Institut Eurécom
Digital Communications

Final Examination

Date: Feb. 9, 2011

Duration: 2 hours

Answer any 3 out of 4 questions. All questions will be graded equally, All documents are allowed.

1 QPSK with IQ Gain Error

Consider an ideal QPSK system with a received signal

$$y = \sqrt{E_s}x + z$$

where x is a symbol from a QPSK alphabet and z is AWGN with variance N_0. In many analog systems the receiver electronics induce a distortion termed *IQ Gain Error* whereby the quadrature mixer in the receiver does not perform and ideal complex multiplication (i.e. by sine and cosine signal generators) to generate the complex baseband signal. Instead the amplitude difference between the two signals is $\delta_{\mathrm{IQ}} < 1$ which is an unknown imbalance that the receiver must estimate or ignore. This distortion results in the channel model

$$\mathrm{Re}(y) = \sqrt{E_s}\mathrm{Re}(x) + \mathrm{Re}(z), \mathrm{Im}(y) = \sqrt{\delta_{\mathrm{IQ}}E_s}\mathrm{Im}(x) + \mathrm{Im}(z),$$

1. Give an upper-bound or exact expression for the symbol error rate as a function of δ_{IQ} under the assumption that the maximum-likelihood receiver for $\delta_{\mathrm{IQ}} = 1$ is used (i.e. neglecting the imbalance). In other words, the decision regions remain the four quadrants of the real-imaginary plane.
2. What would the ML receiver and its performance be in the case of perfect knowledge of δ_{IQ}?

2 Multiple-symbol Detection of M-PSK

Consider the following $N-$dimensional detection problem

$$y_n = \sqrt{E_s} e^{j2\pi\theta} x_n + z_n, n = 0, 1, \cdots, N-1,$$

where θ is a random phase offset uniformly distributed on $[0, 2\pi)$, x_n is a modulated M-PSK information sequence with $x_0, x_1, \cdots, x_{p-1} = 1$ and z_n is a complex zero-mean Gaussian random sequence with variance N_0.

1. What is the maximum-likelihood (non-coherent) receiver for x_p in this system?
2. What is the relationship between this detector and the coherent detector for x_p?
3. Suppose $N \gg p$. Argue that the performance of the non-coherent detector for $x_p, \cdots, x_{N-1}$ should approach that of the symbol-by-symbol coherent detector.

3 OFDM

Consider a receiver for OFDM system to be designed on a wireless channel. The sampling rate of the system is 15.36 Ms/s. The number of carriers per OFDM symbol is $N_c = 1024$. Two cyclic prefix lengths are used for every group of seven symbols. $N_{p0} = 72$ for the first symbol and $N_p = 80$ for the remaining 6 symbols.

1. Under the assumption that 300 carriers are used in the positive part of the spectrum, 300 carriers are used in the negative part of the spectrum and that the DC carrier is skipped, what is the occupied bandwidth?
2. What is the maximum channel duration that the system can cope with and explain in words what effect a longer channel would have on the system performance?
3. Assuming we use 16-QAM modulation, what is the spectral-efficiency of the system (spectral efficiency is measured in bits/s/Hz), including the overhead due to the cyclic prefix?

4 Trellis Diagrams and the Viterbi Algorithm

A BPSK signal with symbol energy E_s is generated using an arbitrary pulse of duration T seconds, It is transmitted across a dispersive channel for which the complete channel response (transmit filter, receive filter and channel) is represented by the composite impulse response

$$h(t) = \begin{cases} e^{-2t/T} & t \in (0, 2.5T), \\ 0 & \text{otherwise.} \end{cases}$$

This yields the received signal

$$r(t) = \sqrt{E_s} \sum_n a_n * h(t - nT) + z(t)$$

where a_n is the QPSK information sequence and $z(t)$ is AWGN with power spectral density N_0.

1. What is the autocorrelation sequence (g_n) of the sampled cascaded channel $h(t)$.
2. How many states does the correspoding state-space representation (Ungerboeck form) have?
3. Draw the trellis.
4. What is the maximum-likelihood update rule in the Viterbi algorithm for this example?