Institut Eurécom
Digital Communications

Midterm Examination

Date: Dec. 15, 2010

Duration: 2 hours

Answer any 3 out of 4 questions

1 Problem 1

Consider the three signals,

$$\gamma_0(t) = \begin{cases} \sqrt{\beta E} e^{-\alpha t}, & t \in [0, T/2] \\ 0 & \text{otherwise} \end{cases}, \gamma_1(t) = \gamma_0(t - T/4), \gamma_2(t) = \gamma_0(t - 3T/4)$$

(a) Find the value of β for which the average energy of this signal set is E.

(b) Find an orthonormal basis for this set of signals.

(c) Draw a constellation representing this signal set on the basis you found

(d) What is the minimum distance in terms of E.

2 Problem 2

Consider the ternary set of signals, $x_m = e^{j2\pi m/3}, m = 0, 1, 2$ transmitted over an AWGN channel

$$y = \sqrt{E} x_m + z$$

where z is a zero-mean circularly-symmetric complex Gaussan random variable with variance N_0.

(a) What is the information-rate in bits per symbol?

(b) What is the union-bound on the probability of error for this signal set assuming an additive white-Gaussian noise channel model with power-spectral density N_0 and normalized signal energy.

(c) Find an exact expression (even in the form of an integral) for the probability of error under the same channel model.

3 Problem 3

Consider the binary communication problem consisting of vectors:

$$\mathbf{y} = \sqrt{E}\begin{pmatrix}1\\1\end{pmatrix}x + \begin{pmatrix}z_1\\z_2\end{pmatrix}$$

where $x \in \{-1, 1\}$ (antipodal modulation), z_i are real, zero-mean Gaussian random variables with correlation matrix

$$\mathrm{E}\begin{pmatrix}z_1\\z_2\end{pmatrix}\begin{pmatrix}z_1 & z_2\end{pmatrix} = \sigma^2\begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}$$

and $0 \leq \rho \leq 1$.

(a) What is the ML receiver for this problem?

(b) Give a bound (or exact expression) for the error probability as a function of E,σ^2 and ρ.

4 Problem 4

Consider the binary communication problem consisting of vectors:

$$\mathbf{y} = \sqrt{E}\begin{pmatrix}h_1\\h_2\end{pmatrix}x + \begin{pmatrix}z_1\\z_2\end{pmatrix}$$

where $x \in \{0, \sqrt{2}\}$ (on-off modulation), z_i are zero-mean independent complex circularly symmetric Gaussian random variables with variance N_0, and h_i are zero-mean independent complex circularly symmetric Gaussian random variables with variance 1.

(a) What is the ML receiver (non-coherent) for this problem?

(b) Give a bound (or exact expression) for the error probability.