Institut Eurécom
Digital Communications

Midterm Examination

Date: Dec. 16, 2013

Duration: 2 hours

Answer any 3 out of 4 questions

1 Problem 1

Consider the three signals,

$$p_0(t) = \begin{cases} \sqrt{\beta E}\exp(-t), & t \in [0, T/2] \\ 0 & \text{otherwise} \end{cases}, p_1(t) = 0, p_2(t) = p_0(t - T/2)$$

(a) Find the value of β for which the average energy of this signal set is E.

(b) Find an orthonormal basis for this set of signals.

(c) Draw a constellation representing this signal set on the basis you found

(d) What is the minimum distance in terms of E.

2 Problem 2

Consider the quaternary set of signals, $x_m = \sqrt{\beta E}e^{j2\pi m/3}, m = 0, 1, 2, x_3 = 0$ transmitted over an AWGN channel

$$y = \sqrt{E}x_m + z$$

where z is a zero-mean circularly-symmetric complex Gaussan random variable with variance N_0.

(a) What is the value of β for which the average signal energy is E given that all signals are equally likely?

(b) Draw the decision regions for this constellation

(c) What is the union-bound on the probability of error for this signal set assuming an additive white-Gaussian noise channel model with power-spectral density N_0 and normalized signal energy?

(d) What is the coding gain/loss with respect to QPSK modulation?

3 Problem 3

Consider the 4-PAM contellation in Figure 1. Here the modulated symbols are mapped to the 2-bit sequences $(b_0 b_1)$ as shown using so-called *Gray Coding*.

(a) Assuming that we observe the transmitted symbols in white Gaussian noise with variance $N_0/2$ as $y = s_m + z, 0 \leq m \leq 3$, write the likelihood functions $p(y|b_0)$ and $p(y|b_1)$.

(b) What is the simplest detection rule for b_0?

(c) Use the approximation $\log(e^a + e^b) \approx \max(a, b)$ to approximate the detection rule for b_1 as $b_1 = .5(1 + \text{sign}(|y| - (2\sqrt{E_s/5})))$.

(d) Give a numerical expression (or even a bound) for the error probability for detecting b_0.

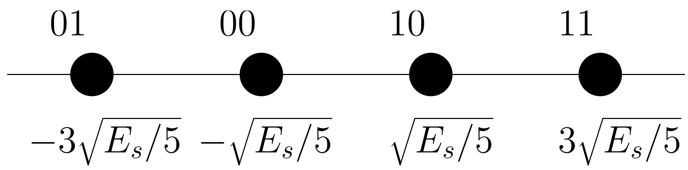

Figure 1: Constellation for Problem 3

4 Problem 4

Consider the binary communication problem consisting of vectors:

$$\mathbf{y} = \sqrt{E}h \begin{pmatrix} 1 \\ x \end{pmatrix} + \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

where $x \in \{-1, 1\}$ (antipodal or BPSK modulation), z_i are zero-mean independent complex circularly symmetric Gaussian random variables with variance N_0, and h is a zero-mean complex circularly symmetric Gaussian random variables with variance 1.

(a) What is the ML receiver (non-coherent) for this problem?

(b) Give a bound (or exact expression) for the error probability.