NORGES TEKNISK-
NATURVITENSKAPELIGE UNIVERSITET
Institutt for elektronikk og telekommunikasjon

Contact during the exam:
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EXAM IN COURSE
TTT4234 SPACE TECHNOLOGY I

Wednesday December 5, 2012
Time: 1500 - 1900

English version

Permitted material: Calculator, of a make according to a list approved by NTNU. Printed material: formula sheet attached to the exam.

Answers should be short and concise.

The results will be announced at the latest January 4th, 2013.

Exercise 1: Geostationary orbit

One of the most commonly used satellite orbits is called the geostationary orbit.

a) What is a geostationary orbit?

Answer:

An equatorial, circular orbit, with an orbital period equal to 24 hours (actually not exactly, as it is a sidereal day and night) so that the satellite appears as stationary above earth.

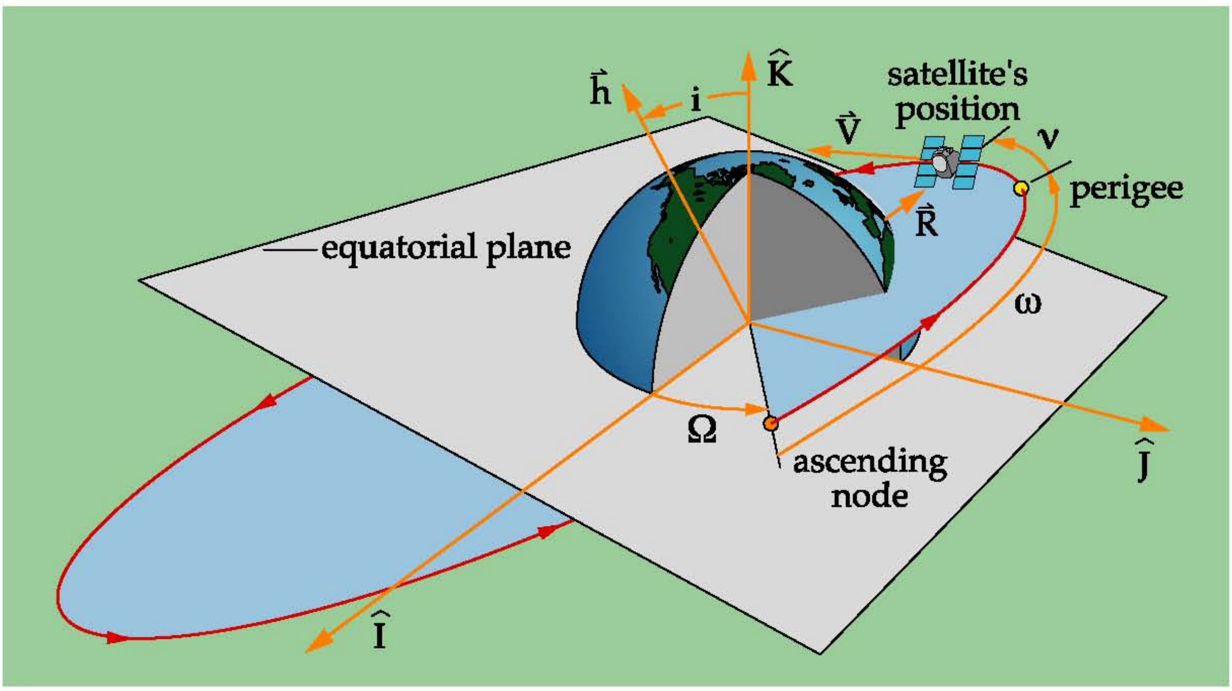
b) Give the six Classical Orbital Elements (COE). Start by defining the co-ordinate system. What values do they take for a geostationary orbit?

Answer:

The co-ordinate system is defined with the z/K-axis from the centre of the earth through the north pole along the rotational axis of the earth. The x/I-axis is defined in the equatorial plane pointing towards the stellar constellation Aries, which is also the direction of the sun at vernal equinox. The y/J-axis is given by the two others, and is also in the equatorial plane.

The six classical orbital elements are: a, e, i, Ω, ω, ν. a is the semimajor axis in an elliptical orbit, e is the eccentricity, i is the inclination , Ω is the right ascension of the ascending node, i.e. the angle between the x/I-axis and the point where the satellite brakes through the equatorial plane on its way into the northern hemisphere, ω is the argument/length of the perigee, i.e. the angle between the ascending node and the perigee, ν is the true anomaly, i.e. the angle between the perigee and the point in the orbit where the satellite is located at any moment. true anomaly is the only time varying parameter out of the six.

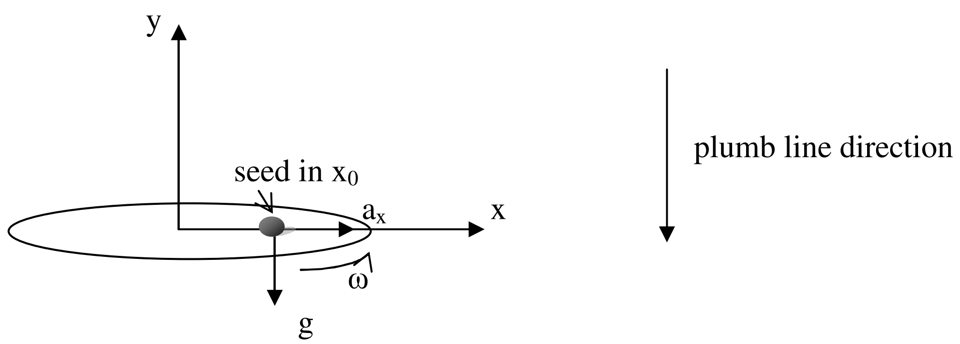
For a geostationary orbit a=radius in a circle (of approx. 42000km), e=0, i=0, and the three last parameters are undefined.



c) Develop Kepler's third law, and use this to calculate at which distance from the earth's centre a geostationary satellite must be placed in order to be actually geostationary.

Answer:
Kepler's third law can be deduced by equalling the centripetal force with the gravitational force.
Centripetal force: $F = ma = mv^2/r = mr\omega^2$ med $\omega=2\pi/T$
Gravitational force: $F=\mu m/r^2$
Giving Kepler's third law: $T^2 = 4\pi^2r^3/\mu$.
With T=24hours (again actually a bit less as one sidereal day&night should be used), you obtain that r, the distance from earths centre to the geostationary satellite orbit must be approx. 42000km.

Exercise 2. Plant growth and microgravity.
A circular, round plate is rotating with relatively low angular speed, *ω*, around a vertical axis (much like a horizontal gramophone record on a gramophone player). Soil has been put on the plate with one plant seed. With watering, a plant shoot appears and grows. The growth can be regarded as restricted to the tip of the shoot stem. In our case it turns out that the growth speed at every moment is proportional to the acceleration both in the plumb line direction (i.e. the vertical direction) and in the horizontal direction. The *direction* of growth is negative with respect to the acceleration vector - they are e.g. growing against the gravity vector.



In the plumb line direction the acceleration is, of course, constant and the growth velocity, therefore, constant. But in the horizontal direction the position of the shoot tip is also determining the acceleration.

a) Departing from the following expressions for the growth speed:
$v_x = -k_x \cdot a_x$, and $v_y = -k_y \cdot a_y$, where k_x and k_y are just constants, and a_x and a_y are the accelerations in the x and y directions respectively, deduce an expression of growth length in the x direction as a function of growth length in the y direction and the parameters given in the figure above and the constants k_x and k_y.

b) Sketch the curve, in the xy-plane, which a shoot tip will follow when the plate rotates with a fixed *ω*. How will the curve change if another value for *ω* is chosen?

c) Describe what the growth curves would look like if the experiment was carried out in free fall.

Answer:

a) The centrifugal acceleration $a = r\cdot\omega^2 = x\cdot\omega^2$

The exercise says that the velocity is proportional with acceleration: $\vec{v} = -k\cdot\vec{a}$, where k is a constant, and k>0.

$$\begin{cases} v_x = -k_x\cdot a_x = -k_x\cdot x\cdot \omega^2 \\ v_y = -k_y\cdot a_y = k_y\cdot g \end{cases}$$

Thus:

$$\begin{cases} v_x = dx/dt = -k_x\cdot x\cdot \omega^2 \\ v_y = dy/dt = k_y\cdot g \end{cases}$$

Integrating, we obtain:

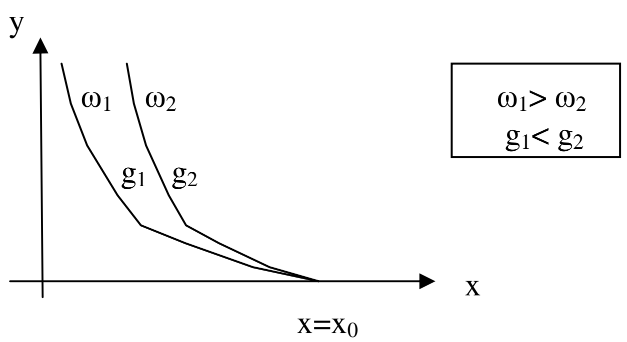
$$\begin{cases} x = x_0\cdot \exp(-k_x\cdot \omega^2\cdot t), \text{ with } x(t=0)=x_0 \text{ and } x(t=\infty)=0 \\ y = k_y\cdot g\cdot t \end{cases}$$

Hence $t = y/k_y\cdot g$ and then:

$x = x_0\cdot \exp(-k_x\cdot \omega^2\cdot t) = x_0\cdot \exp(-k_x\cdot \omega^2\cdot y/k_y\cdot g)$

b) Interpretation:

- y increases linearly with time.
- A plant experiencing an increasing angular speed, will approach the y-axis faster than with a smaller speed. If g decreases, the effect will be similar.



c) If this is in free fall, g tends towards zero, and the plant will grow along the x-axis towards the center of the rotating disk.

Exercise 3: Orbit correction

A satellite manoeuvre in space requires a velocity increase of 5m/s. It must be performed during one single combustion procedure over a period of 5 minutes.

a) How much fuel must be used in order to perform the manoeuvre?

b) How much force is required to perform the manoeuvre, when the initial satellite mass is 400 kg, and the thrusters' specific impulse is I_{sp}=300s?

Answer:

a) You may use the given rocket equation, which after some straight forward transformation becomes an expression for the reduction in mass: $\Delta m=m_0(1-\exp(-\Delta v/g_0 I_{sp}))$, giving $\Delta m=400(1-\exp(-5/(9.81\cdot 300)))=0.68$kg

You may also use series expansion of $e^x \sim 1+x$ n the above expression to obtain the same answer.

b) You may use the definition of the specific impulse: $I_{sp}=F/(g_0\cdot dm/dt)$, dvs. at $F=I_{sp}(g_0\cdot dm/dt)$, $F=300(9{,}81\cdot 0{,}68/(5\cdot 60))=6.7$N.
or just $F=ma= m\Delta v/\Delta t = 400\cdot 5/(5\cdot 60)=6.7$N.

Exercise 4: Additive, White, Gaussian Noise

The signal to noise ratio at the reception of a satellite signal is given by:

$$S/N= EIRP/L_0 \cdot G_r/T \cdot 1/kB \cdot 1/L_a$$

a) In this expression there are parameters that together define the Additive, White, Gaussian Noise, N. Which parameters in the expression above give N, and what is their interpretation?

b) What is mainly the origin of this noise?

c) The spectral density of the noise is denoted by N_0. How is N_0 related to N?

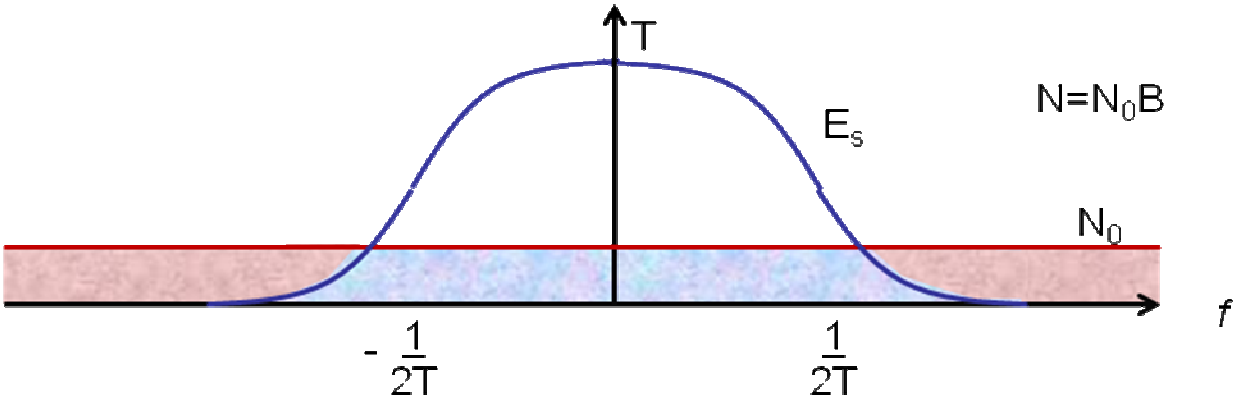
d) Explain why the noise is defined as
- additive
- white
- gaussian

Answer:

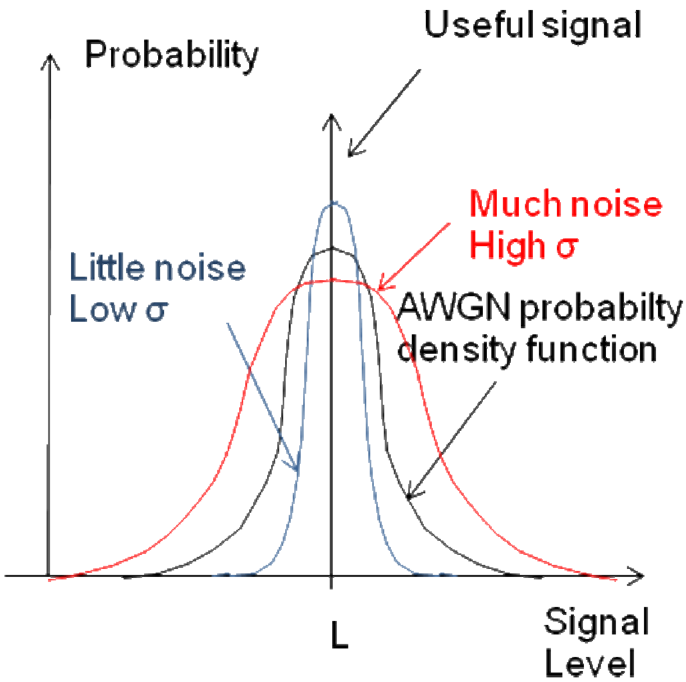
a) N=kTB, k is Boltzmann's constant, T the system (noise) temperature, B is the signal bandwidth, or more precisely the inverse of the symbol rate.

b) Thermal noise, expressed by the system temperature. It is a combination of the noise from space (low), the temperature of the earth (high), the temperature of the sun if the antenna points towards it (very high) and the temperature of the electronics (high).

c) $N= N_0B= N_0/T$ where T is now the symbol rate (not the temperature).



d) additive: the noise adds to the signal, i.e. if the modulated signal can be expressed by x(t), the received signal can be written r(t)=x(t)+n(t), where n(t) is the noise
white: because the spectral density of the signal is N_0, is constant for all frequencies (by equivalence to white light).
gaussian: because the probability that a noise signal of a certain amplitude is added to the information signal has a gaussian probability distribution around the information signal.



Exercise 5: Description, choose one of the two topics below

Choose one of the two possible topics below, i.e. 5a) or 5b):

5a) Describe

- which forces that can act on a satellite in order to de-orbit it
- how the changes in orbit can be detected
- how a satellite can be stabilised

Answer:

Forces acting on a satellite:

- gravitational, from earth, moon, sun,... variable and depending on where the different masses are located at any moment. The gravitational force from the earth is not uniformly distributed on the earth either
- drag forces from the (thin) atmosphere for LEO satellites
- electromagnetic forces (may be variations in earth's magnetic field while e.g. a LEO satellite in polar orbit moves from a pole towards the equator).
- the push from the photons from the sun light. Small force, but may act with some significance over time (ref. solar sails)

Can be detected by sensors:

- Sun sensor
  - Measures the solar intensity with solar cells. The cell output current is proportional to the solar incidence angle
- Earth sensor
  - Infrared sensors detecting the difference in temperature between the earth and cold space
  - Infrared sensors on spinning satellites detecting the crossing of the earth horizon
- Star sensor
  - Compares the measured star constellation with a stored star map
- Gyroscope sensor
  - Uses the principle of conservation of momentum
  - Ring-laser gyroscope measures the frequency shift between two laser beams in a rotating cavity
- GPS and other triangularation methods

Stabilisation/orbit control:

- Gravity-gradient stabilization: work with the torque, not against it
- Spin stabilized: profit from gyroscopic stiffness; the faster an object spins, the more stable it is
- Thrusters: fire off engines to exercise opposite torque
- Momentum-control: conservation of angular momentum

5b) Describe different types of fuel that can be used in rockets and satellites. Explain what are the main advantages and disadvantages for the different types.

These are the most common. At least the solid and the liquid should be mentioned, and preferably at least one of the others. Some might also remember to mention solar sails.

- Solid propellant
  - Fuel, oxidizer and binding substance
  - Simple construction, reliable, stable
  - Cannot be stopped and restarted
  - Moderate specific impulse, high trust
- Liquid propellant
  - Monopropellant, or bipropellant (one, or two tanks)
  - Simple construction, reliable, unstable

  - Can be stopped and started
  - Moderate specific impulse, moderate trust
- Hybrid propellant
  - Solid fuel with liquid oxidizer
  - As flexible as liquid, as simple as solid, reliable, stable
  - Can be stopped and restarted
  - Moderate specific impulse, falls in between solid and liquid
- Ion propellant
  - Acceleration of positive ions
  - Complicated construction, reliable, stable
  - Can be stopped and restarted
  - High specific impulse, low thrust
- Nuclear propellant
  - Thermodynamic propulsion of gas heated by a nuclear reaction
  - Long-term energy supply, can be re-fuelled and re-used.
  - Political problem
  - High specific impulse, high thrust