Channel Coding Theory: Homeworks 09/04/2008

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A. *Exercise A*

A $(5,2)$ linear block code is defined by the following table

u_1	u_2	v_1	v_2	v_3	v_4	v_5
0	0	0	0	0	0	0
0	1	0	1	1	0	1
1	0	1	0	1	1	1
1	1	1	1	0	1	0

1) Find the generator matrix and the parity check matrix of the code.
2) Build the standard array and the decoding table to be used on a BSC.
3) What is the probability of making errors in decoding a codeword assuming an error detection strategy of the decoder?
4) What if we assume error correction capability?

B. *Exercise B*

Optional

Given the $(7,4)$ Hamming code generated by the polynomial

$$g(X) = X^3 + X + 1$$

obtain the code generated by

$$g(X) = (X+1)(X^3 + X + 1).$$

1) How it is related to the original $(7,4)$ code?
2) What is its minimum distance?
3) Show that the new code can correct all single errors and simultaneously detect all double errors.

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C. *Exercise C*

Optional warming up exercise. Recommended before Exercise D. Note it is not compulsory.

Draw the FSFG for the Markov chain

$$p_{XYZ}(x, y, z) = p_X(x)p_{Y|X}(y|x)p_{Z|Y}(z|y).$$

Compute the messages at each node to compute all marginals.

D. *Exercise D*

Consider a quantizer. Let $\mathcal{X}$ be the finite input alphabet and $\mathcal{Y}$ be the finite output alphabet. Let q be the quantization function $q : \mathcal{X} \rightarrow \mathcal{Y}$. Draw the corresponding FSFG. Starting from the general message passing rule and assuming that the incoming messages are $\mu_{xq}(x)$ and $\mu_{yq}(y)$, respectively what are the outgoing messages $\mu_{qx}(x)$ and $\mu_{qy}(y)$?