Institut Eurécom
Digital Communications

Midterm Examination

Date: Dec. 20, 2012

Duration: 2 hours

Answer any 3 out of 4 questions

1 Problem 1

Consider the three signals,

$$p_0(t) = \begin{cases} \sqrt{\beta E}\left(\frac{T}{4} - t\right), & t \in [0, T/2] \\ 0 & \text{otherwise} \end{cases}, p_1(t) = 0, p_2(t) = p_0(t - T/2)$$

(a) Find the value of β for which the average energy of this signal set is E.

(b) Find an orthonormal basis for this set of signals.

(c) Draw a constellation representing this signal set on the basis you found

(d) What is the minimum distance in terms of E.

2 Problem 2

Consider the 8-point QAM constellation in Figure 1.

(a) If all symbols are transmitted equally often and are statistically independent, what should the value of E be such that the average symbol energy is E_s.

(b) What is the minimum squared Euclidean distance of the constellation in terms of E_s

(c) Give an upper-bound on the probablility of symbol-error as a function of E_b/N_0 assuming that transmission is carried out on an circularly-symmetric additive white Gaussian noise channel with variance N_0.

(d) What is the coding gain with respect to 8-PSK?

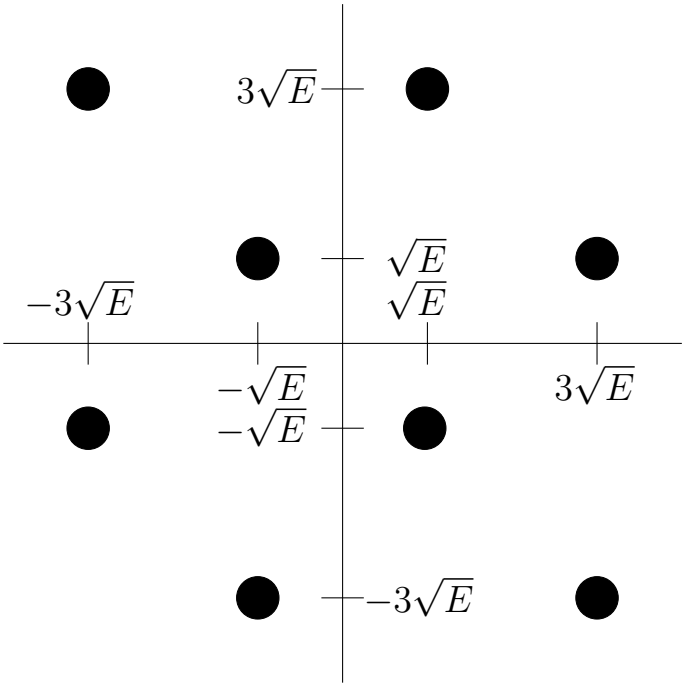

Figure 1: Constellation for Problem 2

3 Problem 3

Consider the binary communication problem consisting of waveforms:

$$s_0(t) = \begin{cases} s_p(t), t \in [0, T_p) \\ -s(t), t \in [T_p, T] \end{cases} \quad s_1(t) = \begin{cases} s_p(t), t \in [0, T_p) \\ s(t), t \in [T_p, T] \end{cases} \tag{1}$$

where $s_p(t)$ is some waveform with energy E_p and $s(t)$ is a waveform with energy E_s. The signal is transmitted over and AWGN channel with noise power spectral density N_0 and an unknown phase shift, so that the received signal is given by

$$y(t) = e^{j\phi} s_m(t) + \nu(t) \tag{2}$$

(a) Give a set of basis functions for this signal set (hint: it is two-dimensional)

(b) Is it an orthogonal signal-set?

(c) What is the ML non-coherent receiver for this signal set and give its block diagram.

(d) Give an expression for the error probability in terms of E_b/N_0 for this signal set (Hint: an appropriate adaptation of eq. 2.52 in the notes suffices)

(e) Explain how the performance depends on the relationship between E_p and E_s and T_p and T.

(f) What is the relationship of this receiver to an ideal coherent ML receiver (Hint: try to manipulate the decision rule so that it looks like an imperfect coherent receiver).

4 Problem 4

Consider the binary communication problem consisting of vectors:

$$\mathbf{y} = \sqrt{E}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} x + \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

where $x \in \{0, \sqrt{2}\}$ (on-off modulation), z_i are zero-mean independent complex circularly symmetric Gaussian random variables with variance N_0, and h_i are zero-mean independent complex circularly symmetric Gaussian random variables with variance 1.

(a) What is the ML receiver (non-coherent) for this problem?

(b) Give a bound (or exact expression) for the error probability.