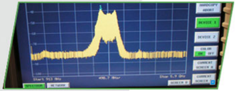


Radio Engineering

Lecture 4: Wideband channels

Florian Kaltenberger

Last lecture

5. Statistical description of fading
   - Equivalent baseband representation
   - Small scale fading without a dominat component
   - Small scale fading with a dominat component
   - Doppler spectra
   - Temporal dependence of fading
   - Large-scale fading

Repetition: Path loss

Path loss is the attenuation of the signal between transmitter and receiver.

$$P_{\mathrm{RX}} = \frac{P_{\mathrm{TX}}}{PL}$$
$$P_{\mathrm{RX}}|_{\mathrm{dB}} = P_{\mathrm{TX}}|_{\mathrm{dB}} - PL|_{\mathrm{dB}}$$

It contains the following factors

$$\mathrm{PL} = \mathrm{PL}(d) \times \mathrm{SF} \times \mathrm{SSF}$$
$$\mathrm{PL}|_{\mathrm{dB}} = \mathrm{PL}(d)|_{\mathrm{dB}} + \mathrm{SF}|_{\mathrm{dB}} + \mathrm{SSF}|_{\mathrm{dB}}$$

- $\mathrm{PL}(d)$ deterministic path loss
- SF large-scale (shadow) fading
- SSF small scale fading

Deterministic Path Loss

$$PL(d) = \left(\frac{4\pi d}{\lambda}\right)^2 \quad 0 \leq d \leq d_{\text{break}}$$

$$PL(d) = PL(d_{\text{break}}) \left(\frac{d}{d_{\text{break}}}\right)^n \quad d > d_{\text{break}}$$

- distance dependent loss in signal energy
- proportional to d^n, where d is the distance and n is the path loss exponent
- typicall values $n \in [1.5, 6]$, depending on terrain and foilage

Large Scale fading

- Deviation of average received signal energy from deterministic path loss
- Averaging done over a small area (a few wavelengths)
- Is usually attributed to multiple interactions of the signal with the enviroment
- A large number of these interactions results in a log-normal distribution

$$\mathrm{pdf}(L|_{\mathrm{dB}}) = \frac{1}{\sqrt{2\pi}\sigma_F|_{\mathrm{dB}}} \exp\left(-\frac{L^2|_{\mathrm{dB}}}{2\sigma_F^2|_{\mathrm{dB}}}\right) \tag{1}$$

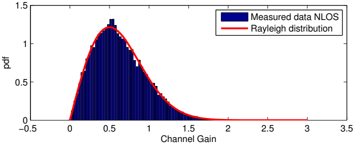
- This distribution matches measurements very well with $\sigma_F|_{\mathrm{dB}} \approx 4 \ldots 10$ dB.

Small Scale Fading

- Small scale fading results from constructive and destructive combination of multipaths
- No dominant component $\Rightarrow$ Rayleigh Fading

$$\text{pdf}(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad 0 \leq r < \infty$$

$$\text{cdf}(r) = 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)$$



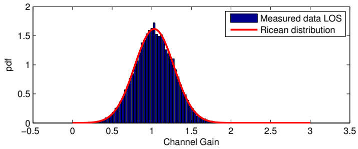
Small Scale Fading

- Dominant component (LOS) $\Rightarrow$ Ricean fading

$$\text{pdf}(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{rA}{\sigma^2}\right), \quad 0 \leq r < \infty$$

$$\text{cdf}(r) = 1 - Q_M\left(\frac{A}{\sigma}, \frac{r}{\sigma}\right)$$

- The ratio of the power in the LOS component and the diffuse component $K = \frac{A^2}{2\sigma^2}$ is called the Ricean factor
- Q_M is Marcum's Q function and I_n is the modified Bessel function of the first kind, order n.



Example: Ricean Fading Margin

Compute the fading margin for a Rice distribution with $K_r = 0.3, 3,$ and 20 dB so that the outage probability is less than 5%.

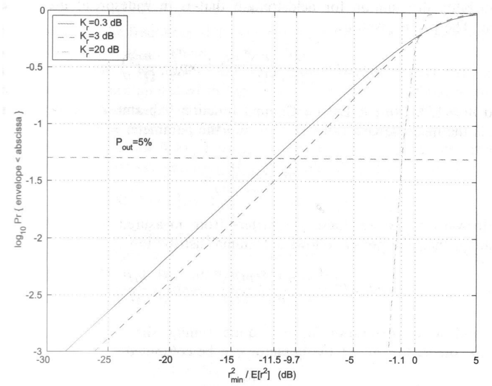


Figure 5.21 The Rice power *cdf*, $\sigma = 1$.

$$M = \frac{r_{\text{rms}}^2}{r_{\text{min}}^2} = \frac{2\sigma^2(1+K_r)}{r_{\text{min}}^2}$$

$$= 11.5, 9.7, 1.1\text{dB}$$

Example: Outage probablity

Assume that, at a certain distance, we have a deterministic propagation loss of $L_0 = 127$ dB and larg-scale fading, which is log-normal distributed with $\sigma_F = 7$ dB.

1. How large is the outage probability $P_{\text{out}} = \Pr\{L_0 + L \geq L_{\max}\}$ (due to large-scale fading) if our system is designed to handle a maximum propagation loss of $L_{\max} = 135$ dB?
2. Which of the following methods can be used to lower the outage probability? Why (not)?
   - (a) Increase the transmit power
   - (b) Decrease the deterministc path loss
   - (c) Change the antennas
   - (d) Lower σ_F
   - (e) Build a better receiver

This lecture

6. Wideband channels
   - Wideband vs narrowband channels
   - System theoretic description of wideband channels
   - WSSUS model
   - Condesed parameters
   - Direction channel description

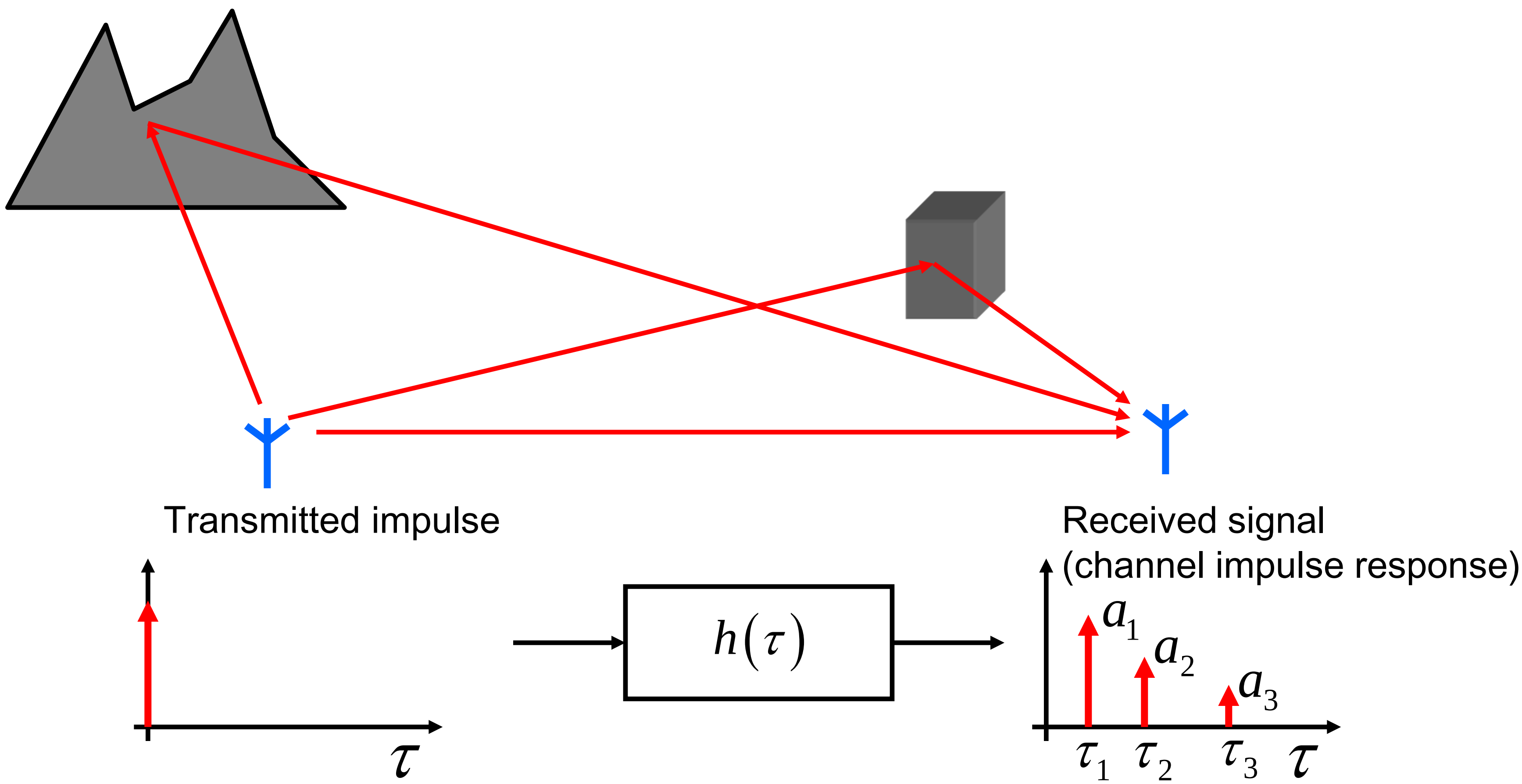
Wideband channels: definition

- So far we only looked at narrowband systems:
  - The symbol duration T_s is *larger* than the maximum delay in the channel $\Delta\tau$
  - $\Rightarrow$ Receiver cannot distinguish different echos
- A communication system is wide-band if
  - The symbol duration T_s is *smaller* than the maximum delay in the channel $\Delta\tau$
  - $\Rightarrow$ One transmitted symbol can spread over more than one symbol at the receiver

Chapter 6

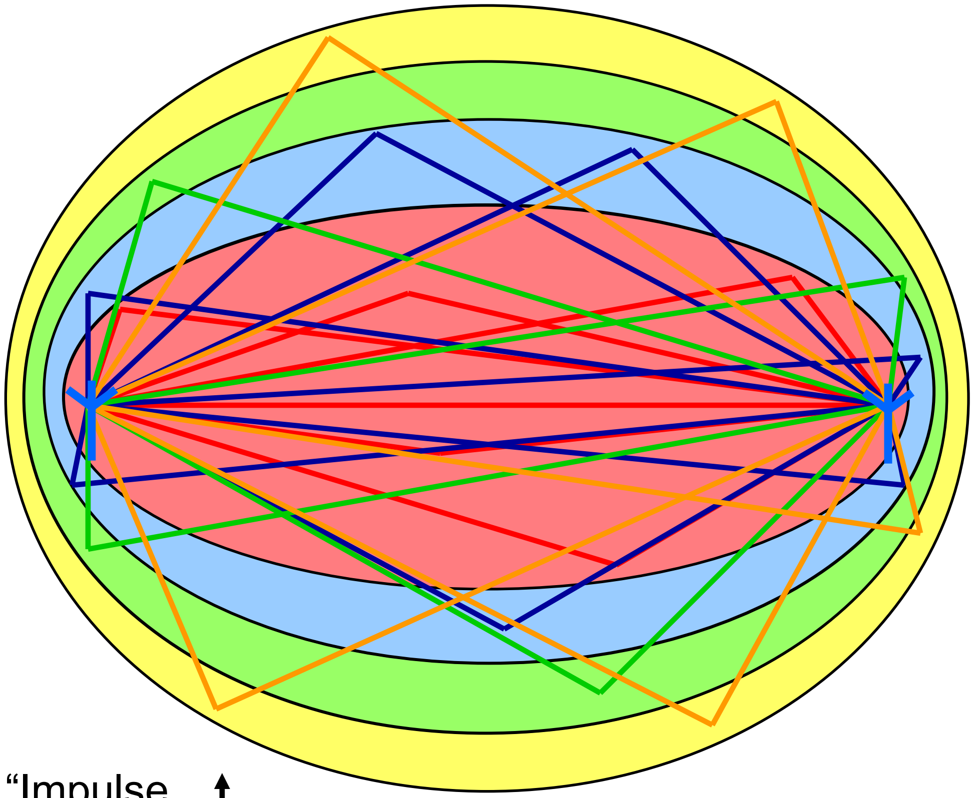
Wideband channels

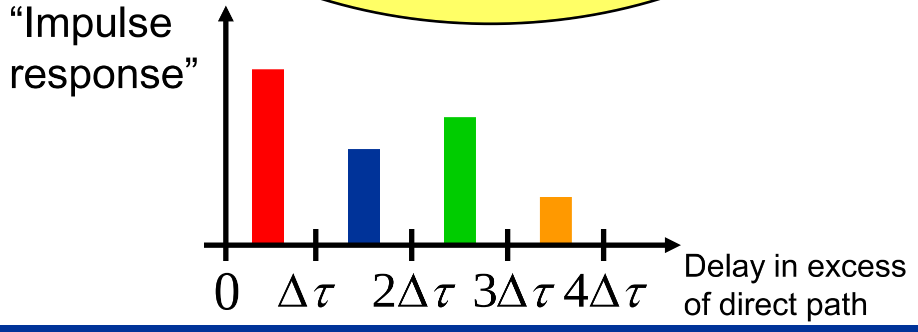
Delay (time) dispersion
A simple case



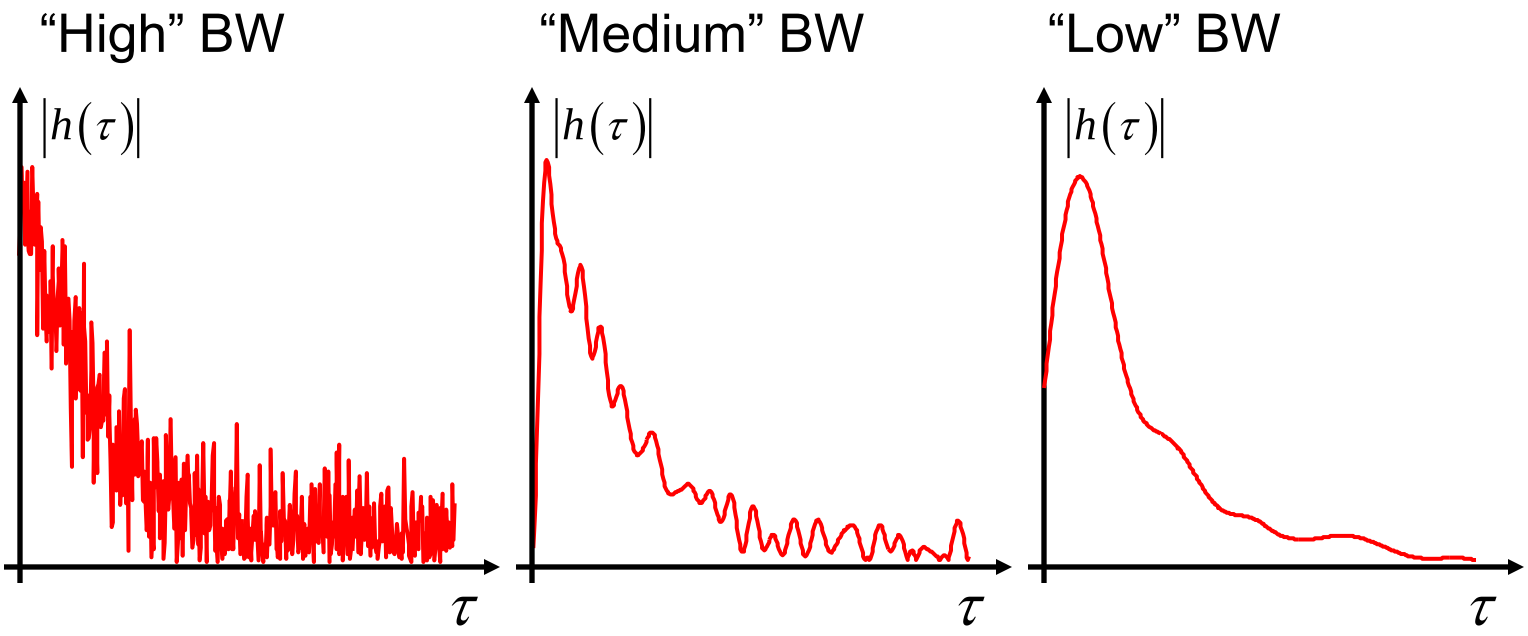
$$h(\tau) = a_1\delta(\tau - \tau_1) + a_2\delta(\tau - \tau_2) + a_3\delta(\tau - \tau_3)$$

Delay (time) dispersion
One reflection/path, many paths

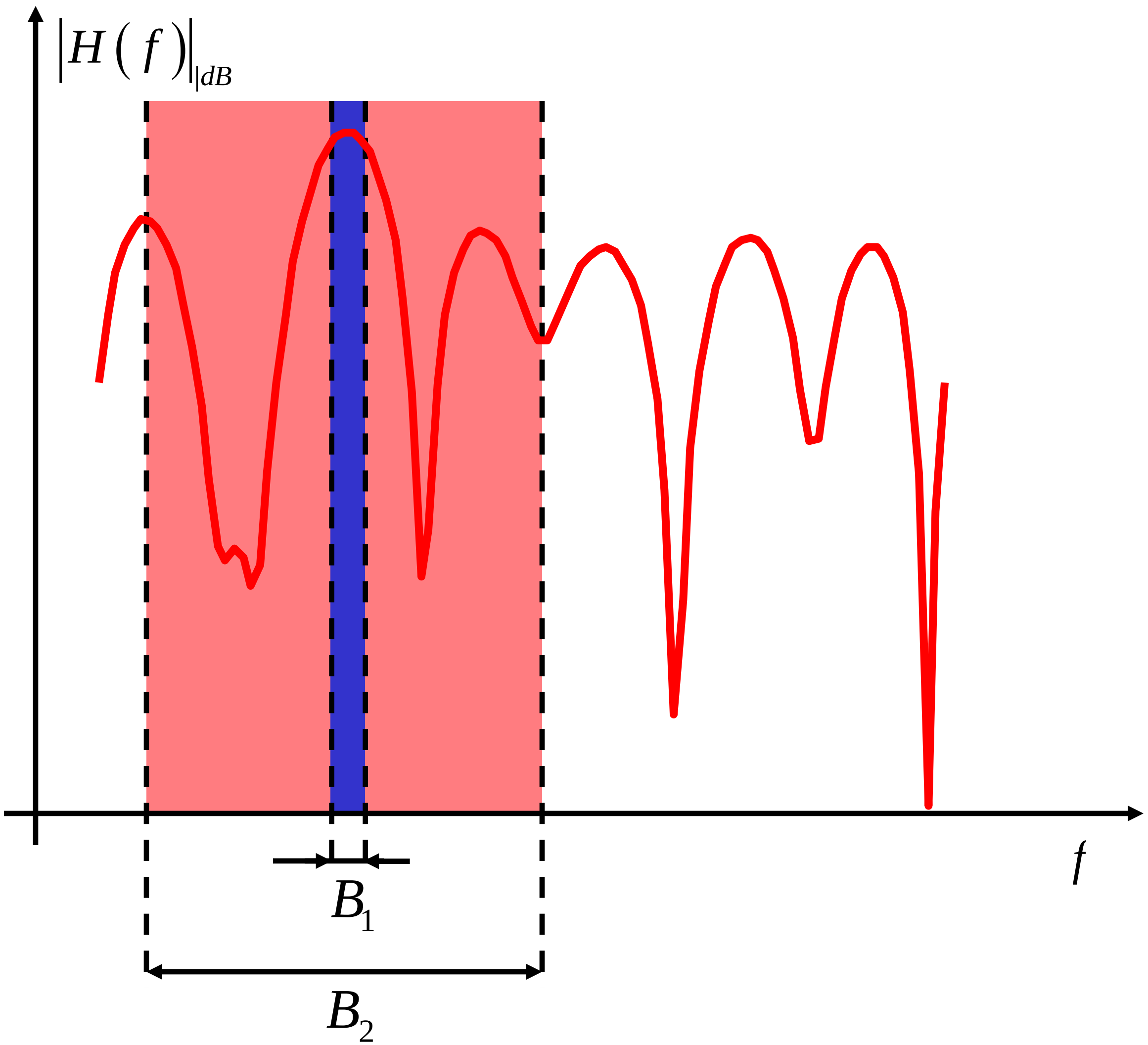




Narrow- versus wide-band
Channel impulse response



Narrow- versus wide-band Channel frequency response



System functions (1)

- Time-variant impulse response $h(t,\tau)$
  - Due to movement, impulse response changes with time
  - Input-output relationship:

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(t,\tau)d\tau$$

- Time-variant transfer function *H(t,f)*
  - Perform Fourier transformation with respect to τ

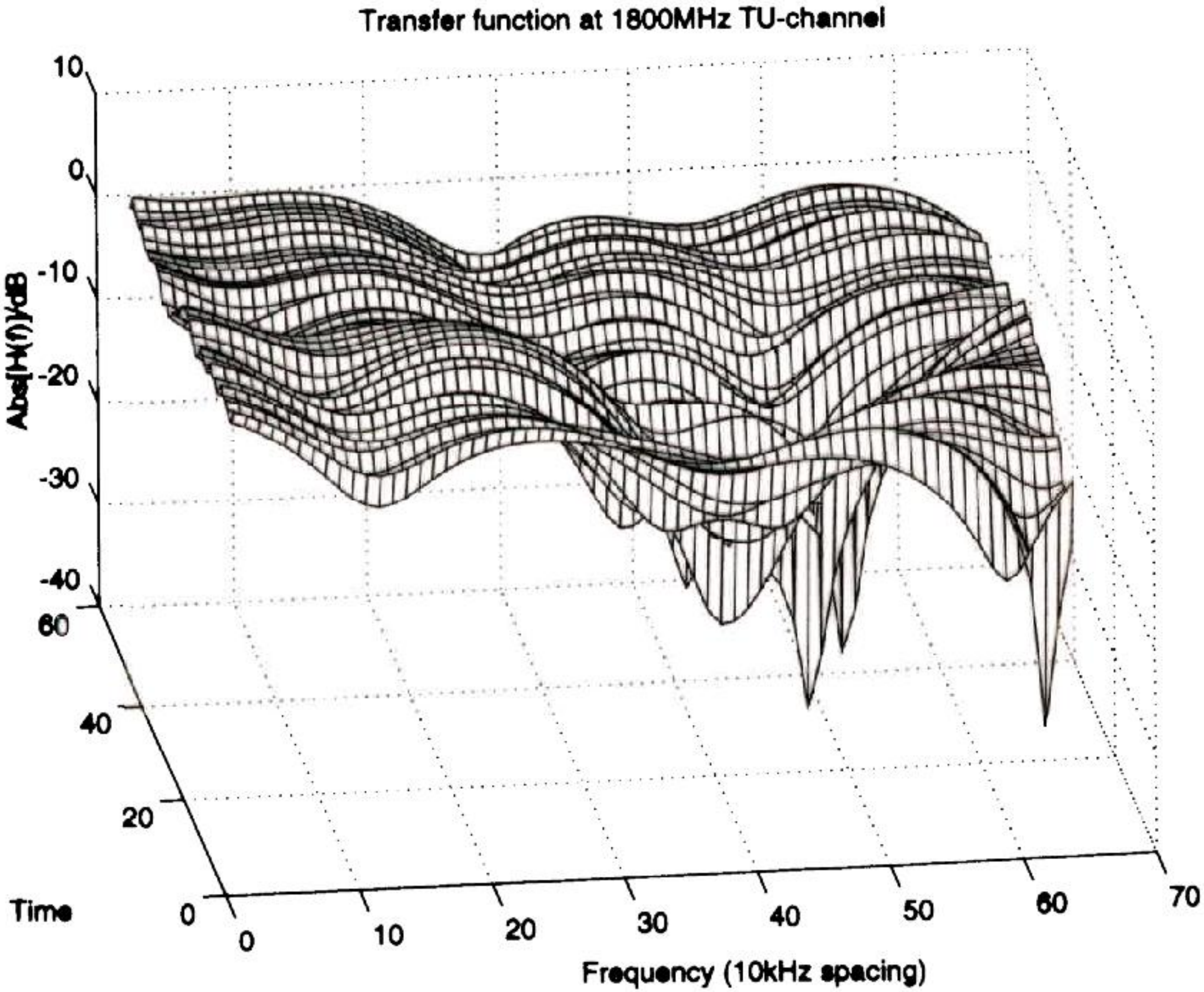
$$H(t,f) = \int_{-\infty}^{\infty} h(t,\tau)\exp(-j2\pi f\tau)d\tau$$

  - Input-output relationship

$$Y(\tilde{f}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(f)H(t,f)\exp(j2\pi ft)\exp(-j2\pi\tilde{f}t)dfdt$$

becomes *Y(f)=X(f)H(f)* only in *slowly* time-varying channels

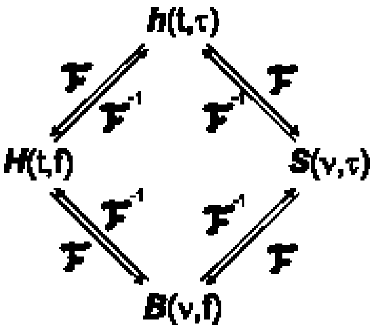
Transfer function, Typical urban



System functions (2)

- Further equivalent system functions:
  - Since impulse response depends on two variables, Fourier transformation can be done w.r.t. each of them

  -> *four* equivalent system descriptions are possible:
    - Impulse response
    - Time-variant transfer function
    - Spreading function $S(\nu,\tau) = \int_{-\infty}^{\infty} h(t,\tau)\exp(-j2\pi\nu t)dt$
    - Doppler-variant spreading function $B(\nu,f) = \int_{-\infty}^{\infty} S(\nu,\tau)\exp(-j2\pi f\tau)d\tau$



Stochastic system functions

- autocorrelation function (second-order statistics)

$$R_h(t,t',\tau,\tau') = E\{h^*(t,\tau)h(t',\tau')\}$$

- Input-output relationship:

$$R_{yy}(t,t') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{xx}(t-\tau,t'-\tau')R_h(t,t',\tau,\tau')d\tau d\tau'$$

The WSSUS model: mathematics

- If WSSUS is valid, ACF depends only on two variables (instead of four)
- ACF of impulse response becomes

$$R_h(t, t+\Delta t, \tau, \tau') = \delta(\tau - \tau') P_h(\Delta t, \tau)$$

$$P_h(\Delta t, \tau) \text{....} \textit{Delay cross power spectral density}$$

- ACF of transfer function

$$R_H(t+\Delta t, f+\Delta f) = R_H(\Delta t, \Delta f)$$

- ACF of spreading function

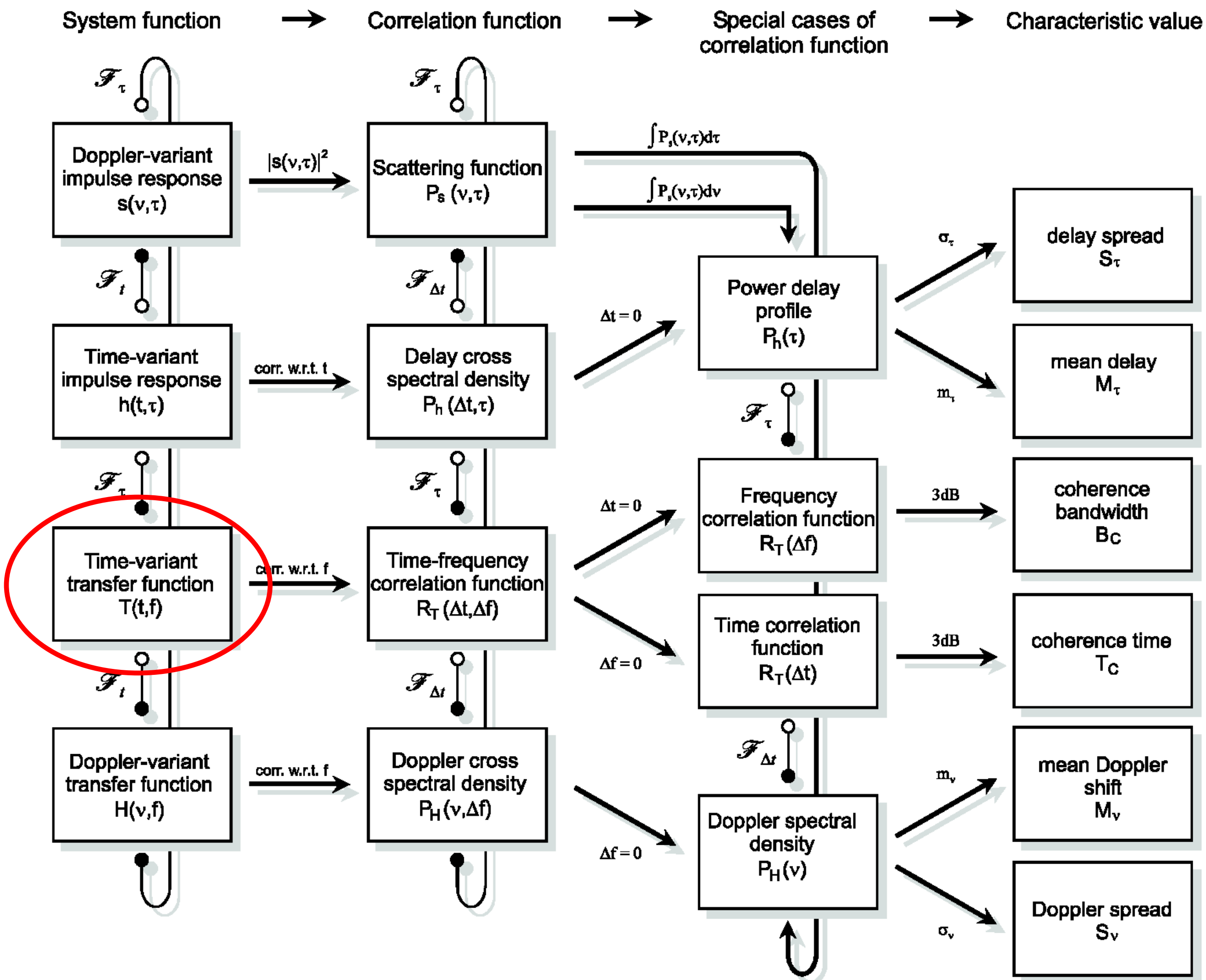
$$R_s(\nu, \nu', \tau, \tau') = \delta(\nu - \nu')\delta(\tau - \tau') P_s(\nu, \tau)$$

$$P_s(\nu, \tau) \text{.......} \textit{Scattering function}$$

Condensed parameters

- Correlation functions depend on two variables
- For concise characterization of channel, we desire
  - A function depending on one variable or
  - A single (scalar) parameter
- Most common condensed parameters
  - Power delay profile
  - Rms delay spread
  - Coherence bandwidth
  - Doppler spread
  - Coherence time

Channel measures



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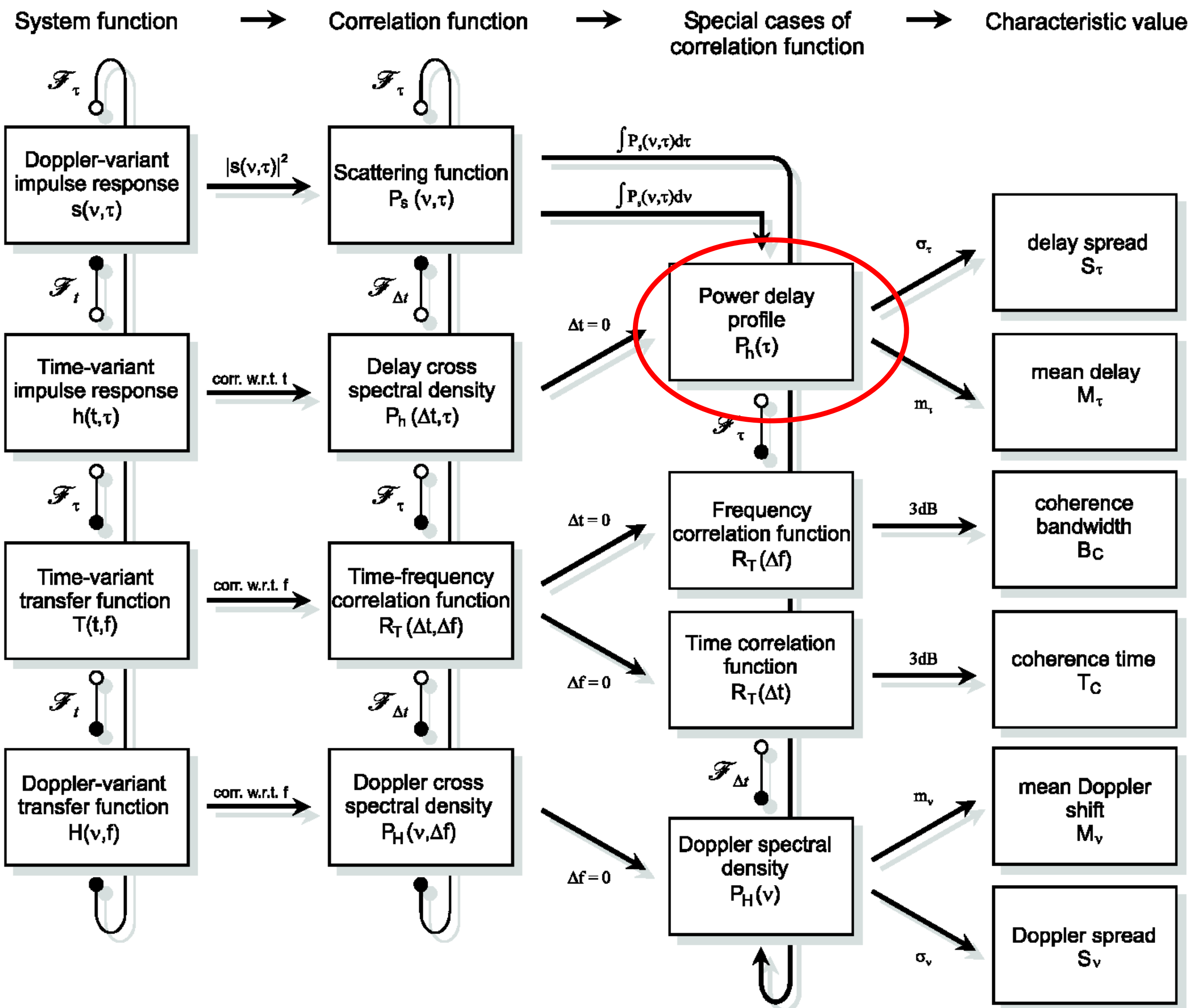
Channel measures



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Condensed parameters
Power-delay profile

One interesting channel property is the **power-delay profile** (PDP), which is the expected value of the received power at a certain delay:

$$P(\tau) = \mathrm{E}_t\left[\left|h(t,\tau)\right|^2\right]$$

E_t denotes expectation over time.

For our tapped-delay line we get:

$$P(\tau) = \mathrm{E}_t\left[\left|\sum_{i=1}^{N} \alpha_i(t)\exp\left(j\theta_i(t)\right)\delta(\tau-\tau_i)\right|^2\right]$$

$$= \sum_{i=1}^{N} \mathrm{E}_t\left[\alpha_i^2(t)\right]\delta(\tau-\tau_i) = \sum_{i=1}^{N} 2\sigma_i^2\delta(\tau-\tau_i)$$

Average power of tap i.

Condensed parameters
Power-delay profile (cont.)

We can “reduce” the PDP into more compact descriptions of the channel:

Total power (time integrated):

$$P_m = \int_{-\infty}^{\infty} P(\tau)\, d\tau$$

Average mean delay:

$$T_m = \frac{\int_{-\infty}^{\infty} \tau P(\tau)\, d\tau}{P_m}$$

Average rms delay spread:

$$S = \sqrt{\frac{\int_{-\infty}^{\infty} \tau^2 P(\tau)\, d\tau}{P_m} - T_m^2}$$

For our tapped-delay line channel:

$$P_m = \sum_{i=1}^{N} 2\sigma_i^2$$

$$T_m = \frac{\sum_{i=1}^{N} \tau_i 2\sigma_i^2}{P_m}$$

$$S = \sqrt{\frac{\sum_{i=1}^{N} \tau_i^2 2\sigma_i^2}{P_m} - T_m^2}$$

Condensed parameters
Frequency correlation

A property closely related to the power-delay profile (PDP) is the **frequency correlation** of the channel. It is in fact the Fourier transform of the PDP:

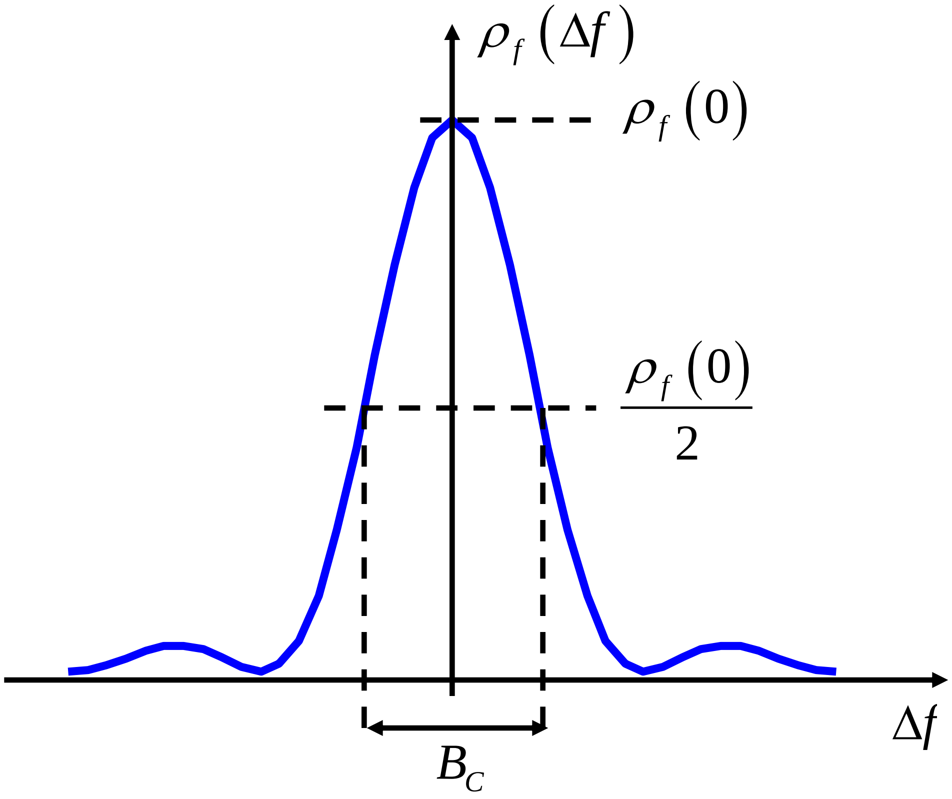
$$\rho_f(\Delta f) = \int_{-\infty}^{\infty} P(\tau) \exp(-j2\pi\Delta f \tau) \, d\tau$$

For our tapped delay-line channel we get:

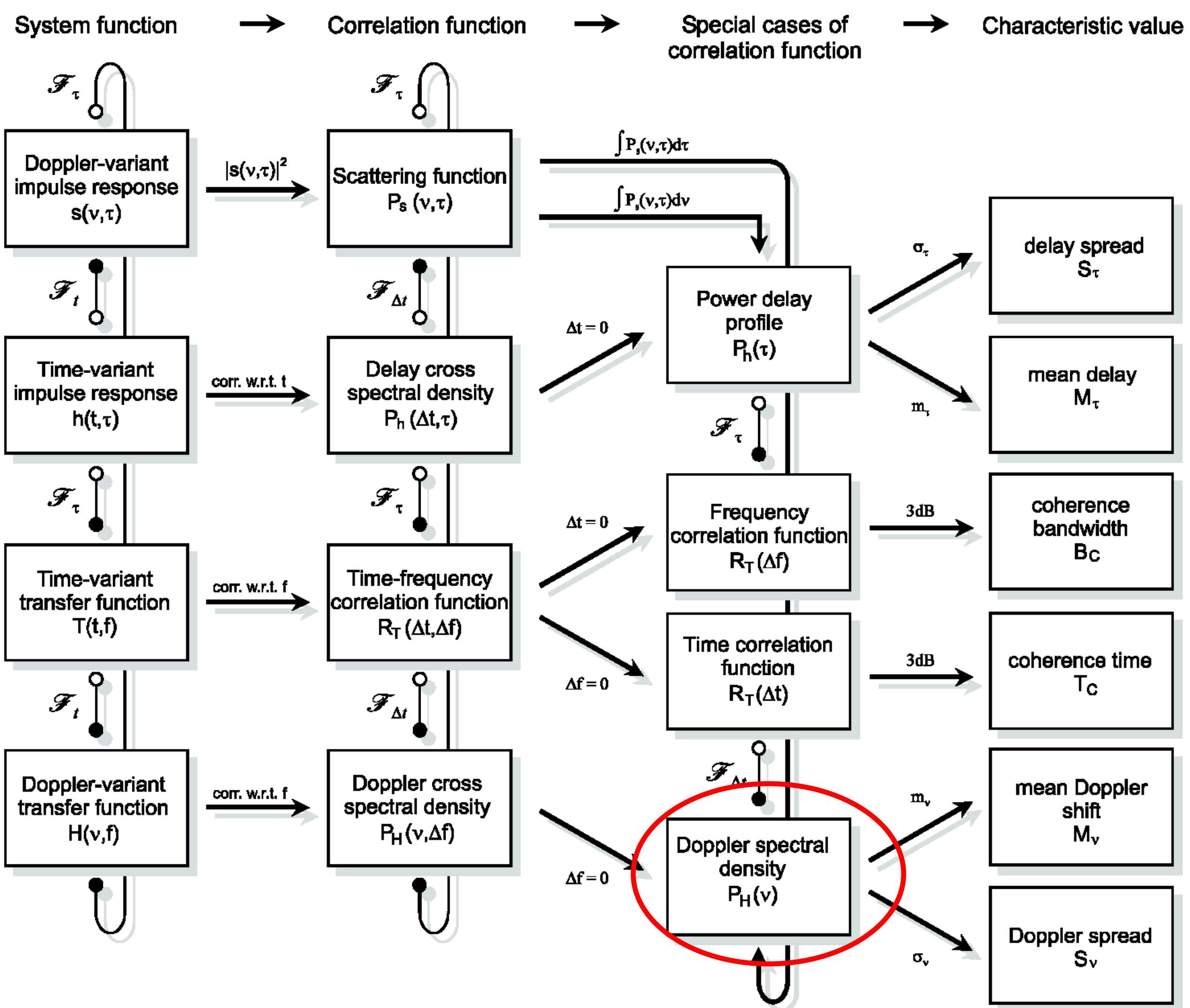
$$\rho_f(\Delta f) = \int_{-\infty}^{\infty} \left(\sum_{i=1}^{N} 2\sigma_i^2 \delta(\tau - \tau_i) \right) \exp(-j2\pi\Delta f \tau) \, d\tau$$

$$= \sum_{i=1}^{N} 2\sigma_i^2 \exp(-j2\pi\Delta f \tau_i)$$

Condensed parameters
Coherence bandwidth



Channel measures



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Condensed parameters
The Doppler spectrum

Given the scattering function P_s (doppler spectrum as function of delay) we can calculate a total **Doppler spectrum** of the channel as:

$$P_B(\nu) = \int P_S(\nu, \tau) \, d\tau$$

For our tapped delay-line channel, we have:

Doppler spectrum of tap *i*.

$$P_S(\nu, \tau) = \frac{2\sigma_i^{\ 2}}{\pi\sqrt{\nu_{i,\max}^{\ 2} - \nu^2}} \delta(\tau - \tau_i)$$

$$P_B(\nu) = \int_{-\infty}^{\infty} \frac{2\sigma_i^{\ 2}}{\pi\sqrt{\nu_{i,\max}^{\ 2} - \nu^2}} \delta(\tau - \tau_i) \, d\tau$$

$$= \sum_{i=1}^{N} \frac{2\sigma_i^{\ 2}}{\pi\sqrt{\nu_{i,\max}^{\ 2} - \nu^2}}$$

Condensed parameters
The Doppler spectrum (cont.)

We can "reduce" the Doppler spectrum into more compact descriptions of the channel:

Total power (frequency integrated):

$$P_{B,m} = \int_{-\infty}^{\infty} P_B(\nu)\, d\nu$$

Average mean Doppler shift:

$$T_{B,m} = \frac{\int_{-\infty}^{\infty} \nu P_B(\nu)\, d\nu}{P_{B,m}}$$

Average rms Doppler spread:

$$S_B = \sqrt{\frac{\int_{-\infty}^{\infty} \nu^2 P(\nu)\, d\nu}{P_{B,m}} - T_{B,m}^2}$$

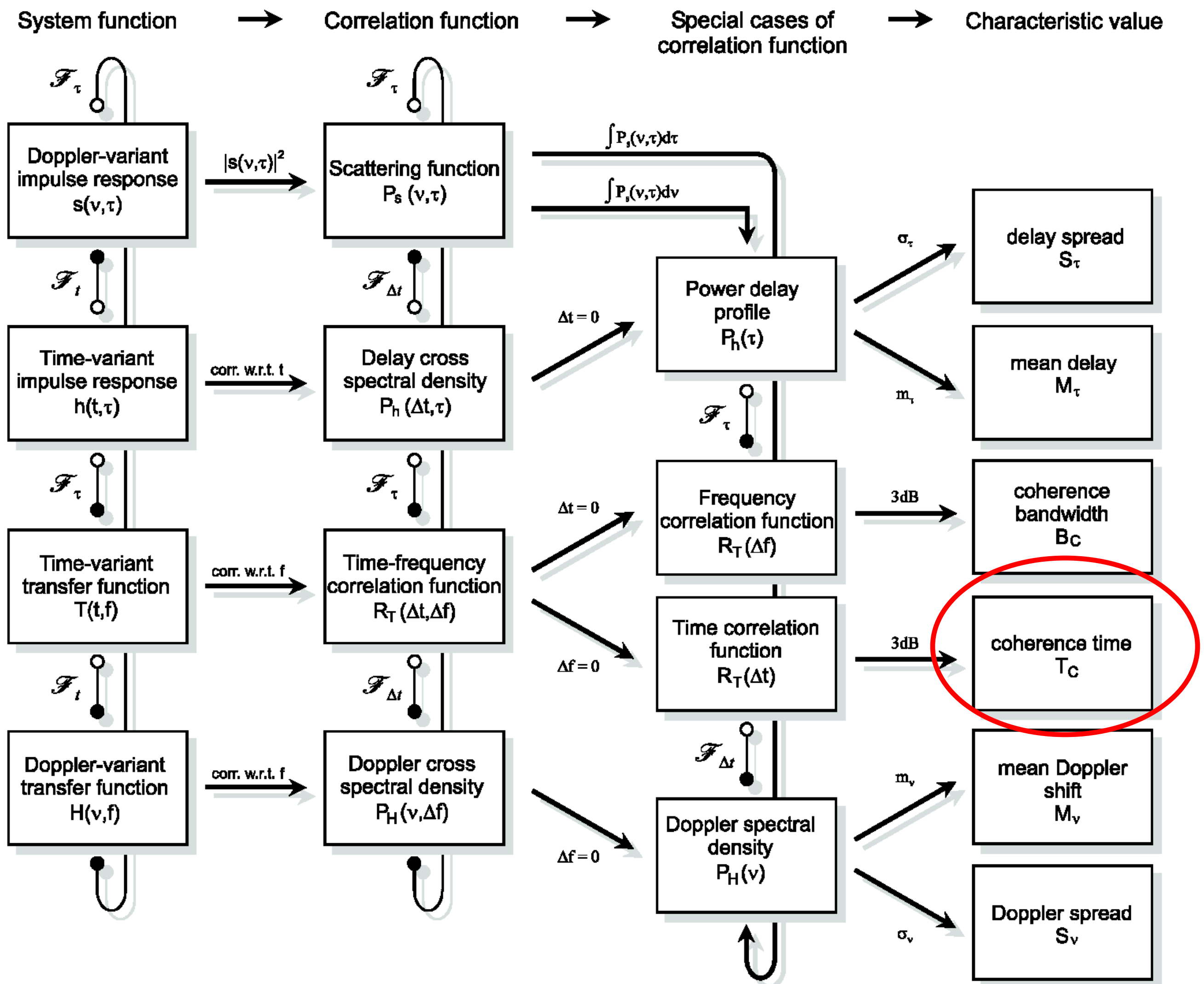
For our tapped-delay line channel:

$$P_{B,m} = \sum_{i=1}^{N} 2\sigma_i^{\,2}$$

$$T_{B,m} = 0$$

$$S_B = \sqrt{\frac{\sum_{i=1}^{N} \sigma_i^{\,2} \nu_{i,\max}^{\,2}}{P_{B,m}}}$$

Channel measures



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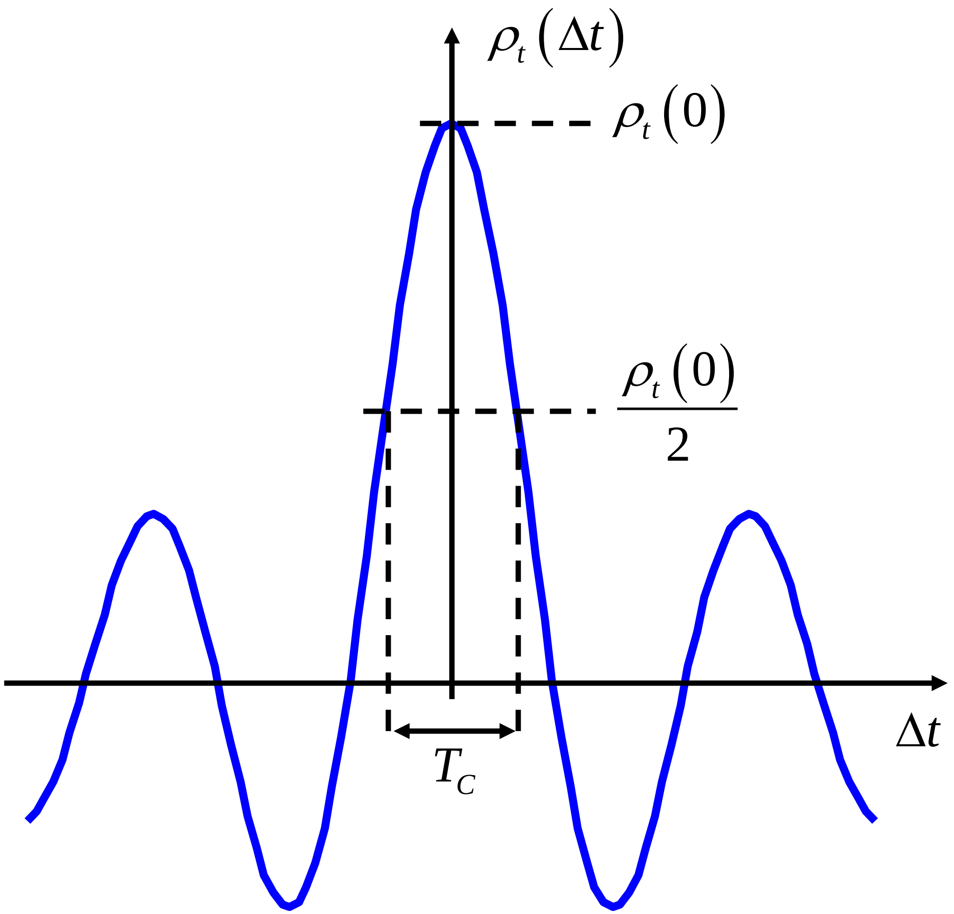
Condensed parameters
Coherence time

Given the time correlation of a channel, we can define the **coherence time** T_C:



Condensed parameters
The time correlation

A property closely related to the Doppler spectrun is the **time correlation** of the channel. It is in fact the inverse Fourier transform of the Doppler spectrum:

$$\rho_t(\Delta t) = \int_{-\infty}^{\infty} P_B(\nu)\exp(j2\pi\nu\Delta t)\,d\nu$$

For our tapped-delay line channel we get

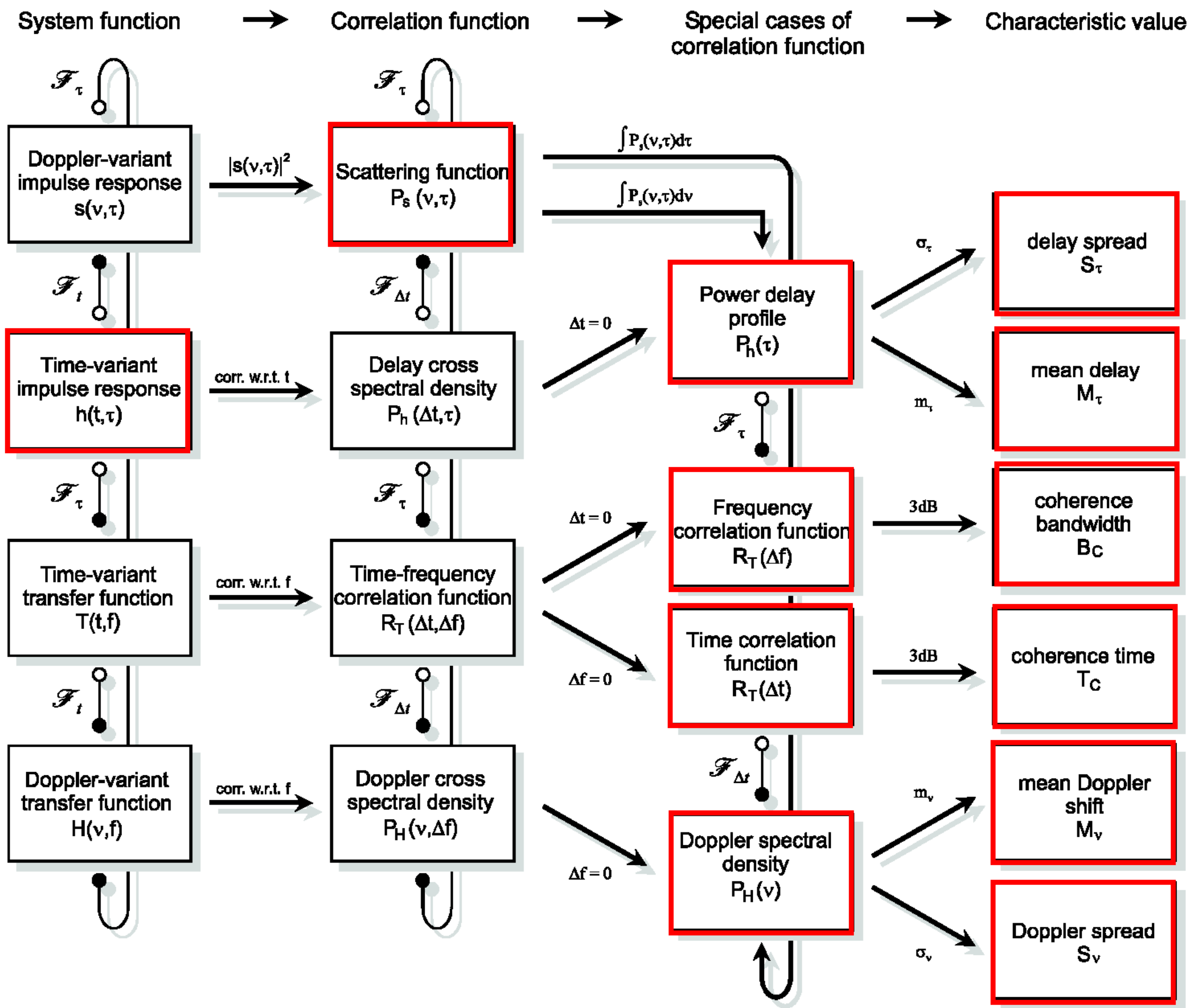
$$\rho_t(\Delta t) = \int_{-\infty}^{\infty} \sum_{i=1}^{N} \frac{2\sigma_i^{\,2}}{\pi\sqrt{\nu_{i,\max}^{\,2} - \nu^2}} \exp(j2\pi\nu\Delta t)\,d\nu$$

$$= \sum_{i=1}^{N} \int_{-\infty}^{\infty} \frac{2\sigma_i^{\,2}}{\pi\sqrt{\nu_{i,\max}^{\,2} - \nu^2}} \exp(j2\pi\nu\Delta t)\,d\nu$$

$$= \sum_{i=1}^{N} 2\sigma_i^{\,2} J_0(2\pi\nu_{i,\max}\Delta t)$$

Sum of time correlations for each tap.

It's much more complicated than what we have discussed!



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Properties of Wideband Channels

- In wideband communication systems, multipath propagation results in inter-symbol interference (ISI)
- Wideband channels fade also in frequency, causing *frequency-selective* fading
- Wideband channels can be mathematically described as linear time-variant systems (LTV)
- Wideband channels are wide-sense stationary (WSS) with uncorrelated scattering (US) iff
    1. their second order statistics (autocorrelation function) do not change over time
    2. contributions with different delays are uncorrelated

WSS-US channels: discussion

- The WSSUS assumption greatly simplifies many calculations, and is thus very popular
- However, it should be used with great care and applicability should always be checked
- It assumes that channels are *ergodic* (i.e., ensemble average can be interchanged with time average)
- Real channels are never wide-sense stationary

Double directional impulse response

TX position

RX position

number of multipath components for these positions

$$h(t, \vec{r}_{\mathrm{TX}}, \vec{r}_{\mathrm{RX}}, \tau, \Omega, \Psi) = \sum_{\ell=1}^{N(\vec{r})} h_\ell(t, \vec{r}_{\mathrm{TX}}, \vec{r}_{\mathrm{RX}}, \tau, \Omega, \Psi)$$

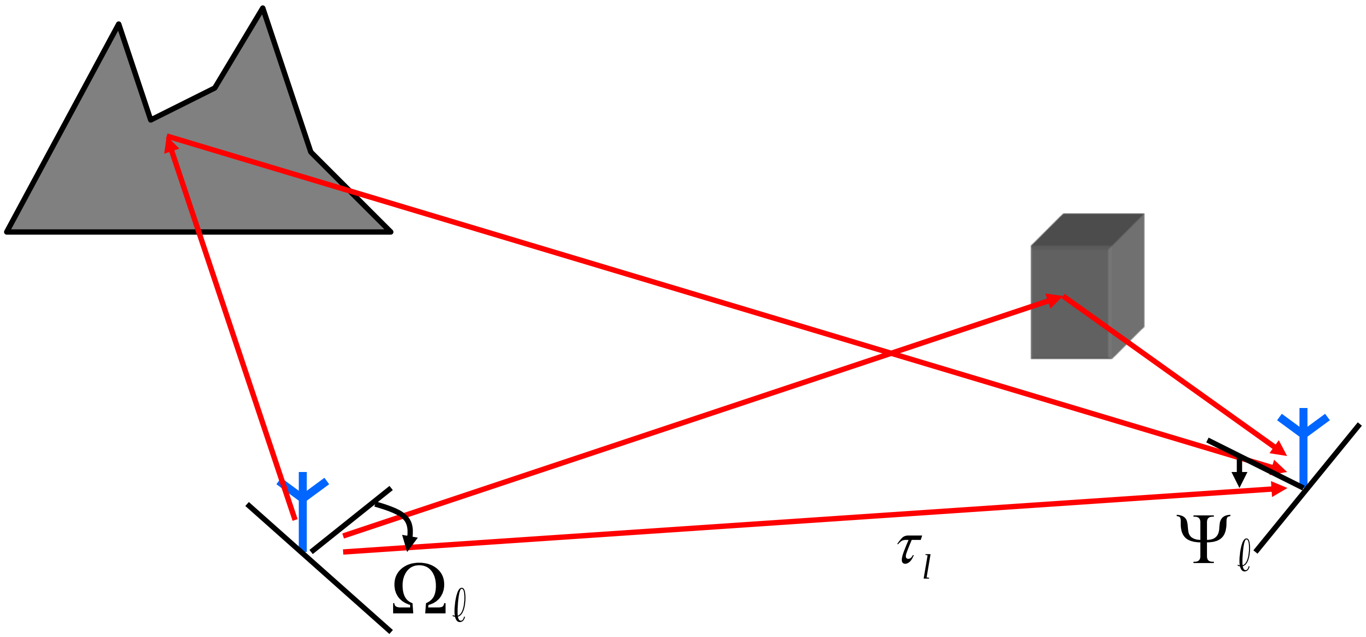
direction-of-arrival

delay

direction-of-departure

$$h_\ell(t, \vec{r}_{\mathrm{TX}}, \vec{r}_{\mathrm{RX}}, \tau, \Omega, \Psi) = |a_\ell| e^{j\varphi_\ell} \delta(\tau - \tau_\ell) \delta(\Omega - \Omega_\ell) \delta(\Psi - \Psi_\ell)$$

Physical interpretation



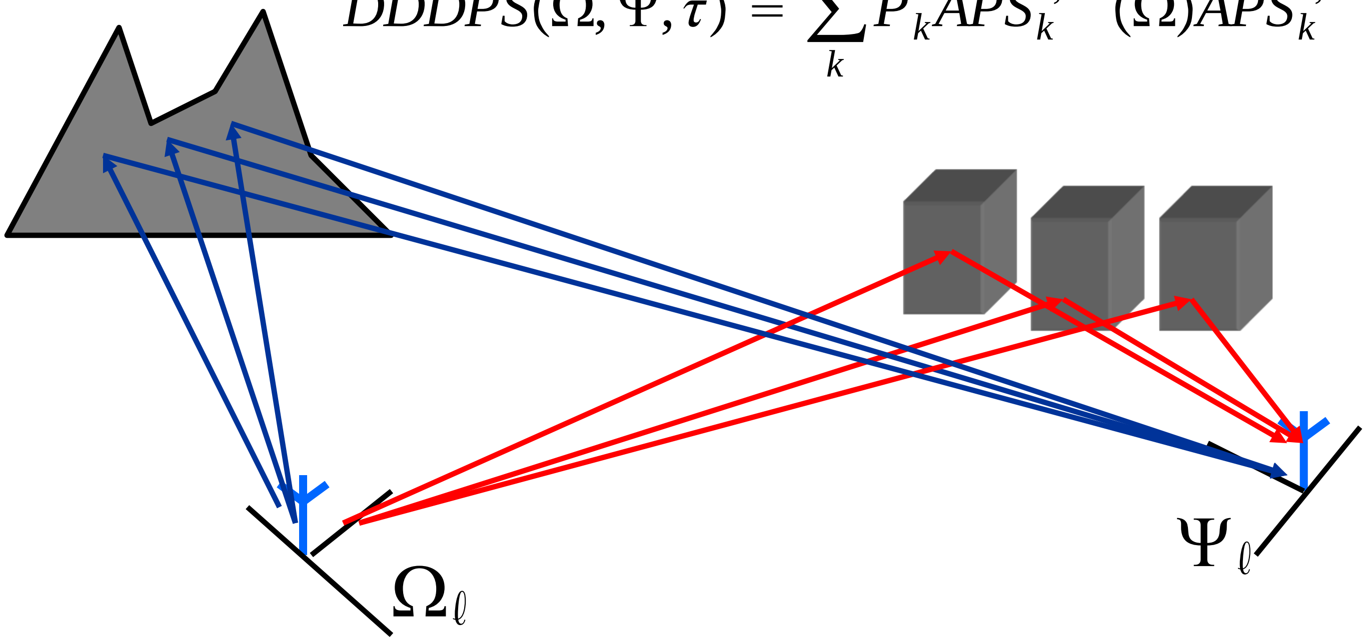
Directional models

- The double directional delay power spectrum is sometimes factorized w.r.t. DoD, DoA and delay.

$$DDDPS(\Omega,\Psi,\tau) = APS^{\mathrm{BS}}(\Omega)APS^{\mathrm{MS}}(\Psi)PDP(\tau)$$

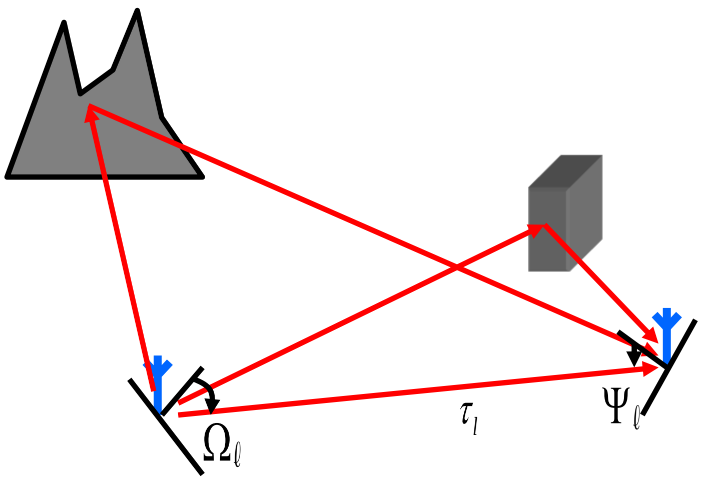
- Often in reality there are groups of scatterers with similar DoD and DoA – clusters

$$DDDPS(\Omega,\Psi,\tau) = \sum_k P_k^{\mathrm{c}} APS_k^{\mathrm{c,BS}}(\Omega)APS_k^{\mathrm{c,MS}}(\Psi)PDP_k^{\mathrm{c}}(\tau)$$



Angular spread

$$E\{s^*(\Omega,\Psi,\tau,\nu)s(\Omega',\Psi',\tau',\nu')\} = P_s(\Omega,\Psi,\tau,\nu)\delta(\Omega-\Omega')\delta(\Psi-\Psi')\delta(\tau-\tau')\delta(\nu-\nu')$$



double directional delay power spectrum

$$DDDPS(\Omega,\Psi,\tau) = \int \mathrm{P_s}(\Psi,\Omega,\tau,\nu)\mathrm{d}\nu$$

angular delay power spectrum

$$ADPS(\Omega,\tau) = \int DDDPS(\Psi,\Omega,\tau)G_{\mathrm{MS}}(\Psi)d\Psi$$

angular power spectrum

$$APS(\Omega) = \int \mathrm{APDS}(\Omega,\tau)\mathrm{d}\tau$$

power

$$P = \int \mathrm{APS}(\Omega)\mathrm{d}\Omega$$

References

P. Bello,
“Characterization of randomly time-variant linear channels,”
Communications Systems, IEEE Transactions on, vol. 11, no. 4, pp. 360 –393, 1963.