Institut Eurécom
Digital Communications

Midterm Examination

Date: Dec. 16, 2009

Duration: 2 hours

Answer Problem 1 and only one out of Problems 2 and 3.

1 Problem 1 (12 points)

Consider the set of signals in Figure 1.

(a) What is the average energy of the signal set in terms of d?

(b) What is the information-rate in bits per symbol?

(c) What is the union-bound on the probability of error for this signal set assuming an additive white-Gaussian noise channel model with power-spectral density N_0 and normalized signal energy.

(d) Find an exact expression for the probability of error under the same channel model.

2 Problem 2 (8 points)

Consider the binary communication problem consisting of vectors:

$$H_0 : \mathbf{x} = \mathbf{s}_0 + \mathbf{n}$$
$$H_1 : \mathbf{x} = \mathbf{s}_1 + \mathbf{n}$$

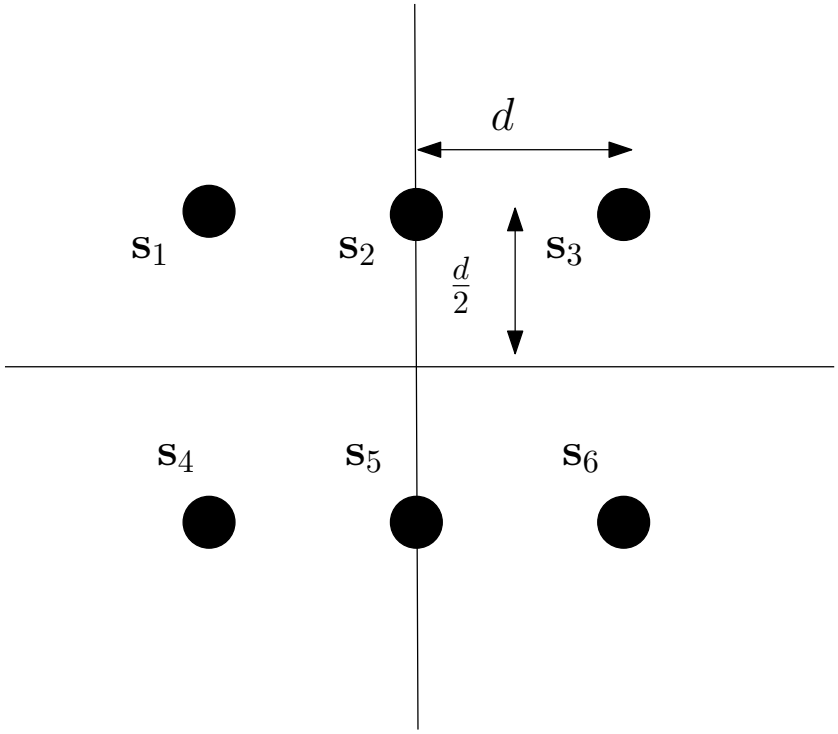

Figure 1: Signal Set for Problem 1

where $\mathbf{s}_0 = (s_0, 0)^T$, $\mathbf{s}_1 = (0, s_1)^T$ and $\mathbf{n}$ is a zero-mean real Gaussian random vector with covariance matrix

$$\mathcal{K}_n = \begin{pmatrix} \sigma_0^2 & 0 \\ 0 & \sigma_1^2 \end{pmatrix}$$

- What is the receiver that minimizes the probability of error under equal *a priori* probabilities on the two hypotheses.
- Write expressions to characterize the two probabilities of error, i.e. $\Pr(\text{Error}|H_0)$ and $\Pr(\text{Error}|H_1)$.

3 Problem 3 (8 points)

Consider the binary communication problem consisting of vectors:

$$H_0 : x = mb + n$$
$$H_1 : x = n$$

where b and n are zero-mean real Gaussian random variables with variances σ_b and σ_n and m is a constant. Assuming equiprobable hypotheses,

(a) Find and sketch the optimum receiver

(b) Calculate the minimum-error probability for the receiver

(c) Repeat (a) and (b) when b has mean m_b