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Orbits

Space technology I

Vendela Paxal

Overview

- Newton and Kepler
- Coordinate definition
- Orbital elements
- Orbit types
- Cosmic speeds
- Velocity changes, and Hohmann transfer
- Orbital perturbations
- Special orbit types
- Case example

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History

- Space has been observed since the first signs of civilization
- Many observations led Kepler to state the movements of celeste bodies
- Newton continued, and then many after that...
- Commercial use of space
- Scientific use of space, applications that can not be performed from earth
- Colonization of space

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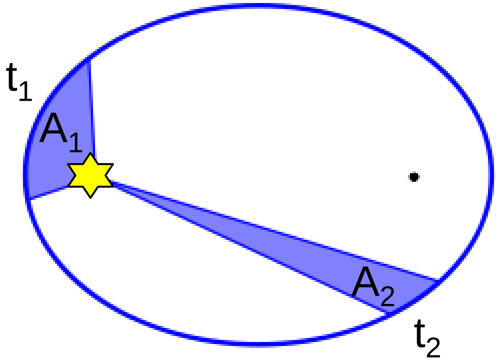
Kepler's laws

Johannes Kepler (1571-1630)

Kepler's 1st law: The orbits of the planets are ellipses with the sun at one focus.

Kepler's 2nd law: The line joining a planet to the sun sweeps out equal areas in equal times.
If $t_1=t_2 \Leftrightarrow A_1=A_2$



Kepler's 3rd law: The square of the orbital period is directly proportional to the cube of the average distance between the planet and the sun: $T^2 \propto a^3$

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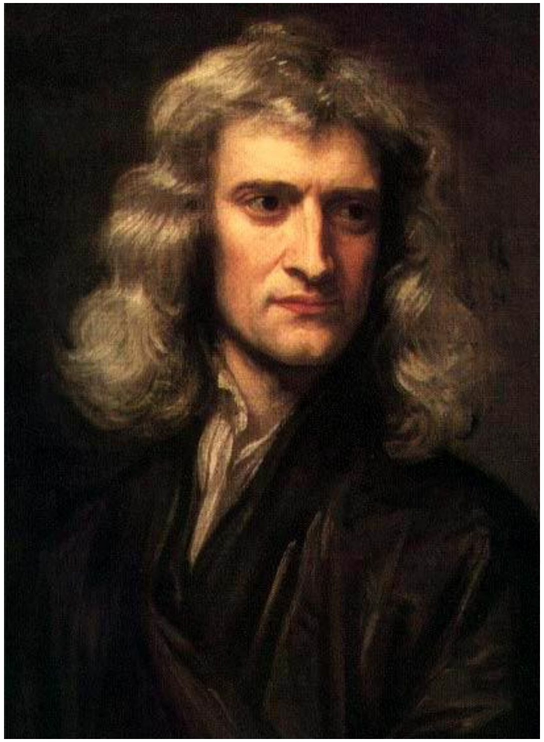
Newton's laws

Newton's 1st law of motion: A body continues in its state of rest, or in uniform motion in a straight line, unless compelled to change that state by forces impressed upon it.

Newton's 2nd law of motion: The time rate of change of an object's momentum equals the applied force. $F=\Delta p/\Delta t=ma$.

Newton's 3rd law of motion: When body A exerts a force on body B, body B will exert an equal, but opposite, force on body A.



Isaac Newton 1642-1726

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Newton's law of universal gravitation

The force of gravity between two bodies is directly proportional to the product of the two masses and inversely proportional to the square of the distance between them.

$F=Gm_1m_2/R^2$

G is the universal gravitational constant, $G = 6.67 \cdot 10^{-11} Nm^2/kg^2$.

If e.g. m_1 is the earth, it is usually combined with G, giving a new constant $Gm_1 = \mu = 3.986 \cdot 10^5\ km^3/s^2$, then called the gravitational parameter of the earth.

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The centripetal and gravitational forces for a circular orbit

Using Pythagoras for the centripetal force:

$(r+h)^2=(vt)^2+r^2$

$r^2 + 2rh + h^2 = v^2t^2 + r^2$

$h(2r + h) = v^2t^2$

and $h << r$

$2rh = v^2t^2$

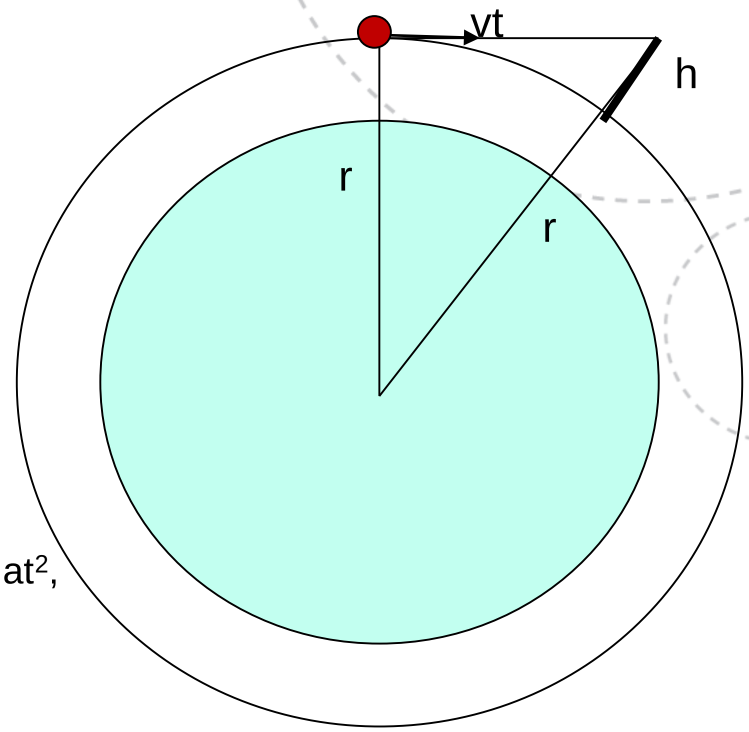
$h= ½ (v^2/r) t^2$

Comparing this with the constant acceleration expression $h = ½ at^2$, gives $a=v^2/r$

With $F = ma = mv^2/r$

Newton's law of universal gravitation gives $F = \mu m/r^2$. For an object in orbit around the earth, the centripetal force is equal to the gravitational force, hence $mv^2/r = mr\omega^2= \mu m/r^2$ since $v=2\pi r/T$ and $\omega=2\pi/T$

From this, we may also deduce Kepler's third law: $T^2 = 4\pi^2r^3/\mu$



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Newton's lunear calculation

Shows that the gravitational acceleration behaves like the inverse square of the distance between two bodies.

Moon: $a_{Moon} = r\omega^2 = r\cdot 4\pi^2/T^2 = 2.72\cdot 10^{-3} m/s^2$
As $T = 27.32$ days and $r = 3.83\cdot 10^8 m = 60R_E$

An apple on earth has a gravitational acceleration of
$a_{Apple} = g_0 = 9.81 m/s^2$

Hence: $a_{Apple}/a_{Moon} = 9.81 / 2.72\cdot 10^{-3} = 3600$

And $(60R_E)^2 / R_E^2 = 3600$

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Newton's law of universal gravitation proves Kepler's laws

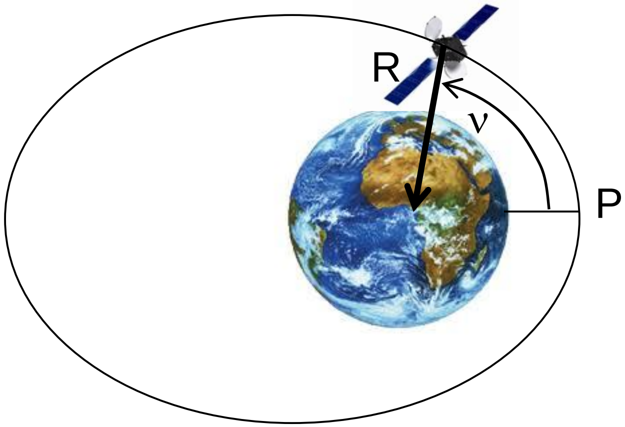
We reduce the forces acting on a spacecraft to a single force: Earth's gravity

=> The two-body equation (see chapter 4.1.3.3 in the Handbook)

$$\underline{F}_g = (-\,\underline{R}/R)\cdot(\mu m/R^2) = m\underline{a} = m\cdot\partial^2\underline{R}/\partial t^2$$

$$\partial^2\underline{R}/\partial t^2 + (\underline{R}/R)\cdot(\mu/R^2) = 0$$

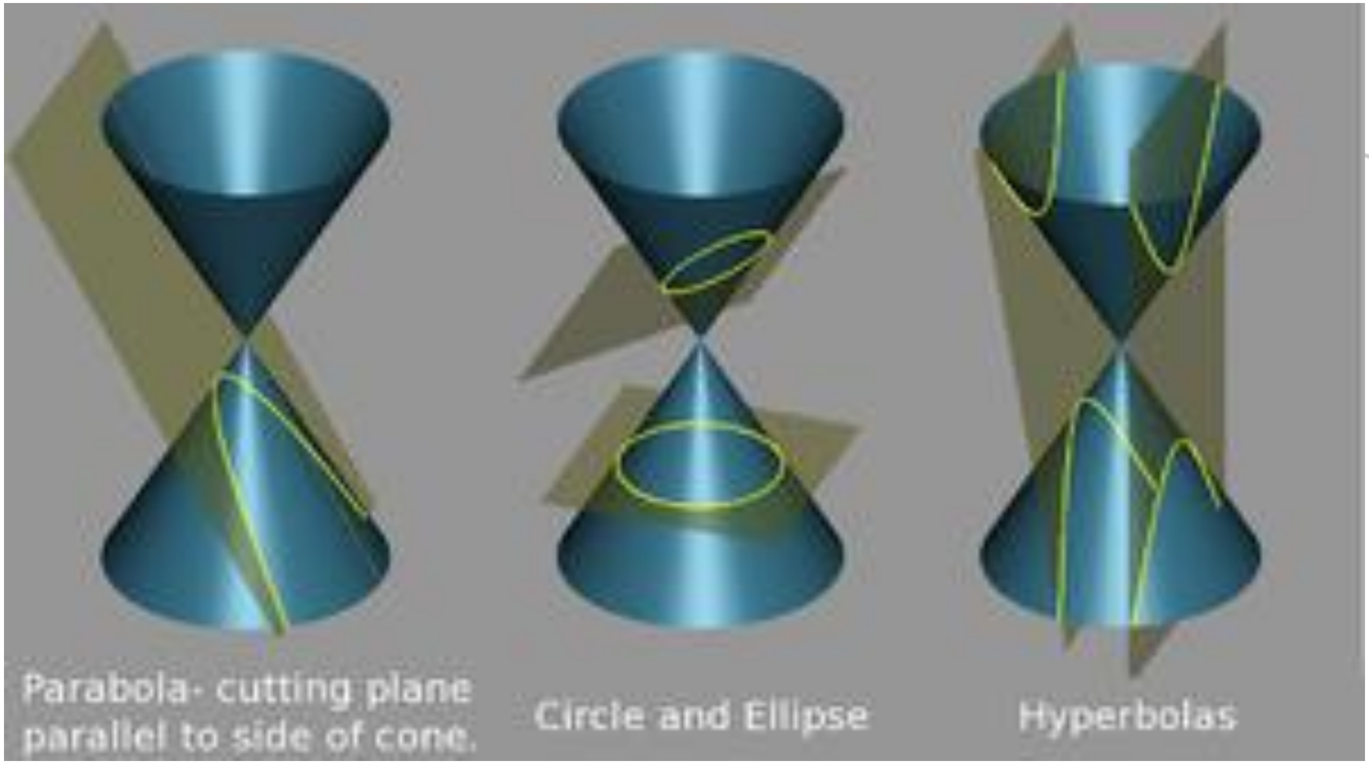
$$R = a(1-e^2)/(1+e\cdot\cos\nu)$$



This equation proves Kepler's laws of planetary motion, as it describes the relationship for conic sections, a being the semi-major axis, and e being the eccentricity of an ellipse.

ν is called the true anomaly

Conic sections



Circle e=0
Ellipse 0<e<1
Parabola e=1
Hyperbola e>1

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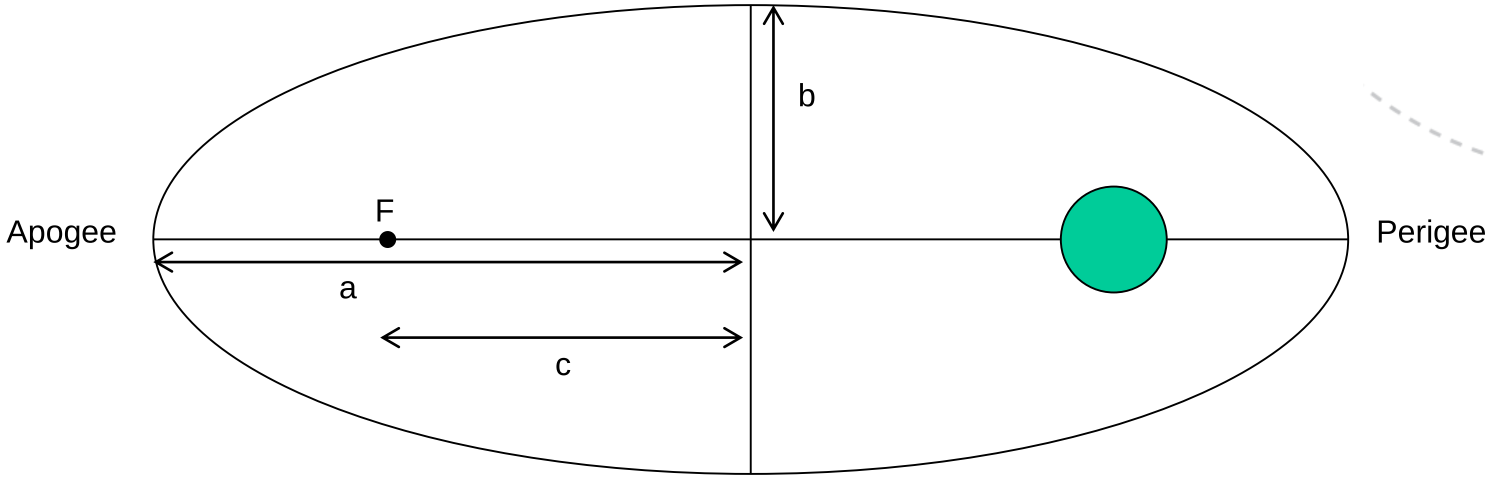
The ellipse

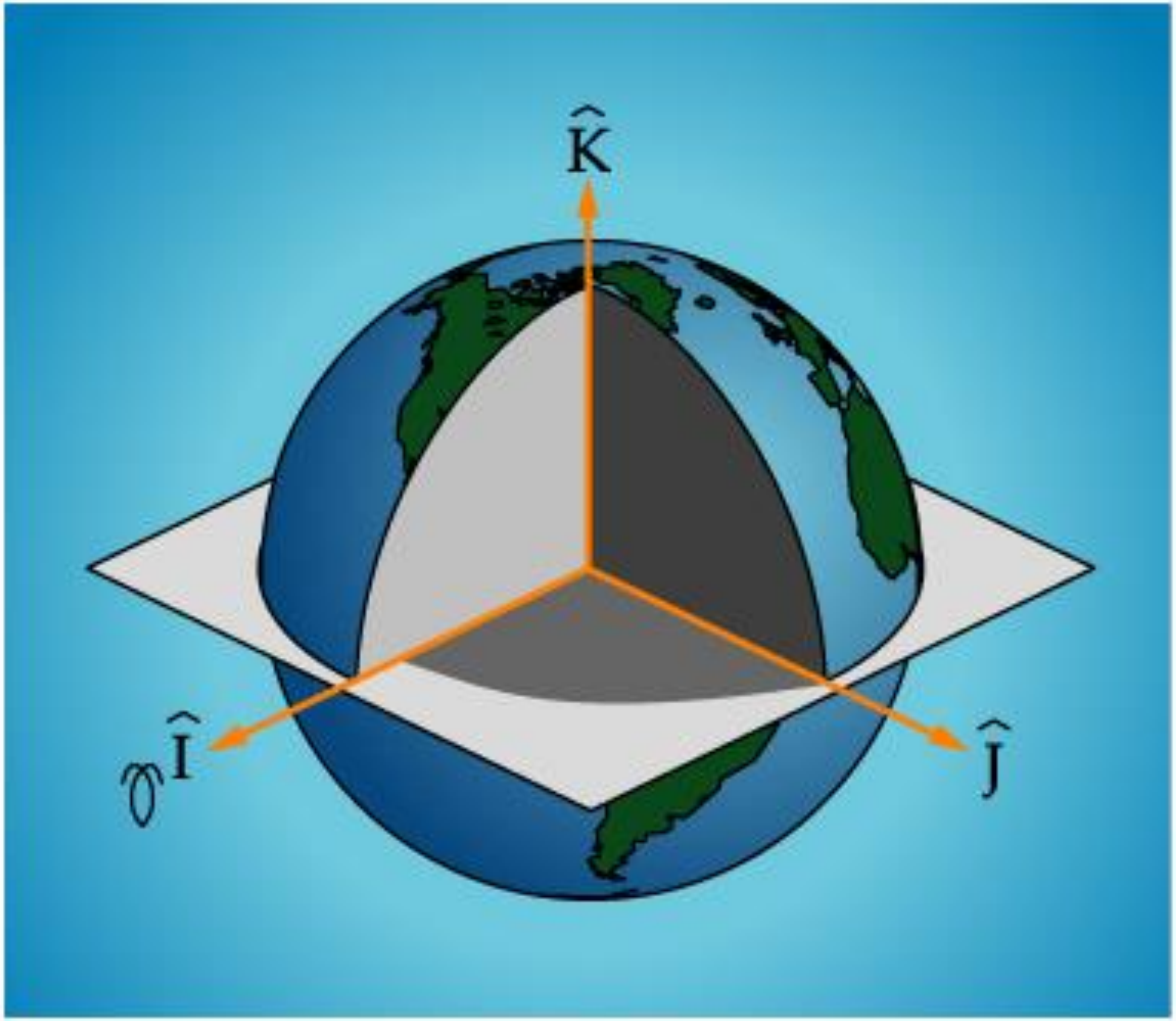


2a is the major axis
a is the semimajor axis

e= c/a is the eccentricity
$c^2 = a^2 - b^2$

Circle e=0
Ellipse 0<e<1
Parabola e=1
Hyperbola e>1

Defining a reference frame



♈ is the vernal equinox direction, or towards the constellation Aries (Væren). Obtained by drawing a line from the earth centre through the sun on the first day of spring.

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The six classical orbital elements (COE)

a = the semi-major axis
e = the eccentricity
i = the inclination
Ω = the right ascension of the ascending node
ω = the argument of perigee
ν = the true anomaly (time varying)

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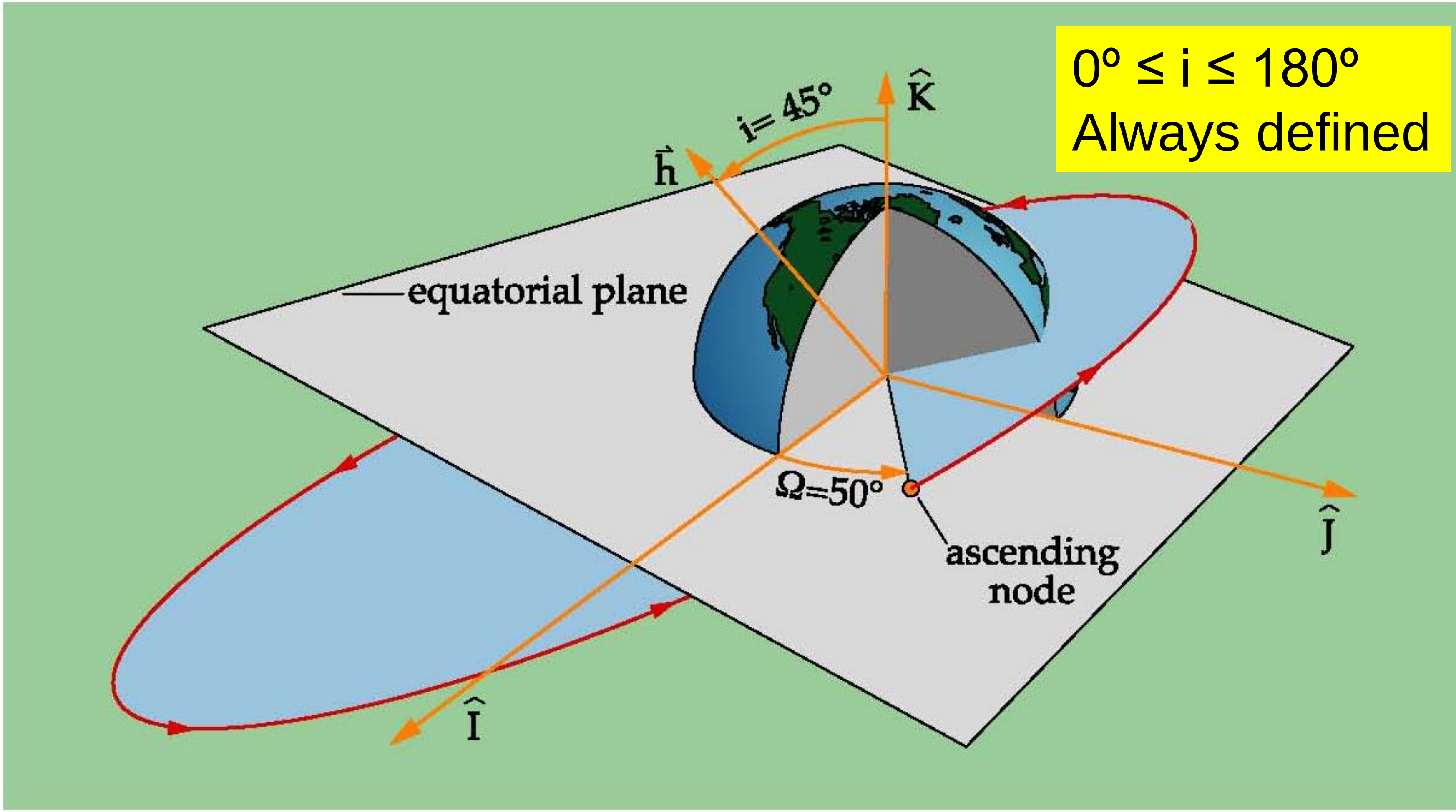
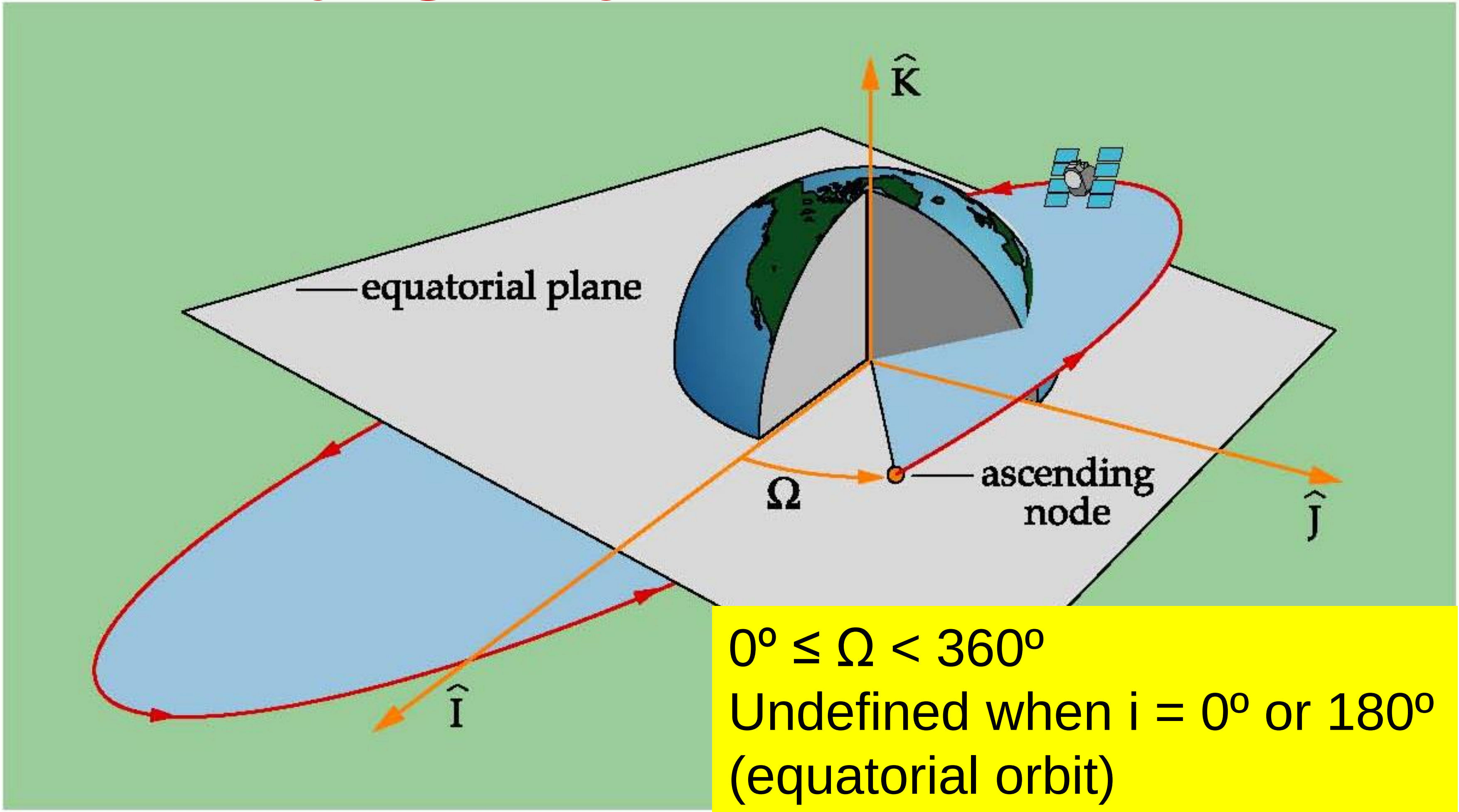
The inclination, i (degrees)



0º ≤ i ≤ 180º
Always defined

The right ascencion of the ascending node, Ω (degrees)



$0^{\circ} \leq \Omega < 360^{\circ}$
Undefined when $i = 0^{\circ}$ or 180°
(equatorial orbit)

På norsk: rektasensjon for oppadstigende knute

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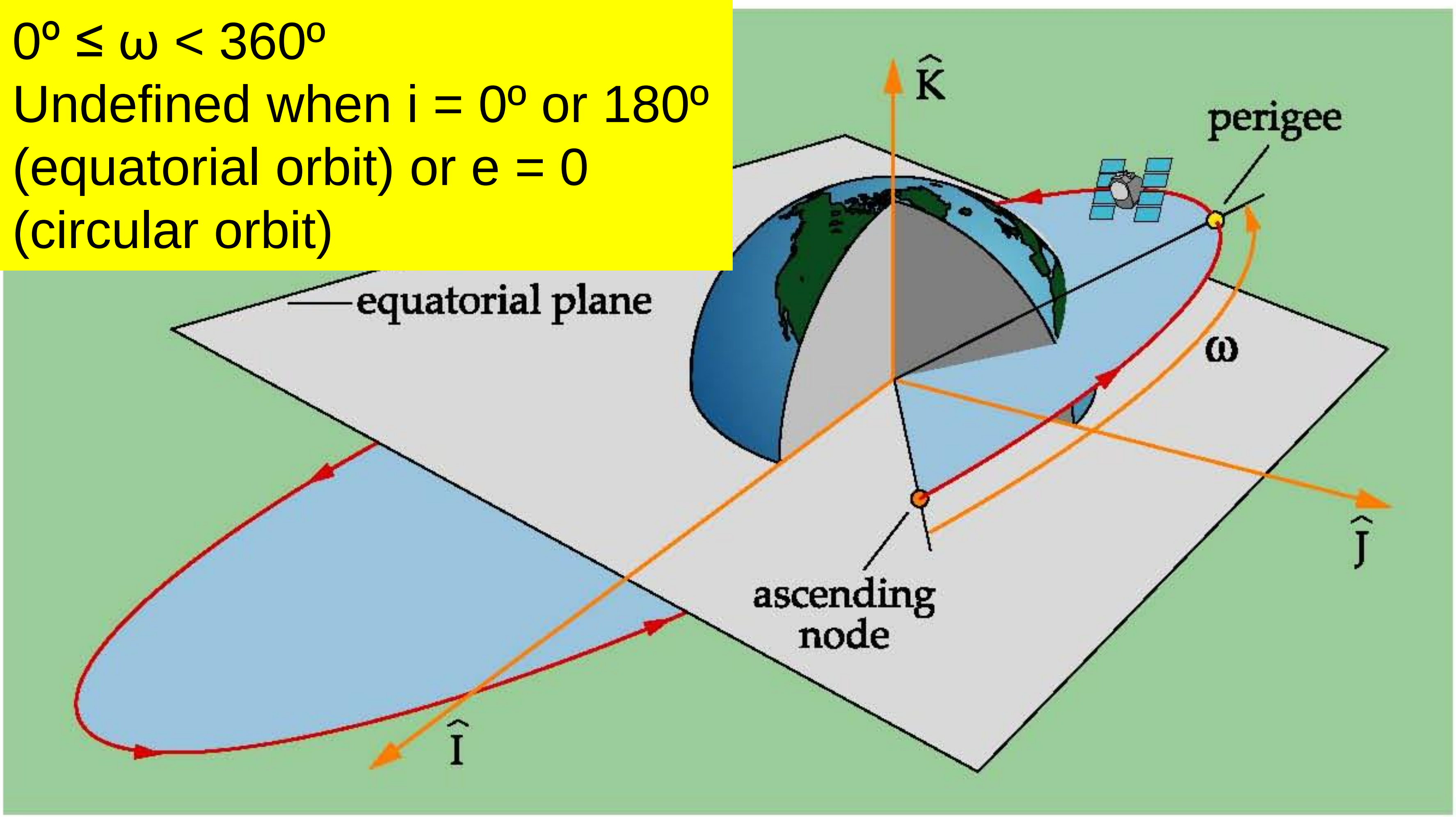
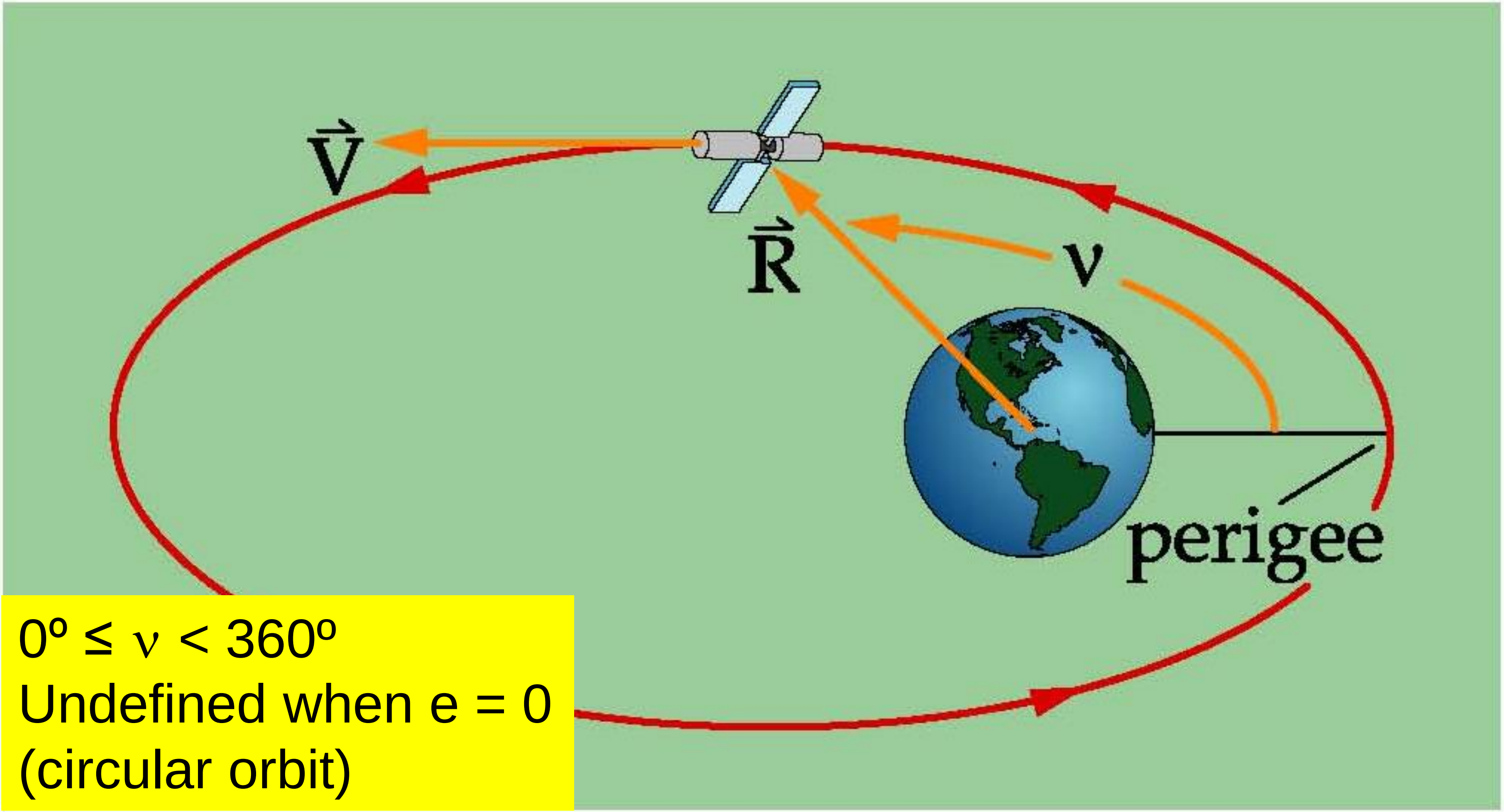
The argument of the perigee, ω (degrees)

$0^o \leq \omega < 360^o$
Undefined when $i = 0^o$ or 180^o (equatorial orbit) or $e = 0$ (circular orbit)



The true anomaly, ν (degrees)



$0^{\circ} \le \nu < 360^{\circ}$
Undefined when e = 0
(circular orbit)

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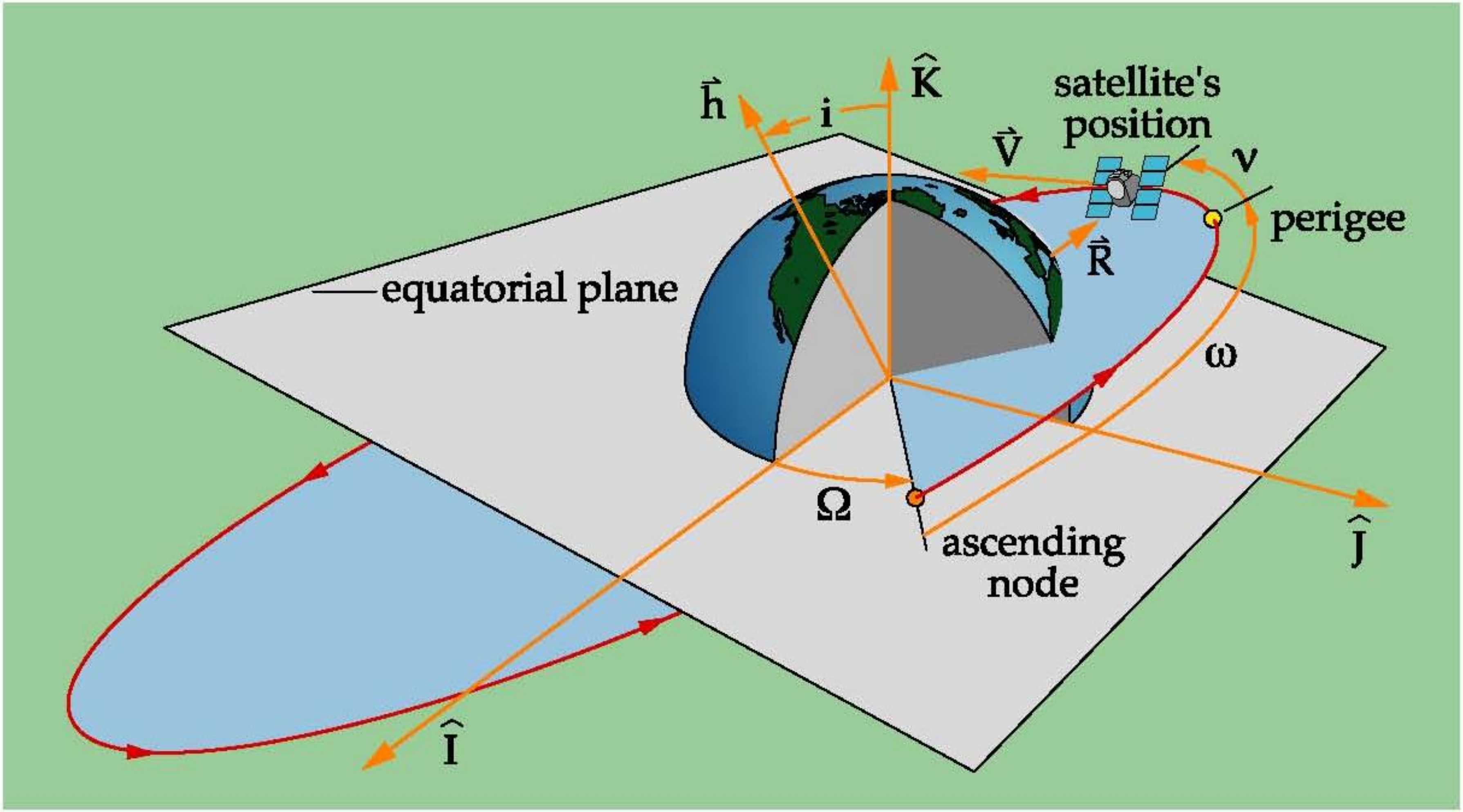
Four of the Classical Orbital Elements



The two last define the ellipse; a and e

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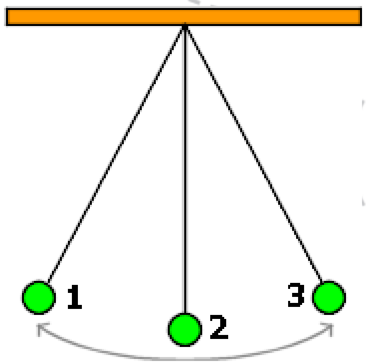
Kinetic and potential energy -> Finding a spacecraft's velocity

The mechanical energy of a body is the sum of the kinetic and the potential energy:

$$E = KE + PE \quad [kg\ m^2 / s^2]$$



$$KE = ½\ mv^2$$

$$PE = mah = -m\mu/R$$

By convention PE = 0 when R=∞, diving into a potential well as approaching earth, and at the centre of earth, with R=0, PE= - ∞

$$E = ½\ mv^2 - m\mu/R$$

In a conservative field, like a gravitational field, the total energy is conserved, i.e. E is constant.

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Finding a spacecraft's velocity (cont'd)

Departing from the expression of total mechanical energy:

$E = \frac{1}{2} mv^2 - m\mu/R$

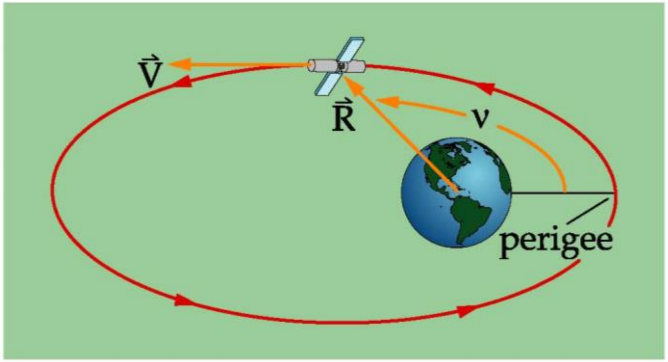
$v^2 = 2(\mu/R + E/m)$

It is possible to show (see Handbook chapter 4.1.4.2-3) that:

$E/m = - \mu/2a$

where a is the semimajor axis of an ellipse

$v^2 = 2\mu(1/R-1/2a)$



R is the distance from the spacecraft to the earth's center at any moment

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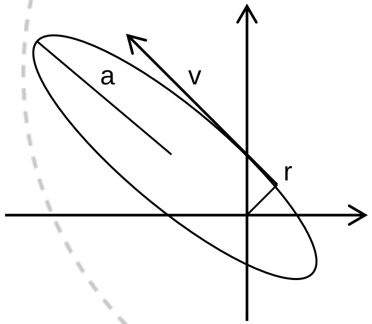
The two cosmic speeds



The first cosmic speed is the speed needed to set a spacecraft into orbit just above the earth's surface (assuming no atmosphere).

The second cosmic speed is the speed needed for a spacecraft to escape from earth, in order to never return.

We depart from the expression of the velocity: $v^2 = 2\mu(1/r-1/2a)$

1st cosmic speed: $a=r=R_E=6378$ km and $\mu=3.986 \cdot 10^5$ km^3/s^2
Giving $v_{C1}= 7.9$ km/s

2nd cosmic speed: $a \rightarrow \infty$ and $r=R_E$
Giving $v_{C2} = 11.2$ km/s

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Placing a spacecraft in orbit

Due to the speed needed to lift from earth and go into space, and the limitations of today's propellants:

- it is not yet possible to go into space with a one-stage rocket
- most of a rocket's weight at lift-off is propellant, the payload usually counts for just a few %
- propellant is therefore a very expensive part of the mission's budget and must be used with extreme care
- propellant is usually the limiting factor for a satellites life-time in space
- placing a spacecraft into orbit must then also be performed in the most economic way
- changes in speed are necessary, but changes in speed direction should be minimized or eliminated

The rocket equation: $\Delta v = I_{sp}\, g_0 \ln(m_i/m_f)$

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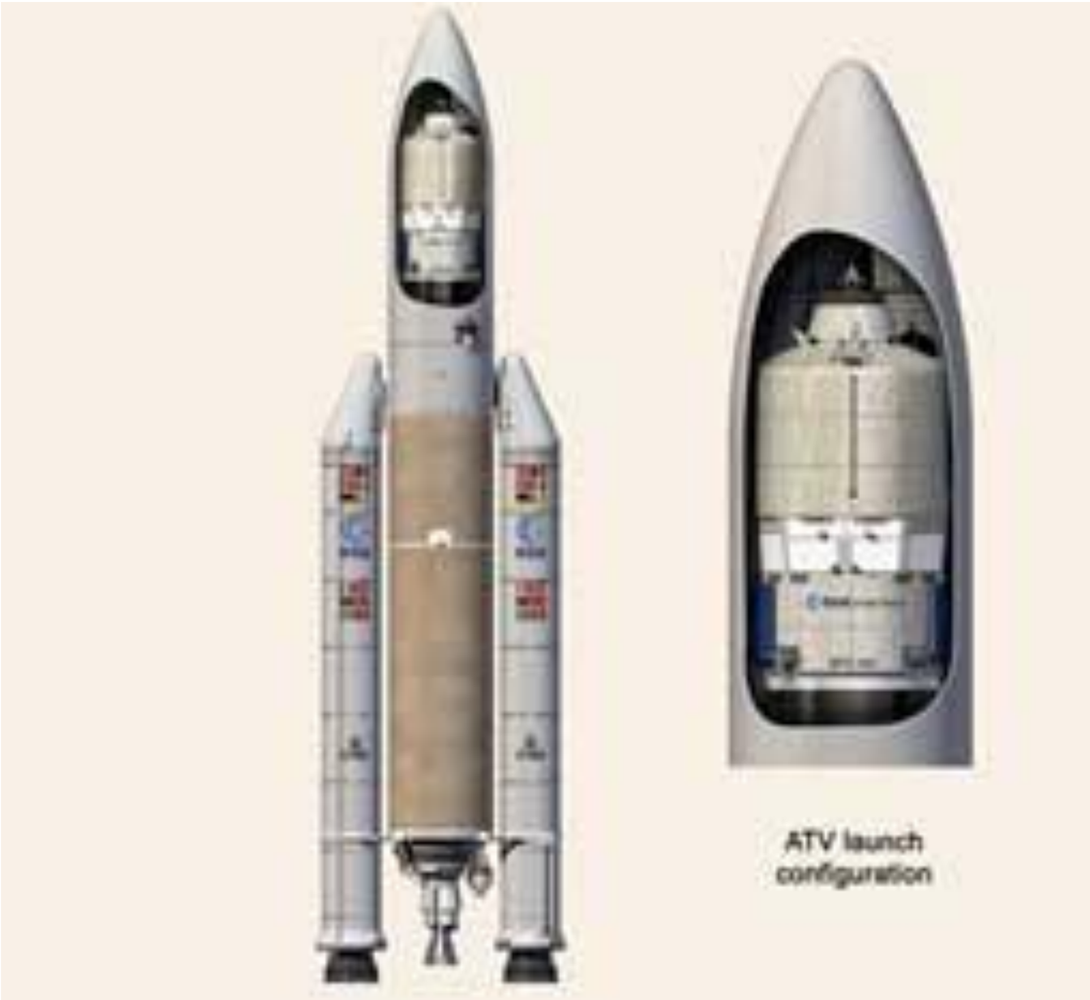
Ariane 5 example



Example:
With a conventional propellant, I_{sp} is about 400s.
The satellite mass is 2 tons. In order to obtain Δv = 10km/s, two stages are needed, and the total mass at launch is calculated to be about 125 tons, so the payload only counts for about 2% of the total mass.

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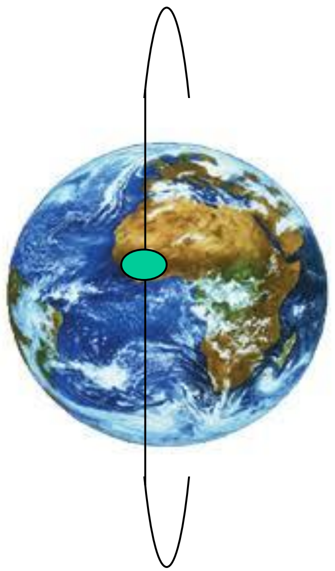
When and where to launch?



An orbit is fixed in space, the earth rotates below it. For a polar orbit, the same spot on earth will be right below the orbit twice per day, so in order to launch into this polar orbit from that place, there will be a launch window twice per day, twelve hours apart.

Or actually not twelve hours apart, but twelve sidereal hours apart.

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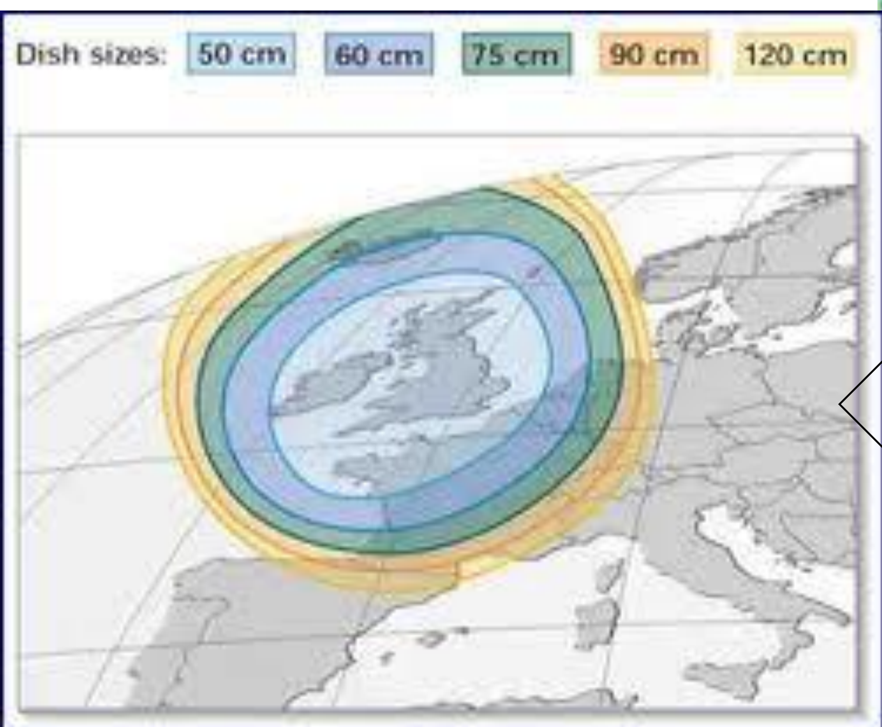
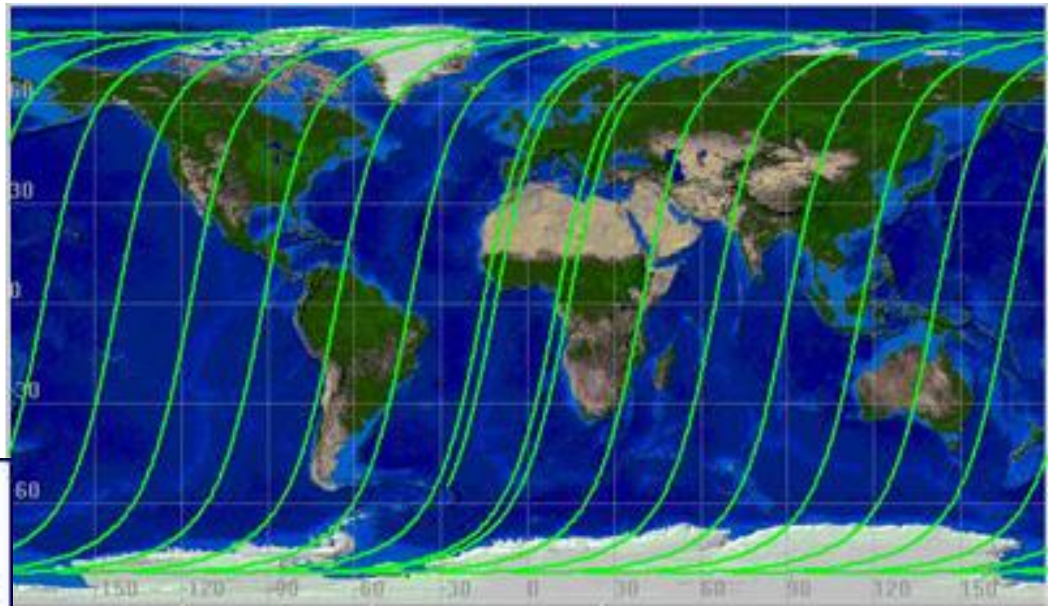
Satellite ground track and footprint definitions

Footprint





Groundtrack

NB: Dish size indicates the dish size needed for reception of the satellite signal on earth as you move towards the edge of the footprint, not the footprints as a function of different transmitter antenna sizes on the satellite.

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http://spot5.cnes.fr/gb/systeme/310.htm

Launch window

In order to launch ”directly” into orbit, your launch site must be situated directly below the satellite orbit. Therefore you may only launch satellites into orbits with higher inclination than your launch site's latitude;

$$L_0 \leq i$$

If $L_0 < i$, then you have two launch windows, the ascending and the descending opportunity.

If $L_0 = i$, then you have only one launch window, at 90º from the ascending node.

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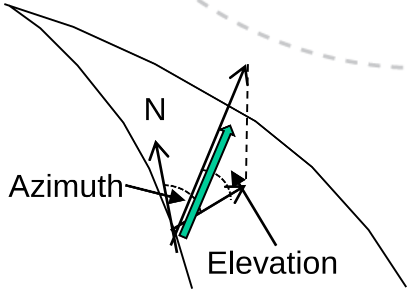
The influence of earth’s rotation

In addition to the sidereal times giving the launch windows, the earth’s rotation must be taken into account in order to calculate the right launch angles (elevation and azimuth) to get the spacecraft into orbit.



It is advantageous to launch GEO satellites from the equator, as you get ”free” help from earth’s rotation: 0.464 km/s from Kourou at L_0= 4º, vs. 0.4087 km/s from Kennedy Space Center at L_0= 28.5º.

In case you are not able to launch directly into orbit, you must use fuel to adjust the orbit. One clever way to optimize such adjustments, is by using transfer orbits.

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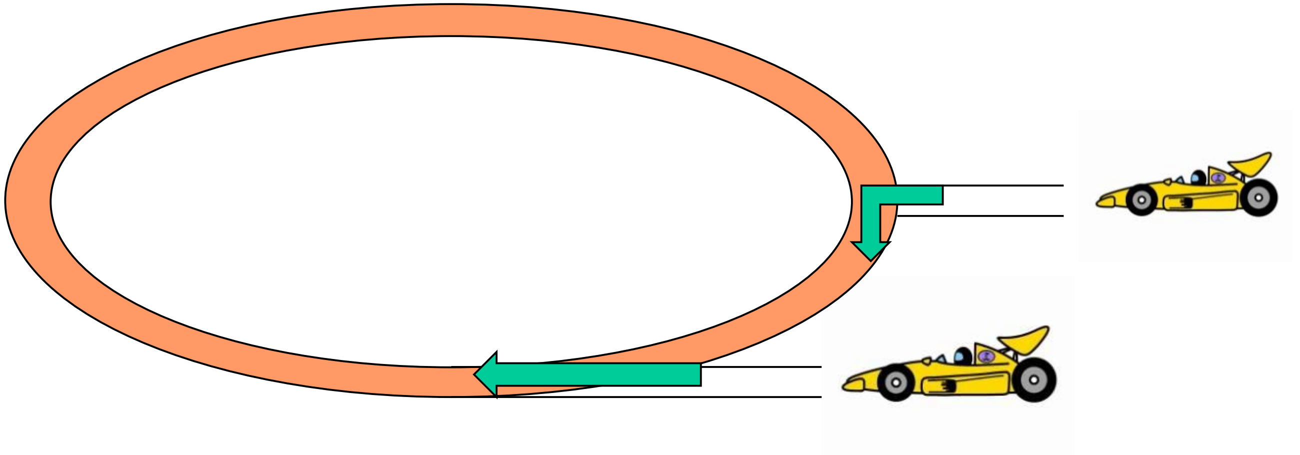
What is the best way of going into, or changing orbit?



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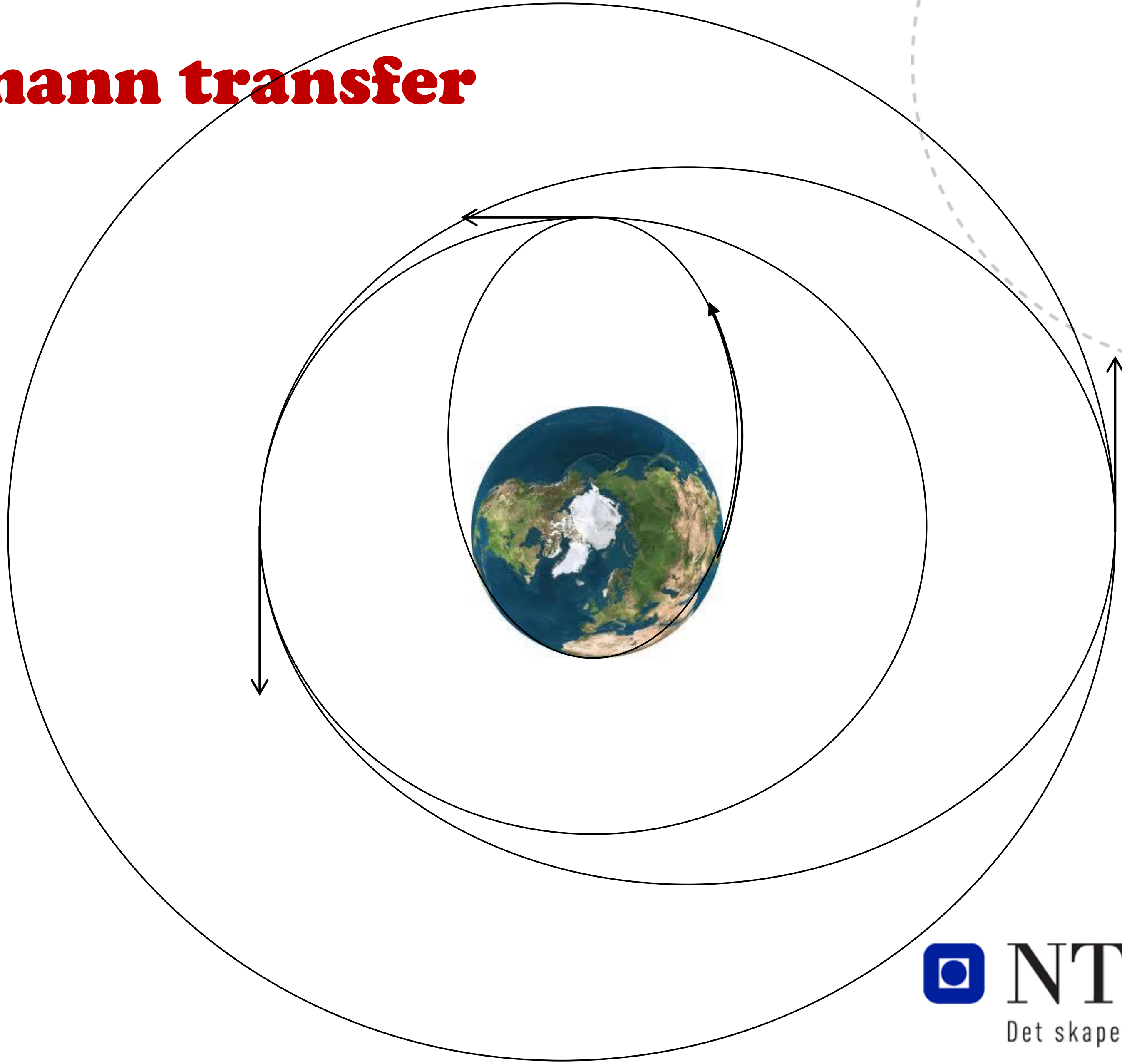
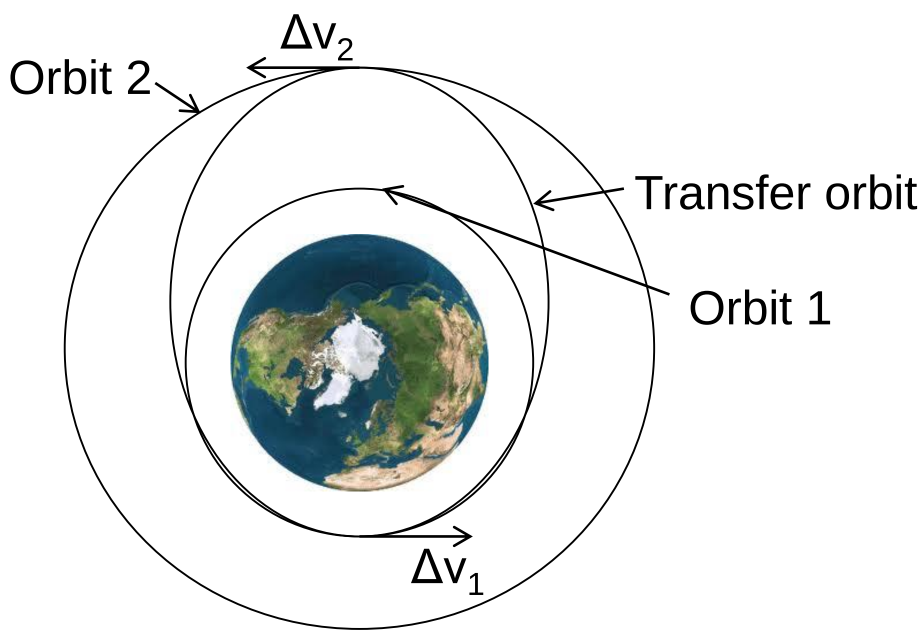
Hohmann transfer



Velocity change for transfer



$$\Delta v_1 = v_{to} - v_{o1}$$

$$\Delta v_2 = v_{o2} - v_{to}$$

$$\Delta v_{total} = \Delta v_1 + \Delta v_2$$

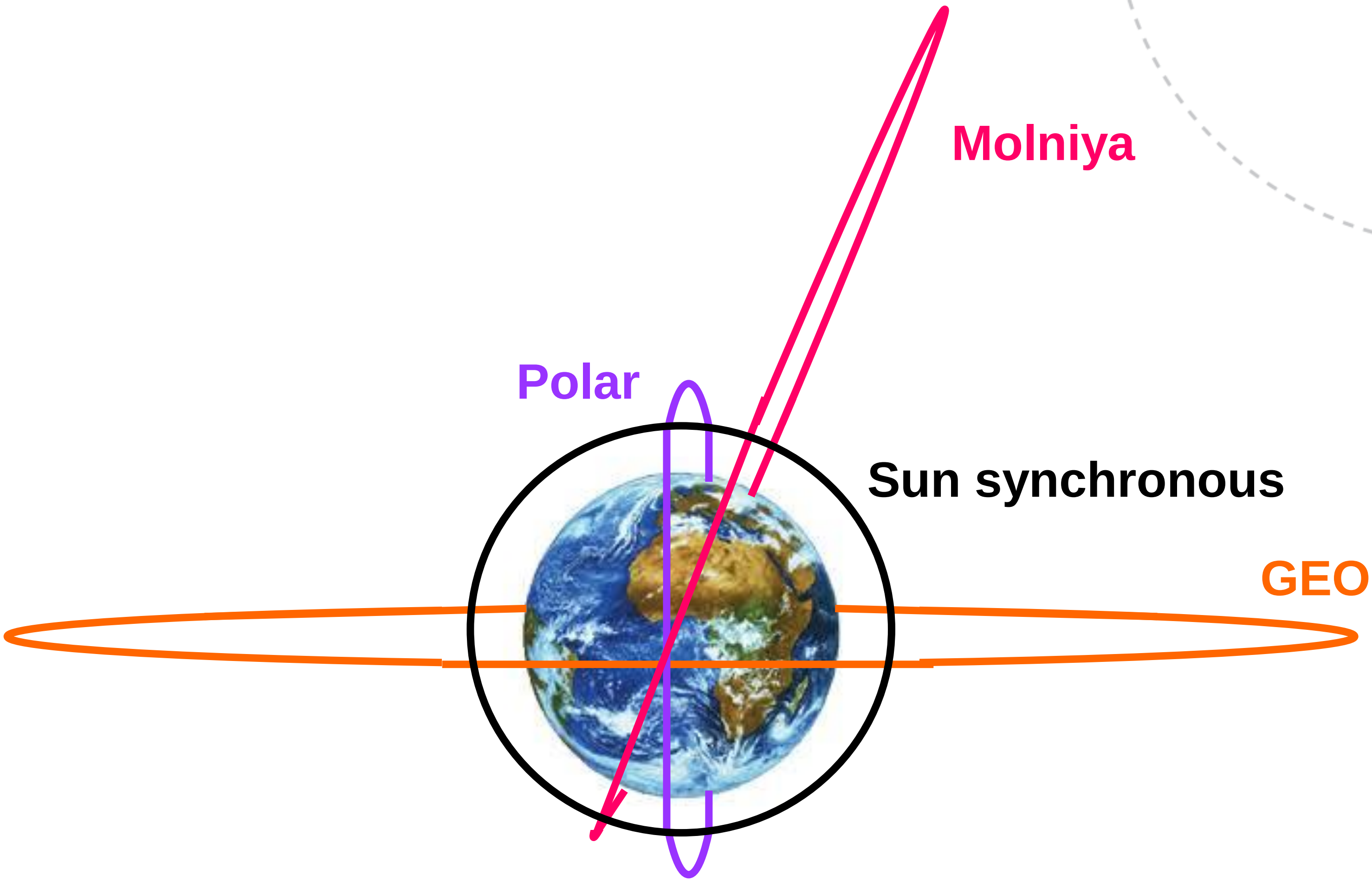
$$v^2 = 2\mu(1/R - 1/2a)$$

R is the distance from the spacecraft to the earth's center at any moment and a is the semimajor axis

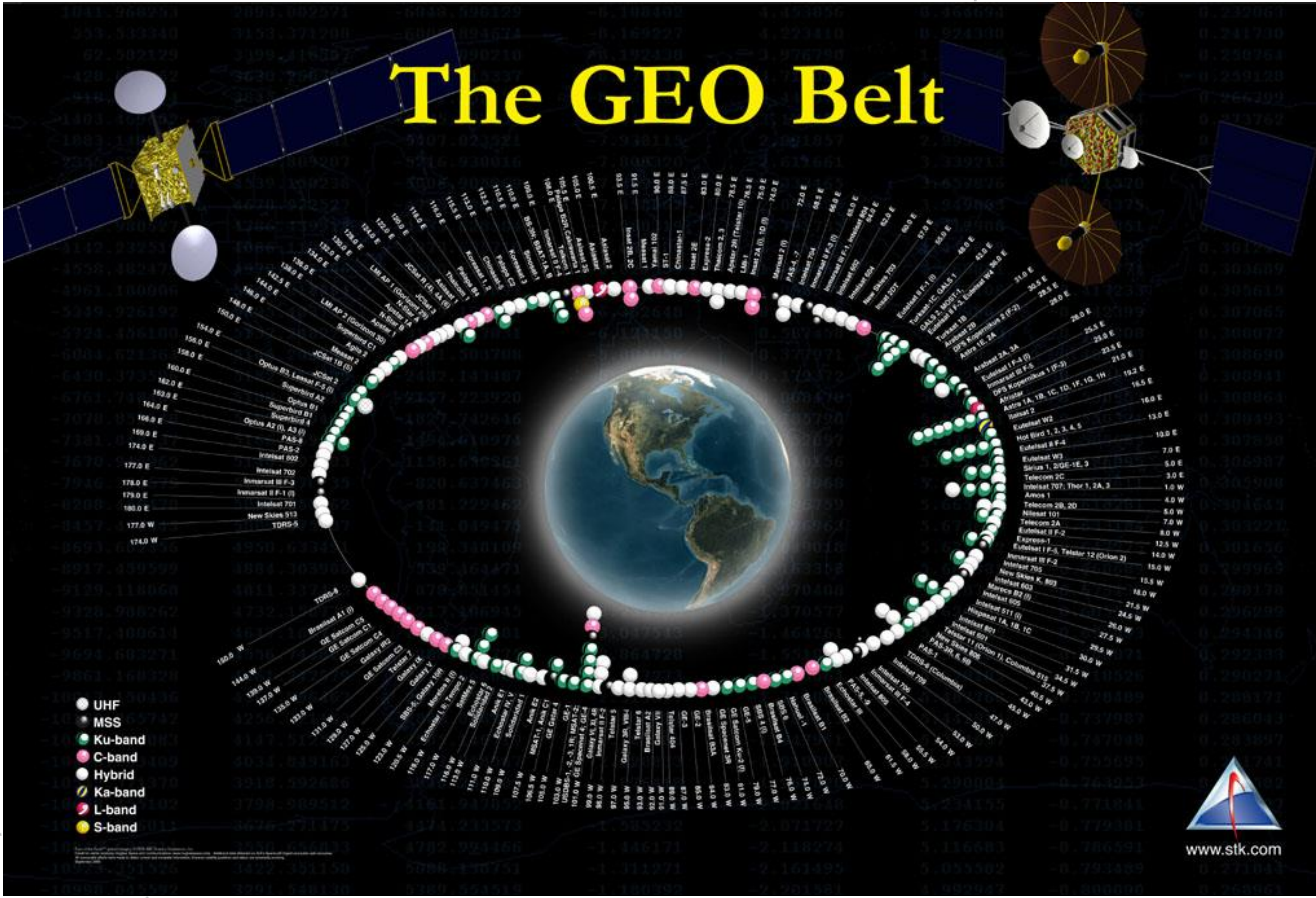
Then you must use the rocket equation to find out how much fuel you need for transfer:

$\Delta v = I_{sp}\, g_0 \ln(m_i/m_f)$, as $m_i = m_f + m_p$

Some particular orbit types



The GEO orbit is a limited natural resource

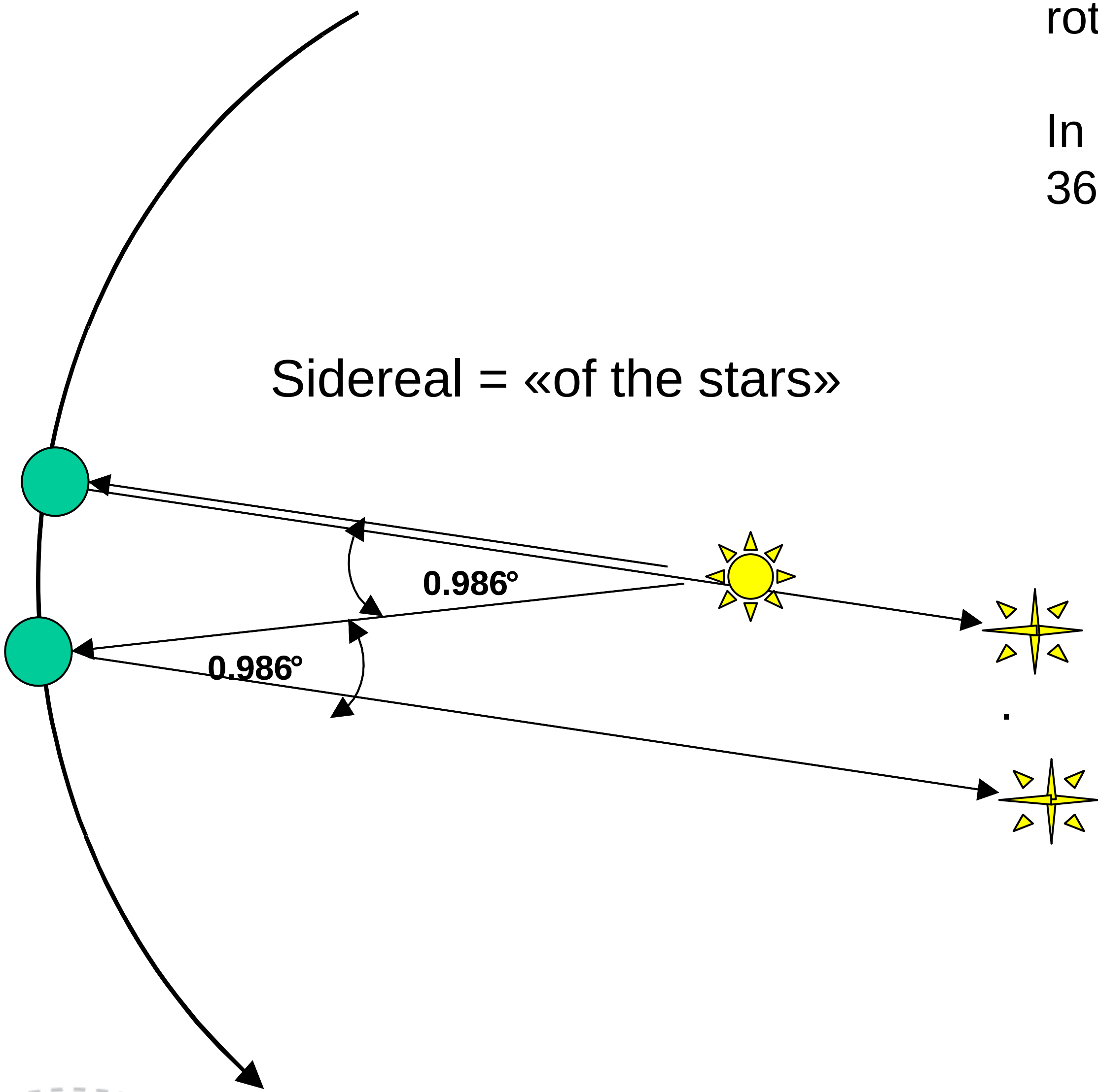


Sidereal time

In one sidereal day, the eart must rotate 360º

In one solar day, the earth must rotate 360º + 0.986º

Sidereal = «of the stars»



The orbital period of a GEO satellite is one sideral day, T=86164 s.

$$a = \sqrt[3]{\mu\left(\frac{T}{2\pi}\right)^2} = 42164km$$

$$v = \sqrt{\frac{\mu}{42164}} = 3.075km/s$$

with μ = $3.986 \cdot 10^5$ km^3/s^2

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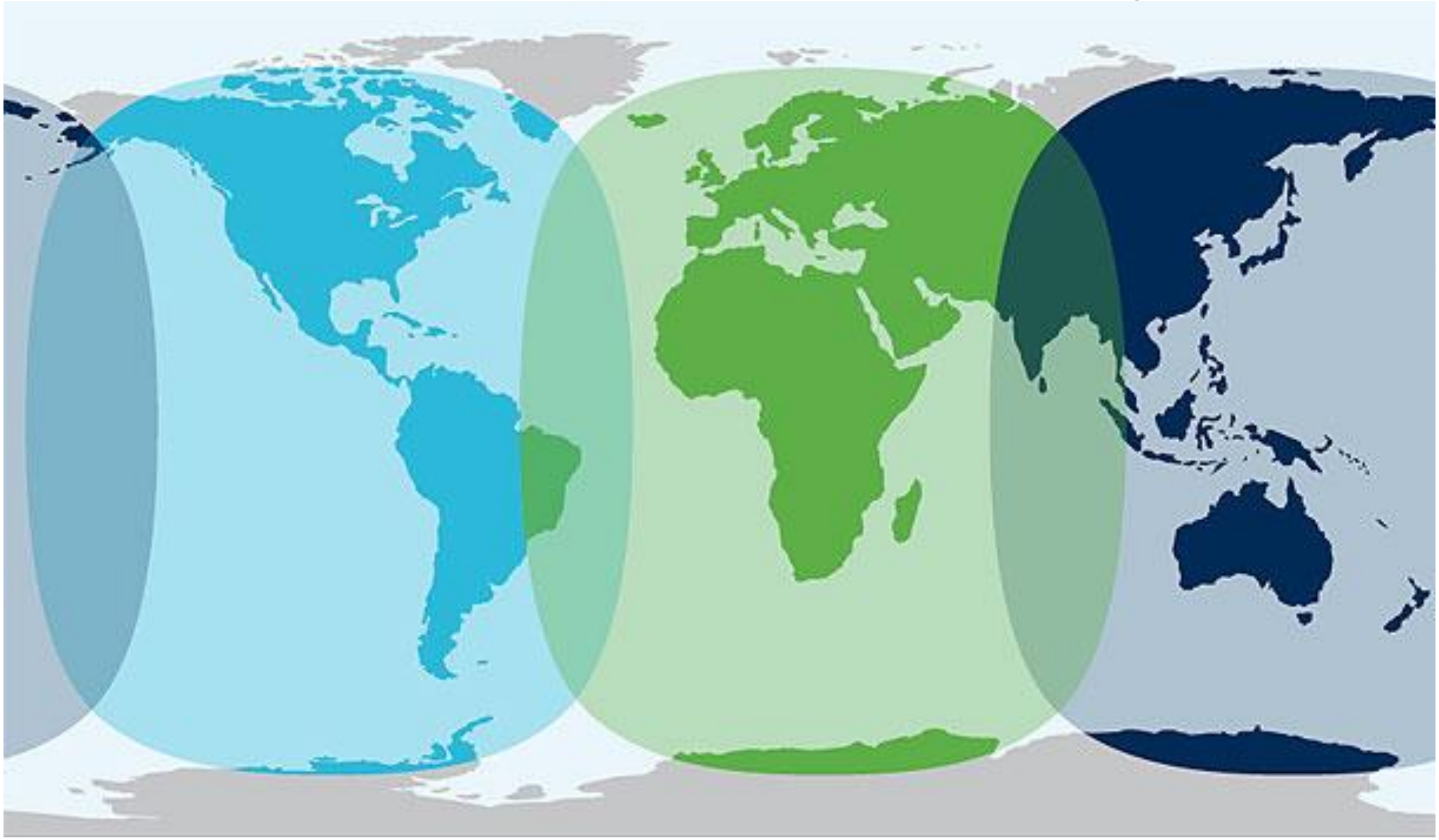
Typical coverage from GEO (example: Inmarsat)



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Orbital perturbations

- gravitational effect – third body (sun, moon,…)
- gravitational effect – oblateness of the earth
- atmospheric drag for LEO
- solar radiation pressure
- unexpected thrusting – outgassing or malfunctioning

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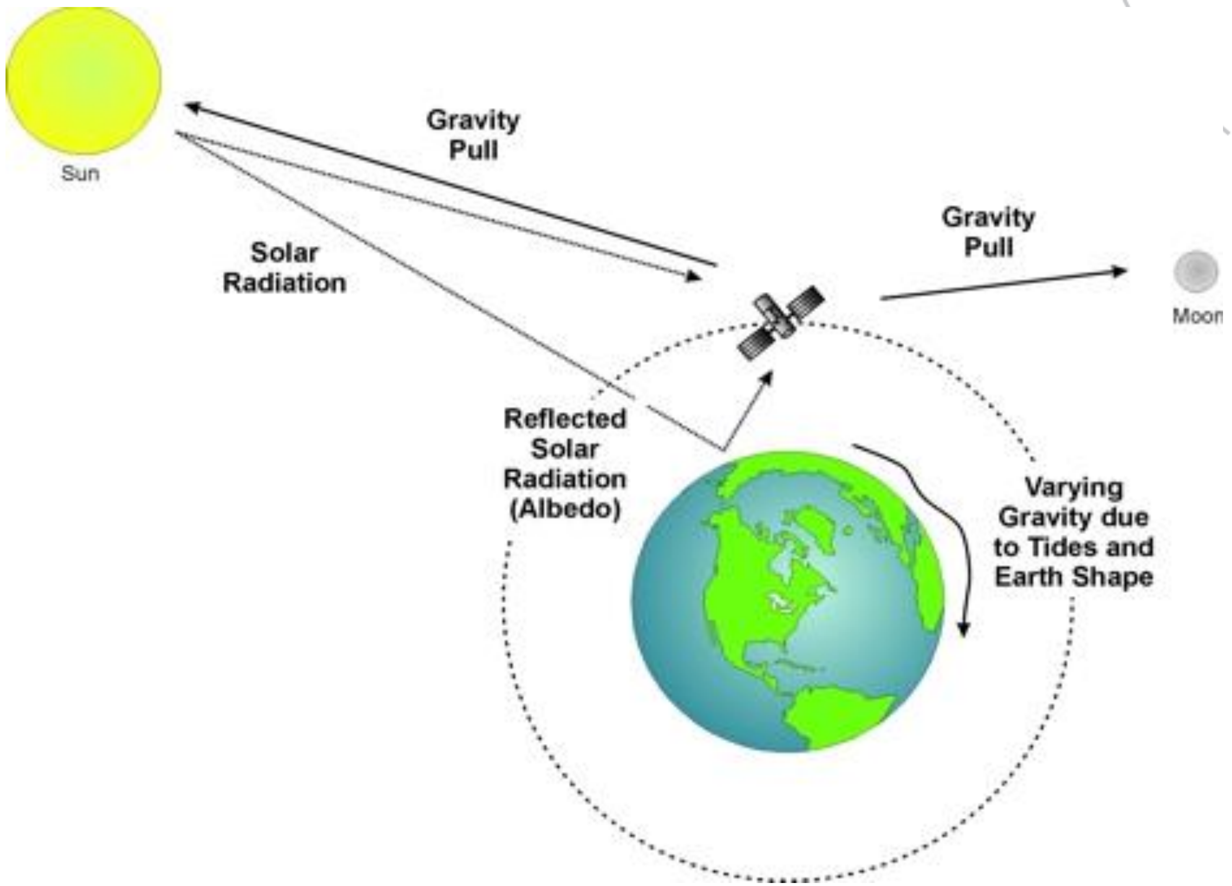
Gravitational effects



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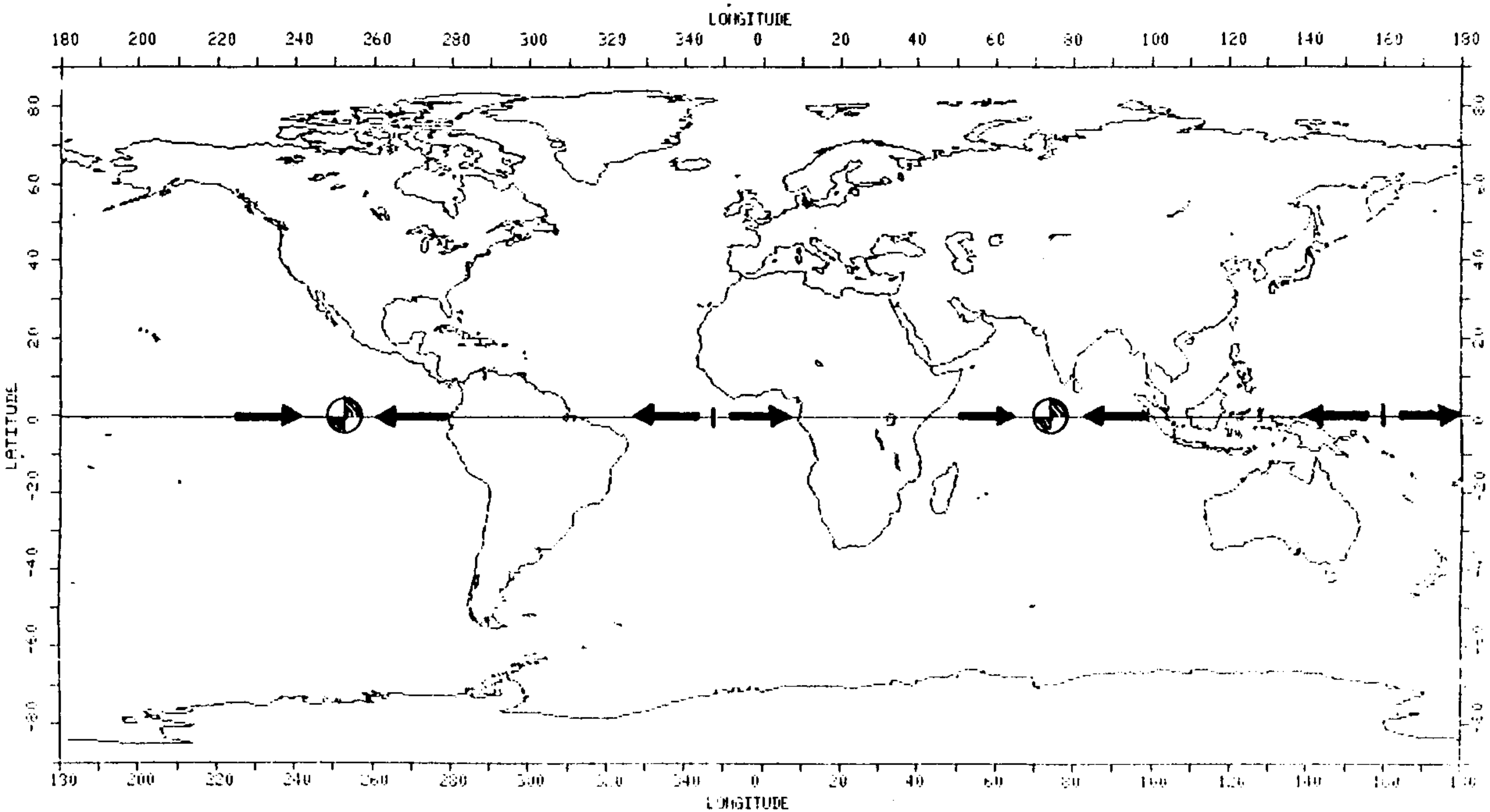
Effect on a GEO satellite



Due to the uneven repartition of the earth's mass, there will be tangential components giving an east-west drift to the satellites in GEO orbit. South of India, and in the east part of the Pacific, the positions are stable, whereas they are unstable above the Atlantic and north of New Zealand.

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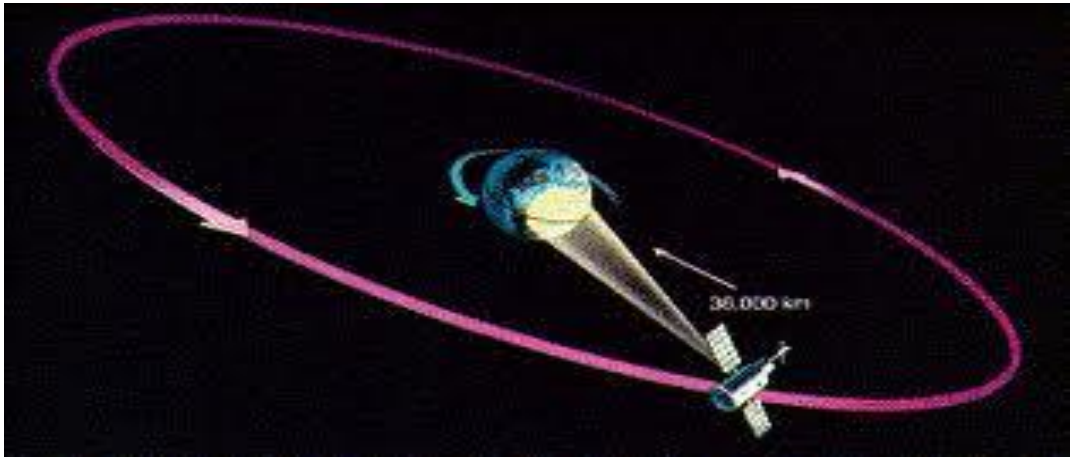
Changes in the inclination, i

8 if the orbit is circular

if the orbit is elliptic

A typical shift in inclination is about 1º per year.

Geosynchronous orbit (GEO)



Summary of the GEO orbit:

- The satellite is above a fixed point on earth at any time
- Then T is fixed to one sidereal day, giving only one possible R (~42000km from earth's center)
- The COE are a=r, and e=0. i=0º and the last three parameters are not defined
- The orbit is subject to nodal regression giving east-west drift
- A drift in the inclination will give an apparent 8-movement from earth and will lead to pointing error

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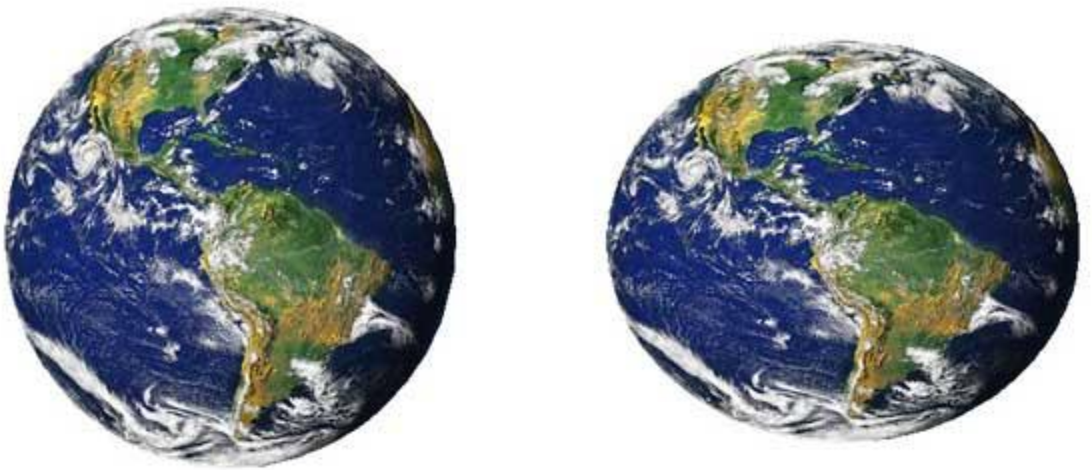
Earth's oblateness



Due to earth's rotation, and earth not being a solid, stiff ball, the radius at equator is larger than at the poles, by approximately 22 km. The effect is described mathematically as the J2 effect.

Effects on a satellite (on the 6 COE):

- the gravitational field is conservative, so the mechanical energy will remain constant
- hence, since $E/m = -\mu/2a$ and m is constant, then a remains constant
- e will remain constant (more difficult to show)
- i remains constant
- but Ω and ω will change

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http://www.jpl.nasa.gov/releases/2002/release_2002_217.cfm

The gravitational potential

The deviation from sphere symmetry can be expressed as:

$$U = -\frac{\mu}{r}\sum_{n=2}^{\infty} J_n \left(\frac{R_E}{r}\right)^n P_n(\cos\phi)$$

The most significant element for the deviation is for n=2:

$$J_2 = 1{,}0826\times 10^{-3} \qquad P_2(\cos\phi) = \frac{1}{2}\left[3\sin^2(\varphi+\omega)\sin^2 i - 1\right]$$

The consequence of the earth's oblateness then becomes:

$$\boxed{\frac{d\Omega}{dt} = -\frac{3}{2}J_2\left[\frac{R_E}{a(1-e^2)}\right]^2\cdot\sqrt{\frac{\mu}{a^3}}\cdot\cos i} \qquad \boxed{\frac{d\omega}{dt} = -\frac{3}{4}J_2\left[\frac{R_E}{a(1-e^2)}\right]^2\cdot\sqrt{\frac{\mu}{a^3}}\cdot\left(5\cos^2 i - 1\right)}$$

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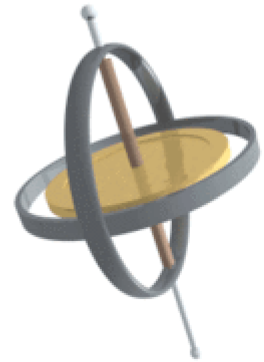
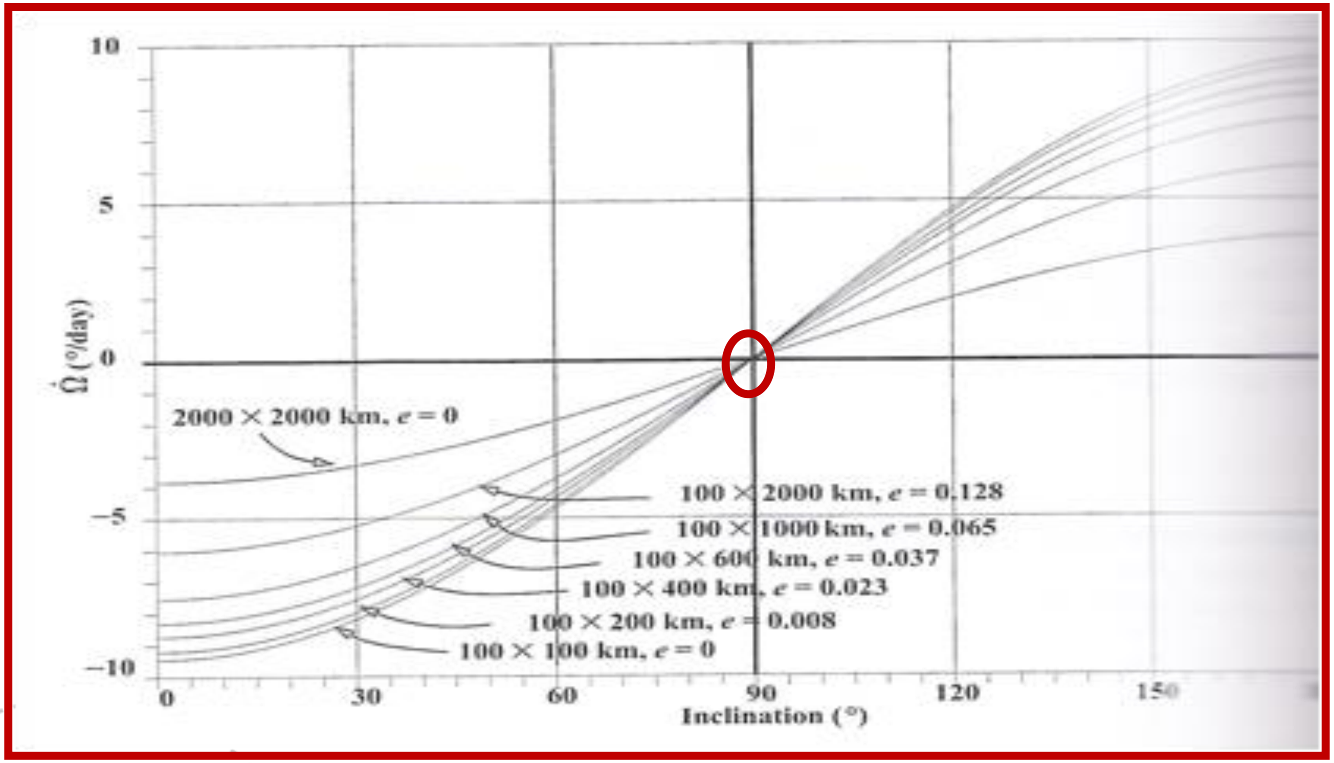
Effect on Ω

As the gravitational force does not come exactly from the earth's center, the orbit will start precessing, resulting in a change of the ascending node, a $\Delta\Omega(t)$.
The change is westward for i<90º, zero for i= 90º and eastward for i>90º.





The nodal regression rate $d\Omega/dt = f(i,a,e)$

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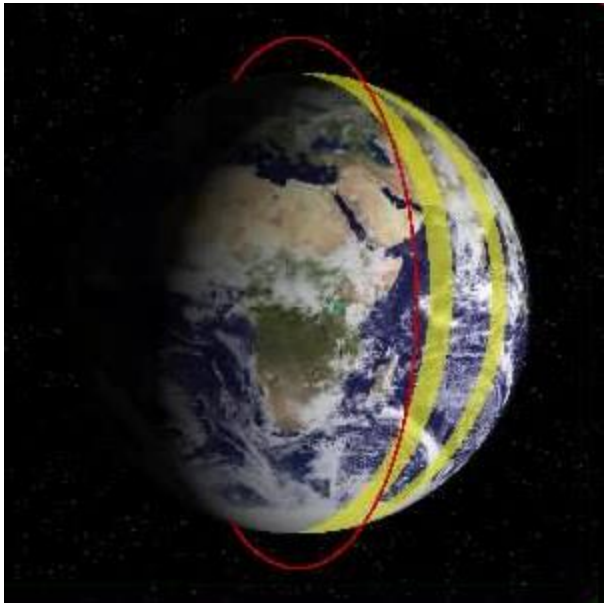
Polar orbit



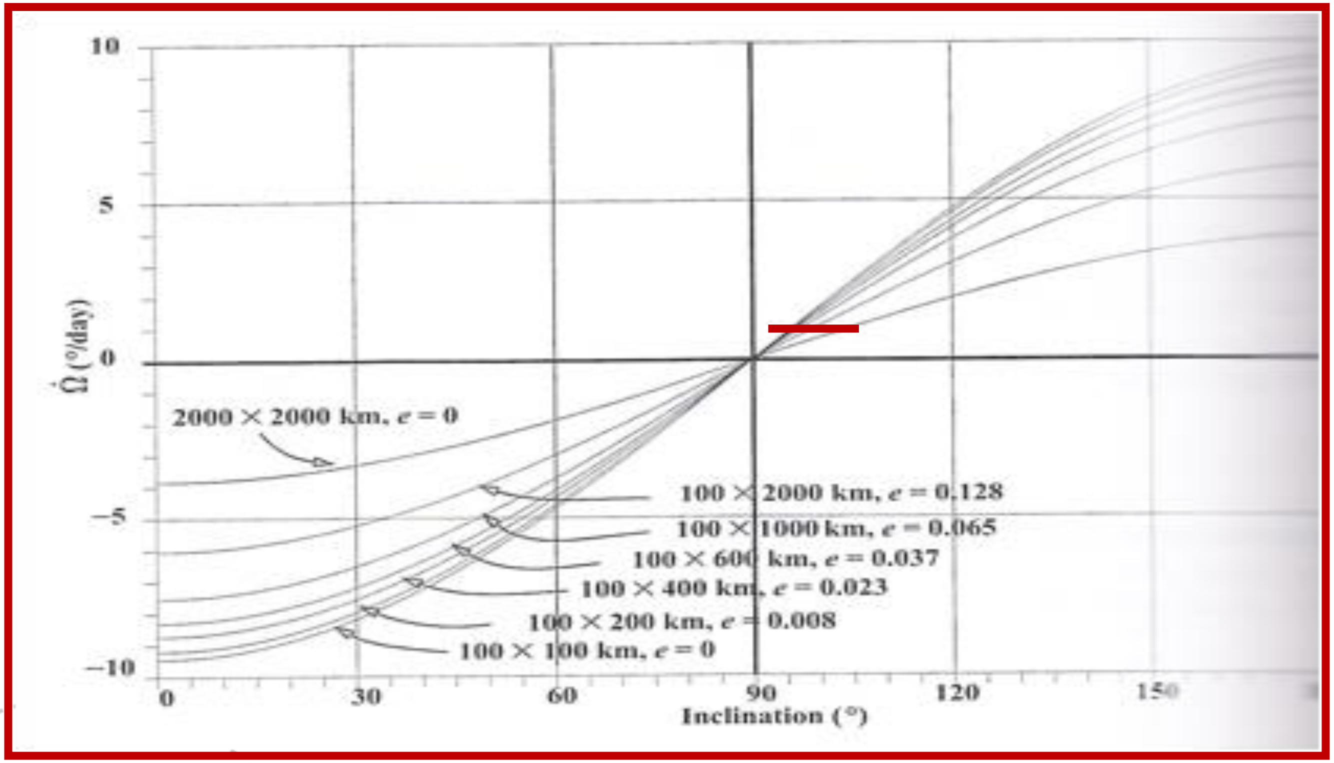
Key points on the Polar orbit:

- Will sweep the entire surface of the earth as the earth rotates below
- Any T can be chosen
- The COE are a=r, and e=0. i=90º. Ω can take any value, but ω and ν are not defined
- The orbit is not subject to nodal regression as:

$$\frac{d\Omega}{dt} = -\frac{3}{2} J_2 \left[\frac{R_E}{a(1-e^2)}\right]^2 \cdot \sqrt{\frac{\mu}{a^3}} \cdot \cos i \;\; = 0 \; for \; i = 90^o$$

Effect on Ω

As the gravitational force does not come exactly from the earth's center, the orbit will start precessing, resulting in a change of the ascending node, a $\Delta\Omega(t)$.
The change is westward for i<90º, zero for i= 90º and eastward for i>90º.



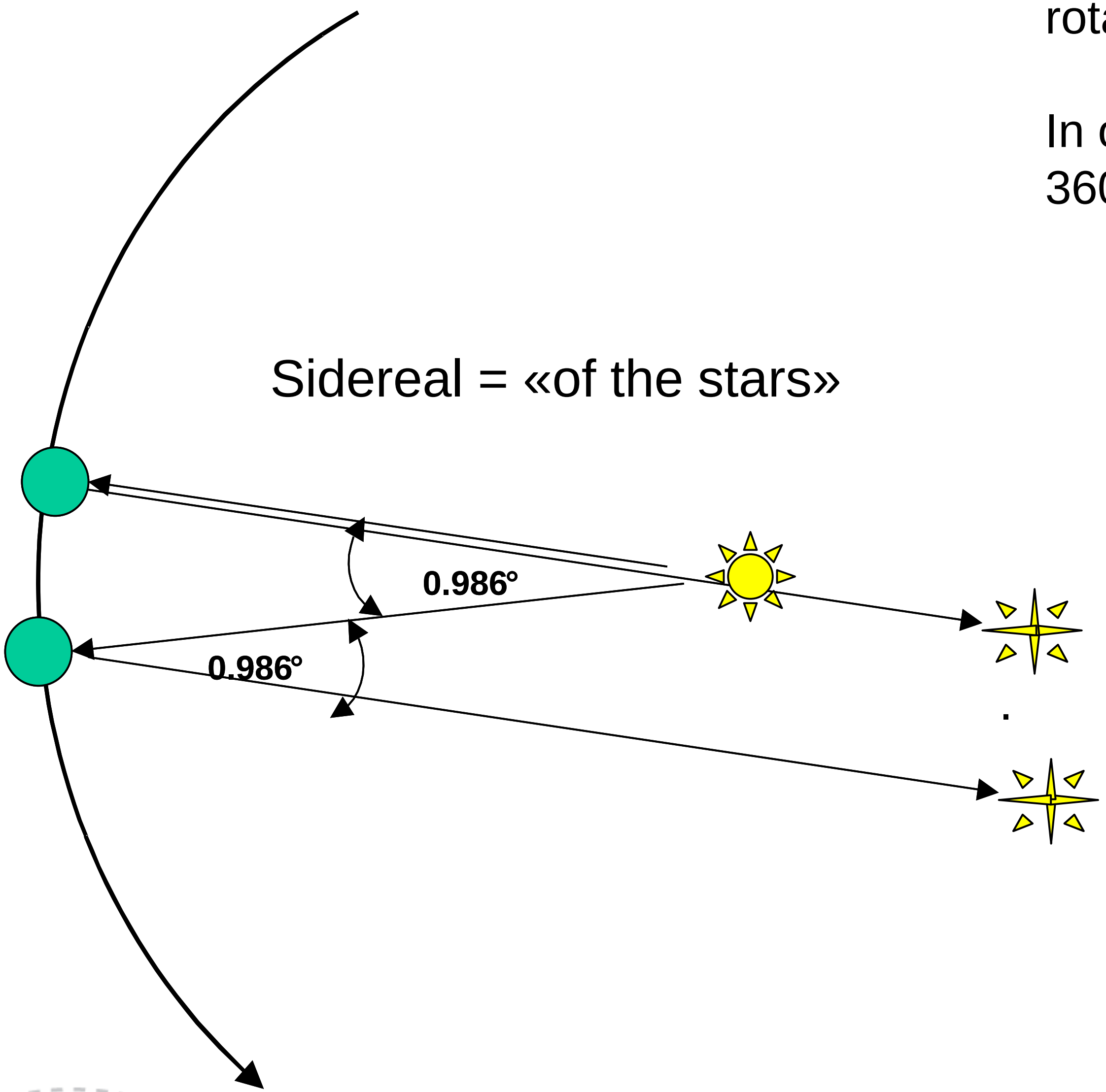
The nodal regression rate $d\Omega/dt = f(i,a,e)$

Sidereal time

In one sidereal day, the eart must rotate 360º

In one solar day, the earth must rotate 360º + 0.986º

Sidereal = «of the stars»



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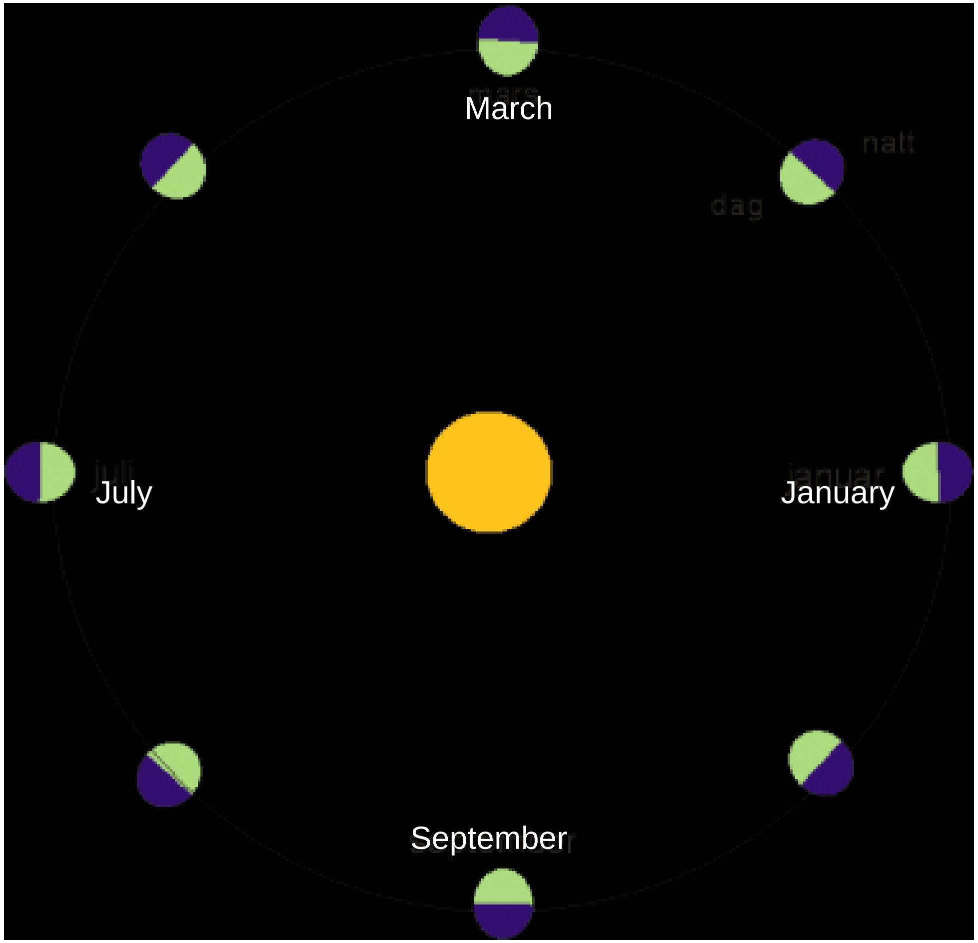
Sun synchronous orbit over a year



Two extreme types:

Morning/evening

Day/night

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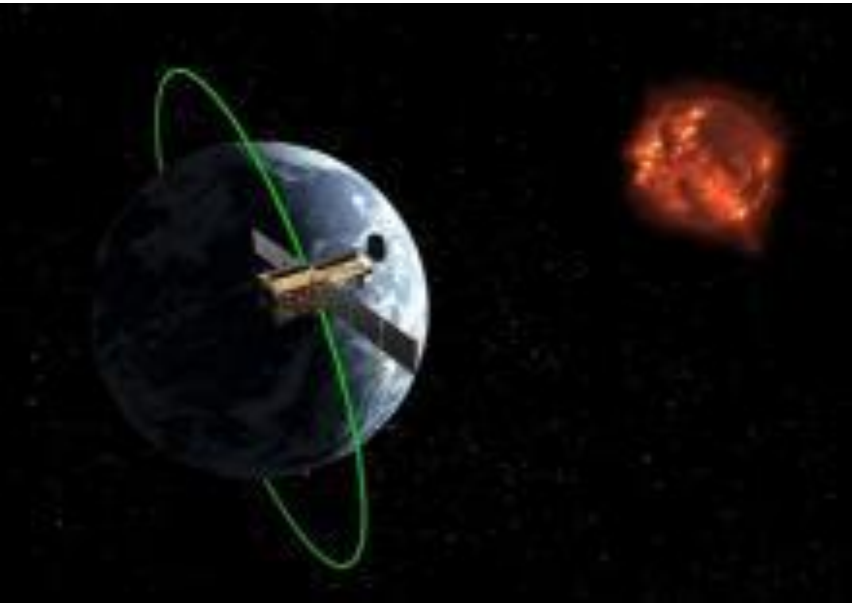
Sun synchronous orbit

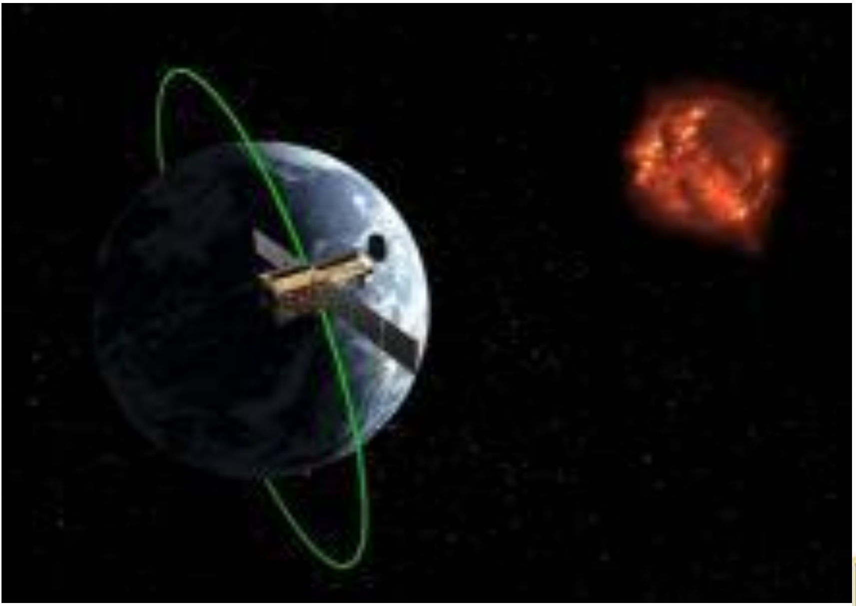


Key points on the Sun synchronous orbit:

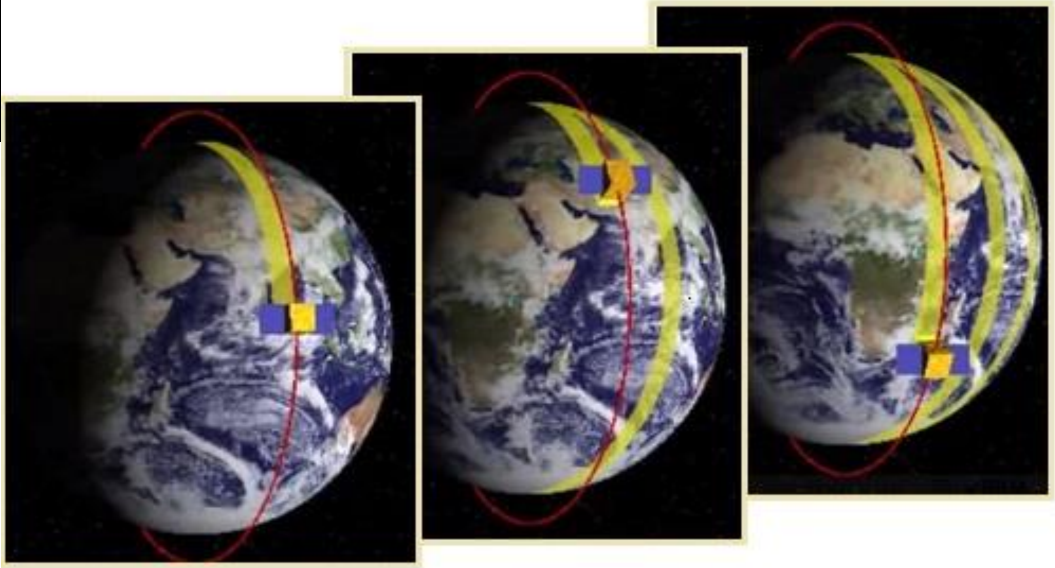
- The satellite should be above the same point on earth at the same local time at certain intervals (e.g. every day)
- This is obtained by choosing the nodal regression rate $d\Omega/dt = 0.986^{o}$, in this way the orbit will drift just as much as the earth has to turn in addition to the 360^{o} per day in order to always face the sun.
- Then i has to be approximately 98^{o}.
- The choice of the period T, will give the orbital element a=R.

- In addition, a Sun synchrounous satellite can be placed in a morning-evening orbit never entering the earth shadow

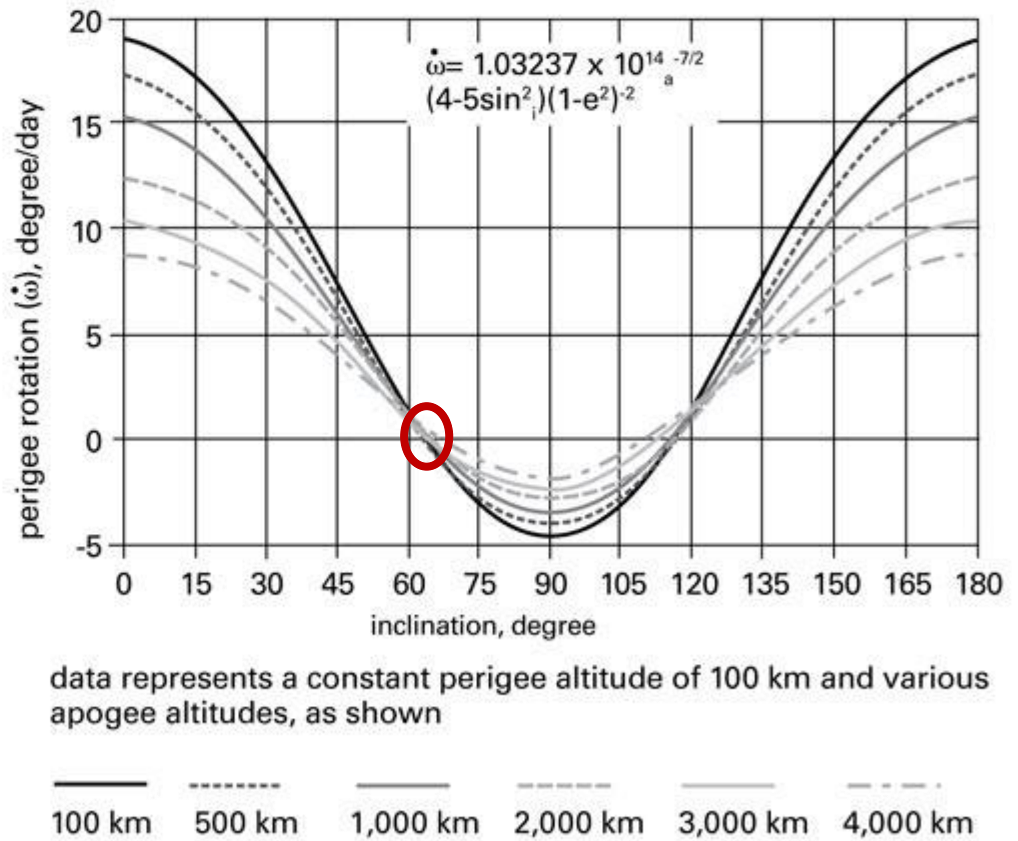
Morning-evening orbit



Day-night orbit



Effect on ω, the argument of the perigee



Source: Jerry J. Sellers et al., *Understanding Space: An Introduction to Astronautics*, 3d ed. (New York: McGraw-Hill, 2005), figure 8–11.

Note: it is possible to design particular orbits from these effects; the Polar orbit, the Sun-synchronous orbit and the Molniya orbit.

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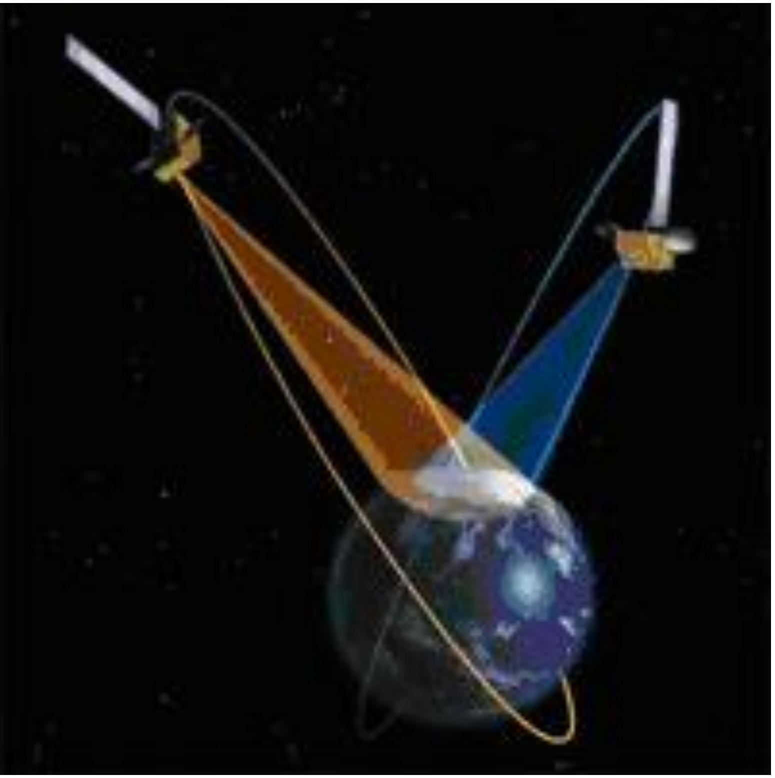
Molniya orbit



Key point on the Molniya orbit:

- The satellite should be above a fixed point on earth for a long time (~11h); pseudo-stationary
- It is a special case of a HEO, a Highly Elliptical Orbit
- It was “invented” by the Russians (molniya=lightning)
- One must then choose an orbit with no perigee drift, Δω/Δt=0, i.e. i=63.4º
- Then T must be chosen to be 12 sidereal hours, then the orbital parameter are given (the apogee is then ~46000km from earth’s center, and the perigee ~7000km)

$$\frac{d\omega}{dt} = -\frac{3}{4} J_2 \left[\frac{R_E}{a(1-e^2)} \right]^2 \cdot \sqrt{\frac{\mu}{a^3}} \cdot \left(5\cos^2 i - 1 \right) = 0 \text{ for } i = 63.4^o$$

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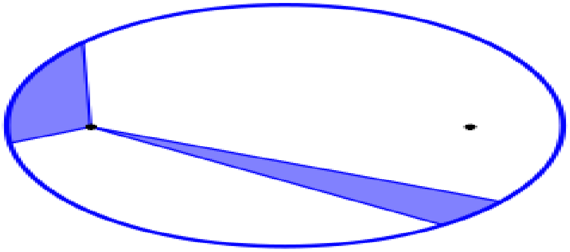
http://www.asc-csa.gc.ca/images/recherche/photo.aspx?id=1825&format=2&search=pcw&l=eng

Molniya orbit ground track



$$v^2 = 2\mu(1/R-1/2a)$$

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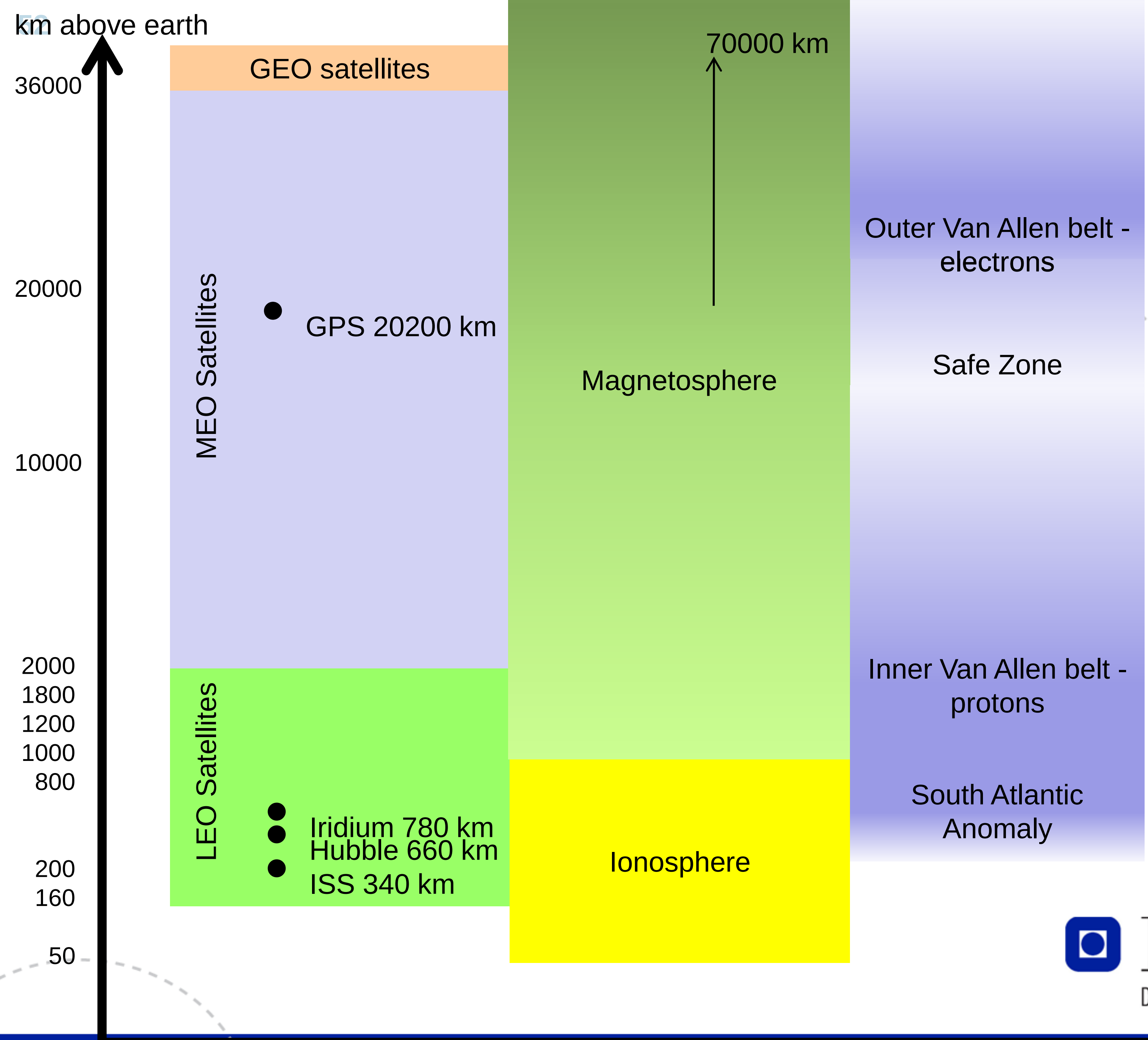

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Case example: Communication in the arctic

Satellite orbit types: LEO, MEO, GEO, HEO

GEO satellites are usually chosen for communication:

- Stable above the earth, no handover between satellites necessary
- One satellite can be equipped with large antennas and high power, gives large bandwidths
- Long lifetime

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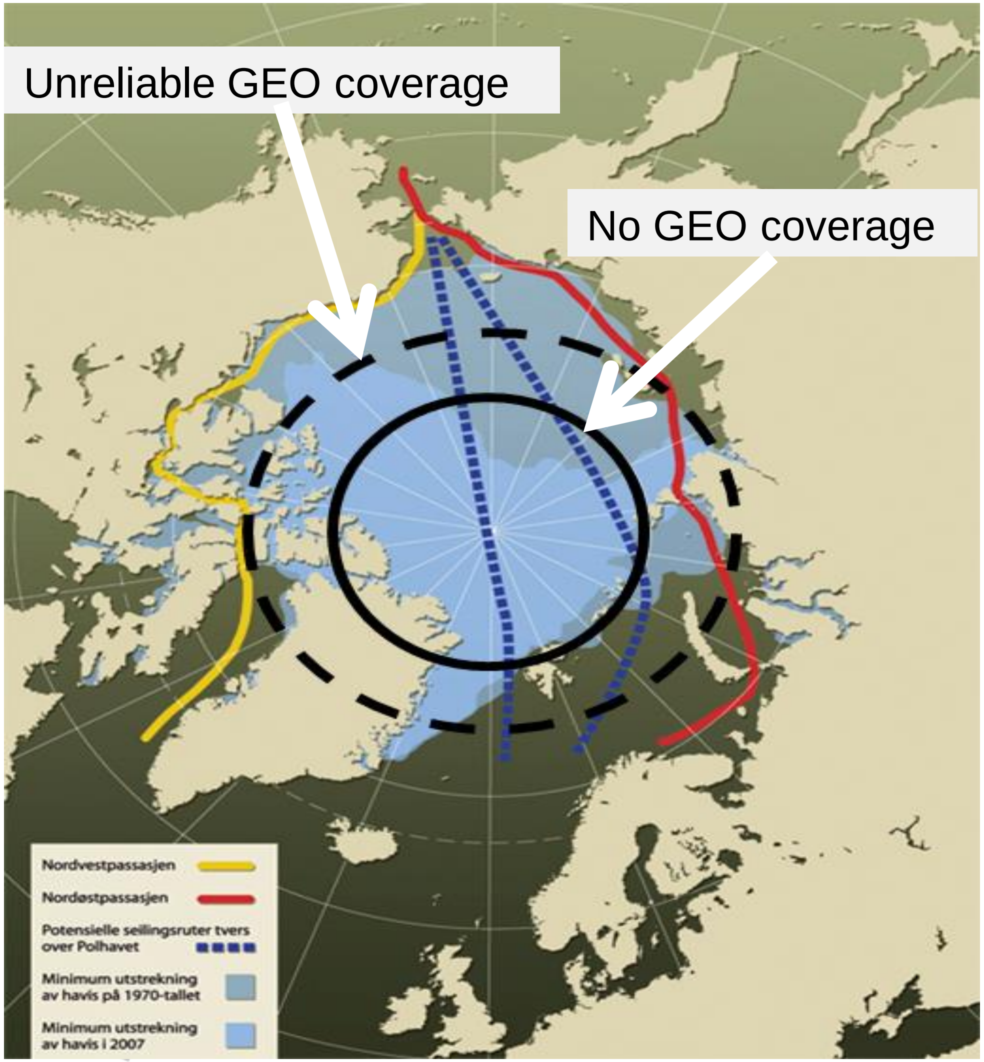
GEO coverage



But in polar regions, GEO satellites will not provide coverage

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LEO communication satellites

How shall a constellation be designed if you need continuous availability globally, also in polar regions?

- Start with the radius
- Calculate the time a satellite is visible
- Determine the number of satellites necessary in one orbit
- Select inclination, and select number of orbits necessary to cover the earth surface continuously
- Use link budget calculations to select data rate and necessary parameter settings (power, antenna size, carrier frequency etc.)
- All within a certain budget

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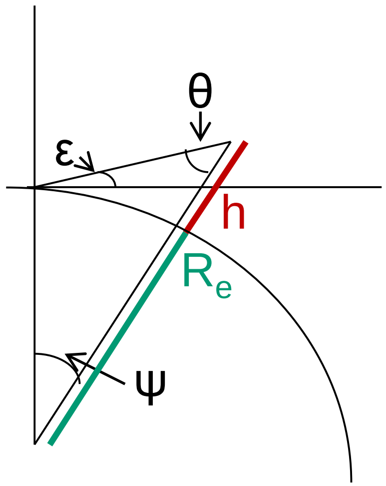
Satellite visibility

A typical earth station antenna will track the satellite down to an elevation angle, ε, of 10 degrees. Let us select an orbit 1000km above the earth surface, a typical LEO orbit.

Orbital period $T_o=2\pi\cdot\sqrt{a^3/\mu}$

where $\mu=3.986\cdot10^5 km^3/s^2$, $R_e=6378km$, $h=1000km$ => $a=7378km$ =>

$T_o=6306s=105.12min$ ~1t45m



Rule of sines:

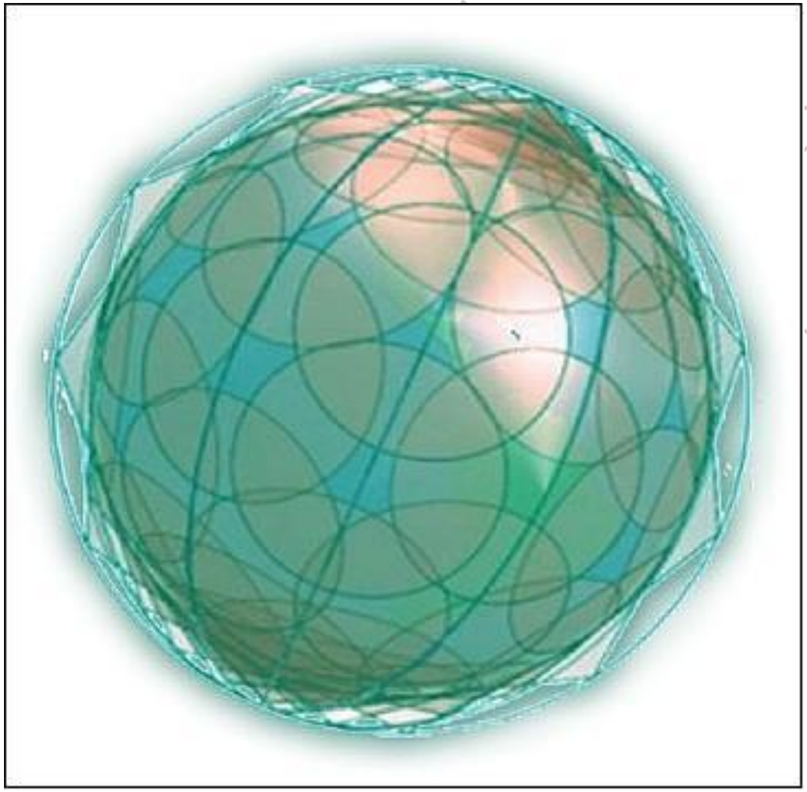
$\sin(\varepsilon+90^\circ)/(R_e+h)= \sin(\theta)/R_e$

$\Rightarrow$ $\theta=56.6^\circ$ => $\psi=180-105-56.6=18.4^\circ$, in total $2\cdot\psi=36.8^\circ$

$\Rightarrow$ so time visible is $T_v=105.12min\cdot36.8/360=\underline{\underline{10.75min}}$

Iridium

It takes about 105 minutes before the same satellite comes back, and the time of visibility is about 10 minutes, therefore 11 satellites pr. orbit is necessary.



Beam size on earth depends on carrier frequency and antenna size.

Iridium architecture:
L band 1616 - 1626.5 MHz
Spot diameter 250 miles = 400 km
6 orbital planes 30 degrees apart
Polar orbit; inclination i = 86.4°

This gives global coverage,
but 66 satellites are necessary.

LEO constellation choice

Pros:
- Global, including, polar coverage
- Close to earth surface
- Low cost for launch
- No need for equatorial launch site
- Low power/small aperture

Cons:
- Need for a high number of satellites
- Handover
- Intersatellite links (ISL)
- Low bandwidth
- Short lifetime
- Inner Van Allen belt, South Atlantic Anomaly
- Extensive house keeping

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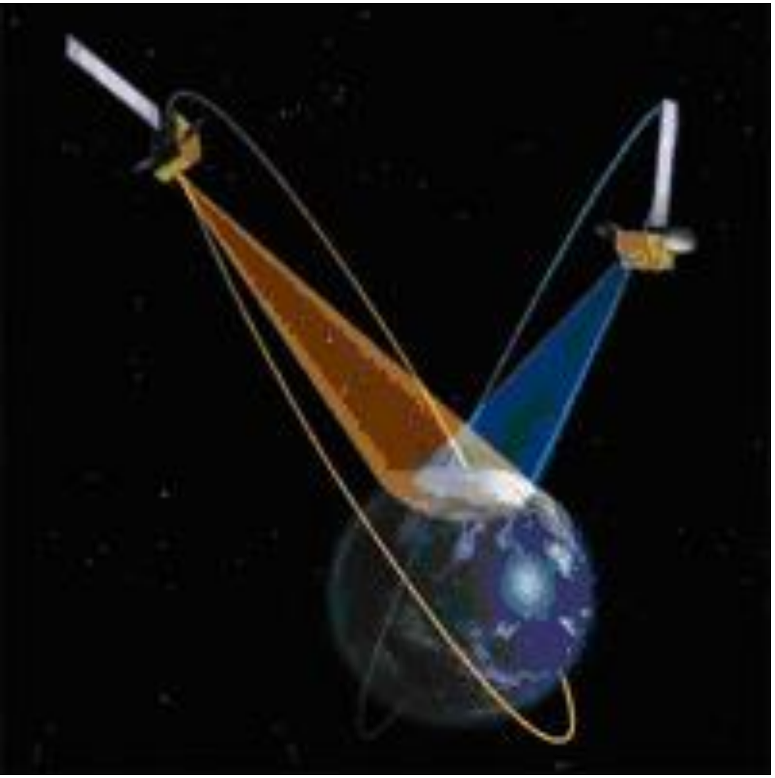
HEO orbit



- The satellite will stay above a fixed point on earth for a long time (~11h); pseudo-stationary
- Polar coverage with i = 63.4º
- T = 12 sidereal hours, continuous coverage with 2 satellites, usually 3 are chosen

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http://www.asc-csa.gc.ca/images/recherche/photo.aspx?id=1825&format=2&search=pcw&l=eng

HEO orbit footprint

HEO communication satellites

Pros:
- More GEO-type satellite, quasi stationary, larger satellites, more power and larger antennas, higher bandwidths
- No need for equatorial launch sites

Cons:
- With an apogee of 46000km from earth's center, and a perigee of 7000km, the satellite will pass two times trough the outer Van Allen belt per orbital period; satellite wear
- High free space loss, varying
- High cost

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Summary

- Kepler's 3 laws
- Newton's 3 laws
- Newton's universal law of gravity
- The six orbital elements
- The rocket equation
- The two cosmic speeds
- Orbit types

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