

Newton's 1st law of motion:

A body continues in its state of rest, or in uniform motion in a straight line, unless compelled to change that state by forces impressed upon it.

Newton's 2nd law of motion:

The time rate of change of an object's momentum equals the applied force. $F = \Delta p / \Delta t = ma$ if $\Delta m = 0$.

Newton's 3rd law of motion:

When body A exerts a force on body B, body B will exert an equal, but opposite, force on body A.

Newton's law of universal gravitation:

The force of gravity between two bodies is directly proportional to the product of the two masses and inversely proportional to the square of the distance between them.

$$F = Gm_1m_2/R^2$$

$$\text{Universal gravitational constant } G = 6,67 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$m_1 = M_E = M = 6 \cdot 10^{24} \text{ kg}$$

$$\text{Earth's gravitational parameter } \mu = M \cdot G = 4 \cdot 10^5 \text{ km}^3/\text{s}^2$$

$$R_E = 6400 \text{ km}$$

Kepler's 1st law:

The orbits of the planets are ellipses with the sun at one focus.

Kepler's 2nd law:

The line joining a planet to the sun sweeps out equal areas in equal times.

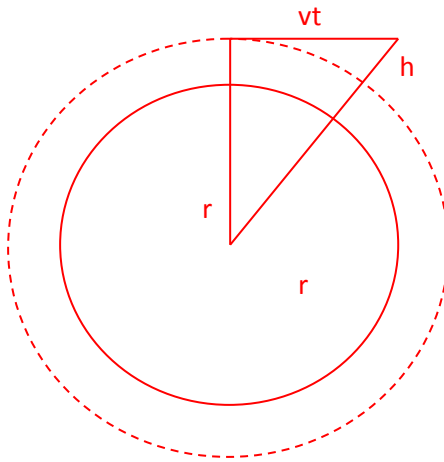
Kepler's 3rd law:

The square of the orbital period is directly proportional to the cube of the average distance between the planet and the sun.

$$T^2 = 4\pi^2 a^3 / \mu$$

Show that $mv^2/r = \mu m/r^2$, i.e. the centripetal force equals the gravitational force for a satellite in earth orbit.

A geometrical approach to find the centripetal force (Tipler):



$$(r+h)^2 = (vt)^2 + r^2$$

$$r^2 + 2rh + h^2 = v^2 t^2 + r^2$$

$$h(2r + h) = v^2 t^2$$

and $h \ll r$

$$2rh = v^2 t^2$$

$$h = \frac{1}{2} (v^2/r) t^2$$

Comparing this with the constant acceleration expression $h = \frac{1}{2} at^2$, gives $a = v^2/r$.

With $F = ma = mv^2/r$.

Newton's law of universal gravitation gives $F = \mu m/r^2$. For an object in orbit around the earth, the centripetal force is equal but opposite to the gravitational force, hence $mv^2/r = \mu m/r^2$

Show Kepler's third law, i.e. that r^3 is proportional to T^2 . Show that the centripetal force can be expressed as $m\omega^2 r$.

$$mv^2/r = \mu m/r^2$$

$$v^2 = \mu/r$$

$$(2\pi r/T)^2 = \mu/r$$

$$\underline{4\pi^2 r^3 = \mu T^2, \text{ hence } r^3 \text{ is proportional to } T^2}$$

$$mv^2/r = (m/r)(2\pi r/T)^2 = \underline{m\omega^2 r} \quad \text{as } \omega = 2\pi f = 2\pi/T$$