Institut Eurécom
Digital Communications

Final Examination

Date: Feb. 14, 2008

Duration: 2 hours

Answer any 3 out of 4 questions. All questions will be graded equally, All documents are allowed.

1 QPSK with a DC offset

Consider a QPSK system with a received signal

$$y = \sqrt{E_s}x + z + s_{\text{off}}$$

where x is a symbol from a QPSK alphabet, $s_{\text{off}} = |s|e^{\jmath\theta}$ is some unknown complex offset unknown to the receiver and z is complex white Gaussian noise with variance N_0. We can safely assume that $|s|^2 \ll E_s$.

1. Give an upper-bound or exact expression for the symbol error rate as a function of $|s|$ and θ under the assumption that the maximum-likelihood receiver for $|s| = 0$ is used (i.e. neglecting the DC-offset). In other words, the decision regions remain the four quadrants of the real-imaginary plane.
2. Can you say something about the average error rate as a function of $|s|$ (i.e. when θ is averaged out by assuming it is uniform)?

2 Doubly Differential Detection

Consider the following $N-$dimensional detection problem

$$y_n = \sqrt{E_s} e^{j(2\pi f n+\theta)} x_n + z_n$$

where f is some unknown uniform random frequency offset known to lie in the interval $-f_d \leq f \leq f_d$, θ is a random phase offset uniformly distributed on $[0, 2\pi)$, x_n is a modulated M-PSK information sequence with $x_0 = x_1 = 1$ and z_n is a complex zero-mean Gaussian random sequence with variance N_0.

1. Show that the ML detector can be simplified to

$$\hat{x} = \underset{\mathbf{x}}{\operatorname{argmax}} \int_{-f_d}^{f_d} I_0 \left(\frac{2}{N_0} |\mathbf{y}^H \mathbf{D}(f)\mathbf{x}| \right) df$$

with $\mathbf{D}(f) = \operatorname{diag}(e^{j2\pi f n}), n = 1, 2, \cdots, N$. To our knowledge, it cannot be simplified further without approximations.

2. Assume we use a *doubly-differential encoder* $x_n = x_{n-1}x'_n$ and $x'_n = x'_{n-1}u_n$ where u_n is an M-PSK information sequence. Note that with respect to x'_n, x_n is simply differential encoding. Argue that performing doubly-differential detection defined by

$$\hat{u_n} = y'_n y'^*_{n-1}, y'_n = y_n y^*_{n-1}$$

removes the unknown phase(θ) and frequency (f) offsets.

3. Do you expect noise enhancement to be more of a problem than in regular differential detection and why?

3 OFDM

Consider a receiver for OFDM system to be designed on a wireless channel with maximum channel duration of 1μ s. The maximum sampling rate of the system is 30.72 Ms/s and occupied channel bandwidth can be $1.25k$ MHz ($k = 1, 2, 4, 8, 16$). The number of carriers per OFDM symbol is denoted N_c, the length of the cyclic-prefix is N_p and the number of useful carriers (i.e. those that are non-zero) is N_u.

1. Assuming we use a DFT size of 2048 samples, what should the number of useful carriers be to occupy the channel bandwidth for each value of k?

2. Assuming we use 16-QAM modulation what is the spectral-efficiency of the system as a function of k (spectral efficiency is measured in bits/s/Hz)?

3. What disadvantage do you see in using the minimum size cyclic-prefix in the case of an unknown time-delay between the transmitter and receiver.

4 Trellis Diagrams and the Viterbi Algorithm

A BPSK (2-AM) signal with symbol energy E_s is generated using a square-pulse of duration T seconds,

$$p_T(t) = \begin{cases} \sqrt{\frac{1}{T}}, t \in [0, T) \\ 0, t \notin [0, T). \end{cases}$$

It is transmitted across a dispersive channel $h(t) = h_0\delta(t) + h_1\delta(t - 1.25T)$ yielding the received signal

$$r(t) = \sqrt{E_s} \sum_n a_n p(t - nT) * h(t) + z(t)$$

where a_n is the BPSK information sequence (i.e. $a_n \in \{-1, 1\}$).

1. What is the autocorrelation sequence (g_n) of the cascaded channel $p_T(t) * h(t)$.

2. How many states does the correspoding state-space representation (Ungerboeck form) have?

3. Draw the trellis

4. What is the maximum-likelihood update rule in the Viterbi algorithm for this example?