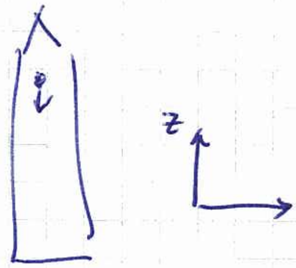
# Drop tower.




a) without catapult system

$v_0 = 0$

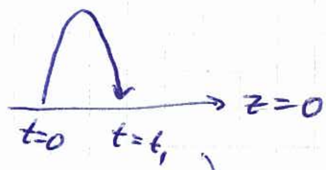
$z_0 = 122\ m$

$$\ddot{z} = -g \quad \longrightarrow \quad z = -gt^2/2 + v_0 t + z_0$$

$$0 = -gt^2/2 + 0 + 122$$

$$\Rightarrow \underline{\underline{t \approx 5\,s}}$$

with catapult




$$0 = -gt^2/2 + \frac{175000}{3600} \cdot t + 0$$

$t_1$ equal solutions

$$t_1^2 = \frac{2 \cdot 175000}{3600 \cdot g} \quad \Rightarrow t_1 \approx \underline{\underline{9.9\,s}}$$

b) speed : $\frac{175000}{3600}$ m/s   assume constant acceleration a $\Rightarrow$

$$a = \frac{175000}{3600 \cdot \Delta t} \quad \Rightarrow a = \frac{175000}{3600 \cdot \underset{0.25}{\Delta t}} = 194.4\ m/s^2$$

$$\approx \underline{\underline{19.8\,g}}$$

c) $\approx 10^{-6}\, g$   from kompendium.

Gimbaling the massive engines to change their thrust direction requires large hinges, hydraulic arms, and supporting structure.

Launch-vehicle designers have the challenge of carefully integrating all of these structures and mechanisms with the engines, tanks, and other subsystems, to create a compact, streamlined vehicle. Sadly, for expendable vehicles, all the painstaking design and expensive construction and testing to build a reliable launch vehicle burns up or drops in the ocean within 10 minutes after launch! Figure 14-62 shows a cut-away view of the Ariane V launch vehicle. As you can see, the majority of the structure is propellant tanks and engines. All the other subsystems squeeze into small boxes, tucked into the secondary structure. Notice there are several sets of engines on this, and all launch vehicles currently in use. Each set comprises a separate stage. Next, we'll see why all these stages are needed to get a spacecraft to orbit.

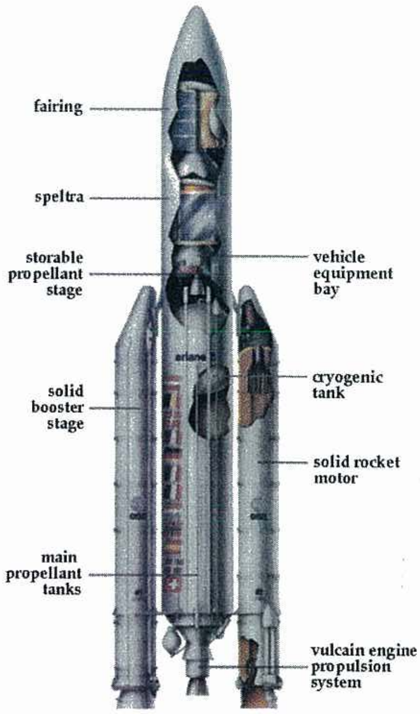



**Figure 14-62. Ariane V Cut-away.** Most of the mass and volume of this giant booster consists of propellant tanks. *(Courtesy of Arianespace/European Space Agency/Centre National D'Etudes Spatiales)*

## Staging

Getting a payload into orbit is not easy. As we showed in Section 14.2, the state-of-the-art in chemical rockets (the only type currently available with a thrust-to-weight ratio greater than 1.0) can deliver a maximum $I_{sp}$ of about 470 s. Given the velocity change, $\Delta V$, needed to get into orbit, as we determined in Chapter 9, and the hard realities of the rocket equation, designers must create a launch vehicle that is mostly propellant. In fact, over 80% of a typical launch vehicle's lift-off mass is propellant. Large propellant tanks, that also add mass, contain all of this propellant. Of course, the larger the mass of propellant tanks, and other subsystem, the less mass is available for payload. One way of reducing the vehicle's mass on the way to orbit is to get rid of stuff that's no longer needed. After all, why carry all that extra tank mass along when the rocket engines empty the tanks steadily during launch? Instead, why not split the propellant into smaller tanks and then drop them as they empty? Fighter planes, flying long distances, use this idea in the form of "drop tanks." These tanks provide extra fuel for long flights and can be dropped when they are empty, to lighten and streamline the plane. This is the basic concept of staging.

*Stages* consist of propellant tanks, rocket engines, and other supporting subsystems that are discarded to lighten the launch vehicle on the way to orbit. As the propellant in each stage is used up, the stage drops off, and the engines of the next stage ignite (hopefully) to continue the vehicle's flight into space. As each stage drops off, the vehicle's mass decreases, meaning a smaller engine can keep the vehicle on track into orbit. Figure 14-63 shows an artist's concept of the Saturn I vehicle staging on the way to orbit.

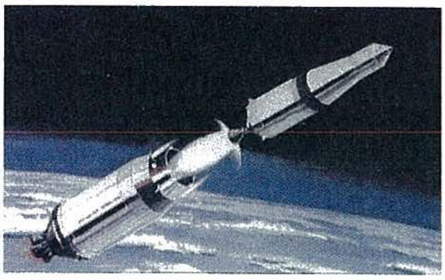
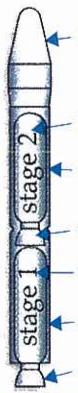


**Figure 14-63. Saturn I during Staging.** When a launch vehicle, such as the Saturn I shown here in an artist's concept, stages, it shuts off the lower-stage rocket, separates it, and ignites the rocket on the next stage to continue into orbit. *(Courtesy of NASA/Kennedy Space Center)*

Table 14-11 gives an example of how staging can increase the amount of payload delivered to orbit. For this simple example, notice the two-stage vehicle can deliver more than twice the payload to orbit as a similar-sized, single-staged vehicle with the same total propellant mass—even after adding 10% to the structure's overall mass to account for the extra engines and plumbing needed for staging. This added payload-to-orbit capability is why all launch vehicles currently rely on staging.

In Table 14-11, for both cases, the mass of the payload delivered to orbit compared to the mass of the entire launch vehicle is pretty small—5% or less. About 80% of a typical vehicle is propellant. The other 15% or so includes structure, tanks, plumbing, and other subsystems. Obviously, we could get more payload into space if the engines were more efficient. However, with engines operating at or near the state-of-the-art, the only other option, as the examples show, is to shed empty stages on the way into orbit.

**Table 14-11. Comparing Single-stage and Two-stage Launch Vehicles.**

| Launch Vehicle | Parameters | Payload to Orbit |
|---|---|---|
| **Single Stage**   | $\Delta V_{design}$ = 8000 m/s<br>$I_{sp}$ = 480 s<br>$m_{structure}$ = 250 kg<br>$m_{propellant}$ = 1500 kg | $m_{payload}$ = 84 kg |
| **Two Stage**   | $\Delta V_{design}$ = 8000 m/s<br>**Stage 2**<br>$I_{sp}$ = 480 s<br>$m_{structure}$ = 140 kg<br>$m_{propellant}$ = 750 kg<br>**Stage 1**<br>$I_{sp}$ = 480 s<br>$m_{structure}$ = 140 kg<br>$m_{propellant}$ = 750 kg | $m_{payload}$ = 175 kg |

Now let's see how we use the rocket equation to analyze the total $\Delta V$ we get from a staged vehicle. We start with

$$\Delta V = I_{sp} g_o \ln\left(\frac{m_{initial}}{m_{final}}\right)$$

Recognize that, for a staged vehicle, each stage has an initial and a final mass. Also, the $I_{sp}$ may be different for the engine(s) in different stages. To get the total $\Delta V$ of the staged vehicle, we must add the $\Delta V$ for each stage.

This gives us the following relationship for the ΔV of a staged vehicle with n stages.

$$\Delta V_{total} = \Delta V_{stage\ 1} + \Delta V_{stage\ 2} + \ldots + \Delta V_{stage\ n} \tag{14-40}$$

$$\boxed{\begin{aligned}\Delta V_{total} = {} & I_{sp\ stage\ 1} g_o \ln\left(\frac{m_{initial\ stage\ 1}}{m_{final\ stage\ 1}}\right) \\ & + I_{sp\ stage\ 2} g_o \ln\left(\frac{m_{initial\ stage\ 2}}{m_{final\ stage\ 2}}\right) \\ & \vdots \\ & + I_{sp\ stage\ n} g_o \ln\left(\frac{m_{initial\ stage\ n}}{m_{final\ stage\ n}}\right)\end{aligned}} \tag{14-41}$$

where

$\Delta V_{total}$ = total ΔV from all stages (m/s)
$I_{sp\ stage\ n}$ = specific impulse of stage n (s)
$g_o$ = gravitational acceleration at sea level (9.81 $m/s^2$)
$m_{initial\ stage\ n}$ = initial mass of stage n (kg)
$m_{final\ stage\ n}$ = final mass of stage n (kg)

What is the initial and final mass of stage 1? The initial mass is easy; it's just the mass of the entire vehicle at lift-off. But what about the final mass of stage 1? Here we have to go to our definition of final mass when we developed the rocket equation. Final mass of any stage is the initial mass of that stage (including the mass of subsequent stages) less the propellant mass burned in that stage. So for stage 1

$$m_{final\ stage\ 1} = m_{initial\ vehicle} - m_{propellant\ stage\ 1}$$

Similarly, we can develop a relationship for the initial and final mass of stage 2, stage 3, and so on.

$$m_{initial\ stage\ 2} = m_{final\ stage\ 1} - m_{structure\ stage\ 1}$$

$$m_{final\ stage\ 2} = m_{initial\ stage\ 2} - m_{propellant\ stage\ 2}$$

Example 14-4 shows how to compute the total ΔV for a staged vehicle. Overall, staging has several unique advantages over a one-stage vehicle. It

- Reduces the vehicle's total mass for a given payload and ΔV requirement
- Increases the total payload mass delivered to space for the same-sized vehicle
- Increases the total velocity achieved for the same-sized vehicle
- Decreases the engine efficiency ($I_{sp}$) required to deliver a same-sized payload to orbit

But, as the old saying goes, "There ain't no such thing as a free lunch" (or launch)! In other words, all of these staging advantages come with a few drawbacks. These include

- Increased complexity because of the extra sets of engines and their plumbing
- Decreased reliability because we add extra sets of engines and the plumbing
- Increased total cost because more complex vehicles cost more to build and launch

Another interesting limitation of staging has to do with the law of diminishing returns. So far, you may be ready to conclude that if two stages are good, four stages must be twice as good. But this isn't necessarily the case. Although a second stage significantly improves performance, each additional stage enhances it less. By the time we add a fourth or fifth stage, the increased complexity and reduced reliability offsets the small performance gain. That's why most launch vehicles currently in use have only three or four stages.

As we'll see in Chapter 16 in more detail, getting into space is expensive. In some cases, the price per kilogram to orbit is more than the price per kilogram of gold! In an ongoing effort to reduce the cost of access to space, researchers are looking for ways to make launch vehicles less expensive. One of the most promising ways is to make the entire vehicle reusable. One company, Kistler Aerospace, is attempting to do this with a two-stage vehicle design (see the Mission Profile at the end of this chapter).

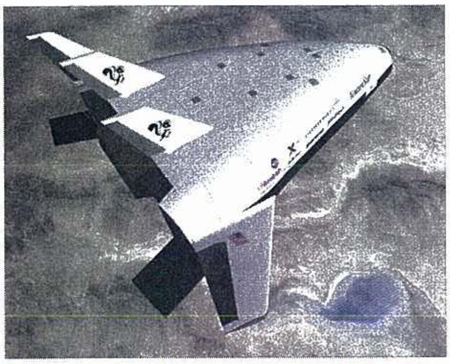


**Figure 14-64. Single-stage-to-orbit (SSTO).** The X-33 is a prototype SSTO vehicle that promises to revolutionize access to space. *(Courtesy of NASA/Marshall Space Flight Center)*

However, the ultimate goal would be a single-stage-to-orbit vehicle that could take off and land as a single piece, offering airline-like operations. However, the technical challenges in propulsion and materials to overcome the limitations of a single stage are formidable. The goal of NASA's X-33 program, shown in Figure 14-64, is to push the state of the art in rocket engines (the aerospike design described earlier), materials, computer-aided design and fabrications, and operations. One day, the successors to this pioneering program may give all of us the ability to live and work in space routinely.