold off
ylim([-1 1])
grid
legend
```

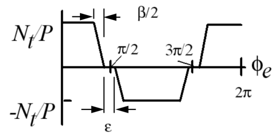


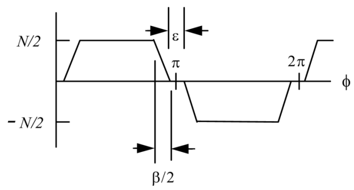
Problem 2: Magnetizing Inductance

Calculate the magnetizing inductance of the above winding (fractional pitch, uniformly distributed 2-pole winding).

Textbook pg. 31

Winding Type	Winding Function	Magnetizing Inductance

d) Fractional pitch uniformly distributed		$\mu_o \frac{rl}{g}(2\pi)\left(\frac{N_t}{P}\right)^2\left(1-\frac{2\beta}{3\pi}-\frac{\varepsilon}{\pi}\right)$

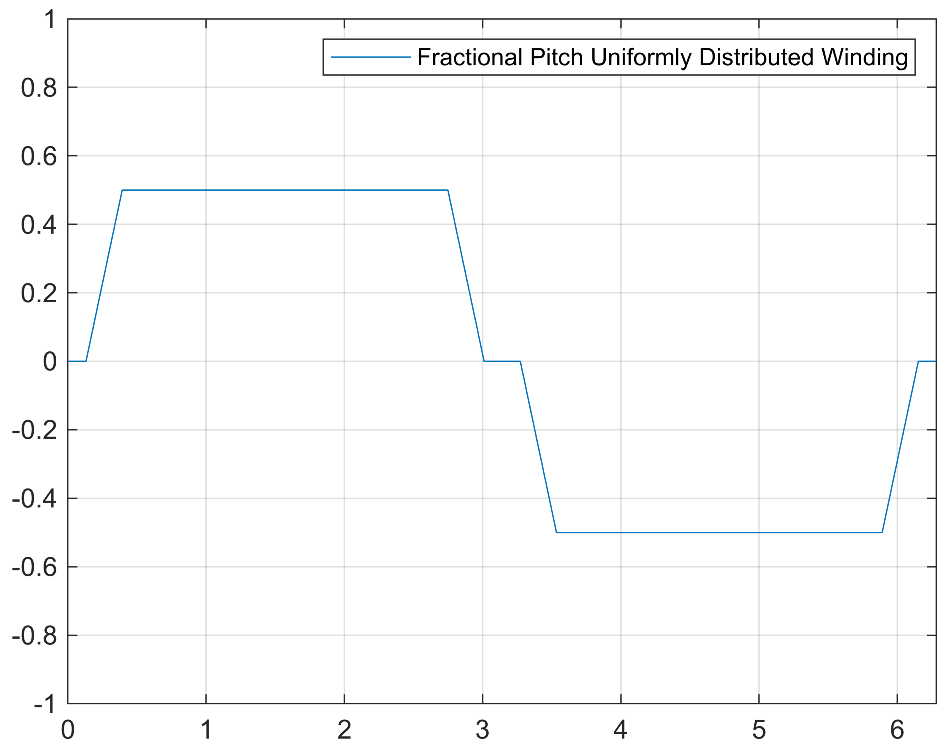
Winding Type	**Winding Function**	**Harmonic Winding Factor** k_h **(*h* odd)**
g) Fractional Pitch, Uniformly Distributed		$\frac{\sin\left(\frac{h\beta}{4}\right)}{\frac{h\beta}{4}}\cos\left[h\left(\frac{\beta}{4}+\frac{\varepsilon}{2}\right)\right]$

Winding Function

```
syms epsilon P
windingFun_star = piecewise( ...
    phi >= 0                     & phi < epsilon/2, 0, ...
    phi >= epsilon/2             & phi < epsilon/2 + beta/2, (N_t/P)/(beta/
2)*(phi-epsilon/2), ...
    phi >= epsilon/2 + beta/2      & phi < pi - epsilon/2 - beta/2, N_t/P, ...
    phi >= pi - epsilon/2 - beta/2 & phi < pi - epsilon/2, -(N_t/P)/(beta/2)*(phi-
(pi-epsilon/2)), ...
    phi >= pi - epsilon/2          & phi < pi, 0 ...
);

windingFun_star = piecewise( ...
    phi >= 0  & phi <= T/2, windingFun_star, ...
    phi > T/2 & phi <= T,  -subs(windingFun_star, phi, phi-pi) ...
);
```

(Figure) Winding Function

```
syms mu_o r l g

windingFun_star = subs( windingFun_star, [beta epsilon], [pi/6 pi/12] );

figure
fplot(subs(windingFun_star, [N_t P], [1 2]), [0 2*pi], ...
    "DisplayName", "Fractional Pitch Uniformly Distributed Winding")
hold off
ylim([-1 1])
grid
legend
```



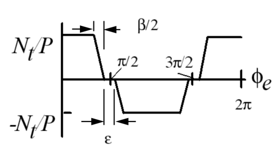
Magnetizing Inductance

My Result

```
LAA = mu_o*r*l/g * int( windingFun_star^2, phi, [0 2*pi] )
```

LAA =

$$\frac{29\,\pi\,N_t^2\,l\,\mu_o\,r}{18\,P^2\,g}$$

Winding Type	Winding Function	Magnetizing Inductance
d) Fractional pitch uniformly distributed		$\mu_o \frac{rl}{g}(2\pi)\left(\frac{N_t}{P}\right)^2\left(1-\frac{2\beta}{3\pi}-\frac{\varepsilon}{\pi}\right)$

Textbook Result

```
LAA_ref = subs( (mu_o*r*l/g) * (2*pi) * (N_t/P)^2 * (1 - 2*beta/(3*pi) - epsilon/
pi), ...
              [beta epsilon], ...
              [pi/6 pi/12] )
```

LAA_ref =

$$\frac{29\,\pi\,N_t^2\,l\,\mu_o\,r}{18\,P^2\,g}$$

Problem 3: Mutual Inductance

Verify the mutual inductance between a (full pitch) uniformly distributed and a sinusoidally distributed winding each with P poles.

Winding Function of Sinusoidally Distributed P-Pole Winding

```
% syms phi
syms N_sin
windingFun_sin = N_sin/P * sin(P/2*phi)
```

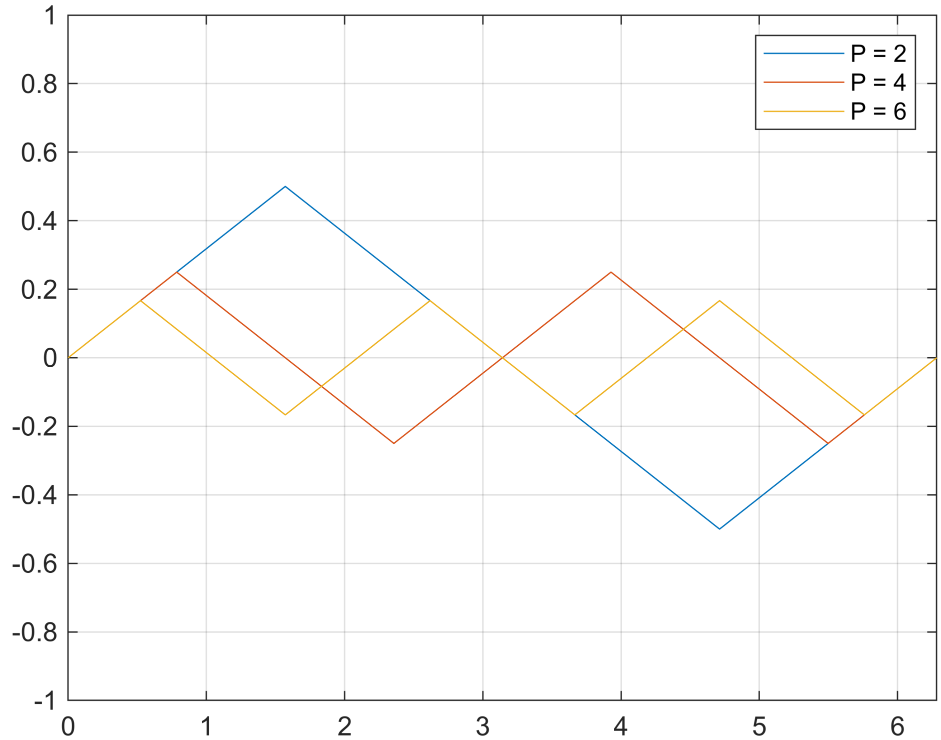
windingFun_sin =

$$\frac{N_{\sin}\,\sin\left(\frac{P\,\phi}{2}\right)}{P}$$

Winding Function of (Full Pitch) Uniformly Distributed P-Pole Winding

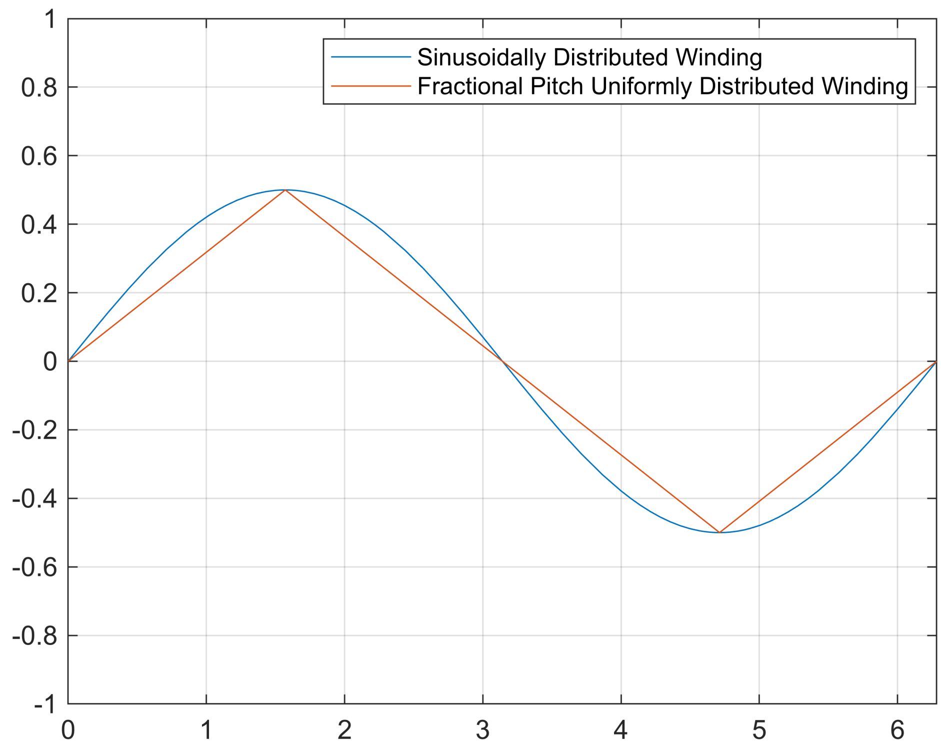
```
intv = T/(2*P);
slope = (N_t/P)/intv;
t = 4*intv;

windingFun_fp_uni = piecewise( ...
    phi >= 0        & phi < intv,    slope * (phi), ...
    phi >= intv     & phi < 3*intv,  -slope * (phi-intv) + N_t/P, ...
    phi >= 3*intv   & phi < t,  slope * (phi-3*intv) - N_t/P ...
);

windingFun_fp_uni_ext = piecewise( ...
    phi >= 0 & phi < t, windingFun_fp_uni, ...
    phi >= t, subs(windingFun_fp_uni, phi, phi-t), ...
    phi >= 2*t, subs(windingFun_fp_uni, phi, phi-2*t) ...
);

figure

for numberOfPoles = 2:2:6
    % Substitute P directly in the symbolic expression
    fplot(subs(windingFun_fp_uni_ext, [N_t P], [1 numberOfPoles]), [0 2*pi], ...
    'DisplayName', sprintf('P = %d', numberOfPoles))  % Use sprintf to format
DisplayName
    hold on
end

ylim([-1 1])
```

```
hold off
grid
legend
```



(Figure) Winding Functions of Interest

```
figure
fplot(subs(windingFun_sin, [N_sin P], [1 2]), [0 2*pi], ...
    "DisplayName", "Sinusoidally Distributed Winding")
hold on
fplot(subs(windingFun_fp_uni_ext, [N_t P], [1 2]), [0 2*pi], ...
    "DisplayName", "Fractional Pitch Uniformly Distributed Winding")
hold off
ylim([-1 1])
grid
legend
```



Mutual Inductance

My Result

$$L_{AB} = \frac{\mu_0 r l}{g}\left(\frac{P}{2}\right)\int_0^{2\pi} N_A(\phi_e) N_B(\phi_e) d\phi_e \qquad (1.85)$$

```
% Use equation 1.85
assume(P >= 2 & mod(P, 2) == 0);
L_uniform_sin = (mu_o*r*l/g) * (P/2) * int( windingFun_sin*windingFun_fp_uni, ...
                                            phi, [0 2*pi]);
```

```
Warning: Unable to check whether the integrand exists everywhere on the integration interval.
```

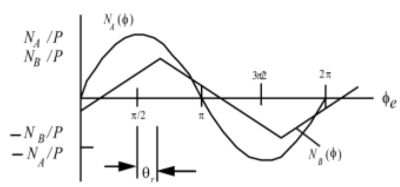
```
L_uniform_sin = simplify(L_uniform_sin)
```

L_uniform_sin =

$$\frac{8\, N_{\sin}\, N_t\, l\, \mu_o\, r}{P^2\, g\, \pi}$$

Textbook Result

Winding Types	Winding Functions	Mutual Inductance

d) Uniformly distributed and sinusoidally distributed windings		$L_{AB} = \frac{\mu_o r l}{g}\left(\frac{N_A N_B}{p^2}\right)\left(\frac{8}{\pi}\right)\cos\theta_r$

```
syms theta_r
L_uniform_sin_ref  = (mu_o*r*l/g) * (N_t*N_sin/P^2) * (8/(sym(1)*pi)) *
subs(cos(theta_r), theta_r, 0)
```

L_uniform_sin_ref =

$$\frac{8\,N_{\sin}\,N_t\,l\,\mu_o\,r}{P^2\,g\,\pi}$$

Problem 4

A **uniform air-gap** machine has an axial length of 1m, a rotor radius of 0.5 m, and a gap length of 0.50 cm. The rotor and the stator are each wound with a sinusoidally distributed 4-pole winding with 50 turns per pole. If the rotor and stator coil axes are aligned and the two windings connected in series, how much current should be passed through the windings to produce a peak air-gap flux density of 0.8 weber/m2?

Parameters

```
LENGTH = 1;       % m
RADIUS = 0.5;     % m
GAP    = 0.5e-2; % m

N_POLE       = 4;
N_TURN_POLE  = 50;
N_TURN_TOTAL = N_POLE*N_TURN_POLE;

B_PEAK = 0.8;     % Weber/m^2
```

- sinusoidally distributed 4-pole winding
- uniform air-gap
- rotor and stator coil axes are aligned => theta_r = 0, cos(theta_r) = 1
- two windings connected in series (, respectively?)
- Do I need to calculate magnetizing and mutual inductance?

```
uo = 4*pi*1e-7  % H/m
```

uo = 1.2566e-06

```
% magnetic flux density  : T = Wb/m^2 = N/(A·m)
% permeability           : H/m N/A^2
% magnetic field strength: A/m
H_PEAK = B_PEAK / uo     % A/m = N/(A·m)/(N/A^2)
```

```
H_PEAK = 6.3662e+05
```

```
syms N A m
N/(A*m)/(N/A^2)
```

```
ans =
```

$$\frac{A}{m}$$

```
MMF_PEAK = H_PEAK*GAP
```

```
MMF_PEAK = 3.1831e+03
```

Winding Functions

```
% Stator
% winding function
stator_windingFun_sin = subs( windingFun_sin, [N_sin P], [N_TURN_TOTAL N_POLE] )
```

stator_windingFun_sin = $50\sin(2\,\phi)$

```
% Rotor winding function
rotor_windingFun_sin  = stator_windingFun_sin;
```

*Inductances

```
% magnetizing inductance
stator_Lm = mu_o*r*l/g * (N_POLE/2) * int( stator_windingFun_sin^2, phi, [0 2*pi] );
stator_Lm = vpa( subs(stator_Lm, [l mu_o r g], [LENGTH uo RADIUS GAP]) ) % H
```

stator_Lm = 1.974

```
rotor_Lm = stator_Lm
```

rotor_Lm = 1.974

```
% stator-rotor mutual inductance
L_stator_rotor = mu_o*r*l/g * (N_POLE/2) *
int( stator_windingFun_sin*rotor_windingFun_sin, phi, [0 2*pi] );
L_stator_rotor = vpa( subs(L_stator_rotor, [l mu_o r g], [LENGTH uo RADIUS GAP]) )
% H
```

L_stator_rotor = 1.974

Composite Airgap MMF

```
syms I
stator_MMF = stator_windingFun_sin * I;
rotor_MMF  = rotor_windingFun_sin * I;
composite_MMF  = stator_MMF + rotor_MMF
```

composite_MMF = $100\,\mathrm{I}\sin(2\,\phi)$

Find the location where MMF reaches its peak value

```
[peak_phi, peak_MMF] = fminbnd(@(phi) -sin(2*phi), 0, 2*pi); % Find the minimum of the negative function
peak_MMF = -peak_MMF  % Get the maximum value
```

```
peak_MMF = 1.0000
```

```
% MATLAB does not have an fmaxbnd function because finding a maximum is simply a variant of finding a minimum, which is already handled by the function fminbnd.
% To find the maximum, you can negate the function and use fminbnd to minimize the negated function.
```

Find peak MMF (symbolic)

```
total_MMF_peak = vpa( subs(composite_MMF, phi, peak_phi) )
```

total_MMF_peak = $100.0\,\mathrm{I}$

Current

```
current = solve(total_MMF_peak==MMF_PEAK) % A
```

current = 31.83