



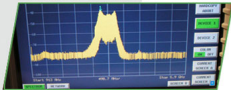
# EURECOM

S o p h i a   A n t i p o l i s

## Radio Engineering

### *Lecture 6: Channel Models*

Florian Kaltenberger



## 6 Multiple-Input Multiple-Output (MIMO) channels

- Definitions
- System model
- Mutual coupling and correlation
- Double directional channel characterization
- Angular power spectra

## 7 Channel Sounding

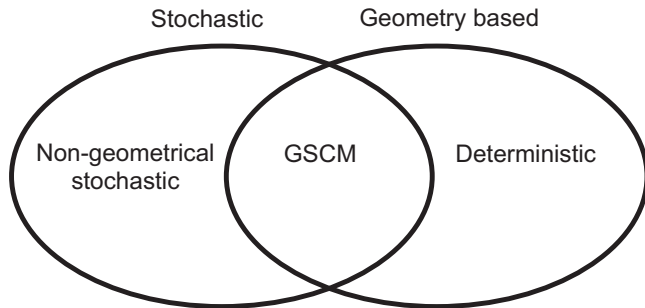
- Time and frequency domain sounding
- Directionally resolved measurements
- Parameter estimation methods

## 8 Channel Models

- Overview
- Stochastic models
- Geometry based models

## 9 Channel Simulation

- Sampled channels
- Correlation simulation
- Geometry based simulation



- Non-geometrical stochastic describe the statistics of the channel via power spectra or correlation function
- Geometry based stochastic channel models (GSCM) describe the environment (scatterer, etc.) in a stochastic way
- Deterministic channel models are either based on Maxwells equations or on stored impulse responses

# Narrowband models

## Review of properties

Narrowband models contain "only one" attenuation, which is modeled as a propagation loss, plus large- and small-scale fading.

Path loss: Often proportional to  $1/d^n$ , where  $n$  is the propagation exponent. ( $n$  may be different at different distances)

Large-scale fading: Log-normal distribution (normal distr. in dB scale)

Small-scale fading: Rayleigh, Rice, Nakagami distributions ... (not in dB-scale)

# Okumura's measurements

Extensive measurement campaign in Japan in the 1960's.

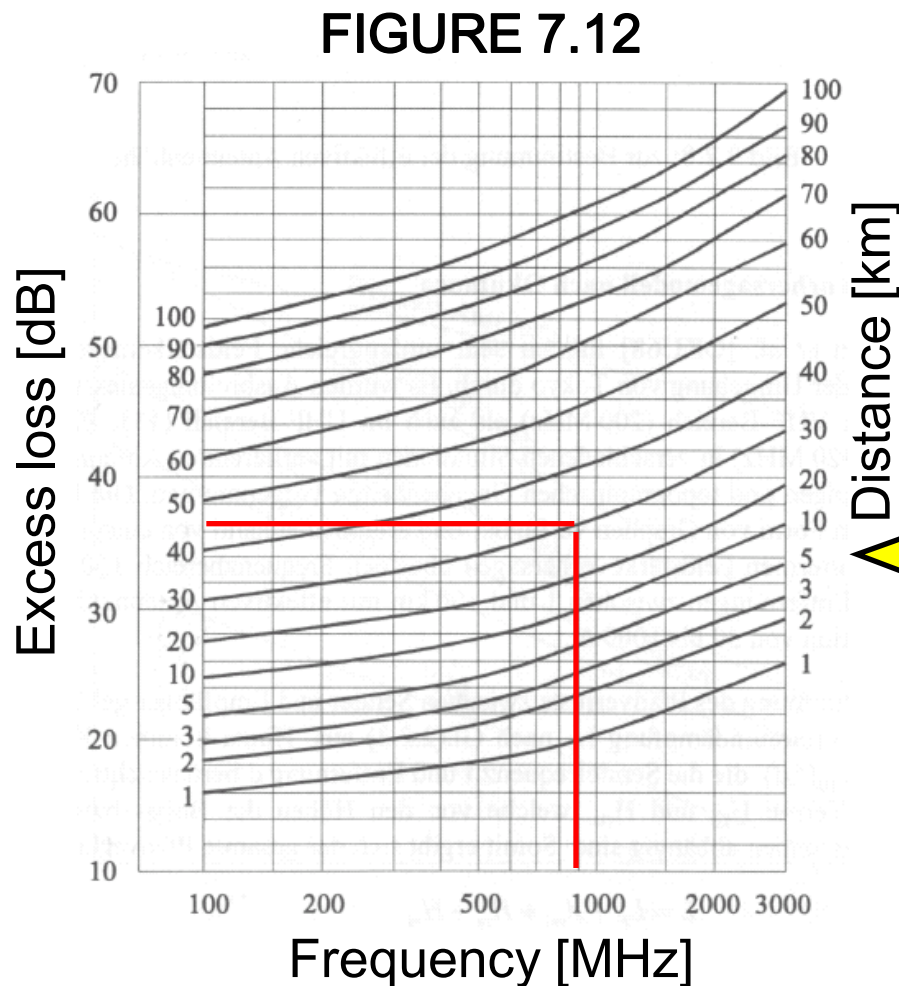
Parameters varied during measurements:

Frequency	100 – 3000 MHz
Distance	1 – 100 km
Mobile station height	1 – 10 m
Base station height	20 – 1000 m
Environment	medium-size city, large city, etc.

Propagation loss is given as **median** values (50% of the time and 50% of the area).

# Okumura's measurements excess loss

Example



These curves  
are only for  
 $h_b=200$  m and  
 $h_m=3$  m

900 MHz and  
30 km distance

From [Okumura et al.]

# The Okumura-Hata model

## How to calculate prop. loss

$$L_{O-H} = A + B \log(d_{|km}) + C$$

$h_b$  and  $h_m$   
in meter

$$A = 69.55 + 26.16 \log(f_{0|MHz}) - 13.82 \log(h_b) - a(h_m)$$

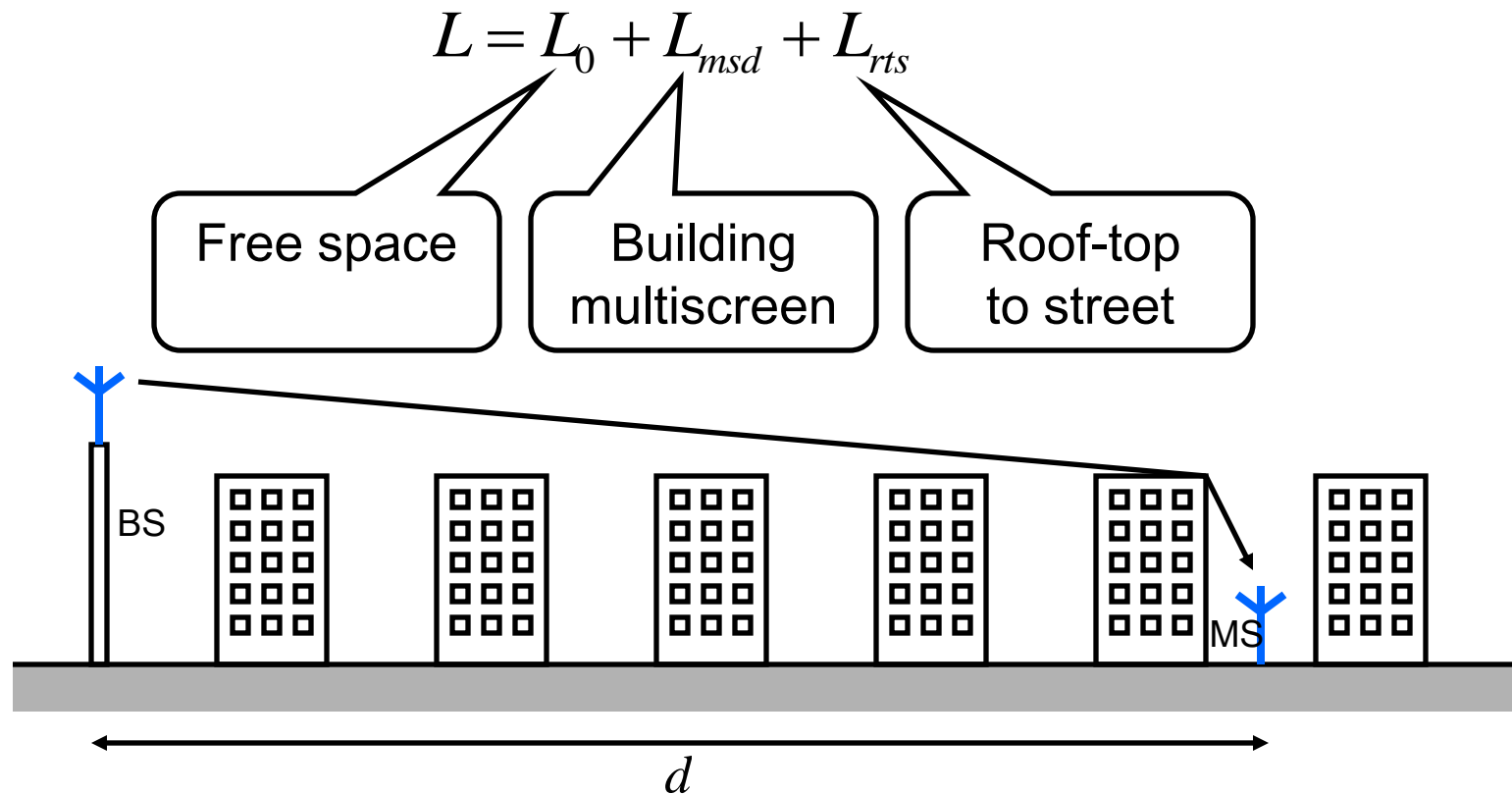
$$B = 44.9 - 6.55 \log(h_b)$$

	$a(h_m) =$	$C =$
Metropolitan areas	$8.29(\log(1.54h_m))^2 - 1.1$ for $f_0 \leq 200$ MHz $3.2(\log(11.75h_m))^2 - 4.97$ for $f_0 \geq 400$ MHz	0
Small/medium-size cities	$(1.1 \log(f_{0 MHz}) - 0.7)h_m -$ $(1.56 \log(f_{0 MHz}) - 0.8)$	0
Suburban environments		$-2[\log(f_{0 MHz} / 28)]^2 - 5.4$
Rural areas		$-4.78[\log(f_{0 MHz})]^2 + 18.33 \log(f_{0 MHz}) - 40.94$



# The COST 231-Walfish-Ikegami model

## How to calculate prop. loss



Details about calculations can be found in the textbook, Section 7.6.2.

# Motley-Keenan indoor model


For indoor environments, the attenuation is heavily affected by the building structure, walls and floors play an important rule

$$PL = PL_0 + 10n \log(d/d_0) + F_{\text{wall}} + F_{\text{floor}}$$

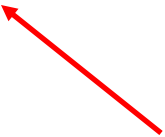
distance dependent  
path loss



sum of attenuations  
from walls, 1-20  
dB/wall



sum of attenuation from the  
floors (often larger than wall  
attenuation)



site specific, since it is valid for a particular case

# Wideband models

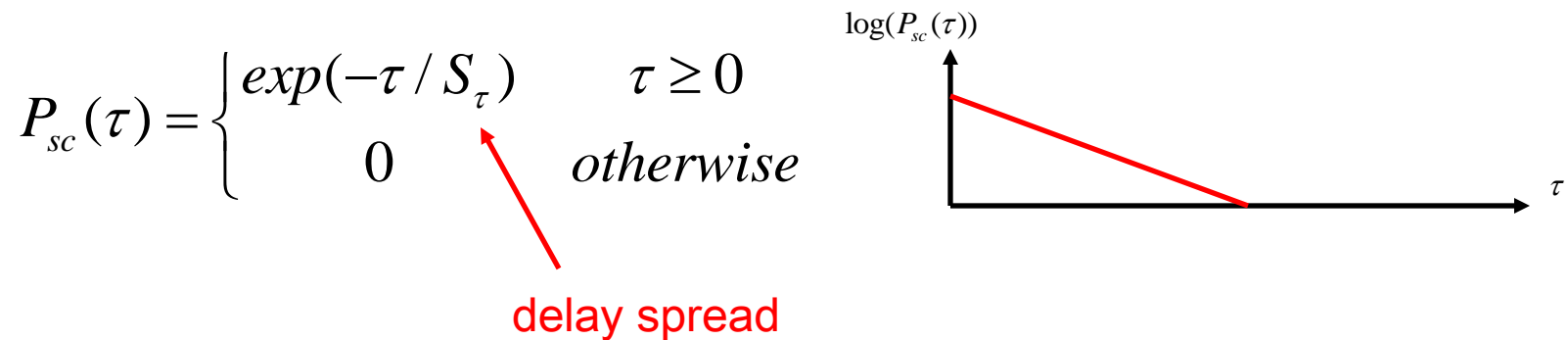
- Tapped delay line model often used

$$h(t, \tau) = \sum_{i=1}^N \alpha_i(t) \exp(j\theta_i(t)) \delta(\tau - \tau_i)$$

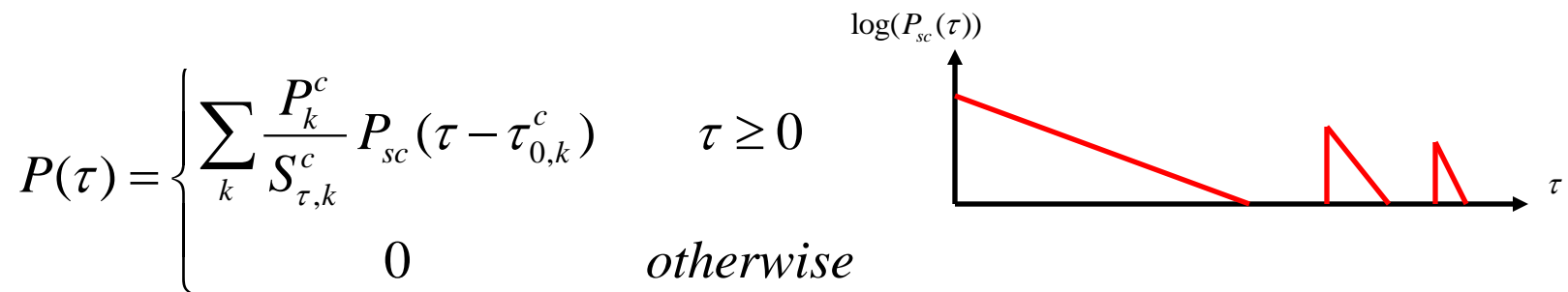
- Often Rayleigh-distributed taps, but might include LOS and different distributions of the tap values
- Mean tap power determined by the power delay profile

# Power delay profile

- Often described by a single exponential decay



- though often there is more than one “cluster”



# arrival time

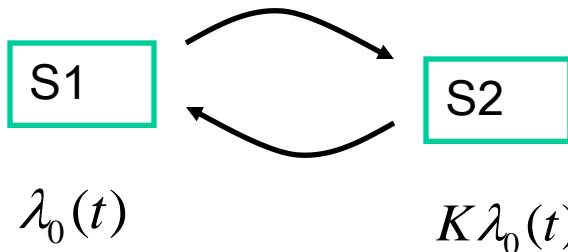
- If the bandwidth is high, the time resolution is large so we might resolve the different multipath components
- Need to model arrival time
- The Saleh-Valenzuela model:

$$h(\tau) = \sum_{l=0}^L \sum_{k=0}^K \alpha_{k,l}(\tau) \delta(\tau - T_l - \tau_{k,l})$$

ray arrival time (Poisson)

cluster arrival time (Poisson)

- The  $\Delta$ -K-model:



arrival rate:

$$\lambda_0(t)$$

$$K\lambda_0(t)$$

# Wideband models

## COST 207 model for GSM

The COST 207 model specifies:

FOUR power-delay profiles for different environments.

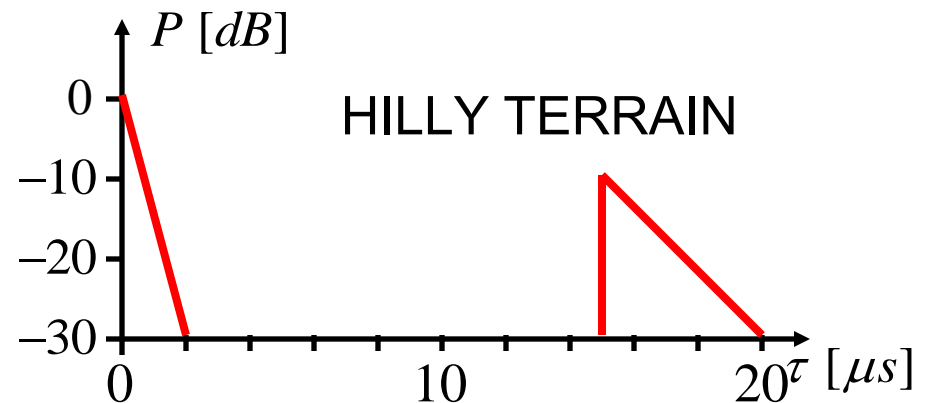
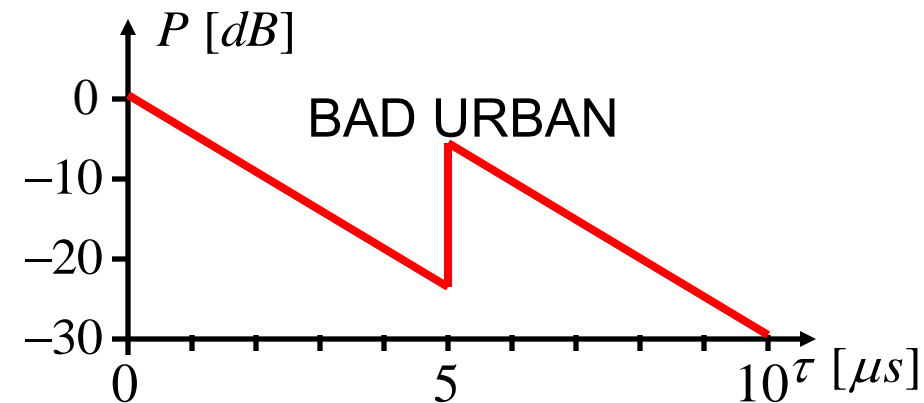
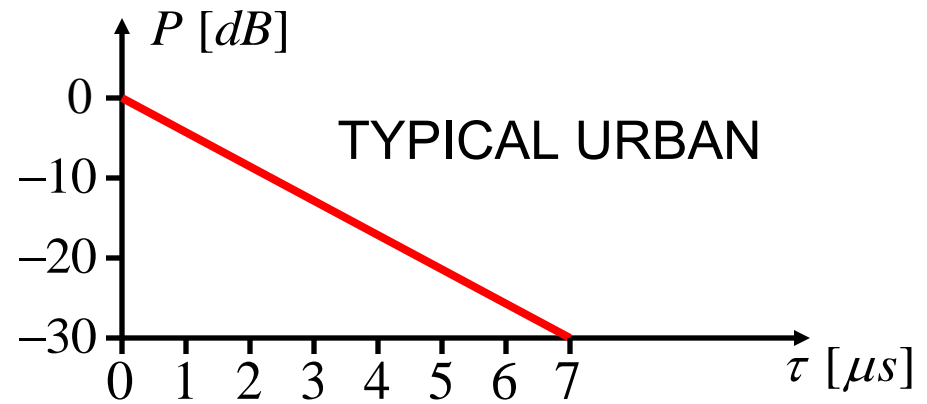
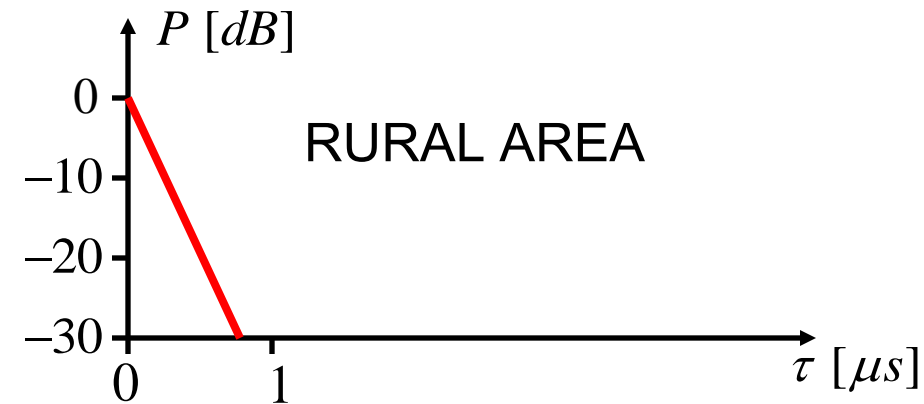
FOUR Doppler spectra used for different delays.

**IT DOES NOT SPECIFY PROPAGATION LOSSES FOR THE DIFFERENT ENVIRONMENTS!**

# Wideband models

## COST 207 model for GSM

Four specified power-delay profiles



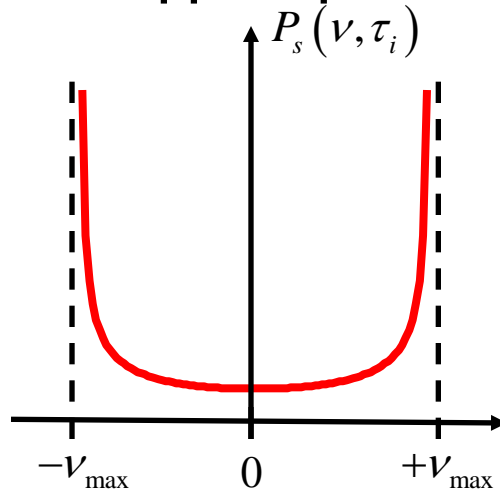
# Wideband models

## COST 207 model for GSM

### Four specified Doppler spectra

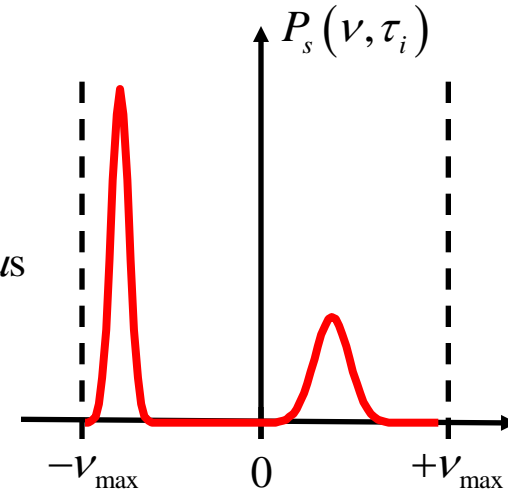
CLASS

$$\tau_i \leq 0.5 \mu\text{s}$$



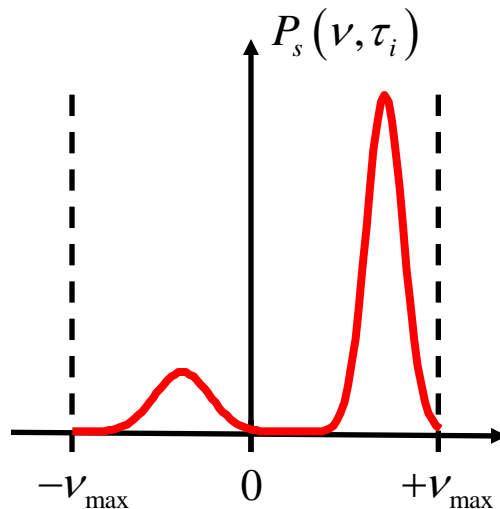
GAUS1

$$0.5 \mu\text{s} < \tau_i \leq 2 \mu\text{s}$$



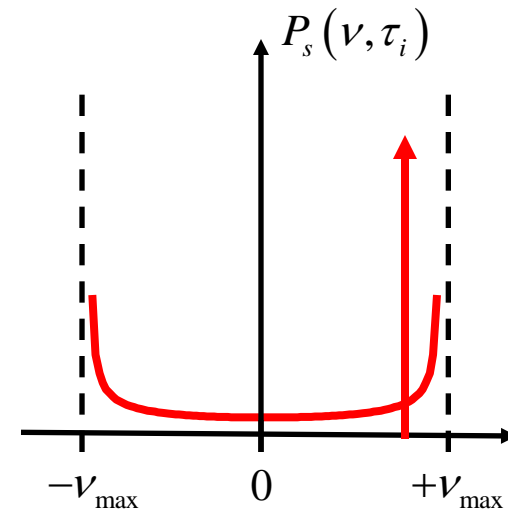
GAUS2

$$\tau_i > 2 \mu\text{s}$$



RICE

Shortest  
path in  
rural areas

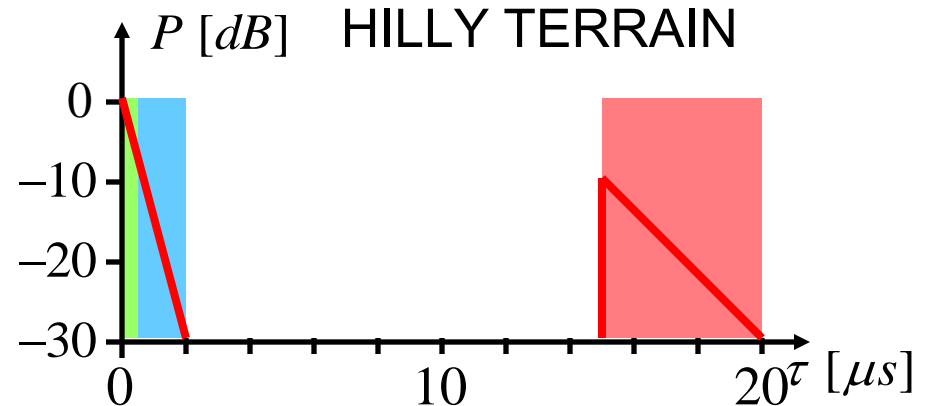
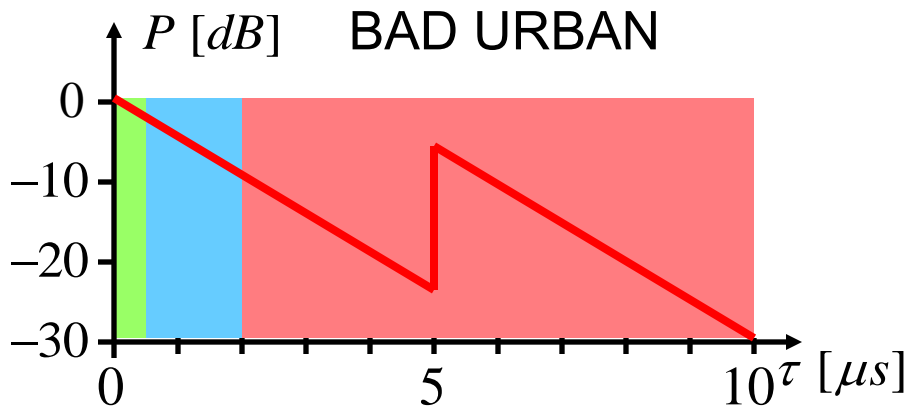
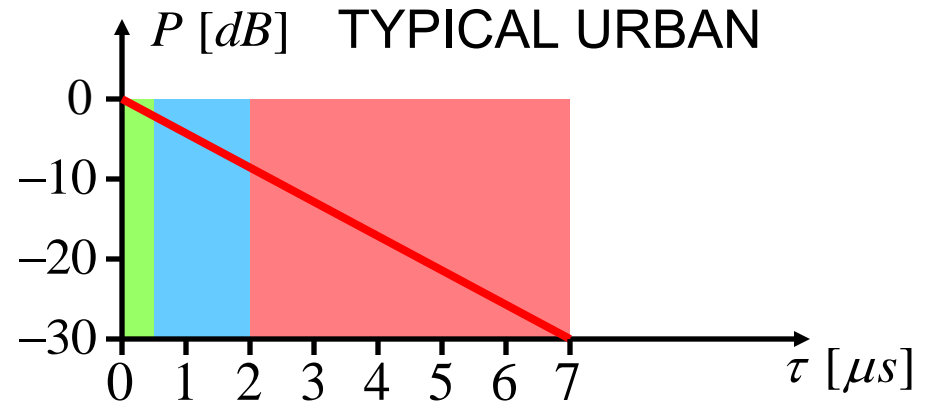
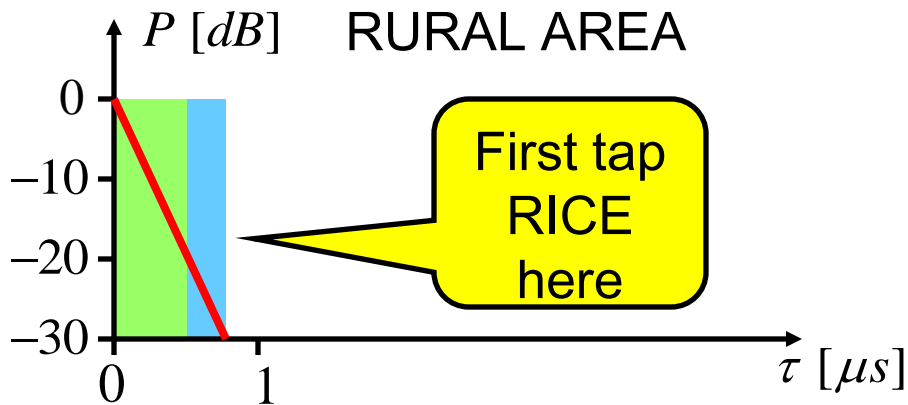




# Wideband models

## COST 207 model for GSM

Doppler spectra: CLASS GAUS1 GAUS2



# Wideband models

## ITU-R model for 3G

Tap No.	delay/ns	power/dB	delay/ns	power/dB
<b>INDOOR</b>	<b>CHANNEL A (50%)</b>		<b>CHANNEL B (45%)</b>	
1	0	0	0	0
2	50	-3	100	-3.6
3	110	-10	200	-7.2
4	170	-18	300	-10.8
5	290	-26	500	-18.0
6	310	-32	700	-25.2
<b>PEDESTRIAN</b>	<b>CHANNEL A (40%)</b>		<b>CHANNEL B (55%)</b>	
1	0	0	0	0
2	110	-9.7	200	-0.9
3	190	-19.2	800	-4.9
4	410	-22.8	1200	-8.0
5			2300	-7.8
6			3700	-23.9
<b>VEHICULAR</b>	<b>CHANNEL A (40%)</b>		<b>CHANNEL B (55%)</b>	
1	0	0	0	-2.5
2	310	-1	300	0
3	710	-9	8900	-12.8
4	1090	-10	12900	-10.0
5	1730	-15	17100	-25.2
6	2510	-20	20000	-16.0

- Stochastic MIMO models model the correlation matrix  $\mathbf{R}_h$
- Can be combined with models for wideband and time-variant channels
- Number of antennas and antenna geometry is predetermined
- Combined modeling of spatial correlation and mutual coupling
- Well suited for testing signal processing algorithms

- iid model (“canonical model”)

$$\mathbf{R}_h = \mathbf{I}$$

- Kronecker model

$$\mathbf{R}_h = \mathbf{R}_{Rx} \otimes \mathbf{R}_{Tx}$$

where  $\mathbf{R}_{Rx}$  and  $\mathbf{R}_{Tx}$  are the Rx and Tx correlation matrices

- Weichselberger model

$$\mathbf{R}_h = \sum_{i=1}^{N_{Tx}} \sum_{j=1}^{N_{Rx}} \omega_{ji} (\mathbf{u}_{Tx,i} \otimes \mathbf{u}_{Rx,j}) (\mathbf{u}_{Tx,i} \otimes \mathbf{u}_{Rx,j})^H$$

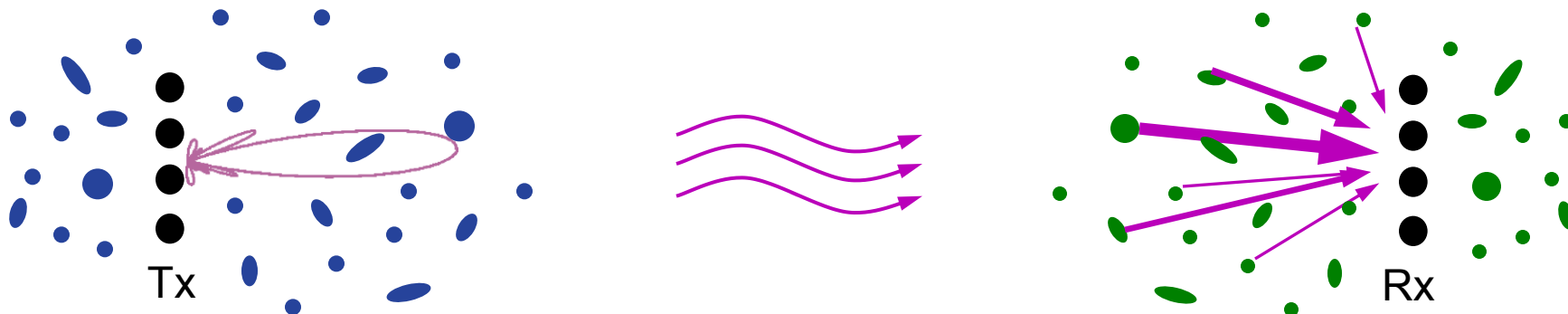
where  $\mathbf{u}_{Tx,i}$  and  $\mathbf{u}_{Rx,j}$  are the eigenvectors of the Tx and the Rx correlation matrices and  $\omega_{ji}$  are the elements of the coupling matrix

- Full-correlation model

- Simple but not realistic: assume that all elements of  $\mathbf{H}(t, \tau)$  are identically and independently distributed (i.i.d)
- Any channel model can be used for the elements (Rayleigh, Rician, etc.)
- Problems with this model:
  - ignores effects of correlation and mutual coupling
  - overestimates capacity ( $\text{rank}(\mathbf{H}) = \min(N_{\text{Tx}}, N_{\text{Rx}})$  with probability 1)
  - not verified by measurements

- Treats correlation independently at Tx and Rx
- Transmit correlation matrix:  $\mathbf{R}_{Tx} = \mathcal{E} \{ \mathbf{H}^H \mathbf{H} \}$
- Receive correlation matrix:  $\mathbf{R}_{Rx} = \mathcal{E} \{ \mathbf{H} \mathbf{H}^H \}$

# Kronecker model



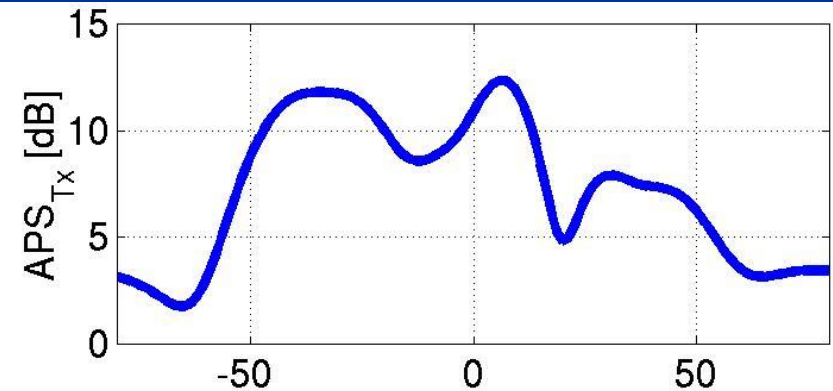
- The spatial structure of the MIMO channel is neglected.
- The MIMO channel is described by separated link ends:

$$\mathbf{R}_H = c \cdot \mathbf{R}_{\text{Tx}} \otimes \mathbf{R}_{\text{Rx}} \quad \mathbf{H} = \mathbf{R}_{\text{Rx}}^{1/2} \mathbf{G} \mathbf{R}_{\text{Tx}}^{T/2}$$

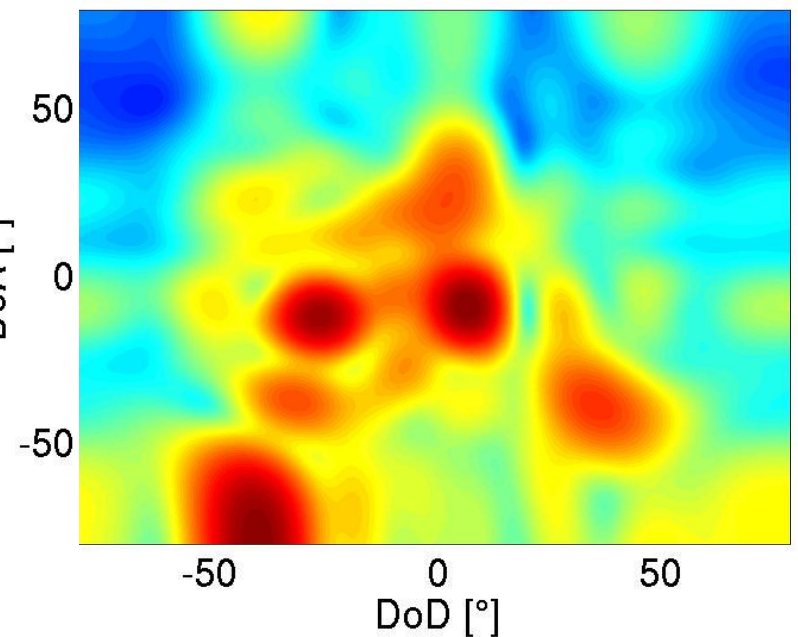
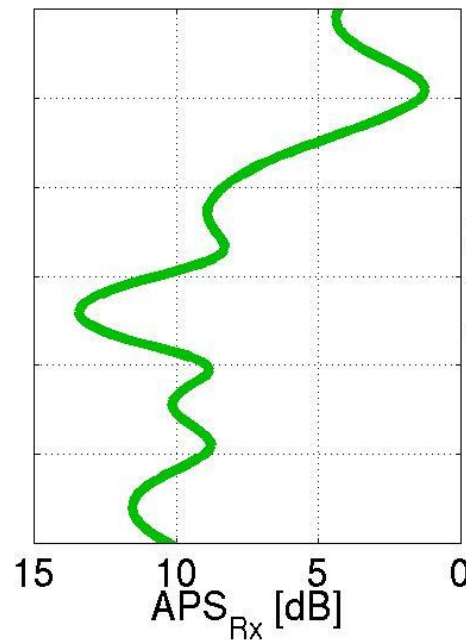
**Any** transmit signal results in one  
and the same receive correlation!

# Kronecker model (cont.)

Joint APS is the product of marginal Rx- and Tx-APS.



measurement

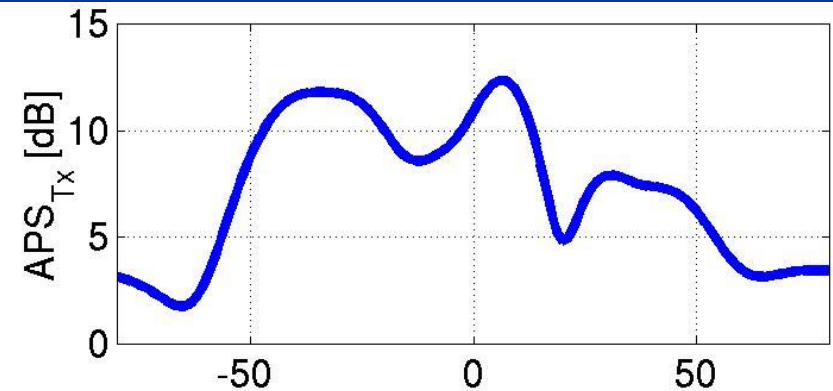


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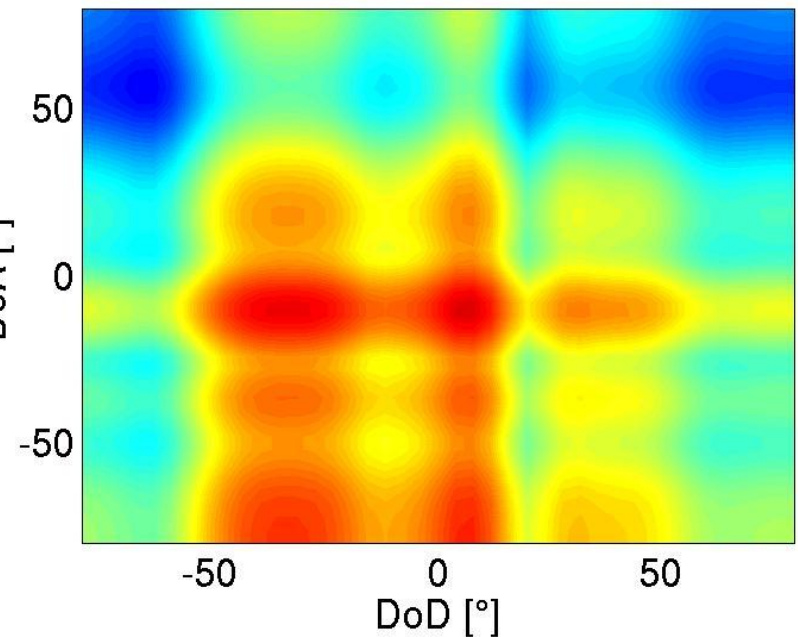
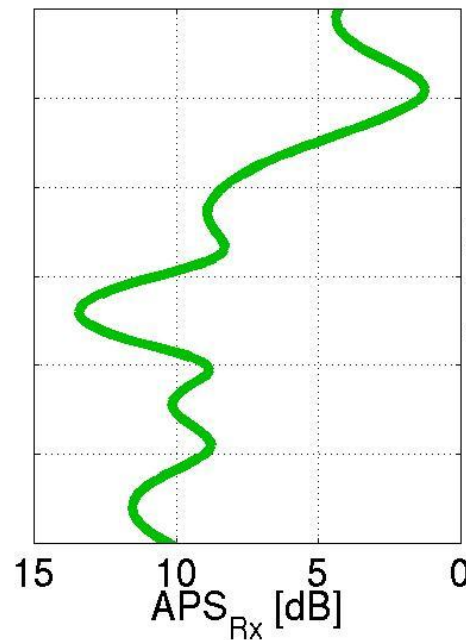


# Kronecker model (cont.)

Joint APS is the product of marginal Rx- and Tx-APS.



Kronecker approximation



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# Weichselberger Model – Definition

- Relaxes assumptions of Kronecker model by using **power coupling of Tx and Rx eigenmodes**, defined by

$$\mathbf{R}_{\text{Tx}} = \underbrace{\mathbf{U}_{\text{Tx}}}_{\text{Tx eigenmodes}} \mathbf{\Lambda}_{\text{Tx}} \mathbf{U}_{\text{Tx}}^H$$

$$\mathbf{R}_{\text{Rx}} = \underbrace{\mathbf{U}_{\text{Rx}}}_{\text{Rx eigenmodes}} \mathbf{\Lambda}_{\text{Rx}} \mathbf{U}_{\text{Rx}}^H$$

- Power coupling of eigenmodes is described by **coupling matrix**

$$\mathbf{\Omega}_{\text{WB}} = \mathbb{E} \left\{ \left( \mathbf{U}_{\text{Rx}}^H \mathbf{H} \mathbf{U}_{\text{Tx}}^* \right) \odot \left( \mathbf{U}_{\text{Rx}}^T \mathbf{H} \mathbf{U}_{\text{Tx}} \right) \right\}$$

- Channel correlation matrix** is modelled as

$$\mathbf{R}_{\text{h}} = \sum_{i=1}^{M_{\text{T}}} \sum_{j=1}^{M_{\text{R}}} \omega_{ji} (\mathbf{u}_{\text{Tx},i} \otimes \mathbf{u}_{\text{Rx},j}) (\mathbf{u}_{\text{Tx},i} \otimes \mathbf{u}_{\text{Rx},j})^H, \quad \text{with } \omega_{ji} = (\mathbf{\Omega}_{\text{WB}})_{j,i}$$

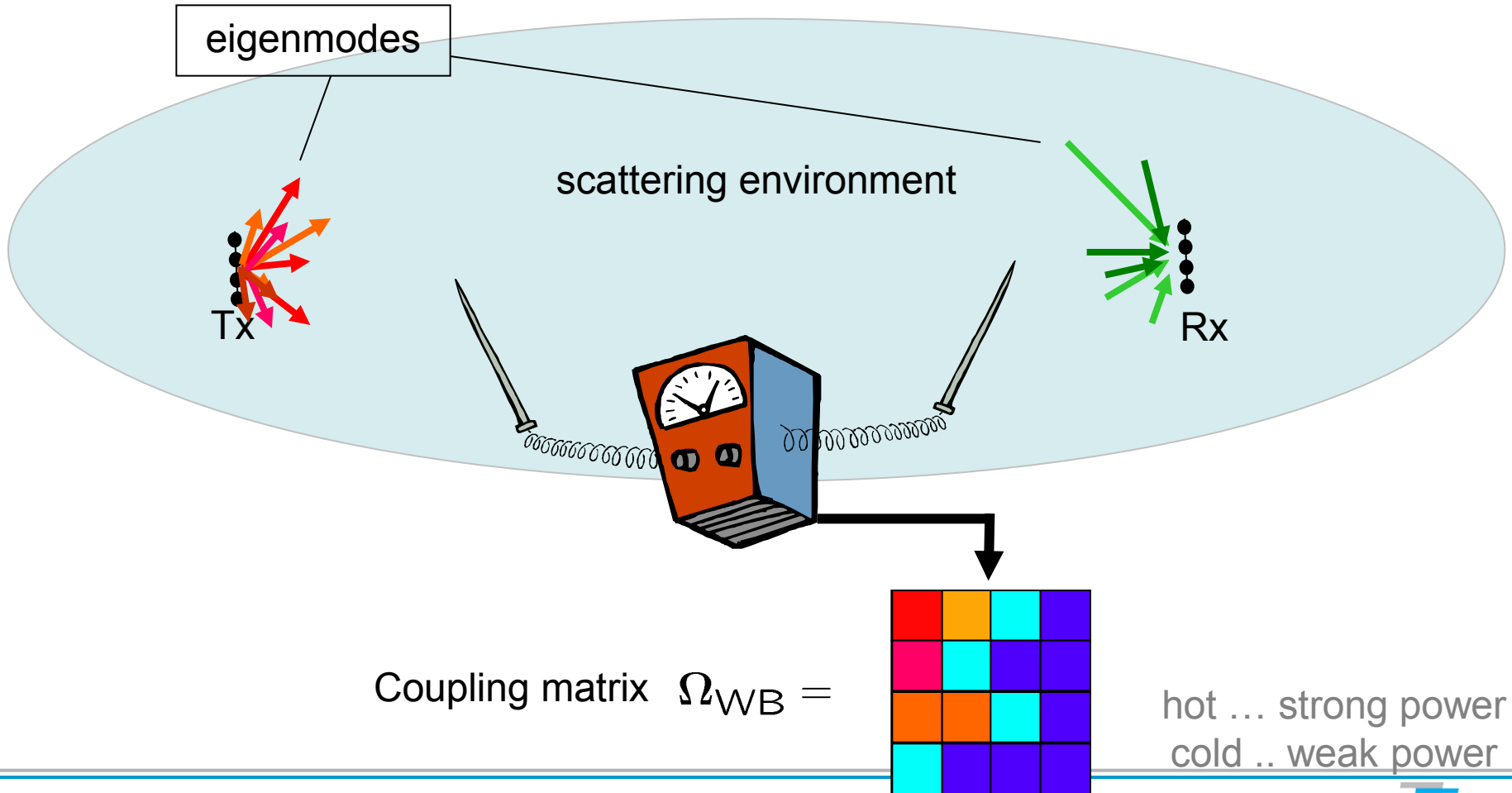
- Channel realizations** can be generated by

$$\mathbf{H} = \mathbf{U}_{\text{Rx}} (\tilde{\mathbf{\Omega}}_{\text{WB}} \odot \mathbf{G}) \mathbf{U}_{\text{Tx}}^T,$$

where  $\tilde{\mathbf{\Omega}}_{\text{WB}}$  is element-wise square-root of  $\mathbf{\Omega}_{\text{WB}}$ , and  $\mathbf{G} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$

# Weichselberger Model – Parameters

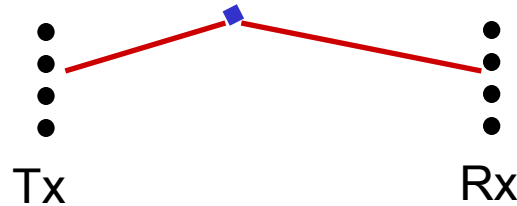
What are “eigenmodes” and the coupling matrix  $\Omega_{WB}$ ?



# Weichselberger Model – Coupling Matrix (1)

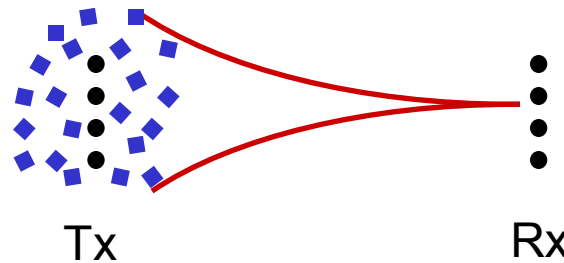
Structure of  $\Omega_{WB}$  strongly depends on the environment:

$$\Omega_{WB} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix}$$



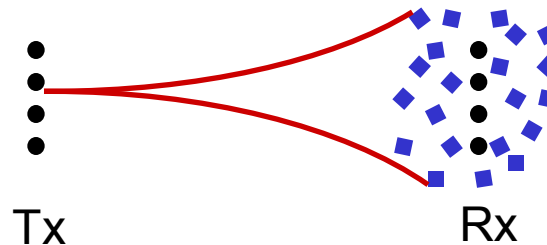
low-rank  
channel

$$\Omega_{WB} = \begin{bmatrix} \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix}$$



low rank,  
Tx diversity

$$\Omega_{WB} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \blacksquare & \square & \square & \square \end{bmatrix}$$

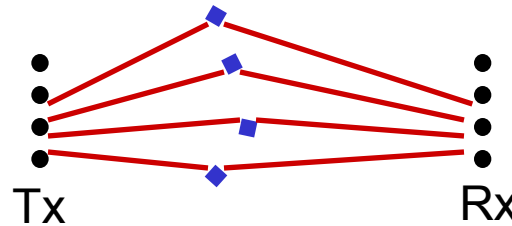


low rank,  
Rx diversity

# Weichselberger Model – Coupling Matrix (2)

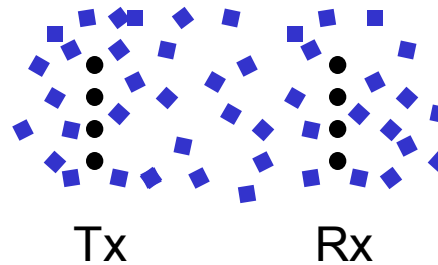
Structure of  $\Omega_{WB}$  strongly depends on the environment:

$$\Omega_{WB} = \begin{bmatrix} \text{orange} & \text{white} & \text{white} & \text{white} \\ \text{white} & \text{orange} & \text{white} & \text{white} \\ \text{white} & \text{white} & \text{orange} & \text{white} \\ \text{white} & \text{white} & \text{white} & \text{orange} \end{bmatrix}$$



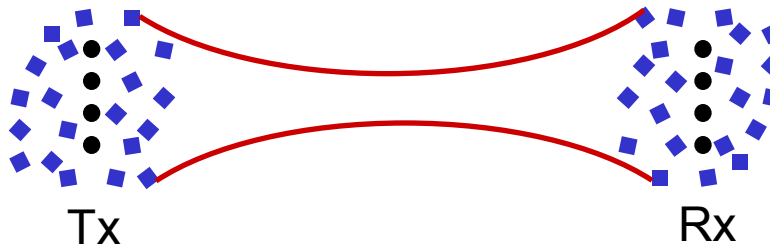
full rank,  
no diversity

$$\Omega_{WB} = \begin{bmatrix} \text{blue} & \text{blue} & \text{blue} & \text{blue} \\ \text{blue} & \text{blue} & \text{blue} & \text{blue} \\ \text{blue} & \text{blue} & \text{blue} & \text{blue} \\ \text{blue} & \text{blue} & \text{blue} & \text{blue} \end{bmatrix}$$



full rank,  
full diversity

$$\Omega_{WB} = \begin{bmatrix} \bullet & \text{cyan} & \text{cyan} & \text{cyan} & \text{cyan} \\ \text{cyan} & & & & \\ \text{cyan} & & & & \\ \text{cyan} & & & & \\ \text{cyan} & & & & \end{bmatrix}$$



full rank,  
Kronecker

- Full-correlation model
  - Very complex
  - Most accurate
- Weichselberger model
  - Good approximation
  - Good performance-complexity compromise
- Kronecker model
  - Separates channel into Tx and Rx sides
  - Very popular
  - Very limited validity
- iid model
  - Most simple
  - No physical relevance

- 3GPP specifies four simplified Spatial Channel Models (SCM) for link level evaluations
- See 3GPP TR 25.814 V7.1.0, Section 7.1.3

# Deterministic modeling methods

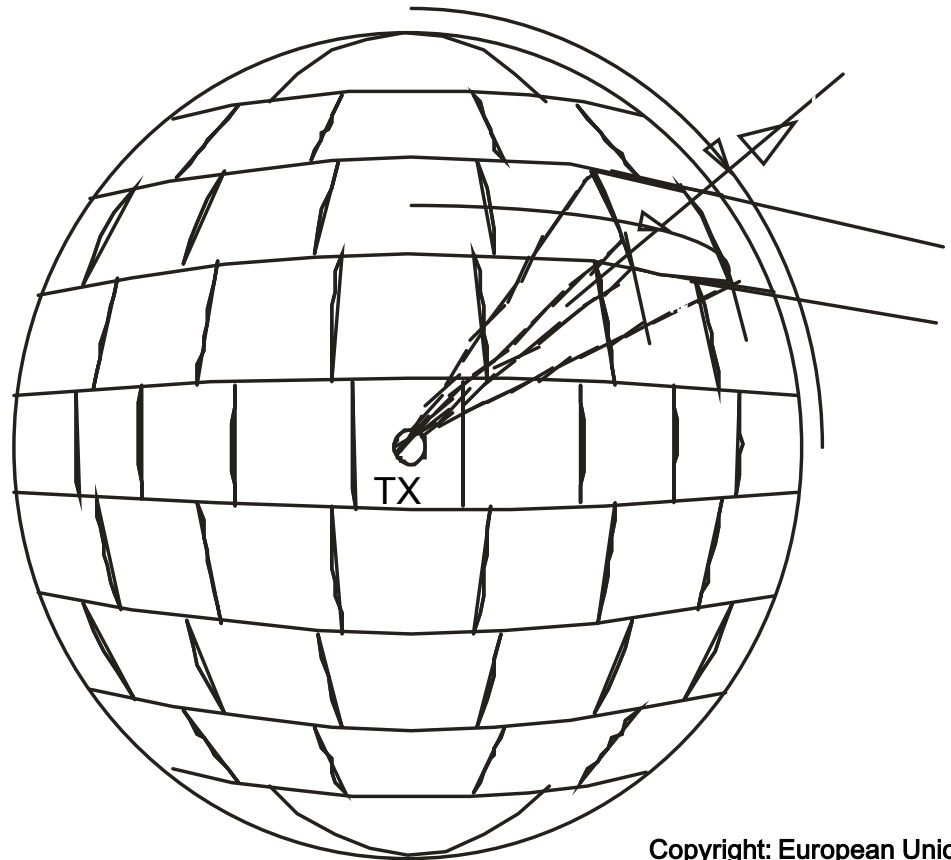
- Solve Maxwell's equations with boundary conditions
- Problems:
  - Data base for environment
  - Computation time
- “Exact” solutions
  - Method of moments
  - Finite element method
  - Finite-difference time domain (FDTD)
- High frequency approximation
  - All waves modeled as rays that behave as in geometrical optics
  - Refinements include approximation to diffraction, diffuse scattering, etc.



# Ray launching

Rays are launched from TX  
into discrete directions

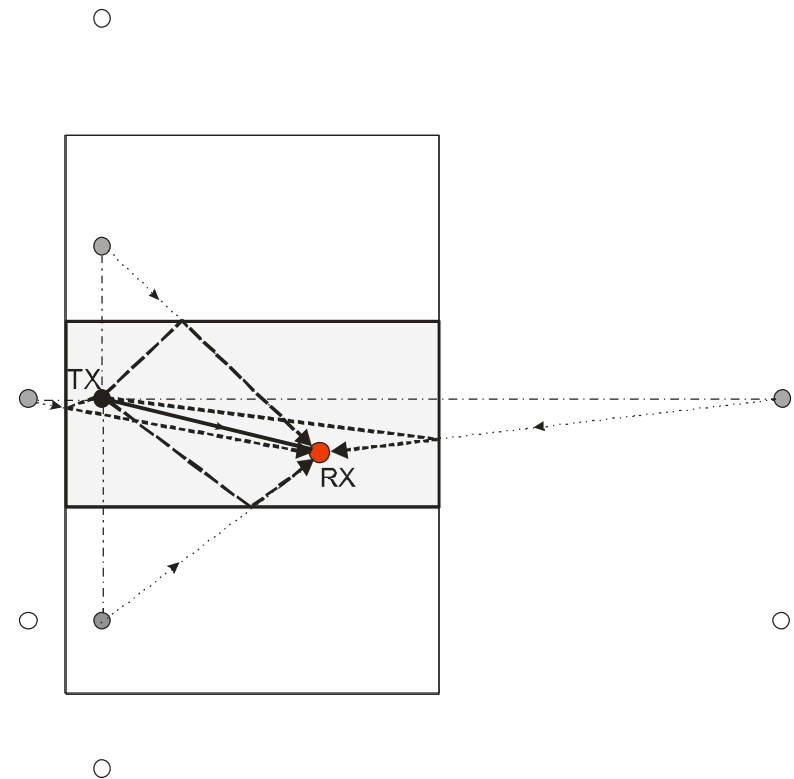
Rays are followed until their  
energy is below a threshold  
or if they get in the vicinity of  
the receiver.



Copyright: European Union

# Ray tracing

- Determines rays that can go from one TX position to one RX position
  - Uses imaging principle
  - Similar to techniques known from computer science
- Then determine attenuation of all those possible paths



- Geometry-based models use the theory of electromagnetic wave propagation (Maxwell equations) to characterize wireless channels.
- Solutions can be written as a sum of plane waves

$$h(t, f, \vec{x}, \vec{y}) = \sum_p \beta_p e^{2\pi j(\phi_p + \langle \vec{\zeta}_p, \vec{x} \rangle - \langle \vec{\xi}_p, \vec{y} \rangle - f\tau_p + t\omega_p)},$$

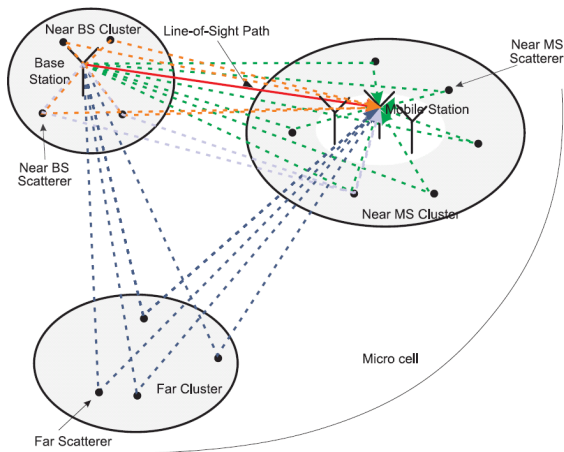
where

- $\beta_p$  and  $\phi_p$  are the amplitude and phase
- $\vec{\zeta}_p$  and  $\vec{\xi}_p$  are the Tx and Rx wavevectors
- $\tau_p$  and  $\omega_p$  are the delay and the Doppler shift

of path  $p$ .

- GSCM model the environment by placing clusters of scatterers in space
- Distribution of clusters and scatterers within clusters modeled stochastically
- Amplitude and phases of scatterers are modeled stochastically
- Use simplified ray tracing to rest of the path parameters
- Single bounce or multiple bounce

# Geometry Based Stochastic Models (GSCM)



## Advantages of GSCM

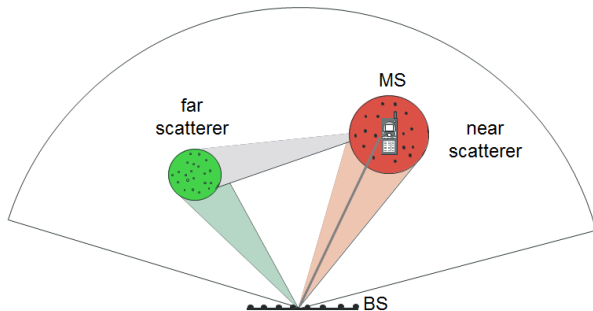
- Simpler than fully deterministic models
- More realistic than stochastic models
- Implicitly models mobility, MIMO, multi-user, etc
- Valid for regions

## Disadvantages of GSCM

- Very hard to parameterize
- Can still be computationally expensive

- For every GSCM statistics can be computed
- Some GSCMs and stochastic models are equivalent
- Example: One ring model
  - Scatterers uniformly distributed on a ring around Rx
  - Equivalent to Rayleigh fading
- More sophisticated models (including MIMO)
  - COST 259 and 273
  - WINNER models
  - 3GPP spatial channel model

- Single-bounce model, no scatterers around BS
- Fixed relationship between AOD, AOA, and delay
- Well suited for smart antenna systems, but not MIMO



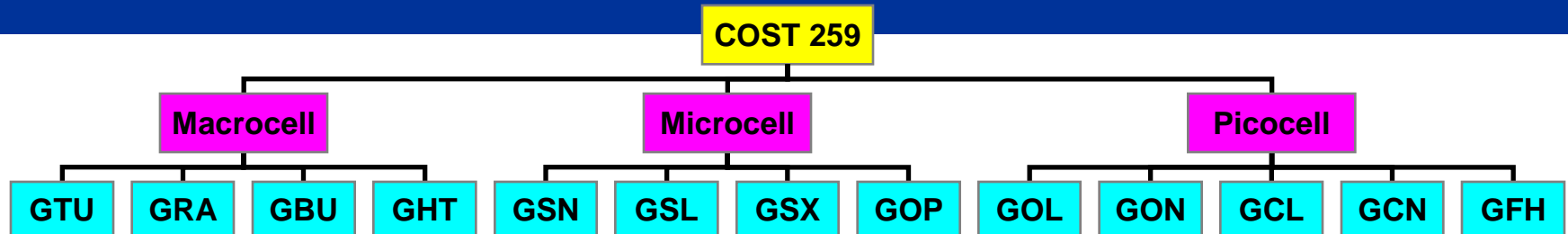
AOA: angle of arrival; AOD: angle of departure; BS/MS: base/mobile station



# COST 259 DCM - Philosophy

- Parametric approach, WSSUS not required
- No statement about implementation method (stochastic or GSCM)
- Based on clustering approach
- Multi-layer approach:
  - Radio environments
  - Large-scale effects
  - Small-scale effects

# Radio environments



GTU Generalized Typical Urban

GRA Generalized Rural Area

GBU Generalized Bad Urban

GHT Generalized Hilly Terrain

GSN Generalized Street NLOS

GSL Generalized Street Canyon LOS

GSX Generalized Street Crossing

GOP Generalized Open Place

GOL Generalized Office LOS

GON Generalized Office NLOS

GCL Generalized Corridor LOS

GCN Generalized Corridor NLOS

GFH Generalized Factory Hall



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# COST 259 DCM - Simulation procedure

Simulation steps:

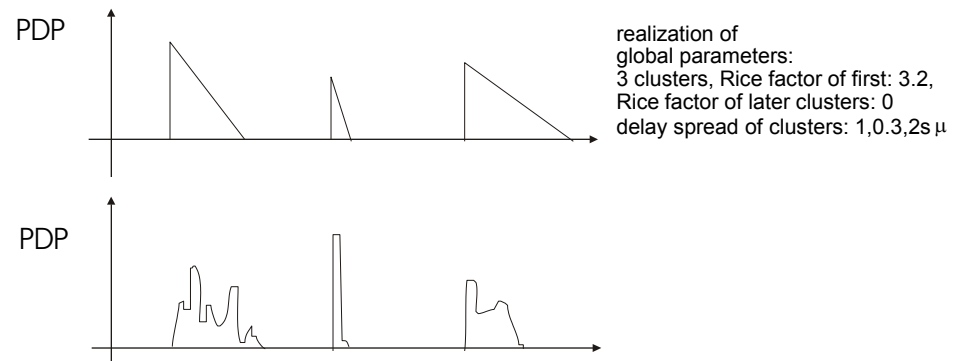
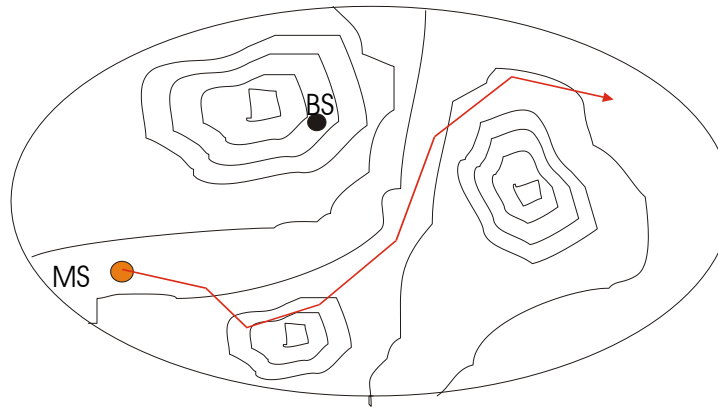
- 1) select scenario
- 2) select global parameters  
(number of clusters,  
mean Rice factor,....)

### 3) REPEAT

compute one realization of global parameters. This realization prescribes small-scale averaged power profiles (ADPS)

create many instantaneous complex impulse responses from this average ADPS

## Generalized Hilly Terrain (GHT)

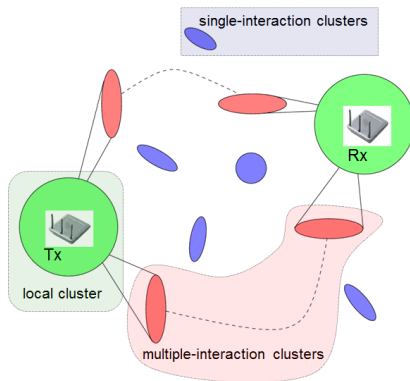


# COST 259 DCM - Important features

- **Very realistic !**
- Distinguishes 13 different radio environments
- Treats large-scale and small-scale variations
- Far scatterer clusters included, with birth/death process
- Delay spread and angular spread treated as (correlated) random variables
- Angular spectra are functions of delay
- Azimuth and elevation

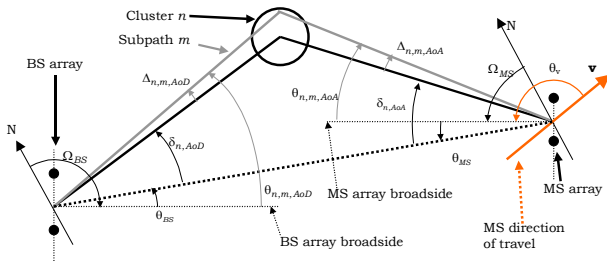
# The COST 273 Approach (1)

- Model based on clusters
- 3 different cluster types
- Clusters are placed geometrically and stochastically



- Local clusters around MS and/or BS may occur, depending on the scenario
- Any combination of delay and angles can be modelled, not limited to double scattering
- All parameters are given per cluster; there are no global spreads
- Direct coupling between AOAs and AODs; no “Kronecker” structure

- 3 different scenarios:
  - Suburban Macro,
  - Urban Macro,
  - Urban Micro
- Otherwise similar to COST 259
- Originally for 5MHz bandwidth, extensions to 100MHz exist



- Based on the outcomes of the WINNER project
- 13 different scenarios (also indoor)
- Introduces cross-correlation between large-scale parameters



- Sampled channels
- Correlation-based simulation
- Geometry based simulation

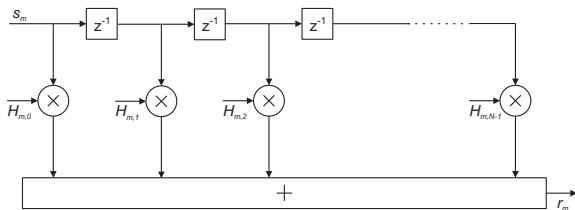
- Only sampled channels can be simulated ( $T_S$  is the sampling rate):

$$h[m, n] = h(mT_S, nT_S)$$

- Input-output relation

$$r[m] = \sum_{n=0}^{N-1} h[m, n]s[m - n] + n[m]$$

- Can be implemented as a tapped delay line



- Special case: narrowband channel:  $h[m, n] = 0$  for  $n \neq 0$
- Special case: static channel  $h[m, n] = h[n]$

- We wish to generate samples of a WSS process  $h[m]$  with autocorrelation function (ACF)

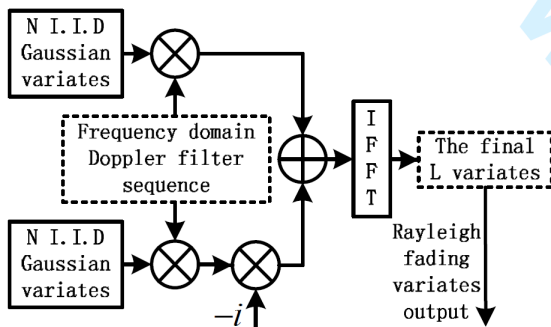
$$R_h(m - m') = \mathcal{E}\{h[m]h^*[m']\}$$

or equivalently with power spectrum density (PSD)

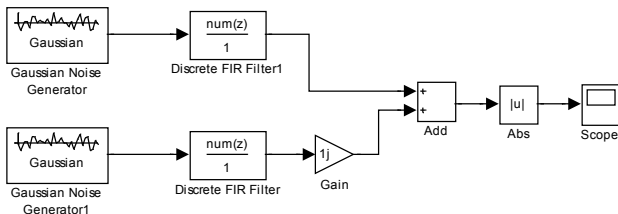
$$S_h(\nu) = \text{DFT}\{R_h(\Delta m)\}$$

- Two main methods exist
  - Frequency-domain filtering
  - Time-domain filtering
  - Sum-of-sinusoids

- First generate  $N$  i.i.d. Gaussian random variates (real and imaginary)
- Shape them with a frequency domain filter corresponding to the desired PSD
- Apply an IFFT



- First generate i.i.d. Gaussian random variates (real and imaginary)
- Pass them through a time-domain filter corresponding to the desired ACF
- Advantage: non-block based, no discontinuities



- The Doppler fading process is usually highly oversampled.
- Example: Sampling rate 7.68MHz, max. Doppler 500Hz
- For the frequency-domain method
  - ⇒ Only a small part of the PSD is non-zero
  - ⇒ large number of samples  $N$  required for accuracy
  - ⇒ Large IFFT has high memory and complexity requirements
- For the time-domain method
  - ⇒ A large number of filter coefficients required
  - ⇒ High complexity
- Both methods can be improved by generating a correlated process with a lower sampling rate and then using interpolation

- We wish to generate an US process  $h[n]$ , with a certain power delay profile (PDP = PSD)
- Same problem as before, but simpler since
  - Samples can be generated directly in the delay domain ( $n$ )
  - Process is sampled at lower rate
- However, interpolation might be necessary if the delays of taps are not multiples of the sampling rate



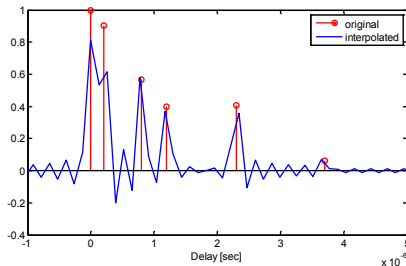
# Example: sinc interpolation

- Impulse response

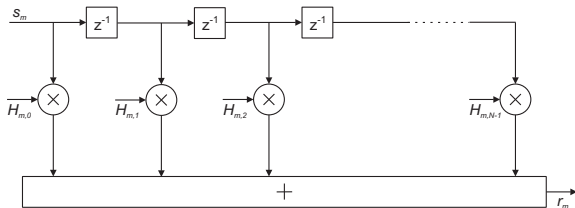
$$h(\tau) = \sum_{p=0}^{P-1} a_p \delta(\tau - \tau_p)$$

- Sampled at rate  $T_S$ :

$$h[n] = \sum_{p=0}^{P-1} a_p \operatorname{sinc}\left(\tau - \frac{\tau_p}{T_S}\right)$$



- Each tap of the tapped delay line is a time-correlated sequences multiplied with the weight of the tap



- Example: 3GPP channel model in Matlab

- We wish to generate a MIMO channel  $\mathbf{H}$  with a certain correlation matrix  $\mathbf{R} = \mathcal{E} \{ \text{vec}(\mathbf{H}) \text{vec}(\mathbf{H})^H \}$
- Since the number of correlated samples is finite and small, we can use the direct method
- Let  $\mathbf{R}^{\frac{1}{2}}$  be the matrix square root of  $\mathbf{R}$  (can be computed with the Cholesky factorization) and  $\mathbf{G}$  a matrix of i.i.d. Gaussian random variates. Then

$$\mathbf{H} = \text{unvec} \left( \mathbf{R}^{\frac{1}{2}} \text{vec}(\mathbf{G}) \right)$$

## Simplified MIMO models

- Weichselberger model

$$\mathbf{H} = \mathbf{U}_{Rx}(\mathbf{\Omega} \odot \mathbf{G})\mathbf{U}_{Tx}^T$$

- Kronecker model

$$\mathbf{H} = \mathbf{R}_{Rx}^{\frac{1}{2}} \mathbf{G} (\mathbf{R}_{Tx}^{\frac{1}{2}})^T$$

- i.i.d. model

$$\mathbf{H} = \mathbf{G}$$

- Based on and used for geometry based model

$$h(t, f, \vec{x}, \vec{y}) = \sum_p \beta_p e^{2\pi j(\phi_p + \langle \vec{\zeta}_p, \vec{x} \rangle - \langle \vec{\xi}_p, \vec{y} \rangle - f\tau_p + t\omega_p)},$$

- Parameters for each path are either taken from a random distribution or from geometrical calculations
- In general more realistic (especially for MIMO) but also the most computationally complex

- For a narrowband SISO channel

$$h_m = \sum_p \beta_p e^{2\pi j(\phi_p + m\nu_p)},$$

where  $\nu_p = \omega_p T_S$  is the normalized Doppler shift of path  $p$

- If
  - $\beta_p = 1/\sqrt{P}$ ,
  - $\nu_p = \nu_{\max} \cos \psi_p$  where  $\nu_{\max}$  is the maximum Doppler shift and  $\psi_p$  is the AoA of path  $p$
  - $\phi_p$  and  $\psi_p$  are mutually independent and uniformly distributed in  $[-\pi, \pi)$
- Then, as  $P \rightarrow \infty$ , the spectrum of  $h_m$  approaches

$$S_h(\nu) = \frac{1}{\pi \nu_{\max} \sqrt{1 - \left(\frac{\nu}{\nu_{\max}}\right)^2}}$$