TTT 26 Radar Solutions to problems

Probl 2.5

a)

$$P_{fa} = \frac{1}{T_{fa} \cdot B} = \frac{1}{30 \cdot 60 \cdot 0{,}4 \cdot 10^6} = \underline{\underline{1{,}4 \cdot 10^{-9}}}$$

b)

$$P_{fa} = e^{-\frac{V_T^2}{2\psi_0}}$$

$$\Downarrow$$

$$\frac{V_T^2}{\psi_0} = -2 \ln P_{fa} = -2 \ln 1{,}4 \cdot 10^{-9} = \underline{\underline{40{,}8}}$$

c)

One year T_{fa}:

$$P_{fa} = \frac{1}{8760 \cdot 3600 \cdot 0{,}4 \cdot 10^6} = \underline{\underline{7{,}9 \cdot 10^{-14}}}$$

$$\frac{V_T^2}{\psi_0} = -2 \ln 7{,}9 \cdot 10^{-14} = \underline{\underline{60{,}4}}$$

d)

0,3 dB lower threshold to noise ratio

$$\frac{V_T^2}{\psi_0} = 10^{\frac{-0{,}3}{10}} \cdot 40{,}8 = \underline{38{,}1}$$

$$T_{fa} = \frac{1}{B} e^{\frac{V_T^2}{2\psi_0}} = \frac{1}{0{,}4 \cdot 10^6} \cdot e^{\frac{38{,}1}{2}} = 469{,}1\ s$$

$\Downarrow$: $\underline{\underline{7{,}8\ \text{min}}}$

e)

0,3 dB higher threshold to noise ratio

$$\frac{V_T^2}{\psi_0} = 10^{\frac{0{,}3}{10}} \cdot 40{,}8 = \underline{43{,}7}$$

$$T_{fa} = \frac{1}{0{,}4 \cdot 10^6} \cdot e^{\frac{43{,}7}{2}} = \underline{7713{,}9\ s}$$

$\Downarrow$: $\underline{\underline{2t\ 8{,}5\text{min}}}$

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Probl 2.5 cont.

f) A difference of only 0.6 dB in the threshold increases or decreases the false alarm time with as much as a factor 16 i.e. it is not easy to adjust the threshold to give a precisely specified false alarm time.

Probl. 2.8

a)

$$\Theta_B = \frac{1{,}2\,\lambda}{D} = \frac{1{,}2\,\frac{c}{f}}{D} = \frac{1{,}2\,\frac{3\cdot 10^8}{1{,}35\cdot 10^9}}{32\cdot 0{,}3048} = 0{,}0273 \text{ rad}$$

$$\hat{=}\ \underline{1{,}6^\circ}$$

Pulse repetition frequency:

$$f_p = \frac{c}{2\cdot R_u} = \frac{3\cdot 10^8}{2\cdot 2204852} = \underline{368{,}2\ Hz}$$

Number of pulses per beamwidth

$$n_B = \frac{\Theta_B}{360^\circ}\cdot T_s \cdot f_p = \frac{1{,}6}{360}\cdot 10 \cdot 368{,}2 = \underline{\underline{16{,}4 \text{ pulses}}}$$

b)

From fig 2.7 a:

16 pulses integrated, $P_d = 0{,}9$, $n_f = 10^4$

gives approx $I_i(16) \approx \underline{\underline{8}}$

From fig 2.7 b:

Integration loss approx. $L_i(16) = \underline{\underline{2{,}5\ dB}}$

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Probl. 2.13

In the Rayleigh region the wavelength is much larger than the physical size of the object, and the RCS drops rapidly as the size of the object gets smaller.

In the resonance region the wavelength and the physical size of the object is of the same order of magnitude, and the RCS will oscillate around its value in the optical region as the size varies.

In the optical region the wavelength is much smaller than the physical object and the RCS is constant. For a sphere, the RCS in the optical region corresponds to its projected area.

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Probl. 2.14

RCS of flate plate A in the optics region:

$$\sigma = \frac{4\pi A^2}{\lambda^2} \quad (A - \text{area})$$

$$A = \sqrt{\frac{\sigma \cdot \lambda^2}{4\pi}} = \sqrt{\frac{1 \cdot (0{,}032)^2}{4\pi}} = \underline{0{,}009\ m}$$

Side of plate

$$a = \sqrt{0{,}009} \approx \underline{\underline{9{,}5\ cm}}$$

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Probl. 2.18

a) From fig 2.6:

$P_d = 0{,}5$, $P_{fa} = 10^{-6}$ requires $S/N \approx \underline{\underline{11{,}3\,dB}}$

(single pulse).

b) $P_d = 0{,}99$, $P_{fa} = 10^{-6}$ requires $S/N \approx \underline{\underline{14{,}5\,dB}}$

c) SW I results in "fluctuation loss", L_f, which can be found for the single pulse case in fig 2.23.

Required S/N

$P_d = 0{,}5$: $S/N + L_f = 11{,}3\,dB + 1{,}5\,dB = \underline{12{,}7\,dB}$

$P_d = 0{,}99$: $S/N + L_f = 14{,}5\,dB + 17\,dB = \underline{31{,}5\,dB}$

d)

The fluctuation loss is much more severe at high P_d.

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Probl 2 24

a)

$$R^4_{max} = \frac{P_{av} G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 F_n B \tau (S/N)_1 L_f^{1/n_e} L_s f_p}$$ Radar range eq. (eq 2.51 incl. fluct. and system loss)

$\frac{S}{N}$ will be a function of P_d and P_{fa}

The single pulse S/N can be found from Albersheim formula (2.30) or from fig. 2.6, but needs to be modified with integration gain, which can be found from fig. 2.7 a).

P_d is a variable, P_{fa} is calculated from the T_{fa} as

$$P_{fa} = \frac{1}{T_{fa} B} = \frac{1}{4 \cdot 3600 \cdot 15 \cdot 10^6} = \underline{4{,}6 \cdot 10^{-12}}$$

Number of pulses within 3dB beamwidth:

$$n_B = \frac{\Theta_B}{360^\circ} \cdot \frac{60}{RPM} f_p = \frac{0{,}8^\circ}{360^\circ} \cdot \frac{60}{20} \cdot 4000 \approx \underline{27 \text{ pulses}}$$

Fig 2.7 shows that there is little variation with P_d and P_{fa} (n_f), so the integration gain found for 27 pulses, $P_d = 0{,}5$ and $n_f = 10^{12}$, is used as an average integration gain for our case of $0{,}3 < P_d < 0{,}99$ and $P_{fa} = 4{,}6 \cdot 10^{-12}$.

From fig 2.7:

$$I_i(27) \approx 14$$

⟹ Equivalent number of pulses integrated: $n_{eq} = n E_i(n) = \underline{14}$

TTT 26 Radar Solutions to problems

Prob. 2.24 a) cont.:

The $(S/N)_{min}$ to go into the radar-range eq., is the single pulse S/N from fig. 2.6. The n_{eq} (or $n E_i(n)$) is included in the numerator.

The effective aperture is calculated from the antenna gain:

$$A_e = \frac{G\lambda^2}{4\pi} = \frac{10^{\frac{33}{10}} \cdot \left(\frac{3 \cdot 10^8}{9410 \cdot 10^6}\right)^2}{4\pi} = \underline{0,16\ m^2}$$

For steady target the fluctuation loss $L_f = 1$.

With S/N from fig 2.6, all parameters in the radar eq are known and a R_{max} can be calculated:

P_d	0,99	0,9	0,8	0,7	0,6	0,5	0,4	0,3
$(S/N)_1$	47,6	36,1	32,2	29,7	27,6	25,6	23,7	21,6
R_{max} [nmi]	8,1	8,6	8,9	9,1	9,3	9,4	9,6	9,8

($(S/N)_1$ is single pulse S/N in linear values)

b) If the target is fluctuating scan to scan according to the Swerling I model, the S/N to go into the radar eq. must be modified with the "fluctuation loss", L_f, which can be found in fig 2.23. For Swerling I the pulses are completely correlated puls to puls, i.e. $n_e = 1$, and $L_f^{\frac{1}{n_e}} = L_f$.

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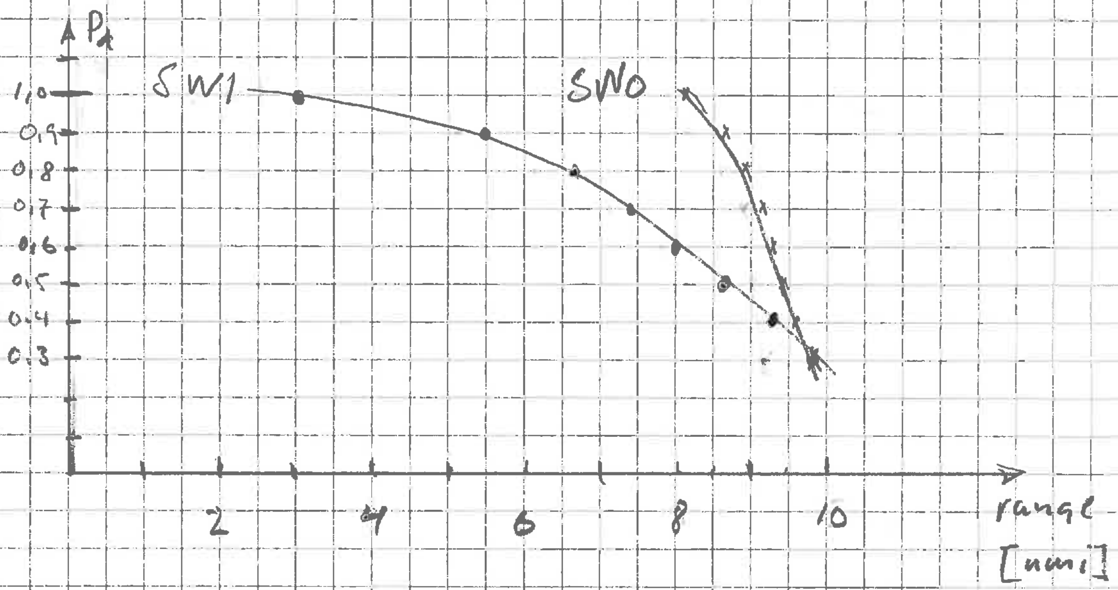
Probl. 2.24 b) cont.

If the buoy is fluctuating according to Swerling 1, we get the following detection ranges

P_d	0,99	0,9	0,8	0,7	0,6	0,5	0,4	0,3
$(S/N) \cdot L_t$	2383	221,8	106,8	66,4	49,0	36,2	26,6	21,6
R_{max} [nmi]	3,0	5,5	6,6	7,4	8,0	8,6	9,3	9,8



c) If the buoy is a SW 1 target (conservative estimate), it can be detected with a $P_d = 0,8$ at over 6 nmi, which will provide the navigator with a reasonable ahead warning of the buoy, if the ship is not moving <u>very</u> fast.

d) The wide elevation pattern on ship mounted radar antennas is to compensate for any pitch and roll of the ship.

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Probl. 2.29

The improvement in S/N is due to the reduction in the fluctuation loss, $L^{\frac{1}{n_e}}$, when n_e is the number of independent samples. For two independent samples, the fluctuation loss is simply $L^{\frac{1}{2}}$.

L can be found in fig. 2.23 for the different combinations of P_d and Swerling cases.

a) $P_d = 0{,}95$, Sw1:

From fig 2.23 $L_{dB} = 11\,dB$

$$\left(L^{\frac{1}{n_e}}\right)_{dB} = \frac{L_{dB}}{n_e} = \frac{11}{2} = \underline{\underline{5{,}5\,dB}}$$

b) $P_d = 0{,}6$, Sw1:

From fig 2.23 $L_{dB} = 2{,}5\,dB$

$$\left(L^{\frac{1}{n_e}}\right)_{dB} = \frac{L_{dB}}{n_e} = \frac{2{,}5}{2} = \underline{\underline{1{,}25\,dB}}$$

c)

Minimum decorrelation frequency:

$$\Delta f \geq \frac{c}{2D}$$

$$\Downarrow$$

$$\Delta f \geq \frac{3 \cdot 10^8}{2 \cdot 30} = \underline{\underline{5\,MHz}}$$

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Probl 2.30

a) Examples of system losses

- Microwave "plumbing" 4.5 dB
  Tx line, Duplexer, rotary joint..

- Antenna
  Beam shape 2 dB
  Radome 1 dB
  Phased array

- Signal processing
  <u>Ex</u>: Rx filter mismatch 1.5 dB
  CFAR 2 dB
  Integrator 2 dB
  Straddling / sampling 1 dB

Total loss **14 dB**

See pp 80-87.

b) Range with loss

If we divide the radar range equations with loss on the equation without loss we get:

$$10 \log\left(\frac{R}{200}\right)^4 = -14 \text{ dB}$$, R range with loss

⇓

$$R = 200 \cdot 10^{-\frac{14}{40}} = \underline{\underline{89 \text{ nmi}}}$$

i.e. a loss of 14 dB more than halves the detection range.