Institut Eurécom
Digital Communications

Midterm Examination

Date: Dec. 17, 2008

Duration: 2 hours

Answer Problem 1 and only one out of Problems 2 and 3.

1 Problem 1 (12 points)

Consider the set of signals in Figure 1.

1. Find an orthonormal basis for the signal set.
2. What is the average energy of the signal set?
3. Draw the constellation which represents the signal set in terms of the basis that you found.
4. What is the information-rate in bits per symbol?
5. Give a bound on the probability of error for this signal set.

2 Problem 2 (8 points)

Consider the binary communication problem consisting of waveforms:

$$s_0(t) = \begin{cases} s_p(t), t \in [0, T_p) \\ -s(t), t \in [T_p, T] \end{cases} \qquad s_1(t) = \begin{cases} s_p(t), t \in [0, T_p) \\ s(t), t \in [T_p, T] \end{cases} \tag{1}$$

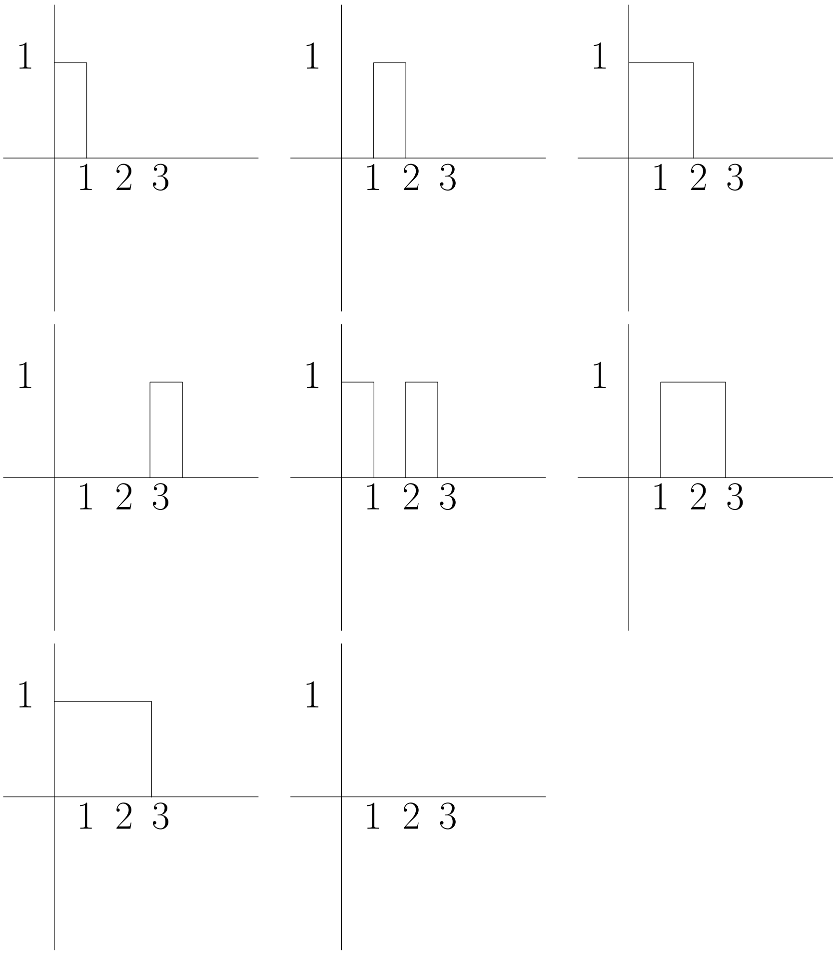

Figure 1: Signal Set for Problem 1

where $s_p(t)$ is some waveform with energy E_p and $s(t)$ is a waveform with energy E_s. The signal is transmitted over and AWGN channel with noise power spectral density N_0 and an unknown phase shift, so that the received signal is given by

$$y(t) = e^{j\phi} s_m(t) + \nu(t) \tag{2}$$

1. Give a set of basis functions for this signal set (hint: it is two-dimensional)
2. Is it an orthogonal signal-set?
3. What is the ML non-coherent receiver for this signal set and give its block diagram.
4. Give an expression for the error probability in terms of E_b/N_0 for this signal set (Hint: an appropriate adaptation of eq. 2.53 in the notes suffices)
5. Explain how the performance depends on the relationship between E_p and E_s and T_p and T.

6. What is the relationship of this receiver to an ideal coherent ML receiver (Hint: try to manipulate the decision rule so that it looks like an imperfect coherent receiver).

3 Problem 3 (8 points)

Consider an M-ary orthogonal signal set with signals, $s_m(t), m = 1, 2, \cdots, M.$. Now consider a new M-ary signal set (called a M-ary simplex) given by

$$s'_m(t) = s_m(t) - \frac{1}{M}\sum_{m=1}^{M} s_m(t). \tag{3}$$

Here we have just removed the mean value of the signal set

1. Show that the average energy of the signal set is reduced by a factor $1 - \frac{1}{M}$
2. Does the minimum-distance change?
3. What is the dimensionality of the new signal set? (Hint: Think about it in 2 and 3 dimensions and extrapolate)
4. What is the probability of error of the new signal set in terms of E_b/N_0 (Hint: Consider the difference with respect to the original orthogonal signal-set and your answer to question 3.2?)