Exercise sheet 3

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Game Theory, Fall 2015

Exercise 1: Bertrand duopoly

In class, we considered Cournot competition where two firms choose quantities and let the price be fixed by the market. We consider here a different model of competition on prices called Bertrand competition.

Consider 2 firms producing identical products (i.e., that are perfect substitutes) with a constant marginal cost c. Each firm i choose the price $p_i \in [0, 1]$. We assume that the demand curve is linear so that the total demand is $Q(p) = 1 - p$ where $p = \min(p_1, p_2)$ and is entirely directed to the firm with the smallest price. If both prices are equal, the demand is equally shared between the two firms. We denote by q_i the demand to firm i. For instance, the demand to firm 1 is

$$q_1(p_1, p_2) = \begin{cases} 1 - p_1 & \text{if } p_1 < p_2, \\ 0 & \text{if } p_1 > p_2, \\ (1 - p_1)/2 & \text{if } p_1 = p_2. \end{cases}$$

Note that this corresponds to the demand model used in class for Cournot duopoly with $a = b = 1$. Both firms maximize their profit:

$$u_1(p_1, p_2) = p_1 q_1 - c q_1,$$
$$u_2(p_1, p_2) = p_2 q_2 - c q_2.$$

1. Suppose first that firm 1 is alone in the market. What is the optimal price p_1? What is the quantity purchased q_1? Comment.

2. Compute the best response of firm 1 to a price p_2 of firm 2. [hint: no calculation, just reasoning.]

3. Find the Nash equilibria. Comment.

Comments: The result of question 3. is not very realistic. It is due to the modeling assumption that products are perfect substitutes. In the next exercise, we remove this assumption by using the linear city model.

Exercise 2: The linear city model

In this exercise, we consider a duopoly where firms compete on prices (like in Bertrand duopoly) but we relax the assumption that products are identical.

We think of a city as a line of length 1. There are 2 firms, at each end of the line. Each firm produces a product at constant marginal cost c. The firms simultaneously choose the price of their product p_1 and p_2. Potential customers are uniformly distributed along the line, with a total population normalized to 1. Each potential customer buys exactly one unit either from firm 1 or firm 2, so the total demand is 1. Given its position on the line y (i.e., the customer is at distance y from firm 1 and $1 - y$ from firm 2), a customer will buy its unit:

$$\text{from firm 1 if } p_1 + ty^2 < p_2 + t(1 - y)^2$$
$$\text{from firm 2 if } p_1 + ty^2 > p_2 + t(1 - y)^2 \tag{1}$$

and will toss a fair coin is these two quantities are equal.

Conditions (1) translate what the customers care about: they try to buy from the firm with smallest $price + t(distance)^2$. If we think of the line as representing geographical distance, $t(distance)^2$ is a transport cost. However, other interpretations are possible. For instance, we can think of the line as representing some aspect of the product quality. Then, the proximity to a firm represents the preference and the term $t(distance)^2$ measures the inconvenience of having to move away from the preferred product. Parameter t represents how much the products are *differentiated*. If $t = 0$, the products are identical.

The demand resulting from customers choices to firm i is denoted $d_i(p_1, p_2)$. As usual, each firm's objective is to maximize its profit:

$$u_1(p_1, p_2) = p_1 d_1(p_1, p_2) - c d_1(p_1, p_2),$$
$$u_2(p_1, p_2) = p_2 d_2(p_1, p_2) - c d_2(p_1, p_2).$$

1. Will either firm i ever set a price $p_i < c$? Why?

2. Suppose that firm 2 sets a price p_2. What is the maximum price p_1 for which firm 1 captures the entire market (i.e., has demand 1)?

3. Suppose that prices p_1 and p_2 are close enough that each firm has positive demand. Find the position of the customer who is indifferent between buying from firm 1 and buying from firm 2. Compute the demand to firm 1 $d_1(p_1, p_2)$.

4. Suppose that prices p_1 and p_2 are close enough that each firm has positive demand. Compute the best-response of firm 1 to p_2.

5. Draw the complete best response diagram (best careful while plotting $BR_1(p_2)$ when $p_2 < c - t$ and $p_2 > 2t + c$ and similarly for BR_2).

6. Find the Nash equilibrium.

7. What is the equilibrium when $t = 0$? Comment.

Comments: Products differentiation allows firms to charge higher prices and make more profit.