Exercise 1: Sun synchronuous orbit

1a) What is meant by sun synchronous orbit, and which of the classic orbital elements will vary with time for such an orbit, and what will be their rate of change?

1b) Why are these orbits attractive for earth observation satellites?

1c) Which are the two main types of sun synchronous orbits, with respect to when they pass above a certain point on earth? What are the advantages and disadvantages for each of these types?

Answer: from Stette's compendium p.34, and from Understanding Space p. 275.

a) 10 points. For certain combinations of the orbital radius and inclination (and due to the oblateness of the earth that leads to nodal drift), the right ascension of the ascending node, Ω, will increase with 360 degrees in one year, or approximately one degree per day. For this orbit, the inclination must be more than 90 degrees. The satellite orbital plane will then be fixed related to the sun. The orbital plane must then rotate in order to always be fixed related to the sun, i.e. 360 degrees eastward per year, or $d\Omega/dt=0.9856$ degrees per day.
b) 5 points. A sun synchronous orbit will always be located above the same spots on earth at the same local time throughout the year. It is possible to choose the orbit so that the satellite can orbit in LEO without ever being in the earth shadow. This is called a morning-evening-orbit. In addition it is possible to make the orbit synchronous with the earth's rotation, so that the satellite is above the same spot on earth with fixed intervals, e.g. every day, or every second day. For earth observation this can be very beneficial, as it will be possible to compare observations over time under the same light conditions, e.g. will the shadows be identical which may have importance for intelligence, weather or resources' observations. For earth observing satellites using visible light instruments, it is also a benefit to always have sun light available. This is also an advantage for the solar cell panels.
c) 8 points. As mentioned in b) there is a morning-evening-orbit, and another much used variant is the midday-midnight-orbit. If it is important to have as short shadows as possible for observations, or to observe the earth by night, the midday-midnight orbit is the best choice.

Exercise 2: Newton's law of gravitation

2a) Starting from Newton's law of universal gravitation, find out how large the relative (in percent) change of the gravitational force will be if the distance from the gravity centre of the earth to a satellite is increased by 2%.

2b) Assuming that the earth radii is R_0=6500km, and that the gravitational acceleration at the earth's sea level is g_0, what will be the relative (in percent) change of the gravitational acceleration (as compared to that of the earth's seal level) for an orbit of 300km height above the earth's sea level (typical height for the space shuttles)?

Answer:

a) 5 points. Newton's gravitational law says: $F=GMm/r^2$.
If F_0 denotes the gravitational force in the original position and F_1 the one after an increase in the distance of 2%, then $F_0 = GMm/r^2$ and $F_1 = GMm/(r\cdot 1.02)^2$, hence $F_1 = 0.96\cdot F_0$, which means that the gravitational force has been reduced by 4%.
b) 7 points. With the same reasoning as in a), you have that the distance has increased by a factor (6500km+300km)/6500km = 1.046, hence 4.6%. Then $F_1 = F_0/1.046^2 = 0.91\cdot F_0$, which means that the gravitational force has been reduced by 9%. The same relation is valid for the change in the gravitational acceleration, as $F=mg(r)$ also, so that F is proportional with g(r).
NB! Those who had used other, and more correct, values for the earth radius were of course not punished for that.

Exercise 3: Hohmann transfer orbit

A satellite is to be launched into earth orbit, and is initially given an elliptical orbit with perigee equal to $2R_0$, and apogee equal to $5R_0$, both related to the earth's surface, and where R_0 is the earth radii.

3a) What will the orbital element "a" be equal to?

The satellite is the lifted up via a Hohmann transfer orbit, into a new orbit, now circular with a distance from the earth surface equal to the apogee. The formula of the orbit velocity for such a transfer is:

$v=(\mu(2/r-1/a))^{1/2}$

3b) Give a description of the different parameters, and find out how large the velocity of the new circular orbit will be.

Unfortunately, the orbital plane of the satellite in the new orbit obtains an inclination with one degree offset from the required equatorial plane, and an orbit correction is required. Δm for such a correction is given by the formula:

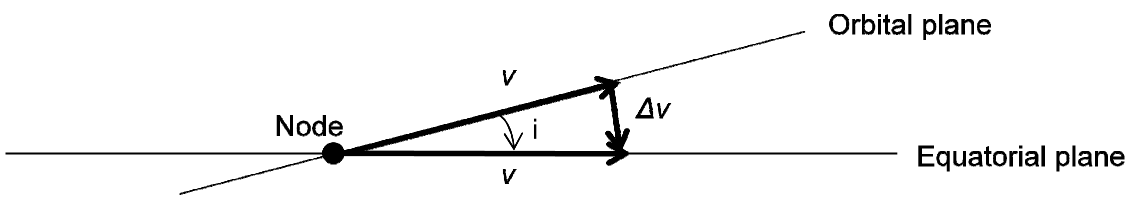
$\Delta m = m_i(1-e^{-\Delta v/g_0 I_{sp}})$

3c) Describe the parameters in the formula, and evaluate the approximate relative change of mass, $\Delta m/m_i$, for the orbit correction by choosing a normal value for I_{sp}.

Answer:

a) 5 points. The orbital element $a = (5R_0+R_0+R_0+3R_0)/2 = 4.5R_0$ (which is the semi major axis)
b) 10 points. v is the orbit velocity in the new orbit, μ is the gravitational parameter = GM, with G = the universal gravitational constant, and M equals the earth's mass. r is the distance from earth's gravitational centre to the point where the satellite is located at any moment, and a is the semi major axis of an elliptical orbit. For a circular orbit r = a, and in this case $r = 6R_0$. The formula then reduces to: $v=(\mu/r)^{1/2}$ (See Understanding Space p.198, $V_{orbit\ 2}$)
With R_0 = 6500km from the previous exercise (again, no problem if other more correct values were used), $M = 5.98\cdot10^{24}$kg, $G = 6.67259\cdot10^{-11}$ Nm^2/kg^2, $\mu = 3.986\cdot10^5$ km^3/s^2, and $r = 6R_0$ you obtain:
$v = (\mu/6R_0)^{1/2} = (3.986\cdot10^5\ km^3/s^2 /(6\cdot6500km))^{1/2} = 3.2km/s.$
c) 10 points. In the rocket equation $\Delta m = m_i(1-e^{-\Delta v/g_0 I_{sp}})$, where Δm is the amount of fuel/propellant used for the orbital correction, and m_0 is the initial mass for the entire satellite construction before correction (i.e. structure, payload and fuel). Δv is the velocity change necessary to perform the orbital correction, g is the gravitational acceleration, and I_{sp} is the specific impulse of the propellant used, giving the efficiency of the propellant. A typical value for I_{sp} would be 300s (should be chosen above 200s, and usually not more than 1000s), and we always use $g_0 = 9.81m/s^2$ in this equation.

The figure below shows the correction to perform:



The necessary velocity change will be: $\Delta v = v\cdot2\cdot\sin(i/2) = 3.2\cdot10^3\cdot2\cdot\sin(0.5°) = 55.8m/s.$

i is so small that using both $\Delta v = v\cdot\sin(i)$ and even $\Delta v = v\cdot i$ (i in radians) is ok.

Then: $\Delta m/m_i = (1-e^{-\Delta v/g_0 I_{sp}}) \approx 1-e^{-55,8/(9,81 \cdot 300)} = 0.019$. I.e. the mass of the satellite has been reduced by approximately 2% by this correction.

Exercise 4: Link budget and antenna gain

The signal to noise ratio at the reception of a satellite signal is given by:
$S/N = EIRP/L_0 \times G_r/T \times 1/kB \times 1/L_a$

4a) In this expression G_r is the antenna gain for the receiver antenna. Which parameter comprises the antenna gain of the transmitter antenna?

The general expression for the antenna gain is: $G = \eta \times 4\pi A / \lambda^2$ and the 3dB beam-width can be expressed as: $\theta_{3dB} = k \times \lambda / A$.

4b) Explain what the parameters are. What are the options usually considered by a satellite system designer wanting to increase the antenna gain?

4c) What are the advantages and disadvantages for different applications when such measures are taken?

Answer:

a) 5 points. The gain of the transmitter antenna, G_t, is comprised in the expression of the EIRP, as $EIRP = P_t G_t$, where P_t is the transmitted power.
b) 5 points. η is the antenna efficiency (0< η <1) depending on the antenna construction and surface coating. A is the antenna surface, and λ is the wavelength of the carrier. A system designer will usually try to increase the antenna surface, or the carrier frequency (shorter λ). k is a coefficient (it is actually not Boltzmann's constant, but I see that by putting this constant on the formula sheet, I might have mislead you unintentionally, so I have not punished for this confusion).
c) 10 points.
   - Coverage area: If the antenna surface is increased the antenna becomes more directive, the main lobe is reduced. Hence the coverage area will decrease if this is done on the satellite. It might be an advantage for spot beam systems, but is not if you want to cover large areas such as for TV broadcasting from satellite.
   - Pointing error: If the antenna surface is increased, this increases the requirements for pointing accuracy as the antenna has become more directive. For TV receivers for instance, this is a drawback, as normal households usually do not have the instruments or the competence for accurate antenna pointing installations. That is why parabolic TV receivers usually are as small as possible. It the frequency is increased, the result will be the same advantages and disadvantages when it comes to coverage and pointing accuracy.
   - The antenna size will usually be limited on a satellite (due to weight, cost, and mechanical limitations). There is more freedom of choice on ground than on the satellite, but even here the antenna size may be limited for reasons such as interference with other systems, local radiation restrictions, restrictions due to buildings and civil constructions, mechanical limitations such as wind resistance, and solid foundations for fixing the antenna.
   - Increasing the frequency will increase the free space loss L_0, and the attenuation in the atmosphere will increase due to clouds and water vapour.

Exercise 5: Description, choose one of the two topics below

Choose one of the two possible topics below, i.e. 5a) or 5b):

5a) Molniya orbit
- Give a description of the Molniya orbit.
- Describe the advantages of such an orbit

    - compared to a circular polar orbit
    - compared to a geostationary orbit

5b) Astronauts onboard the ISS (International Space Station) will, as known, stay in a state of weightlessness for a shorter or longer period. They will work hard inside the station, but may also for some periods of time work outside the station in what is called EVA (Extra Vehicular Activity). Give a short description of the different physiological effects on the astronauts during short and long term missions, both inside the station and while working outside the station.

Answer: 20 points.
5a)
Description:

- A Molniya orbit is a highly elliptical orbit, a HEO.
- The special inclination angles of 63.4º results in a drift of the perigee equal to zero.
- The orbital period of the satellite is usually chosen to be 12 hours (which will determine the value of a), as the satellite then will be geo synchronous and the footprint will be fixed on earth.
- The satellite is then what is called pseudo stationary, and will spend most of the orbital period above the intended coverage area, but will also be seen when on the ”night side”. E.g. a satellite orbit constructed for coverage above Siberia at day-time will also to a high extent cover Siberia when located above Canada at night time. The satellite will only spend one of the 12 orbital period hours over the southern hemisphere (ref. Kepler's 2. law). It will pass it's perigee at a distance of 548 km above the earths surface, while the apogee is at 39957 km above the earth's surface.
- This orbit was widely in use by the Soviet union, as they did not posses launching sites close to the equator (which is highly beneficial for geostationary satellites, but not so for Molniya), and they wanted to be independent of other countries. In addition they obtained the coverage at high latitudes.

- The advantage of a Molniya orbit when compared to a normal circular polar orbit is that the Molniya satellite is pseudo stationary, which a polar orbit satellite will not be.
- The advantage of a Molniya orbit when compared to a GEO-satellite, is the coverage of the northern/polar regions, where the GEO-satellites are limited due to low elevation angles towards the north(south), and even no coverage in the polar regions above 70-80 degrees north(south). In addition, launch near the equator is not necessary.

5b) Five effects of weightlessness on the human body should be mentioned:

- Space sickness and the reasons for it (the balance system, sedimentation etc.)
- Muscle changes (atrophy)
- Skeleton changes (calcium)
- Heart problems (due to muscle atrophy and changes in liquid distribution etc.)
- Particle radiation and cell changes (ionising radiation)

A certain division into short term and long term effects should be given.
In any case, radiation should be mentioned.