Institut Eurécom
Digital Communications

Final Examination

Date: Feb. 11, 2009

Duration: 2 hours

Answer any 3 out of 4 questions. All questions will be graded equally, All documents are allowed.

1 QPSK with a residual phase offset

Consider a QPSK system with a received signal

$$y = \sqrt{E_s} e^{\phi} x + z$$

where x is a symbol from a QPSK alphabet, ϕ is some unknown complex phase offset unknown to the receiver and z is complex white Gaussian noise with variance N_0. You can assume that ϕ is in the range $[-\phi_0 \quad \phi_0]$ with $\phi_0 \ll 1$.

1. Give an upper-bound or exact expression for the symbol error rate as a function of ϕ under the assumption that the maximum-likelihood receiver for $\phi = 0$ is used (i.e. neglecting the phase offset). In other words, the decision regions remain the four quadrants of the real-imaginary plane.
2. Can you say something about the average error rate as a function of ϕ (i.e. when ϕ is averaged out by assuming it is uniform on $[-\phi_0 \quad \phi_0]$)? Approximations like $\sin \phi \approx \phi$ and $\cos \phi \approx 1 - \phi$ are acceptable.

2 Orthogonal Modulation on Wideband Fading Channels

Consider the following orthogonal modulation system

$$y(t) = \sqrt{E_s} h_m \xi_m(t) + z(t), m = 0, 1$$

where $\{\xi_m(t), m = 0, 1\}$ for an orthonormal set. h_m are zero-mean unit-variance and independent complex Gaussian random variables. This would typically represent a wideband FSK system with large frequency spacing and small symbol time in a rich multipath environment. Assume that the h_m are unknown to the receiver.

1. What are the basis functions for ML detection and what is the dimension of the signal-space?
2. Derive the ML receiver for this general non-coherent detection problem.
3. Give a bound or exact expression for the probability of error.

3 OFDM

Consider a receiver for an OFDM system with a sampling rate of 6.5 Ms/s and based on a 256-point DFT. The cyclic prefix length is 64 samples.

1. What is the efficiency of the system in terms of the ratio of data samples to total samples per OFDM symbol.
2. Suppose we use the 160 lowest frequency carriers (80 on each end around DC) out of the 256 and the rest are set to zero. What is the bandwidth of the transmitted waveform?
3. Assuming we use 16-QAM modulation what is the spectral-efficiency of the system?
4. What is the longest length channel response (in μs) that the OFDM can tolerate under the assumption that that we cannot allow intersymbol-interference.

5. Suppose we have a two-way system where the receiver synchronizes its transmit signal to the strongest peak in the received channel response from the other-end (i.e. in the channel estimate that you saw in class). Further assume that the strongest peak is the first sample of the channel. If the channel lasts at most 1μs, how much propagation delay can the system tolerate (i.e. the sum of the forward and return links) so that the receiver on the other end does not suffer from intersymbol interference.

4 Trellis Diagrams and the Viterbi Algorithm

A BPSK (2-AM) signal with symbol energy E_s is generated using a square-pulse of duration T seconds,

$$p_T(t) = \begin{cases} \sqrt{\frac{1}{T}}, t \in [0, T) \\ 0, t \notin [0, T). \end{cases}$$

It is transmitted across a dispersive channel $h(t) = h_0\delta(t) + h_1\delta(t - .75T)$ yielding the received signal

$$r(t) = \sqrt{E_s} \sum_n a_n p(t - nT) * h(t) + z(t)$$

where a_n is the BPSK information sequence (i.e. $a_n \in \{-1, 1\}$).

1. What is the autocorrelation sequence (g_n) of the cascaded channel $p_T(t) * h(t)$.
2. How many states does the correspoding state-space representation (Ungerboeck form) have?
3. Draw the trellis
4. What is the maximum-likelihood update rule in the Viterbi algorithm for this example?