Institut Eurécom
Digital Communications

Final Examination

Date: Feb. 8, 2012

Duration: 2 hours

Answer any 3 out of 4 questions. All questions will be graded equally, although one bonus mark will be given for anybody attempting *all three of the LTE questions*. All documents are allowed.

1 Orthogonal Modulation with a Cyclic Prefix (LTE PRACH)

Consider the following detection problem

$$y(n) = x_p(n) * h(n) + z(n), n = 0, \cdots, N_x + N_h - 1$$

where $x_p(n)$ is a time-domain signal of length $N_x = N_c + N_p$ samples depending on a message p taking on values $p \in \{0, 1, \cdots, P-1\}$ where P can be made variable, $h(n)$ is a known time-limited channel of duration N_h samples and $z(n)$ is a zero-mean circularly symmetric complex Gaussian noise sequence with variance N_0. A cyclic prefix of length N_p is present in $x_p(n)$ and has duration $N_p > N_h$, so that we have the relationship

$$x_p(n) = x_p(n \mod N_c)$$

as in OFDM transmission. Furthermore we have that

$$x_p(n \mod N_c) = x((n - p\Delta) \mod N_c)$$

so that messages differ in their *cyclic* shifts of a known sequence $x(n)$.

1. For what value of Δ can the P signals at the receiver be considered orthogonal (i.e. provide the relationship between Δ and P). Explain.

2. Suppose we remove the first N_p samples at the receiver and transform the signal to the frequency-domain using an N_c-point DFT (i.e. like in OFDM) and assume that Δ and P are chosen such that the signals at the receiver for different values of p are orthogonal. Propose an optimal non-coherent receiver structure based on $X(k) = \mathrm{DFT}(x(n))$. As a guide, think about projecting (like in OFDM) in the frequency-domain using $X(k)$ and then going back to the time-domain to perform detection. You should make use of the cyclic-shifts in the time domain in the detection process (think about detection of orthogonal signals based on pulse-position modulation as we did in class).

2 Multiple-symbol Detection of BPSK (LTE PUCCH)

Consider the following $N \times N_2-$dimensional detection problem

$$y_{n,i} = \sqrt{\frac{E_s}{N}} h_n x_i + z_{n,i}, n = 0, 1, \cdots, N-1, i = 0, \cdots, N_2 - 1$$

where h_n is an unknown random circularly symmetric Complex Gaussian zero-mean N-dimensional vector with i.i.d. unit-variance components (i.e. $\mathrm{E}(h_n^* h_m) = \delta_{n,m}$), x_n is a BPSK information sequence with $x_0 = 1$,$x_1 = x_2 = \cdots = x_{N_2-1} = x$, with x to be detected, and $z_{n,i}$ is a circulary-symmetric complex zero-mean Gaussian random sequence with variance N_0.

1. What is the maximum-likelihood (non-coherent) receiver for x in this system?
2. Consider the case where $N_2 = 1$. What is the relationship between this detector and the coherent detector for x (i.e. when h_n is known)?

3 OFDM (LTE 20MHz)

Consider a receiver for OFDM system to be designed on a wireless channel. The sampling rate of the system is 30.72 Ms/s. The number of carriers per OFDM symbol is $N_c = 2048$. Two different transmission formats are used. First (normal prefix format) two different cyclic prefix lengths are used for every group of seven symbols, $N_{p0} = 160$ for the first symbol and $N_p = 144$ for the remaining 6 symbols. Second (extended prefix format), only one cyclic prefix length is used for every symbol, namely $N_p = 512$.

1. Under the assumption that 600 carriers are used in the positive part of the spectrum, 600 carriers are used in the negative part of the spectrum and that the DC carrier is skipped, what is the occupied bandwidth?

2. What is the maximum channel duration that the system can cope with (i.e. in both formats) and explain in words which format should be used as a function of channel conditions.

3. Assuming we use 64-QAM modulation, what is the spectral-efficiency of the system (spectral efficiency is measured in bits/s/Hz), including the overhead due to the cyclic prefix in both cases?

4 Trellis Diagrams and the Viterbi Algorithm

A QPSK signal with symbol energy E_s is generated using an arbitrary pulse of duration T seconds, It is transmitted across a dispersive channel for which the complete channel response (transmit filter, receive filter and channel) is represented by the composite impulse response

$$h(t) = \begin{cases} e^{-2t/T} & t \in (0, 1.5T), \\ 0 & \text{otherwise.} \end{cases}$$

This yields the received signal

$$r(t) = \sqrt{E_s} \sum_n a_n * h(t - nT) + z(t)$$

where a_n is the QPSK information sequence and $z(t)$ is AWGN with power spectral density N_0.

1. What is the autocorrelation sequence (g_n) of the sampled cascaded channel $h(t)$.

2. How many states does the correspoding state-space representation (Ungerboeck form) have?

3. Draw the trellis.

4. What is the maximum-likelihood update rule in the Viterbi algorithm for this example?