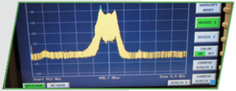

# Radio Engineering

## *Lecture 3*
## *Statistical Channel Characterization*

**Florian Kaltenberger**

# Last lecture

4. Antennas and Propagation
    - Maxwell equations
    - Plane waves
    - Linear and circular polarization

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4. Antennas and Propagation
    - Maxwell equations
    - Plane waves
    - Linear and circular polarization
    - Free space loss
    - Reflection and transmission
    - Diffraction
    - Scattering

# This lecture

5. Statistical description of fading
    - Equivalent baseband representation
    - Small scale fading without a dominat component
    - Small scale fading with a dominat component
    - Doppler spectra
    - Temporal dependence of fading
    - Large-scale fading

# Equivalent Baseband Representation (1)

- A signal is *bandpass* if the bandwidth of the signal is small wrt the carrier frequency.
- Most signals used in wireless communication are bandpass

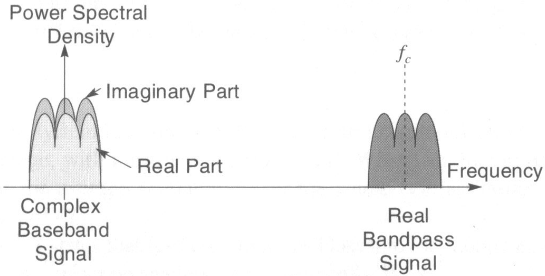



**Figure 10.1**: Complex baseband representation of signal spectrum

# Equivalent Baseband Representation (2)

- A bandpass signal can be written as

$$s(t) = A(t)\cos\left(2\pi f_c t + \Phi(t)\right)$$

- Complex baseband representation

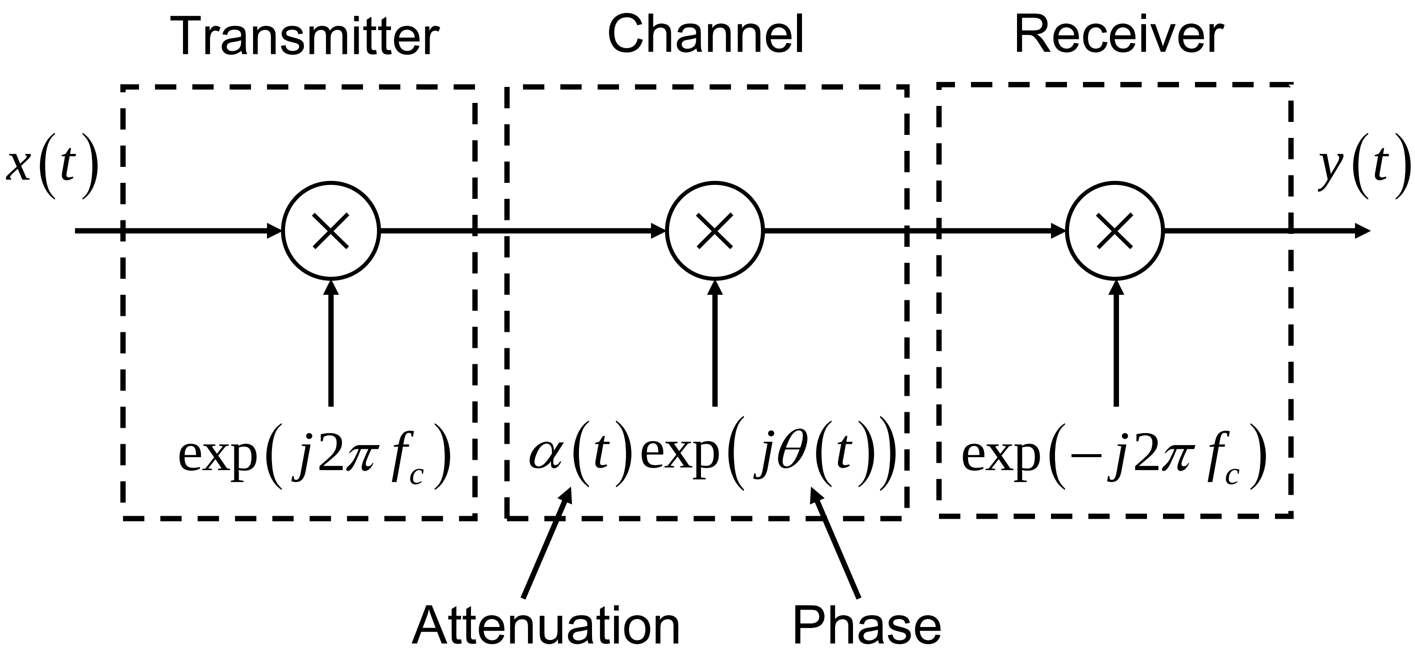
$$\begin{aligned} X(f) &= S(f + f_c) \Leftrightarrow \\ x(t) &= s(t)\exp\left(-j2\pi f_c t\right) \\ &= A(t)\exp\left(j\Phi(t)\right) \end{aligned}$$

  $A(t)$ . . . Amplitude, $\Phi(t)$ . . . Phase

- Bandpass signal can be recovered by

$$s(t) = \Re\{x(t)\exp\left(j2\pi f_c t\right)\}$$

# A narrowband system described in complex notation (noise free)




In: $$x(t) = A(t)\exp(j\phi(t))$$

Out: $$y(t) = A(t)\exp(j\phi(t))\cancel{\exp(j2\pi f_c t)}\alpha(t)\exp(j\theta(t))\cancel{\exp(-j2\pi f_c t)}$$
$$= A(t)\alpha(t)\exp\left(j\left(\phi(t)+\theta(t)\right)\right)$$

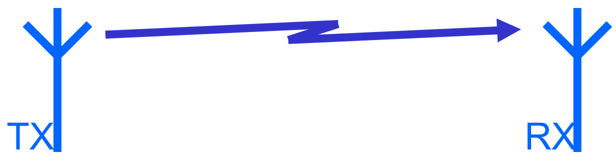
It is the behavior of the channel attenuation and phase we are going to model.

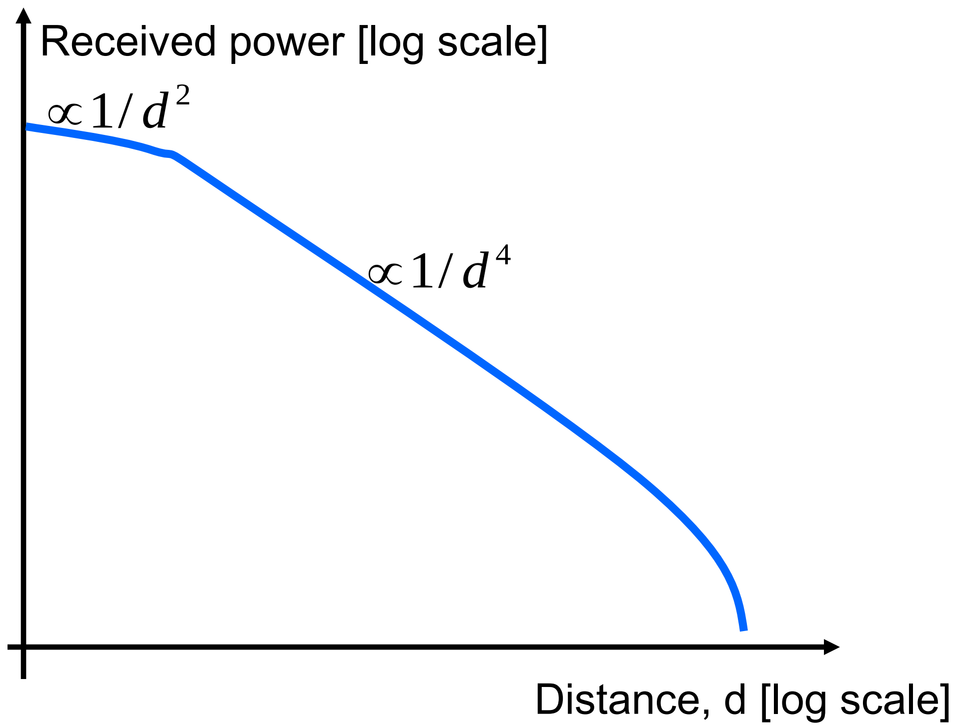
# Statistical Characterization of the Radio Channel

Multipath propagation causes *fading*, which can be categorized as

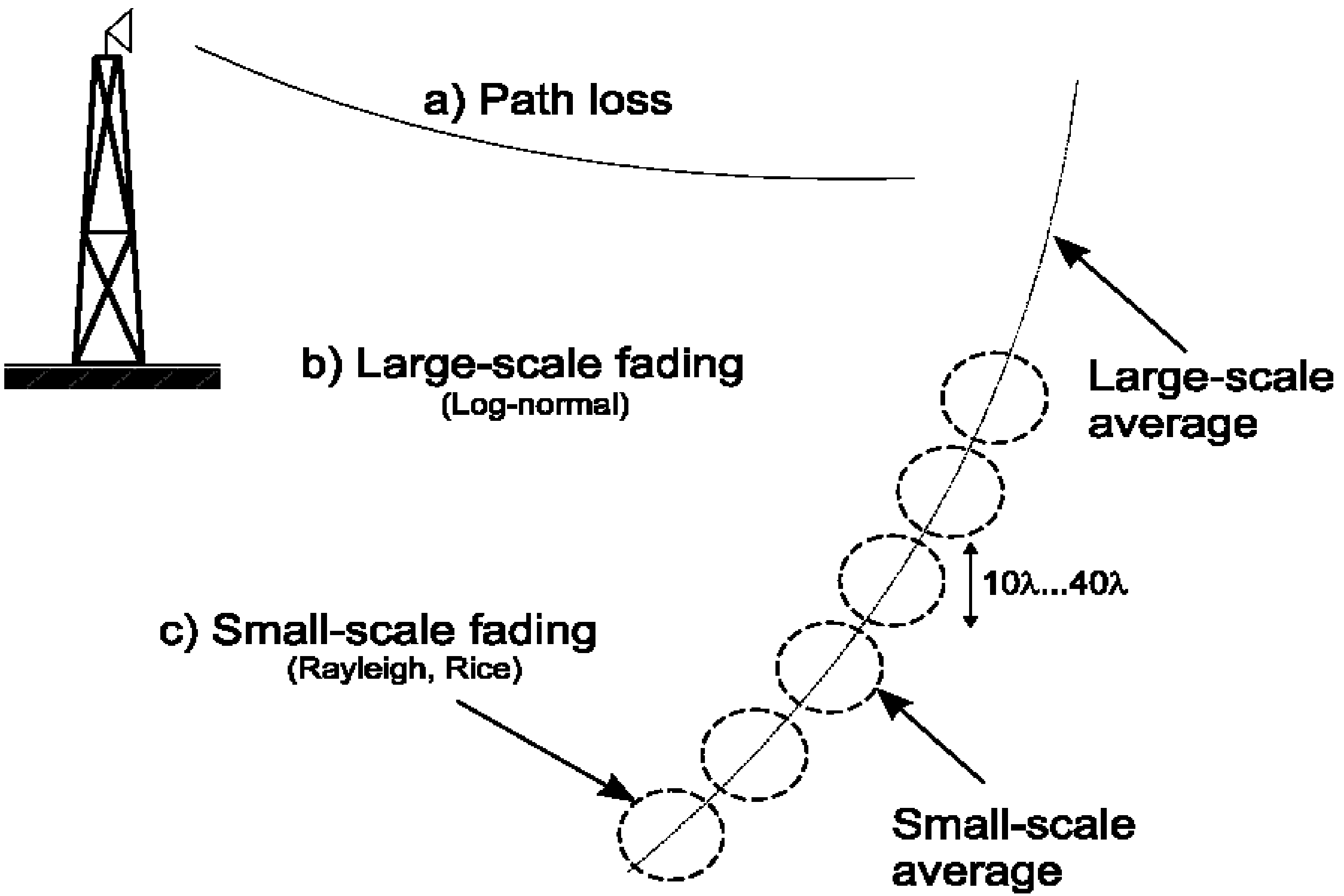
- mean path loss:
  - distance dependent loss in signal energy
  - proportional to $d^{-n}$, where $d$ is the distance and $n$ is the path loss exponent
  - typicall values $n \in [1.5, 6]$, depending on terrain and foilage
- large-scale (shadow) fading
  - Deviation of received signal energy from path loss
  - Caused by obstruction
- small scale fading
  - Rest of constructive and destructive combination of multipaths

# THE RADIO CHANNEL
## Path loss

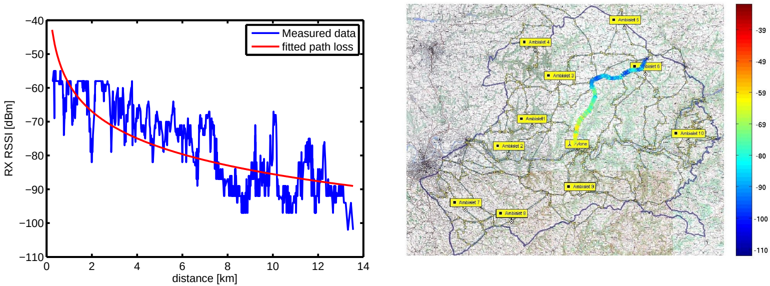


# What is large scale and small scale?



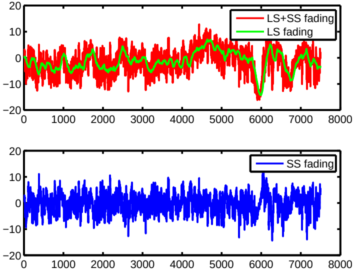
# Example: Path loss




Received power of a terminal in a rural area[1].

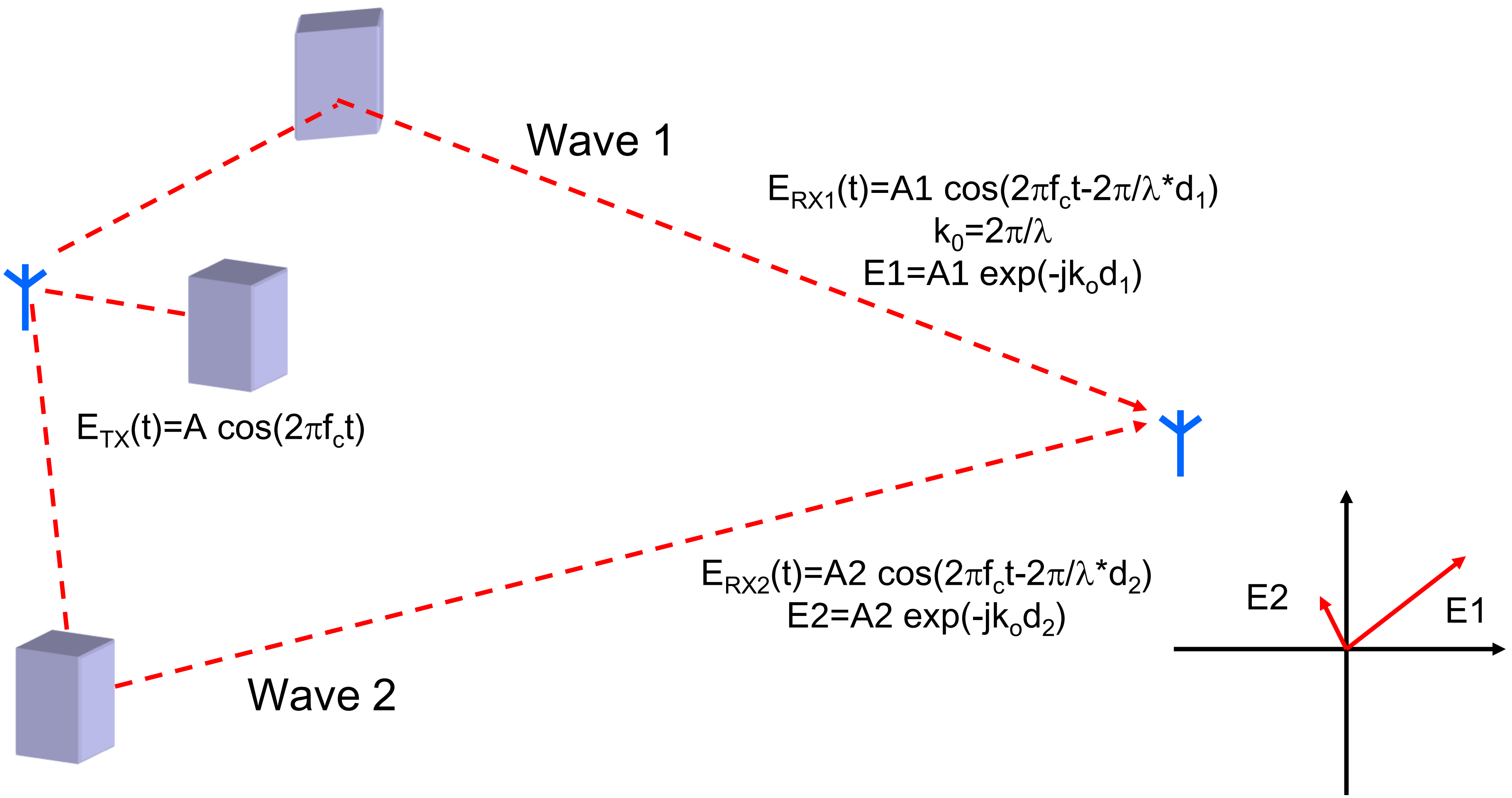
[1] Measurements were taken with OpenAirInterface.org platform at 859.5MHz close to Ambialet, France in collaboration with the CNES

# Example: Large scale and small scale fading

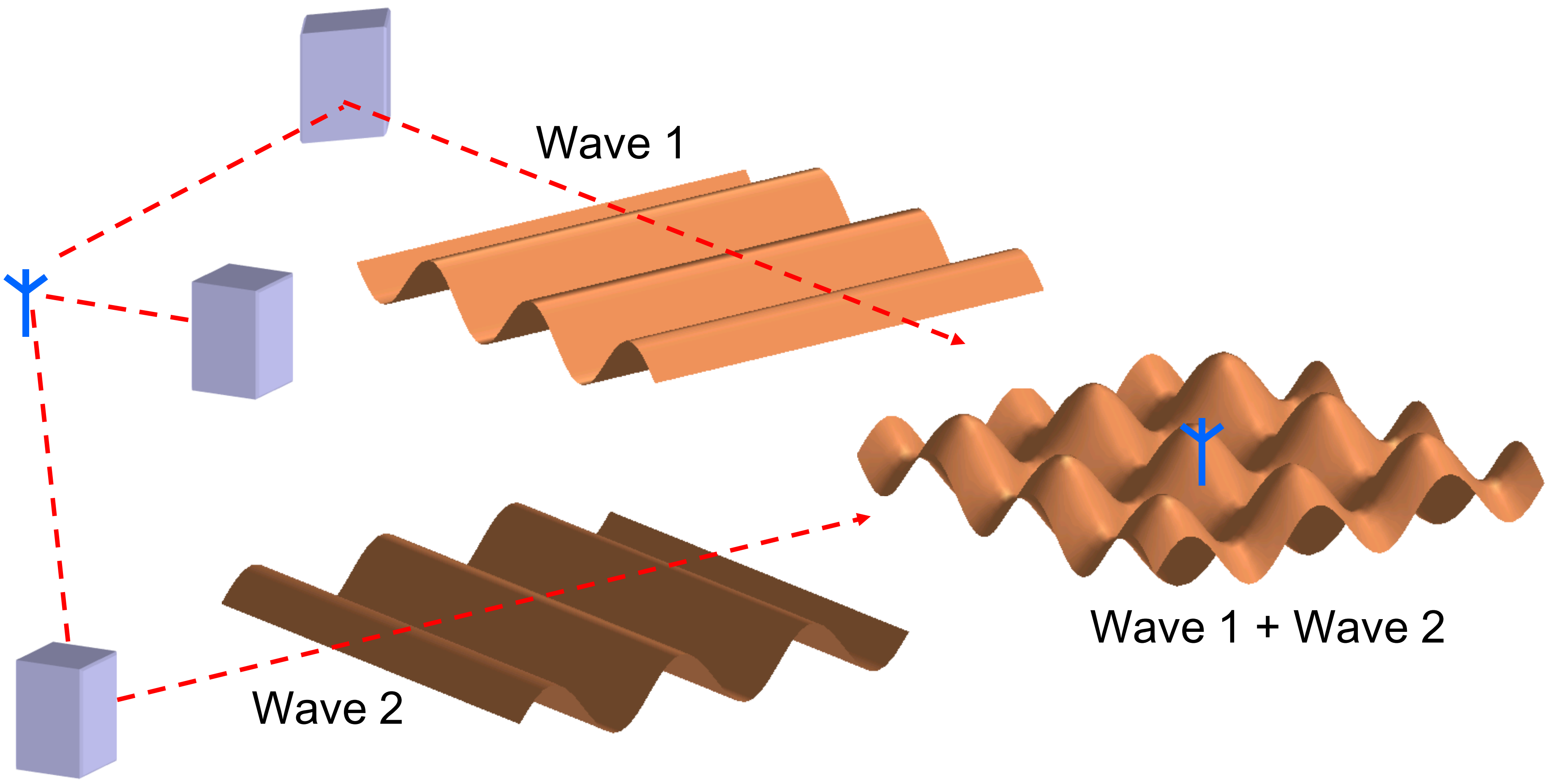



Large scale fading was obtained by applying a moving average filter over 0.25 s $\approx$ 2.5 m (at 10m/s) $\approx 8\lambda$

# Small-scale fading
# Two waves



# Small-scale fading
# Two waves



# THE RADIO CHANNEL
## Small-scale fading (cont.)

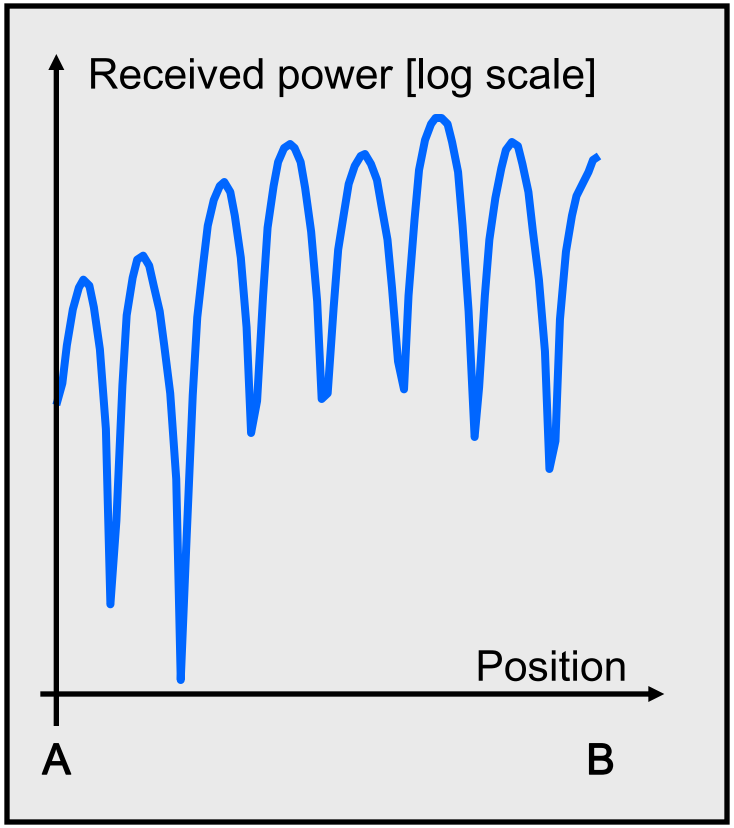
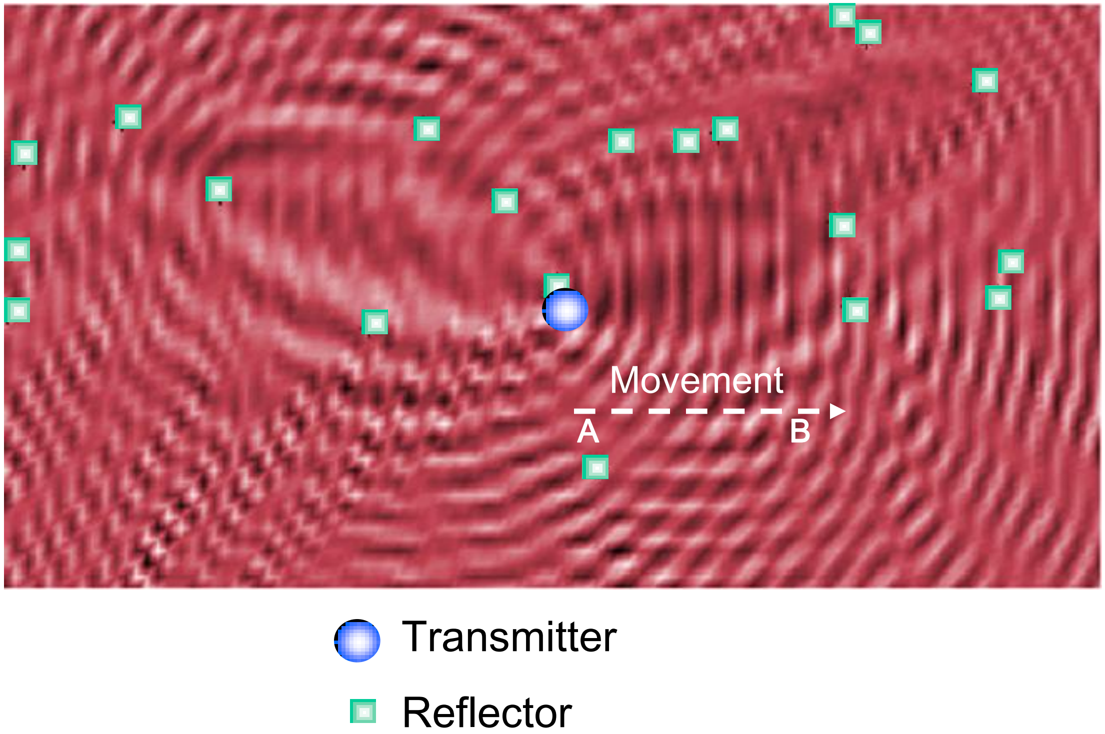
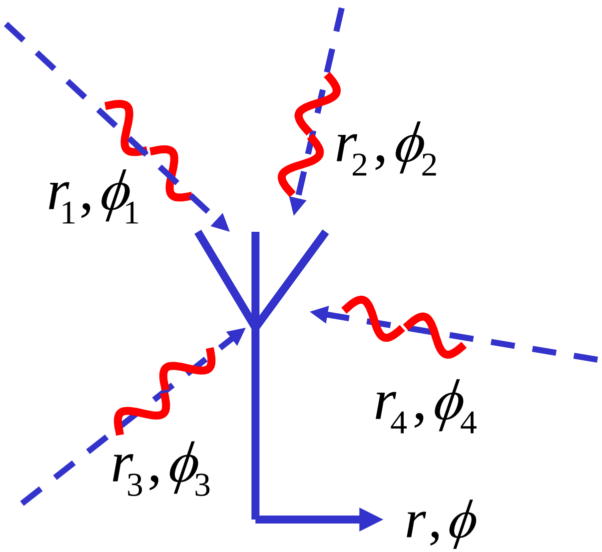



Illustration of interference pattern from above

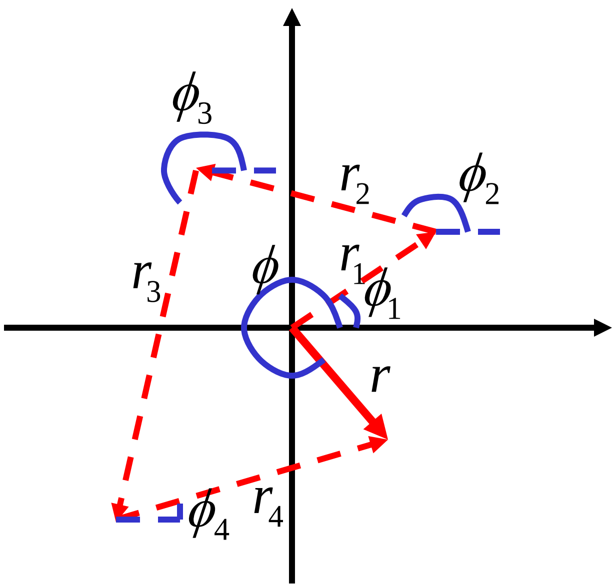


# Small-scale fading
# Many incoming waves

Many incoming waves with independent amplitudes and phases




Add them up as phasors




$$r \exp(j\phi) = r_1 \exp(j\phi_1) + r_2 \exp(j\phi_2) + r_3 \exp(j\phi_3) + r_4 \exp(j\phi_4)$$

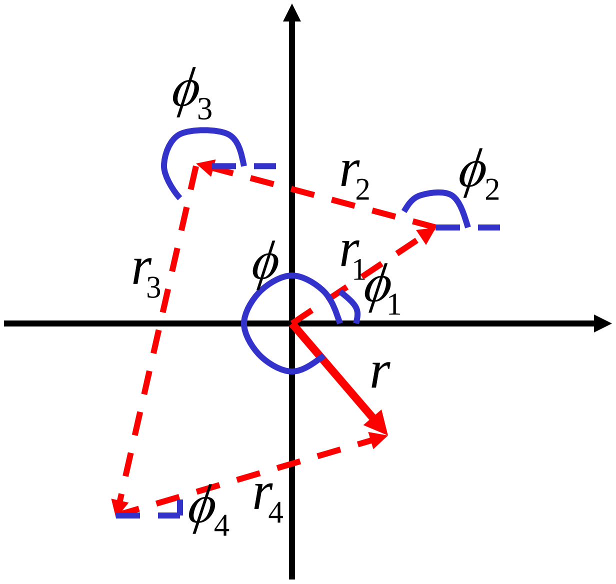
# Small-scale fading
# Many incoming waves

Re and Im components are sums of many independent equally distributed components

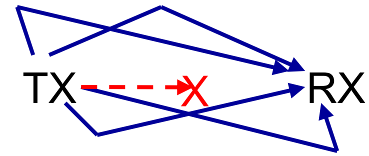
$$\mathrm{Re}(r) \in N(0, \sigma^2)$$

Re(r) and Im(r) are independent

The phase of r has a uniform distribution



# Small-scale fading
# Rayleigh fading

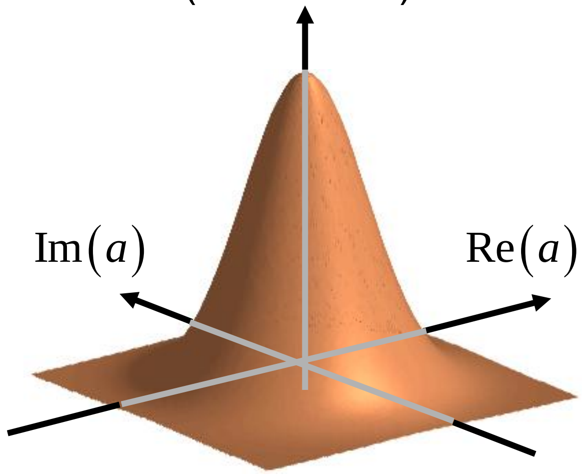



## No dominant component
## (no line-of-sight)

**Tap distribution**

2D Gaussian

(zero mean)

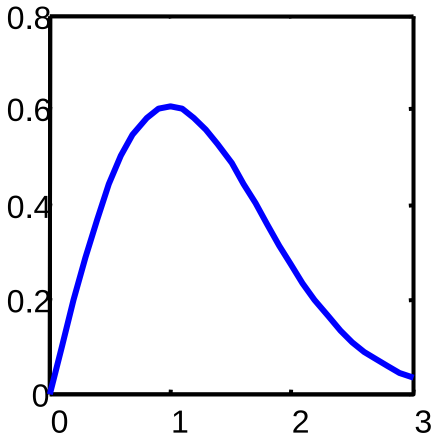



No line-of-sight component

$$r = |a|$$

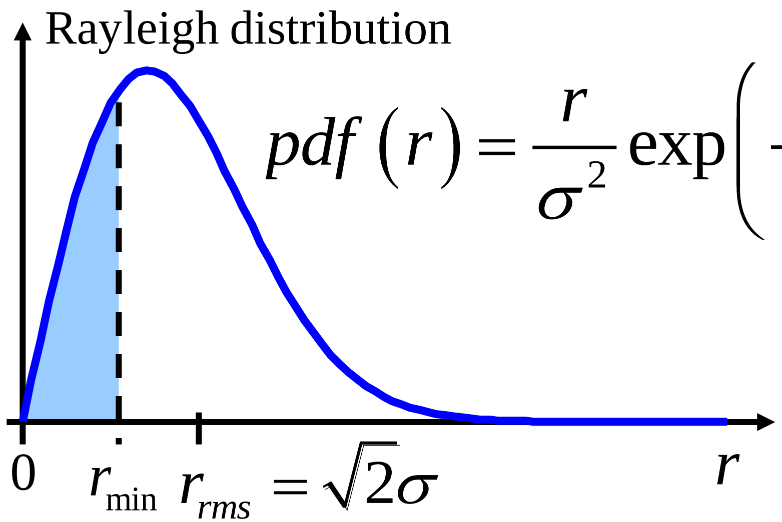
**Amplitude distribution**

Rayleigh




$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

# Small-scale fading
# Rayleigh fading




$$pdf\left(r\right) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

$$\Pr\left(r < r_{\min}\right) = \int_0^{r_{\min}} pdf\left(r\right)dr = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right)$$

# Example: Fading Margin

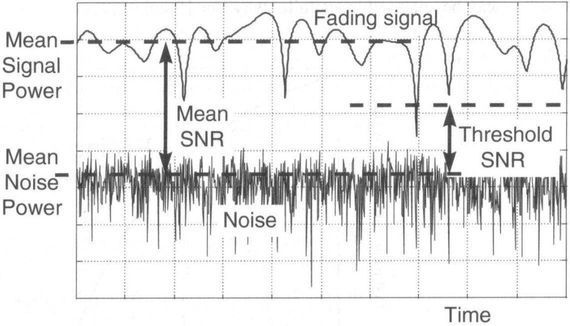
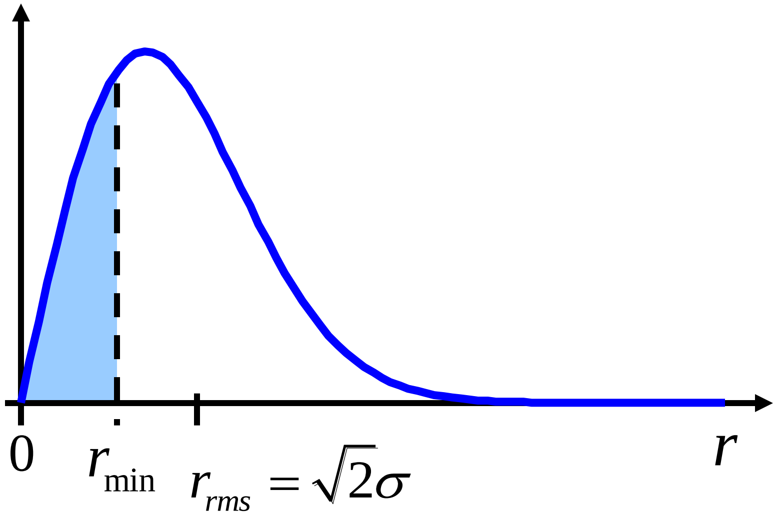



**Figure 10.12**: Variation of instantaneous SNR relative to mean value

# Small-scale fading
# Rayleigh fading – fading margin

$$M = \frac{r_{rms}^{\;2}}{r_{\min}^{\;2}}$$

$$M_{|dB} = 10\log_{10}\left(\frac{r_{rms}^{\;2}}{r_{\min}^{\;2}}\right)$$



# Small-scale fading
# Rayleigh fading – fading margin

How many dB fading margin, against Rayleigh fading, do we need to obtain an outage probability of 1%?

$$\Pr\left(r < r_{\min}\right) = 1 - \exp\left(-\frac{r_{\min}^{2}}{r_{rms}^{2}}\right) = 1\% = 0.01$$

Some manipulation gives

$$1 - 0.01 = \exp\left(-\frac{r_{\min}^{2}}{r_{rms}^{2}}\right) \Rightarrow \ln\left(0.99\right) = -\frac{r_{\min}^{2}}{r_{rms}^{2}}$$

$$\Rightarrow \frac{r_{\min}^{2}}{r_{rms}^{2}} = -\ln\left(0.99\right) = 0.01 \quad \Rightarrow M = \frac{r_{rms}^{2}}{r_{\min}^{2}} = 1/0.01 = 100$$

$$\Rightarrow \boxed{M_{|dB} = 20}$$

# Small-scale fading
# Rayleigh fading – signal and interference

- What is the probability that the instantaneous SIR will be below 0 dB if the mean SIR is 10 dB when both the desired signal and the interferer experience Rayleigh fading?

$$\Pr\left(r < r_{\min}\right) = 1 - \frac{\overline{\sigma}^2 r_{\min}}{(\overline{\sigma}^2 + r_{\min}^2)} = 1 - \frac{10}{(10+1)} \approx 0.09$$

# Small-scale fading
# one dominating component

In case of Line-of-Sight (LOS) one component dominates.

- Assume it is aligned with the real axis

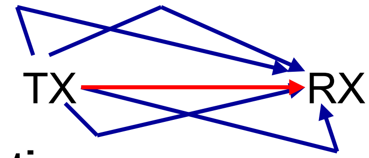
$$\mathrm{Re}(r) \in N(A, \sigma^2) \quad \mathrm{Im}(r) \in N(0, \sigma^2)$$

- The received amplitude has now a Ricean distribution instead of a Rayleigh

- The ratio between the power of the LOS component and the diffuse components is called Ricean K-factor

$$k = \frac{\text{Power in LOS component}}{\text{Power in random components}} = \frac{A^2}{2\sigma^2}$$

# Small-scale fading
# Rice fading

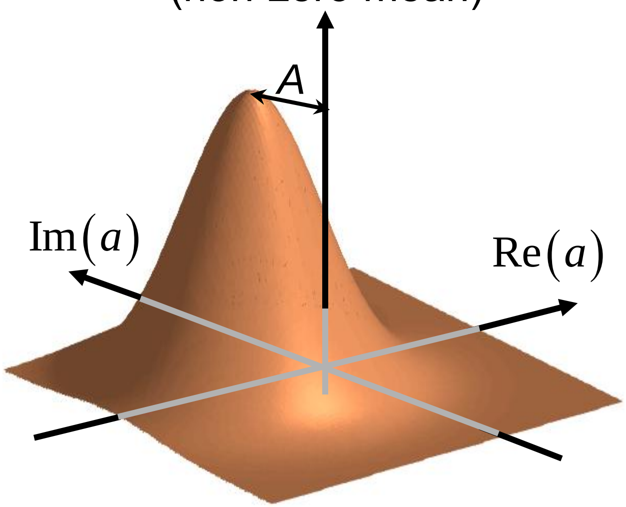



## A dominant component (line of sight)

**Tap distribution**

2D Gaussian

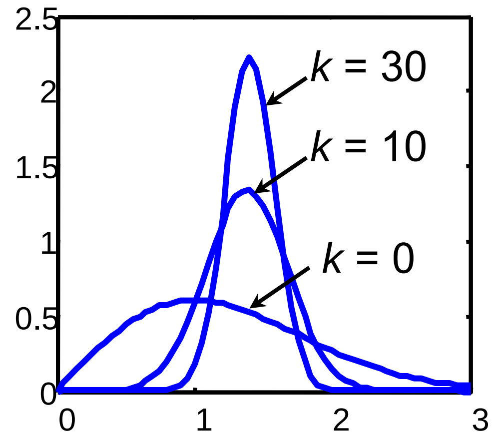
(non-zero mean)




Line-of-sight (LOS) component with amplitude *A*.

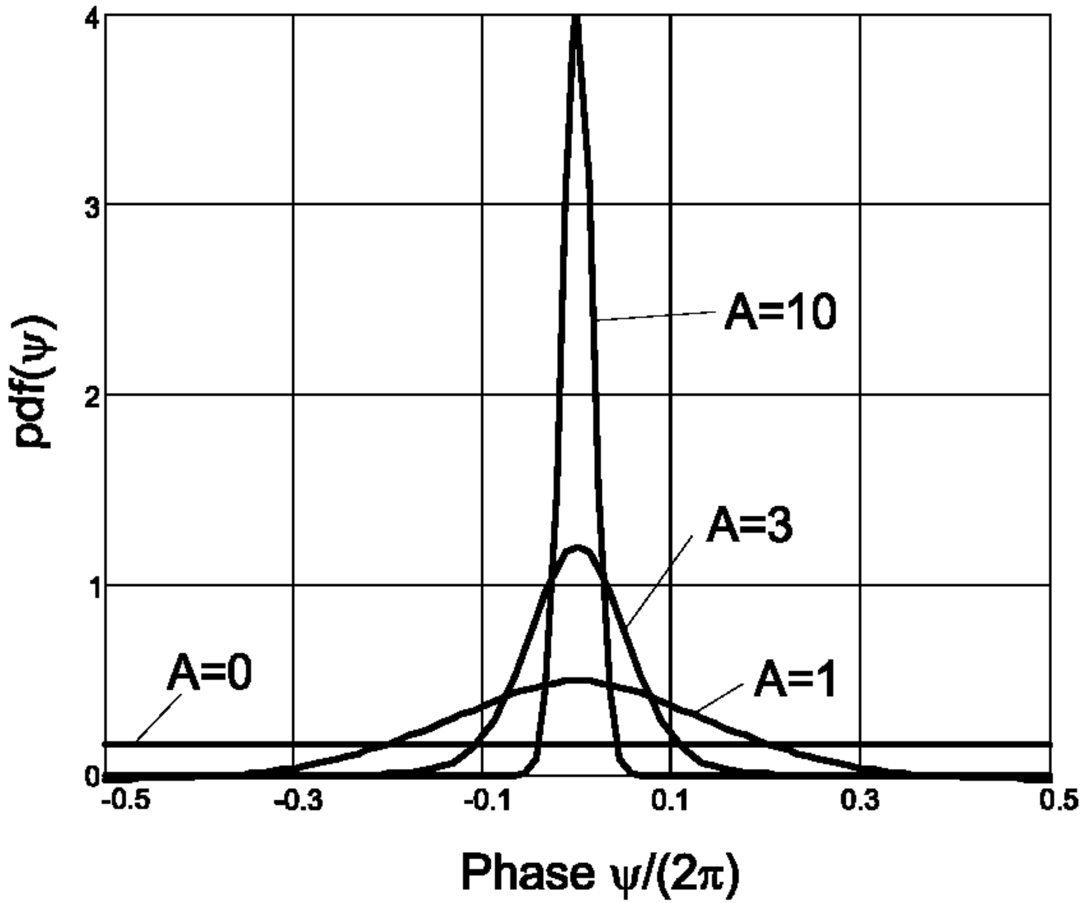
$$r = |\alpha|$$

**Amplitude distribution**




$$pdf\,(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{rA}{\sigma^2}\right)$$

# Small-scale fading
# Rice fading, phase distribution



# Small Scale Fading: Rice fading

- Probability density function, cumulative distribution function, and mean square value of Ricean distribution

$$\mathrm{pdf}(r) = \frac{r}{\sigma^2} \exp - \frac{r^2 + A^2}{2\sigma^2} I_0 \left( \frac{rA}{\sigma^2} \right), \quad 0 \leq r < \infty,$$

$$\mathrm{cdf}(r) = 1 - Q_M \left( \frac{A}{\sigma}, \frac{r}{\sigma} \right)$$

$$\bar{r^2} = 2\sigma^2 + A^2$$

where $I_0$ is the modified Bessel function of the first kind, order 0 and $Q_M$ is Marcum's Q function

# Example: Ricean Fading Margin

Compute the fading margin for a Rice distribution with $\sigma = 1$ and $K_r = 0.3, 3$, and 20 dB so that the outage probability is less than 5%.

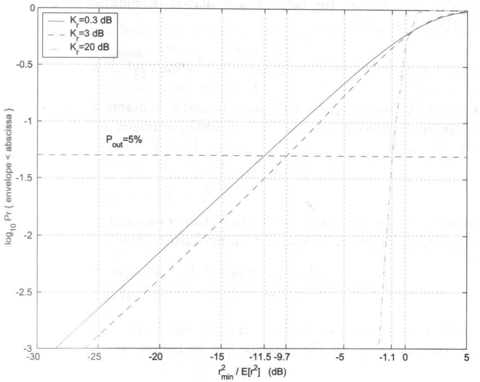



**Figure 5.21** The Rice power *cdf*, $\sigma = 1$.

$$M = \frac{r_{\text{rms}}^2}{r_{\text{min}}^2} = \frac{2\sigma^2(1+K_r)}{r_{\text{min}}^2}$$
$$= 11.5, 9.7, 1.1\text{dB}$$

# Small-scale fading
# Nakagami distribution

- In many cases the received signal can not be described as a pure LOS + diffuse components
- The Nakagami distribution is often used in such cases

$$pdf(r) = \frac{2}{\Gamma(m)} (\frac{m}{\Omega})^m r^{2m-1} \exp(-\frac{m}{\Omega} r^2)$$

$\Gamma(m)$ is the gamma function

$$\Omega = \overline{r^2}$$

$$m = \frac{\Omega^2}{(r^2 - \Omega)^2}$$

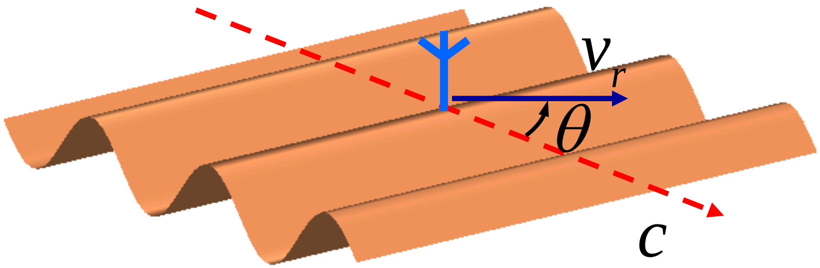
with m it is possible to adjust the dominating power

# Second-order fading statistics

- Capture dynamic effects of the channel (evolution over time, rate of change)
- Let $x(t)$ be a stochastic process, then the *autocorrelation of x* is defined as
$$R_{xx}(t_1, t_2) = \mathcal{E}\{x(t_1)x^*(t_2)\}$$
- Iff $R_{xx}(t_1, t_2) = R_{xx}(t_1 - t_2)$, $x$ is *wide sense stationary* (WSS)
- The *power spectrum* of a WSS process is given by

$$S(f) = \mathcal{F}\{R_{xx}(\tau)\} = \int R_{xx}(\tau)e^{-j2\pi f\tau}\mathrm{d}\tau$$

# Small-scale fading
# Doppler shifts




Receiving antenna moves with speed $v_r$ at an angle $\theta$ relative to the propagation direction of the incoming wave, which has frequency $f_0$.

Frequency of received signal:

$$f = f_0 + \nu$$

where the Doppler shift is

$$\nu = -f_0 \frac{v_r}{c} \cos(\theta)$$

The maximal Doppler shift is

$$\nu_{\max} = f_0 \frac{v}{c}$$

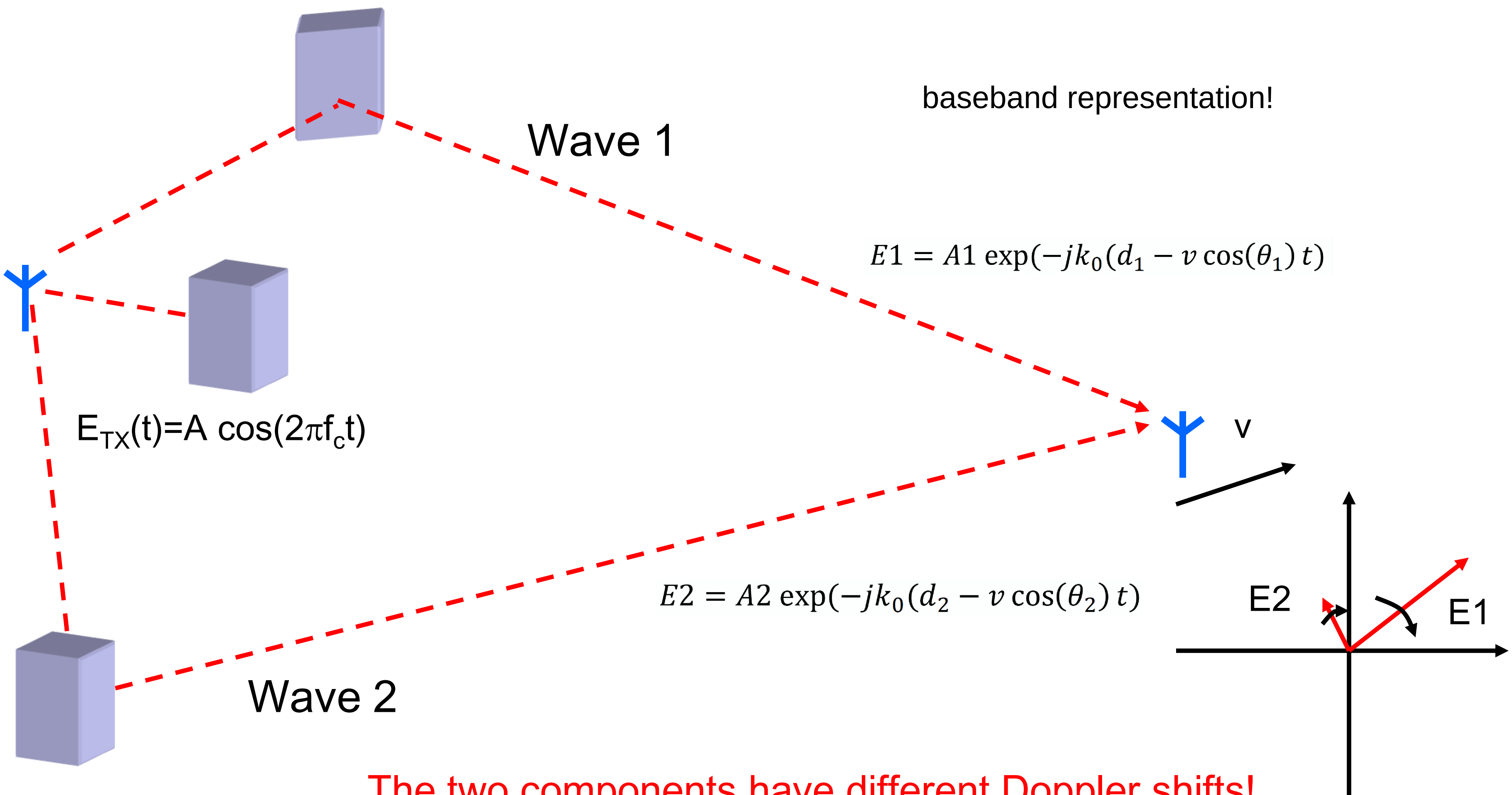
# Small-scale fading
# Doppler shifts

How large is the maximum Doppler frequency at pedestrian speeds for 5.2 GHz WLAN and at highway speeds using GSM 900?

$$\nu_{\max} = f_0 \frac{v}{c}$$

- $f_0$=5.2 $10^9$ Hz, v=5 km/h, (1.4 m/s) ⇨ 24 Hz
- $f_0$=900 $10^6$ Hz, v=110 km/h, (30.6 m/s) ⇨ 92 Hz

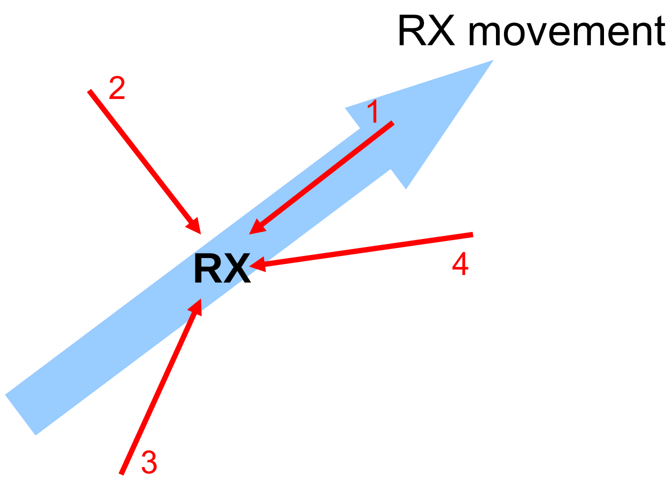
# Small-scale fading
# Doppler spectra




The two components have different Doppler shifts!
The Doppler shifts will cause a random frequency modulation

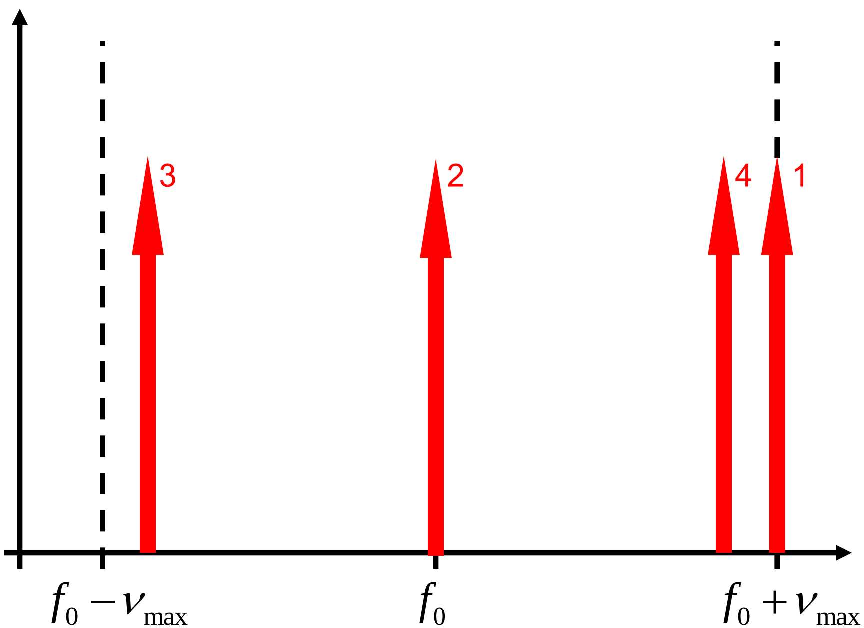
# Small-scale fading
# Doppler spectrum

Incoming waves from several directions
(relative to movement or RX)




All waves of equal strength in
this example, for simplicity.

**Spectrum of received signal**
**when a $f_0$ Hz signal is transmitted.**



# Small-scale fading
# Doppler spectrum

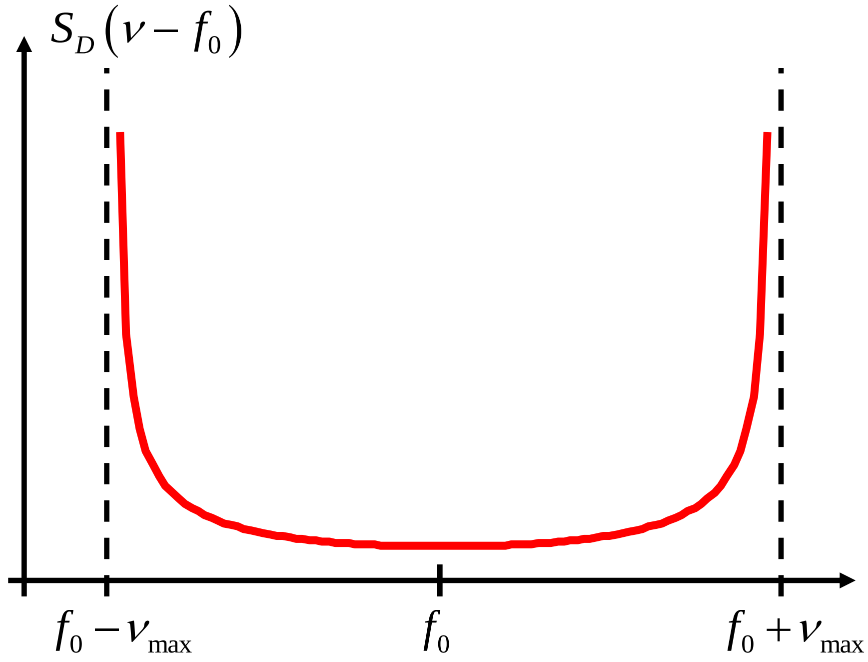
AoA are uniformly distributed

$$S_D(\nu) = \int \rho(\Delta\tau) e^{-j2\pi\nu\Delta\tau} d\Delta\tau$$

$$\propto \frac{1}{\pi\sqrt{\nu_{\max}{}^2 - \nu^2}}$$

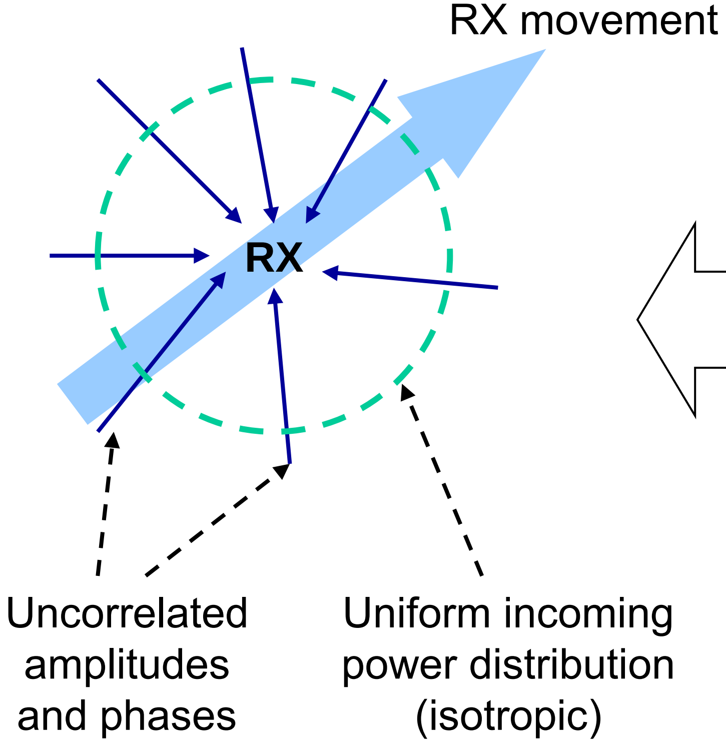
for $-\nu_{\max} < \nu < \nu_{\max}$

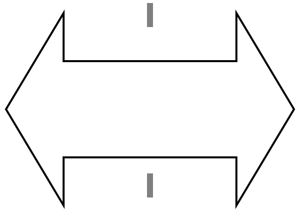
Doppler spectrum
at center frequency $f_0$.



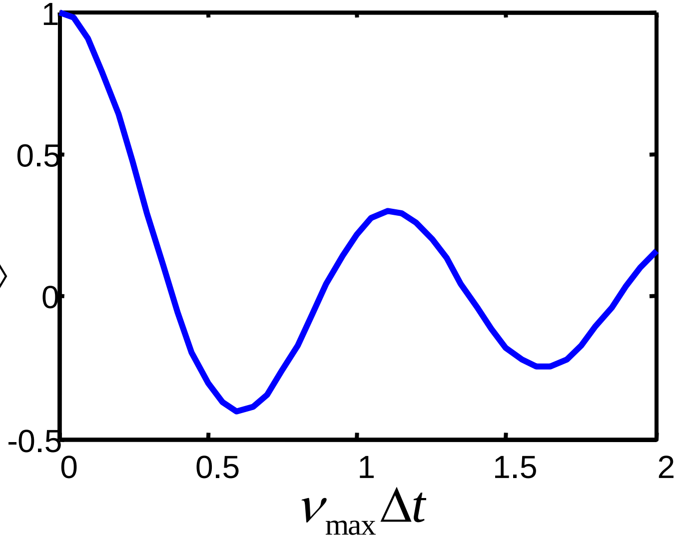
# Small-scale fading
# Doppler spectrum

**Isotropic uncorrelated scattering**



**Time correlation**

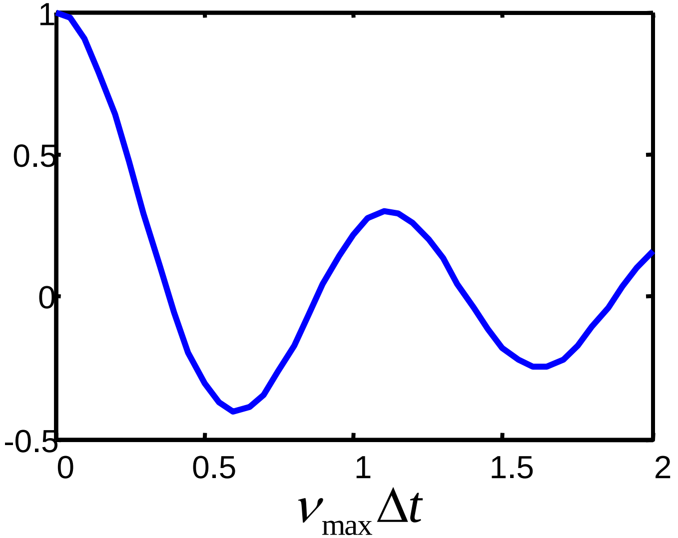


# Small-scale fading
# Doppler spectrum

How static is the channel?

- Time correlation of in-phase and quadrature components*

$$\rho(\Delta t) = E\{a(t)a^*(t+\Delta t)\} \propto J_0(2\pi \nu_{\max} \Delta t)$$



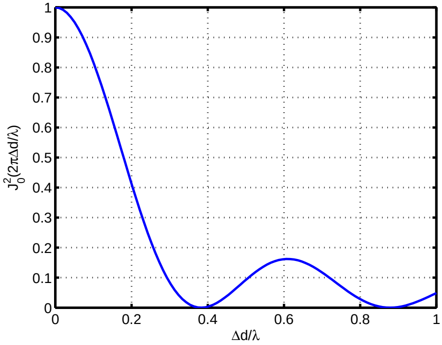

- The time correlation for the amplitude is

$$\rho(\Delta t) \propto J_0^2(2\pi \nu_{\max} \Delta t)$$

* correlation between in-phase and quadrature is 0!

# Example: Autocorrelation

Assume that the mobile is in a fading dip. On average, what minimum distance should the user move, so that it is no longer influenced by this fading dip?

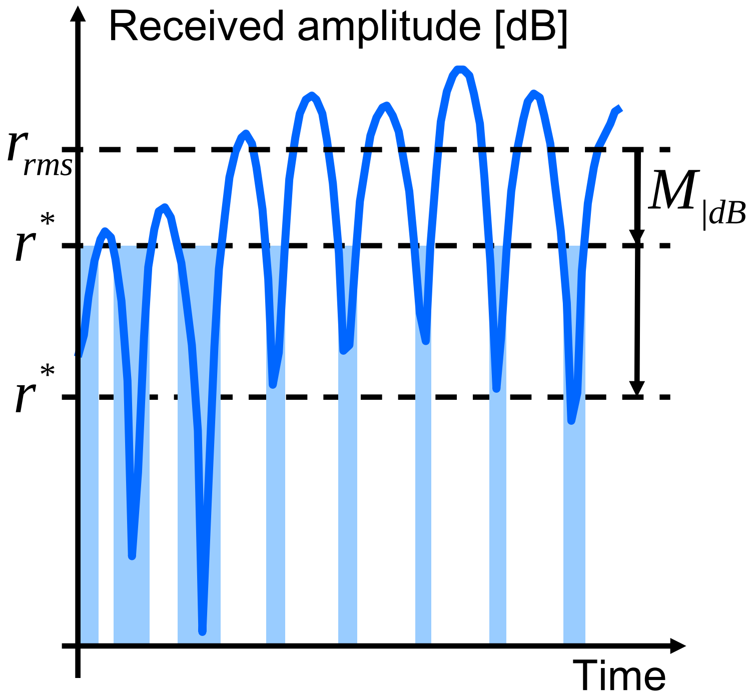



Note: this measure is strongly related to the coherence time (later).

# Small-scale fading
# Fading dips

What about the length and the frequency of fading dips ?

Level crossing rate: how often does the signal cross the level r*?
Average duration of fade: how long does the signal stay below r*?



# Level crossing rate and avarage duration of fades

- Level crossing rate

$$N_R(r) = \int_0^\infty \dot{r} \cdot \mathrm{pdf}(r, \dot{r}) \mathrm{d}\dot{r}$$
$$= \sqrt{\frac{\Omega_2}{\pi\Omega_0}} \frac{r}{\sqrt{2\Omega_0}} \exp\left(-\frac{r^2}{2\Omega_0}\right)$$

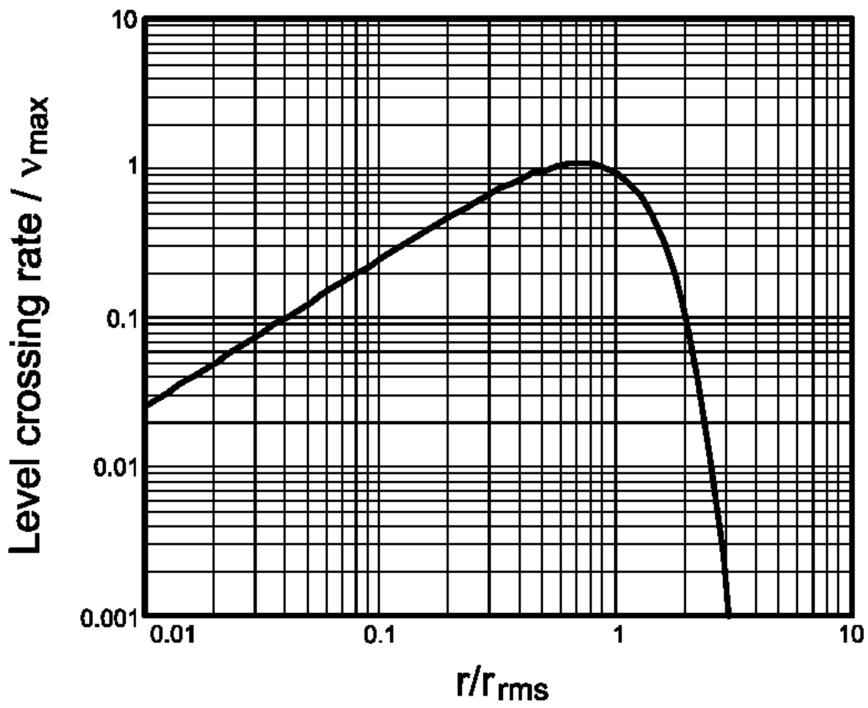
  where $\Omega_n$ is the $n$-th moment of the Doppler power spectrum ($r_{rms} = \sqrt{\Omega_0}$)

- Average duration of fade

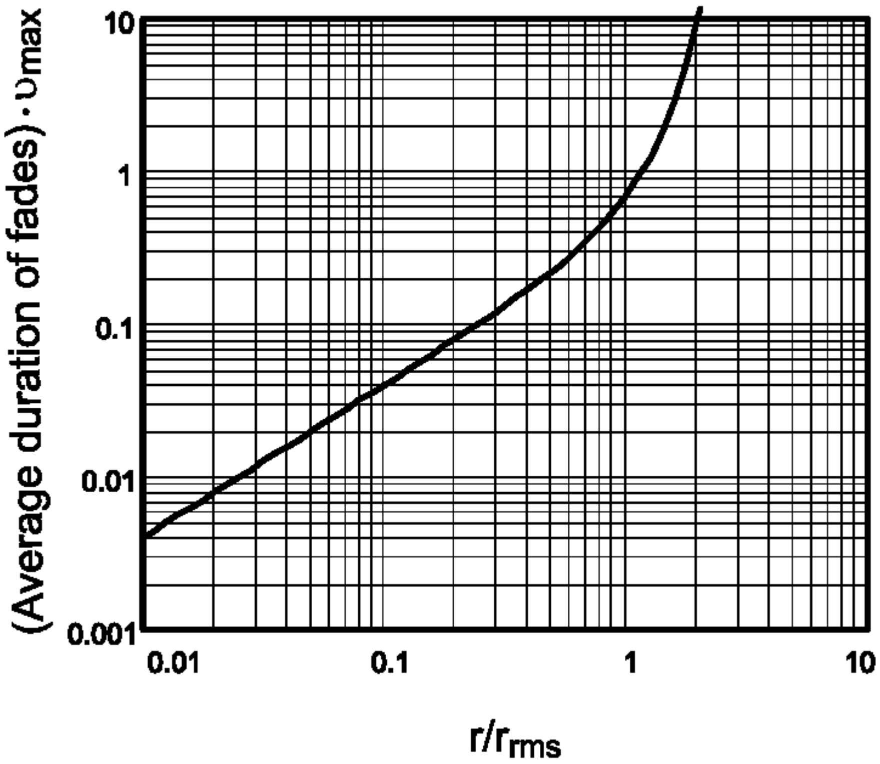
$$ADF(r) = \frac{\mathrm{cdf}(r)}{N_R(r)}$$

# Small-scale fading
# Statistics of fading dips

Frequency of the fading dips
(normalized dips/second)



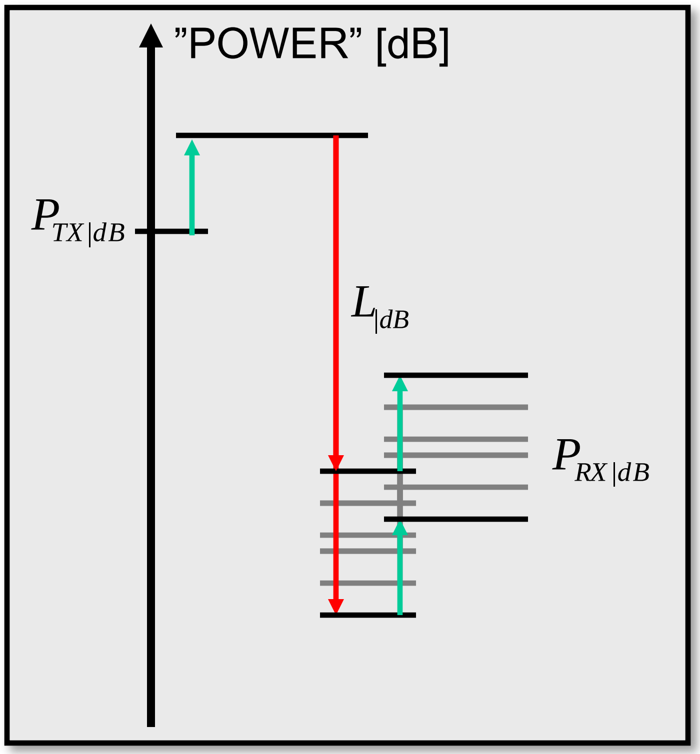

Length of fading dips
(normalized dip-length)

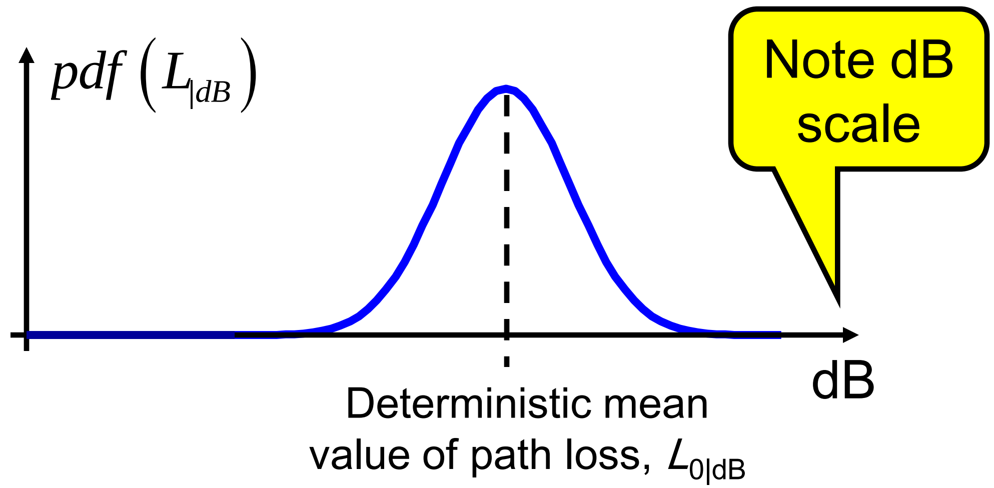


# Example: GSM system

Consider a GSM system at $f_c = 900$ MHz and a maximum user speed of $v_{\max} = 100$ km/h. Assume that the channel has a classical Doppler spectrum. What is the Doppler bandwidth and the coherence time? What is the level crossing rate and the average fade duration given a fading margin of 10 and 20 dB respectively.
Discuss the implication of the finding under the consideration that GSM has a burst duration of 0.5 ms.

# Large-scale fading
# Log-normal distribution




$$pdf\left(L_{|dB}\right) = \frac{1}{\sqrt{2\pi}\sigma_{F|dB}} \exp\left(-\frac{\left(L_{|dB} - L_{0|dB}\right)^2}{2\sigma_{F|dB}^{\ 2}}\right)$$

Standard deviation $\sigma_{F|dB} \approx 4 \ldots 10 \text{ dB}$

# Large-scale fading
# Basic principle