Channel Coding Theory

Homework: 02/04/2008

A. Exercise 1

Given the (21,16) cyclic code $\mathcal{C}$ with generator polynomial $g(x) = x^5 + x^4 + 1$ determine:

1) The generator polynomial of the dual code.
2) The WEF of the dual code.
3) The WEF of the original code.
4) The probability of undetected errors assuming a binary symmetric channel (BSC) with $\varepsilon = 10^{-2}$.
5) Find the fraction of undetectable bursts of length 10 in $\mathcal{C}$ and motivate the result.

Hints: In order to determine the WEF work on the WEF of the dual code. Take into account the fundament properties of linear cyclic codes. Take into account that the data words in polynomial notation $a_1(x) = 1$, $a_2(x) = 1 + x + x^2$, *and* $a_3(x) = 1 + x^2 + x^3$ *are encoded by the dual code into 3 codewords that are* ***not*** *one the cyclic shifted version of the others.*

Recall: $(a+b)^n = \sum_{j=0}^{n} \begin{pmatrix} n \\ j \end{pmatrix} a^{n-j} b^j$.

B. Exercise 2

Given the cyclic code $\mathcal{C}$ in Exercise 1 draw an implementation of the encoder with $n-k = 5$ shift registers. Explain how to construct an encoder with $k = 16$ shift registers.