

### **Solution exercise 1:**

a)

$$S/N = EIRP/L_o \cdot G_r/T \cdot 1/kB \cdot 1/L_a$$

$$S/N_{dB} = EIRP_{dB} - L_{o,dB} + G/T_{dB} - k_{dB} - B_{dB} - L_{a,dB}$$

$$L_o = (4\pi d/\lambda)^2 = (4\pi \cdot 3.9 \cdot 10^7 \cdot 14.4 \times 10^9 / 3 \times 10^8)^2 = 5.53 \cdot 10^{20} \quad (\text{as } c = \lambda f)$$

$$L_{o,dB} = 207\text{dB}$$

$$B_{dB} = 10 \log(30 \cdot 10^6) = 74.77\text{dB}$$

$$\text{Additional loss } L_a = 2.0 + 0.9\text{dB}$$

$$S/N_{dB} = 40 - 207 + 20 + 228.6 - 74.77 - 2.9 = \underline{3.93\text{dB}}$$

b)

$L_o$  is free space loss, i.e. the loss a signal with wavelength  $\lambda$  will experience in free space (vacuum). It has nothing to do with atmospheric loss or rain fade.

$G/T$  is the "figure of merit" for the receiver antenna, and indicates how much the receiver antenna can amplify the signal in a system where the system temperature  $T$  will contribute with white gaussian noise (AWGN).

c)

$$S/N = EIRP/L_o \cdot G_r/T \cdot 1/kB \cdot 1/L_a = P_t G_t/L_o \cdot G_r/T \cdot 1/kB \cdot 1/L_a$$

$$= P_t \cdot \eta \cdot 4\pi A_t/\lambda^2 \cdot (\lambda/4\pi d)^2 \cdot \eta \cdot 4\pi A_r/\lambda^2 \cdot 1/(kTB) \cdot 1/L_a$$

Simplification shows that  $S/N$  is proportional with  $P_t$ ,  $A_t$ ,  $A_r$ ,  $f^2$

The effect of the frequency on  $S/N$  is:

- The uplink sender will have easy access to power, and can have a large transmitter antenna. In total,  $S/N$  will increase with the square of the frequency, and this is most important on the uplink, as the receiver antenna on the satellite has limited size.
- The downlink has a limited transmit power and antenna size, but the receiver antenna on ground can be dimensioned to compensate for this, and therefore it is not so important (as for the uplink) to optimize the link budget by increasing the frequency.

Additional aspects that might be mentioned:

Higher frequency also gives a smaller beam width, as the main lobe is proportional to  $1/f$  and  $1/A$ , and hence better directivity. Thereby, you will obtain a narrow beam by choosing high frequency and a large antenna, whereas the beam will be much wider with lower frequency and a small antenna. This may also be an advantage for satellite coverage, e.g. for broadcasting.

Finally,  $L_a$  may depend on the frequency through atmospheric attenuation or rain fade.  $L_a$  will usually increase with the frequency. However, these effects are seldom taken directly into consideration for link budget optimization.

d)

The radio delay between the earth and the satellite is:  $t = d/c = 3.9 \times 10^7 / 3 \times 10^8 = 130 \text{ ms}$

The total delay will in best case be: (player-satellite-earth station-satellite-player):

$dT = 2 + 130 + 10 + 130 + 70 + 130 + 10 + 130 + 15 = 627 \text{ ms}$  if the command from the player hits the start of a TDMA slot and is sent right away.

In the worst case scenario, the command will have to wait 400ms for the next slot i.e.  $dT = 627 \text{ ms} + 400 \text{ ms} = 1027 \text{ ms}$ .

In the worst case scenario, this is more than a second. Usually, this will be experienced as too slow, and new moves may have been performed before the reaction is seen from the previous move. Some possible counter measures are:

- Shorter time gap between the slots
- Use another access method than TDMA, e.g. CDMA or FDMA

But still, with both the above solutions we still have at least 627ms delay, and this is still a lot.

- Caching or download of the play on the receiver side may be a solution. Then the interaction will be local, and reduce the delay.
- Finally, the satellite medium is not so well suited for time critical interactive applications, so it could be an idea to use other types of network.

## **Solution exercise 2:**

a)

Start finding  $\Delta V$  of the second stage:

$$\begin{aligned} \Delta V_2 &= I_{sp2} \cdot g_0 \cdot \ln(M_{\text{initial stage 2}}/M_{\text{final Total}}) \\ &= 400s \cdot 9.81M/s^2 \cdot \ln((12000+1800)/(12000+1800-9000)) \end{aligned}$$

$$= 4144 \text{ m/s}$$

$\Delta V$  for the first stage must then be:

$$\Delta V_1 = \Delta V_{\text{design}} - \Delta V_2 = 10000 - 4144 = 5856 \text{ m/s.}$$

$$\Delta V_1 = I_{\text{sp}1} g_0 \ln(M_{\text{initial Total}}/M_{\text{final stage 1}}) = I_{\text{sp}1} g_0 \ln(M_{\text{initial Total}}/(M_{\text{structure 1}} + M_{\text{initial stage 2}}))$$

$\Rightarrow$  Solving the equation with respect to  $M_{\text{initial Total}}$  gives the total mass at take off:

$$\begin{aligned} M_{\text{initial Total}} &= (M_{\text{structure 1}} + M_{\text{initial stage 2}}) e^{\Delta V_1 / (I_{\text{sp}1} g_0)} \\ &= (7800 + 12000 + 1800) e^{5856 / (370 \cdot 9.81)} = \underline{108424 \text{ kg}} \end{aligned}$$

b)

The amount of propellant is then:

$$M_{\text{fuel}} = M_{\text{initial Total}} - M_{\text{structure 1}} - M_{\text{initial stage 2}} = 108424 - 7800 - (12000 + 1800) = \underline{86824 \text{ kg.}}$$

c)

$$1800/108424 = 0.017 \Rightarrow \underline{1.7\% \text{ of the total mass at lift off is payload.}}$$

### **Solution exercise 3:**

The figure shows the Iridium orbit with a height above the earth surface of 780 km. The orbit radius is then

$$r = 6378 + 780 = 7158 \text{ km}$$

The orbital speed is given by:

$$v = \sqrt{\frac{\mu}{r}}$$

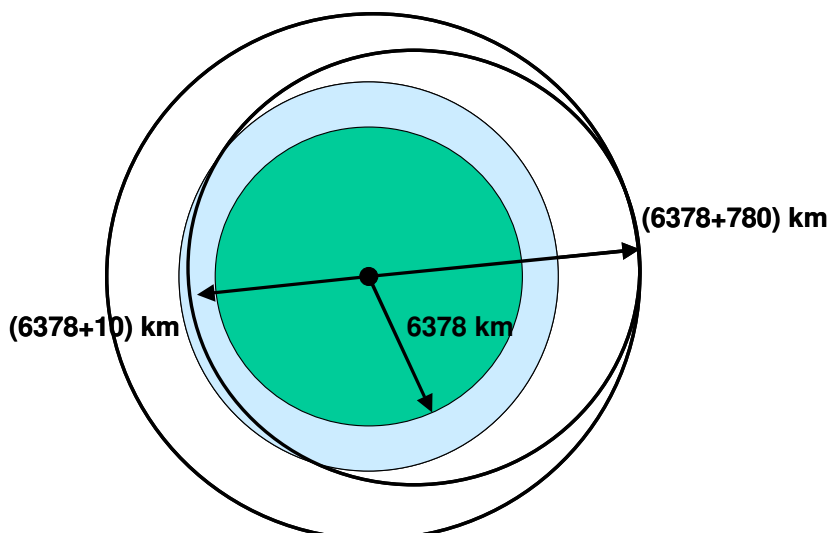
$$\text{where } \mu = 398603.2 \text{ km}^3/\text{s}^2$$

This gives a speed of 7.462 km/s.

In order to lead the satellite down into the atmosphere, the orbit must be transformed into an elliptic one, and perigee must be lowered to a height of 10 km above the earth surface. The distance to perigee becomes:

$$p = 6378 + 10 = 6388 \text{ km}$$

The distance to the center of earth from apogee is 7158 km



and to perigee 6388 km. The length of the semi major axis is:

$$a = \frac{1}{2} (7158 + 6388) = 6773 \text{ km}$$

The speed in apogee for the elliptic orbit is given by:

$$v_2 = \sqrt{\mu \cdot \left( \frac{2}{r} - \frac{1}{a} \right)}$$
$$v_2 = \sqrt{398603.2 \cdot \left( \frac{2}{7158} - \frac{1}{6773} \right)}$$

This gives  $v_2 = 7.247 \text{ km/s}$

The necessary reduction in speed is then:

$$\Delta v = (7.462 - 7.247) \text{ km/s} = \underline{0.215 \text{ km/s}}$$

#### **Key words for exercise 4:**

The best answer was given top score and the others ranked after that. No-one was meant to remember everything.

##### a) Earth observation

- General description: Aperture, resolution, and atmospheric window should be mentioned. Slides 6, 9, 10 and 11 from the Earth observation presentation.
- Methods
  - To improve the resolution:
    - Synthetic aperture
    - Chirp signals
    - Pulse compression by pulse coding
  - Methods for observation:
    - Interferometry (active)
    - Radar altimeter (active)
    - Bragg scattering (active)
    - Radiometry (passive)
    - Gravitational (passive)
    - Spectral measurements of stars (passive)
    - Infrared measurements/photos (passive)
- Application examples mentioned in the lectures:
  - Earth quakes (interferometry)
  - Volcanoes (interferometry)
  - Relief (interferometry)
  - Sea level (radar altimeter)

- Gravitational differences (radar altimeter and twin satellites)
- Tsunami (radar altimeter)
- Wind speed over the ocean, clouds, rain (Bragg scattering)
- Ice drift and coverage in the arctic (Bragg scattering)
- Oil spill (Bragg scattering)
- Ozon layer (spectrometry of star light)
- Vegetation (infrared photography)
- Corall reefs (infrared measurements of sea temperature)

b) Environment in space

Radiation from the sun:

- Solar flares
- Solar pressure
- Thermal

Protection by earth magnetosphere – origin and effect of Van Allen belts

Vulnerable to radiation:

- solar cells
- integrated circuits
- sensors
- humans
  - Bit flip - SEU
  - Material damage
  - Sputtering
  - Electronic components damage
  - Solar cells damage
  - Secondary radiation and interference
  - Electric discharge

Radiation from space:

- Cosmic rays
- Thermal

Radiation from the earth:

- Albedo, reflected from the sun
- Thermal from the earth

Magnetic field effect:

- Charging spinning satellites

Graviational effects:

- Sun, moon, earth, tides, geoides,....

Vacuum:

- Material effects like outgassing in vacuum
- Atomic oxygen erosion when not in complete vacuum

Mechanical:

- Space debris
- Reentry in the atmosphere

Humans:

- Radiation
- Free fall/weightlessness
- Gravitational at lift-off
- Psychological