Channel Coding Theory — Homework I

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Strict deadline: 16/04/2012 at 18:30

Deliver your homework in Cottatellucci's pigeon hole located in level -2 in front of the offices 2-03, 2-5, and 2-07.

The solutions must be handwritten and thoroughly justified.

An exercise with final result but without discussion of the solving procedure it is considered as not solved.

For Exercise 3, the plots can be printed but the approach to derive them has to be detailed.

Advices: If you consider necessary to make some assumption, state it and develop the solution consistently. Read carefully the provided hints.

Exercise 1

Let S denote the memoryless binary erasure channel with input alphabet $F = \{0, 1\}$ and output alphabet $\Phi = F \bigcup \{\Delta\}$, and erasure probability $p = 0.1$. A codeword of the binary $(4, 3)$ parity check code $\mathcal{C}$ is transmitted through S and the following decoder $\mathcal{D} : \Phi^4 \to \mathcal{C} \bigcup \{\boldsymbol{\Delta}\}$ is applied to the received word

$$\mathcal{D}(\mathbf{y}) = \begin{cases} \mathbf{c}, & \text{if } \mathbf{y} \text{ agrees with exactly one } \mathbf{c} \in \mathcal{C} \text{ on the entries in F;} \\ \boldsymbol{\Delta}, & \text{otherwise.} \end{cases}$$

1) Compute the probability that $\mathcal{D}$ produces $\boldsymbol{\Delta}$.
2) Does this probability depend on which codeword is transmitted?
3) Compute the probability that $\mathcal{D}$ produces $\boldsymbol{\Delta}$ when instead of using the parity check code the code $\mathcal{C} = \{0000, 0111, 1011, 1101\}$ is adopted. Does this probability depend on which codeword is transmitted?

Hints: (1) For the first two items you may make use of the properties of the parity check code that we studied. (2) For the third item, observe that the code is not linear and you need to make specific observations on the structure of the codewords.

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Exercise 2

The polynomial $X^{15}+1$ when factored yields

$$X^{15}+1=(X^4+X^3+1)(X^4+X^3+X^2+X+1)(X^4+X+1)(X^2+X+1)(X+1).$$

Consider the generator polynomial

$$g(X)=(X^4+X^3+X^2+X+1)(X^4+X+1)(X^2+X+1).$$

1) Write the generator matrix of a systematic (15,5) code with generator polynomial $g(X)$.
2) Is the polynomial $X^{13}+X^{12}+X^9+X^8+X^7+X^4+X^2+1$ a code polynomial? What is its syndrome?
3) Determine the weight distribution of the code. What is the minimum distance of the code?
4) Assume that the code is used for transmission over a binary symmetric channel with error probability p. Determine the probability that transmission errors are not detected.
5) Assume that the code is used for transmission over a BIAWGN channel. The symbols are transmitted with energy E and the additive white Gaussian noise channel has variance σ^2. Provide an upper bound of the error correction probability.
6) Consider the burst correction capabilities of the code. How many error bursts of length ℓ, with $\ell = 1,\dots 15$ can be detected?

Hints: (1) For item 3, it is not required to list all the codewords. Properties of the cyclic codes can be efficiently applied to determine the weight distribution of the code.

Exercise 3

Consider the communication system shown in Figure 1.

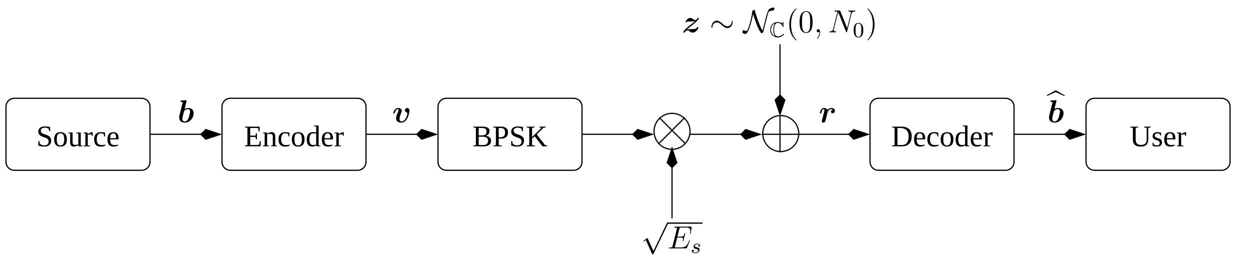

Fig. 1. Communication system

A source generates sequences of $K=5$ information bits $b[i]$ which are encoded with the binary cyclic linear code in Exercise 2.

The encoded output sequence $\boldsymbol{v}[i]$ is sent to a BPSK modulator with mapping rule $\mu : \{0,1\} \to \{-1,+1\}$ and then transmitted through an AWGN channel with power spectral density N_0. Let $\boldsymbol{r}[i] = \sqrt{E_s}\mu(\boldsymbol{v}[i]) +$

$\boldsymbol{z}[i]$ be the received sequence where $\boldsymbol{z}[i] = [z_1[i], z_2[i], \dots z_n[i]]$ is the noise with $z_j[i], j = 1, \dots n$, i.i.d. and $z_j[i] \sim \mathcal{N}(0, N_0)$. The received sequence $\boldsymbol{r}$ is the input of a decoder. The decoder provides the user with the decoded information sequence $\widehat{\boldsymbol{b}}$.

- Plot the capacity of the BIAWGN channel as a function of [1] $\frac{E_b}{N_0}$ (expressed in dB) in the range $\frac{E_b}{N_0}|_{dB} \in [-3, 12]$.
- Determine graphically the value $\frac{E_b}{N_0}$ (expressed in dB) for which the channel has capacity equal to the code rate.
- On the same figure, plot the undetected error probability of an error detection decoder versus $\frac{E_b}{N_0}$ (expressed in dB) in the range $\frac{E_b}{N_0}|_{dB} \in [-3, 12]$ for the proposed encoder.

[1] Not $\frac{E_s}{N_0}$! Note that $\frac{E_s}{N_0}$ and $\frac{E_b}{N_0}$ are related by $\frac{E_b}{N_0} = \frac{1}{R}\frac{E_s}{N_0}$ where R is the rate of the code, for a modulation set of cardinality 2.