Institut Eurécom
Digital Communications

Midterm Examination

Date: Dec. 16, 2011

Duration: 2 hours

Answer any 3 out of 4 questions

1 Problem 1

Consider the three signals,

$$p_0(t) = \begin{cases} \sqrt{\beta E}t, & t \in [0, T/2] \\ 0 & \text{otherwise} \end{cases}, p_1(t) = p_0(t - T/4), p_2(t) = p_0(t - T/2)$$

(a) Find the value of β for which the average energy of this signal set is E.

(b) Find an orthonormal basis for this set of signals.

(c) Draw a constellation representing this signal set on the basis you found

(d) What is the minimum distance in terms of E.

2 Problem 2

Consider the quaternary set of signals, $x_m = \sqrt{\beta E}e^{j2\pi m/3}, m = 0, 1, 2, x_3 = 0$ transmitted over an AWGN channel

$$y = \sqrt{E}x_m + z$$

where z is a zero-mean circularly-symmetric complex Gaussan random variable with variance N_0.

(a) What is the value of β for which the average signal energy is E given that all signals are equally likely?

(b) Draw the decision regions for this constellation

(c) What is the union-bound on the probability of error for this signal set assuming an additive white-Gaussian noise channel model with power-spectral density N_0 and normalized signal energy?

(d) What is the coding gain/loss with respect to QPSK modulation?

3 Problem 3

Consider the M-ary communication problem consisting of vectors:

$$\mathbf{y} = \sqrt{E}\begin{pmatrix} e^{j\phi} \\ e^{j\phi}x \\ e^{j\phi} \end{pmatrix} + \begin{pmatrix} z_{-1} \\ z_0 \\ z_1 \end{pmatrix}$$

where $x \in S$, with S being an arbitrary constellation where $|x|^2 = 1, \forall x \in S$, z_i are independent complex, zero-mean and circularly-symmetric Gaussian random variables with variance N_0, and ϕ is uniformly distributed on $[0, 2\pi)$ and unknown to the receiver.

(a) What is the ML receiver for this problem?

(b) Give a bound (or exact expression) for the error probability as a function of E_s/N_0 when x is BPSK modulation.