Electromechanical Energy Conversion

Prof. Hamid A. Toliyat
toliyat@tamu.edu

Advanced Electric Machines & Power Electronics (EMPE) Lab

Department of Electrical and Computer Engineering
Texas A&M University

Introduction

- Electromechanical energy conversions
  - use a magnetic field as the medium of energy conversion

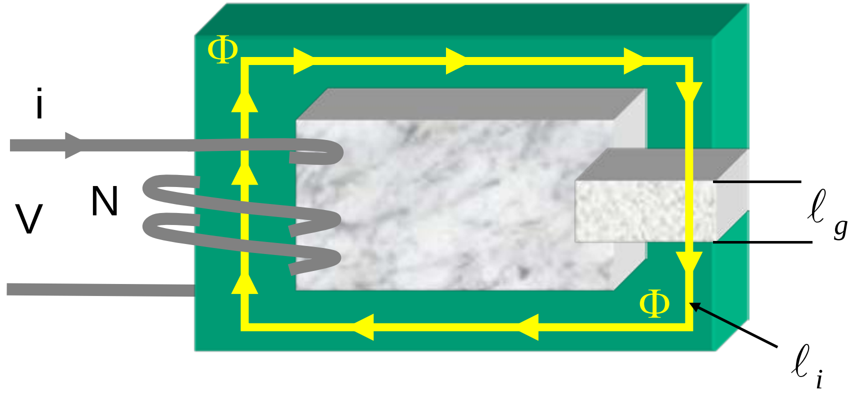
- Electromechanical energy conversion device
  - Converts electrical energy into mechanical energy or
  - Converts mechanical energy into electrical energy.

- Three categories of electromechanical energy conversion devices:
  - Transducers (for measurement and control)- small motion
    - Transform the signals of different forms;
    - Examples: Microphones, sensors and speakers.
  - Force producing devices (translational force)- limited mechanical motion
    - Produce forces mostly for linear motion drives;
    - Examples: Actuators, relays, solenoids and electromagnets.
  - Continuous energy conversion equipment
    - Operate in rotating mode;
    - Examples: Motors and generators.

Magnetic Circuits

- Consider the following coil:

$$B = \frac{\Phi}{A} \qquad B = \mu H$$



B =	Flux density in the core (Weber/m^2)
Φ =	Total flux in the core (Weber=Joules/Ampere)
ℓ_i =	Length of flux path in the core
ℓ_g =	Air gap length
μ =	Permeability of core
H =	Field intensity
A_i =	Core cross section area

Recall that Ampere's Law states that the line integral of the field intensity "**H**" about a closed path is equal to the net current enclosed within this closed path of integration, that is,

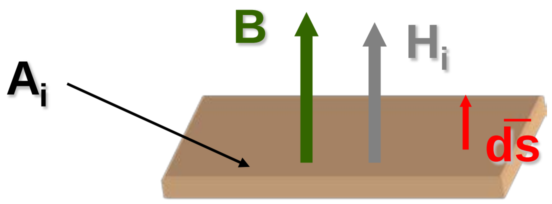
$$\oint H \cdot d\ell = Ni_{enclosed}$$

$$\int_a^b H_i d\ell + \int_b^a H_g d\ell = Ni$$

$$H_i \ell_i + H_g \ell_g = Ni$$

Magnetic Circuits

- Consider a cross section of the magnetic circuit depicted before,

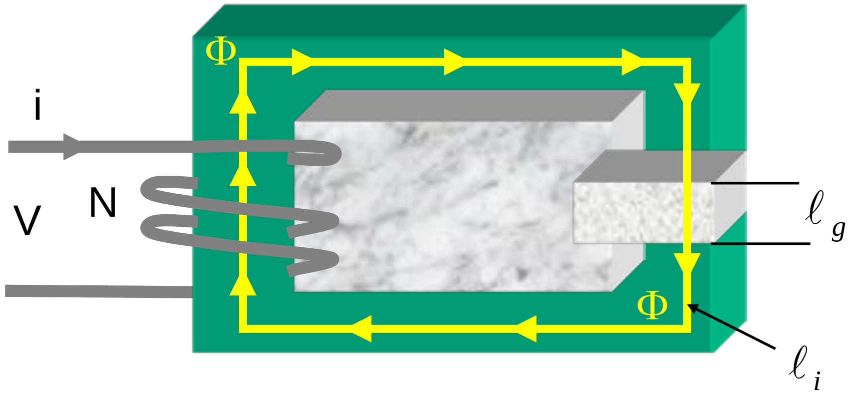


- The flux density "B" is related to the field intensity "H " by, $B = \mu\ H \quad \left[\frac{Weber}{m^2}\right]$
- The permeability of the medium is usually represented by μ.
- The surface integral of the flux density is equal to the flux Φ, $\Phi = \int_A \overline{B} \cdot d\ \overline{s} \qquad \left[Weber = \frac{Joules}{amperes}\right]$

Magnetic Circuits

- If a uniform flux density is assumed, the following relationships can be obtained,

$$\Phi_i = B_i \cdot A_i \;\; , \;\; \Phi_g = B_g \cdot A_g \;\; where \;\; A_g = A_i \;\; and \;\; \Phi_g = \Phi_i = \Phi$$

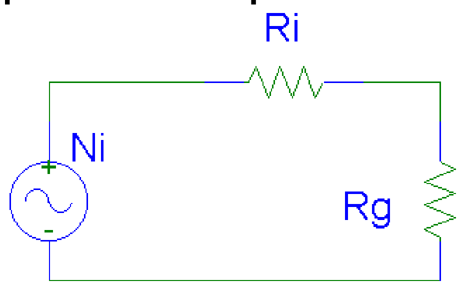


- Substituting for H and B, we will have the following expression:

$$\frac{B_i}{\mu_i} l_i + \frac{B_g}{\mu_g} l_g = Ni \qquad\qquad \frac{l_i}{\mu_i A_i}\Phi + \frac{l_g}{\mu_g A_g}\Phi = Ni$$

$$where \quad \mu_i = 500 \rightarrow 4000 \quad and \quad \mu_g = \mu_o = 4\pi \times 10^{-7} \frac{H}{m}$$

- The above equations represents the following circuit:



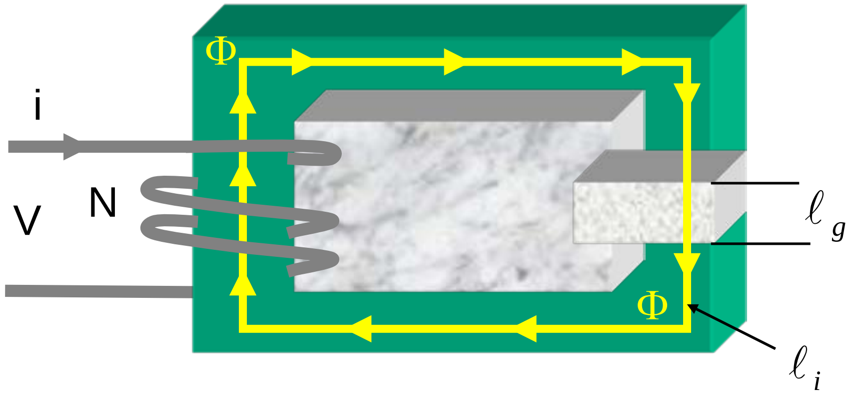
$$\Re_i = \frac{l_i}{\mu_i A_i} \quad , \quad \Re_g = \frac{l_g}{\mu_g A_g} \quad \Rightarrow \quad (\Re_i + \Re_g)\Phi = Ni$$

- The coil inductance is given by,

$$\boxed{L = \frac{N\Phi}{i} = \frac{N^2}{(\Re_i + \Re_g)}}$$

Magnetic Circuits

- Example: Assume ℓ_g =10-3m, ℓ_i =0.06x 10- 6m, Ai= Ag=0.1 m2, μi=500, μ0=4π x 10-7H/m



$$\Re_i = \frac{l_i}{\mu_i A_i} \quad , \quad \Re_g = \frac{l_g}{\mu_g A_g}$$

$$\Re_i = \frac{0.06\times 10^{-3}}{(4\pi \times 10^{-7} \times 500)0.01} \qquad \Re_g = \frac{10^{-3}}{(4\pi \times 10^{-7})0.01}$$

$$\Re_i = 0.96\times 10^4 \frac{1}{H} \qquad \Re_g = 7.95\times 10^4 \frac{1}{H}$$

$$\boxed{L = \frac{N\Phi}{i} = \frac{N^2}{(\Re_i + \Re_g)}}$$

$$\Re_g > \Re_i$$

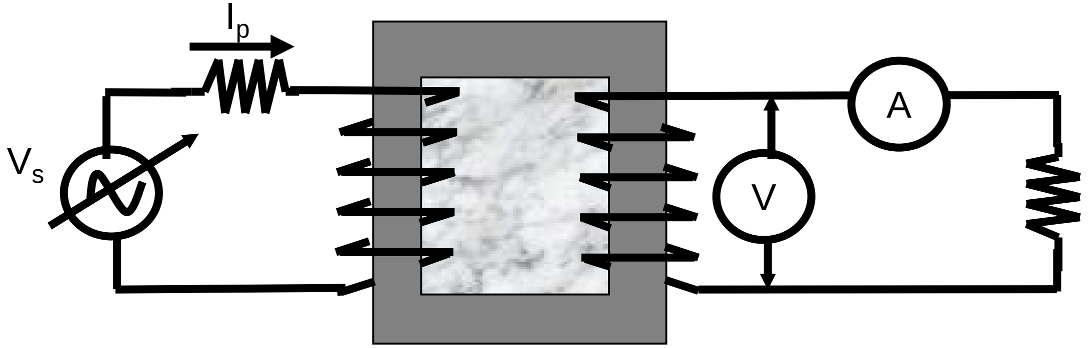
$\ell_g \uparrow$ larger (length of the airgap) $\Rightarrow$ L $\downarrow$ smaller

$A_i \uparrow$ larger (cross sectional area) $\Rightarrow$ L $\uparrow$ *bigger*

$\Re_g \downarrow$ (smaller) and $\Re_i \downarrow$ (*smaller*)

Hysteresis Loop

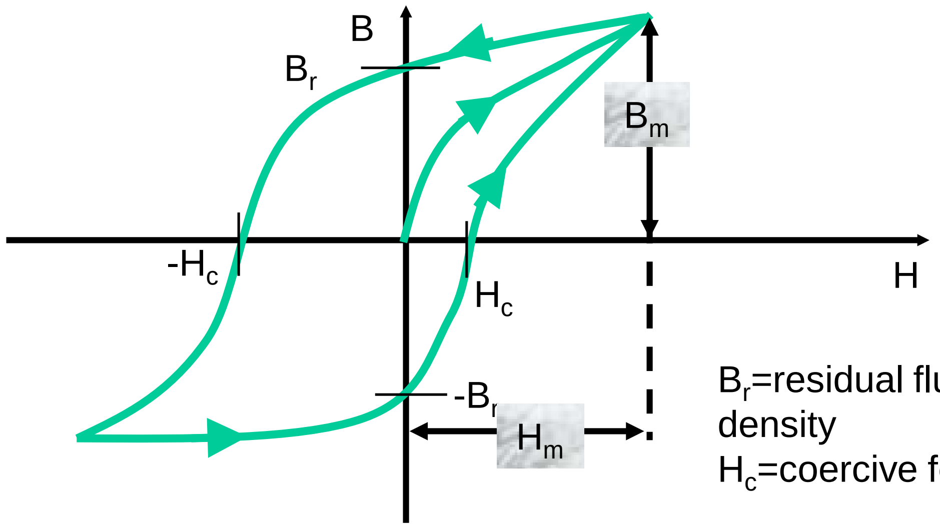
- Consider the following magnetic circuit:



$$NI = H\ell_i \quad , \quad B = \mu H \quad , \quad \Phi = BA$$

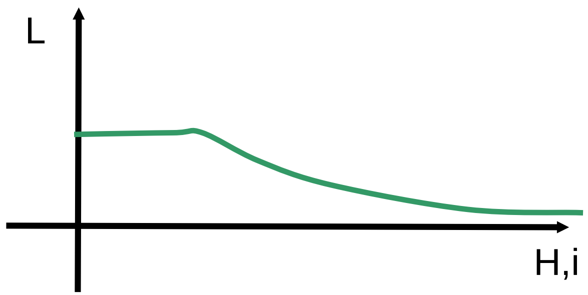
$$\lambda = N\Phi \quad , \quad V_s = \frac{d\lambda}{dt} \Rightarrow \lambda = \int V_s dt$$

- Magnetization curve:



B_r=residual flux density
H_c=coercive force

- Inductance versus current variations



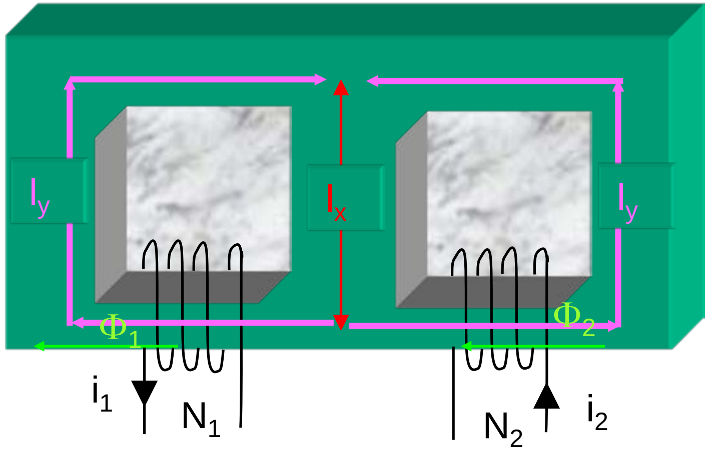
$$L = \mu \frac{N^2 A}{\ell_i}$$

$$L = \frac{N^2}{\Re}$$

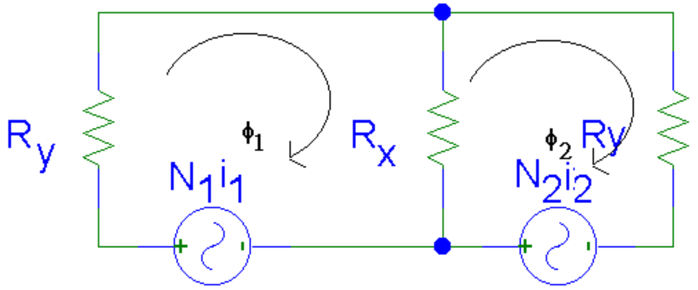
$$\Re = \frac{\ell_i}{\mu A}$$

Magnetic Circuits

- Problem: Consider the magnetic system shown below,



- Compute L_{11} and L_{12}.
- Solution: The magnetic circuit can be modeled as,



- The loop equations are:

$$N_1 i_1 = \Phi_1 \Re_y + \Re_x (\Phi_1 - \Phi_2)$$
$$N_2 i_2 = \Phi_2 \Re_y + \Re_x (\Phi_2 - \Phi_1)$$

- The flux ϕ_1 is given by,

$$\Phi_1 = \frac{(\Re_y + \Re_x) N_1 i_1 - (-\Re_x) N_2 i_2}{(\Re_y + \Re_x)^2 - \Re_x^{\ 2}}$$

- The inductances L_{11} and L_{12} can be found from:

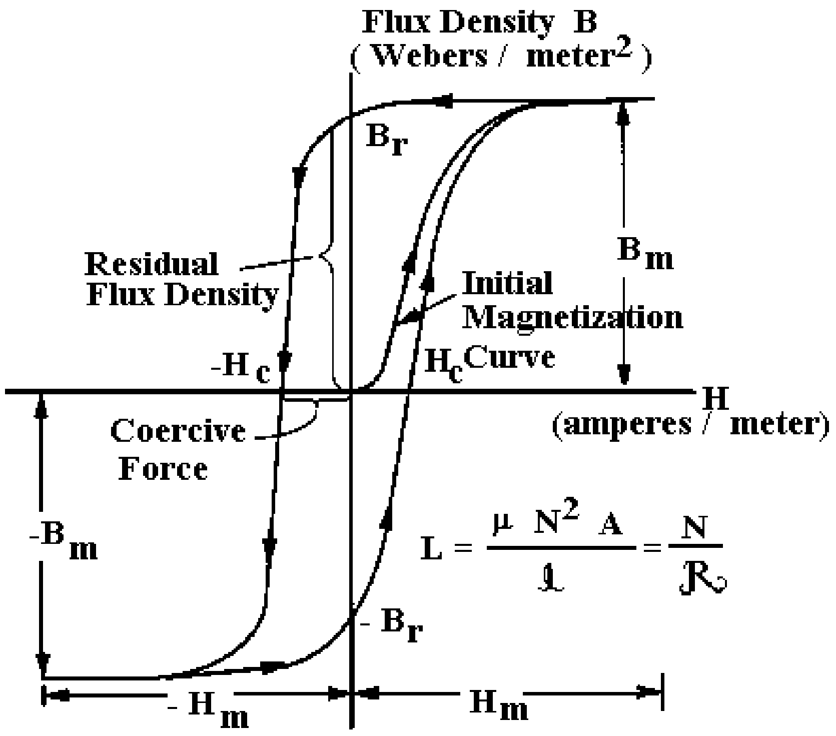
$$\lambda_1 = N_1 \Phi_1$$
$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$
$$L_{11}\big|_{i_2=0} = \frac{(\Re_y + \Re_x) N_1^{\ 2} i_1}{\Re_y^{\ 2} + 2\Re_x \Re_y}$$
$$L_{12}\big|_{i_1=0} = \frac{\Re_x N_2 N_1 i_2}{\Re_y^{\ 2} + 2\Re_x \Re_y}$$

Magnetic Circuits

- Permanent Magnet Materials:

Materials	*Retentivity* Webers / meters^2	*Coercivity* Amp / meter	BH_{max} joules/meter^2
Chrome steel (98 Fe, 0.9 Cr, 0.6 C, 0.4 Mn)	**1.0**	**4 000**	**1 600**
Oxide (57 Fe, 28 O, 15 Co)	**0.2**	**72 000**	**4 800**
Alnico 2 (55 Fe, 12 Co, 17 Ni, 10 Al, 6 Cu)	**0.7**	**44 800**	**13 600**
Platinum cobalt (77 Pt, 23 Co)	**0.4**	**200 000**	**30 400**
Alnico 5 (Alcomax) (51 Fe, 24 Co, 14 Ni, 8 Al, 3 Cu)	**1.25**	**44 000**	**36 000**

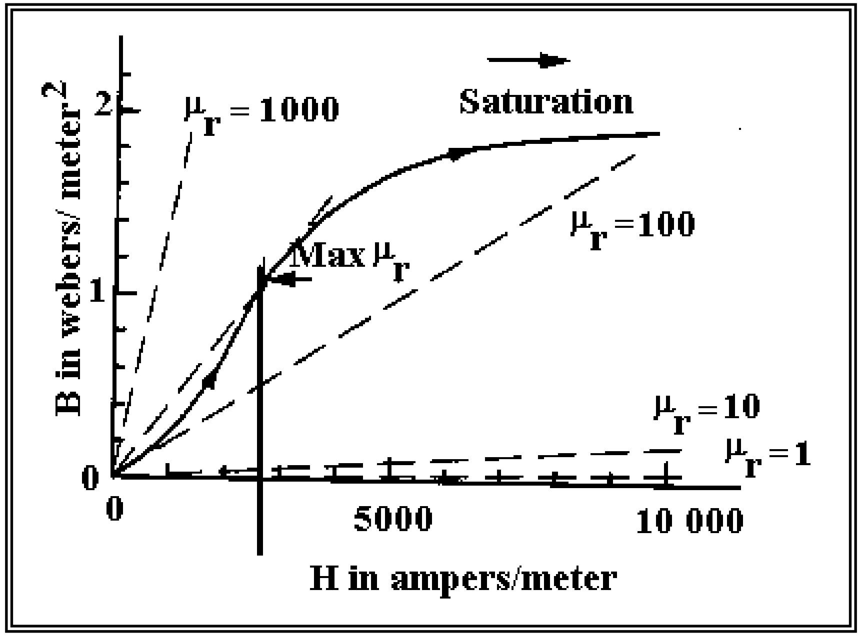
- Hysteresis Loop



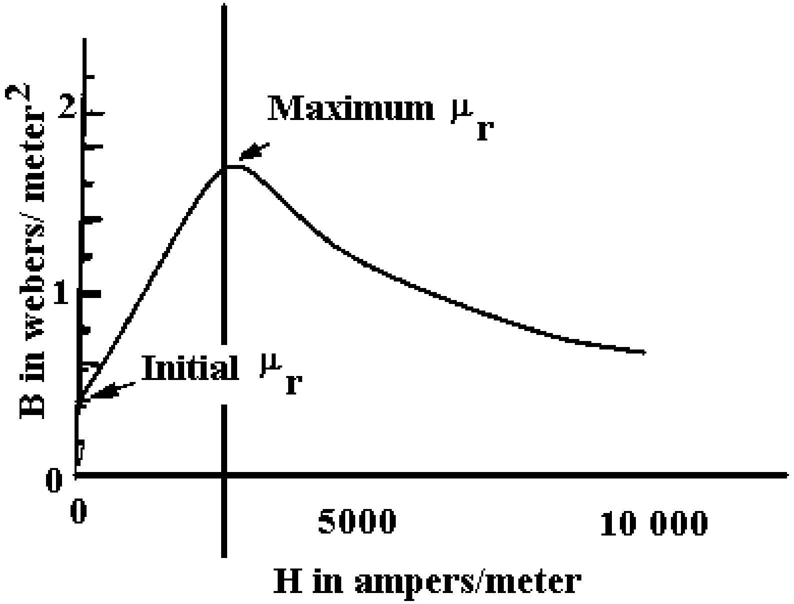
$$L = \frac{\mu N^2 A}{l} = \frac{N}{\mathcal{R}}$$

Magnetic Circuits

- Typical magnetization curve

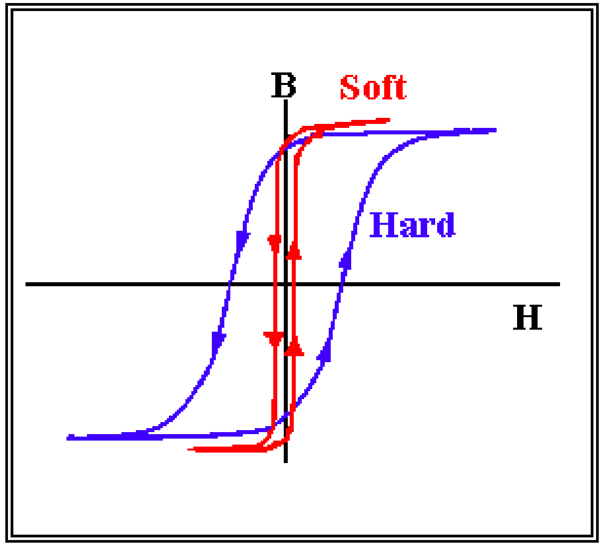


- Corresponding relation of relative permeability to an applied field H

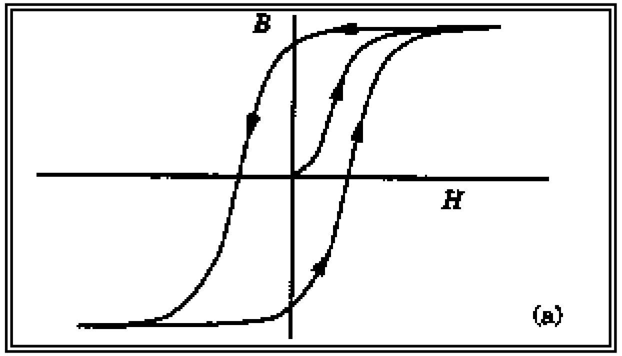


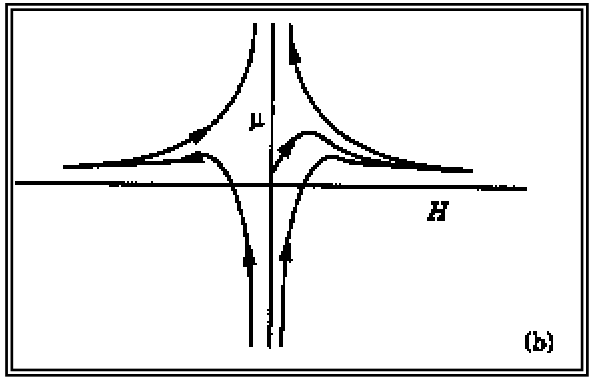
Hysteresis Loops for Soft and Hard Magnetic Materials

- Hysteresis loops for soft and hard (i.e. permanent magnet) magnetic materials



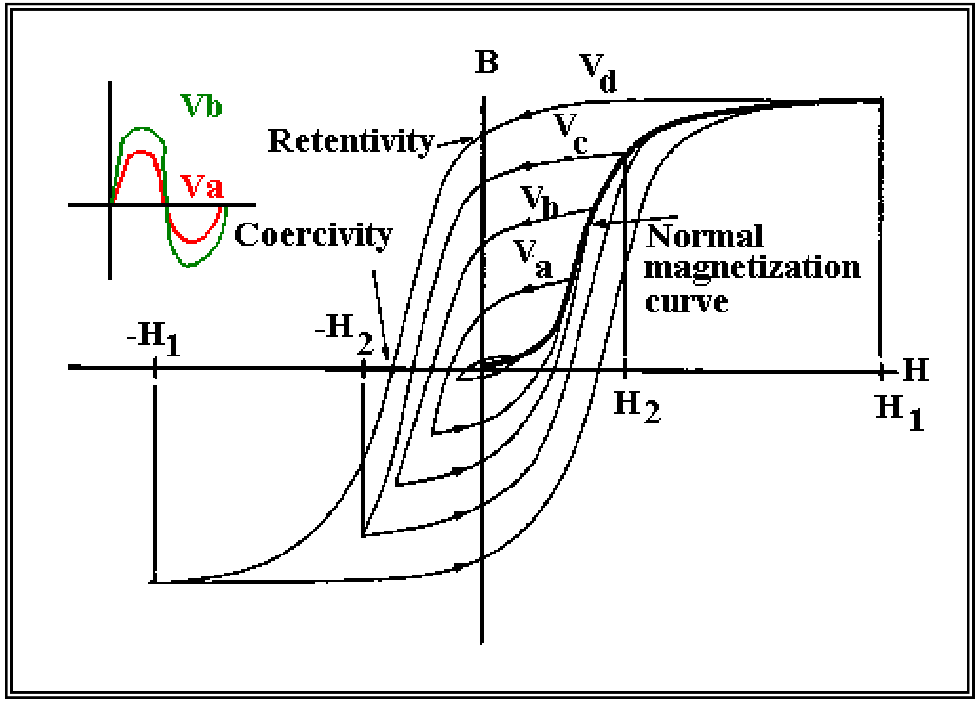
- (a) Hysteresis Loop,
- (b)Corresponding permeability curve





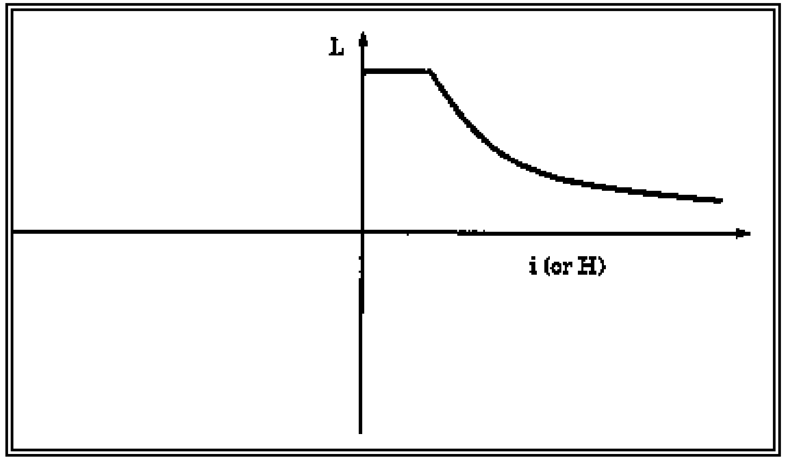
Normal Magnetization Curve

- Normal Magnetization curve with relation to the hysteresis loop



- Definition of inductance:

$$L = \mu \frac{N^2 A}{l} = \frac{N^2}{\Re} \quad ; \quad \Re = \frac{l}{\mu A} \quad ; \quad \mu = \frac{\mathrm{B}}{\mathrm{H}}$$

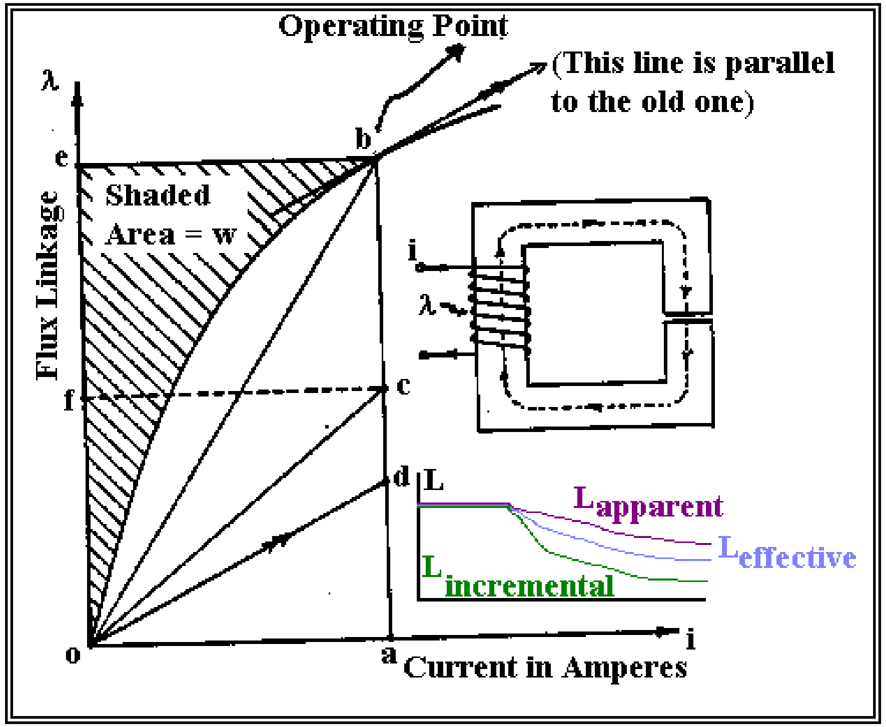


Operating Point of a Given Magnetic Circuit

- Point b represents an operating point of the given magnetic circuit.
- The shaded area *obe* represents the energy W stored in the magnetic field.
- The point c is located on the line *ab* such that

  W = Area *ocf* = Area *obe* (Shaded)

- The slope *ad/oa* equals the slope of the tangent to the magnetization curve at the point b, which represents the rate of change of the flux linkage per ampere, i.e., $L_{incremental}$.
- For unsaturated coils, the three definitions of inductances give the same value, i.e. $L_{app}=L_{eff}=L_{inc}$
- For saturated coils, we will have the following $L_{app}>L_{eff}>L_{inc}$



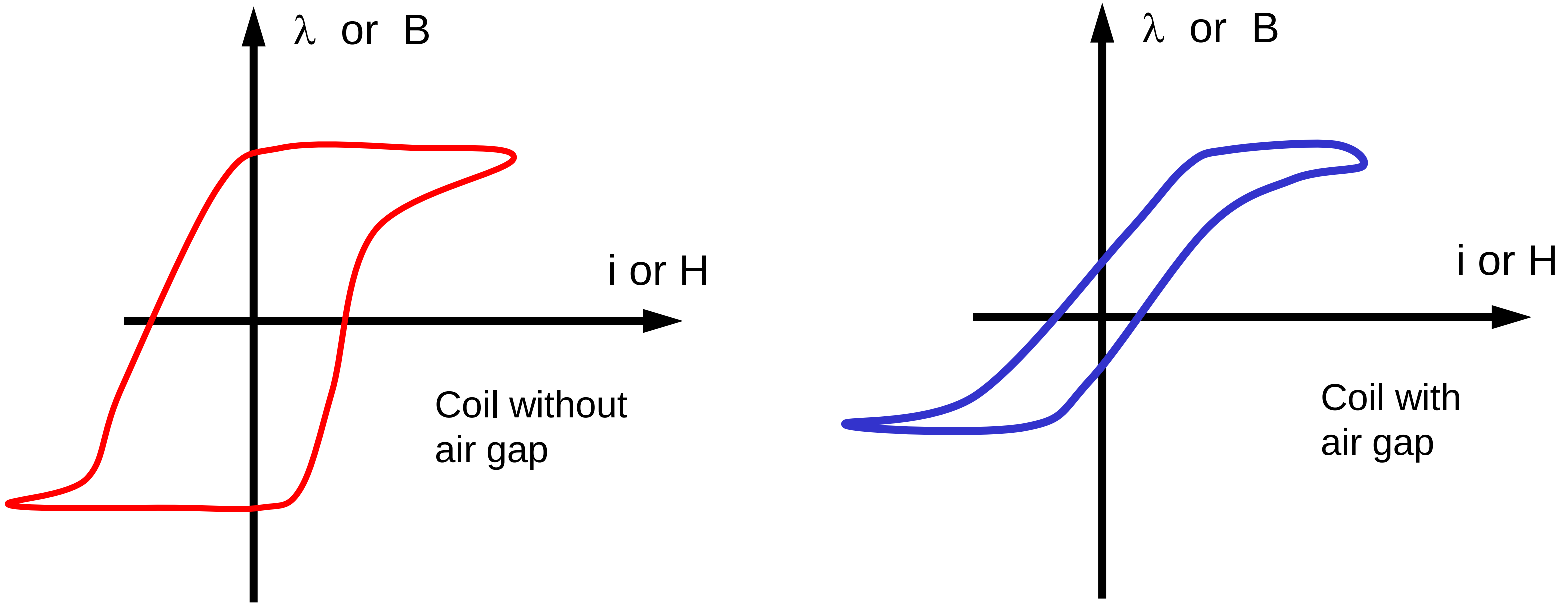
$$1-L_{apparent}=\frac{\lambda}{i}=\frac{ab}{oa}$$

$$2-L_{effective}=\frac{2\omega}{i^2}=\frac{ac}{oa}$$

$$3-L_{incremental}=\frac{\partial\lambda}{\partial i}=\frac{ad}{oa}$$

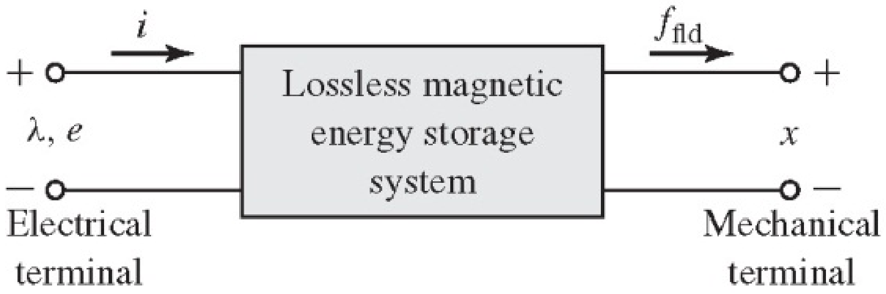
Magnetic Circuits

- When the air gap is introduced in a coil, the hysteresis loop becomes more linear, as shown in the figure below.
- As the air gap of a coil is increased, the linear inductance region of the coil will also increase, and the coil will not saturate for the same excitation current; which would cause saturation if the air gap was not included.

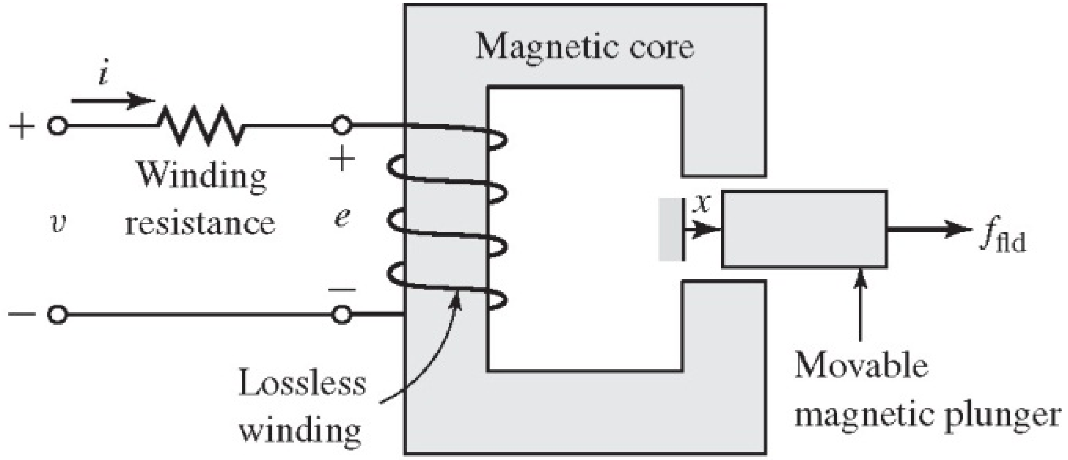


Energy Conversion Process

- The principle of conservation of energy:
  - Energy can neither be created nor destroyed. It can only be changed from one form to another. Therefore total energy in a system is constant
- An electromechanical converter system has three essential parts:
  - An electrical system (electric circuits such as windings)
  - A magnetic system (magnetic field in the magnetic cores and air gaps)
  - A mechanical system (mechanically movable parts such as a rotor in an electrical machine).

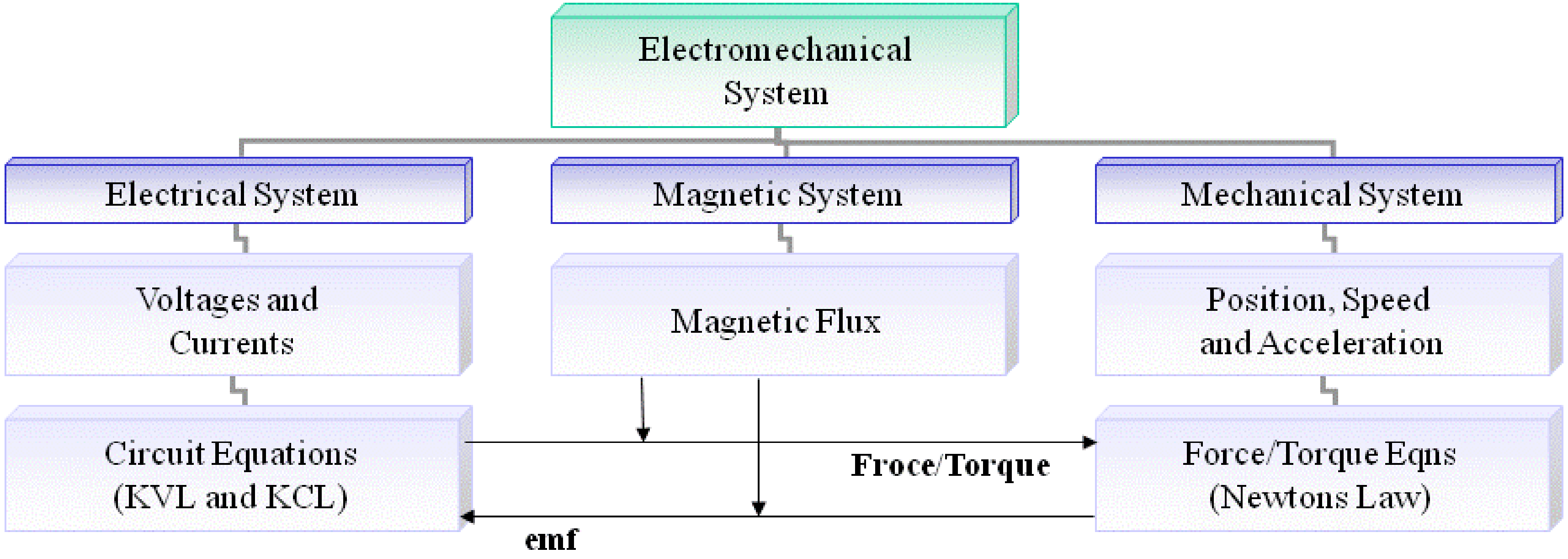


(a)



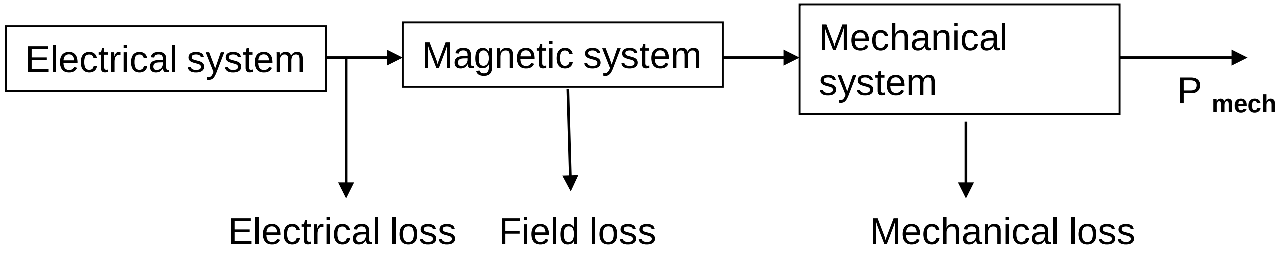
(b)

Energy Conversion Process



Concept of electromechanical system modeling

Energy Conversion Process



The energy transfer equation is as follows:

$$\begin{pmatrix} \text{Energy input} \\ \text{from electric} \\ \text{sources} \end{pmatrix} = \begin{pmatrix} \text{Mechanical} \\ \text{energy} \\ \text{output} \end{pmatrix} + \begin{pmatrix} \text{Increase in} \\ \text{energy stored in} \\ \text{magnetic field} \end{pmatrix} + \begin{pmatrix} \text{Energy} \\ \text{converted} \\ \text{to heat} \end{pmatrix}$$

The energy balance can therefore be written as:

$$\begin{pmatrix} \text{Electrical energy} \\ \text{input from sources} \\ -\text{resistance loss} \end{pmatrix} = \begin{pmatrix} \text{Mechanical energy} \\ \text{output} + \text{friction} \\ \text{and windage loss} \end{pmatrix} + \begin{pmatrix} \text{Increase in} \\ \text{stored field} \\ \text{energy} + \text{core loss} \end{pmatrix}$$

For the lossless magnetic energy storage system in differential form,

$$dW_e = dW_m + dW_{fld}$$

$dW_e = i\,d\lambda$ = differential change in electric energy input

$dW_m = f_m\,dx$ = differential change in mechanical energy output

dW_{fld} = differential change in magnetic stored energy

Energy Conversion Process

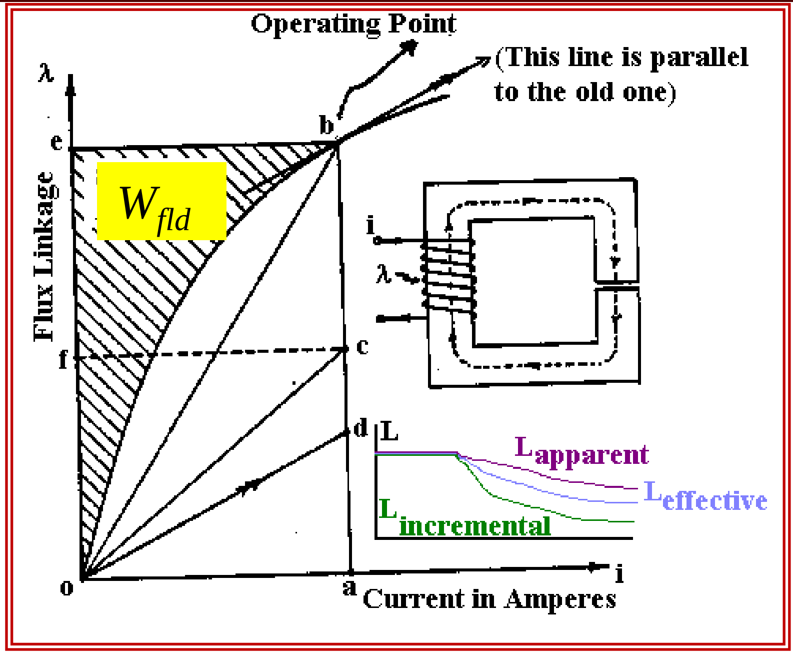
For the lossless magnetic energy storage system in differential form,

$$dW_e = dW_m + dW_{fld}$$

$dW_e = i\,d\lambda$ = differential change in electric energy input

$dW_m = f_{fld}\,dx$ = differential change in mechanical energy output

dW_{fld} = differential change in magnetic stored energy



However,

$$dW_e = ei\,dt \quad ; \quad e = \frac{d\lambda}{dt}$$

e: voltage induced in the electric terminals by changing magnetic stored energy.

thus

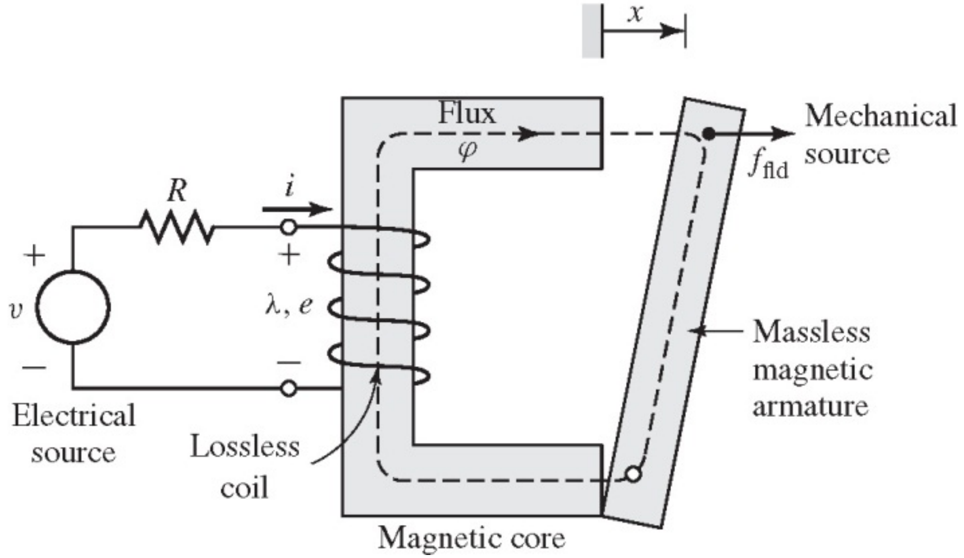
$$dW_e = \frac{d\lambda}{dt} i dt = i d\lambda$$

Together with Faraday's law for induced voltage, form the basis for the energy method.

Singly Excited System
Energy, Coenergy and Force or Torque

Energy in Magnetic Systems

Consider the following electromechanical system:



$$dW_e = dW_m + dW_{fld}$$

The mechanical force f_m is defined as acting from the relay upon the external mechanical system and the differential mechanical energy output of the relay is

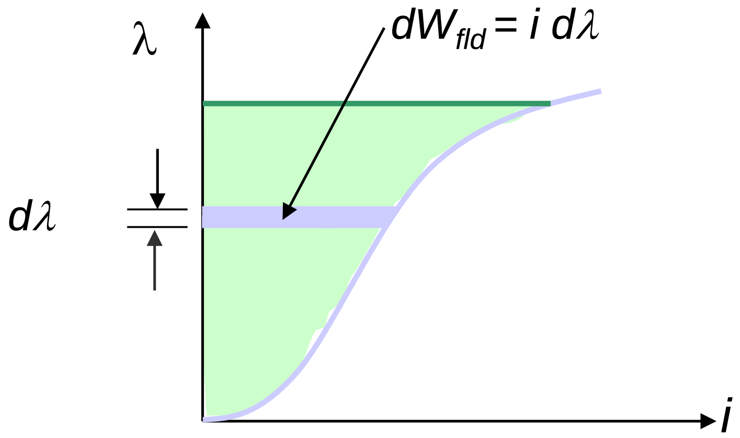
$$dW_m = f_{fld}\, dx$$

Substituting $dW_e = i\, d\lambda$, gives

$$dW_{fld} = i\, d\lambda - f_{fld}\, dx$$

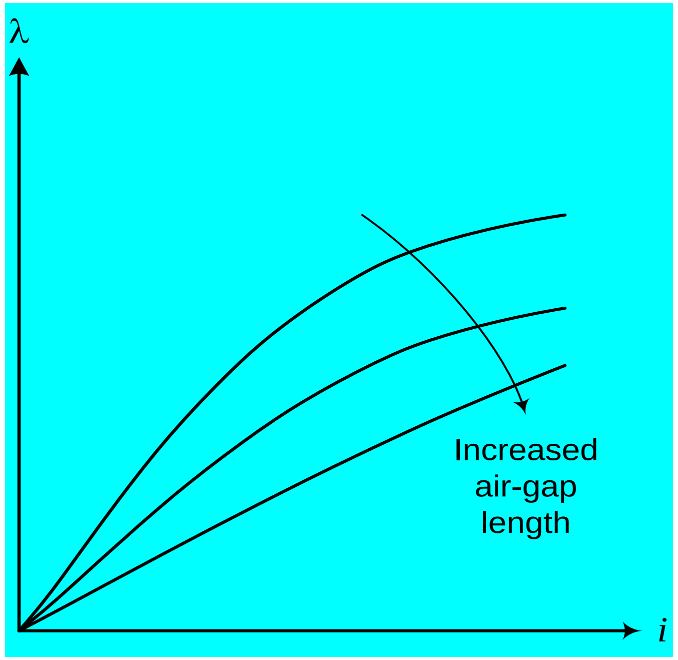
Value of W_{fld} is uniquely specified by the values of λ and x, since the magnetic energy storage system is lossless.

Energy in Magnetic Systems



$$W_{fld} = \int i\, d\lambda$$

dW_{fld} = differential change in magnetic stored energy



The λ-*i* characteristics of an electromagnetic system depends on the air-gap length and B-H characteristics of the magnetic material. For a larger air-gap length the characteristic is essentially linear. The characteristic becomes non linear as the air-gap length decreases.

Energy and Coenergy

- For a particular value of air-gap length, the *field energy* is represented by the red area between λ– axis and λ-*i* characteristic.

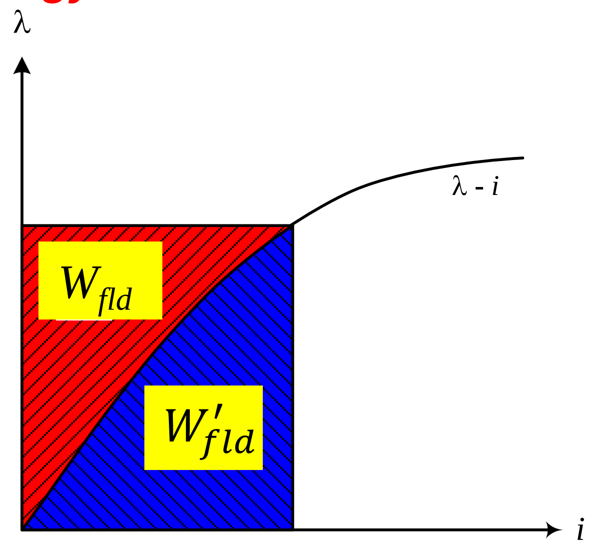
- The blue area between *i*- axis and λ-*i* characteristic is known as the *coenergy.*

- The coenergy is defined as:

$$W'_{fld} = \int_0^i \lambda \, di$$

- From the λ-*i* characteristic,

$$W_{fld} + W_{fld}' = \lambda i$$



Note that $W_{fld}' > W_{fld}$ if the λ - *i* characteristic is non-linear and $W_{fld}' = W_{fld}$ if it is linear.

The quantity of coenergy has no physical significance. However, it can be used to derive expressions for force (torque) developed in an electromagnetic system.

Determination of Force from Energy

- The magnetic stored energy W_{fld} is a state function, determined uniquely by the independent state variables λ and x. This is shown explicitly by, $dW_{fld}(\lambda,x) = id\lambda - f_{fld}\,dx$

- For any function of two independent variables $F(x_1,x_2)$, the total differential equation of F with respect to the two state variables x_1 and x_2 can be written as:

$$dF(x_1,x_2) = \left.\frac{\partial F(x_1,x_2)}{\partial x_1}\right|_{x_2} dx_1 + \left.\frac{\partial F(x_1,x_2)}{\partial x_2}\right|_{x_1} dx_2$$

Therefore, for the total differential of W_{fld}

$$dW_{fld}(\lambda,x) = \frac{\partial W_{fld}(\lambda,x)}{\partial \lambda} d\lambda + \frac{\partial W_{fld}(\lambda,x)}{\partial x} dx$$

However, $dW_{fld}(\lambda,x) = id\lambda - f_{fld}\,dx$

By matching both equations, the current: $i = \dfrac{\partial W_{fld}(\lambda,x)}{\partial \lambda}$

where the partial derivative is taken while holding x constant and the mechanical force:

$$f_{fld} = -\frac{\partial W_{fld}(\lambda,x)}{\partial x}$$

where the partial derivative is taken while holding λ constant.

Determination of Force from Energy: Linear System

For a linear magnetic system for which $\lambda = L(x)\, i$:

$$W_{fld}(\lambda, x) = \int_0^{\lambda} i(\lambda, x)\, d\lambda = \int_0^{\lambda} \frac{\lambda}{L(x)}\, d\lambda = \frac{1}{2}\frac{\lambda^2}{L(x)}$$

and the force, f_{fld} can be found directly:

$$f_{fld} = -\frac{\partial W_{fld}(\lambda, x)}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{1}{2}\frac{\lambda^2}{L(x)}\right] = \frac{\lambda^2}{2L(x)^2}\frac{dL(x)}{dx}$$

For a system with a rotating mechanical terminal, the mechanical terminal variables become the angular displacement θ and the torque *T*. Therefore, the equation for the torque is given by,

$$T = -\frac{\partial W_{fld}(\lambda, \theta)}{\partial \theta}$$

where the partial derivative is taken while holding λ constant.

Determination of Force from Coenergy

The coenergy W_{fld}' is defined as: $W'_{fld}(i,x) = i\lambda - W_{fld}(\lambda, x)$

and the differential coenergy dW_{fld}': $dW'_{fld}(i,x) = d(i\lambda) - W_{fld}(\lambda, x)$

However, $dW_{fld}(\lambda, x) = i\,d\lambda - f_{fld}dx$

By expanding *d(iλ)*: $d(i\lambda) = id\lambda + \lambda di$

So, the differential coenergy dW_{fld}': $dW'_{fld}(i,x) = d(i\lambda) - dW_{fld}(\lambda, x) = id\lambda + \lambda di - \left(id\lambda - f_{fld}dx\right) = \lambda di + f_{fld}dx$

By expanding $dW_{fld}'(i,x)$: $dW'_{fld}(i,x) = \dfrac{\partial W'_{fld}(i,x)}{\partial i}di + \dfrac{\partial W'_{fld}(i,x)}{\partial x}dx$

and, from the previous result: $dW'_{fld}(i,x) = \lambda\,di + f_{fld}\,dx$

Determination of Force from Coenergy

By matching both equations, λ: $\lambda = \dfrac{\partial W'_{fld}(i,x)}{\partial i}$

where the partial derivative is taken while holding x constant and the mechanical force:

$$f_{fld} = \frac{\partial W'_{fld}(i,x)}{\partial x}$$

where the partial derivative is taken while holding *i* constant.

For a linear magnetic system for which $\lambda = L(x)i$:

$$W'_{fld}(i,x) = \int_0^i \lambda(i,x)\, di = \int_0^i L(x) i\, di = L(x)\frac{i^2}{2}$$

and the force, f_{fld} can be found directly:

$$f_{fld} = \frac{\partial W'_{fld}(i,x)}{\partial x} = \frac{\partial}{\partial x}\left[L(x)\frac{i^2}{2}\right] = \frac{i^2}{2}\frac{dL(x)}{dx}$$

Determination of Torque from Coenergy

For a system with a rotating mechanical terminal, the mechanical terminal variables become the angular displacement θ and the torque *T*. Therefore, the torque is given by,

$$T = \frac{\partial W'_{fld}(i, \theta)}{\partial \theta}$$

where the partial derivative is taken while holding i constant.

The selection of energy or coenergy as the function to find the force is purely a matter of convenience. They both give the same result, but one or the other may be simpler analytically, depending on the desired result and characteristics of the system being analyzed. Coenergy is often convenient since current is a more easily measured than flux linkage

Direction of Force Developed

1. By using stored energy function:

$$\boxed{f_{fld} = -\frac{\partial W_{fld}(\lambda, x)}{\partial x}}$$

The negative sign shows that the force acts in a direction to decrease the magnetic field stored energy at constant flux.

2. By using coenergy function:

$$\boxed{f_m = \frac{\partial W'_{fld}(i, x)}{\partial x}}$$

The positive sign emphasizes that the force acts in a direction to increase the coenergy at constant current.

3. By using inductance function:

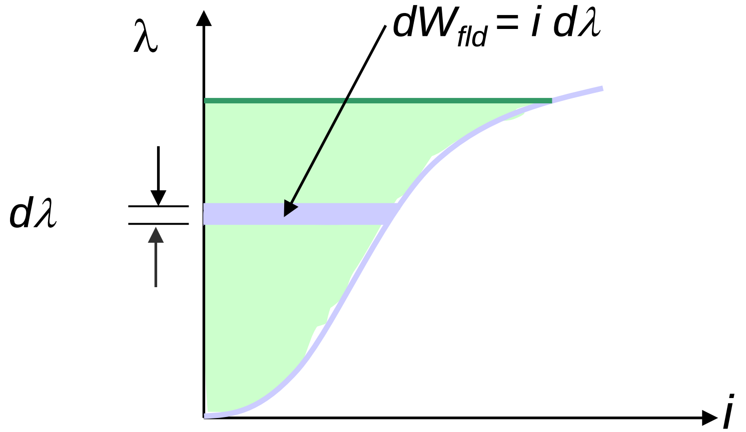
$$\boxed{f_{fld} = \frac{i^2}{2}\frac{dL(x)}{dx}}$$

The positive sign emphasizes that the force acts in a direction to increase the inductance at constant current.

B-H Curve and Energy Density

In a magnetic circuit having a substantial air gap g, and high permeability of the iron core, nearly all the stored energy resides in the gap. Therefore, in most of the cases we just need to consider the energy stored in the gap. The magnetic stored energy,

$$W_{fld} = \int_0^{\lambda} i\, d\lambda$$



in which $i = \dfrac{Hg}{N}$ and $d\lambda = d(N\phi) = d(NAB) = NAdB$

Therefore,

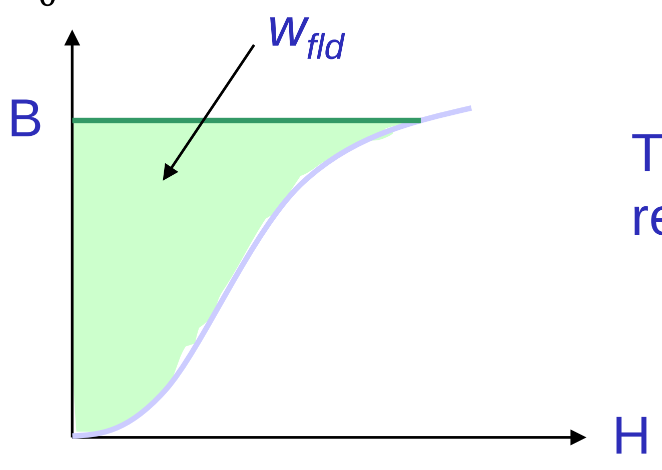
$$W_{fld} = \int_0^{B} \frac{H_g}{N} NA\, dB = A\, g \int_0^{B} H\, dB$$

However, Ag is volume of the air gap. Dividing both sides of the above equation by the volume Ag results in

$$W_{fld} = \frac{W_f}{A\, g} = \int_0^{B} H\, dB$$

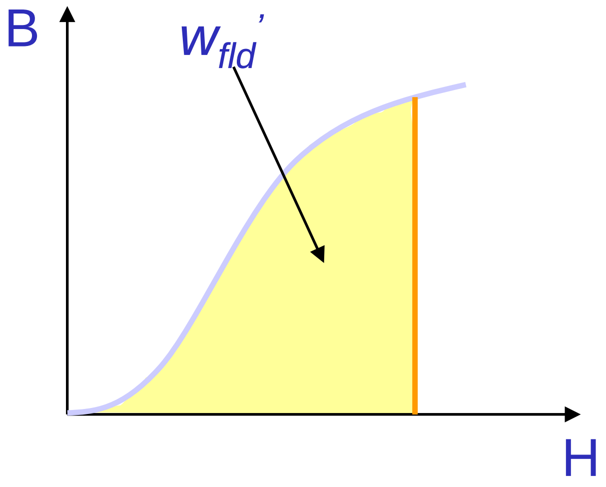
B-H Curve and Energy Density

where $W_{fld} = \int_0^B H\, dB$ is energy per unit volume and w_{fld} is known as energy density.



The area between the B-H curve and B axis represents the energy density in the air gap.

In the same manner, $W'_{fld} = \int_0^H B\, dH$ is coenergy per unit volume.



The area between the B-H curve and H axis represents the coenergy density in the air gap.

B-H Curve and Energy Density

For a linear magnetic circuit, $B = \mu H$ or $H = B/\mu$, energy density:

$$W_{fld} = \int_0^B H\, dB = \int_0^B \frac{B}{\mu} dB = \frac{B^2}{2\mu}$$

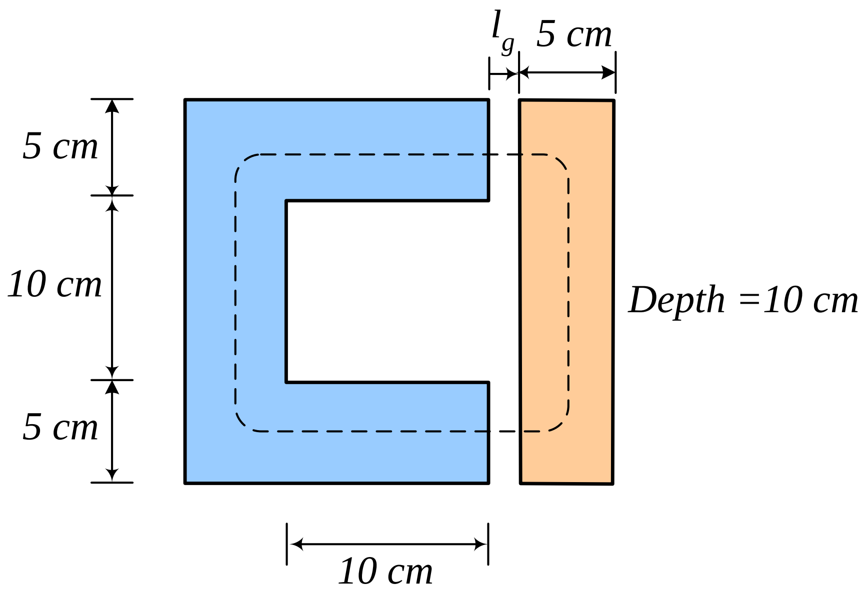
and coenergy density:

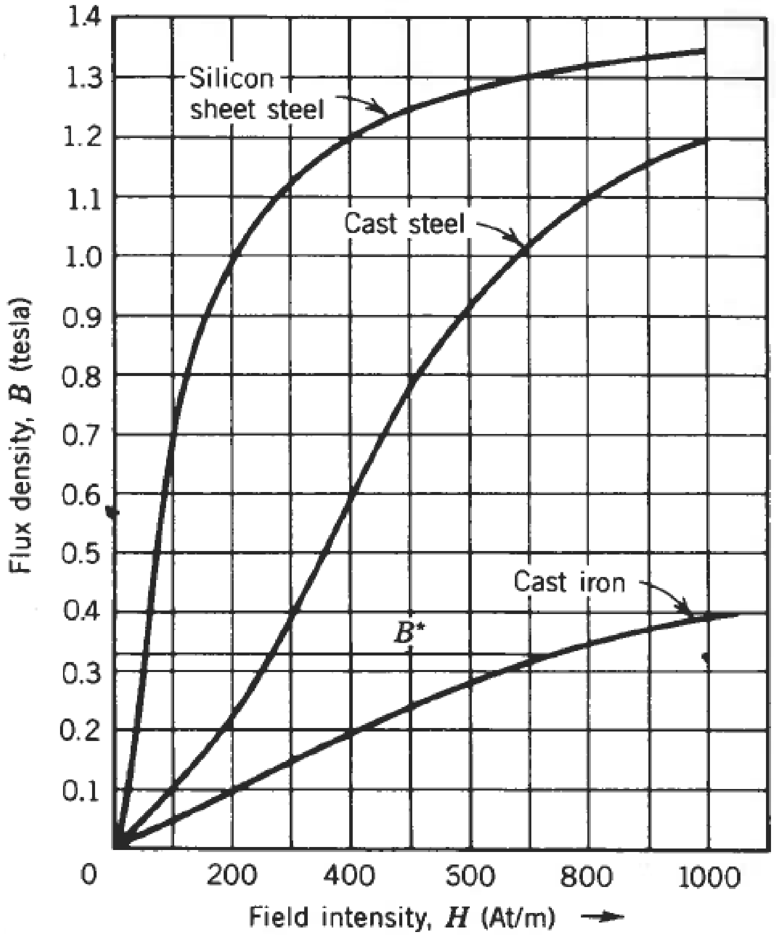
$$W'_{fld} = \int_0^H B\, dH = \int_0^H \mu H dH = \frac{\mu H^2}{2}$$

In this case, it is obvious that $w_{fld} = w_{fld}'$.

Exercise 1

- The dimensions of a relay system are shown in figure below. The magnetic core is made of cast steel whose B-H characteristic is also shown. The coil has 300 turns, and the coil resistance is 6 ohms. For a fixed air-gap length l_g = 4 mm, a dc source is connected to the coil to produce a flux density of 1.1 Tesla in the air-gap. Calculate
  (a) The voltage of the dc source.
  (b) The stored field energy.





Exercise 2

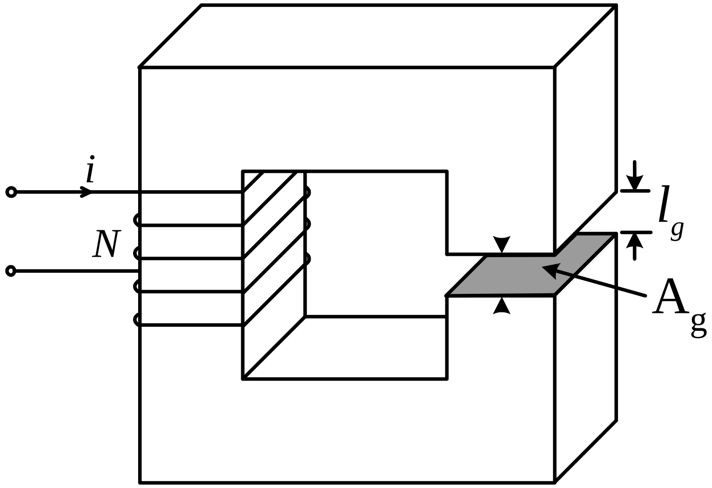
The λ-*i* relationship for an electromagnetic system is given by

$$i = \left(\frac{\lambda g}{0.09}\right)^2$$

which is valid for the limits $0 < i < 4$ A and $3 < g < 10$ cm. For current $i = 3$A and airgap length $g = 5$ cm, find the mechanical force on the moving part using coenergy and energy of the field.

Exercise 3

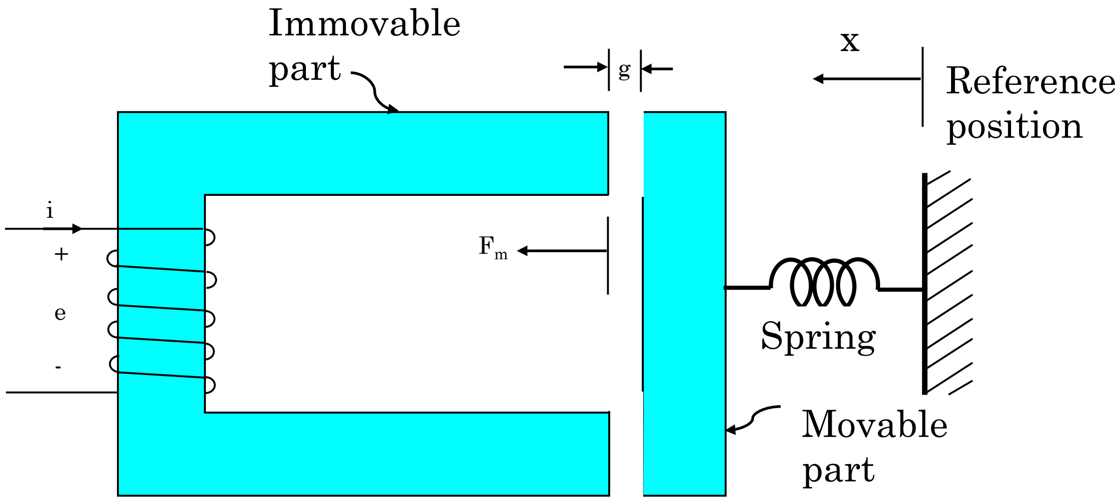
- The magnetic system shown in the figure has the following parameters:
  N = 400, i = 3 A
  Width of air-gap = 2.5 cm
  Depth of air-gap = 2.5 cm
  Length of air-gap = 1.5 mm



- Neglect the reluctance of the core, leakage flux and the fringing flux. Determine:
  (a) The force of attraction between both sides of the air-gap
  (b) The energy stored in the air-gap.
  (c) Coil Inductance

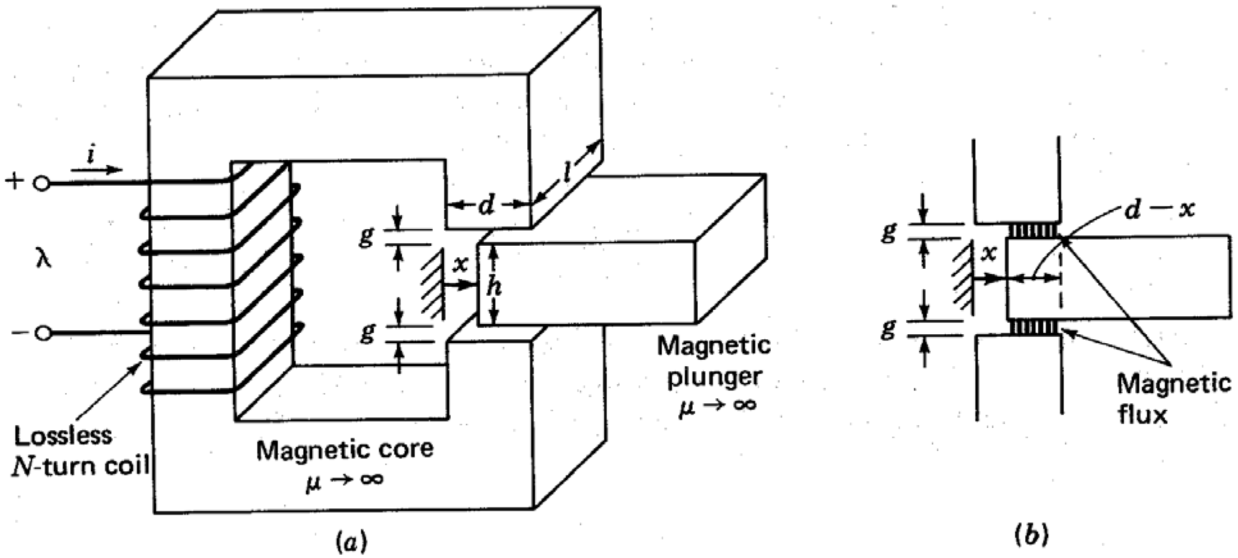
Exercise 4

- The magnetic circuit shown below is made of high permeability steel so that its reluctance can be negligible. The movable part is free to move about an x-axis. The coil has 1000 turns, the area normal to the flux is (5 cm $\times$ 10 cm), and the length of a single air gap is 5 mm.
  - Derive an expression for the inductance, L, as a function of air gap, g.
  - Determine the force, F_m, for the current i =10 A.
  - The maximum flux density in the air gaps is to be limited to approximately 1.0 Tesla to avoid excessive saturation of the steel. Compute the maximum force.



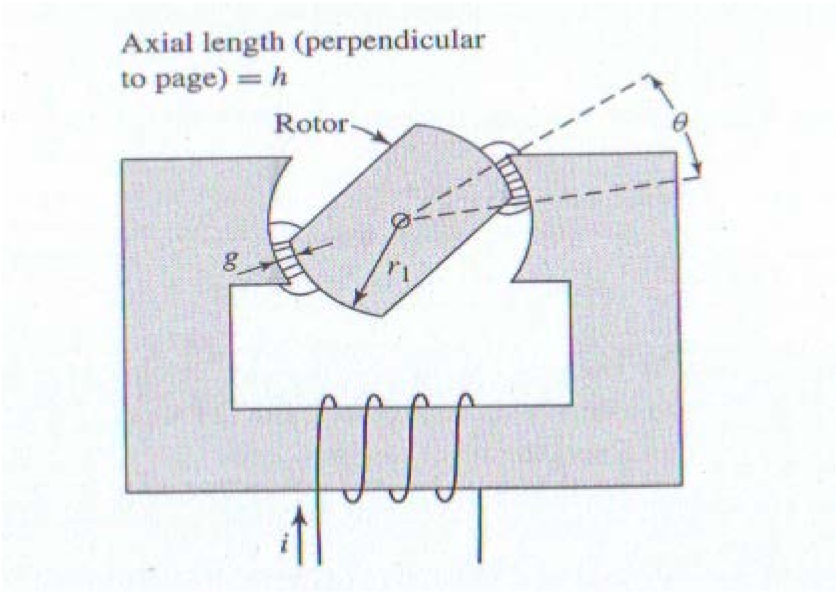
Exercise 5

- Figure below shows a relay made of infinitely-permeable magnetic material with a moveable plunger (infinitely-permeable material). The height of the plunger is much greater than air gap length (h>>g). Calculate
  - The magnetic storage energy W_f as a function of plunger position ($0 < x < d$) for N = 1000 turns, g = 2 mm, d= 0.15 m, ι= 0.1 m and i = 10 A.
  - The generated force, F_m



Exercise 6

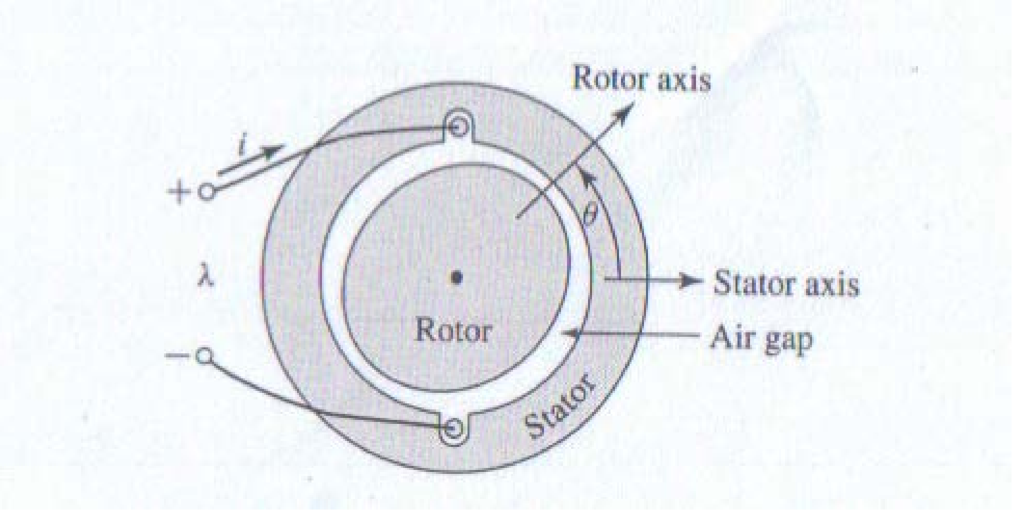
- The magnetic circuit shown is made of high-permeability electrical steel. Assume the reluctance of steel μ -- infinity. Derive the expression for the torque acting on the rotor .



Exercise 7

- The magnetic circuit below consists of a single coil stator and an oval rotor. Because of the air-gap is non uniform, the coil inductance varies with the rotor angular position. Given the coil inductance $L(\theta) = L_o + L_2\cos2\theta$, where L_o= 10.6 mH and L_2= 2.7 mH,

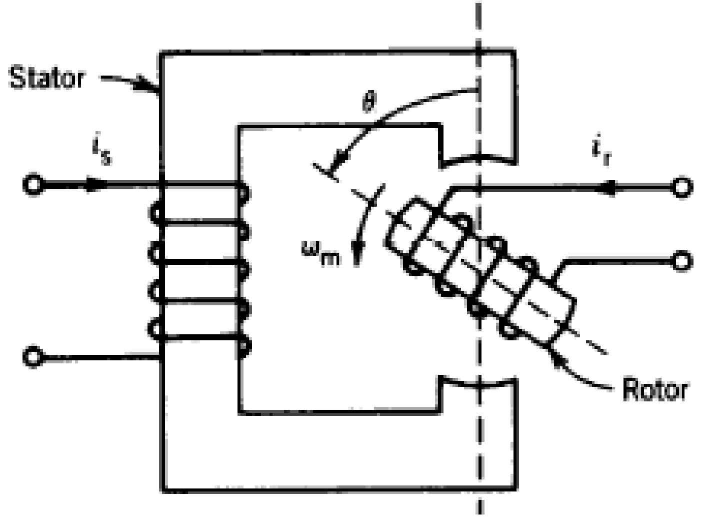
  Find torque as a function of θ for a coil current of 2 A.



Doubly-excited Systems
Energy, Coenergy and Force or Torque

Rotating Machines

- Most of the energy converters, particularly the higher-power ones, produce rotational motion.
- The essential part of a rotating electromagnetic system is shown in the figure.
- The fixed part is called the *stator,* the moving part is called the *rotor.*
- The rotor is mounted on a shaft and is free to rotate between the poles of the stator
- Let's consider general case where both stator & rotor have windings carrying current (i_s and i_r)



Rotating Machines

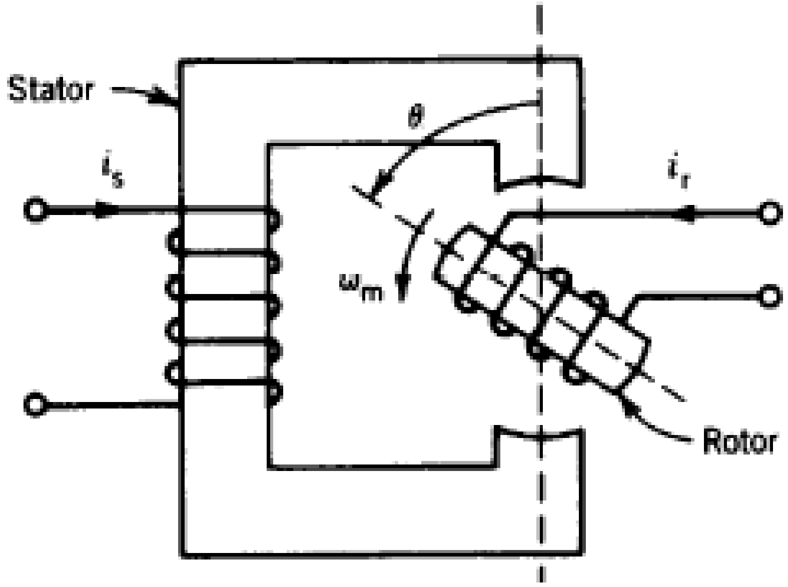
- Assume general case, both stator and rotor have winding carrying currents (non-uniform air gap – silent pole rotor)
- The system stored field energy, W_f can be evaluated by establishing the stator current i_s and the rotor current i_r and let system operate at no-load, i.e. no mechanical output

$$
\begin{aligned}
dW_f &= e_s i_s \, dt + e_r i_r \, dt \\
&= i_s \, d\lambda_s + i_r \, d\lambda_r
\end{aligned}
$$

$$
\begin{aligned}
\lambda_s &= L_{ss} i_s + L_{sr} i_r \\
\lambda_r &= L_{rs} i_s + L_{rr} i_r
\end{aligned}
$$

$$
\begin{vmatrix} \lambda_s \\ \lambda_r \end{vmatrix} = \begin{vmatrix} L_{ss} & L_{sr} \\ L_{sr} & L_{rr} \end{vmatrix} \begin{vmatrix} i_s \\ i_r \end{vmatrix}
$$

Stator and rotor flux linkage λ is expressed in terms of inductances L (which depends on rotor position angle θ, $L(\theta)$)



$$
\begin{aligned}
dW_f &= i_s \, d\left(L_{ss} i_s + L_{sr} i_r\right) + i_r \, d\left(L_{sr} i_s + L_{rr} i_r\right) \\
&= L_{ss} i_s \, di_s + L_{rr} i_r \, di_r + L_{sr} \, d\left(i_s i_r\right)
\end{aligned}
$$

Rotating Machines

- Stored field energy

$$W_f = L_{ss}\int_0^{i_s} i_s\, di_s + L_{rr}\int_0^{i_r} i_r\, di_r + L_{sr}\int_0^{i_s i_r} d(i_s i_r)$$

$$= \frac{1}{2}L_{ss}i_s^2 + \frac{1}{2}L_{rr}i_r^2 + L_{sr}i_s i_r$$

- Torque

$$T = \left.\frac{\partial W_f'(i,\theta)}{\partial \theta}\right|_{i=\text{constant}}$$

In linear system:
coenergy = energy
$W'_f = W_f$

$$T = \frac{1}{2}i_s^2\frac{dL_{ss}}{d\theta} + \frac{1}{2}i_r^2\frac{dL_{rr}}{d\theta} + i_s i_r\frac{dL_{sr}}{d\theta}$$

- First two terms represents reluctance torque; variation of self inductance (exist in both salient stator and rotor, or in either stator or rotor if salient)

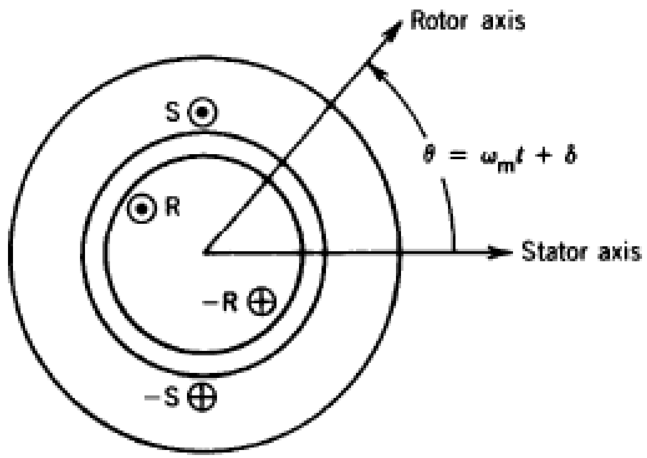
- The third term represents alignment torque; variation of mutual inductance.

Reluctance Torque – It is caused by the tendency of the induced pole to align with excited pole such that the minimum reluctance is produced. At least one or both of the winding must be excited.

Alignment Torque – It is caused by a tendency of the excited rotor to align with excited stator so as to maximize the mutual inductance. Both winding must be excited.

Cylindrical Machines

- A cross sectional view of an elementary two pole cylindrical rotating machine is (uniform air gap) shown.
- The stator and rotor windings are placed on two slots.
- In the actual machine the windings are distributed over several slots.
- If the effects of the slots are neglected, the reluctance of the magnetic path is independent of the position of the rotor.
- Assumed L_{ss} and L_{rr} are constant (i.e. no reluctance torque produced).
- Alignment torque is caused by the tendency of the excited rotor to align with the excited stator, depends on mutual inductance



Cylindrical machines

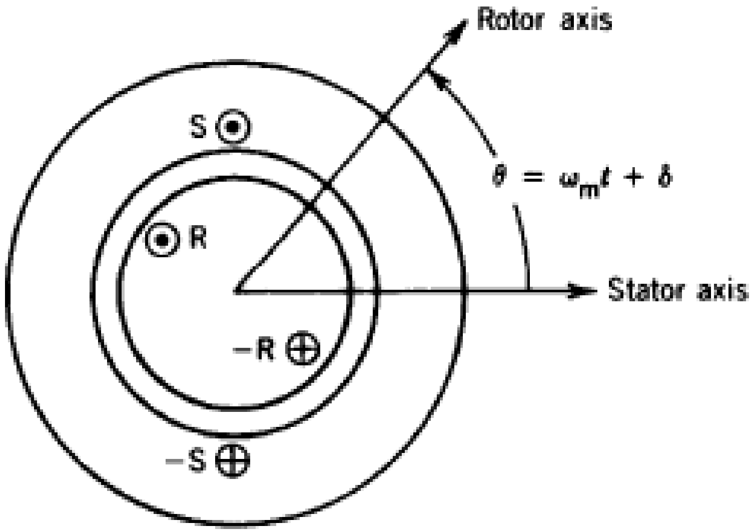
- Torque produced

$$T = i_s i_r \frac{dL_{sr}}{d\theta} = i_s i_r \frac{dM\cos\theta}{d\theta} = M i_s i_r \sin\theta$$

T_{max} when θ=90°

- Mutual inductance

$$L_{sr} = M\cos\theta$$



- Currents

$$i_s = I_{sm}\cos\omega_s t$$

$$i_r = I_{rm}\cos(\omega_r t + \alpha)$$

Where

M = peak value of mutual inductance

θ = the angle between magnetic axis of the stator and rotor windings

ω_m = angular velocity of rotor

- Rotor position

$$\theta = \omega_m t + \delta$$

Cylindrical Machines

$$T = -I_{sm} I_{rm} M \cos\omega_s t \ \cos(\omega_r t + \alpha) \ \sin(\omega_m t + \delta)$$

$$T = -\frac{I_{\mathrm{sm}} I_{\mathrm{rm}} M}{4} \begin{bmatrix} \sin\{(\omega_{\mathrm{m}} + (\omega_{\mathrm{s}} + \omega_{\mathrm{r}}))t + \alpha + \delta\} + \\ \sin\{(\omega_{\mathrm{m}} - (\omega_{\mathrm{s}} + \omega_{\mathrm{r}}))t - \alpha + \delta\} + \\ \sin\{(\omega_{\mathrm{m}} + (\omega_{\mathrm{s}} - \omega_{\mathrm{r}}))t - \alpha + \delta\} + \\ \sin\{(\omega_{\mathrm{m}} - (\omega_{\mathrm{s}} - \omega_{\mathrm{r}}))t + \alpha + \delta\} \end{bmatrix}$$

- Torque in general varies sinusoidally with time
- Average value of each term is zero unless the coefficient of t is zero

Cylindrical Machines

- Non zero average torque exists/develop only if

$$\omega_m = \pm(\omega_s \pm \omega_r) \quad |\omega_m| = |\omega_s \pm \omega_r|$$

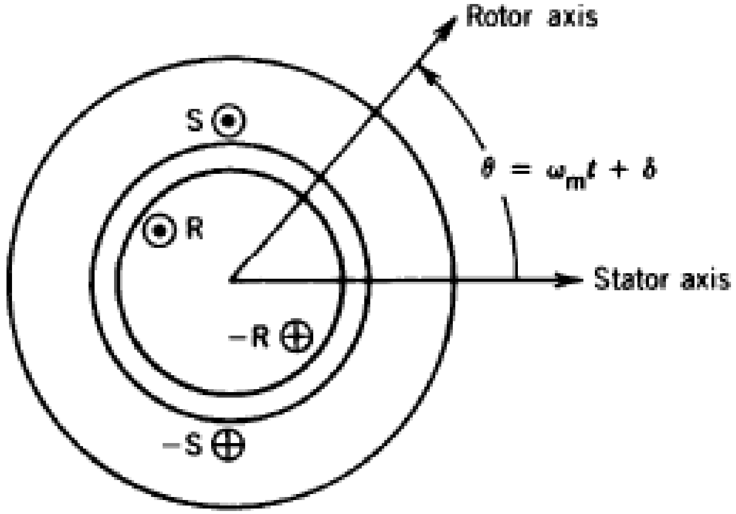
Machine develop torque if sum or difference of the angular speed of the stator and rotor current

Case 1: $\omega_r = 0 \qquad \omega_m = \omega_s \qquad \alpha = 0$

- Synchronous machine $\omega_r = 0$ – I_{dc} in rotor

$$T = -\frac{I_{sm} I_R M}{2}\left\{\sin(2\omega_s t + \delta) + \sin\delta\right\}$$

$$T_{avg} = -\frac{I_{sm} I_R M}{2}\sin\delta$$



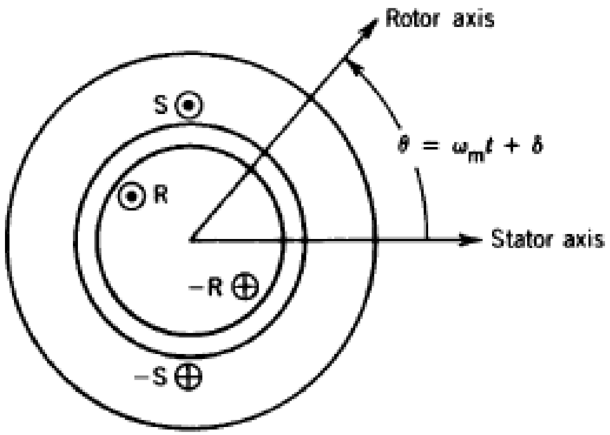
- Single phase machine
  - Pulsating torque
  - Polyphase machine minimize pulsating torque
  - Not self starting ($\omega_m = 0 \rightarrow T_{avg} = 0$

Cylindrical Machines

- Asynchronous machines $\omega_{\mathrm{m}} = \omega_{\mathrm{s}} - \omega_{\mathrm{r}}$ $\omega_{\mathrm{m}} \neq \omega_{\mathrm{r}}$ $\omega_{\mathrm{m}} \neq \omega_{\mathrm{s}}$

$$T = -\frac{I_{\mathrm{sm}} I_{\mathrm{rm}} M}{4} \begin{bmatrix} \sin(2\omega_{\mathrm{s}} t + \alpha + \delta) + \sin(-2\omega_{\mathrm{r}} t - \alpha + \delta) + \\ \sin(2\omega_{\mathrm{s}} t - 2\omega_{\mathrm{r}} t - \alpha + \delta) + \sin(\alpha + \delta) \end{bmatrix}$$

$$T_{\mathrm{avg}} = -\frac{I_{\mathrm{sm}} I_{\mathrm{rm}} M}{4} \sin(\alpha + \delta)$$



- Single phase machine
- Pulsating torque
- Not self starting

- Polyphase machine minimize pulsating torque and self starting

Exercise 8

- In a electromagnetic system, the rotor has no winding (i.e. we have a reluctance motor) and the inductance of the stator as a function of the rotor position θ is

  $L_{ss} = L_0 + L_2 \cos 2\theta$. The stator current is $i_s = I_{sm} \sin \omega t$

(a) Obtain an expression for the torque acting on the rotor

(b) Let $\theta = \omega_m t + \delta$, where ω_m is the angular velocity of the rotor and δ is the rotor position at $t = 0$. Find the condition for the non-zero average torque and obtain the expression for the average torque.

Exercise 9

In a doubly excited rotating actuator shown in figure below, the stator inductances are given as L_{11}= (3+cos2θ) mH, L_{12} = 0.3cosθ, and the rotor inductance is L_{22} = 30+10cos2θ. Find the developed torque in the system for i_1=0.8A and i_2 = 0.01 A.