Institut Eurécom
Digital Communications

Midterm Examination

Date: Dec. 20, 2007

Duration: 2 hours

Answer 3 out of 4 questions. All questions will be graded equally, All documents are allowed.

1 Problem 1

Consider the set of signals in Figure 1.

1. Find an orthonormal basis for the signal set.
2. Find α,β, and γ such that the signals are all equal-energy E_s
3. Draw the constellation which represents the signal set in terms of the basis that you found
4. What is the information-rate in bits per symbol?

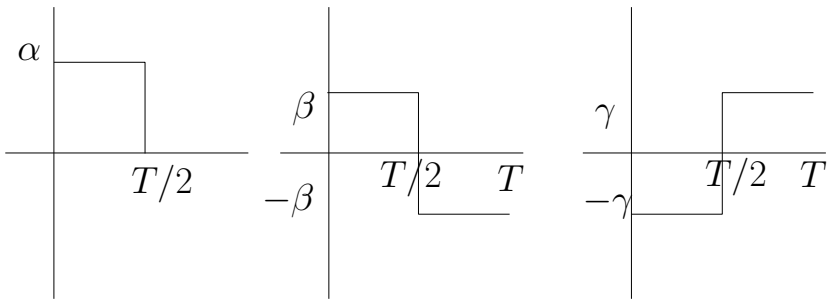

Figure 1: Signal Set for Problem 1

2 Problem 2

Consider the 8-point QAM constellation in Figure 2.

1. If all symbols are transmitted equally often and are statistically independent, what should the value of E be such that the average symbol energy is E_s.
2. What is the minimum squared Euclidean distance of the constellation in terms of E_s
3. Give an upper-bound on the probablility of symbol-error as a function of E_b/N_0 assuming that transmission is carried out on an circularly-symmetric additive white Gaussian noise channel with variance N_0.
4. What is the coding gain with respect to 8-PSK?

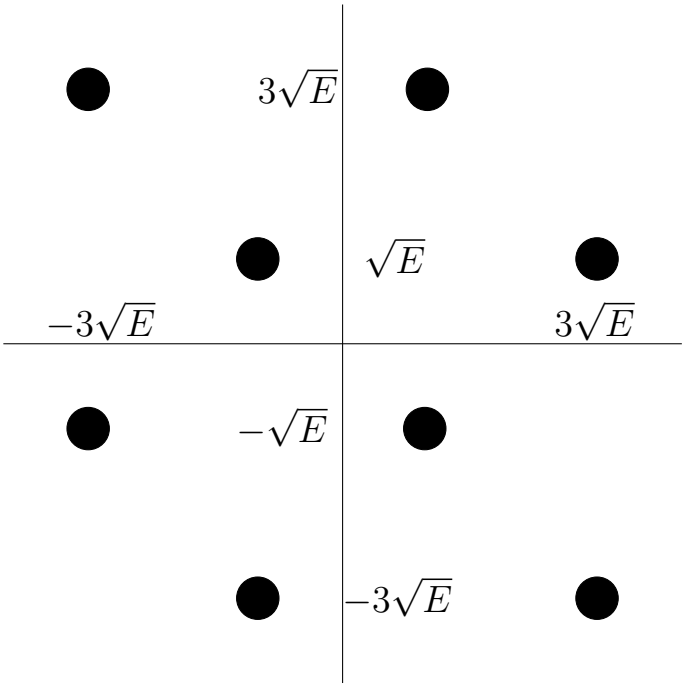

Figure 2: Constellation for Problem 2

3 Problem 3

Consider the 4-PAM contellation in Figure 3. Here the modulated symbols are mapped to the 2-bit sequences $(b_0 b_1)$ as shown using so-called *Gray Coding*.

1. Assuming that we observe the transmitted symbols in white Gaussian noise with variance $N_0/2$ as $y = s_m + z, 0 \leq m \leq 3$, write the likelihood functions $p(y|b_0)$ and $p(y|b_1)$.

2. What is the simplest detection rule for b_0?

3. Use the approximation $\log(e^a + e^b) \approx \max(a, b)$ to approximate the detection rule for b_1 as $b_1 = .5(1 + \text{sign}(|y| - (2\sqrt{E_s/5})))$.

4. Give a numerical expression (or even a bound) for the error probability for detecting b_0.

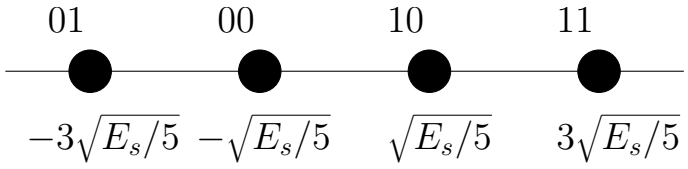

Figure 3: Constellation for Problem 3

4 Problem 4

Consider an on/off transmission system with an unknown multiplicative channel and additive white Gaussian noise

$$y = hx + z, \tag{1}$$

where h is white complex circularly-symmetric Gaussian noise with unit-variance and unknown to the receiver, z is white complex circularly-symmetric Gaussian noise with variance N_0, and x takes on the values $\sqrt{2E}$ and 0 with equal probability.

1. What is the maximum-likelihood detector for x.

2. Express the detection rule in a similar form to the non-coherent detector with only unknown phase (i.e. the one in class).

[BONUS QUESTION](i.e. extra marks added to your final grade!) Give an expression for the error probability in terms of E.