Institut Eurécom
Digital Communications

Final Examination

Date: February 10, 2014

Duration: 2 hours

Chooose and complete any 3 out of the following questions. All questions will be graded equally. All documents are allowed.

1 Problem 1

Consider the set of 4 signals in Figure 1. This is an example of 4-ary pulse-position modulation where a basic signaling pulse $p(t)$ is shifted in time by $mT_s/4, m = 0, 1, 2, 3$ where T_s is parameter to be chosen. The value of m indicates the information that is transmitted. The signaling pulse $p(t)$ is time-limited to T_p seconds, in the sense that $p(t) = 0, t \notin [0, T_p)$. The transmitted signal is sent across a finite-duration time-invariant linear channel $h(t)$ with duration T_c seconds. At the receiver we have

$$r(t) = \sqrt{\alpha E_s} p(t - mT_s/4) * h(t) + z(t)$$

where $z(t)$ is additive white Gaussian noise with power-spectral density N_0. We assume that the receiver has perfect knowledge of both $p(t)$ and $h(t)$ (i.e. they are not random).

1. What is the value of α which guarantees that the average signal energy per symbol is E_s?
2. What is the relationship between T_p, T_c and T_s that makes the 4 signals orthogonal for any channel $h(t)$?
3. For an orthogonal configuration, what is the maximum bit rate that this signal set can achieve. This should be expressed in bits/s as a function of T_p and T_c.
4. What is the maximum-likelihood receiver for this signal set (in general, not necessarily in an orthogonal configuration)?

5. What is the minimum Euclidean distance between the signals as a function of the auto-correlation function of $h(t) * p(t)$, denoted by $\rho(t)$ and what is the resulting upper-bound (i.e. union-bound) on the probability of symbol error.

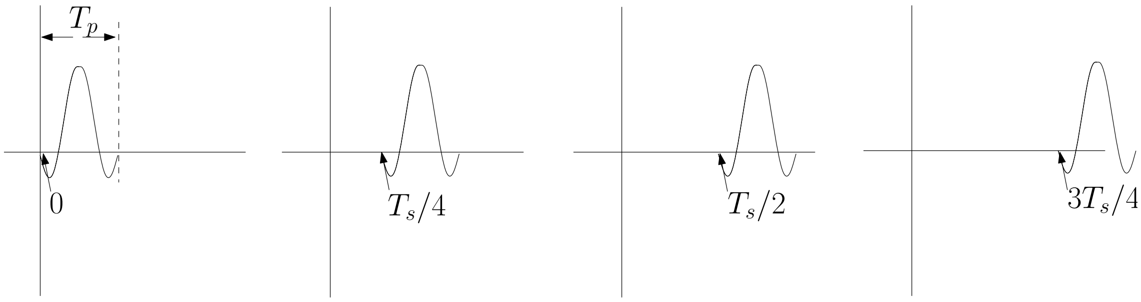

Figure 1: Signal Set for Problem 1

2 Problem 2

Consider the following orthogonal modulation system

$$y(t) = \sqrt{E_s} h_m \xi_m(t) + z(t), m = 0, 1, \cdots M - 1$$

where $\{\xi_m(t), \forall m\}$ forms an orthonormal set. h_m are zero-mean unit-variance and independent circularly-symmetric complex Gaussian random variables and $z(t)$ is a circularly-symmetric complex Gaussian random process with mean 0 and power spectral density N_0. This would typically represent a wideband FSK system with large frequency spacing and small symbol time in a rich multipath environment. Assume that the h_m are unknown to the receiver.

1. What are the basis functions for ML detection and what is the dimension of the signal-space?
2. Derive the ML receiver for this general non-coherent detection problem.
3. Give the union bound for the probability of error.
4. Mimic the derivtion in Section 2.6.1 to find an exact expression for the probability of error.

3 Problem 3

Consider a receiver for OFDM system to be designed on a wireless channel. The sampling rate of the system is 10 Ms/s. The number of carriers per OFDM symbol is denoted $N_c = 64$. The length of the cyclic-prefix is $N_p = 16$. The number of useful carriers (i.e. those that are non-zero) is $N_u = 52$. One of the zeroed carriers is in the DC component (i.e. position 0 in the frequency-domain).

1. What is the occupied bandwidth for the chosen system parameters?
2. What is the maximum channel duration that the system can cope with and explain in words what effect a longer channel would have on the system performance?
3. Assuming we use 16-QAM modulation what is the spectral-efficiency of the system (spectral efficiency is measured in bits/s/Hz)?

4 Problem 4

A BPSK (2-AM) signal with symbol energy E_s is generated using a square-pulse of duration T seconds,

$$p_T(t) = \begin{cases} \sqrt{\frac{1}{T}}, t \in [0, T) \\ 0, t \notin [0, T). \end{cases}$$

It is transmitted across a dispersive channel $h(t) = h_0\delta(t) + h_1\delta(t - .75T)$ yielding the received signal

$$r(t) = \sqrt{E_s} \sum_n a_n p(t - nT) * h(t) + z(t)$$

where a_n is the BPSK information sequence (i.e. $a_n \in \{-1, 1\}$).

1. What is the autocorrelation sequence (g_n) of the cascaded channel $p_T(t) * h(t)$.
2. How many states does the correspoding state-space representation (Ungerboeck form) have?
3. Draw the trellis
4. What is the maximum-likelihood update rule in the Viterbi algorithm for this example?