Channel Coding Theory: Homeworks 30/05/2008

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A. *Exercise A*

Consider the convolutional code defined by the following parameters

Memory	$\nu_\nu = 2$
Rate	$1/n = 1/4$
Generators (octal notation)	(5,5,7,7).

- Write the state diagram of the convolutional code.
- Draw the corresponding encoder.
- Determine the output to the following input sequence 101101100.
- Compute the Input Output Weight Enumerating Function (IO-WEF) making use of the transfer function method.
- Determine the free distance d_{free}.

B. *Exercise B*

Consider the communication system shown in Figure 1.

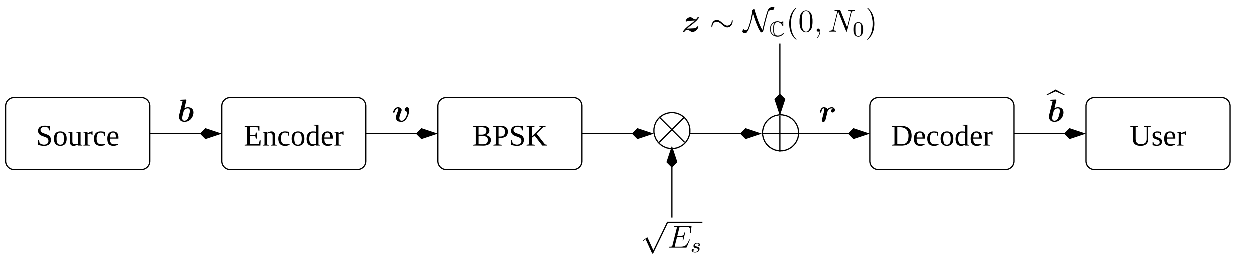

Fig. 1. Communication system

A source generates sequences of information bits $b[i]$ which are encoded with a binary convolutional code. The parameters of the convolutional codes are defined in Table I.

The encoded output sequence $\boldsymbol{v}[i]$, is sent to a BPSK modulator with mapping rule $\mu : \{0, 1\} \rightarrow \{-1, +1\}$ and then transmitted through an AWGN channel with power spectral density N_0. Let $\boldsymbol{r}[i] = \sqrt{E_s}\mu(\boldsymbol{v}[i]) +$

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Memory	Rate	Generators
ν_ν	$1/n$	(octal notation)
2	1/3	(1,5,7)

TABLE I

CODE PARAMETERS

$\boldsymbol{z}[i]$ be the received sequence where $\boldsymbol{z}[i] = [z_1[i], z_2[i], \ldots z_n[i]]$ is the noise with $z_j[i], j = 1, \ldots n$, i.i.d. and $z_j[i] \sim \mathcal{N}(0, N_0)$. The received sequence $\boldsymbol{r}$ is decoded by a Viterbi algorithm. The decoder provides the user with the decoded information sequence $\widehat{\boldsymbol{b}}$.

- Write the state diagram of the convolutional code.
- Consider the transmission of the $K = 3$ bits (101) and assume that $E_s = 1$, $N_0 = 1$, and the received sequence is $\boldsymbol{r} = (0.5674, -0.6656, 1.1253, -0.7123, -2.1465, 2.1909, 2.1892, 0.9624, -0.6727, -0.8254, -1.1867, 1.7258, -1.5883, 3.1832, 0.8636)$. Draw the trellis of the code. At each stage and for each state determine the survivor path and the corresponding metric. The Viterbi decoder decodes correctly the transmitted codeword?
- Compute the bounds on the BER via the symbolic transfer function method described below.

a) Numerical Computation of the Bit Error Probability Bound: In order to compute the upper bound on the bit error probability $P_b(E)$ it is necessary to determine the IOWEF $A(W, D)$. Let us adopt the same approach as in the course handouts. Let $\boldsymbol{x}(W, D)$ be the vector of the nonzero states and $\boldsymbol{x}_0(W, D)$ be the vector of the inputs. As shown in the handout it is possible to write the linear system

$$\boldsymbol{x}(W, D) = \boldsymbol{T}(W, D)\boldsymbol{x}(W, D) + \boldsymbol{x}_0(W, D)$$

and determine the nonzero states as solution of the system. The output-zero state can be reached from the nonzero states through the relation $\boldsymbol{C}(W, D)\boldsymbol{x}(W, D)$ where $\boldsymbol{C}(W, D)$ is row vector whose j-th element is the label of the transition in the modified graph that brings from the j-th nonzero state to the output zero state. Then,

$$A(W, D) = \boldsymbol{C}(W, D)(\boldsymbol{I} - \boldsymbol{T}(W, D))^{-1}\boldsymbol{x}_0.$$

The bit error probabilities $P_b(E)$ is bounded by

$$P_b(E) \leq \frac{1}{k}\frac{\partial}{\partial D}A(D, W)|_{D=\mathrm{e}^{-\frac{RE_b}{N_0}}, W=1}$$

In order to calculate the union bound avoiding the symbolic inversion of the matrix $(\boldsymbol{I} - \boldsymbol{T}(W, D))$ we can approximate the derivative of $A(D, W)$ with respect to D in $D = 1$ by the incremental ratio

$$\frac{\partial}{\partial W}A(W, D)|_{W=1, D=\mathrm{e}^{-\frac{RE_b}{N_0}}} \approx \frac{1}{\epsilon}\left(A(1+\epsilon, \mathrm{e}^{-\frac{RE_b}{N_0}}) - A(1, \mathrm{e}^{-\frac{RE_b}{N_0}})\right)$$

where ϵ is a small number (e.g. $\epsilon = 0.01$).

In any case, either if $\frac{\partial}{\partial W} A(W, D)\Big|_{W=\mathrm{e}^{-\frac{RE_b}{N_0}}, D=1}$ is evaluated in a symbolic form or by the above numerical method, since $A(W, D)$ is a power-series, it is meaningful only when it is evaluated inside the region of convergence. It turns out that for $\frac{E_b}{N_0}$ smaller than a certain threshold the bound on $P_b(E)$ assumes values which may be greater than 1 or negative. In these case we are outside the convergence domain and we shall replace the bound on the BER by the value $\frac{1}{2}$ which is the trivial bound obtained by deciding randomly on every information bit by disregarding the decoder outcome.