# Exercise sheet 4

## Patrick Loiseau

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## Exercise 1

1. Find all pure strategies and mixed strategies Nash equilibria of the following two-players game:

|   | a | b |
|---|---|---|
| A | 2, 1 | 0, 0 |
| B | 1, 0 | 0, 2 |

## Exercise 2:

Consider the following two-persons game:

|   | l | r |
|---|---|---|
| U | 12, 2 | 3, 9 |
| D | 5, 8 | 4, 2 |

1. Find all pure strategies and mixed strategies Nash equilibria.

2. Assume now that $u_2(D, l)$ is reduced from 8 to 6. Find all pure strategies and mixed strategies Nash equilibria.

3. Compare the strategies of player 1 and 2 in the mixed strategy Nash equilibria of questions 1. and 2. Comment.

## Exercise 3:

Suppose that player 1's car is not working properly: it lacks power. He does not know whether it needs a small engine cleaning or a major repair (say, a new engine). The probability that it needs a new laser is $\rho$. At his local garage, he finds that a new engine costs $L$, while a cleaning costs $C$ ($L > C$). He knows that the expert at the garage, player 2, gets the same profit $\pi$, if she charges him for a new engine and indeed fixes the engine, or if she charges him for a cleaning and indeed just cleans it. But she can make more profit, $\Pi > \pi$ if she charges him for a new engine but in fact (secretly) just cleans it. If it only needed a cleaning anyway, then she will get away with this, but she knows she will get sent to jail if she only cleans it when it needed a new engine. The expert is very good at her job, so she knows which is needed.

1. Explain why player 1 should always believe player 2 when she says it just needs a cleaning but why he might be skeptical if she says it needs a new laser.

Player 1 can reject the local expert's advice and get a second opinion from a consultant who never lies. Assume however that, if he does so, he must accept the second expert's advice and accept new repair costs $L' > L$ or $C' > C$. The game is then:

|   | Honesty | Dishonesty |
|---|---|---|
| Always accept advice | $-\rho L - (1-\rho)C, \pi$ | $-L, \rho\pi + (1-\rho)\Pi$ |
| Reject if told 'new engine' | $-\rho L' - (1-\rho)C, (1-\rho)\pi$ | $-\rho L' - (1-\rho)C', 0$ |

2. Explain the terms in the payoff matrix.

3. Assume that $L > \rho L' + (1 - \rho)C'$. Is there a pure strategy Nash equilibrium?

4. Find the mixed strategy Nash equilibrium (as a function of the parameters).

5. As we increase the cost of repair at the local garage $L$, what happens to the equilibrium probability that the expert chooses 'honest'? What happens to the equilibrium probability that player 1 chooses 'Reject if told 'new engine"? Comment.

6. As we increase the profit from lying $\Pi$, what happens to the equilibrium probability that the expert chooses 'honest'? What happens to the equilibrium probability that player 1 chooses 'Reject if told 'new engine"? Comment.

7. It has been said that, in America, when people go to the doctor, they never think they have a cold: they think they have 'mono'. Assuming this is true, why might we expect doctors in America often to act dishonestly? [Hint: think about how the parameter $\rho$ affects the equilibrium in the above model].