Institut Eurécom
Digital Communications

Final Examination

Date: Feb. 7, 2013

Duration: 2 hours

Chooose and complete any 4 out of the following questions. All questions will be graded equally. All documents are allowed.

1 Problem 1

Consider the 4-PAM contellation in Figure 1. Here the modulated symbols are mapped to the 2-bit sequences $(b_0 b_1)$ as shown using so-called *Gray Coding*.

1. Assuming that we observe the transmitted symbols in white Gaussian noise with variance $N_0/2$ as $y = s_m + z, 0 \leq m \leq 3$, write the likelihood functions $p(y|b_0)$ and $p(y|b_1)$.
2. What is the simplest detection rule for b_0?
3. Use the approximation $\log(e^a + e^b) \approx \max(a, b)$ to approximate the detection rule for b_1 as $b_1 = .5(1 + \text{sign}(|y| - (2\sqrt{E_s/5})))$.
4. Give a numerical expression (or even a bound) for the error probability for detecting b_0.

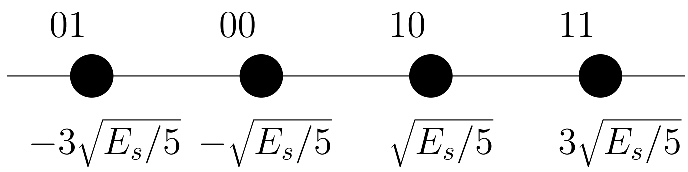

Figure 1: Constellation for Problem 3

2 Problem 2

Consider the binary communication problem consisting of waveforms:

$$s_0(t) = \begin{cases} s_p(t), t \in [0, T_p) \\ -s(t), t \in [T_p, T] \end{cases} \quad s_1(t) = \begin{cases} s_p(t), t \in [0, T_p) \\ s(t), t \in [T_p, T] \end{cases} \tag{1}$$

where $s_p(t)$ is some waveform with energy E_p and $s(t)$ is a waveform with energy E_s. The signal is transmitted over and AWGN channel with noise power spectral density N_0 and an unknown phase shift, so that the received signal is given by

$$y(t) = e^{j\phi} s_m(t) + \nu(t) \tag{2}$$

1. Give a set of basis functions for this signal set (hint: it is two-dimensional)
2. Is it an orthogonal signal-set?
3. What is the ML non-coherent receiver for this signal set and give its block diagram.
4. Give an expression for the error probability in terms of E_b/N_0 for this signal set (Hint: an appropriate adaptation of eq. 2.52 in the notes suffices)
5. Explain how the performance depends on the relationship between E_p and E_s and T_p and T.
6. What is the relationship of this receiver to an ideal coherent ML receiver (Hint: try to manipulate the decision rule so that it looks like an imperfect coherent receiver).

3 Problem 3

Consider an M-ary orthogonal signal set with signals, $s_m(t), m = 1, 2, \cdots, M$.. Now consider a new M-ary signal set (called an M-ary simplex) given by

$$s'_m(t) = s_m(t) - \frac{1}{M} \sum_{m=1}^{M} s_m(t). \tag{3}$$

Here we have just removed the mean value of the signal set

1. Show that the average energy of the signal set is reduced by a factor $1 - \frac{1}{M}$

2. Does the minimum-distance change?
3. What is the dimensionality of the new signal set? (Hint: Think about it in 2 and 3 dimensions and extrapolate)
4. What is the probability of error of the new signal set in terms of E_b/N_0 (Hint: Consider the difference with respect to the original orthogonal signal-set and your answer to question 3.2?)

4 Problem 4

Consider a receiver for OFDM system to be designed on a wireless channel. The sampling rate of the system is 15.36 Ms/s. The number of carriers per OFDM symbol is $N_c = 1024$. Two different transmission formats are used. First (normal prefix format) two different cyclic prefix lengths are used for every group of seven symbols, $N_{p0} = 80$ for the first symbol and $N_p = 72$ for the remaining 6 symbols. Second (extended prefix format), only one cyclic prefix length is used for every symbol, namely $N_p = 256$.

1. Under the assumption that 300 carriers are used in the positive part of the spectrum, 300 carriers are used in the negative part of the spectrum and that the DC carrier is skipped, what is the occupied bandwidth?
2. What is the maximum channel duration that the system can cope with (i.e. in both formats) and explain in words which format should be used as a function of channel conditions.
3. Assuming we use 16-QAM modulation, what is the spectral-efficiency of the system (spectral efficiency is measured in bits/s/Hz), including the overhead due to the cyclic prefix in both cases?

5 Problem 5

Consider the following detection problem

$$y(n) = x_p(n) * h(n) + z(n), n = 0, \cdots, N_x + N_h - 1$$

where $x_p(n)$ is a time-domain signal of length $N_x = N_c + N_p$ samples depending on a message p taking on values $p \in \{0, 1, \cdots, P-1\}$ where P can be made variable, $h(n)$ is a known time-limited channel of duration N_h samples and $z(n)$ is a zero-mean circularly symmetric complex Gaussian noise sequence with variance N_0. A cyclic prefix of length N_p is present in $x_p(n)$ and has duration $N_p > N_h$, so that we have the relationship

$$x_p(n) = x_p(n \mod N_c)$$

as in OFDM transmission. Furthermore we have that

$$x_p(n \mod N_c) = x((n - p\Delta) \mod N_c)$$

so that messages differ in their *cyclic* shifts of a known sequence $x(n)$.

1. For what value of Δ can the P signals at the receiver be considered orthogonal (i.e. provide the relationship between Δ and P). Explain.

2. Suppose we remove the first N_p samples at the receiver and transform the signal to the frequency-domain using an N_c-point DFT (i.e. like in OFDM) and assume that Δ and P are chosen such that the signals at the receiver for different values of p are orthogonal. Propose an optimal non-coherent receiver structure based on $X(k) = \mathrm{DFT}(x(n))$. As a guide, think about projecting (like in OFDM) in the frequency-domain using $X(k)$ and then going back to the time-domain to perform detection. You should make use of the cyclic-shifts in the time domain in the detection process.

6 Problem 6

Consider the following orthogonal modulation system

$$y(t) = \sqrt{E_s} h_m \xi_m(t) + z(t), m = 0, 1$$

where $\{\xi_m(t), m = 0, 1\}$ for an orthonormal set. h_m are zero-mean unit-variance and independent circularly-symmetric complex Gaussian random variables and $z(t)$ is a circularly-symmetric complex Gaussian random process with mean 0 and power spectral density N_0. This would typically represent a wideband FSK system with large frequency spacing and small symbol time in a rich multipath environment. Assume that the h_m are unknown to the receiver.

1. What are the basis functions for ML detection and what is the dimension of the signal-space?

2. Derive the ML receiver for this general non-coherent detection problem.

3. Give a bound or exact expression for the probability of error.