



NTNU – Trondheim
Norwegian University of
Science and Technology

Department of Electronics and Telecommunications

Examination paper for TTT4234 Space Technology I

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Permitted examination support material: Calculator, of a make according to a list approved by NTNU. Printed material: formula sheet attached to the exam.

Other information: Answers should be short and concise.

Language: English

Number of pages: 3

Number of pages enclosed: 1

Checked by:

Date

Signature

Exercise 1. The rocket equation

- a) The rocket equation can be expressed as follows: $\Delta v = v_e \ln (m_i/m_f)$. Explain the physical meaning of this equation, and the significance of the different parameters in the equation.

Answer: Δv = the velocity change induced by the ignition of the propellant (m/s)

v_e is the exhaust velocity of the propellant, and $v_e = I_{sp} g_0$

I_{sp} = propellant's specific impulse (s)

g_0 = standard gravitational acceleration (9.81 m/s²)

m_i = the total initial mass (kg) = $m_s + m_p$

m_f = the total final mass = $m_i - m_p$

m_s = satellite mass (structure + payload)

m_p = propellant mass

In this exercise I had not given the expression $v_e = I_{sp} g_0$, as I consider the rocket equation to be so essential and explained in so many lectures, that this should be known.

CryoSat II is an earth observation satellite, launched from Baikonur in April 2010. The principal mission for this satellite is to monitor ice thickness in polar regions from space. The mass of the satellite is 684 kg. The launcher (rocket) was of the type Dnepr-1, a rocket with mass equal to 211000 kg. It has a specific impulse of 318 s, and the total mass of the fuel was at launch equal to 186550 kg.

- b) Calculate the maximum possible speed of the rocket after launch.

Answer:

$$\Delta v = 318 * 9.81 * \ln ((186550+211000+684)/211000+684)) = 3119,58 *$$

$$\ln(398234/211684)= 1971 \text{ m/s}$$

In this exercise we have also accepted the answer with the fuel as included in the Dnepr-1 mass of 211000 kg giving

$$\Delta v = 318 * 9.81 * \ln ((211000+684)/211000-186550+684)) = 3119,58 *$$

$$\ln(211684/25134)= 6647 \text{ m/s}$$

This question was not entirely clear, and during the exam I gave the information that it should not be considered included, but also that I would accept both answers as many had already solved the equation. Anyway the conclusion in 1d will be the same. I had some remarks that the rocket was excessively heavy for that amount of fuel, which is correctly observed. The Dnepr-1 rocket is a multistage rocket, from 3-5 stages, and this exercise is therefore a constructed example, which will in all cases lead to some strange numbers. However, this was also exactly what should be the conclusion in 1d.

- c) The first and second cosmic speeds are border cases giving respectively the minimum speed an object needs in order to orbit earth right above the surface of earth (theoretical case with no atmosphere), and the speed an object needs to escape earth's gravitational field. Deduce the expressions of both speeds, and calculate the figures.

Answer:

It is possible to develop this from scratch doing this:

The 1st cosmic velocity, v , is the velocity needed to set an object into orbit at the earth's surface (assuming vacuum):

$$F_{\text{centr}} = F_{\text{grav}} \Rightarrow mv^2/r = mg(r) \Rightarrow v = \sqrt{R_E g_0} = 7.91 \text{ km/s} \sim 8 \text{ km/s}$$

The 2nd cosmic velocity is the velocity needed to escape the gravitational force of the earth. When the kinetic energy of an object is higher or equal to the gravitational potential energy:

$$E_{\text{kin}} = E_{\text{grav}} \Rightarrow mv^2/2 = GMm/r \Rightarrow v = \sqrt{2GM/R_E} = 11.2 \text{ km/s}$$

But it is also possible to take a shortcut, using the given formula from the formula sheet: $v = \sqrt{\mu(2/r - 1/a)}$

1st cosmic speed is when $a=r=R_E$, giving $v=\sqrt{\mu/R_E}= 7.91 \text{ km/s} \sim 8 \text{ km/s}$

2nd cosmic speed when a goes towards infinity, giving $v=\sqrt{2\mu/R_E}=11.2 \text{ km/s}$

- d) Comment on the result from the calculation in b) when compared to c). What will be the consequence for the design of the rocket?

Answer: Cryosat will never get into orbit with the figures from 1b). It is necessary to use a rocket with several stages with today's technology (limited I_{sp}). This will be the conclusion with both calculation possibilities in 1b). And the rocket used is actually a 3 to 5 stage rocket.

Exercise 2. Earth observation

CryoSat II was launched into a close to circular orbit (from this point considered as circular) 725 km over the surface of the earth with an inclination of 92 degrees.

- a) Each of these parameter values give name to orbit designations, which are they, and which are the characteristics of these orbits?

Answer: Low Earth Orbit (LEO) and polar orbit. The parameter 725km indicates LEO. LEO is a typical earth observation orbit, close to earth, so close that air drag will have some influence. It is also within the first Van Allen belt, and will be influenced by charged particles. The parameter 92 degrees indicate a (close to) polar orbit. The polar orbit crosses close to the poles, and will be advantageous for any observation in polar regions as is the mission of this satellite. (Some of you stated that this is a sun synchronous orbit, which is not the case, as the sun synchronous orbit needs an inclination of about 98 degrees in order to obtain a nodal regression rate of approximately 1 degree per day.)

b) Calculate the orbital period and velocity of Cryosat II.

Answer: Use Kepler's 3rd law: $T^2 = 4\pi^2 a^3 / \mu$, $\mu = 3,986 \cdot 10^5 \text{ km}^3/\text{s}^2$, $R_E = 6370 \text{ km}$, $a = R_E + h$.

Then $T^2 = 4 \cdot 3.14^2 \cdot 7095^3 / 3,986 \cdot 10^5 \text{ s}^2 = 35373631 \text{ s}^2$, $\Rightarrow T = 5947 \text{ s}$, i.e. approximately 99 minutes or 1 and a half hour.

The velocity is calculated as: $v = \sqrt{\mu/a} = \sqrt{(3,986 \cdot 10^5 \text{ km}^3/\text{s}^2)/(7095 \text{ km})} = 7.5 \text{ km/s}$.

c) In order to accomplish its mission for earth observation, CryoSat uses an instrument called SAR/Interferometric Radar Altimeters. This instrument uses one of the two atmospheric windows. Describe the meaning of an atmospheric window, and which is the other window.

Answer: radar/radio waves and light are the waves that will pass through the atmosphere with minimum attenuation.

Some of you have said that there are three atmospheric windows as you consider infrared as being a separate one, and that is perfectly ok. I myself consider IR to be so close to the light window, that I usually include it as the same.

d) SAR means Synthetic Aperture Radar. Describe the purpose of using this technique. What would be the resolution in the direction of flight if Cryosat had an exposure time of 6 seconds and a radar frequency of 13.575GHz. And what would have been the resolution if the SAR technique had not been used, when the antenna diameter of CryoSat is 1.1m?

Answer: The purpose of the technique is to improve the resolution of radio wave techniques. Light waves give good resolution, but are impractical to use in cloudy conditions and during night. Radio waves do not have this problem, but have a much poorer resolution than light waves for the same h and D . The SAR technique will use observations of the same object taken over a certain period of time when flying over, and by combining the information from the observations over time, it may be seen as equivalent to having an aperture the size of the flight track.

In 6 seconds, the satellite has moved $7.5 \text{ km/s} \cdot 6 \text{ s} = 45 \text{ km}$ (using $s = v \cdot t$), so it is as if the satellite has an aperture of $D = 45 \text{ km}$. The resolution then becomes:

$$R = 2.44 \cdot (c/f) \cdot h / D = 2.44 \cdot 0.022 \cdot (725/45) = 0.86 \text{ m}$$

$$\text{With } D = 1.1 \text{ m the resolution is: } R = 2.44 \cdot (c/f) \cdot h / D = 2.44 \cdot 0.022 \cdot (725/0.0011) = 35.4 \text{ km}$$

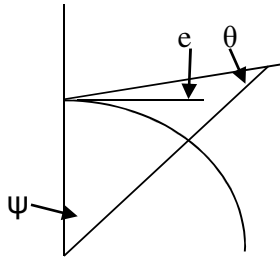
Exercise 3. Link budget

CryoSat II is operated (TT&C) from the ESA/ESOC center in Darmstadt in Germany, but it is at the earth station in Kiruna that the earth observation data is downloaded. The TT&C carrier frequency is in S-band, while the download is performed on an X-band down link with a center frequency of 8.1GHz, and has a download transfer rate of 100Mbit/s.

- a) If we consider a satellite pass with the satellite passing right over the gateway in Kiruna, and the download cannot start before the satellite is 10 degrees above the horizon on its way up, and similarly stops when the satellite is on its way down with an elevation angle of 10 degrees, for how long time will the earth station see the satellite at this pass, and how much data can be downloaded?

Answer:

Use «law of sines»:



$$\sin(e+90^\circ)/(R_e+h) = \sin(\theta)/R_e \Rightarrow \sin(\theta) = 6370 \cdot \sin(100)/(6370+725) = 0.8842$$

$\theta = 62.15^\circ \Rightarrow \psi = 180 - 100 - 62.15 = 18.4^\circ$, in total $2 \cdot \psi = 36.8^\circ$, so the time the satellite is visible is $T_v = 5947s \cdot 36.8/360 = 594.7s$, and the amount of data that can be downloaded is then $100\text{Mb/s} \cdot 594.7s = 59.470\text{Gbits}$.

Many of you were apparently not familiar with the law of sines, I thought it would be better known, as it is a trigonometric equation. But some of you were, so the question has been taken into account for the evaluation.

There was also a complaint that it was not clear where Kiruna is located geographically, but this is without any importance for these exercises as we say that we consider a pass where the satellite is right over the earth station. It could have been Trondheim, or anywhere on earth. FYI: Kiruna is in northern Sweden, and as a general rule, the further north you place your earth station, the more passes you will see of the polar orbiting satellite.

- b) Calculate the Free space loss in dB for this link, considering the satellite to be right above the earth station.

$$\text{Answer: } L_0 = (4\pi d/\lambda)^2 = (4\pi \cdot 725000 / (3000000000/8100000000))^2 = 6.0509 \cdot 10^{16} \text{ giving } 167.82\text{dB}$$

In this exercise you had to know that $s = v \cdot t$, giving $\lambda = c/f$.

- c) The receiver antenna in Kiruna has a diameter of 13 meters, and an antenna efficiency of 50%.
Calculate the gain of the receiver antenna in dB.

$$\text{Answer: } G = \eta \times 4\pi A / \lambda^2 = 0.5 \times 4\pi \times (\pi \cdot (13/2)^2) / (3000000000/8100000000)^2 = 607975, \text{ giving } 57.84 \text{ dB.}$$

- d) The loss due to pointing/tracking error and other types of loss amounts to as much as 20dB. If the spectral density of the noise is -202.71dBW/Hz, and the noise bandwidth is 80MHz, what will then be the received signal to noise ratio in dB (again with the satellite right above the earth station)?

Answer:

In this exercise, the information EIRP=20dB was lacking. The EIRP was given to be 20dB at the exam.

$$S/N = \text{EIRP} - L_0 + G_r - N_0 - B - L_a = 20 - 167.82 + 57.84 + 202.71 - 10\log(80000000) - 20 = 13.7 \text{ dB.}$$

Exercise 4. Description

Select one of the topics below, and give a short description.

- a) Describe how you think a plant will grow, if the seed is placed near the edge of a rotating disk in weightlessness, e.g. in the international space station. The seed remains in a fixed position on the disk, and has a regular supply of water and nutrition. The light is diffuse and comes from all directions, and has no influence on the direction of growth (it could have been darkness). The answer must be based on reasoning.

Answer: The rotation of the plate will give a centripetal force, acting towards the edge of the plate (outwards along the x-axis) in the plane of the plate:

$$a_x = r \cdot \omega^2 = x \cdot \omega^2$$

What is known from biology, is that plants grow in the opposite direction of any acceleration; e.g. in the opposite direction of the gravitational acceleration on earth. Other conditions also play a role on earth in natural conditions, such as light, but in this exercise, it is given that there are no such other conditions influencing the experiment. A seed that is not placed at the center of the disk will therefore grow in towards the center of the plate.

- b) Kepler and Newton. Give Kepler's and Newton's three laws (not just as formulas), in addition to Newton's law on Universal gravitation. Deduce Kepler's third law from known physical laws.

Answer: See the solutions given to this question at the exercise walk-through last November.