



Orbiting satellites

(Komp. exerc. 1.3)

$$\left. \begin{aligned} M_1 &= \frac{4\pi R_1^3}{3} \cdot \rho_1 \\ M_2 &= \frac{4\pi R_2^3}{3} \cdot \rho_2 \end{aligned} \right\} \text{spheres}$$

Gravitational force $= G \frac{M_1 m}{R_1^2} \equiv m \cdot \frac{v_1^2}{R_1} = \text{centrifugal}$
force; analogously for M_2

We get $v_1^2 = G \cdot \frac{M_1}{R_1}$ and $v_2^2 = G \cdot \frac{M_2}{R_2}$

Orbital period T :

$$v = \frac{2\pi R}{T} \text{ in general}$$

The ratio T_1/T_2 :

$$\begin{aligned} \frac{2\pi R_1}{v_1} / \frac{2\pi R_2}{v_2} &= \frac{R_1}{R_2} \cdot \frac{v_2}{v_1} = \frac{R_1}{R_2} \cdot \frac{\sqrt{G \cdot \frac{M_2}{R_2}}}{\sqrt{G \cdot \frac{M_1}{R_1}}} \\ &= \frac{R_1}{R_2} \sqrt{\frac{R_2^3 \rho_2}{R_2} / \frac{R_1^3 \rho_1}{R_1}} \\ &= \frac{R_1}{R_2} \cdot \frac{R_2}{R_1} \cdot \sqrt{\frac{\rho_2}{\rho_1}} \end{aligned}$$

$$\Rightarrow T_1/T_2 = \sqrt{\frac{\rho_2}{\rho_1}}$$

So, knowing T_1 and ρ_1 for the Earth we can calculate ρ_2 by measuring T_2 ! If $\rho_1 = \rho_2$ then T is independent of R !
(cf. exercise 1.4 in compendium!)