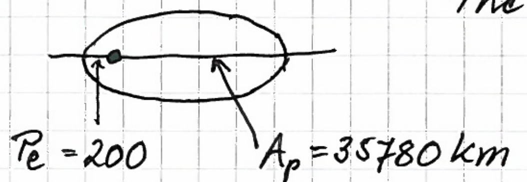
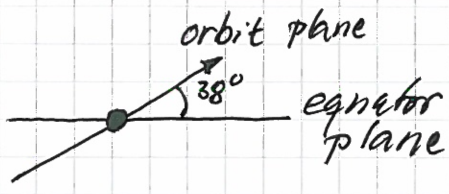
III Inclination correction



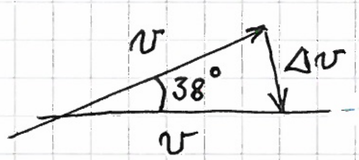
The orbit is elliptical and the major axis, $2a$ $= P_e + A_p +$ diameter of Earth

$$\Rightarrow a = \frac{1}{2}\left(P_e + A_p + 2R\right) = \frac{1}{2}\left(P_e + A_p\right) + R = 24390 \text{ km}$$

If we look at the orbit along the major axis the ellipse is tilted $38°$ with respect to the equator plane.



What Δv is required to change the orbit $38°$ without any other parameter change? Velocity vectors:



The inclination $i = 38°$ is rather big. Use the cosine theorem to calculate Δv!

$$\Delta v^2 = v^2 + v^2 - 2 \cdot v \cdot v \cdot \cos 38°$$

In order to calculate Δv we need to know v at the two nodes, v_{A_p} and v_{P_e}.

$$v_{A_p} = \sqrt{\mu\left(\frac{2}{R_{A_p}} - \frac{1}{a}\right)}$$

Here R_{A_p} is the distance from the mass point

of the Earth to apogeum $= 35780 + 6400$ km
$= 42180$ km

$$\Rightarrow \quad v_{Ap} = \sqrt{398600\left(\frac{2}{42180} - \frac{1}{24390}\right)} = 1.599 \text{ km/s}$$

$\Rightarrow \quad \Delta v_{Ap}$ from the cosine formula

$$= v_{Ap}^2 + v_{Ap}^2 - 2 v_{Ap}^2 \cdot \cos 38^\circ$$

$$\Delta v_{Ap} = 1.042 \text{ km/s}$$

$$\Delta m_{Ap} = m\left(1 - e^{-\Delta v_{Ap}/I_{sp} \cdot g}\right) \approx 4000\left(1 - e^{-\frac{1042}{300 \cdot 10}}\right)$$

$$= 1173 \text{ kg}$$

for perigeum – the alternative – we get

$$v_{Pe} \approx 10.24 \text{ km/s}$$

$$\Delta v_{Pe} \approx 6.67 \text{ km/s}$$

$$\Delta m_{Pe} \approx 3585 \text{ kg}$$

Since the speed is higher close the Earth (perigeum) the correction requires higher Δv and more fuel.