Institut Eurécom
Digital Communications

Final Examination

Date: Feb. 10, 2010

Duration: 2 hours

Answer any 3 out of 4 questions. All questions will be graded equally, All documents are allowed.

1 QPSK with IQ Imbalance

Consider an ideal QPSK system with a received signal

$$y = \sqrt{E_s}x + z$$

where x is a symbol from a QPSK alphabet and z is AWGN with variance N_0. In many radio system the receiver electronics induces a distortion termed *IQ Imbalance* whereby the quadrature mixer in the receiver does not perform and ideal complex multiplication (i.e. by sine and cosine signal generators) to generate the complex baseband signal. Instead the phase difference between the two signals is $\pi/2 + \alpha$ where α is an unknown imbalance that the receiver must estimate or ignore. This distortion results in the approximate channel model

$$\mathrm{Re}(y) = \sqrt{E_s}\mathrm{Re}(x) + \mathrm{Re}(z), \mathrm{Im}(y) = \sqrt{E_s}(\mathrm{Im}(x) + \alpha\mathrm{Re}(x)) + \mathrm{Re}(z),$$

where some of the energy from the imaginary component "spills" into the real component.

1. Give an upper-bound or exact expression for the symbol error rate as a function of α under the assumption that the maximum-likelihood receiver for $\alpha = 0$ is used (i.e. neglecting the imbalance). In other words, the decision regions remain the four quadrants of the real-imaginary plane.
2. What would the ML receiver be in the case of perfect knowledge of α?

2 Doubly Differential Detection

Consider the following $N-$dimensional detection problem

$$y_n = \sqrt{E_s} e^{j(2\pi f n + \theta)} x_n + z_n$$

where f is some unknown uniform random frequency offset known to lie in the interval $-f_d \leq f \leq f_d$, θ is a random phase offset uniformly distributed on $[0, 2\pi)$, x_n is a modulated M-PSK information sequence with $x_0 = x_1 = 1$ and z_n is a complex zero-mean Gaussian random sequence with variance N_0.

1. Show that the ML detector can be simplified to

$$\hat{x} = \underset{\mathbf{x}}{\operatorname{argmax}} \int_{-f_d}^{f_d} I_0 \left(\frac{2}{N_0} |\mathbf{y}^H \mathbf{D}(f) \mathbf{x}| \right) df$$

   with $\mathbf{D}(f) = \operatorname{diag}(e^{j2\pi f n}), n = 1, 2, \cdots, N$. To our knowledge, it cannot be simplified further without approximations.

2. Assume we use a *doubly-differential encoder* $x_n = x_{n-1} x'_n$ and $x'_n = x'_{n-1} u_n$ where u_n is an M-PSK information sequence. Note that with respect to x'_n, x_n is simply differential encoding. Argue that performing doubly-differential detection defined by

$$\hat{u_n} = y'_n y'^*_{n-1}, y'_n = y_n y^*_{n-1}$$

   removes the unknown phase(θ) and frequency (f) offsets.

3. Do you expect noise enhancement to be more of a problem than in regular differential detection and why?

3 OFDM

Consider a receiver for OFDM system to be designed on a wireless channel. The sampling rate of the system is 10 Ms/s. The number of carriers per OFDM symbol is denoted $N_c = 64$. The length of the cyclic-prefix is $N_p = 16$. The number of useful carriers (i.e. those that are non-zero) is $N_u = 52$.

1. What is the occupied bandwidth for the chosen system parameters?

2. What is the maximum channel duration that the system can cope with and explain in words what effect a longer channel would have on the system performance?

3. Assuming we use 16-QAM modulation what is the spectral-efficiency of the system (spectral efficiency is measured in bits/s/Hz)?

4 Trellis Diagrams and the Viterbi Algorithm

A QPSK signal with symbol energy E_s is generated using an arbitrary pulse of duration T seconds, It is transmitted across a dispersive channel for which the complete channel response (transmit filter, receive filter and channel) is represented by the composite impulse response

$$h(t) = \begin{cases} e^{-2t/T} & t \in (0, 1.5T), \\ 0 & \text{otherwise.} \end{cases}$$

This yields the received signal

$$r(t) = \sqrt{E_s} \sum_n a_n * h(t - nT) + z(t)$$

where a_n is the QPSK information sequence and $z(t)$ is AWGN with power spectral density N_0.

1. What is the autocorrelation sequence (g_n) of the sampled cascaded channel $h(t)$.

2. How many states does the correspoding state-space representation (Ungerboeck form) have?

3. Draw the trellis.

4. What is the maximum-likelihood update rule in the Viterbi algorithm for this example?