I Repetition Cosmic speeds

1. kosmisk hastighet (bane rundt jorda)

Fra $v_1 = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$ får vi for $a = r$

$$v_1 = \sqrt{\frac{\mu}{r}} \quad \text{og} \quad \sqrt{\frac{398600}{6378}} \approx 8 \text{ km/s}$$

eller $\frac{mv^2}{R_0} = m \cdot g \Rightarrow$

$$v = \sqrt{g \cdot R_0} \approx 8 \text{ km/s}$$

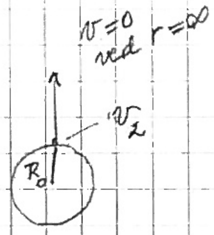
2. Kosmiske hastighet (ut fra jorda)

La a (halve storaksen) $\to \infty$

$$v_2 = \sqrt{\frac{2\mu}{r}} \approx 8 \cdot \sqrt{2} \approx 11.2 \text{ km/s}$$

alternative on the next page.

Alternative to get 2^{nd} cosmic velocity



Kinetisk energi $\frac{m v_2^2}{2}$

skal overvinne potensiell energi

$$\int_0^\infty m \cdot g \cdot ds$$

$$F = G \cdot \frac{M \cdot m}{r^2} = \underbrace{G \frac{M}{R_0^2}}_{g} \cdot \frac{R_0^2 \cdot m}{r^2}$$

$$= \int_{R_0}^\infty m \cdot \frac{R_0^2}{r^2} \cdot g \cdot dr$$

$$= m R_0^2 g \left(-\frac{1}{r}\right)\Big|_{R_0}^\infty = m R_0^2 g \left(+\frac{1}{R_0}\right) = mg R_0$$

$$\therefore \quad m v_2^2 \geq 2\, mg R_0$$

$$\Rightarrow \quad v_2^2 = 2 g R_0 \qquad \text{og} \quad v_2 = \sqrt{2 \cdot g\, R_0} = 11{,}2\ \mathrm{km/s}$$

II Orbit adjustment

Assume #1 and #2 in sequence

$$\Delta m_1 = m_1 - m_1 \cdot e^{-\Delta v_1 / g I_{sp}}$$

$$\Delta m_2 = m_2 - m_2 \cdot e^{-\Delta v_2 / g I_{sp}}$$

$$m_2 = m_1 - \Delta m_1$$

$\Delta m_{tot} = \Delta m_1 + \Delta m_2$ independent of order?

$$\Delta m_{tot} = m_1 - m_1 \cdot e^{-\alpha_1} + m_2 - m_2 e^{-\alpha_2}$$

$$= m_1 - m_1 e^{-\alpha_1} + m_1 - \Delta m_1 - m_1 \cdot e^{-\alpha_2} + \Delta m_1 \cdot e^{-\alpha_2}$$

$$= m_1 - \cancel{m_1 e^{-\alpha_1}} + \cancel{m_1} - \cancel{m_1} + \cancel{m_1 e^{-\alpha_1}} - \cancel{m_1 e^{-\alpha_2}} + \\ + \cancel{m_1 e^{-\alpha_2}} - m_1 \cdot e^{-\alpha_2 - \alpha_1}$$

$$= m_1 - m_1 e^{-(\alpha_1 + \alpha_2)} = m_1 \left(1 - e^{-(\alpha_1 + \alpha_2)}\right)$$

For n stages

$$\Delta m_{tot} = m_1 \left(1 - e^{-(\alpha_1 + \alpha_2 \cdots + \alpha_n)}\right)$$

$$= m_1 \left(1 - e^{-(\Delta v_1 + \Delta v_2 \cdots + \Delta v_n)/g I_{sp}}\right)$$

The order of the Δv's is unimportant

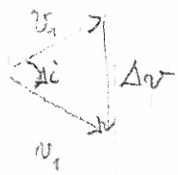
III Inclination correction

To change i we give the thrust at the equator (where apogee and perigee are located in this case)

The orbit is elliptical and $a = \frac{1}{2}(A_p + P_e) + R_{earth}$
$= 24\,390$ km

$$V_{A_p} = \sqrt{\mu\left(\frac{2}{R_{A_p}} - \frac{1}{a}\right)}$$

$$= \sqrt{398600\left(\frac{2}{\underbrace{35780+6400}_{42180}} - \frac{1}{24390}\right)} = 1.599 \text{ km/s}$$



Ola! big angle! $\Delta i = 38^\circ$

Cosine theorem

$$\Delta v^2 = v_1^2 + v_1^2 - 2 \cdot v_1 \cdot v_1 \cdot \cos 38$$

$$\Delta v^2 = v_1^2\,(2 - 2 \cdot 0.788) \Rightarrow \Delta v = 1.042 \text{ km/s}$$

$$\Delta m = 4000\left(1 - e^{-1.042 \cdot 10^3 / 300 \cdot 10}\right) \approx 4000\left(1 - e^{-0.3473}\right)$$

$$\approx 4000\,(1 - 1 + 0.3473) \approx 1173 \text{ kg}$$

$V_{pe} =$ 10.24 km/s

$\Delta v =$ 6.07 km/s

$\Delta m =$ 3585 kg

Best to correct orbit when speed is lowest!