TEST 1

Laplace Transform

a) $F(s) = \int_o^\infty f(t)e^{-st}$

b) Integrate, remove any u(t). Only values on the positive side of x-axis are important. Remove u(t), change integral limits.

c) Linearity: $c_1 f_1(t) + c_2 f_2(t) = c_1 F_1(s) + c_2 F_2(s)$

d) Time Shift: $f(t-t_o)u(t-t_o) = e^{-st_o}F(s)$

d) IVT: Multiply by s first. If $\frac{\infty}{\infty}$, divide top and bottom by s. Solve for $s = \infty$.

e) FVT: Multiply by s first. Solve for $s = 0$.

f) Unit Step: Unit step is the integral of the unit sample. The unit sample is the derivative of the unit step.

Inverse Laplace

1.a) Standard form: decreasing powers of s, coefficient of largest power in den = 1.

1.b) Make Ratio Proper: Power of s in num must be less than or equal to den. If not true, use long division.

1.c) Find Poles (roots of den): Factor or use quadratic formula. Roots can be real/unique, real/repeated, or complex/unique. Real parts for roots should be less than or equal to 0.

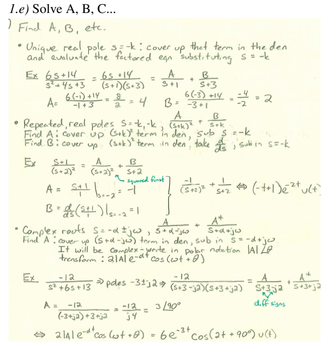
1.d) Break Into Sums:

For real/unique, use sums of $\frac{A}{(s+k)} + \frac{B}{(s+k)}$ ...

For real/repeated, use sums of $\frac{A}{(s+k)^2} + \frac{B}{(s+k)}$ ...

For every complex/unique, $s = -\alpha + -j\omega$ and do the sums of $\frac{A}{s+\alpha-j\omega} + \frac{A^*}{s+\alpha+j\omega}$. For calc use cSolve(blah+x=0,x) to get complex roots.

1.e) Solve A, B, C...



Find A, B, etc.

• Unique real pole $s=-k$: cover up that term in the den and evaluate the factored eqn substituting $s=-k$

Ex $\frac{6s+14}{s^2+4s+3} = \frac{6s+14}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$

$A = \frac{6(-1)+14}{-1+3} = \frac{8}{2} = 4 \quad B = \frac{6(-3)+14}{-3+1} = \frac{-4}{-2} = 2$

• Repeated, real poles $s=-k,-k$, $\frac{A}{(s+k)^2} + \frac{B}{s+k}$

Find A: cover up $(s+k)^2$ term in den, sub $s=-k$

Find B: cover up $(s+k)^2$ term in den, take $\frac{d}{ds}$, sub in $s=-k$

Ex $\frac{s+1}{(s+2)^2} = \frac{A}{(s+2)^2} + \frac{B}{s+2}$

squared first

$A = \frac{s+1}{1}\Big|_{s=-2} = -1$

$B = \frac{d}{ds}\left(\frac{s+1}{1}\right)\Big|_{s=-2} = 1$

$\frac{-1}{(s+2)^2} + \frac{1}{s+2} \Leftrightarrow (-t+1)e^{-2t}u(t)$

• Complex roots $s = -\alpha \pm j\omega$, $\frac{A}{s+\alpha-j\omega} + \frac{A^*}{s+\alpha+j\omega}$

Find A: cover up $(s+\alpha-j\omega)$ term in den, sub in $s=-\alpha+j\omega$

It will be complex - write in polar notation $|A|\angle\theta$

transform: $2|A|e^{-\alpha t}\cos(\omega t+\theta)$

Ex $\frac{-12}{s^2+6s+13} \Rightarrow$ poles $-3 \pm j2 \Rightarrow \frac{-12}{(s+3-j2)(s+3+j2)} = \frac{A}{s+3-j2} + \frac{A^*}{s+3+j2}$

diff signs

$A = \frac{-12}{(-3+j2)+3+j2} = \frac{-12}{j4} = 3\angle 90°$

$\Leftrightarrow 2|A|e^{-\alpha t}\cos(\omega t+\theta) = 6e^{-3t}\cos(2t+90°)u(t)$

Convolution

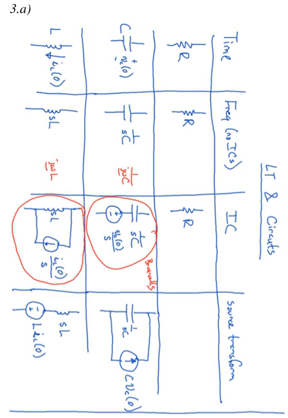
2.a) $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$

2.b) Get rid of u(t). Great new piece-wise function for t $\geqslant 0$ and t $\leqslant 0$.

2.c) Can be replaced by Laplace, via Y(s) = X(s)H(s). Then follow *1.a* to *1.e*.

Circuit Elements and Laplace

3.a)



3.b) Transform the circuit to the frequency domain.

3.c) Use nodal or mesh to solve given problem.

3.d) Reduce and the perform inverse Laplace transform.

TEST 2