Channel Coding Theory

Exam: 06/09/2007

Advices: If you consider necessary to make some assumption, state it and develop the solution consistently. Begin with the easiest questions. Read the provided hints!

Good Luck!

A. *Exercise 1*

Given the (21,16) cyclic code $\mathcal{C}$ with generator polynomial $g(x) = x^5 + x^4 + 1$ determine:

1) The generator polynomial of the dual code.
2) The WEF of the dual code.
3) The WEF of the original code.
4) Find the fraction of undetectable bursts of length 10 in $\mathcal{C}$ and motivate the result.

Hints: In order to determine the WEF work on the WEF of the dual code. Take into account the fundament properties of linear cyclic codes. Take into account that the data words in polynomial notation $a_1(x) = 1$, $a_2(x) = 1 + x + x^2$, *and* $a_3(x) = 1 + x^2 + x^3$ *are encoded by the dual code into 3 codewords that are* ***not*** *one the cyclic shifted version of the others.*

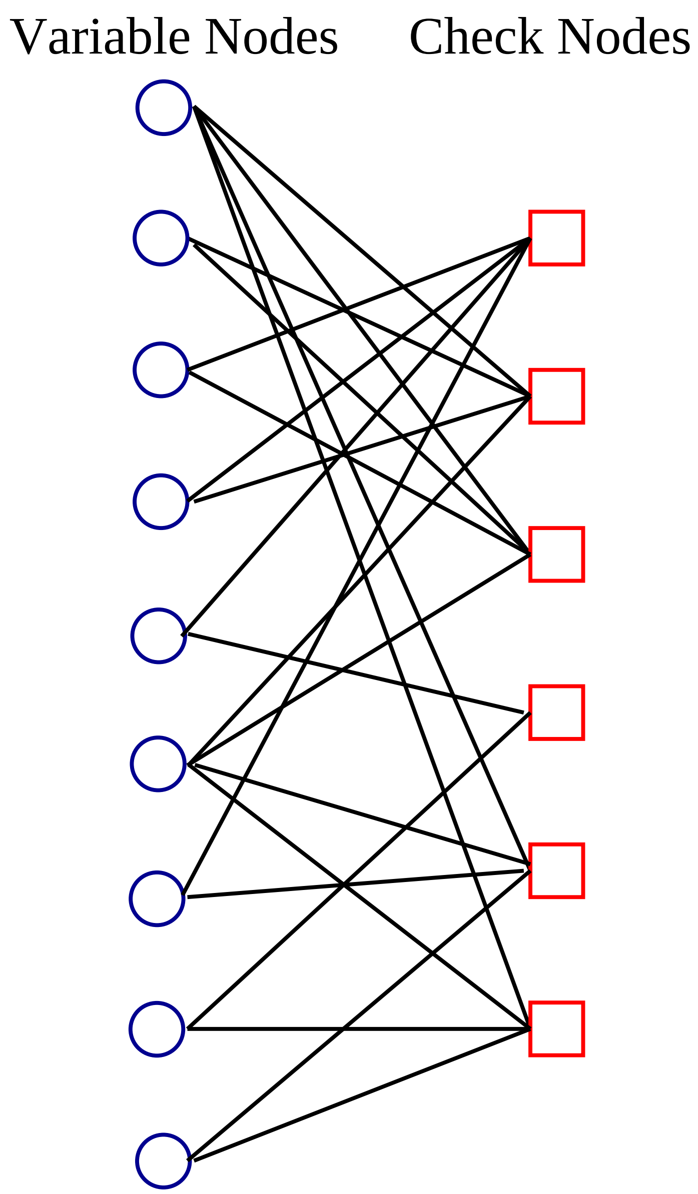
Recall: $(a+b)^n = \sum_{j=0}^{n} \begin{pmatrix} n \\ j \end{pmatrix} a^{n-j} b^j$.

B. *Exercise 2*

Given the cyclic code $\mathcal{C}$ in Exercise 1 draw an implementation of the encoder with $n-k = 5$ shift registers. Explain how to construct an encoder with $k = 16$ shift registers.

C. *Exercise 3*

Given the Tanner graph in the following figure



1) Describe the corresponding LDPC ensemble in terms of the variable and node distributions from an edge perspective $\lambda(x)$ and $\rho(x)$..
2) Determine a stopping set with 2 or 3 variable nodes if there is one.
3) Determine the density evolution function.

D. *Exercise 4*

Consider the convolutional code with rate $\frac{1}{2}$ and memory $\nu = 2$ with polynomial generators in octal notation $(5, 7)$.

- Write the state diagram of the convolutional code.
- Determine the the input output weighting enumerating function (IO-WEF) using the method of the transfer function.
- Describe how to determine a bound on the bit error probability using the IO-WEF.

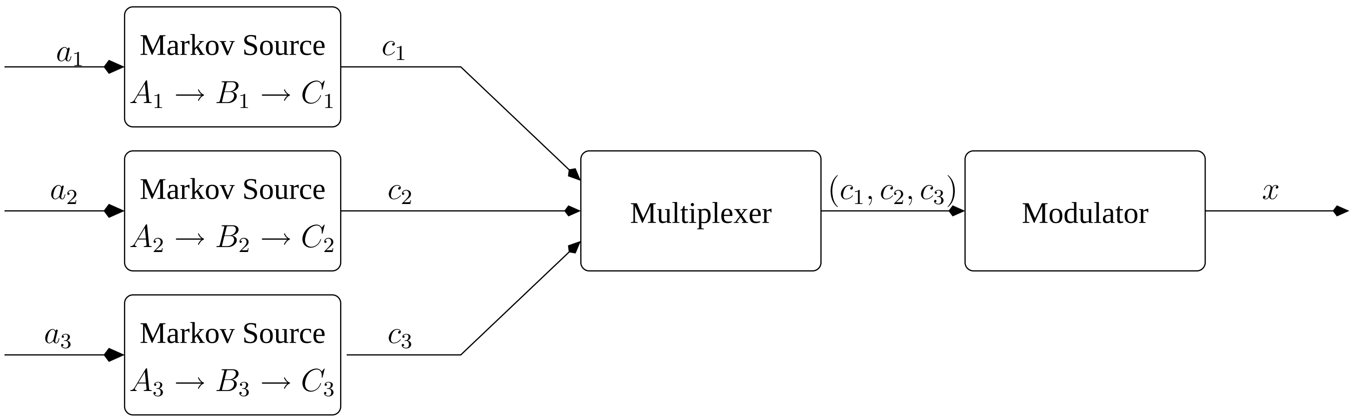


Fig. 1. Modulator of multiplexed Markov sources.

E. Exercise 5

Consider the system shown in Figure 1.

It consists of 3 independent subsystems $\mathcal{S}_i$, $i = 1, ..., 3$. Subsystem $\mathcal{S}_i$ has as input a discrete random variable A_i and as output a binary random variable C_i. C_i and A_i, $i = 1, ...3$, are related by the Markov chain

$$A_i \to B_i \to C_i \tag{1}$$

with joint probability density function $p_{A_i B_i C_i}(a_i b_i c_i) = p_{A_1}(a_1) p_{B_i|A_i}(b_i|a_i) p_{C_i|B_i}(c_i|b_i)$. The variables c_1, c_2, c_3 are multiplexed and mapped by the function $f(c_1, c_2, c_3)$ into an alphabet $\mathcal{X}$ with cardinality $|\mathcal{X}| = 8$. The function $f(c_1, c_2, c_3)$ is presented in the following table

c_1	c_2	c_3	$x = f(c_1, c_2, c_3)$
0	0	0	1
0	1	0	2
0	0	1	3
0	1	1	4
1	0	0	5
1	1	0	6
1	0	1	7
1	1	1	8.

Draw the factor graph to compute all marginals providing the function at each factor node.