# 8. Basic Oligopoly models

## Key terms

- Strategic interaction
- Different oligopoly types
    - Sweezy:
        - kinked demand curve
        - follow price changes in case of decrease
        - ignore price changes in case of increase
        - differentiated products
    - Cournot:
        - Simultaneous decisions by all players
        - Quantity is the strategy
        - Homogeneous or differentiated products
        - Estimate impact of output changes of rivals on market price
        - Reaction / optimal response functions
    - Stackelberg:
        - Leader-follower model
        - Sequential decisions
        - Derivation: first $r_F(Q_L)$, then $Q_L$
        - Quantity is the strategy
        - Homogeneous or differentiated products
    - Bertrand:
        - Price competition
        - Derive reaction function based on the rivals price
        - Homogeneous products
- Conditions
- Isoprofit curves
- Collusion = cooperative behavior
    - Oligopoly firms come together to affect their quantity and manipulate price
    - Cooperative game = cartel
    - Typical example: OPEC (Organization of the Petroleum Exporting Countries)
    - Prone to cheating
        - Not sustainable
        - Anti-competitive
        - Illegal in most jurisdictions

---

## Conditions for Oligopoly

> ⚖️ **Oligopoly**
>
> **Oligopoly market structures** are characterized by a **few firms**, each of which is **large** relative to the total industry.

- Typical number of firms: 2-10
- Products can be
    - identical
    - differentiated

> ⚖️ **Duopoly**
>
> A **duopoly** is a market structure in which two firms dominate the market.

E.g. Boeing and Airbus, Pepsi and Coca-Cola

Difficulties

- **hard to manage**, because managers must consider the reactions of **competitors** to their own actions
- barrier to entry
- barrier to exit: high fixed costs

An oligopoly can be

- **cooperative** (collusion, cartel)
- **non-cooperative**

---

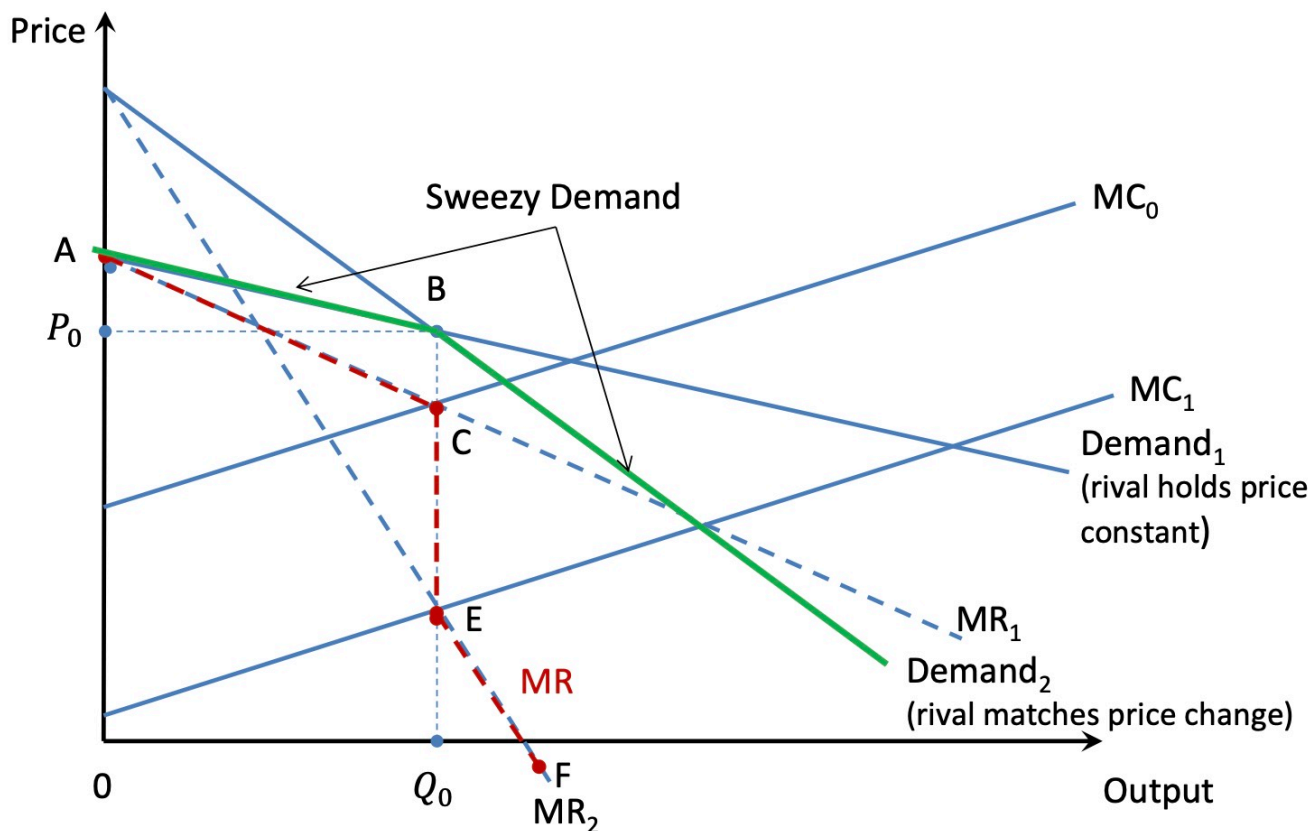# Strategic Interaction

> ⚖️ **Strategic interaction**
>
> **Strategic interaction** occurs when each firm's demand depends on actions of rivals.

---

# Sweezy Oligopoly

> ⚖️ **Sweezy Oligopoly**
>
> In a **Sweezy oligopoly**,
>
> - there are **few firms** in the market, serving **many customers**,
> - the firms produce **differentiated products**,
> - each firm believes the following 2 things:
>   1. if it **lowers the price below the prevailing level**, the competitors will **follow it**
>   2. if it **raises the price above the prevailing level**, the competitors will **not follow it**
> - barriers to entry exist
> - the market demand curve is **kinked** at the prevailing price

## Cournot Oligopoly

> ⚖️ **Cournot Oligopoly**
>
> In a **Cournot oligopoly**,
>
> - there are **few firms** in the market, serving **many customers**,
> - the firms produce either **identical** or **differentiated products**,
> - each firm believes rivals will hold their **output constant** if it changes its own output,
> - barriers to entry exist

### Reaction Functions

Each firm makes an output decision based on the output level under the belief that its rival will hold its output constant when the other firm changes its output.

⇒ Each firm's marginal revenue is impacted by the other firms output decisions.

> ⚖️ **Reaction function / Best response function**
>
> The **best response** or **reaction function** captures the relationship between each firm's profit-maximizing output.

> Σ **Linear inverse demand in Cournot**
>
> $$P = a - b(Q_1 + Q_2)$$

$$C_1(Q_1) = c_1 Q_1 \qquad C_2(Q_2) = c_2 Q_2$$

Where $c_1$ and $c_2$ are **constant marginal costs** for firm 1 and firm 2 respectively.

Σ **Reaction functions in a Cournot duopoly**

The **reaction functions** are

$$Q_1 = r_1(Q_2) = \frac{a - c_1}{2b} - \frac{Q_2}{2}$$
$$Q_2 = r_2(Q_1) = \frac{a - c_2}{2b} - \frac{Q_1}{2}$$

So the output for firm 1 is the reaction of firm 1 to the output of firm 2, and vice versa.

Because

1. $\pi = PQ - C(Q)$, each firm's profit depends on both its own output and the output of its rival.
2. Apply FOC (='First Order Condition: Set MR = MC) to get each firm's optimal output as a function of the other firm's output.

$$\begin{aligned} \pi_1 &= PQ_1 - C_1(Q_1) \\ &= (a - b(Q_1 + Q_2))Q_1 - c_1 Q_1 \\ &= aQ_1 - bQ_1^2 - bQ_1Q_2 - c_1 Q_1 \end{aligned}$$

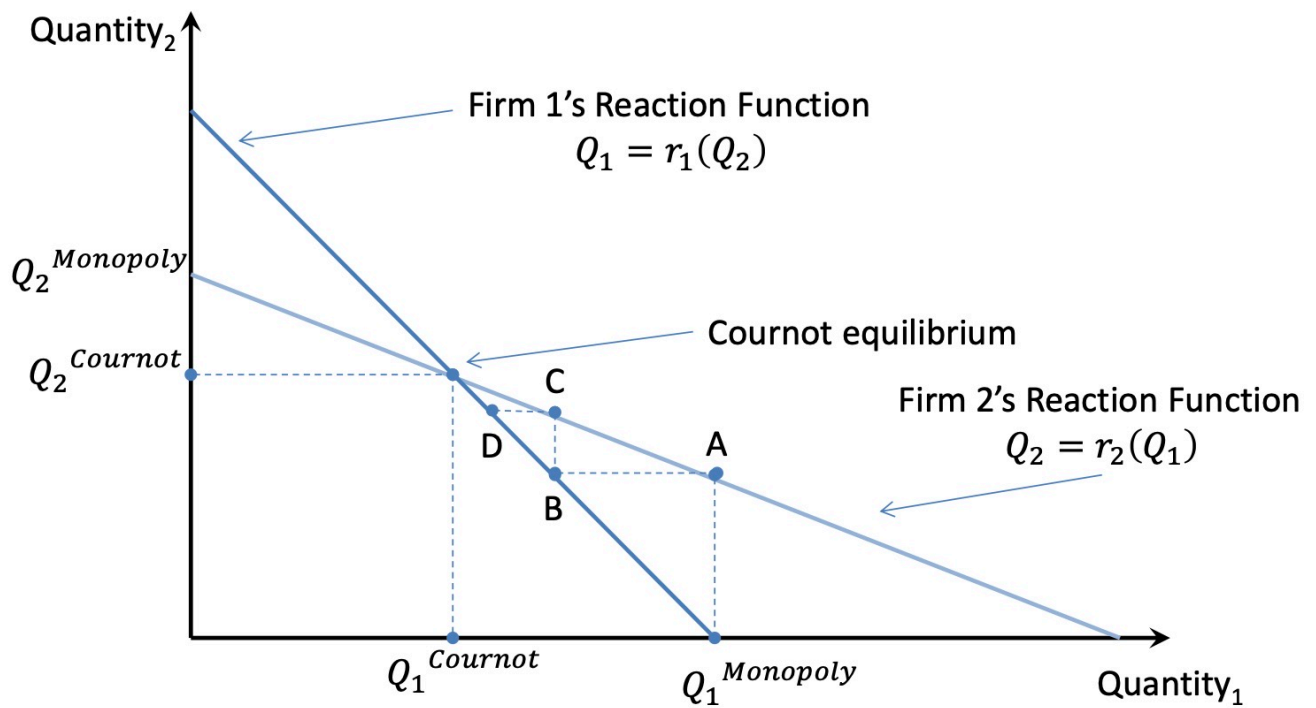FOC: set partial derivative of $\pi_1$ with respect to $Q_1$ equal to zero:

$$\frac{\partial \pi_1}{\partial Q_1} = a - 2bQ_1 - bQ_2 - c_1 = 0$$

Now solve for $Q_1$ to get the reaction function $r_1(Q_2)$.

**Equilibrium in Cournot**

The **Cournot equilibrium** is the intersection of the reaction functions of the firms, where neither firm has an incentive to change its output given the other firm's output.
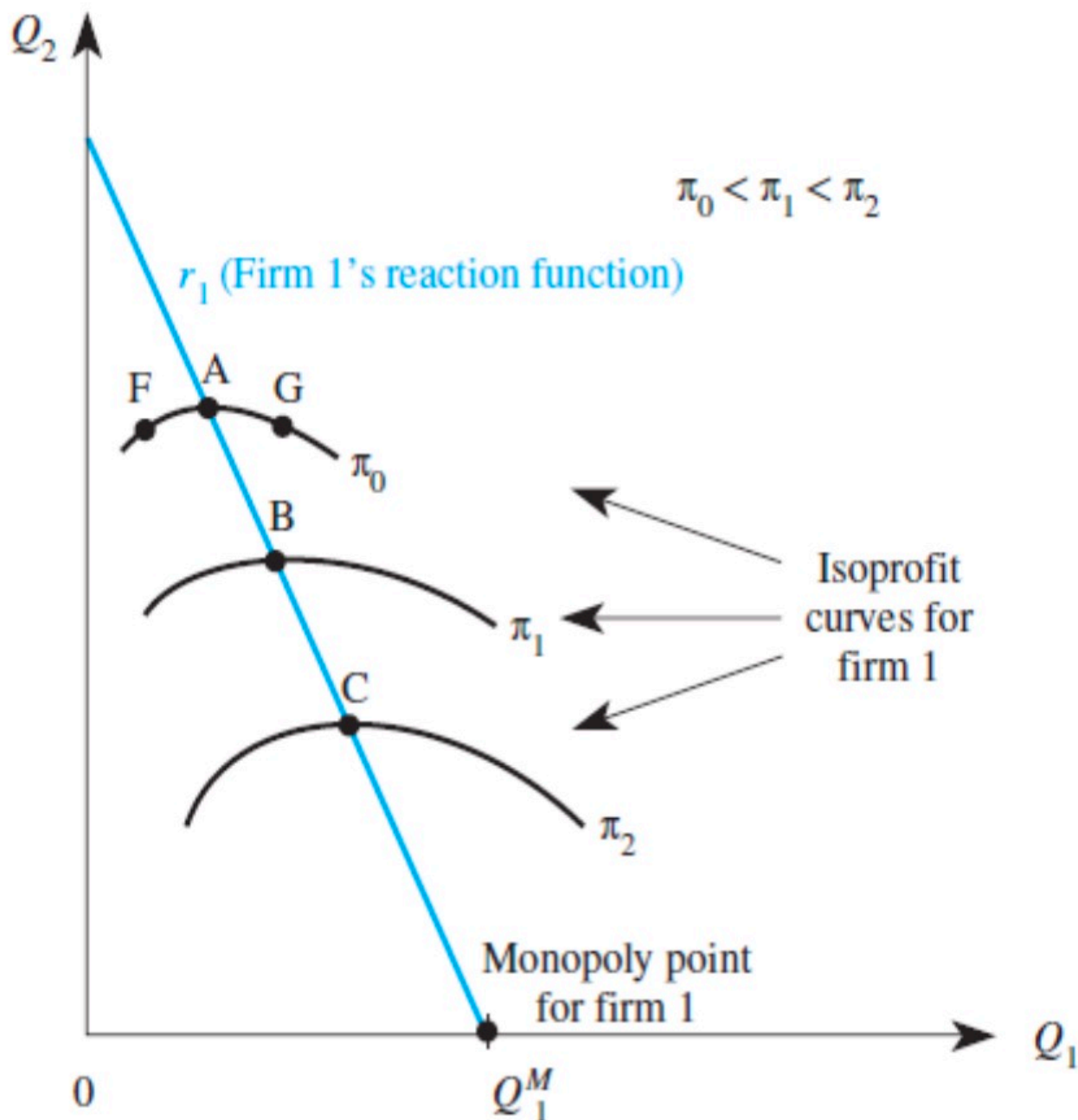
$$Q_1 = Q_2 = Q$$

The figure shows a Cournot duopoly diagram. The vertical axis is labeled Quantity$_2$ and the horizontal axis is labeled Quantity$_1$. Firm 1's Reaction Function $Q_1 = r_1(Q_2)$ and Firm 2's Reaction Function $Q_2 = r_2(Q_1)$ are plotted, intersecting at the Cournot equilibrium. Points labeled $Q_2^{Monopoly}$, $Q_2^{Cournot}$ on the vertical axis and $Q_1^{Cournot}$, $Q_1^{Monopoly}$ on the horizontal axis. Points A, B, C, D are marked near the Cournot equilibrium.

## Isoprofit Curves

> ⚖ **Isoprofit curves**
>
> **Isoprofit curves** are functions that define combinations of outputs produced by **all** firms that yield a given firm the same level of profits.
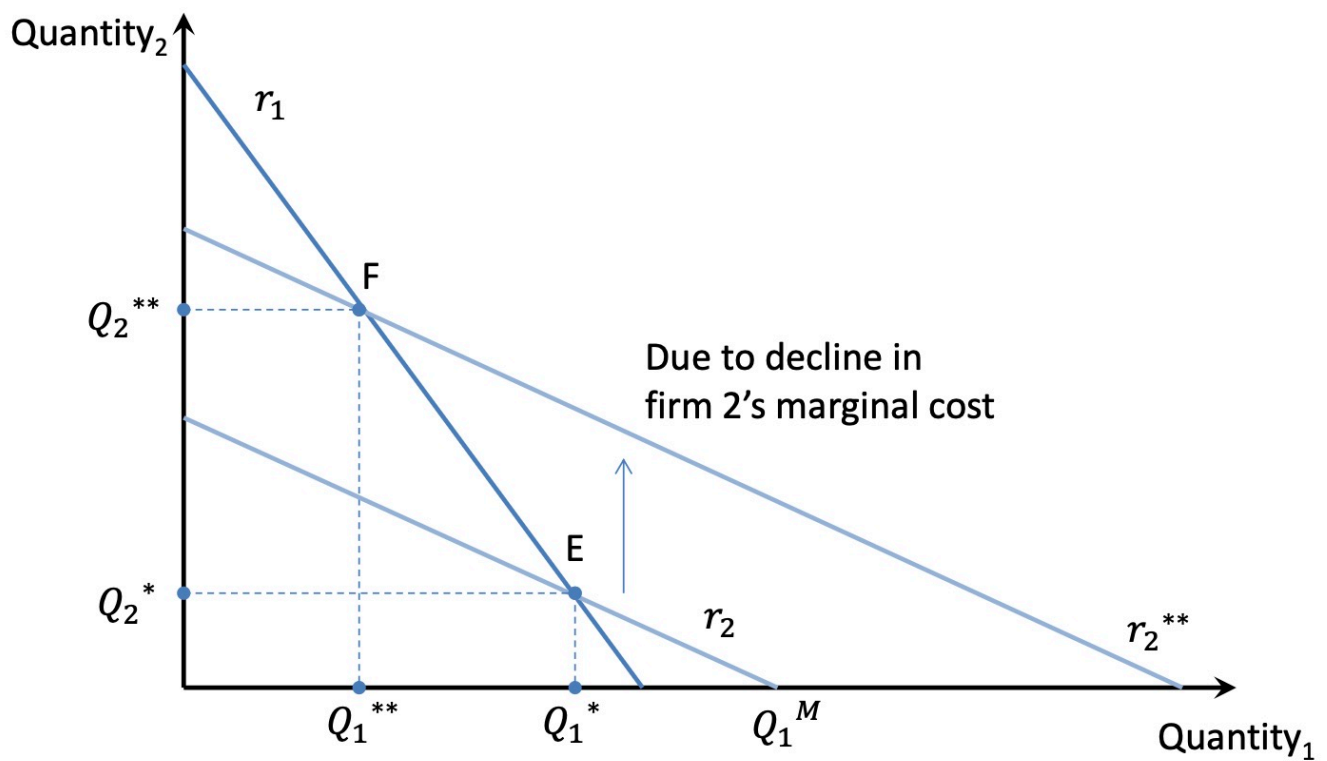
## Characteristics

Isoprofit curves for $\mathrm{firm}_1$:

$$\pi_0 < \pi_1 < \pi_2$$

$r_1$ (Firm 1's reaction function)

Isoprofit curves for firm 1

Monopoly point for firm 1

$Q_1^M$

**Characteristics of isoprofit curves**

1. Every point on a given isoprofit curve yields the same profit for firm 1.
2. Isoprofit curves that **lie closer to firm 1's monopoly output** $Q_M^1$ **yield higher profits** for firm 1.
3. The isoprofit curves for Firm 1 reach their peak where they intersect Firm 1's reaction function $r_1(Q_2)$.
4. The isoprofit curves do not intersect each other.

If $\text{firm}_2$'s marginal cost $c_2$ decreases, its isoprofit curves shift upward, leading to a higher equilibrium output for $\text{firm}_2$ and a lower equilibrium output for $\text{firm}_1$.

## Outcomes

1. Both firms **cooperate**
   - High joint profits
2. One **cheats** while the other **cooperates**
   - Cheater wins big
3. Both **cheat**
   - Low profits for both
   - We're back at Cournot equilibrium

## Collusion in Cournot Oligopoly

Collusion = cartel (OPEC = Organization of the Petroleum Exporting Countries)

> ⚖️ **Collusion**
>
> Markets with only a few dominant firms can coordinate to restrict output to their benefit at the expense of consumers. Restricted output leads to higher market prices and higher profits for the firms. This behavior is called **collusion**.

It is, however, prone to cheating behavior.

---

## Stackelberg Oligopoly

> ⚖️ **Stackelberg Oligopoly**
>
> In a **Stackelberg oligopoly**,
>
> - there are **few firms** in the market, serving **many customers**,
> - the firms produce either **homogeneous** or **differentiated products**,
> - a single firm (the **leader**) chooses its output level **before all other firms** choose their output levels,
> - the other firms (the **followers**) choose outputs that maximize their own profits, given the leader's output,
> - barriers to entry exist

Again, given the lineair demand function

$$P = a - b(Q_1 + Q_2)$$

and the cost functions

$$C_1(Q_1) = c_1 Q_1 \qquad C_2(Q_2) = c_2 Q_2$$

we can derive the reaction function of the follower firm (firm 2) as in the Cournot model.

The follower sets output according to the reaction function $Q_2 = r_2(Q_1)$. Because the follower maximises its profit, we look for $Q_2$ such that $\pi_2$ is maximized, given $Q_1$:

$$\begin{aligned}
\pi_2 &= PQ_2 - C_2(Q_2) \\
&= (a - b(Q_1 + Q_2))Q_2 - c_2 Q_2 \\
&= aQ_2 - bQ_1 Q_2 - bQ_2^2 - c_2 Q_2
\end{aligned}$$

FOC: set partial derivative of $\pi_2$ with respect to $Q_2$ equal to zero:

$$\frac{\partial \pi_2}{\partial Q_2} = a - bQ_1 - 2bQ_2 - c_2 = 0$$

> **Σ Stackelberg FOLLOWER reaction function**
>
> $$\boxed{Q_2 = r_2(Q_1) = \frac{a - c_2}{2b} - \frac{Q_1}{2}} \quad \text{(Follower)}$$

Now, to find the leader's optimal output, we substitute the follower's reaction function into the leader's profit function $\pi_1$:

$$\begin{aligned}
\pi_1 &= PQ_1 - C_1(Q_1) \\
&= (a - b(Q_1 + Q_2))Q_1 - c_1 Q_1 \\
&= (a - b(Q_1 + (\frac{a - c_2}{2b} - \frac{Q_1}{2})))Q_1 - c_1 Q_1 \\
&= (a - b(\frac{a - c_2}{2b} + \frac{Q_1}{2}))Q_1 - c_1 Q_1 \\
&= (a - \frac{a - c_2}{2} - \frac{bQ_1}{2})Q_1 - c_1 Q_1 \\
&= (\frac{a + c_2}{2} - \frac{bQ_1}{2})Q_1 - c_1 Q_1
\end{aligned}$$

FOC: set derivative of $\pi_1$ with respect to $Q_1$ equal to zero:

$$\frac{d\pi_1}{dQ_1} = \frac{a + c_2}{2} - bQ_1 - c_1 = 0$$

> **Σ Stackelberg LEADER reaction function**
>
> $$\boxed{Q_1 = \frac{a + c_2 - 2c_1}{2b}} \quad \text{(eader)}$$

# Bertrand Oligopoly

In a **Bertrand oligopoly**,

- there are **few firms** in the market, serving **many customers**,
- the firms produce **identical products**,
- firms produce **at constant marginal cost**,
- firms engage in **price competition**,
- firms react optimally to price changes of their rivals,
- consumers have **perfect information** and there are **no transaction costs**,
- barriers to entry exist

The conditions for a Bertrand oligopoly imply that firms in this market will **undercut one another** to capture the entire market leaving the rivals with no profit.

All **consumers** will purchase at the **low-price firm**.

The **price war** would come to an end when the price each firm charges equals the marginal cost:

**Bertrand equilibrium price**

$$P_1 = P_2 = MC$$

I.e. the socially efficient level of output.

---

## Comparison of Oligopoly Models

1. As a CEO who only cares about profit, you would prefer **Cournot with Collusion**.
2. As a consumer, you would prefer **Bertrand (price war)**.
3. As a regulator, you would think the most realistic and stable model is **Cournot**.

---

## Contestable Markets

⚖️ **Contestable market**

**Contestable markets** involve strategic interaction among existing firms and potential entrants into the market.

A market is contestable if

- all producers have access to the **same technology**
- **consumers respond quickly** to price changes (high elasticity of demand)
- existing firms cannot respond quickly to entry by lowering price
- there are **no sunk costs**

If these conditions hold, incumbent firms (i.e. firms already in the market) have **no market power** over consumers.

*Example*: The airline industry, where low-cost carriers can enter a market if prices are high, forcing incumbents to keep prices low.