Noise and Interference Limited Systems

Noise Limited - range is signal power limited, BER decreases exponentially with SNR

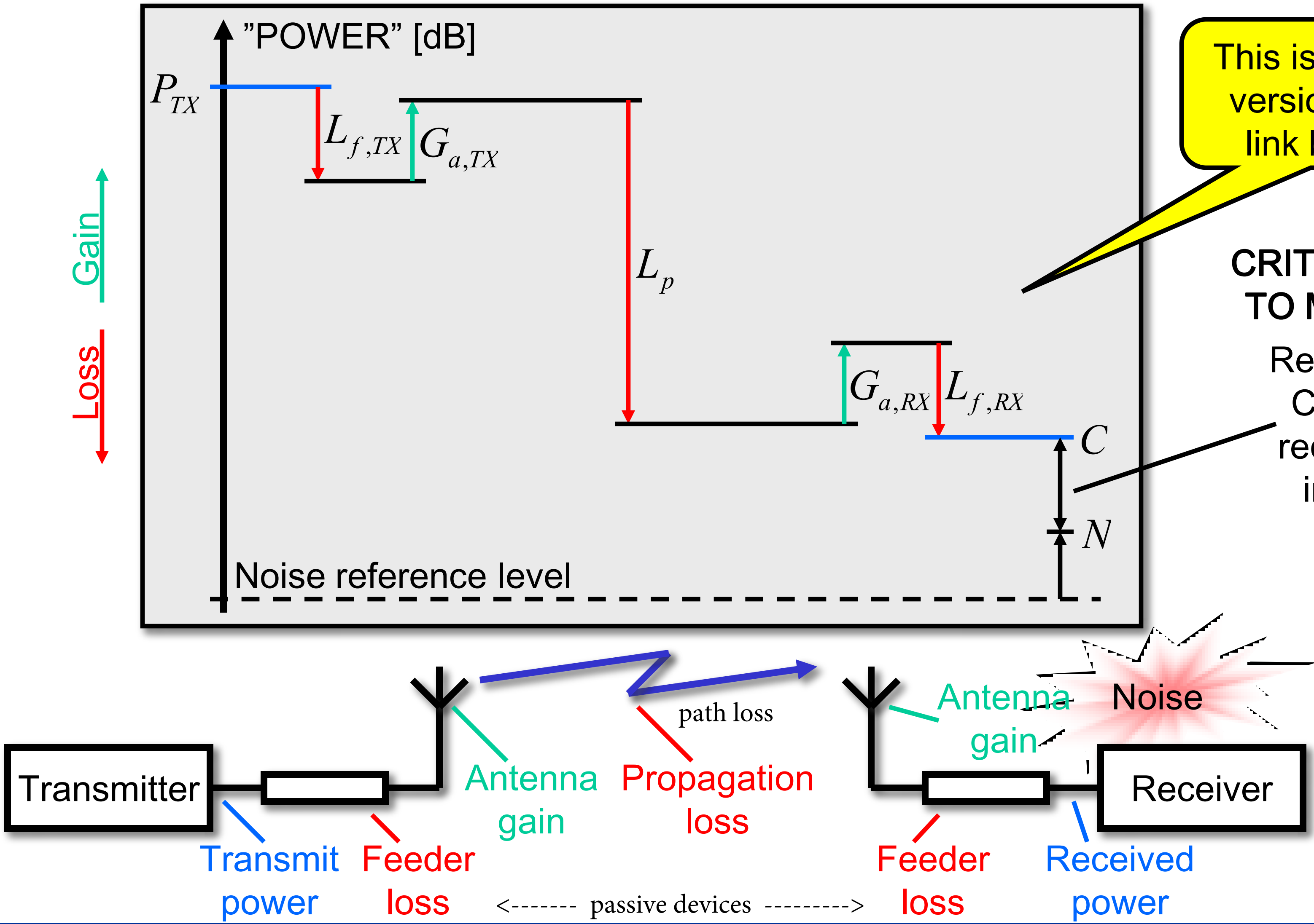
Interference Limited - probablistic based (fading, signal distortion, multipath),
increasing transmit power doesn't improve BER very much since
BER decreases linearly with SNR

Basics of link budgets

- Link budgets show how different components and propagation processes influence the available SNR
- Link budgets can be used to compute required transmit power, possible range of a system or required receiver sensitivity
- Link budgets can be most easily set up using logarithmic power units (dB) $\mathrm{dB} = 10 \log_{10} (\mathrm{Pout}/\mathrm{Pin})$

A logarithmic scheme is a data compression technique scaling what would otherwise be ratios (out/in) of very large or very small quantities. Unfortunately this ends up hiding from our normal perception what are actually very small or very big ratios.

SINGLE LINK
The link budget – a central concept



dB in general

When we convert a measure X into decibel scale, we always divide by a reference value X_{ref}:

$$\frac{X\,|_{non-dB}}{X_{ref}\,|_{non-dB}}$$

Independent of the dimension of X (and X_{ref}), this value is always dimension-less.

The corresponding dB value is calculated as:

$$X\,|_{dB} = 10\log\left(\frac{X\,|_{non-dB}}{X_{ref}\,|_{non-dB}}\right)$$

Note that this ratio has no units, it is dimensionless. It is annotated with dB only to inform us of the mathematics or compression technique that was used on the relative ratio of two numbers of the same units, i.e., apples/apples.

Power

We usually measure power in Watt (W) and milliWatt [mW]

The corresponding dB notations are dB and dBm

	Non-dB	dB
Watt:	$P\|_W$	$P\|_{dB} = 10\log\left(\frac{P\|_W}{1\|_W}\right) = 10\log\left(P\|_W\right)$
milliWatt:	$P\|_{mW}$	$P\|_{dBm} = 10\log\left(\frac{P\|_{mW}}{1\|_{mW}}\right) = 10\log\left(P\|_{mW}\right)$
RELATION:	$P\|_{dBm} = 10\log\left(\frac{P\|_W}{0.001\|_W}\right) = 10\log\left(P\|_W\right) + 30\|_{dB} = P\|_{dB} + 30\|_{dB}$ Change dBw to dBm by adding 30 Change dBm to dBw by subtracting 30	

Decibels (dB) - Details

- $G_{dB} = 10 \log_{10} (P_{out}/P_{in})$ Gain is the inverse of Loss $G = 1/L$

  Gain in dB = - Loss in dB $G_{dB} = - L_{dB}$

- $L_{dB} = - 10 \log (P_{out} / P_{in}) = 10 \log (P_{in} / P_{out})$
- Since $P = V^2/R$ where P = power (Watts) dissipated across R = resistance/impedence where V = voltage across R then

  $$G_{dB} = 10 \log[(V^2_{out}/R) / (V^2_{in}/R)] = 20 \log[V_{out} / V_{in}]$$

  given that the input and output impedances are the same

- 3 dB → power has been doubled (-3 dB is ½ reduction)

  -10 dB → power has been reduced by a factor of 10 (0.1)

- dBW (decibel-Watt) gain referenced to 1 W

  dBm (decibel-milliWatt) gain referenced to 1 mW (10^{-3} W)

- + 30 dBm = 0 dBW 0 dBm = - 30 dBW

Example: Power

Sensitivity level of GSM RX: 6.3×10^{-14} W = -132 dB or -102 dBm

Bluetooth TX: 10 mW = -20 dB or 10 dBm

GSM mobile TX: 1 W = 0 dB or 30 dBm

GSM base station TX: 40 W = 16 dB or 46 dBm

Vacuum cleaner: 1600 W = 32 dB or 62 dBm

Car engine: 100 kW = 50 dB or 80 dBm

ERP – Effective Radiated Power

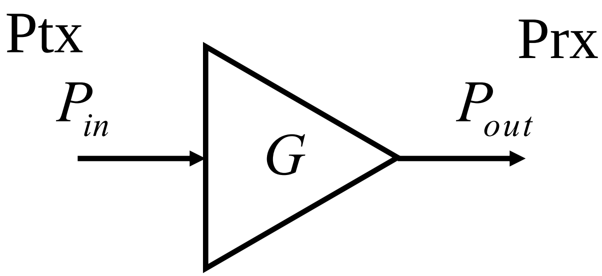
takes antenna gains into account

TV transmitter (Hörby, SVT2): 1000 kW ERP = 60 dB or 90 dBm ERP

Nuclear powerplant (Barsebäck): 1200 MW = 91 dB or 121 dBm

Amplification and attenuation

(Power) Amplification:



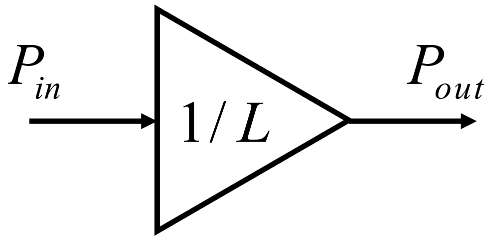
$$P_{out} = GP_{in} \Rightarrow G = \frac{P_{out}}{P_{in}}$$

The amplification is already dimension-less and can be converted directly to dB:

$$G|_{dB} = 10\log_{10} G$$

Note: It doesn't matter if the power is in mW or W. Same result!**

(Power) Attenuation:



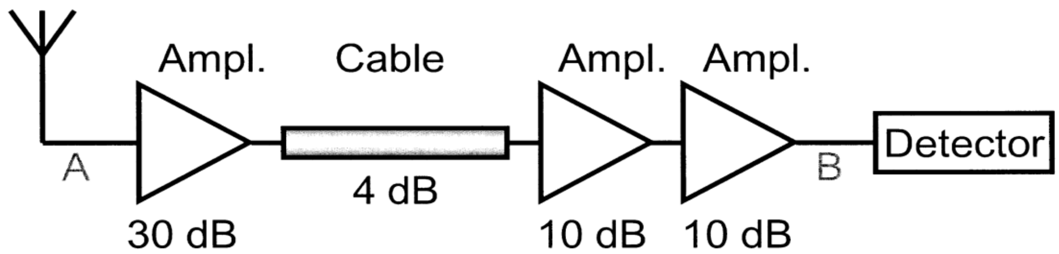
$$P_{out} = \frac{P_{in}}{L} \Rightarrow L = \frac{P_{in}}{P_{out}}$$

The attenuation is already dimension-less and can be converted directly to dB:

$$L|_{dB} = 10\log_{10} L$$

** as long as apples out is the same as apples in

Example: Amplification and attenuation



What does a receiver (detector) sensitivity of -90 dBm represent?

The total amplification of the (simplified) receiver chain (between A and B) is

$$G_{A,B}\,|_{dB} = 30 - 4 + 10 + 10 = 46$$

If 5 mW shows up at A, how much power appears at B? (Hint: either convert 5 mW to dB or convert the G = 46 dB to its relative #. Does it make any difference if the input to A is in dB or dBm?

this is the tricky case	5 mW ---->		----> 199.05 W
just addition for dB	$-23.01\ dB_w$ --->	46 dB	----> $22.99\ dB_w$
just addition for dB_m	$6.99\ dB_m$ ---->		----> $52.99\ dB_m$

Categories of Noise

- Thermal Noise
- Intermodulation noise
- Crosstalk
- Impulse Noise

Noise Terminology

- Intermodulation noise – occurs if signals with different frequencies share the same medium in association with some nonlinear device
  - Interference caused by a signal produced at a frequency that can be multiples of the sum or difference of original frequencies; result of nonlinear devices (a mixer, a diode, a dissimlar junction - just about all electronic devices are nonlinear)
- Crosstalk – unwanted coupling between signal paths (excessive signal strength, no isolation, undesired mutual coupling, etc.)
- Impulse noise – irregular pulses or noise spikes
  - RF Energy of short duration with relatively high amplitudes
  - Caused by external electromagnetic disturbances (lightning), or faults and flaws in the communications system
  - Not a big problem for analog data but the primary error source for digital transmission, may be minimized by the demodulation technique, noise blanker electronic circuits, antenna diversity.

Thermal Noise

- Thermal noise due to agitation of electrons
- Present in all electronic devices and transmission media (white noise)
- Function of temperature
- Cannot be eliminated (except at temperatures of absolute 0^{o}K)
- Particularly significant for satellite communication (since the satellite frequencies don't have many other noise sources, thermal noise is the only normal source of noise)

Thermal Noise

- Amount of thermal noise to be found in a bandwidth of 1 Hz for any device or conductor is:

$$N_0 = \mathrm{k}T\ (\mathrm{W/Hz})$$

  - N_0 = noise power density in watts per 1 Hz of bandwidth
  - k = Boltzmann's constant = 1.3803×10^{-23} J/K
  - T = temperature, in Kelvins (absolute temperature)

Thermal Noise

- Noise is assumed to be independent of frequency
- Thermal noise present in a bandwidth of *B* Hertz (in watts):

$$N = \mathrm{k}TB$$

The larger the bandwidth the larger the noise energy since (white) noise is uniform across the entire spectrum.

or in decibel-watts

$$N_{dBw} = 10 \log \mathrm{k} + 10 \log T + 10 \log B$$

$$= -228.6\ \mathrm{dB_W} + 10 \log T + 10 \log B$$

or in decibel-milliwatts

$$\mathrm{N}_{dBm} = -198.6\ \mathrm{dB_m} + 10 \log \mathrm{T} + 10 \log \mathrm{B}$$

Expression E_b/N_0

a commonly used ratio in digital communications (dimensionless usually in dB)

- Ratio of signal energy per bit to noise power density per Hertz

$$\frac{E_b}{N_0} = \frac{S/R}{N_0} = \frac{S}{\mathrm{k}TR} \qquad \frac{\text{(Signal Power) (Time for 1 bit)}}{\text{Noise Power}}$$

- The bit error rate for digital data is a function of E_b/N_0
  - Given a value for E_b/N_0 to achieve a desired error rate, parameters of this formula can be selected
  - As bit rate R increases, transmitted signal power (S) must increase to maintain required E_b/N_0
- Ratio doesn't depend on bandwidth as does Shannon's channel capacity (It is a normalized SNR measure, a SNR per bit. Used to compare BER for different modulation schemes without taking bandwidth into account.)

Spectral Efficiency for digital signals

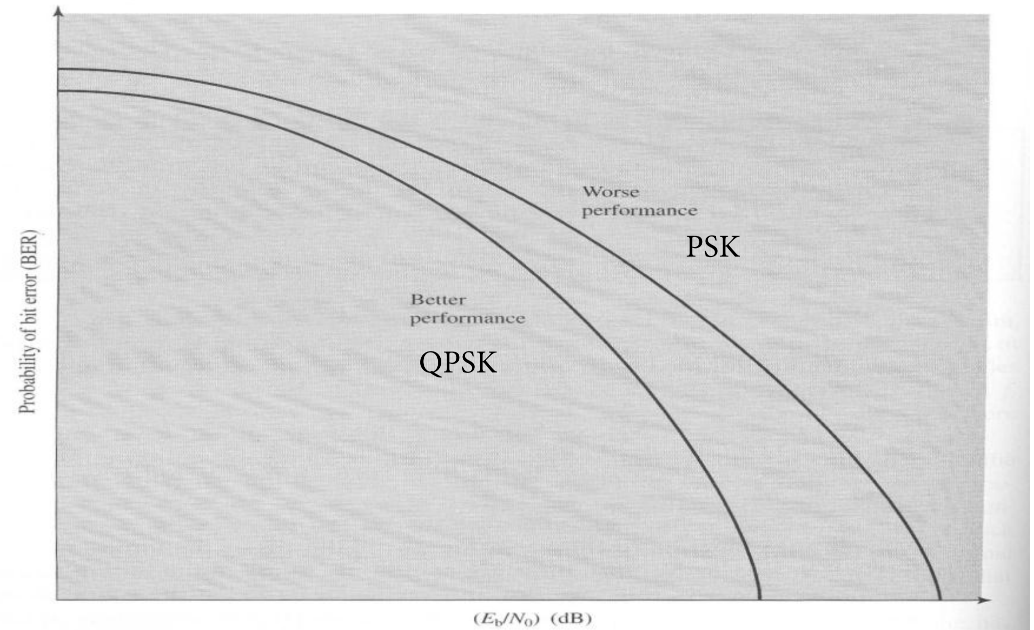


Figure 5.9 General Shape of BER Versus E_b/N_0 Curves

Assumes AWGN Channel with constant noise density N_o and thus E_b/N_o must be used with care since in interference limited channels interference doesn't always meet these assumtions.

Rewriting Shannon's Channel Capacity C wrt SNR (noise)

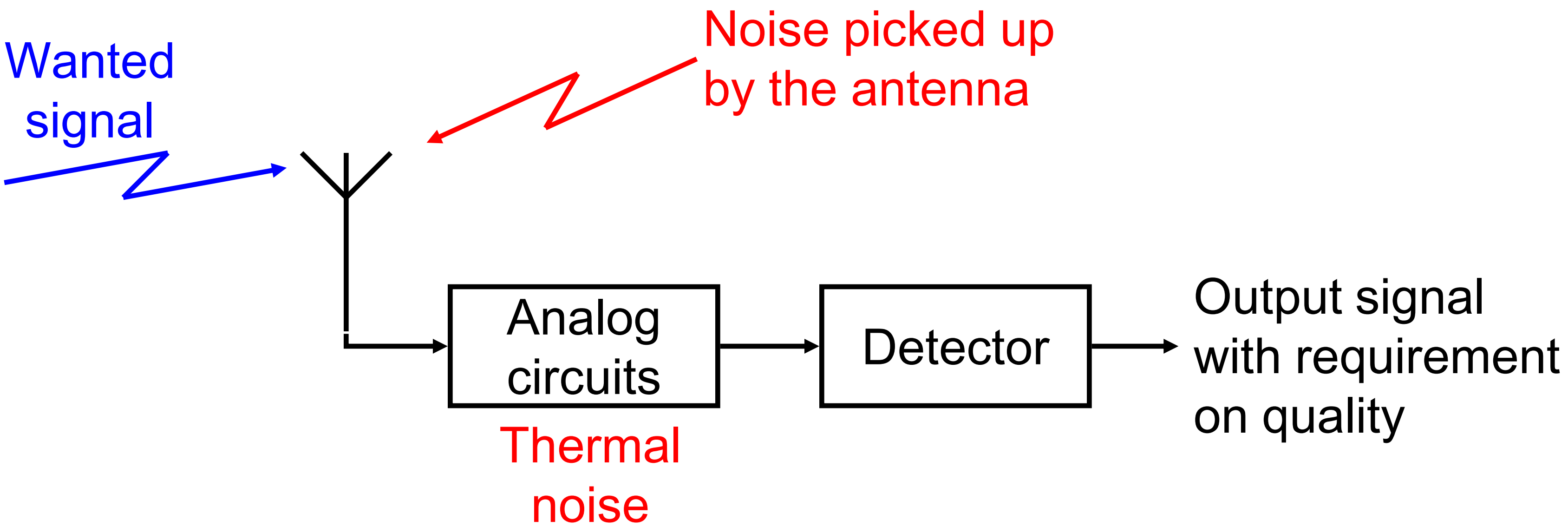
$C = B \log_2(1 + S/N)$ $S/N = 2^{C/B} - 1$ thus

$E_b/N_o = (B/C)(2^{C/B} - 1)$ derivation in Stallings page 113

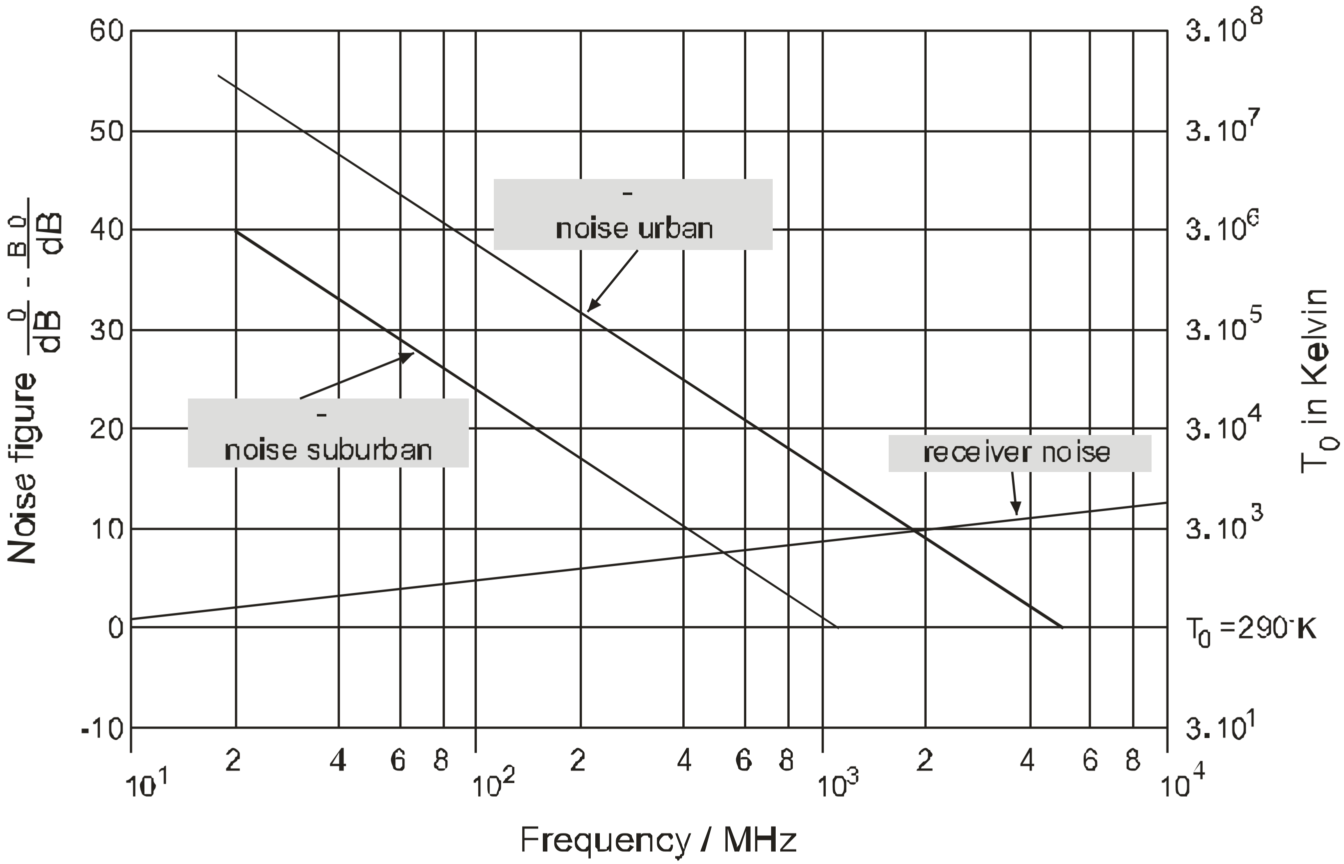
Which allows us to the find the required noise ratio E_b/N_o for a given spectral efficiency C/B Spectral Efficiency = 6 bps/Hz then Eb/No = 10.5 = 10.21 dB

Noise sources

The noise situation in a receiver depends on several noise sources

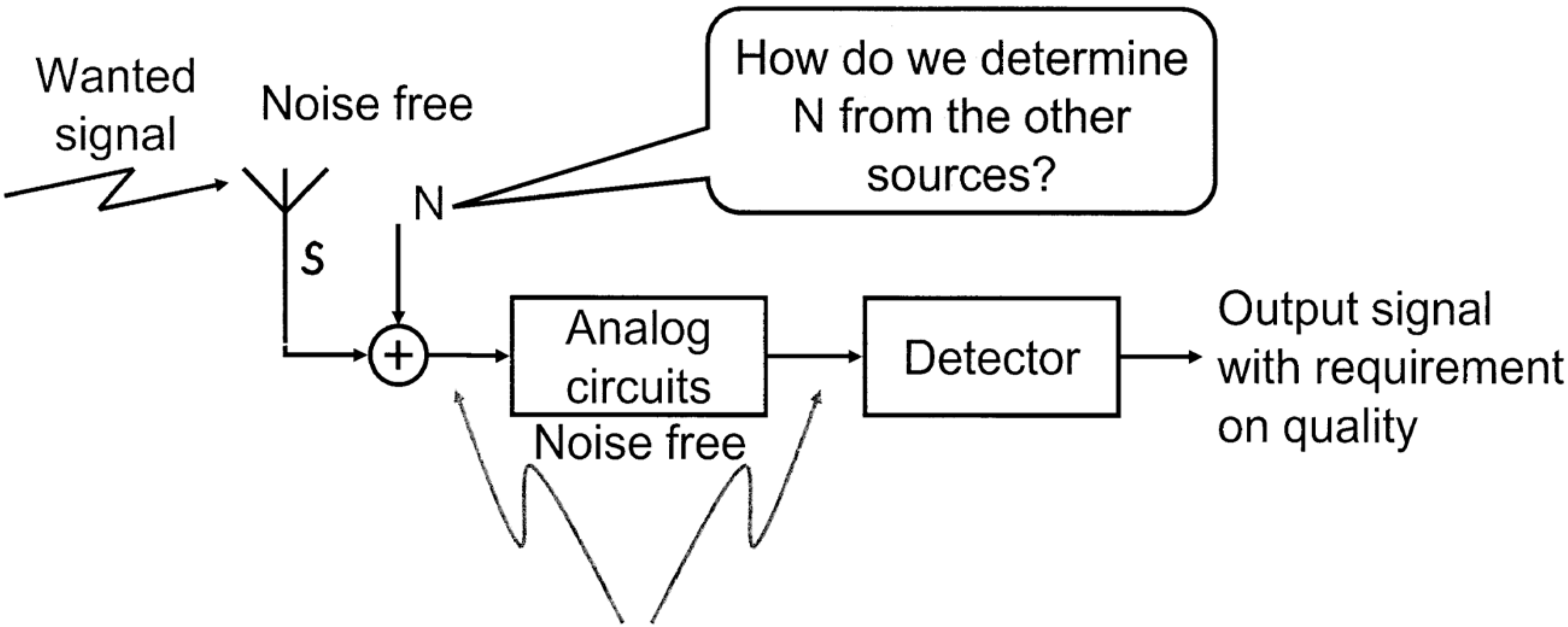


Man-made noise



Receiver Noise: Equivalent Noise Source

To simplify the situation, we replace all noise sources with a single equivalent noise source.



Same "input quality", signal-to-noise ratio, S/N in the whole chain.

Receiver noise: Noise sources

The power spectral density of a noise source is usually given in one of the following three ways:

1) Directly [W/Hz]: N_s

2) Noise **temperature** [Kelvin]: T_s

3) Noise **factor** [no units]: F_s

This one is sometimes called the noise figure when given in dB

The relation between the three is

$$N_s = kT_s = kF_sT_0$$

where k is **Boltzmann's constant** ($1.38\text{x}10^{-23}$ W/Hz) and T_0 is the, so called, **room temperature** of 290 K (17^{o} C).

Noise Figure F

- Noise Figure (F)

$$F = \frac{\text{Measured Noise Power Out of Device at Room Temp}}{\text{Power Out of Device if Device Were Noiseless}}$$

- F is always greater than 1.
- Effective Noise Temperature (T_e)

  $T_e = (F - 1)T_o$ where T_o is ambient room temperature, typically 290 K to 300 K which is 63 ^{0}F to 75^{0}F or 17 ^{0}C to 27 ^{0}C

- A noiseless device has a F = 1 or T_e = 0K

Cascaded System Noise Figure - F_{sys}

- For a cascaded system, the noise figure of the overall system is calculated from the (non-dB) noise figures and gains of the individual components

$$F_{sys} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots\ldots\ldots\ldots\ldots \quad \textit{(F's and G's are \textbf{NOT} in dB's)}$$

which shows that the noise figure of the first active device in a cascaded system (usually F_1) is the most important part of a cascaded system in terms of the F_{sys} noise figure (1st active device's noise figure is far more important than the gain)

- When passive (non-active) components such as transmission lines, attenuators, connectors, etc. are used in cascaded system noise calculations

$\mathbf{F_{db} = L_{db} = - G_{db}}$ or for linear (non-dB) parameters $\mathbf{F = L = 1/G}$

the noise figure F is the same as the loss L

(note that a positive loss/attenuation L in dB represents a negative gain G in dB)

Thus for passive components the Noise Figure = Loss (attenuation) and the corresponding (non-dB) G for the device is 1/L as used in the above F_{sys} equation

(note G and L are not in decibels for F_{sys})

A tool is available at http://www.pasternack.com/t-calculator-noise-figure.aspx

Noise Temperature for a System

- The overall equivalent temperature for a cascaded system has the same relationship as the noise figure, the Te of the system is impacted the most by the first component

$$Te_{sys} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots\ldots\ldots\ldots$$

where the gains are in linear values - NOT dB

Communications System Analysis
Noise Figure and Noise Temperature

- T_e and F are useful since the gains of the receiver stages are not needed to quantify the overall noise amplification of the receiver. If an antenna at room temperature is connected to the input of a receiver having a noise figure F, then the noise power at the output of the receiver referred to the input is simply F times the input noise power or

$$P_{out} = F\,k\,T_o\,B = (1 + Te/To)\,k\,T_o\,B$$

Te - effective noise temperature

To - room temperature

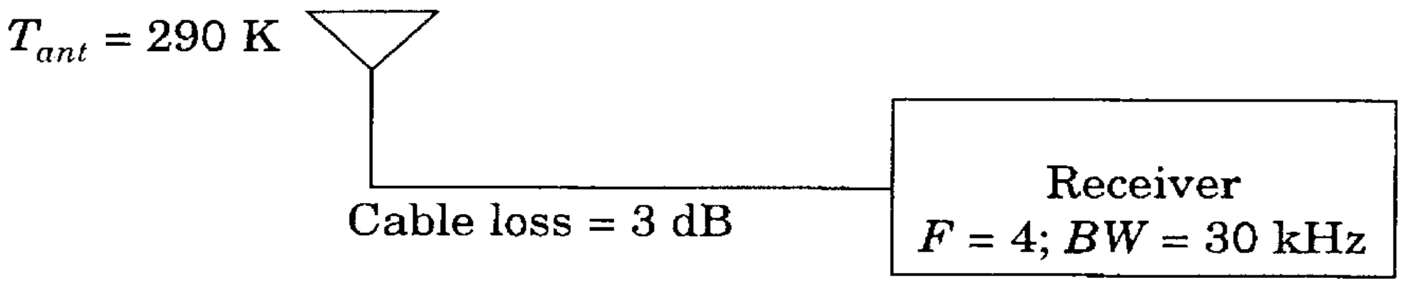
Noise Figure F = 1 + Te / To

Link Budget for a Receiver System

- Consider a cell phone receiver with a noise figure $F = 4$ that is connected to an antenna at room temperature using a coaxial cable with a loss $L = 3$ dB.

  Compute the noise figure of the mobile receiver system as referred to the input of the antenna

Noise Figure of the System



A mobile receiver system with cable losses.

- For non active devices F = L (either dB or linear) the cable (a non-active device) noise factor is F = 3 dB = 2 or with a G = -3 dB = 0.5

- Keeping all values in linear rather than in dB, the receiver system has a noise figure

F1 = 2 or 3 dB   F2 = 4 or 6.02 dB
G1 = 1/F1 = 0.5 or -3.01 dB

$$F_{sys} = 2.0 + (4 - 1)/(0.5) = 8 = 9.03 \text{ dB}$$

Communications Analysis Problems

For the MOBILE RECEIVER SYSTEM, determine the average output thermal noise power, as referred to the input of the antenna terminals. Assume $T_o = 300$ K.

Solution

SINCE $T_e = (F-1)\,T_o$ The cable/receiver system has a noise figure of 9 dB. the effective noise temperature of the system is

$$T_e = (8-1)300 = 2100\text{ K}$$

THE overall system noise temperature due to the antenna is given by

$$T_{total} = T_{ant} + T_{sys} = (290 + 2100)\text{K} = 2390\text{ K}$$

SINCE $P_o = \left(1 + \frac{T_e}{T_o}\right) k T_o B$ then the average output thermal noise power referred to the antenna terminals is given by

$$P_n = \left(1 + \frac{2390}{300}\right)(1.38 \times 10^{-23})(300\text{ K})(30,000\text{ Hz})$$

$$= 1.1 \times 10^{-15}\text{ W} = -119.5\text{ dBm}$$

For the MOBILE RECEIVER SYSTEM, determine the required average signal strength at the antenna terminals to provide a SNR of 30 dB at the receiver output.

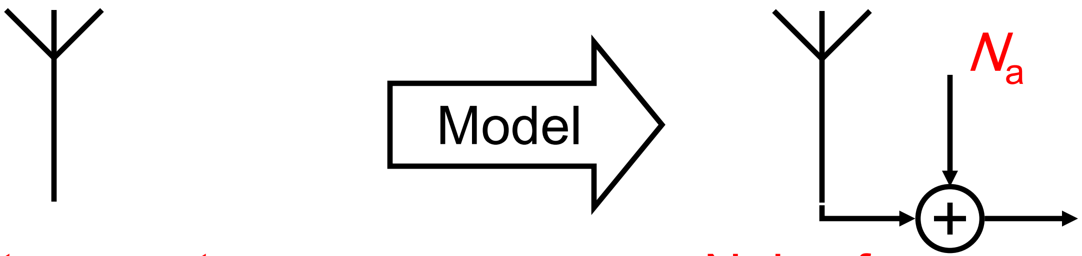
Solution

FROM THE PREVIOUS SOLUTION, the average noise level is -119.5 dBm. Therefore, the signal power must be 30 dB greater than the noise

$$P_s(\text{dBm}) = SNR + (-119.5) = 30 + (-119.5) = -89.5\text{ dBm}.$$

Receiver noise: Noise sources (2)

Antenna example



Noise temperature of antenna 1600 K

Noise free antenna

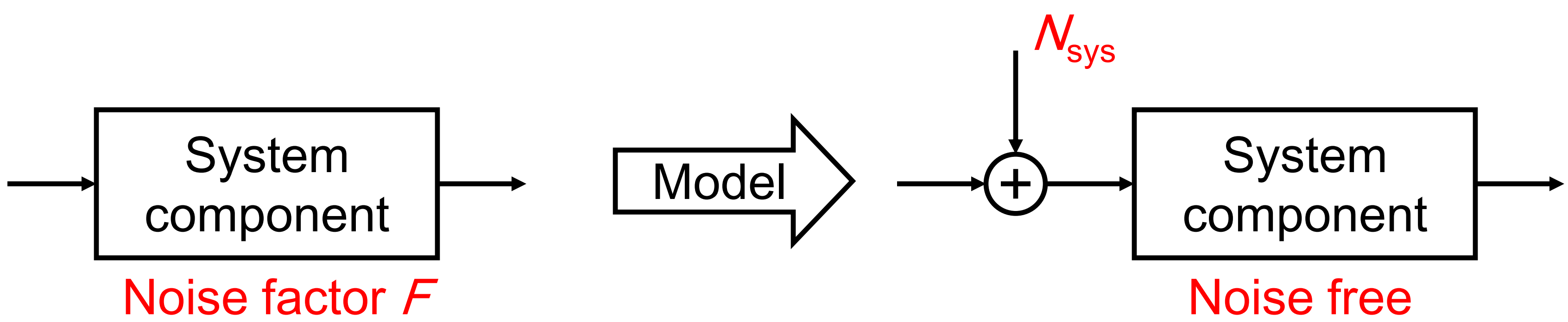
Power spectral density of the antenna noise is

$$N_a = 1.38 \times 10^{-23} \times 1600 = 2.21 \times 10^{-20} \text{ W/Hz} = -196.6 \text{ dB[W/Hz]}$$

and its noise factor is 5.52 or its noise figure is 7.42 dB

$$F_a = 1600/290 = 5.52 = 7.42 \text{ dB}$$ at room temperature 290 K

Receiver noise: System noise



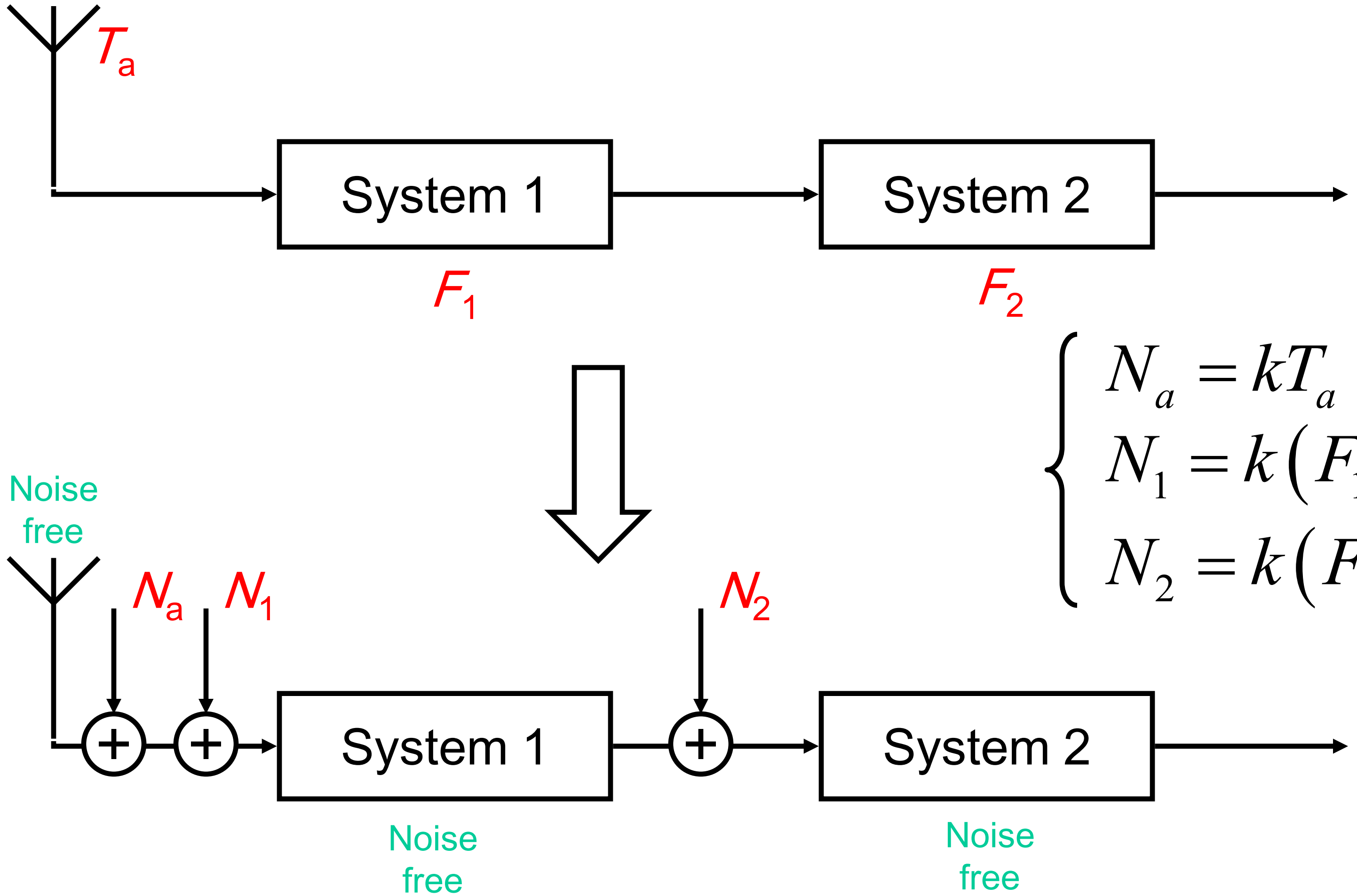
Due to a definition of noise factor (in this case) as the ratio of noise powers on the output versus on the input, when a resistor in room temperature (T_0=290 K) generates the input noise, the PSD of the equivalent noise source (placed **at the input**) becomes

$$N_{sys} = k \underbrace{\left(F-1\right)T_0}_{\text{Equivalent noise temperature}} \text{ W/Hz}$$

Don’t use dB value!

Receiver noise: Several noise sources (1)

A simple example



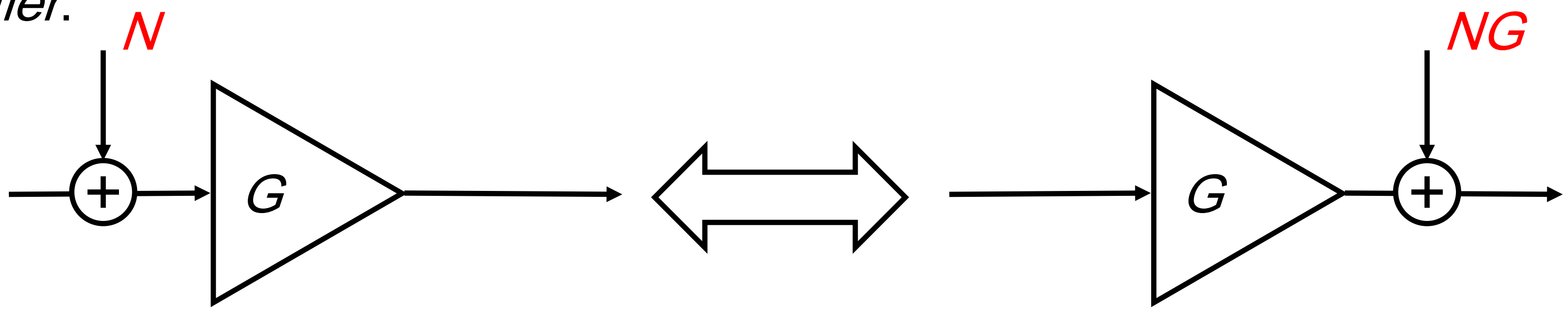
$$\begin{cases} N_a = kT_a \\ N_1 = k\left(F_1 - 1\right)T_0 \\ N_2 = k\left(F_2 - 1\right)T_0 \end{cases}$$

Receiver noise: Several noise sources (2)

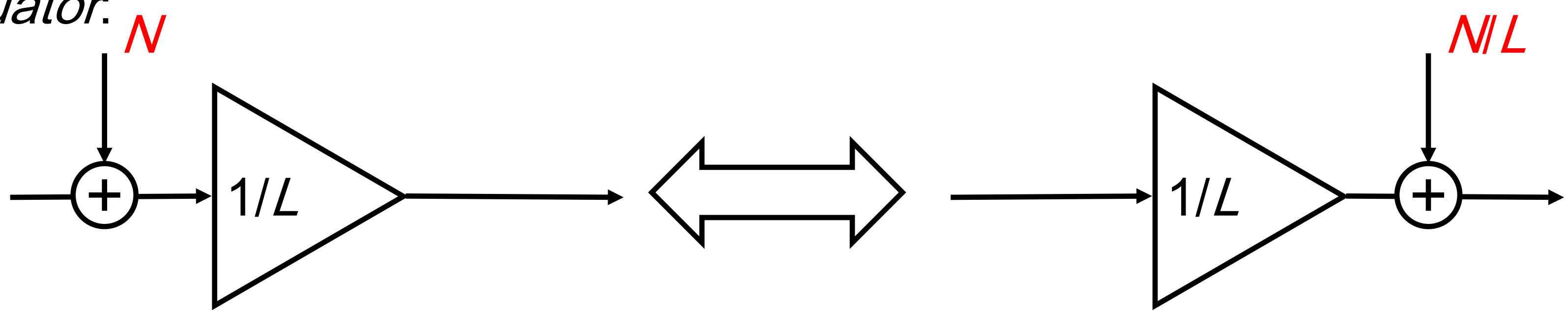
After extraction of the noise sources from each component, we need to move them to one point.

When doing this, we must compensate for amplification and attenuation!

Amplifier:

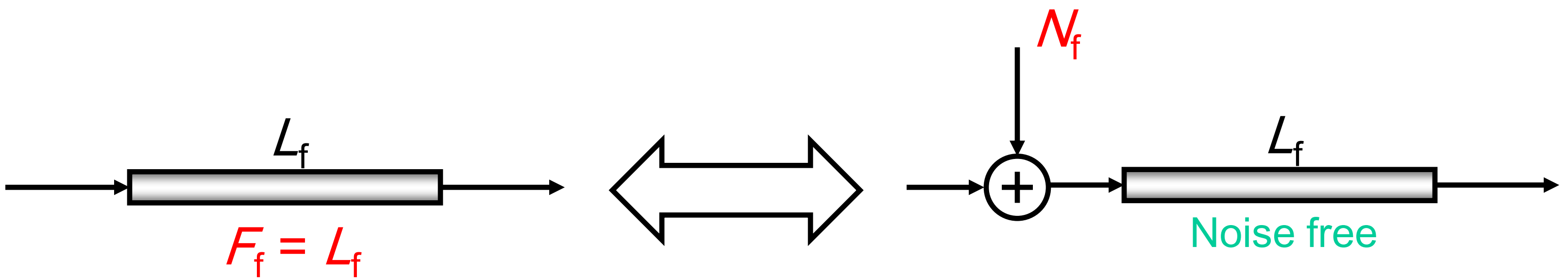


Attenuator:



Pierce's rule

A passive attenuator, in this case a feeder line, has a noise figure equal to its attenuation.



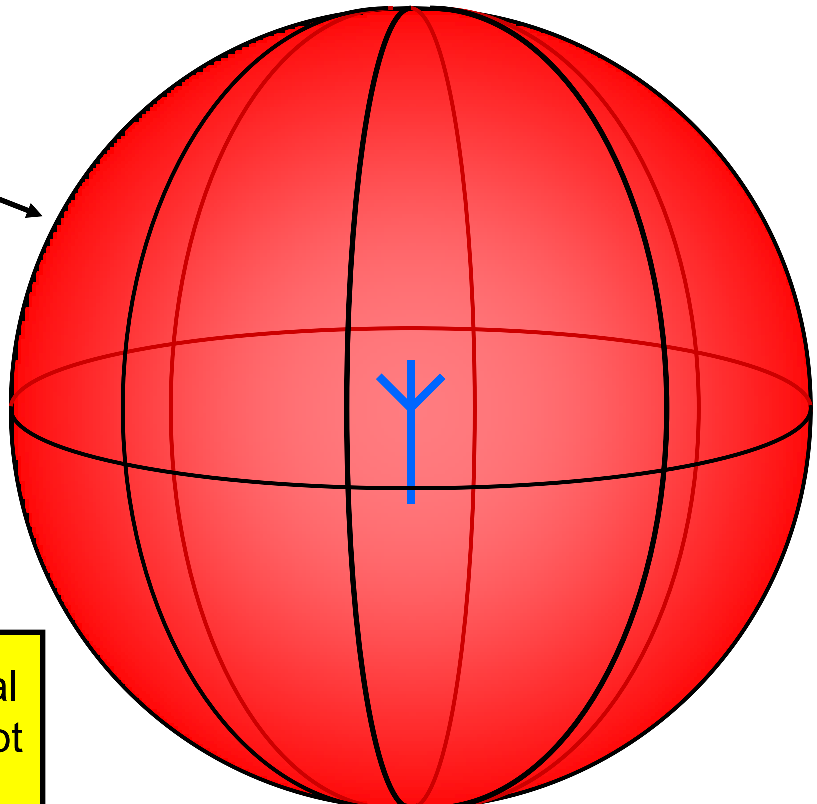
$$N_f = k\left(F_f - 1\right)T_0 = k\left(L_f - 1\right)T_0$$

Remember to convert *from* dB!

The isotropic antenna

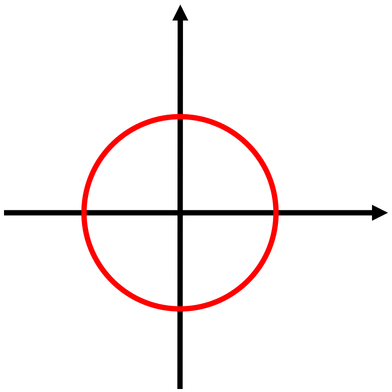
The **isotropic antenna** radiates equally in all directions

Radiation pattern is spherical

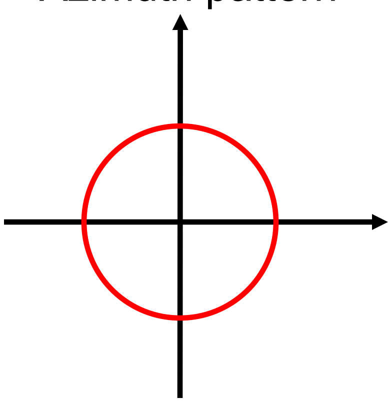


This is a theoretical antenna that cannot be built.

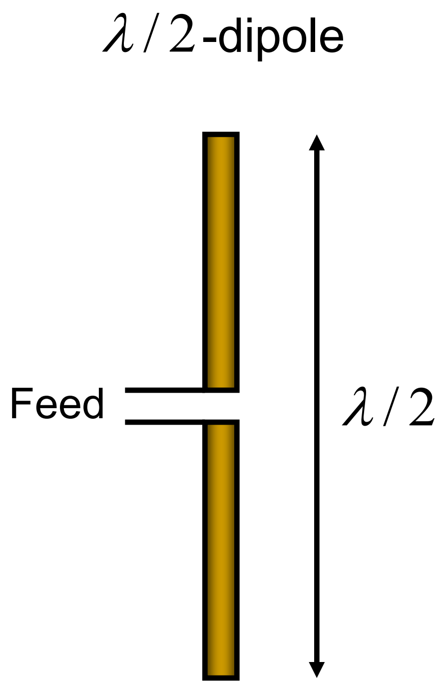
Elevation pattern



Azimuth pattern



The dipole antenna

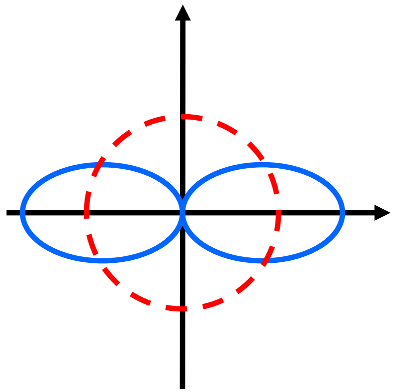


This antenna does not radiate straight up or down. Therefore, more energy is available in other directions.

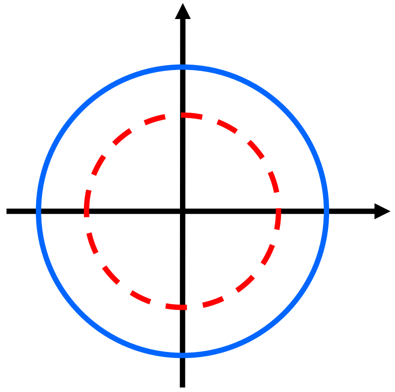
THIS IS THE PRINCIPLE BEHIND WHAT IS CALLED *ANTENNA GAIN*.

A dipole can be of any length, but the antenna patterns shown are only for the λ/2-dipole.

Elevation pattern



Azimuth pattern



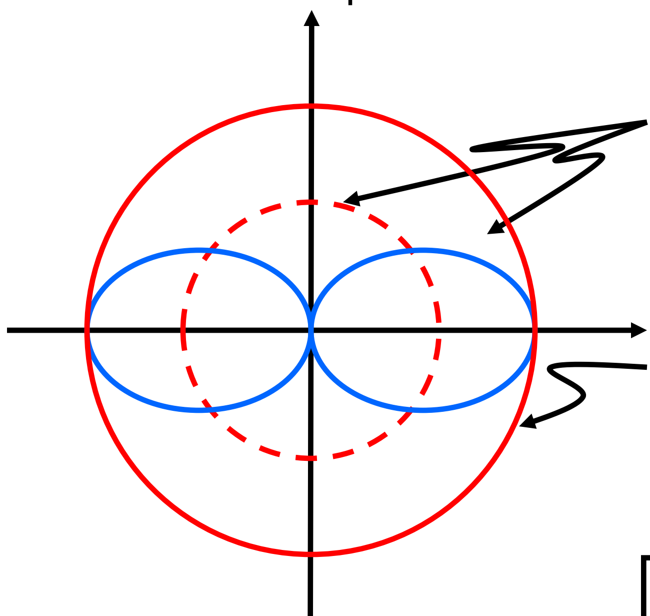
Antenna pattern of isotropic antenna.

Antenna gain (principle)

Antenna gain is a relative measure.

We will use the isotropic antenna as the reference.

Radiation pattern



The amount of increase in input power to the isotropic antenna, to obtain the same maximum radiation is called the **antenna gain**!

Antenna gain of the $\lambda/2$ dipole is **2.15 dB**.

A note on antenna gain

Sometimes the notation **dBi** is used for antenna gain (instead of dB).

The ”**i**” indicates that it is the gain relative to the isotropic antenna (which we will use in this course).

Another measure of antenna gain frequently encountered is **dBd**, which is relative to the λ/2 dipole.

$$G|_{dBi} = G|_{dBd} + 2.15$$

Be careful! Sometimes it is not clear if the antenna gain is given in dBi or dBd.

EIRP: Effective Isotropic Radiated Power

EIRP = Transmit power (fed to the antenna) + antenna gain

$$EIRP\,|_{dB} = P_{TX|dB} + G_{TX|dB}$$

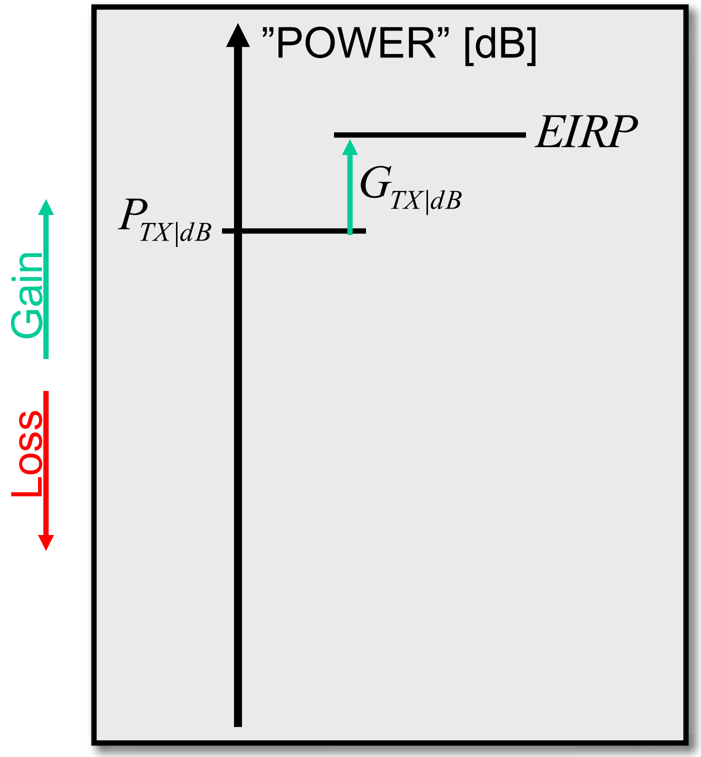
Answers the questions:

How much transmit power would we need to feed an isotropic antenna to obtain the same maximum of radiated power?

How "strong" is our radiation in the maximal direction of the antenna?

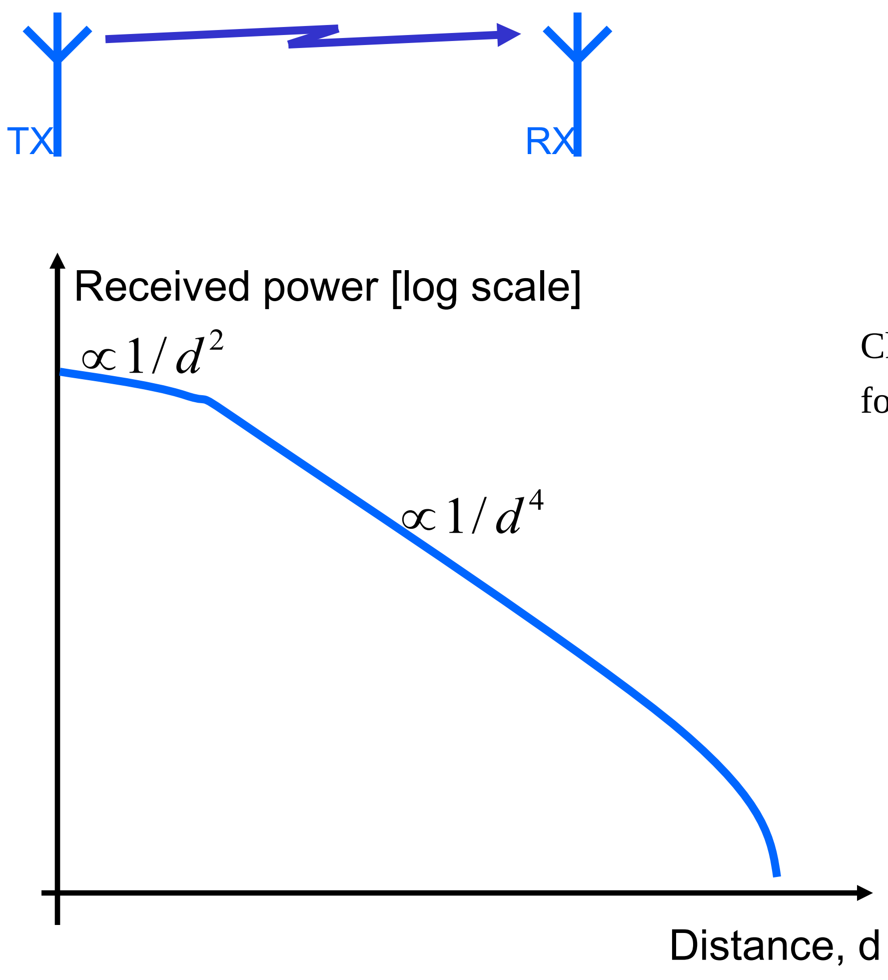
This is the more important one, since a limit on EIRP is a limit on the radiation in the maximal direction.

EIRP and the link budget



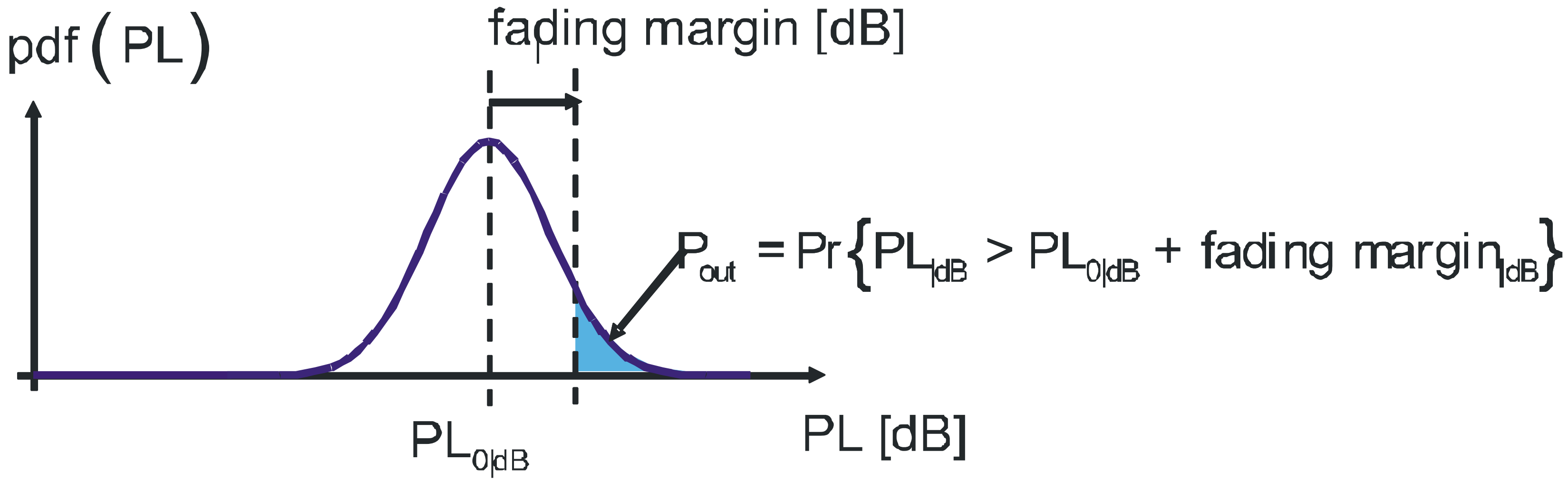
$$EIRP\,|_{dB} = P_{TX|dB} + G_{TX|dB}$$

Path loss

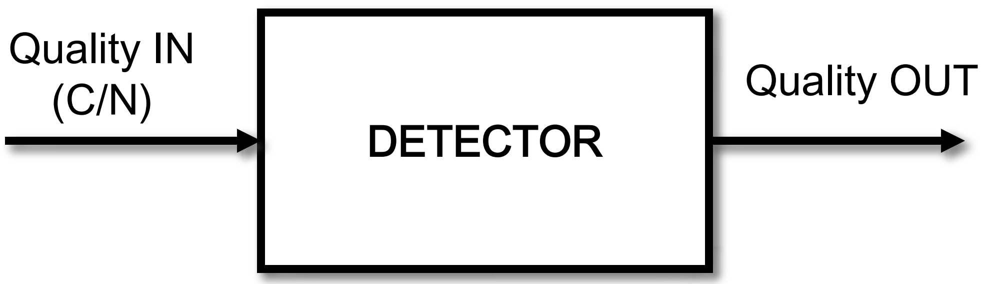


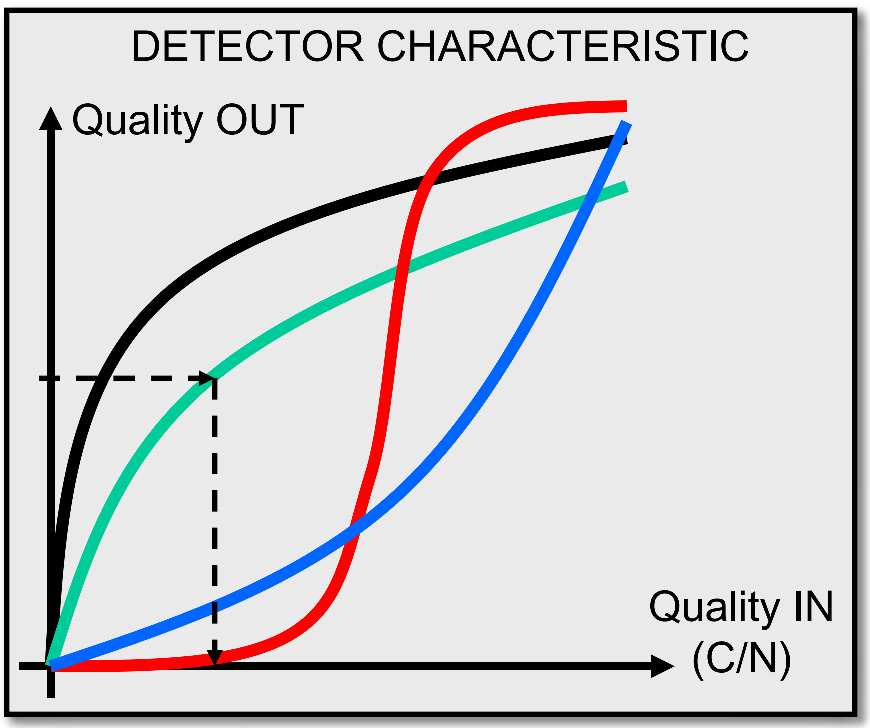
Chapter 4 - Breakpoint Model for path loss Equation 3.8

Fading margin



Required C/N – another central concept



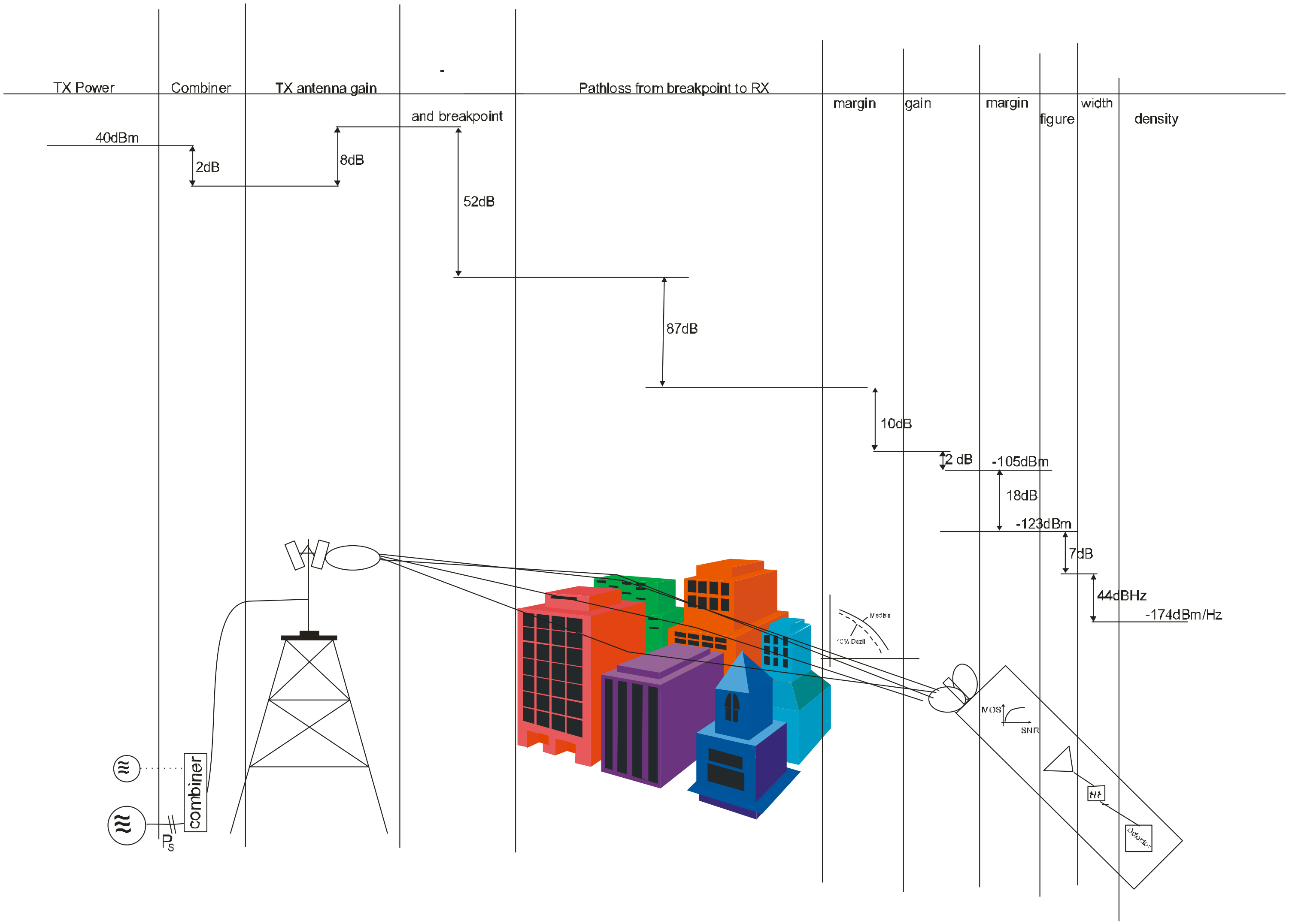


The detector characteristic is different for different system design choices.

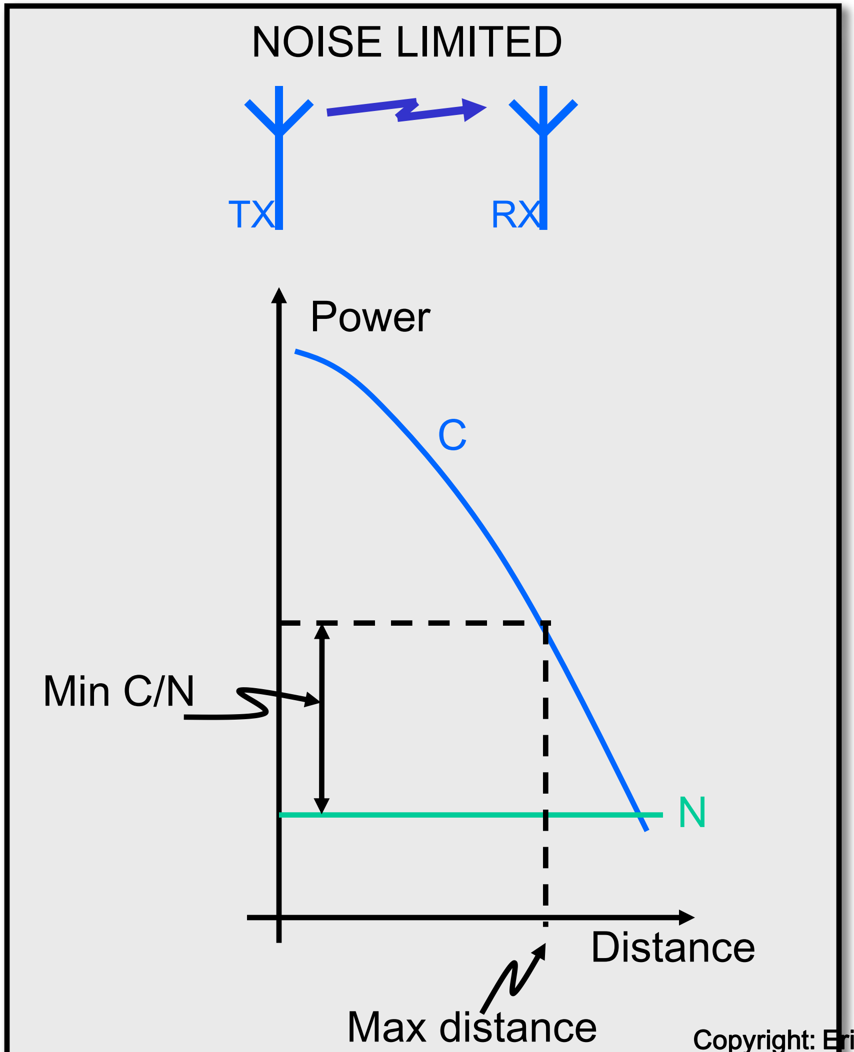
REQUIRED QUALITY OUT:

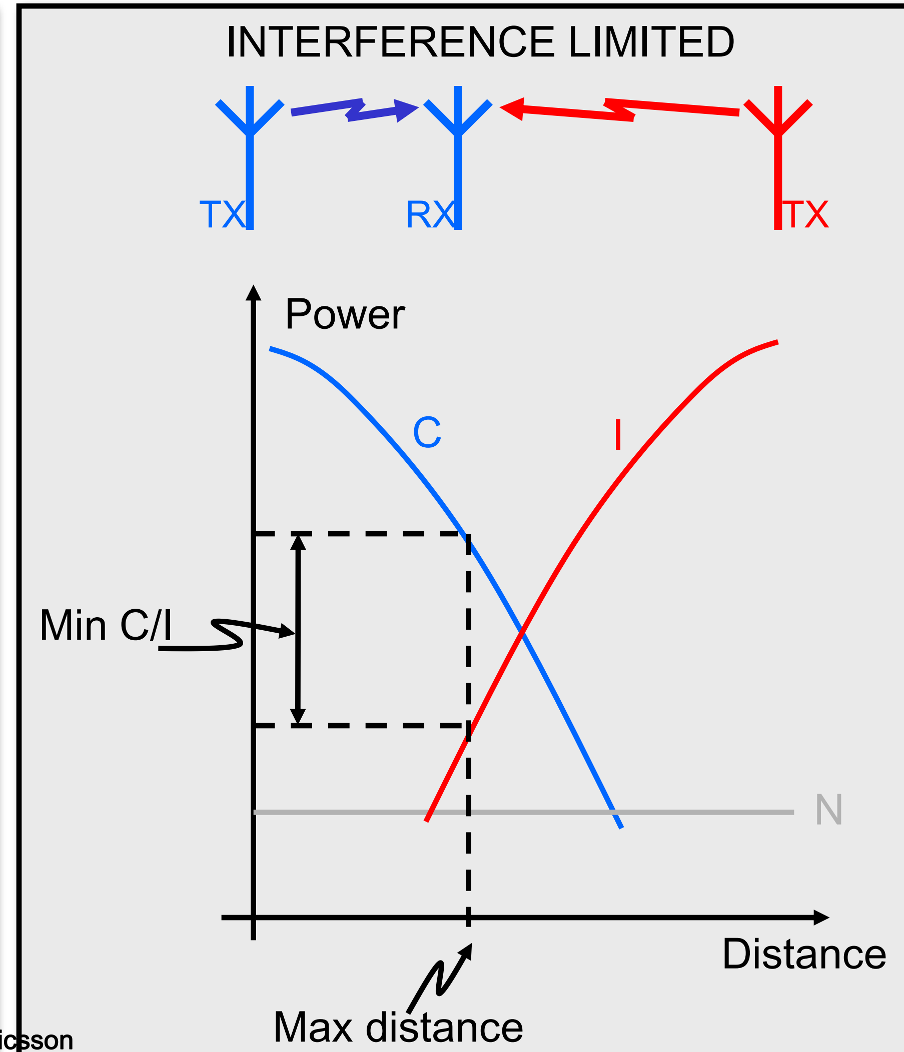
Audio SNR
Perceptive audio quality
Bit-error rate
Packet-error rate
etc.

Example for link budget



Noise and interference limited links





What is the impact of distance between BSs?