4. The Production Process and Costs

Key Concepts

- Production function
- Short run vs. long run
- Total, average and marginal product
- Profit maximization when labor and capital vary in the short run ($VPM_L = w$ and $VMP_K = r$)
- Isoquants and the marginal rate of technical substitution (MRTS)
- Isocost line
- Cost minimization for a given output Q ($\frac{MP_L}{w} = \frac{MP_K}{r}$ or $MRTS_{KL} = \frac{w}{r}$)
- Short-run and long-run cost functions
- Average and marginal costs
- Minimizing ATC where $MC = ATC$
- Fixed vs. sunk costs
- Economies of scale ($LRAC$ decreasing)
- Diseconomies of scale ($LRAC$ increasing)
- Constant returns to scale ($LRAC$ constant)
- Economies of scope ($C(Q_1, 0) + C(0, Q_2) > C(Q_1, Q_2)$)
- Cost complementarities ($\frac{\partial MC_1}{\partial Q_2} < 0$)

Production Function

Production Function

The **production function** is a mathematical function that defines the maximum amount of output that can be produced with a given set of inputs.

- Q = level of **output**
- K = quantity of **capital input**
- L = quantity of **labor input**

∑ Production Function

$$\boxed{Q = f(K, L)}$$

Short Run vs. Long Run

Short Run

The **short run** is the period of time where **some factors of production are fixed** (inputs) and constrain a manager's decisions.

Long Run

The **long run** is the period of time where **all factors of production are variable** (inputs) and a manager can adjust all inputs.

Measures of Productivity

Total product

Total Product (TP)

The **total product** TP is the **total output** Q produced with a given quantity of inputs.

Average product

Average Product (AP)

The **average product** AP is the total product TP **divided by the quantity of inputs used**.

Average Product of labor (AP_L)

$$\boxed{AP_L = \frac{TP}{L}}$$

Average Product of capital (AP_K)

$$\boxed{AP_K = \frac{TP}{K}}$$

Marginal product

Marginal Product (MP)

The **marginal product** MP is the **additional output** produced by adding **one more unit of input**.

Diminishing Marginal Product

As more and more of an input is added, the marginal product of that input eventually declines, holding all other inputs constant.

Marginal Product of labor (MP_L)

$$\boxed{MP_L = \frac{\Delta TP}{\Delta L}}$$

Marginal Product of capital (MP_K)

$$\boxed{MP_K = \frac{\Delta TP}{\Delta K}}$$

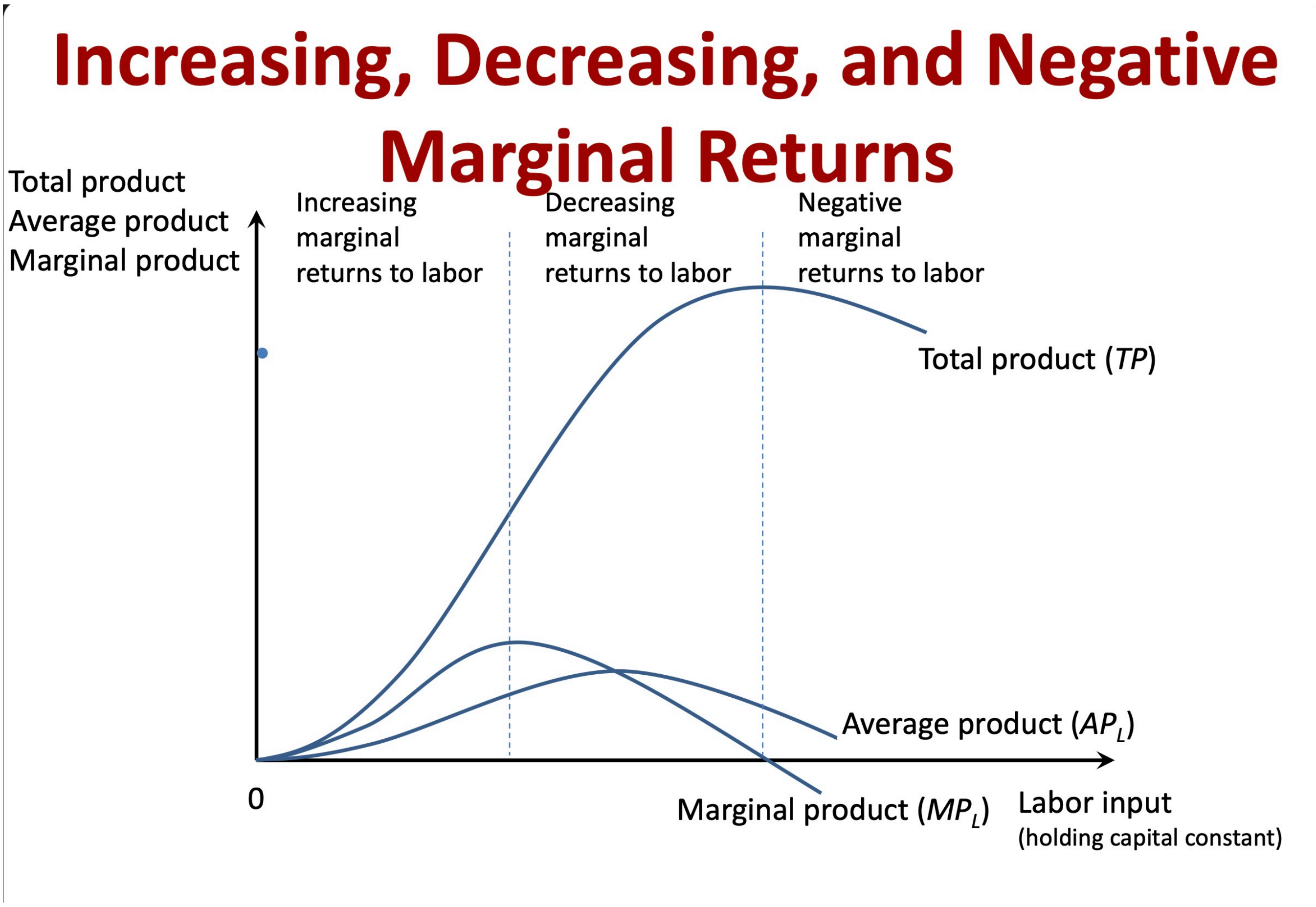
Graphical interpretation

- Total, average and marginal product on the -axis
- Quantity of labor (or capital) on the -axis

Total product:

- Increasing at an increasing rate when $MP > AP$ (and MP is increasing)
  - "Increasing marginal returns to labor"

- Increasing at a decreasing rate when $MP > AP$ (and MP is decreasing)
  - "Decreasing marginal returns to labor"
  - Efficiency of adding more labor is decreasing
- Maximum when $MP = 0$
  - "Maximum total product"
- Decreasing when $MP < 0$
  - "Negative marginal returns to labor"
  - Adding more labor decreases total product, because coordination problems arise.



Maximising profits when labor and capital vary in the *short run*

VMP's for labour and capital

- w = **wage rate** (price of labor)
- r = **rental rate** (price of capital)

∑ **Value of Marginal Product of Labor (VMP_L)**

$$\boxed{VMP_L = P \cdot MP_L}$$

This is the value of the additional output produced by hiring one more unit of **labor**.

∑ **Value of Marginal Product of Capital (VMP_K)**

$$\boxed{VMP_K = P \cdot MP_K}$$

This is the value of the additional output produced by hiring one more unit of **capital**.

Profit-maximization input usage

To maximize profits, use input levels at which **marginal benefits equals marginal costs**.

Profit Maximization Rule for Hiring Labor and Capital

A firm should **hire labor up to the point** where

$$\text{Profit maximied} \iff \boxed{VMP_L = w}$$

and **hire capital up to the point** where

$$\text{Profit maximied} \iff \boxed{VMP_K = r}$$

Algebraic Forms of Production Functions

Linear

∑ Linear

$$\boxed{Q = K + L}$$

Where

- Marginal products:
  - $MP_K =$
  - $MP_L =$
- Average products:
  - $AP_K = \frac{K+L}{K}$
  - $AP_L = \frac{K+L}{L}$

Leontief

∑ Leontief

$$\boxed{Q = \min(K, L)}$$

E.g. $Q = \min(2K, L)$ *means that 2 units of capital and 3 units of labor are needed to produce 1 unit of output. If **either** input is **fixed**, the other input must be adjusted to maintain the desired level of output.*

Cobb-Douglas

∑ Cobb-Douglas

$$\boxed{Q = K^\alpha L^\beta}$$

Where

- Marginal products:
  - $MP_K = \alpha K^{\alpha-1} L^\beta$
  - $MP_L = \beta K^\alpha L^{\beta-1}$
- Average products:
  - $AP_K = K^{\alpha-1} L^\beta$
  - $AP_L = K^\alpha L^{\beta-1}$

Isoquants and the Marginal Rate of Technical Substitution (MRTS)

Isoquant

Isoquants capture the trade-off between combinations of inputs that yield the **same level of output** in the **long run**, when all inputs are variable.

- K on the -axis
- L on the -axis

Marginal Rate of Technical Substitution (MRTS)

The **marginal rate of technical substitution (MRTS)** is the rate at which a producer can **substitute between two inputs and keep the output level constant.**

∑ Marginal Rate of Technical Substitution (MRTS)

$$\boxed{MRTS_{KL} = \frac{MP_L}{MP_K}}$$

and it is the **absolute value of the slope of the isoquant**.

$$\boxed{MRTS_{KL} = \frac{dK}{dL}_{\text{on isouant}}}$$

Diminishing MRTS

Diminishing MRTS

As a producer substitutes labor for capital (moves down along an isoquant), the MRTS

- Decreases
- The isoquant becomes flatter

Isocost

Isocost Line

An **isocost line** shows all **combinations of inputs** that yield the **same cost**.

∑ Isocost Line

$$\boxed{C = wL + rK} \qquad \boxed{K = \frac{C}{r} - \frac{w}{r}L}$$

- For given input prices, isocosts **farther from the origin represent higher cost levels**.
- Changes in input prices **change the slope** of the isocost line.

Cost minimization

Cost Minimization

Producing at the **lowest possible cost** for a **given level of output**.

Cost Minimization Rule

Produce at a **given level of output** where the **marginal product per dollar spent** is **equal** for all inputs:

$$\text{Cost for } Q \text{ minimied } \iff \boxed{\frac{MP_L}{w} = \frac{MP_K}{r}}$$

Or equivalently,

$$\text{Cost for } Q \text{ minimied } \iff \boxed{MRTS_{KL} = \frac{MP_L}{MP_K} = \frac{w}{r}}$$

$\rightarrow$ SO: to minimize the cost of producing a given level of output Q, the firm should use **less of an input** and **more of other** inputs when that input's **price rises**.

Cost Function

Cost Function

Mathematical relationship that relates **cost** to the **cost-minimizing output** associated with an isoquant (=same level of output).

Short-run costs

- Fixed costs FC
- Variable costs $VC(Q)$
- Total costs $TC(Q)$

Σ Short-run Cost Function

$$\boxed{TC(Q) = FC + VC(Q)}$$

Long-run costs

Rule

- In the long run, **all costs are variable**
- Since a manager is **free to adjust levels of all inputs**.
- So $FC = 0$ and $TC(Q) = VC(Q)$.

Average costs

∑ **Average Costs**

$$\textbf{erage fied cost (C)} \quad \boxed{AFC = \frac{FC}{Q}}$$

$$\textbf{erage ariable cost (C)} \quad \boxed{AVC = \frac{VC(Q)}{Q}}$$

$$\textbf{erage total cost (C)} \quad \boxed{ATC = \frac{TC(Q)}{Q} = AFC + AVC}$$

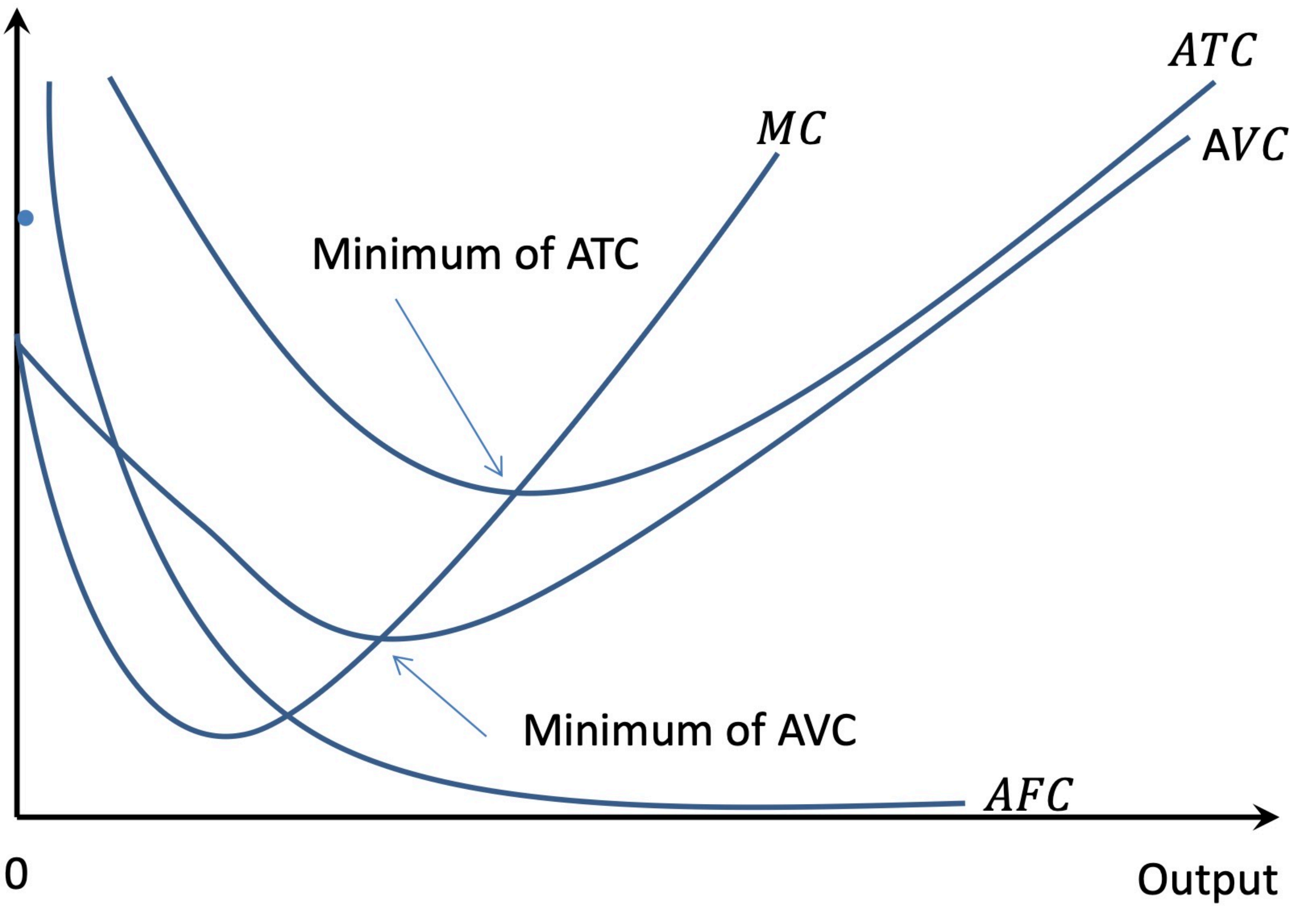
Marginal cost

Marginal Cost (MC)

The incremental cost of producing an additional unit of output.

∑ **Marginal Cost**

$$\boxed{MC = \frac{dC}{dQ} = \frac{dVC(Q)}{dQ}}$$



*Note that the ATC's are **short-run** average total cost curves, because they depend on the level of fixed capital K. When the level of fixed capital K changes, the ATC curve shifts. We can see this on the long-run $LRAC$ curve.*

Minimum average cost

The **marginal cost curve** intersects the **average total cost curve** at its **minimum point**.

$$\boxed{\text{Minimum } ATC \iff MC = ATC}$$

Why?

- $MC < ATC \implies TC$ is decreasing $\implies ATC$ is decreasing.
- $MC > ATC \implies TC$ is increasing $\implies ATC$ is increasing.
- $\frac{dATC}{dQ} = 0 \iff \frac{d}{dQ}\left(\frac{C}{Q}\right) = 0 \iff = -\frac{1}{Q^2}C - \frac{1}{Q}MC = 0 \iff MC = ATC.$

Fixed vs. *Sunk* Costs

Fixed Costs

Fixed costs are costs that **do not vary with the level of output** Q.

Sunk Costs

Sunk costs are costs that are **forever lost** after it has been paid.

E.g. rent paid in advance

Irrelevance of Sunk Costs

A decision maker should **ignore sunk costs to maximize profits or minimize losses**.

Cubic Cost function

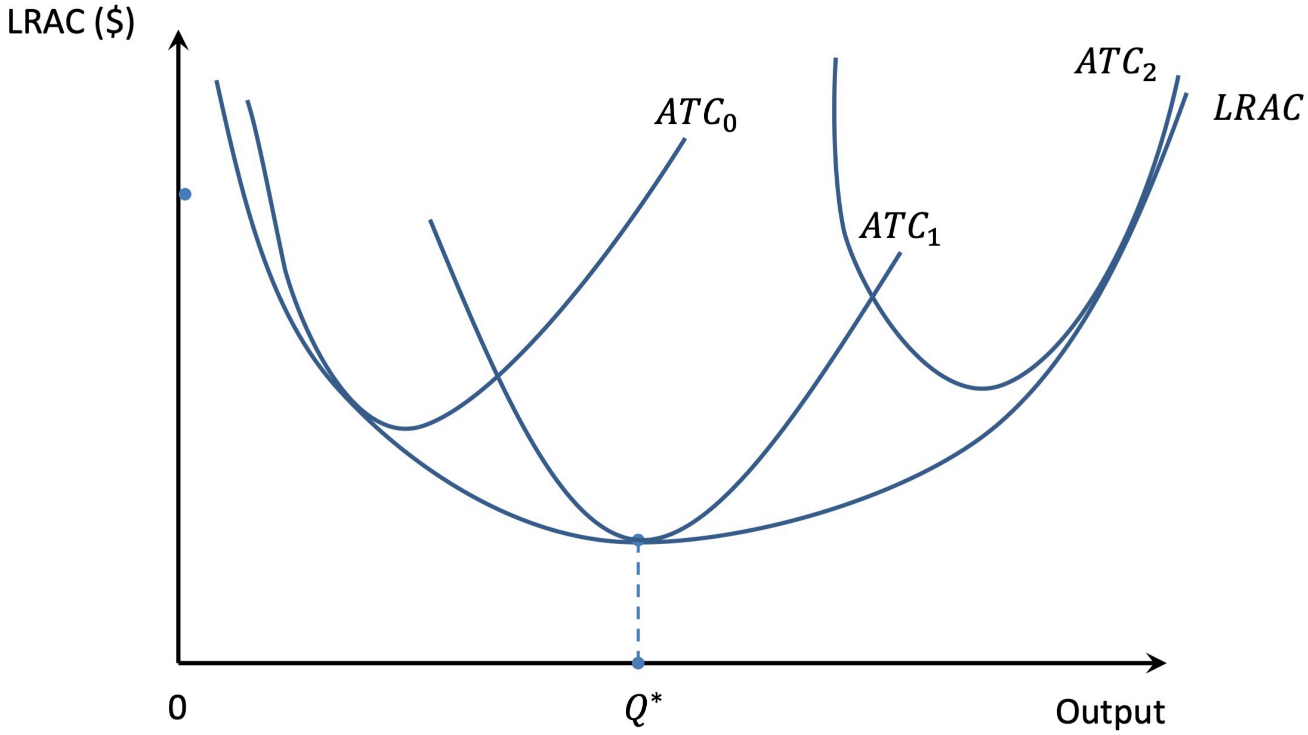
$$\boxed{C(Q) = F + Q + Q^2 + Q}$$

where F represents **fixed costs**, has marginal cost

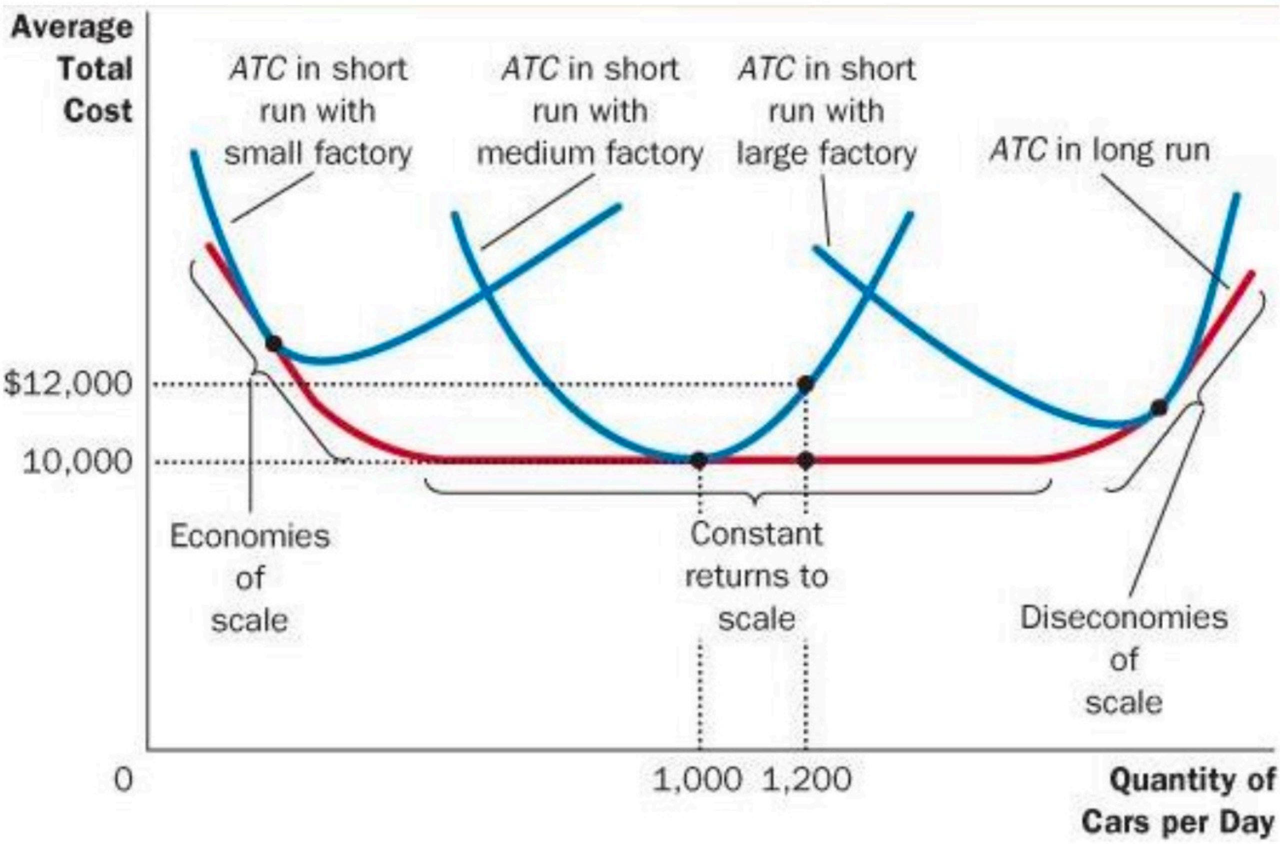
$$MC(Q) = \ + 2Q + Q^2$$

Long-Run Average Cost

For the ATC in the **long run**, we denote it by $LRAC$.



- $LRAC$ **curve** is **U-shaped**.
- ATC_0, ATC_1, ATC_2 are **short-run average total cost curves** for different levels of fixed capital K_0, K_1, K_2.
- The $LRAC$ shows the lowest average cost of producing each level of output when the firm can choose the level of capital.
- All the short-run curves lie on or above the long-run curve because firms have **greater flexibility in the long run.**



Economies of Scale

Economies of Scale

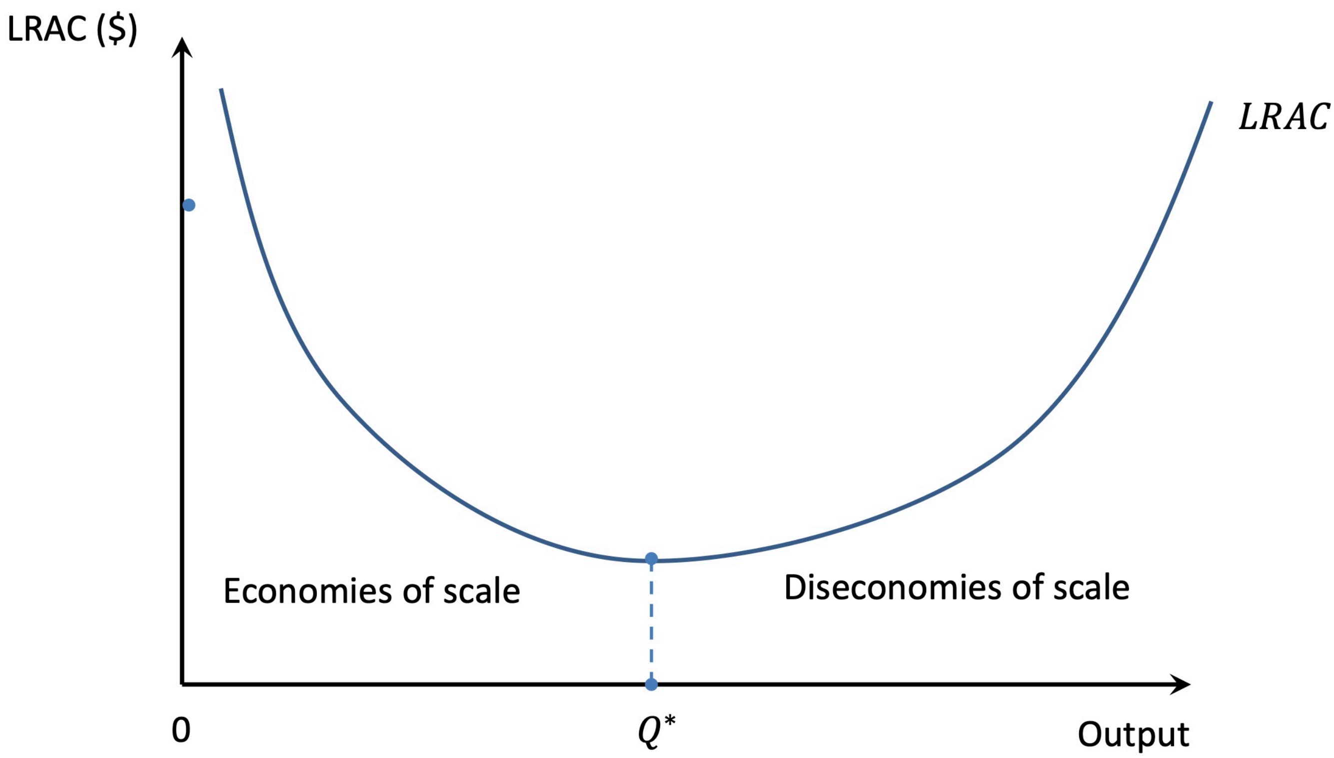
Declining portion of the long-run average cost curve $LRAC$ where average cost decreases as output Q increases.

Diseconomies of Scale

Rising portion of the long-run average cost curve $LRAC$ where average cost increases as output Q increases.

Constant Returns to Scale

Portion of the long-run average cost curve that remains constant as output Q increases.



Economies of Scope

Economies of Scope

Exist when the total cost of producing Q_1 and Q_2 together is less than the total cost of producing them separately:

$$\text{Economies of cope} \iff \boxed{C(Q_1, 0) + C(0, Q_2) > C(Q_1, Q_2)}$$

Cost Complementary

Cost Complementary

Exist when the MC of producing one good decreases as the output of another good increases:

$$\text{Cost Complementary} \iff \boxed{\frac{\partial MC_1(Q_1, Q_2)}{\partial Q_2} < 0 \text{ or } \frac{\partial MC_2(Q_1, Q_2)}{\partial Q_1} < 0}$$

Algebraic Form of a Multiproduct Cost Function

$$\boxed{C(Q_1, Q_2) = f + Q_1Q_2 + (Q_1)^2 + (Q_2)^2}$$

where f represents **fixed costs**, has marginal costs

$$MC_1(Q_1, Q_2) = Q_2 + 2Q_1$$

$$MC_2(Q_1, Q_2) = Q_1 + 2Q_2$$

$$\begin{aligned} < 0 &\implies \text{increase in } Q_2 \text{ decreases } MC_1 \\ &\implies \text{cost function exhibits cost complementary} \end{aligned}$$

Cost Complementary

$$\boxed{< 0} \implies \text{cost function exhibits } \boxed{\textbf{cost complementary}}$$

$$\boxed{> 0} \implies \text{there are } \boxed{\textbf{no cost complementarities}}$$

Economies of Scope

Economies of scope exist if

$$\boxed{f > Q_1Q_2}$$