Institut Eurécom
Digital Communications

Midterm Examination

Date: Dec. 14, 2006

Duration: 2 hours

Answer 3 out of 4 questions. All questions will be graded equally, All documents are allowed.

1 Problem 1

Consider the set of signals in Figure 1.

1. Find an orthonormal basis for the signal set.
2. Find α,β, and γ such that the signals are all equal-energy E_s
3. Draw the constellation which represents the signal set in terms of the basis that you found
4. What is the information-rate in bits per symbol?

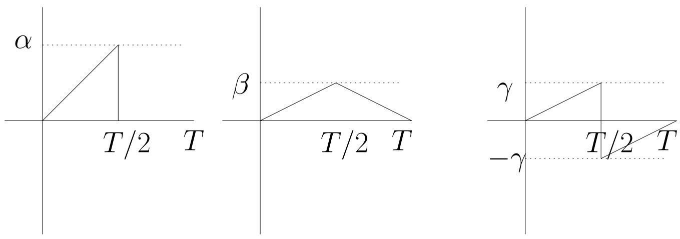

Figure 1: Signal Set for Problem 1

2 Problem 2

Consider the 5-point QAM constellation in Figure 2.

1. If all symbols are transmitted equally often and are statistically independent, what should the value of E be such that the average symbol energy is E_s.
2. What is the minimum squared Euclidean distance of the constellation in terms of E_s
3. Give an upper-bound on the probablility of symbol-error as a function of E_b/N_0 assuming that transmission is carried out on an circularly-symmetric additive white Gaussian noise channel with variance N_0.

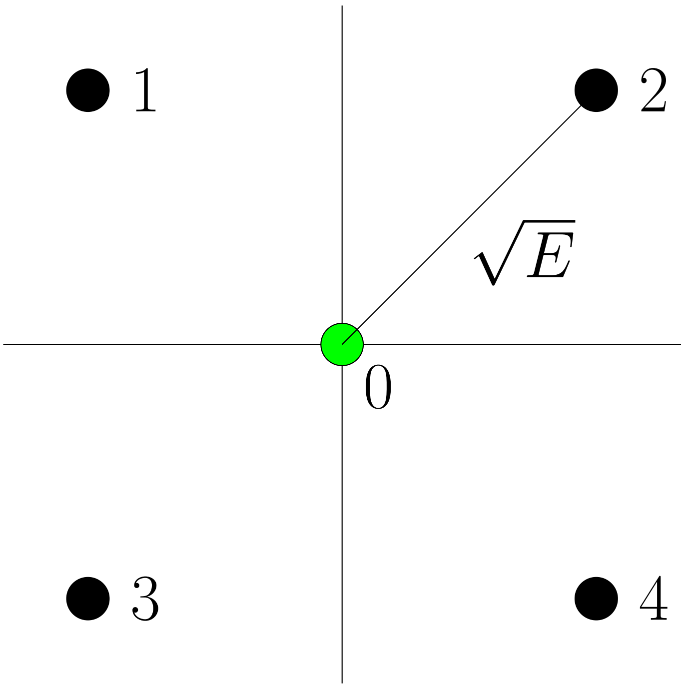

Figure 2: Constellation for Problem 2

3 Problem 3

Consider the contellation in Figure 3. Here this is two superimposed QPSK constellations with energy E_1 and E_2 respectively. Suppose that this constellation is used to transmit two separate data streams x_1 (large font) and x_2 (small font), so that

$$y = x_1 + x_2 + z \tag{1}$$

where $\mathrm{E}(|x_i|^2) = E_i$, each x_i is a QPSK symbol and z is complex, circularly-symmetric, additive white Gaussian noise with variance N_0. We would like to determine the detection rules and error probabilities for detection of the two streams. This is known as unequal error protection. Define $E = E_1 + E_2$, and suppose $E_1 > E_2$. For receivers with high signal to noise ratios E/N_0 we attempt to detect both streams, although independently. For receivers with low signal-to-noise ratios, we detect only x_1, since we imagine that the probability of error for detecting x_2 will be too large.

1. Write the maximum-likelihood detectors for x_1 and x_2, assuming that x_1 is detected first with minimum error-probability (i.e. in the presense of x_2 as additive noise).
2. Show that the error probability for detecting x_2 can be lower-bounded as

$$Pe,2 \geq Q\left(\sqrt{\frac{2E_2}{N_0}}\right) \tag{2}$$

   and when will this bound be tight?
3. Give a numerical expression (or even a bound) for the error probability for detecting x_1.

4 Problem 4

Consider a dual-antenna receiver

$$\mathbf{y}_i = e^{j\theta_i}\mathbf{x} + \mathbf{z}_i, i = 1, 2 \tag{3}$$

where θ_i is a phase shift unknown to the receiver on each antenna, and $\mathbf{x}$ belongs to some multi-dimensional signal set of dimension N, $\mathcal{S}$. Assume that the phase shifts θ_i are both uniform on $[0, 2\pi)$ and independent and that $||x||^2 = E_s, \forall \mathbf{x} \in \mathcal{S}$.

1. What is the maximum-likelihood detector for $\mathbf{x}$.
2. Express the detection rule in a similar form to the one-dimensional non-coherent detector.

[BONUS QUESTION](i.e. extra marks added to your final grade!) If $\mathbf{x}$ is a binary on-off modulation (i.e. not equal energy anymore), give an expression for the error probability in terms of E_s.

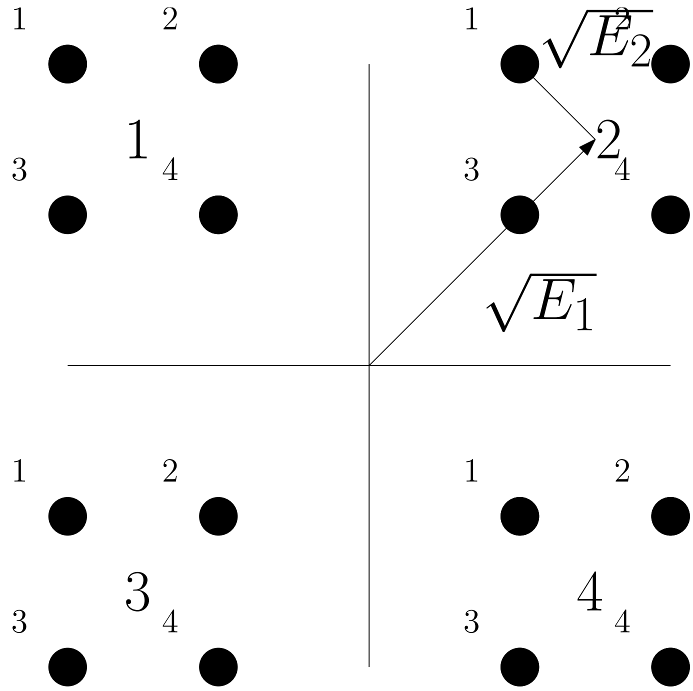

Figure 3: Constellation for Problem 3