Channel Coding Theory: Homeworks 23/04/2008

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A. *Exercise A*

Given the irregular LDPC code with the variable-nodes specified in Table I and the check-nodes specified in Table II:

Degree of variable nodes	Number of variable nodes
2	4448
3	2969
5	676
6	100
7	913
8	99
10	100
12	99
18	100
19	99
30	397

TABLE I

SPECIFICATION OF THE VARIABLE NODES

Degree of check nodes	Number of check nodes
9	1345
10	3464

TABLE II

SPECIFICATION OF THE CHECK NODES

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- Provide the variable and check degree distributions from an edge perspective (λ, ρ).
- Assume that the code is utilized for transmission through a BEC with erasure probability ε. Provide a recursive expression of the erasure probability after k iterations averaged over the standard LDPC ensemble and all possible outputs.
- Plot the density evolution for $\varepsilon = 0.3; 0.57; 0.9$.
- Is it possible to determine all the erasures as $\varepsilon = 0.5$?

B. *Exercise B*

A. Given the LDPC ensemble $(\Lambda(x) = 10x^4, P(x) = 6x^6)$ determine the **design rate**.

B. Consider the realization of the previous LDPC ensemble corresponding to the permutation $\sigma = (24, 31, 8, 30, 25, 15, 27, 21, 39, 26, 2, 29, 22, 33, 19, 36, 10, 16, 7, 35, 4, 38, 28, 3, 6, 40, 14, 32, 11, 12, 23, 9, 37, 5, 20, 18, 13, 34, 17, 1)$ and draw the Tanner graph (clearly otherwise I cannot check it!!!).

C. Determine the generator matrix $\boldsymbol{G}$ and determine the **rate** of the corresponding code.

D. Consider the Tanner graph associated to the generator matrix $\boldsymbol{G}$ (Tanner graph without parallel edges) and determine the variable and node distribution from an edge perspective $\lambda(x)$ and $\rho(x)$.

D. Determine a stopping set with 2 or 3 variable nodes if there is one.

E. Given the LDPC ensemble $(\lambda(x), \rho(x), n)$ determine the **design rate** and the **rate** as $n \to +\infty$ (assume that the conditions on the asymptotic relation between rate and design rate are satisfied).

F. Determine the density evolution function.

C. *Exercise C*

Show that

$$g(X) = 1 + X^2 + X^4 + X^6 + X^7 + X^{10}$$

generates a $(21, 11)$ cyclic code. Devise a syndrome circuit for this code. Let $r(X) = 1 + X^2 + X^{17}$ be a received polynomial. Compute the syndrome of $r(X)$. Display the content of the syndrome circuit after each digit of r has been shifted into the syndrome computation circuit.