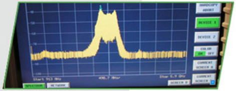


Radio Engineering

Lecture 8: Diversity

Florian Kaltenberger

Last lecture

EURECOM

9. Multiple Access and the Cellular Principle
   - Introduction
   - Network Dimensioning
   - Multiple Access and Duplexing
   - The Cellular Concept
   - Cell planning

This lecture

10. Diversity
    - Introduction
    - Macrodiversity
    - Microdiversity
        - Time
        - Freq
        - Space
    - Correlation coefficients
    - Combination of signals
        - selection combining
        - diversity combining

Reminder: Fading

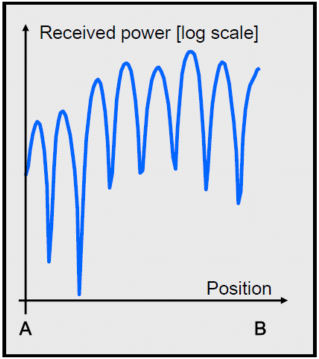
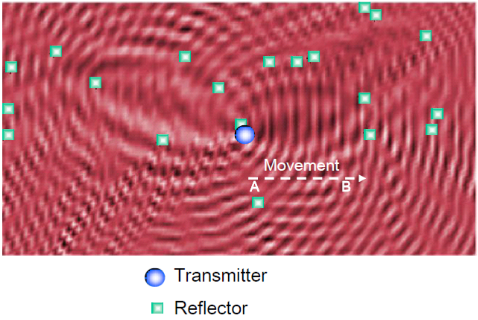


Illustration of interference pattern from above



BER in fading channels

- In an AWGN channel the BER decreases exponentially with SNR
- Example: QPSK

$$\text{BER} = Q(\sqrt{2\gamma})$$

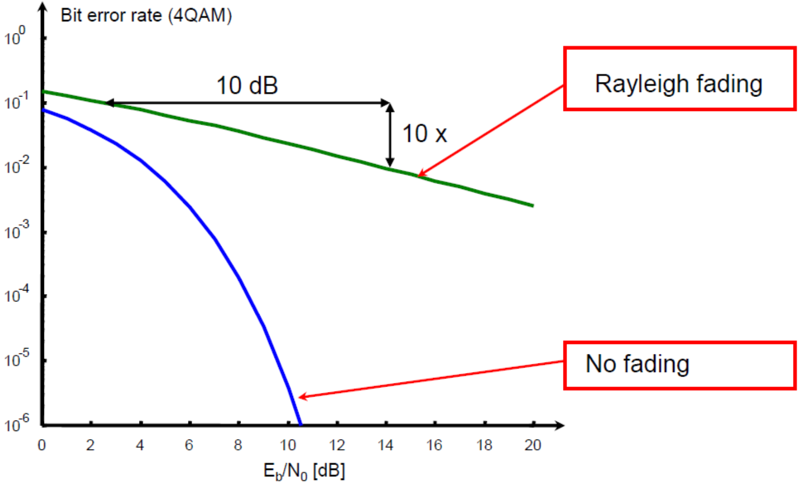
- In a fading channel with SNR distribution pdf(γ) and mean $\bar{\gamma}$

$$\text{BER}_{\text{fading}}(\bar{\gamma}) = \int_0^\infty \text{BER}_{\text{AWGN}}(\gamma)\text{pdf}(\gamma)d\gamma$$

- Example: Rayleigh fading channel

$$\text{pdf}(\gamma) = \frac{1}{\gamma}\exp\left(\frac{-\gamma}{\bar{\gamma}}\right)$$

BER in fading channels



Fading is one of the biggest challenges in wireless communications!

Diversity

- Fading can also be used as an advantage if exploited properly
- If the transmitted signal is available on two or more channels (known as diversity branches), the probability that this signal is affected by a deep fade, occurring simultaneously in all branches, is very low.
- With a convenient algorithm (known as combining method) it is possible to obtain a resulting signal where the effects of fading are minimized.

Diversity: Example

- Consider a fading channel with 2 states:
  - SNR=13.5dB for 90% of the time $\Rightarrow$ BER = 10^{-10}
  - SNR=0dB for 10% of the time $\Rightarrow$ BER = 0.5
- Average BER is $0.9 \cdot 10^{-10} + 0.1 \cdot 0.5 = 0.05$
- For a two antenna receiver employing selection combining
  - SNR=0dB at both chains for 1% of the time
  - SNR=13.5dB at at least one chain for 99% of the time
- Average BER is $0.99 \cdot 10^{-10} + 0.01 \cdot 0.5 = 0.005$

Diversity exponent

More generally we can define

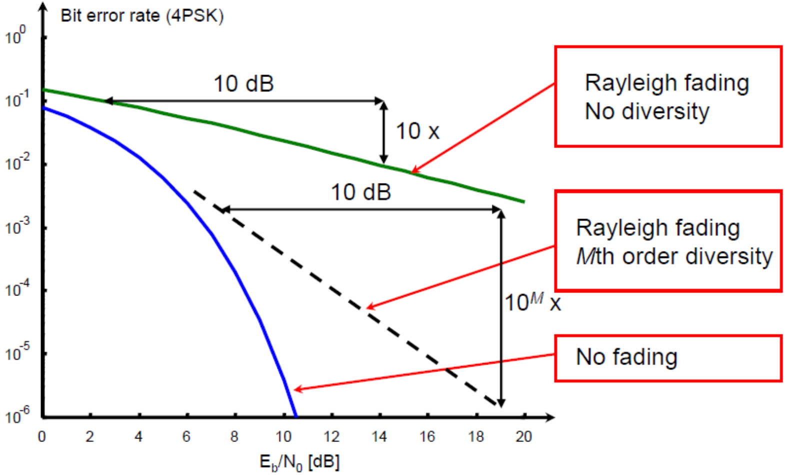
Definition (Diversity exponent)

The diversity exponent is the slope of the Bit Error Rate (BER) for large SNR on a log-log scale

$$d_{\mathrm{div}} = -\lim_{\bar{\gamma}\to\infty} \frac{\log \mathrm{BER}(\bar{\gamma})}{\log \bar{\gamma}}$$

It depends on the transmission method and the receiver used.

Diversity exponent



Types of diversity

Macrodiversity

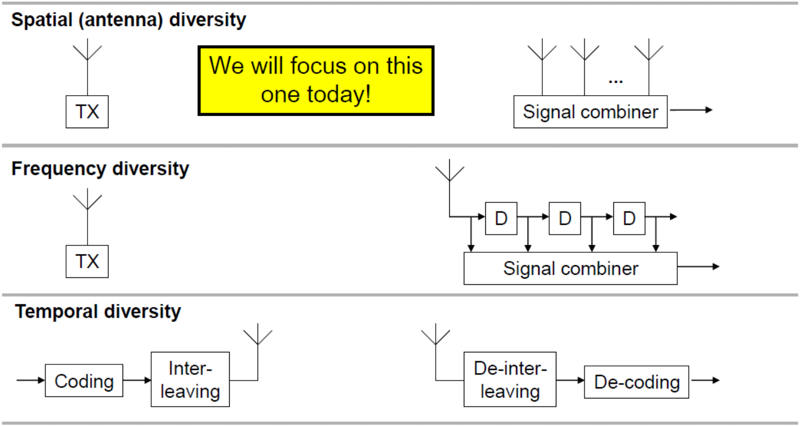
- Macrodiversity tries to counteract large scale fading caused by obstruction etc.
- Use of more than one base station strategically positioned so that the mobiles always have a clear radio path to at least one base station
- Examples: Soft handover, simulcast, coordinated multipoint transmission

Types of diversity

Microscopic diversity

- Microscopic diversity tries to conteract small scale fading caused by multipath propagation
- RX exploits multiple independent copies of the same signal (diversity branches)
- Several methods are availiable
  - Spatial diversity
  - Temporal diversity
  - Frequency diversity
  - Polarization diversity

Microscopic diversity



(We also have angular and polarization diversity)

Correlation coefficient

Diversity can be measured with the *Correlation Coefficient*:

Definition (Correlation Coefficient)

For two signals (from two diversity branches) x and y the correlation coefficient $\rho_{x,y}$ is defined as

$$\rho_{x,y} = \frac{\mathcal{E}\{xy\} - \mathcal{E}\{x\}\mathcal{E}\{y\}}{\sqrt{\left(\mathcal{E}\{x^2\} - \mathcal{E}\{x\}^2\right)\left(\mathcal{E}\{y^2\} - \mathcal{E}\{y\}^2\right)}}$$

x and y are uncorrelated if $\rho_{x,y} = 0$. Practically $\rho_{x,y} < 0.5$ is sufficient.

Correlation coefficient: Example (1)

In a WSSUS channel with isotropic Rayleigh fading and exponential decaying PDP, signals with a temporal separation of τ and a frequency separation of $f_2 - f_1$ have a correlation coefficient of

$$\rho_{x,y} = \frac{J_0^2(k_0 v \tau)}{1 + (2\pi)^2 S_\tau^2 (f_2 - f_1)^2},$$

where v is the speed, $k_0 = 2\pi/\lambda$ is the wavenumber, and S_τ is the rms delay spread.

Correlation coefficient: Example (2)

In a “typical urban” channel (delay spread = 0.977μs), compute the correlation coefficient of two frequencies with separation of 30 kHz, 200 kHz, and 5MHz.

No temporal correlation $\Rightarrow \tau = 0$.

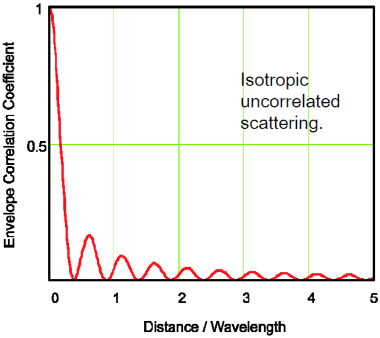
$$\rho_{x,y} = \frac{1}{1 + (2\pi)^2 (0.977 \cdot 10^{-6})^2 (f_2 - f_1)^2}$$
$$= \begin{cases} 0.97 & f_1 - f_2 = 30\,kHz, \\ 0.4 & f_1 - f_2 = 200\,kHz \\ 10^{-3} & f_1 - f_2 = 5\,MHz. \end{cases}$$

Correlation Coefficient: Example (3)

What is the minimum distance so that the signals received by two isotropic antennas is uncorrelated?

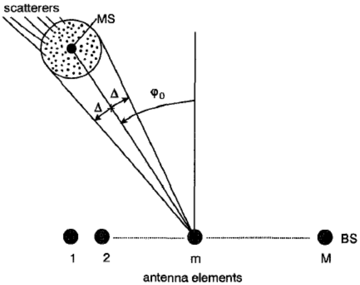
No frequency correlation $\Rightarrow f_1 = f_2$.

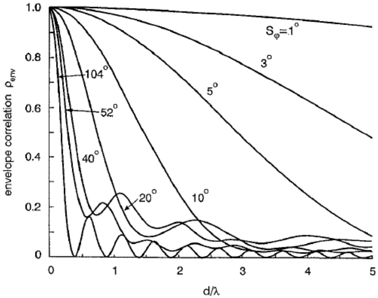
$$\rho_{x,y} = J_0^2(k_0 v \tau) = J_0^2(k_0 d)$$



Correlation Coefficient: Example (4)

Correlation between antenna elements for non-isotropic power distribution with $\varphi_0 = 60^\circ$ and linear antenna array [1]





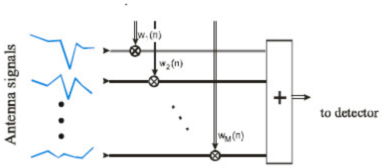
Spatial Diversity: RX

- Signal model:

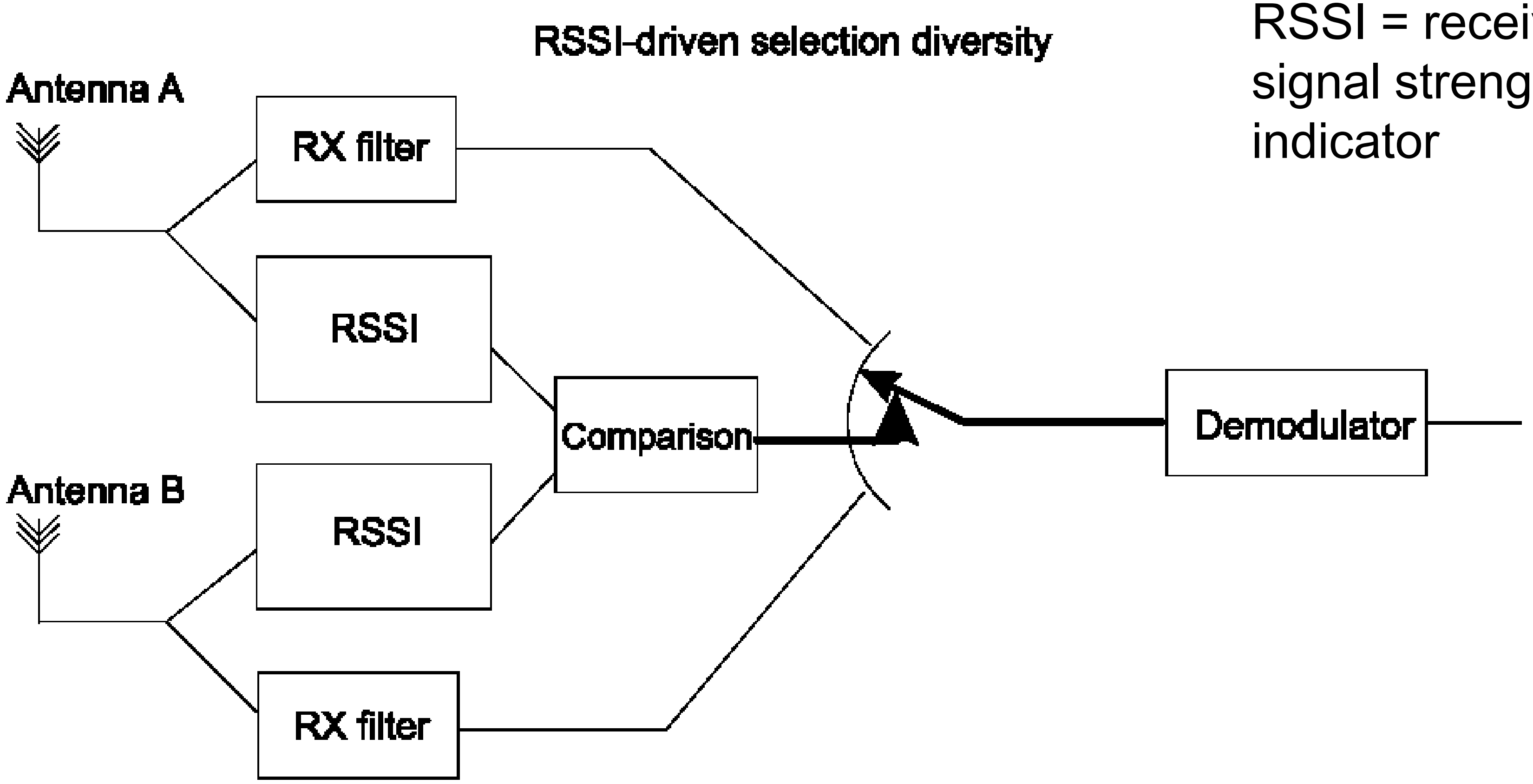
$$\mathbf{r} = \mathbf{h}s + \mathbf{n},$$

  where
  - s is the transmitted signal,
  - $\mathbf{h} = [h_0, \ldots, h_{N-1}]^T$ is the (narrowband) channel response at antenna elements $0, \ldots, N-1$,
  - $\mathbf{n} = [n_0, \ldots, n_{N-1}]^T$ is the i.i.d. noise (AWGN) with variance σ_n^2 and
  - $\mathbf{r} = [r_0, \ldots, r_{N-1}]^T$ is the received signal.
- Basic principle of diversity combining

$$y = \mathbf{w}^T\mathbf{r}$$

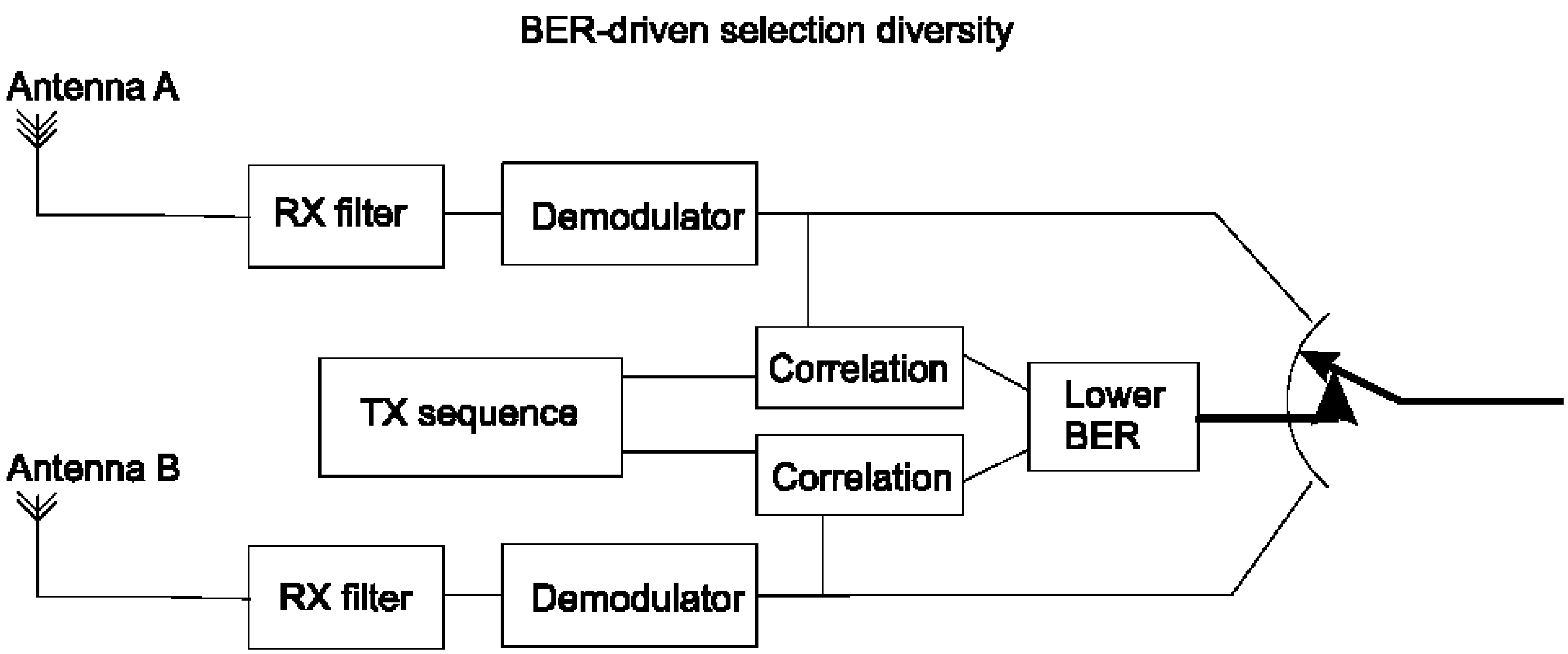


Spatial (antenna) diversity
Selection diversity

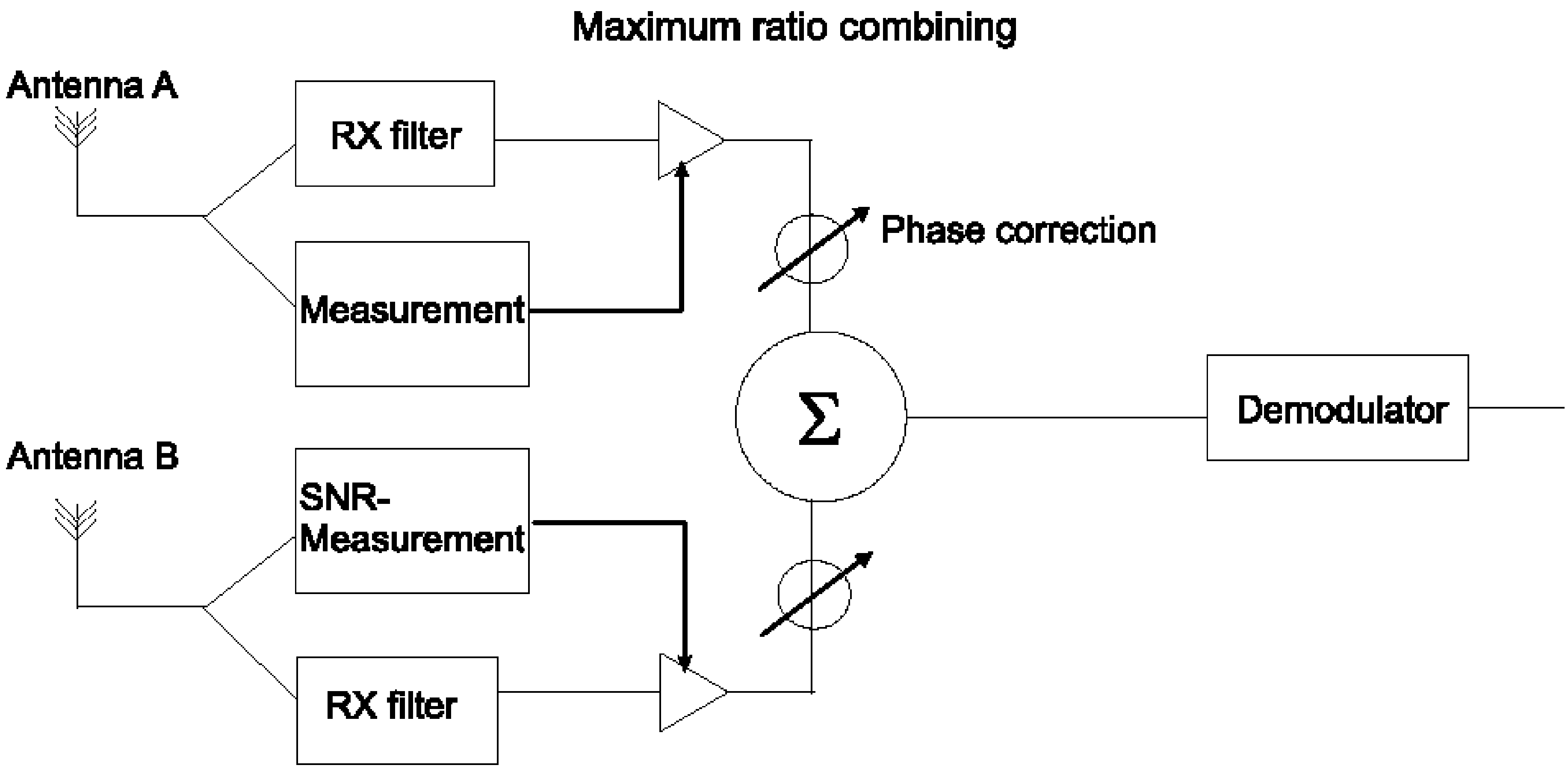


RSSI = received signal strength indicator

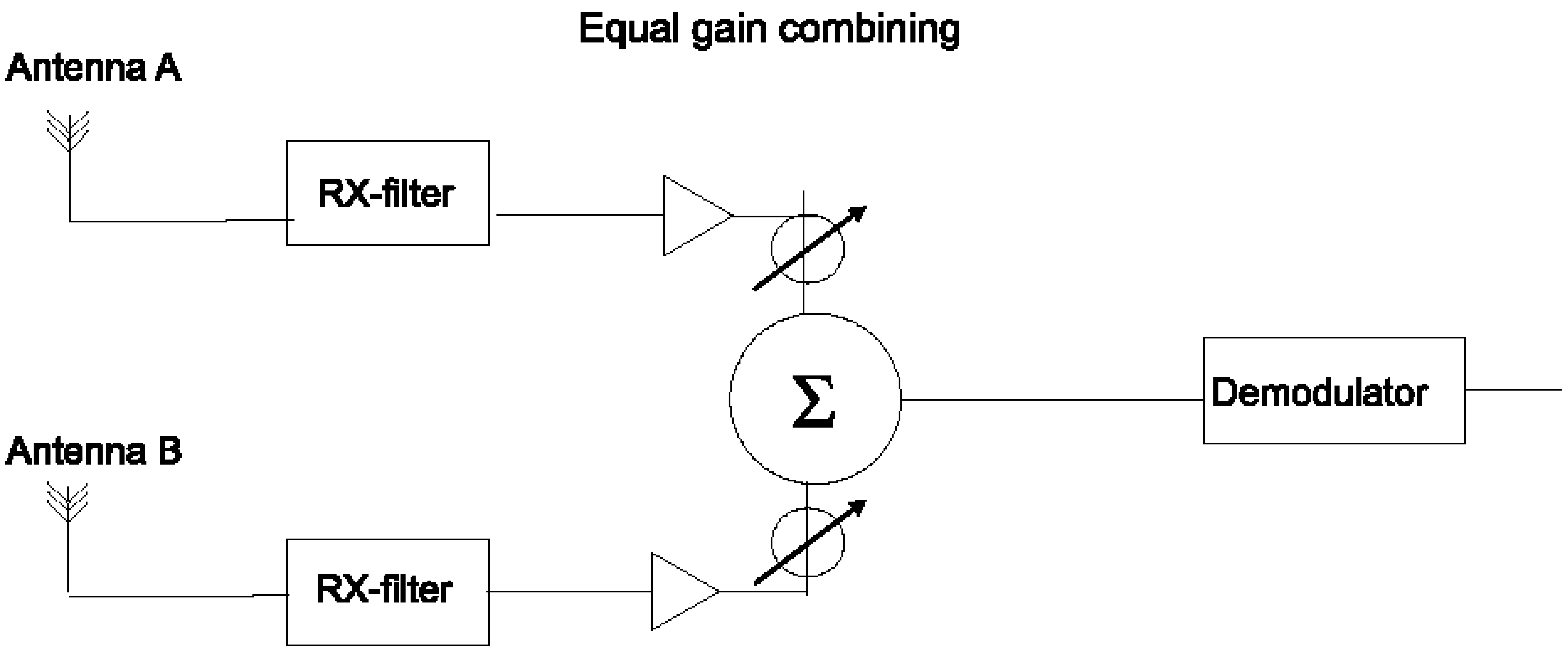
Spatial (antenna) diversity Selection diversity, cont.



Spatial (antenna) diversity
Maximum ratio combining



Spatial (antenna) diversity
Equal gain combining



Selection combining (SC)

- Select diversity branch which is “better”
- Possilbe metrics: BER, RSSI
- Combined SNR $\gamma_{\mathrm{SC}} = \max \gamma_n$

If each branch has Rayleigh distribution with mean $\bar{\gamma}$,

- Combined cdf

$$\mathrm{cdf}_{\mathrm{SC}}(\gamma) = \left(1 - \exp\left(-\frac{\gamma^2}{\bar{\gamma}^2}\right)\right)^N$$

Maximum Ratio Combining (MRC)

- Optimal combining when only disturbance is AWGN
- Each branch is phase corrected and weighted by amplitude
- Combiner weights $\mathbf{w}_{\text{MRC}} = \mathbf{h}^*$
- Combined SNR $\gamma_{\text{MRC}} = \sum_{n=0}^{N-1} \gamma_n$

If each branch has Rayleigh distribution with mean $\bar{\gamma}$,

- Combined pdf

$$\text{pdf}_{\text{MRC}}(\gamma) = \frac{1}{(N-1)!} \frac{\gamma^{N-1}}{\bar{\gamma}^N} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

- Combined cdf

$$\text{cdf}(\gamma) = 1 - \exp\left(-\frac{\gamma}{2\bar{\gamma}}\right) \sum_{i=0}^{N-1} \frac{1}{i!} \left(\frac{\gamma}{2\bar{\gamma}}\right)^i$$

- Combined mean SNR $\bar{\gamma}_{\text{MRC}} = N\bar{\gamma}$

Equal gain combining (EGC)

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- Like MRC, but each branch is only phase corrected (not weighted)
- Combiner weights $\mathbf{w}_{\text{EGC}} = [h_0^*/|h_0|, \ldots, h_{N-1}^*/|h_{N-1}|]$
- Combined SNR $\gamma_{\text{EGC}} = \frac{1}{N}\left(\sum_{n=0}^{N-1} \sqrt{\gamma_n}\right)^2$

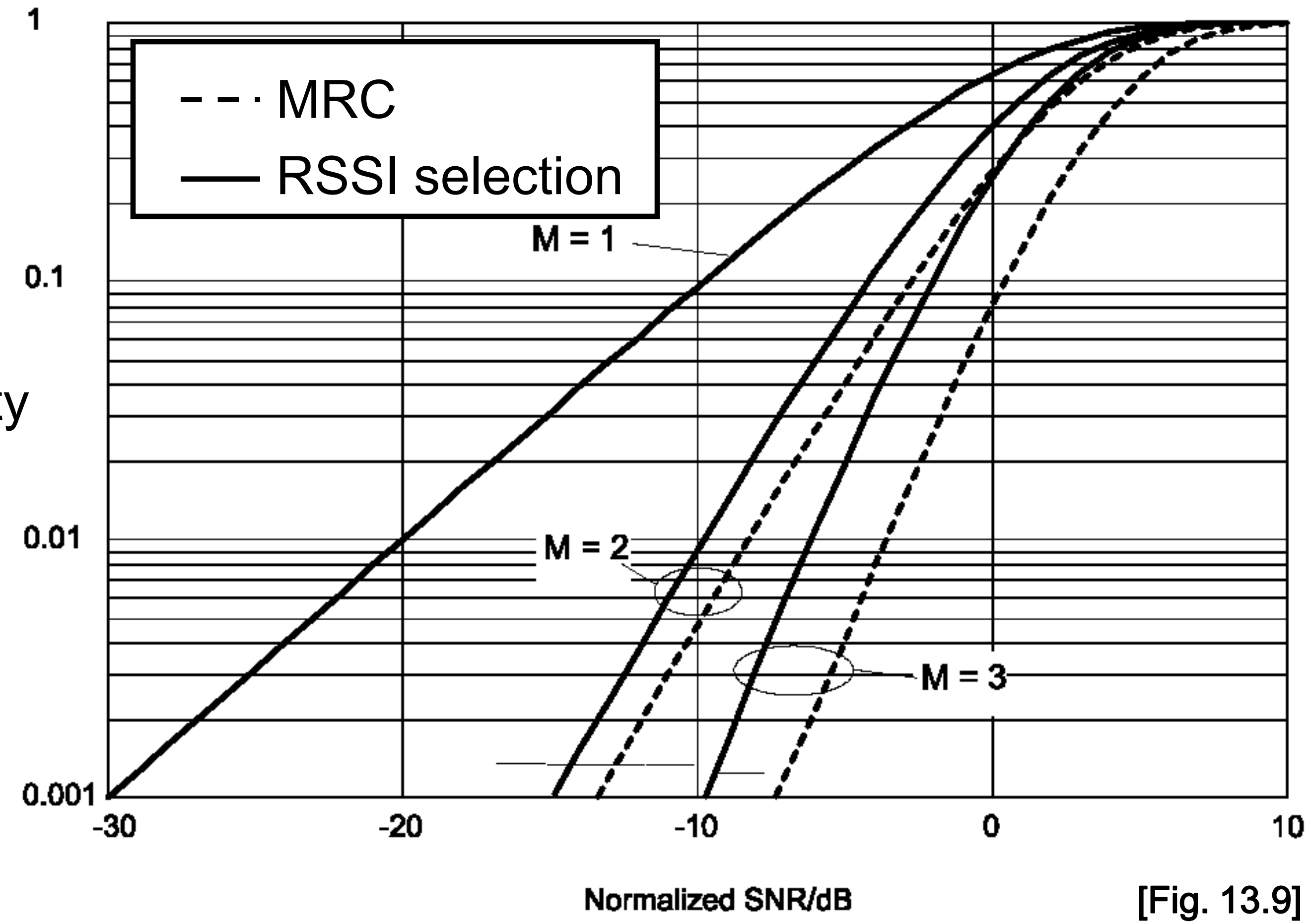
If each branch has Rayleigh distribution with mean $\bar{\gamma}$,

- Combined mean SNR $\bar{\gamma}_{\text{EGC}} = \bar{\gamma}\left(1 + (N-1)\frac{\pi}{4}\right)$

Spatial (antenna) diversity Performance comparison

Comparison of SNR distribution for different number of antennas *M* and two different diversity techniques.

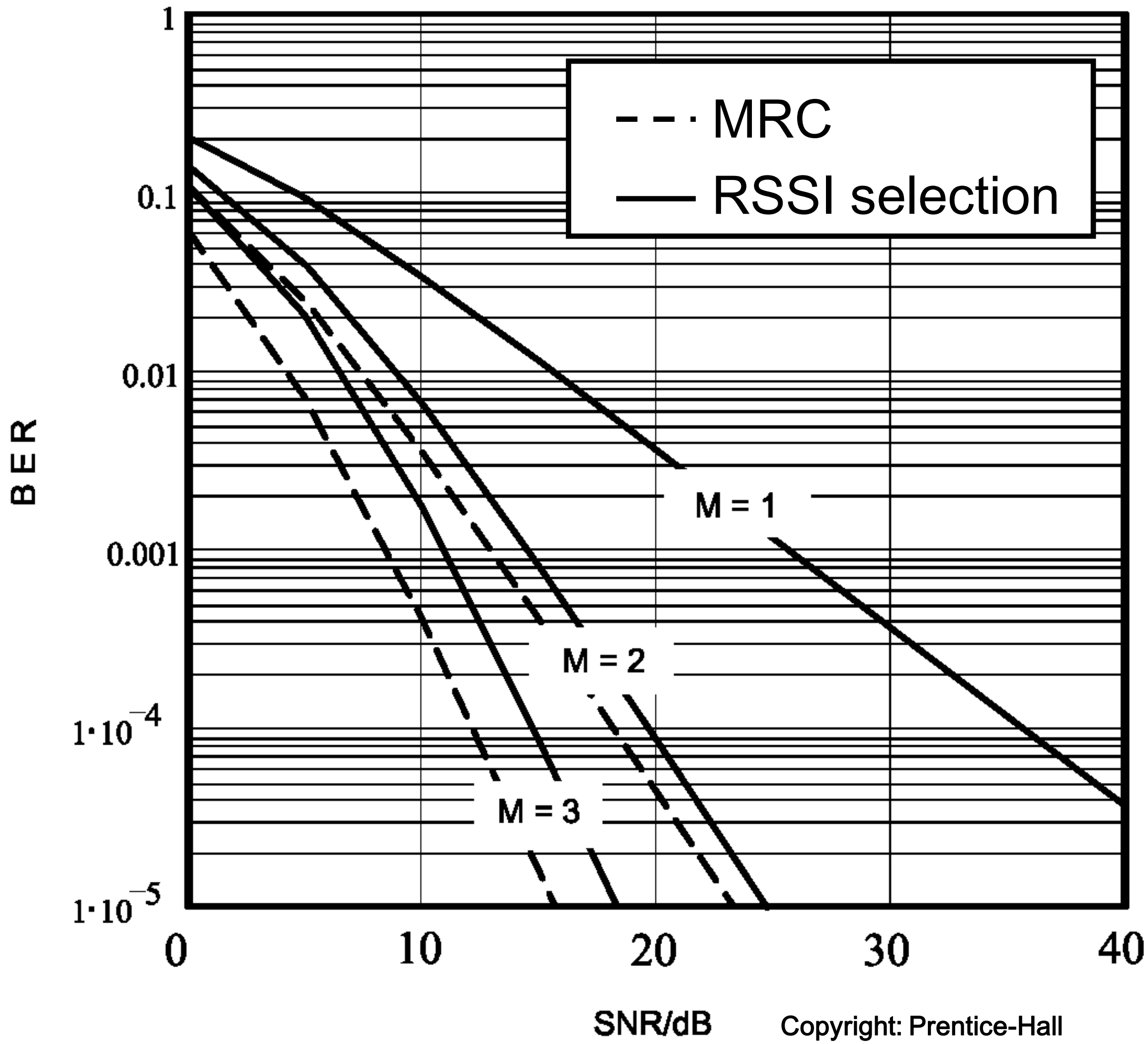
Cumulative distribution of SNR



[Fig. 13.9]

Spatial (antenna) diversity
Performance comparison, cont.

Comparison of 2ASK/2PSK BER for different number of antennas *M* and two different diversity techniques.



Example: Fading margin with SC and MRC

- Consider a receiver with N receive antennas that uses either selection combining or maximum ratio combining. Compute the fading margin for an outage probability of 1% in an uncorrelated Rayleigh fading environment for $N = 1, 2, 4$.

Optimum combining (OC)

- Most systems interference limited
- OC reduces not only fading but also interference
- Each antenna can eliminate one interferer or give one diversity degree for fading reduction (zero-forcing)
- Signal model

$$\mathbf{y}(t) = \mathbf{h}_0 x_0(t) + \sum_{k=1}^{K} \mathbf{h}_k x_k(t) + \mathbf{n}$$

  with $\mathcal{E}\{\mathbf{n}\mathbf{n}^H\} = \sigma^2 \mathbf{I}$
- Computation of weights for combining

$$\mathbf{w}_{\text{opt}} = \mathbf{R}^{-1}\mathbf{h}, \quad \mathbf{R} = \sigma_n \mathbf{I} + \sum_{k=1}^{K} \mathcal{E}\left\{\mathbf{h}_k \mathbf{h}_k^H\right\},$$

Spatial diversity: TX

- Method depends on Channel State Information at the Transmitter (CSIT)
- If available, equivalent methods as for RX can be used
  - Antenna selection
  - Beamforming (Equal gain combining)
  - Maximum Ratio Combining
- If no CSIT is available,
  - Alamouti precoding
  - Cyclic delay diversity

Alamouti precoding

- Alamouti code: transmission of two symbols s_1, s_2 over two transmit antennas and two time instances (rate 1 code)

$$\mathbf{x}_1 = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} -s_2 \\ s_1^* \end{pmatrix}$$

- Received signal

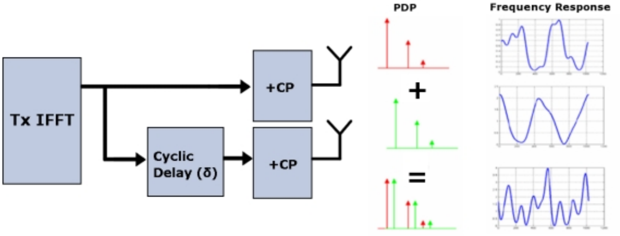
$$y_1 = \mathbf{h}\mathbf{x}_1 + n_1 = s_1 h_1 + s_2 h_2 + n_1, \quad y_2 = \mathbf{h}\mathbf{x}_2 + n_2 = -s_2 h_1 + s_1^* h_2 + n_2$$

- Diversity combiner

$$\hat{s}_1 = (h_1^* y_1 + h_2 y_2^*)/2, \quad \hat{s}_2 = (h_2^* y_1 + h_1 y_2^*)/2$$

Cyclic Delay Diversity

- Requires OFDM with cyclic prefix
- Transforms spatial diversity into frequency diversity



Temporal and frequency diversity

- Main idea: use coding and interleaving to spread information over multiple symbols
- The symbols can be distributed in time and/or frequency
- Classical frequency diversity methods
  - Spreading (Used in CDMA systems)
  - Frequency hopping (Used in FDMA/TDMA systems)

Temporal diversity

- Transmission over different (uncorrelated) time instances
- Repetition coding: bandwidth inefficient
- Automatic repeat request (ARQ):
  - RX informs TX about (un-)successful reception (ACK/NACK); TX repeats if necessary
  - Requires feedback channel
  - Introduces delay (however, several ARQ processes can be pipelined)
- Hybrid ARQ: exploit redundancy in channel code for different retransmissions
- Can also be combined with frequency diversity schemes

References

J. Fuhl, A.F. Molisch, and E. Bonek,
"Unified channel model for mobile radio systems with smart antennas,"
Radar, Sonar and Navigation, IEE Proceedings, vol. 145, no. 1, pp. 32–41, Feb. 1998.