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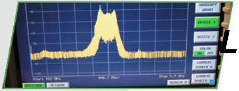


Radio Engineering

Lecture 5: MIMO Channel Characterization

Florian Kaltenberger

Last lecture

6. Wideband channels
   - Wideband vs narrowband channels
   - System theoretic description of wideband channels
   - WSSUS model
   - Condesed parameters
   - Direction channel description

Wideband channels: definition

- A communication system is narrowband if
  - The symbol duration T_s is *larger* than the maximum delay (or the delay spread) in the channel $\Delta\tau$
  - $\Rightarrow$ Receiver cannot distinguish different echos
- A communication system is wide-band if
  - The symbol duration T_s is *smaller* than the maximum delay (or the delay spread) in the channel $\Delta\tau$
  - $\Rightarrow$ One transmitted symbol can spread over more than one symbol at the receiver

Stochastic channel characterization

- The *autocorrelation function* of a stochastic process $h(t)$ is defined as
$$R_h(t,t') = \mathcal{E}\{h(t)h^*(t')\}$$
- A stochastic process $h(t)$ is *wide-sense stationary (WSS)* iff
$$R_h(t,t') = R_h(t-t') = R_h(\Delta t)$$
- The power spectrum $S_h(\nu)$ of a WSS process $h(t)$ is given by the Fourier transform of the autocorrelation function
$$S_h(\nu) = \mathcal{F}(R_h(\Delta t))$$

Stochastic channel characterization

- Autocorrelation function of $H(\nu) = \mathcal{F}(h(t))$

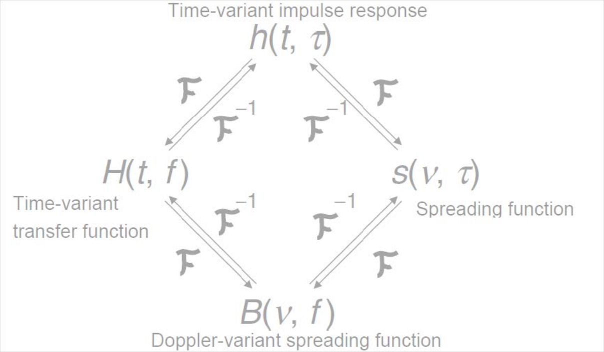
$$R_H(\nu, \nu') = \mathcal{E}\{H(\nu)H^*(\nu')\}$$

- Iff $h(t)$ is WSS then $H(\nu)$ is *uncorrelated scattering (US)*

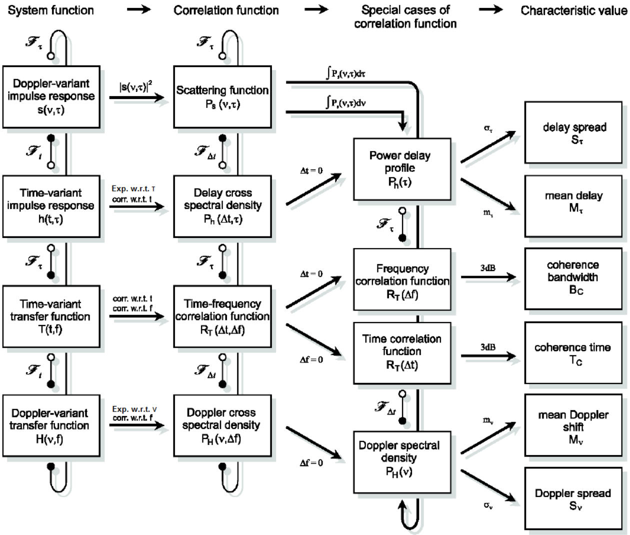
$$R_H(\nu, \nu') = \delta(\nu - \nu')S_h(\nu)$$

Linear time-variant system

- Linear time-variant systems are characterized by one of the four system functions



Correlation functions and condensed parameters



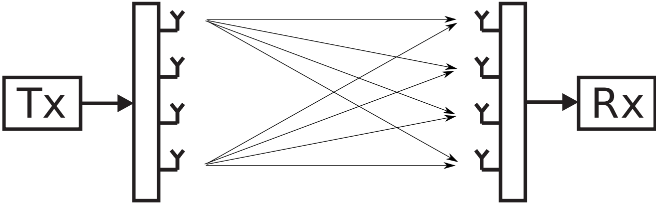
Summary

- In wideband communication systems, multipath propagation results in inter-symbol interference (ISI)
- Wideband channels fade also in frequency, causing *frequency-selective* fading
- Wideband channels can be mathematically described as linear time-variant systems (LTV)
- Wideband channels are wide-sense stationary (WSS) with uncorrelated scattering (US) iff
  1. their second order statistics (autocorrelation function) do not change over time
  2. contributions with different delays are uncorrelated

This lecture

1. Multiple-Input Multiple-Output (MIMO) channels
   - Definitions
   - System model
   - Mutual coupling and correlation
   - Double directional channel characterization
   - Angular power spectra
2. Channel Sounding
   - Time and frequency domain sounding
   - Directionally resolved measurements
   - Parameter estimation methods

Definitions



- SISO: Single-Input Single-Output
- SIMO: Single-Input Multiple-Output
- MISO: Multiple-Input Single-Output
- MIMO: Multiple-Input Multiple-Output

System Model

MIMO input-output realtion

$$\mathbf{y}(t) = \int_{-\infty}^{\infty} \mathbf{H}(t,\tau)\mathbf{x}(t-\tau)\mathrm{d}\tau + \mathbf{n}(t),$$

where

- $\mathbf{x}(t) = [x_0(t), \dots, x_{N_{\mathrm{TX}}-1}(t)]^T$ is the transmitted signal
- $\mathbf{n}(t) = [n_0(t), \dots, n_{N_{\mathrm{RX}}-1}(t)]^T$ is the AWGN (i.i.d.)
- $\mathbf{y}(t) = [r_0(t), \dots, r_{N_{\mathrm{RX}}-1}(t)]^T$ is the received signal
- $\mathbf{H}(t,\tau) = \begin{bmatrix} h_{0,0}(t,\tau) & \dots & h_{0,N_{\mathrm{TX}}-1}(t,\tau) \\ \vdots & \ddots & \vdots \\ h_{N_{\mathrm{RX}},0}(t,\tau) & \dots & h_{N_{\mathrm{RX}}-1,N_{\mathrm{TX}}-1}(t,\tau) \end{bmatrix}$
  is the MIMO channel response

Benefits of MIMO

- Array Gain
  - Increase Power (RX)
  - Beamforming (TX)
- Diversity
  - Mitigate Fading
  - Space-Time Coding
- Spatial Multiplexing
  - Multiply Data Rates
  - Spatially Orthogonal Codes

Capacity of a MIMO channel

- Capacity of a MIMO channel

$$C = \log_2 \left[\det \left(\mathbf{I}_{N_{\mathrm{Tx}}} + \frac{\bar{\gamma}}{N_{\mathrm{Tx}}} \mathbf{H} \mathbf{R}_x \mathbf{H}^H \right) \right],$$

  where $\bar{\gamma}$ is the mean SNR and $\mathbf{R}_x$ is the correlation matrix of the transmitted data

- If channel is know at transmitter, $\mathbf{R}_x$ can be matched to the channel and

$$C = \sum_{k=1}^{\min(N_{\mathrm{Tx}}, N_{\mathrm{Rx}})} \log_2 \left[1 + \frac{P_k}{\sigma_n^2} \sigma_k^2 \right],$$

  where P_k is the results of the power allocation (waterfilling), σ_n^2 is the noise variance and σ_k are the singular values of the channel $\mathbf{H}$

Capacity of a MIMO channel (2)

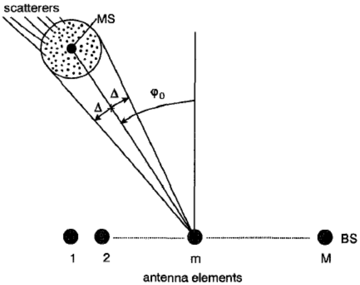
- Capacity of the channel is proportional to the rank (=number of non-zero singular values) of the channel matrix $\mathbf{H}$
- In the ideal case (full channel rank), capacity thus scales with $\min(N_{\mathrm{Tx}}, N_{\mathrm{Rx}})$

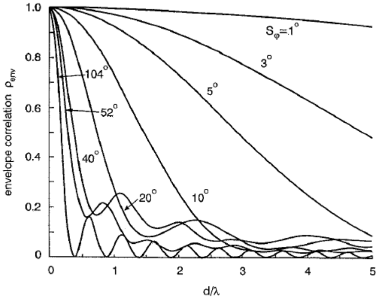
Correlation and Mutual coupling

- Correlation:
  - Signals received at different antenna elements are correlated due to propagation (scattering, etc) [1]
  - Property of the antenna geometry and the environment
- Mutual coupling:
  - radiation pattern of each single antenna is influenced by neighboring antennas [2]
  - property of the antenna array only

Correlation

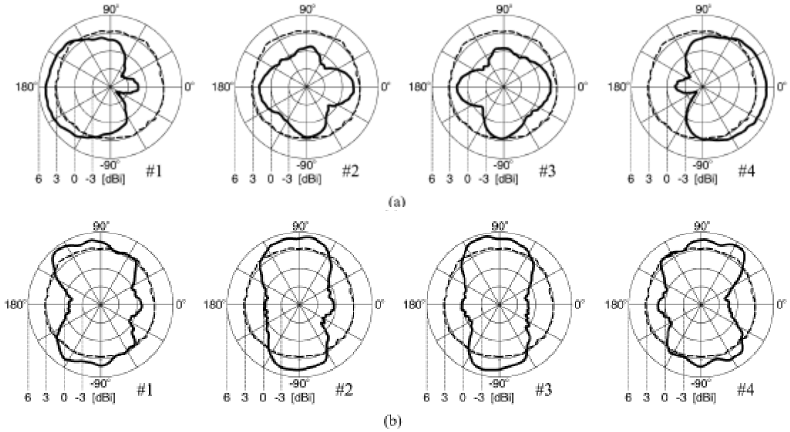
Correlation between antenna elements for $\varphi_0 = 60°$, uniform spectrum, linear antenna array [1]





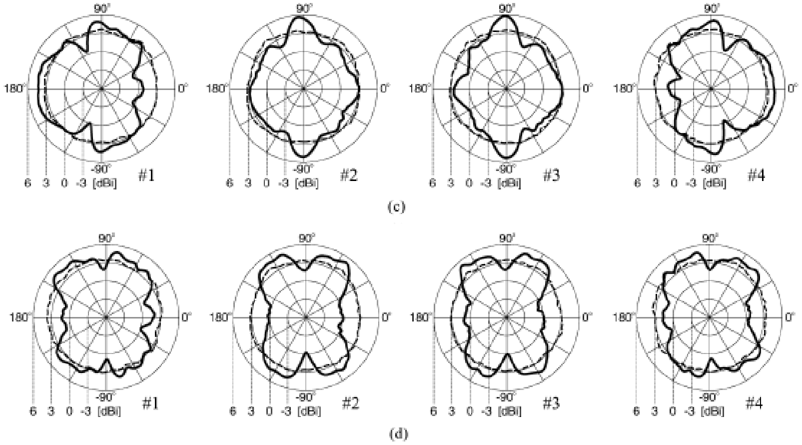
Mutual coupling

Individual antenna patterns influenced by mutual coupling for antenna spacings $d = \lambda/4$ and $d = \lambda/2$ [2]



Mutual coupling (2)

Individual antenna patterns influenced by mutual coupling for antenna spacings $d = 3\lambda/4$ and $d = \lambda$ [2]



Correlation Matrices

- Autocorrelation function of a SISO channel

$$R_h(t,t',\tau,\tau') = \mathcal{E}\left\{h(t,\tau)h^*(t',\tau')\right\}$$

- Autocorrelation function of a MIMO channel

$$R_h(t,t',\tau,\tau',n,n',m,m') = \mathcal{E}\left\{h_{n,m}(t,\tau)h^*_{n',m'}(t',\tau')\right\}$$

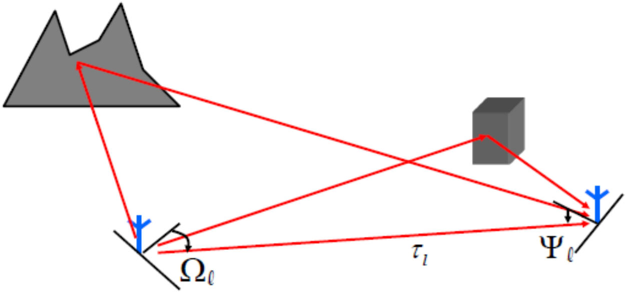
- Can also be written as a *correlation matrix*

$$\mathbf{R}(t,t',\tau,\tau') = \mathcal{E}\left\{vec(\mathbf{H}(t,\tau))\, vec(\mathbf{H}(t',\tau'))^H\right\}$$

The double-directional channel description

- Correlation matrices fully describe the second order statistics of the channels
- However, they are dependent on the antenna geometry
- It is desirable to have an antenna-independent description of the channel

The double-directional channel description (2)



$$h(t, \tau, \Omega, \Psi) = \sum_{l=0}^{N-1} h_l(t, \tau, \Omega, \Psi)$$

$$h_l(t, \tau, \Omega, \Psi) = |a_l| e^{j\varphi_l} \delta(\tau - \tau_l)\delta(\Omega - \Omega_l)\delta(\Psi - \Psi_l)$$

where Ω is the angle of departure and Ψ is the angle of arrival

The double-directional channel description (3)

- The MIMO channel matrix can be calculated from the double-directional channel description
- First include the antenna patterns (including mutual coupling)

$$\bar{h}_{n,m}(t,\tau,\varphi,\psi) = G_{\text{Tx}}^{(m)}(\varphi)h(t,\tau,\varphi,\psi)G_{\text{Rx}}^{(n)}(\psi),$$

  where
  - $G_{\text{Tx}}^{(m)}(\varphi)$ is the antenna pattern of the *n*-th transmit antenna and
  - $G_{\text{Rx}}^{(n)}(\psi)$ is the antenna pattern of the *m*-th receive antenna.

The double-directional channel description (3)

- Then we transform from the angular to the spatial domain

$$\begin{aligned} h_{n,m}(t,\tau) &= h_{n,m}(t,\tau,\vec{x}_m,\vec{y}_n) \\ &= \iint \bar{h}_{n,m}(t,\tau,\varphi,\psi) e^{2\pi j/\lambda \langle \vec{\zeta},\vec{x}_m \rangle} e^{2\pi j/\lambda \langle \vec{\xi},\vec{y}_m \rangle} \, \mathrm{d}\varphi \, \mathrm{d}\psi, \end{aligned}$$

  where
  - $\vec{\zeta} = \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}$, and $\vec{\xi} = \begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix}$,
  - $\vec{x}_0, \ldots, \vec{x}_{N_{\mathrm{TX}}-1}$ are the transmit antenna locations,
  - $\vec{y}_0, \ldots, \vec{y}_{N_{\mathrm{RX}}-1}$ are the receive antenna locations, and
  - $\langle ., . \rangle$ denotes the scalar product.

Angular Power Spectra

- The full autocorrelation function of a double-direction channel is given by

$$S(t,\tau,\Omega,\Psi,t',\tau',\Omega',\Psi') = \mathcal{E}\left\{h(t,\tau,\Omega,\Psi)h(t',\tau',\Omega',\Psi')^*\right\}$$

- If the channel is WSS-US and also contributions from different directions are uncorrelated[1]

$$S(t,\tau,\Omega,\Psi,t',\tau',\Omega',\Psi') = P(\Delta t,\tau,\Omega,\Psi)\delta(\tau-\tau')\delta(\Omega-\Omega')\delta(\Psi-\Psi')$$

- For $\Delta t = 0$ we get the double directional delay power spectrum

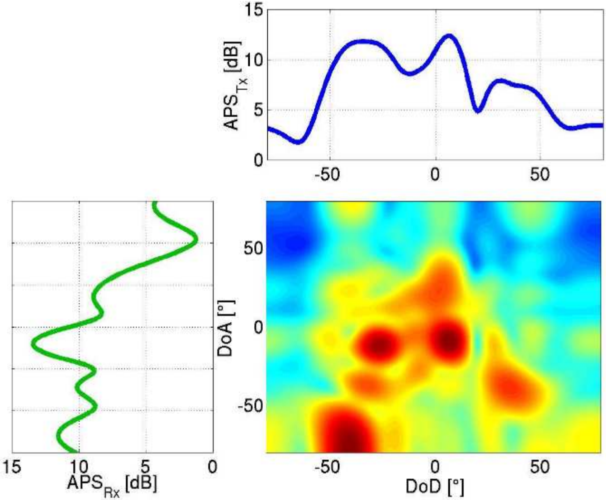
$$DDDPS(\tau,\Omega,\Psi) = P(0,\tau,\Omega,\Psi)$$

- Integrating over τ gives the double directional power spectrum

$$DDPS(\Omega,\Psi) = \int DDDPS(\tau,\Omega,\Psi)\mathrm{d}\tau$$

[1] This assumption is sometimes also called homogeneous

Angular Power Spectra: Example



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Chapter 8

Channel sounding

Channel measurements

In order to model the channel behavior we need to measure its properties

- Time domain measurements
  - impulse sounder
  - correlative sounder
- Frequency domain measurements
  - Vector network analyzer

- Directional measurements
  - directional antennas
  - real antenna arrays
  - multiplexed arrays
  - virtual arrays

Basic identifiability of the channel

- The channel can be measured uniquely only if
  - sampling theorem

$$f_{\mathrm{rep}} \geq 2\nu_{\mathrm{max}}$$

$$\frac{1}{f_{\mathrm{rep}}} \geq \tau_{\mathrm{max}}$$

- Therefore, a channel can only be measured uniquely if it is *underspread*

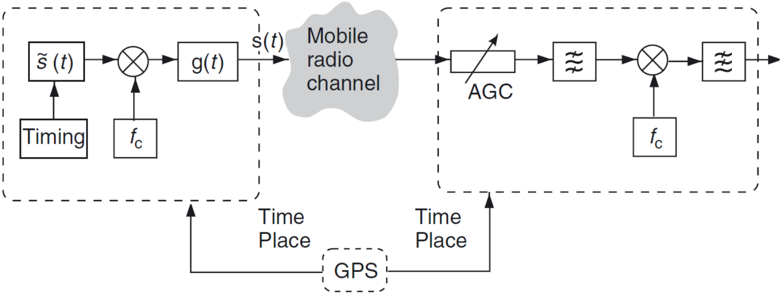
$$2\tau_{\mathrm{max}}\nu_{\mathrm{max}} \leq 1$$

- This condition is fulfilled in all practical wireless applications

Identifiability: Example

A channel sounder is in a car that moves with a speed of 36km/h. The channel sounder operates at a carrier frequency of 2GHz. At what intervals should the channel be measured? What is the maximum excess delay the channel can have to remain underspread?

Generic Sounder Stucture



Sounding signal:

$$s(t) = \sum_{i=0}^{N-1} p_{TX}(t - iT_{rep})$$

Generic Receiver Structure

Received signal:

$$r(t) = s(t) * h(t,\tau) * p_{\mathrm{RX}}(\tau) + n(t) = \sum_{i=0}^{N-1} \underbrace{p(\tau) * h(t_i,\tau)}_{h_{\mathrm{meas}}(t_i,\tau)} + n(t)$$

where

- $t_i = t - iT_{\mathrm{rep}}$,
- $h(t_i,\tau)$ constant during T_{rep}, and
- $p(\tau) = p_{\mathrm{TX}}(\tau) * p_{\mathrm{RX}}(\tau)$

Channel sounder paramters

- *Bandwidth:* determines delay resolution
- *Signal Duration:* T_{rep} should be larger than excess delay plus length of filter $p_{\text{TX}}(t)$, but smaller than coherence bandwidth
- *Time-Bandwidth product:* should be maximized to to increase SNR at RX
- *Power spectral density* of the $p_{\text{TX}}(t)$ should be uniform
- *Crest factor*
- *Correlation properties*

Synchronization

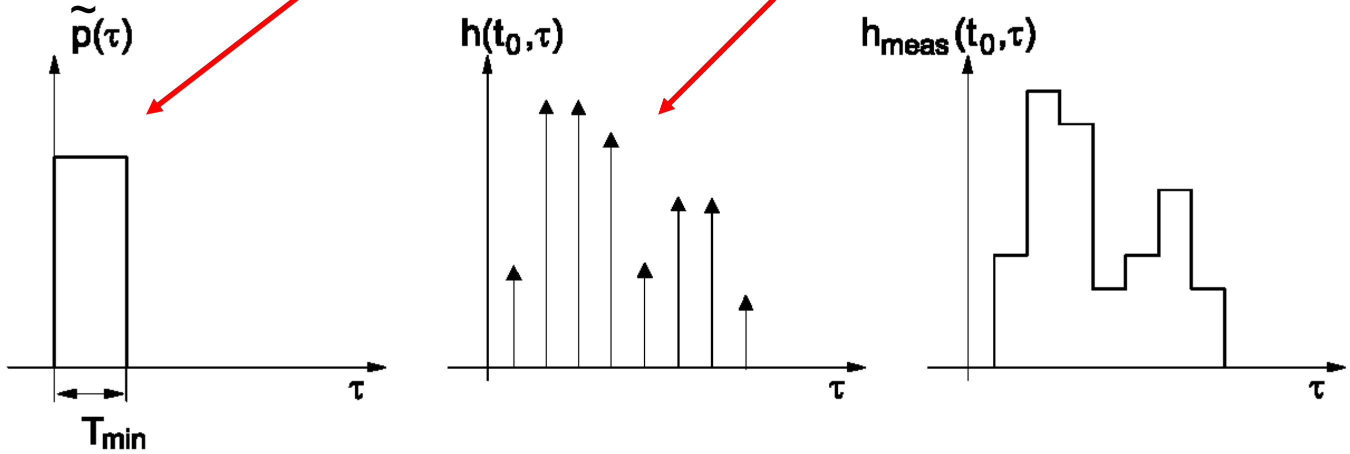
- TX and RX need to be synchronized in time, frequency, and phase!
- Cables: only for short distances
- *GPS:* only outdoor
- *Rubidium clocks:* expensive, need calibration
- *Over-the-air synchronization:* least accurate

Impulse sounder

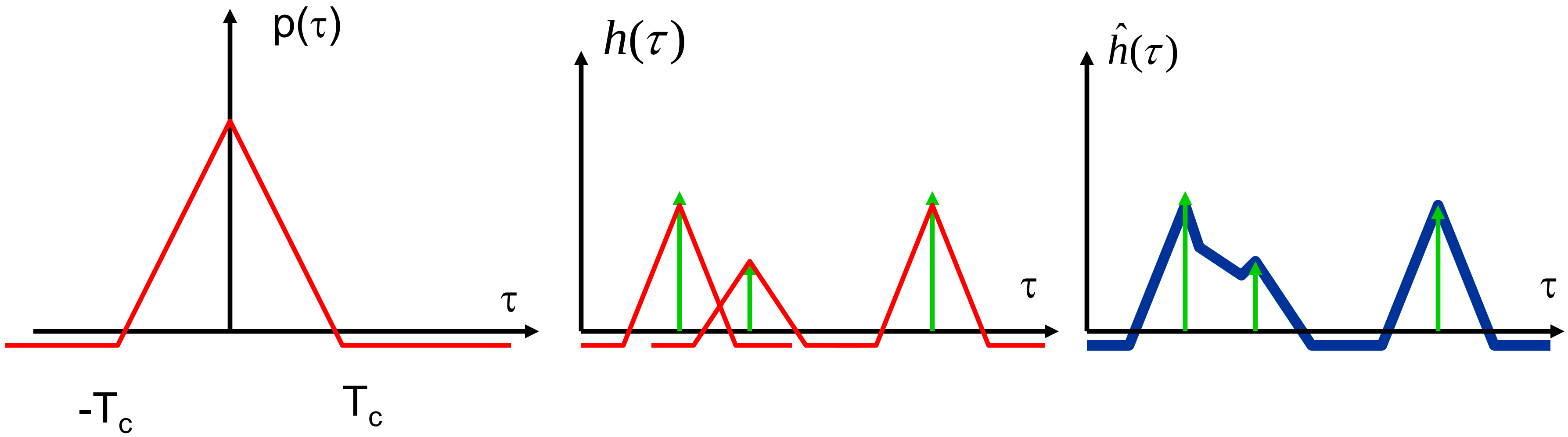
$$h_{\text{meas}}(t_i, \tau) = \widetilde{p}(\tau) * \boxed{h(t_i, \tau)}$$

impulse response of sounder

impulse response of channel



Correlative sounder



correlation peak | impulse response | measured impulse response

Frequency domain measurements

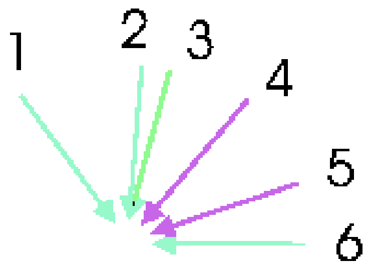
- Use a vector network analyzer or similar to determine the transfer function of the channel

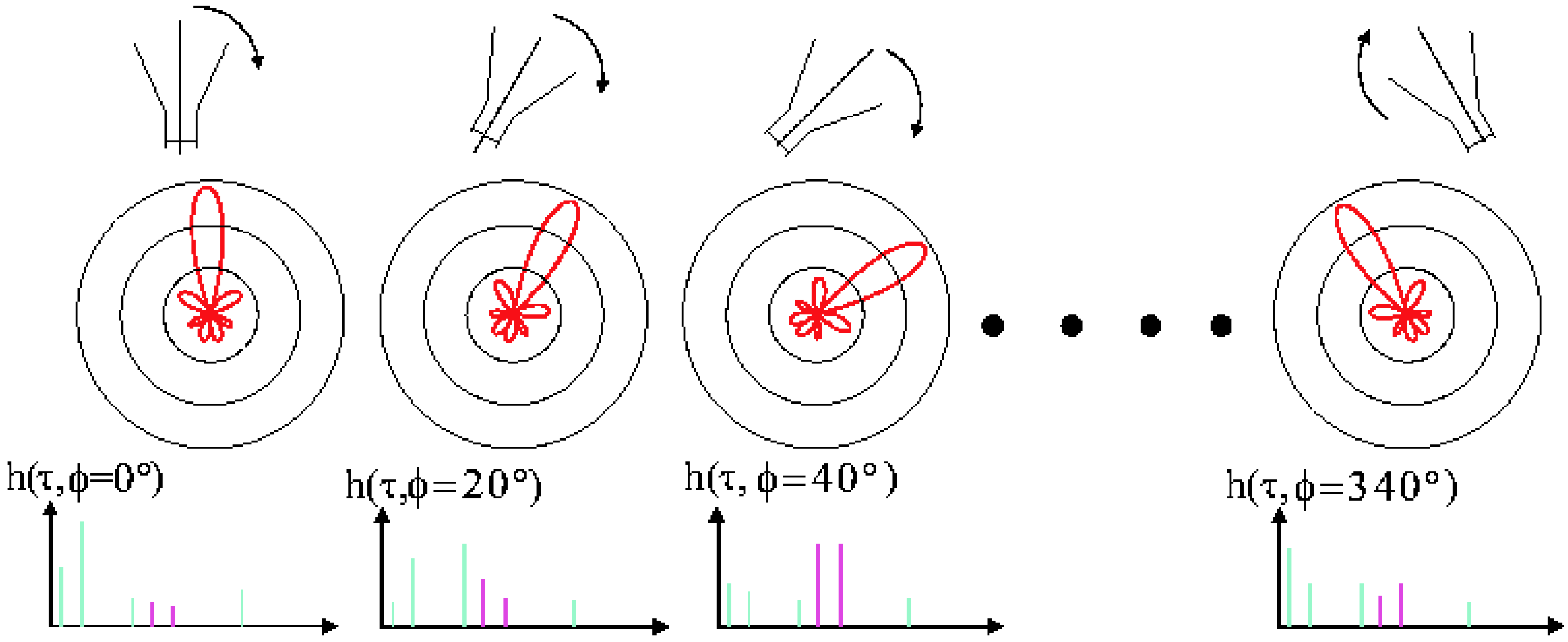
$$H_{meas}(f) = H_{TXantenna}(f) * \boxed{H_{channel}(f)} * H_{RXantenna}(f)$$

- Need to know the influence of the measurement system

Channel sounding – directional antenna

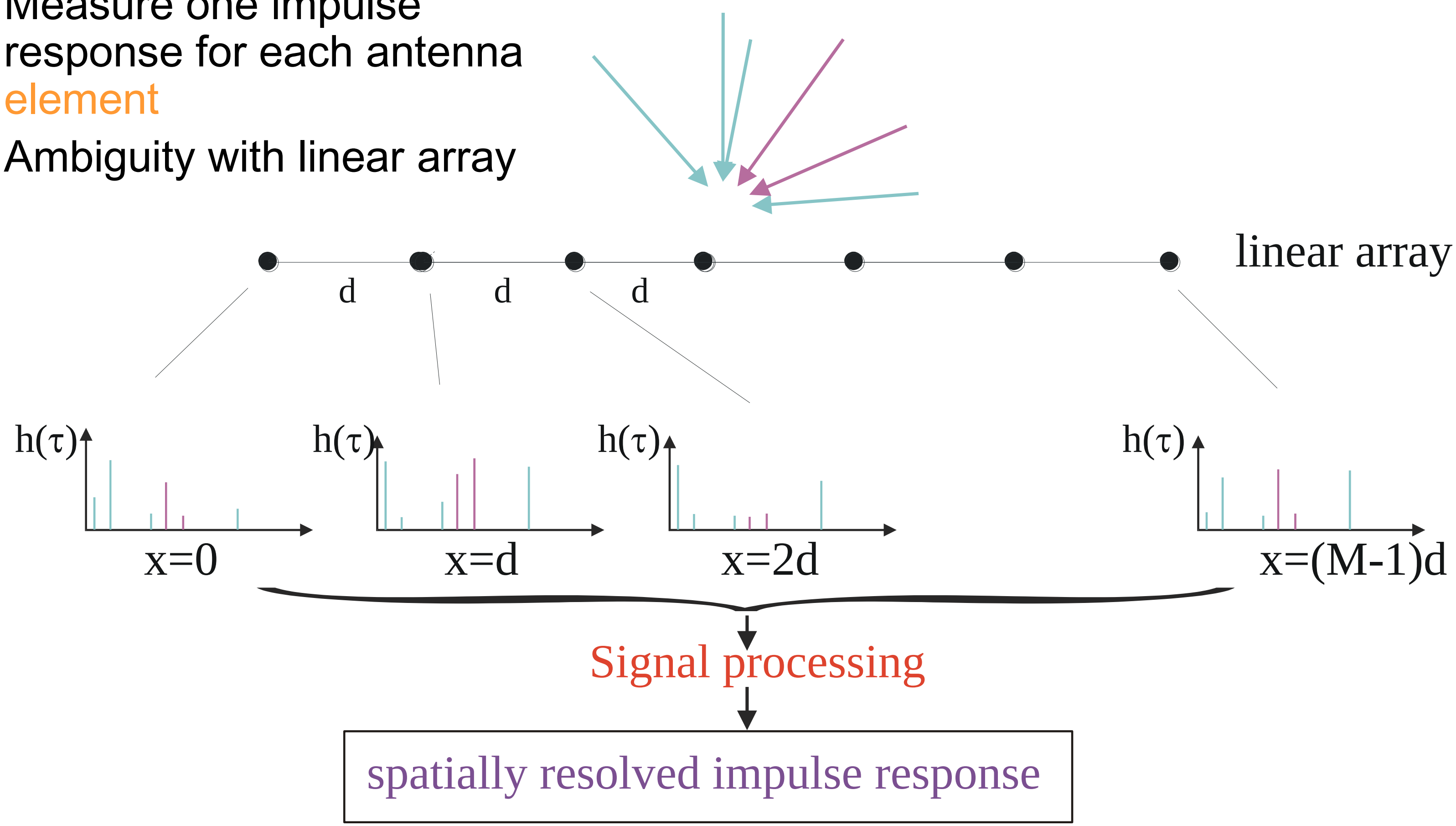
- Measure one impulse response for each antenna orientation





Channel sounding – antenna array

- Measure one impulse response for each antenna element
- Ambiguity with linear array

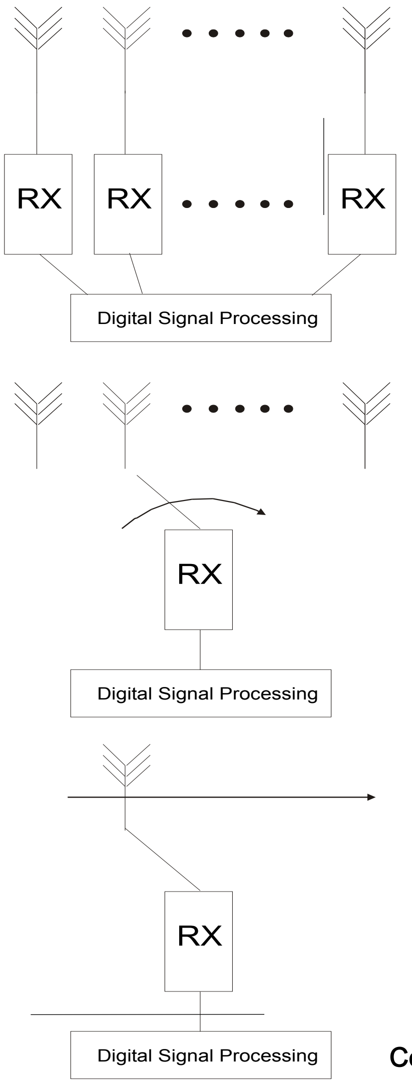


Real, multiplexed, and virtual arrays

- Real array: simultaneous measurement at all antenna elements

- Multiplexed array: short time intervals between measurements at different elements

- Virtual array: long delay
  no problem with mutual coupling



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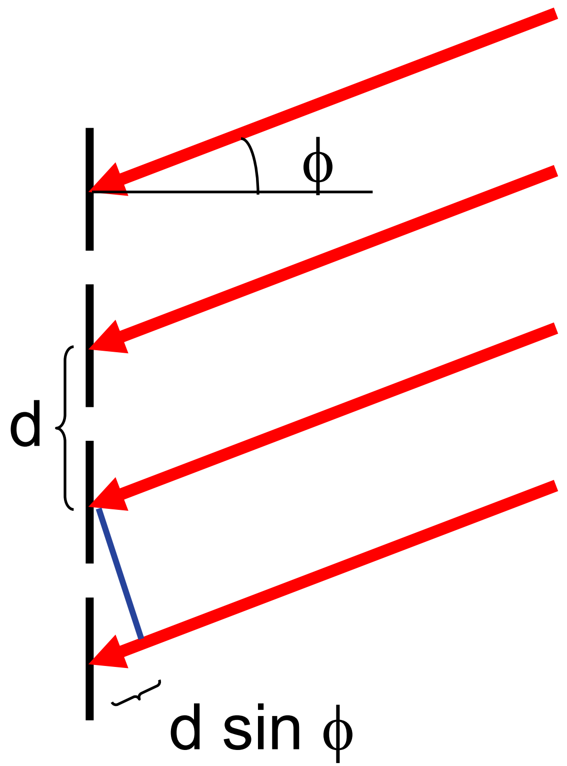
Directional analysis



- The DoA can, e.g., be estimated by correlating the received signals with steering vectors.

$$\vec{a}(\phi) = \begin{pmatrix} 1 \\ \exp(-jk_0 d\cos(\phi)) \\ \exp(-j2k_0 d\cos(\phi)) \\ \vdots \\ \exp(-j(M-1)k_0 d\cos(\phi)) \end{pmatrix}$$

- An element spacing of d=5.8 cm and an angle of arrival of ϕ =20 degrees gives a time delay of $6.6 \cdot 10^{-11}$ s between neighboring elements

High resolution algorithms

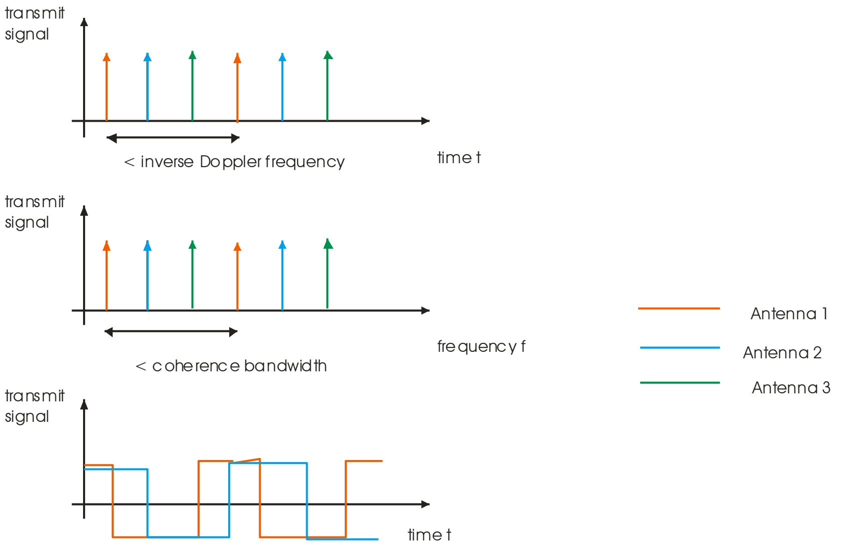
- In order to get better angular resolution, other techniques for estimating the angles are used, e.g.:
  - MUSIC, subspace method using spectral search
  - ESPRIT, subspace method
  - MVM (Capon’s beamformer), rather easy spectral search method
  - SAGE, iterative maximum likelihood method
- Based on models for the propagation
- Rather complex, one measurement point may take 15 minutes on a decent computer

Antenna array TX

- Transmission must be done so that RX can distinguish signals from different TX receivers

  →Transmit signals should be orthogonal

- Orthogonality in time
- Orthogonality in frequency
- Orthogonality in code



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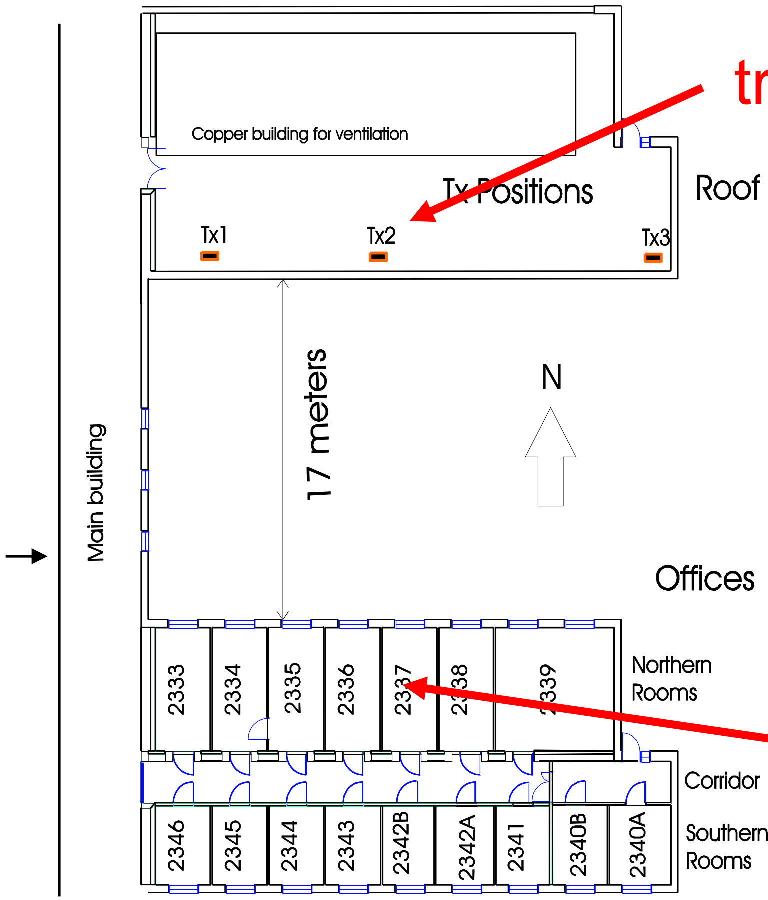
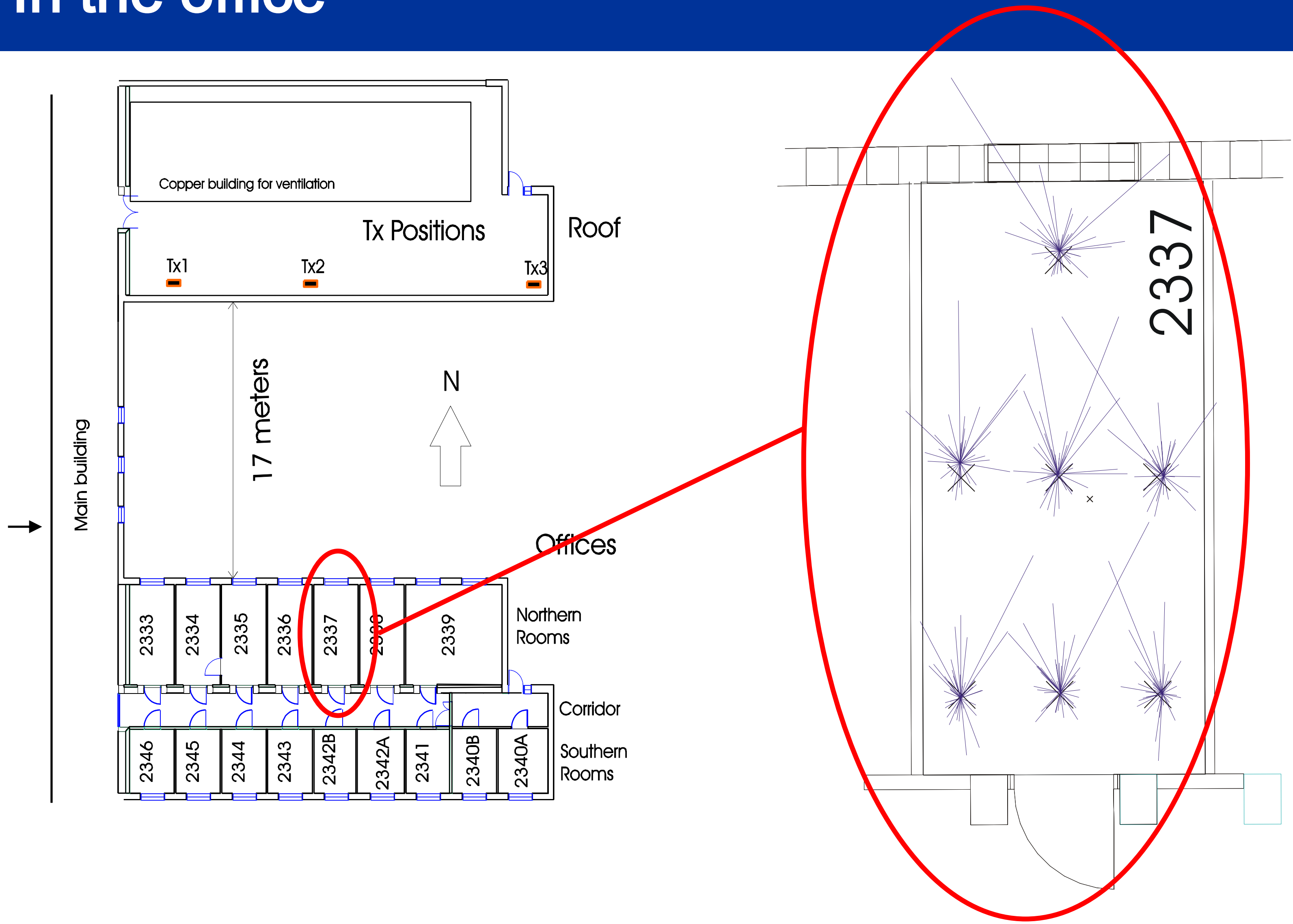
How does the signal reach the receiver Outdoor-to-indoor



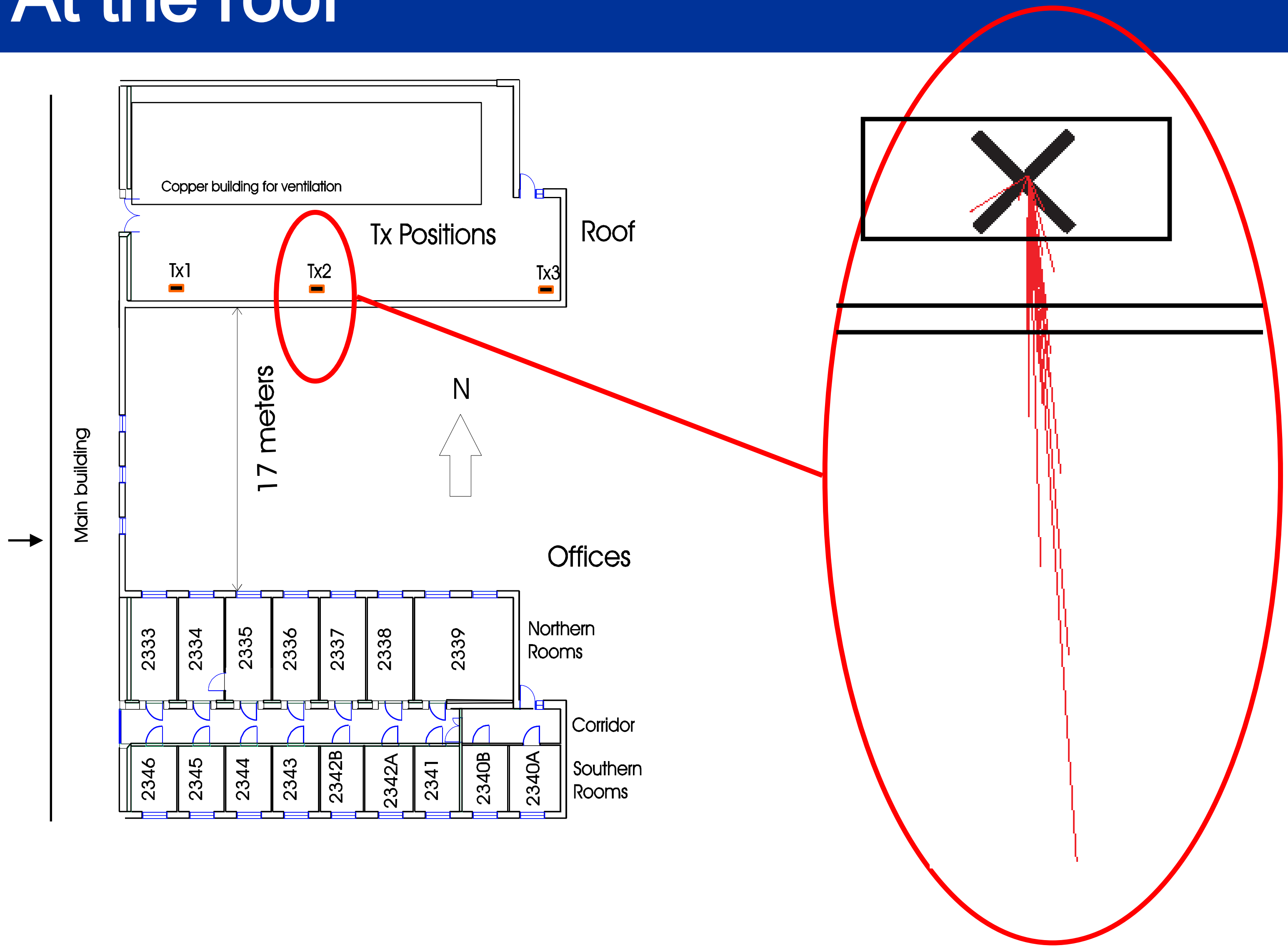
transmitter

receiver

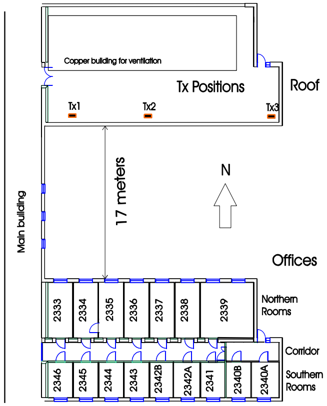
How does the signal reach the receiver In the office

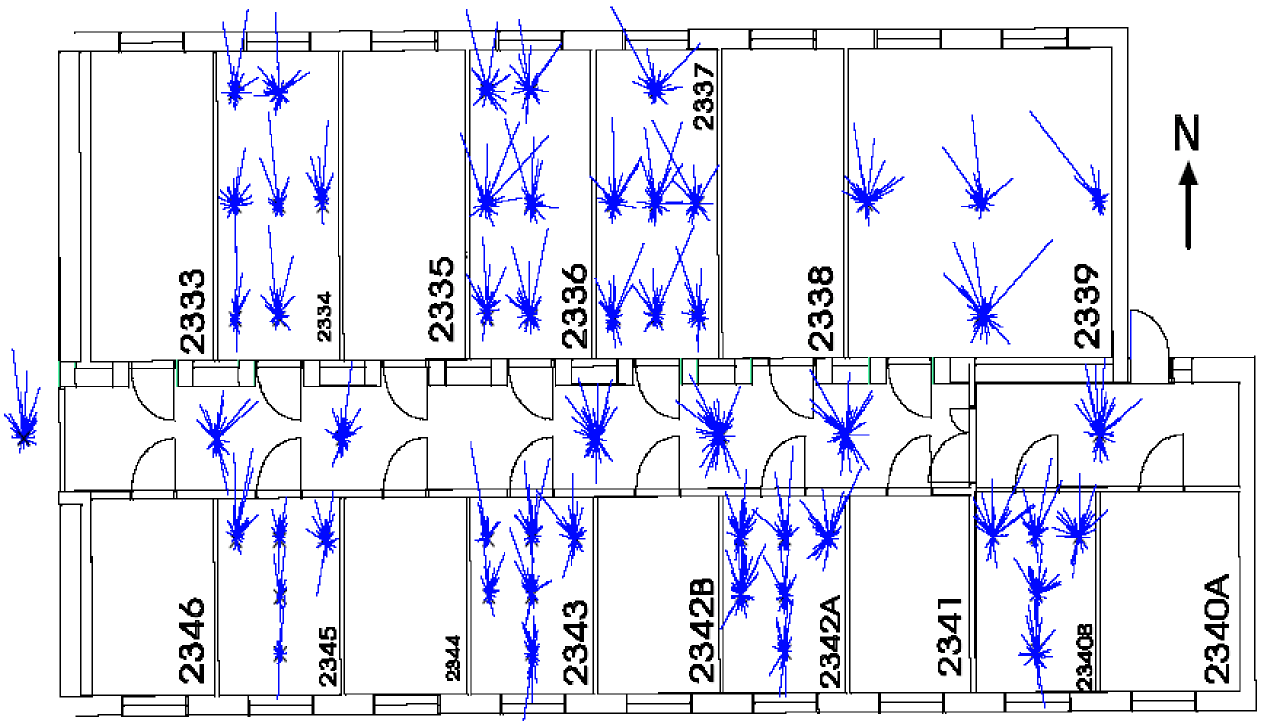


How does the signal leave the transmitter At the roof



In all offices





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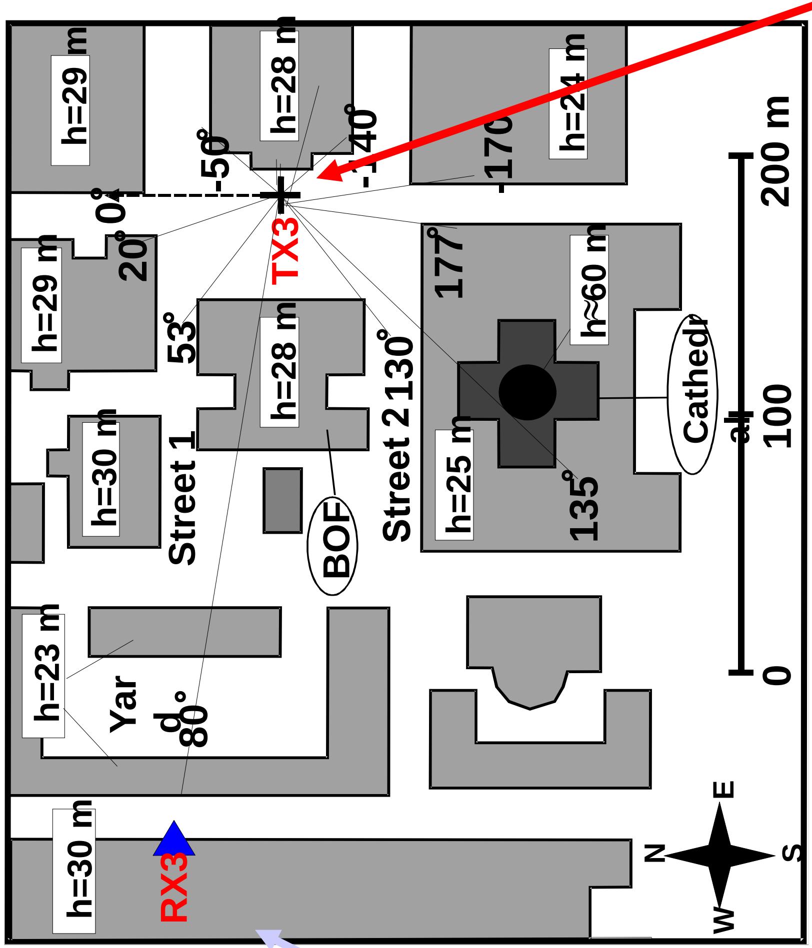
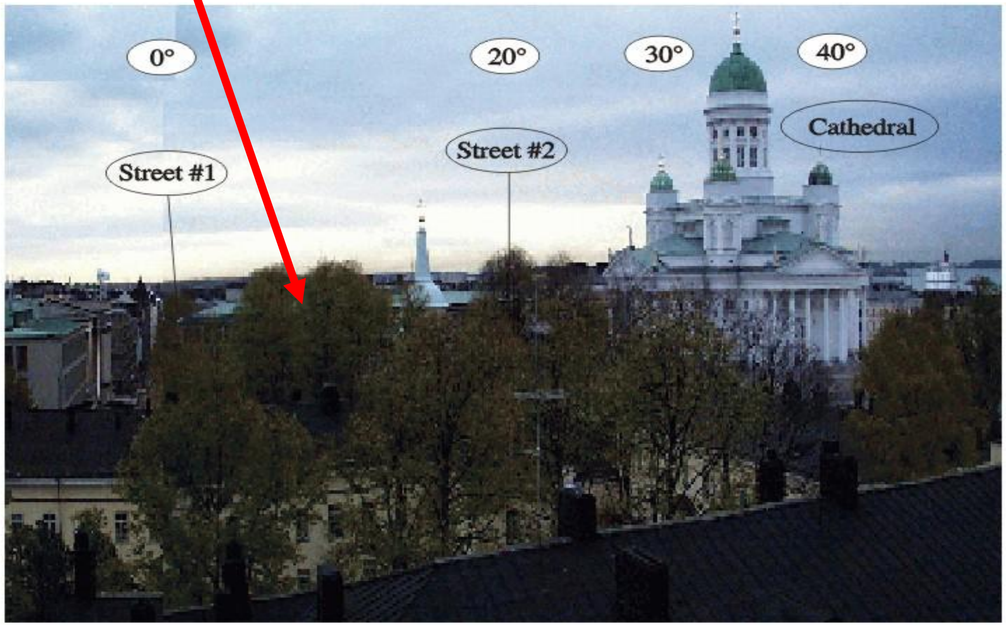
How does the signal reach the receiver outdoor urban

Transmitter





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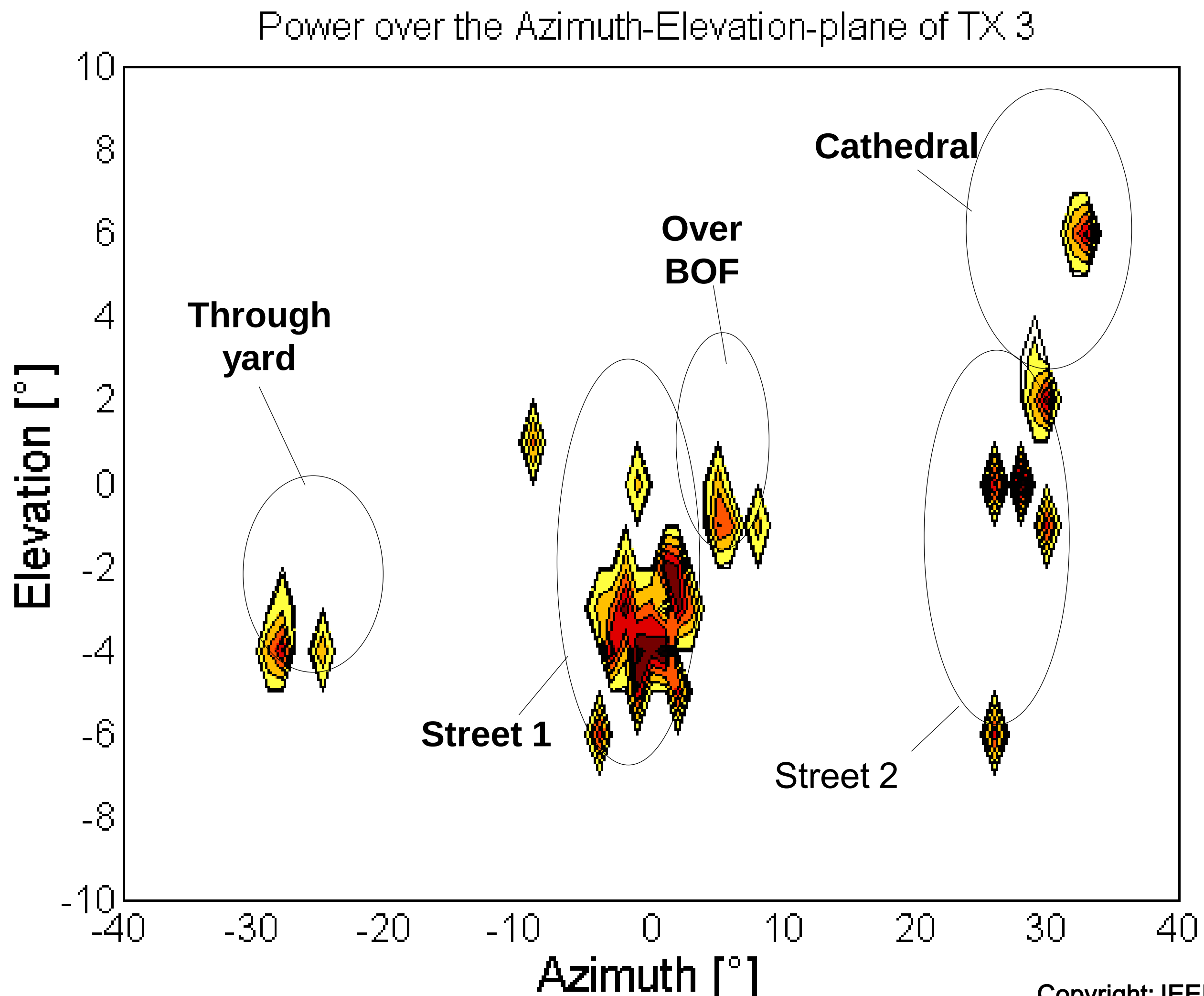
Signal arrives from some specific areas



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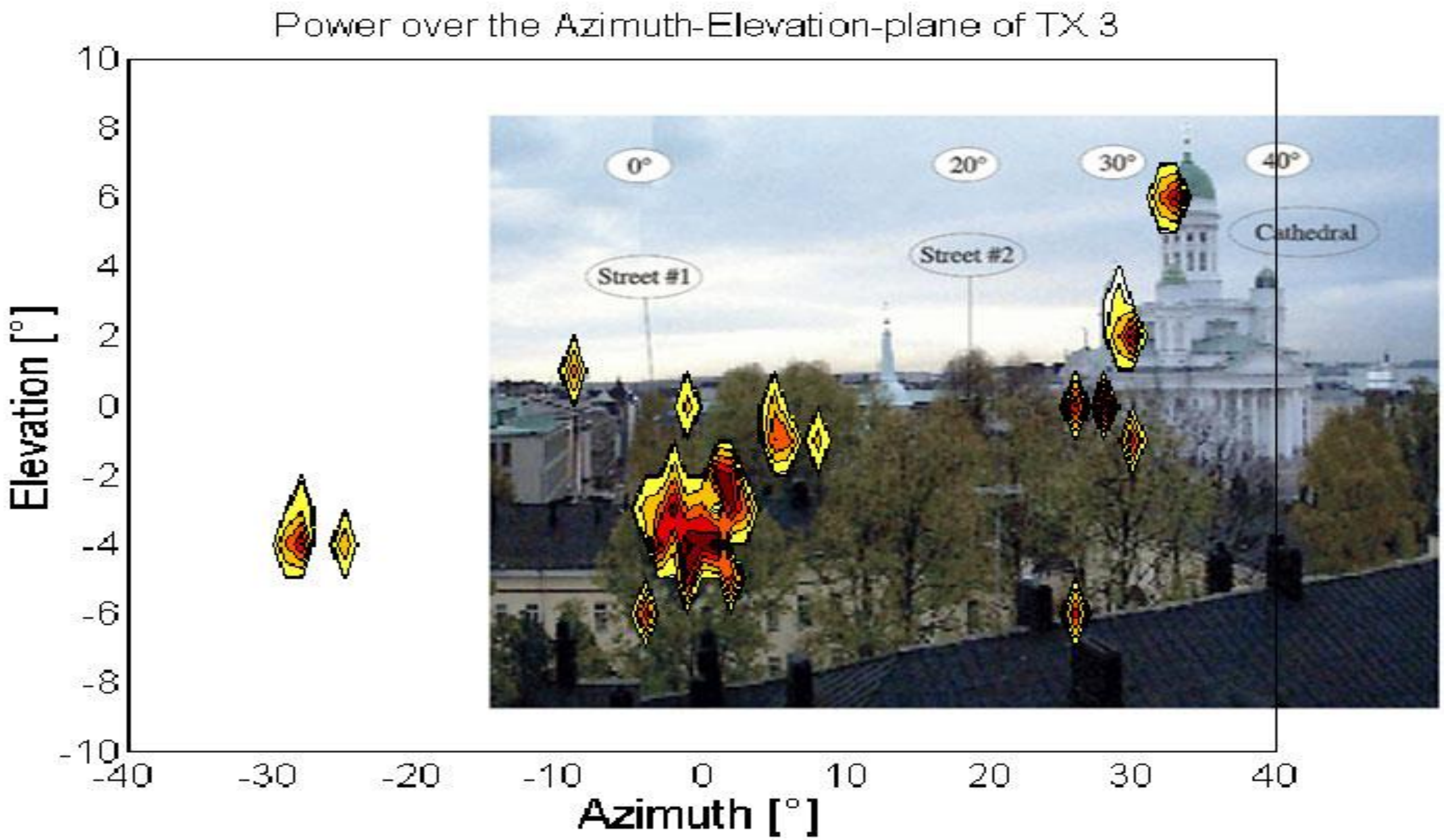
Diffraction, reflection, scattering, transmission



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